Numerical Rayleigh wave propagation on a thin layer

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ABSTRACT

Experiments with numerical Rayleigh waves on shallow geological models show what appear to be realistic and relevant results. It is shown that further testing with vibrator sweeps that are practical for both a vibrator and a finite-difference model would be very valuable. F-D tests with sweeps of two narrow frequency ranges were run on a single layer with uniform properties to show how the velocity of propagation depends on the thickness of the layer. Analysis of the velocity spectra of vibrator sources using different frequencies could then lead to detailed shallow shear wave velocity values.

INTRODUCTION

There have been many suggestions within CREWES that the ground-roll present on most of our seismic records should be analysed to obtain near surface shear wave velocities. Nothing very practical has come of it yet, although Askari(2010) is developing a mathematical analysis. The object of this report is to suggest that a study using 2D forward modelling coupled with vibrator acquisition using sweeps of very limited band width, is a likely step toward an analysis of this sort. Even though the technique might not be used regularly, it should lead to a better understanding of vibrator generated ground-roll.

To date, the most valuable contribution of ground-roll analysis has been in seismology. The Rayleigh waves generated by earthquakes often travel long distances along the surface of the earth and reach many recording stations. The relative travel times to the stations can be used to locate the earthquake source, and the relative travel times of differing frequency components can often be used to identify layers within the earth.

Rayleigh waves have also been used for some engineering purposes, to measure the properties of the sea floor and of pavements for example.

These successful applications usually record events at many wavelengths distance from the source, and the materials in which the waves travel tend to have relatively constant properties. These conditions ensure that the waves act like plane waves on the surface, that the direct arrival is clear of any indirect arrival, and the unique frequencies have time to separate. These conditions do not, however, apply in the case of ground-roll associated with seismic recording.

The conclusions of this report assume the validity of the two-dimensional staggered-grid forward model software developed by the author. The critical part of the model for Rayleigh waves is the coding of the free surface boundary condition. These points are discussed in the author's thesis (Manning 2008).

Recently the author (Manning 2009) has extended the staggered-grid finite-difference concepts to three dimensions. Models created with this software have given results which are effectively identical to the 2D models, except for the obvious differences caused by spreading. In particular, a compact source produces a compact Rayleigh wave which propagates without dispersion on the surface of a uniform velocity medium. Further, a Rayleigh wave on the top of a multi-layered medium may be dispersed, but not reflected.

On the other hand, discontinuities of the elastic properties at the levels of significant Rayleigh wave displacement cause a wave to be reflected and the continuing wave to be altered much like the reflection and transmission of body waves.

The other aspect of the proposed technique is to use specialized vibrator sweeps to create useful Rayleigh waves. The quite short sweeps should range from perhaps 50 Hz down to as low as physically practical, and as close as possible to single frequencies. The frequency range is anticipated to be most useful for the thickness and velocities anticipated for the near surface, and the constant frequency sweeps should be the most easy to analyse. The short sweeps could be repeated if higher signal to noise ratios are required.

GROUND-ROLL MODELS

The basic ground-roll model was developed by Rayleigh(1885) and applies to the free surface of a thick body with constant elastic properties. A Rayleigh wave (ground-roll) may be initiated by a vertical force on the surface of the body (represented by the vector in Figure 1). This type of source may also be used to initiate a finite-difference model if the staggered-grid is used. The force must be varied in time through a reasonably continuous sweep. An example of a sweep is the Ricker wavelet shown in Figure 2. Although this does not have enough power for normal vibroseis purposes, it can work quite well in a finite-difference model.

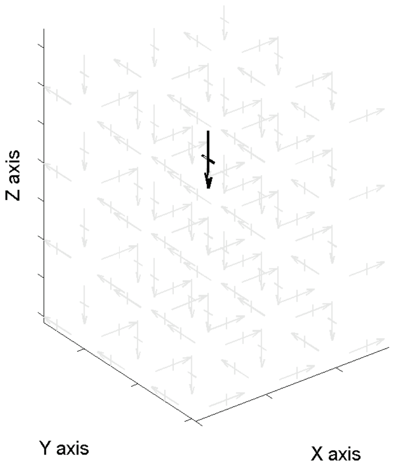

FIG. 1. The Z-monopole shown here is used at the free surface of the model to represent a vibroseis source. The forgiving nature of the staggered grid allows such simple sources.

Figure 3 shows the traces initiated by the wavelet of Figure 2. They were recorded at the surface of a model with a constant shear velocity of 1000 m/sec. The Rayleigh wave propagates without attenuation at a velocity a little slower than the shear wave velocity, all consistent with Rayleigh's original theory.

The display in Figure 3 is flipped from the usual convention in that each trace represents the surface of the earth at a given time. This was done to enable zero crossings to be picked more easily, allowing velocities and spatial wave lengths to be calculated.

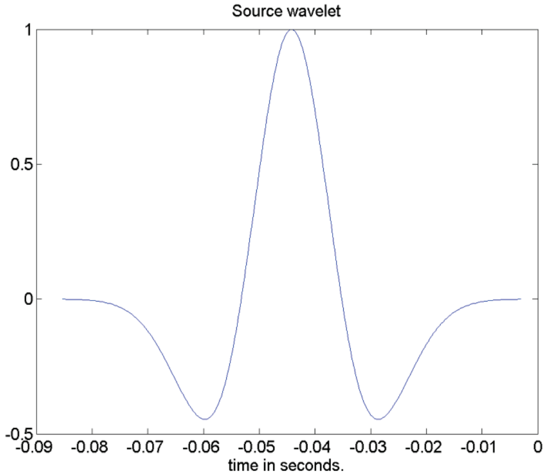

FIG. 2. A Ricker wavelet. This may be used as a surface source to initiate a finite-difference version of a Rayleigh wave.

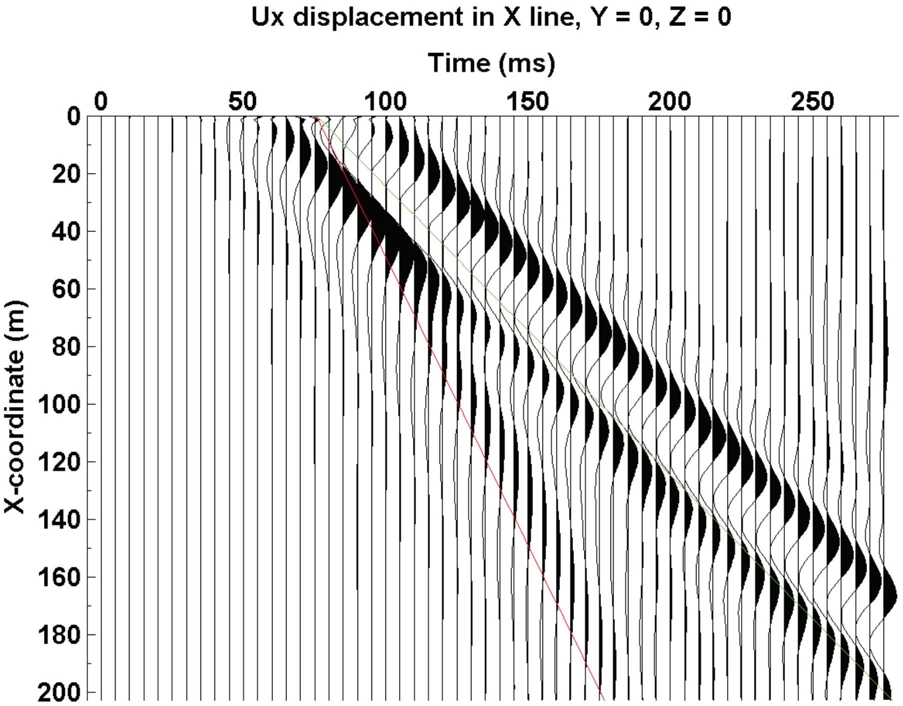

FIG. 3. The Rayleigh wave generated from the wavelet in Figure 2 when applied to the surface of a body with a constant 1000 m/sec shear velocity. After the first break energy peels off at the pressure wave velocity of 2000 m/sec (marked by the red line), the Rayleigh wave propagates unattenuated at a consistent velocity a little slower than the shear wave velocity (marked by the green line). Note that this is not a conventional trace display, the traces being at constant time.

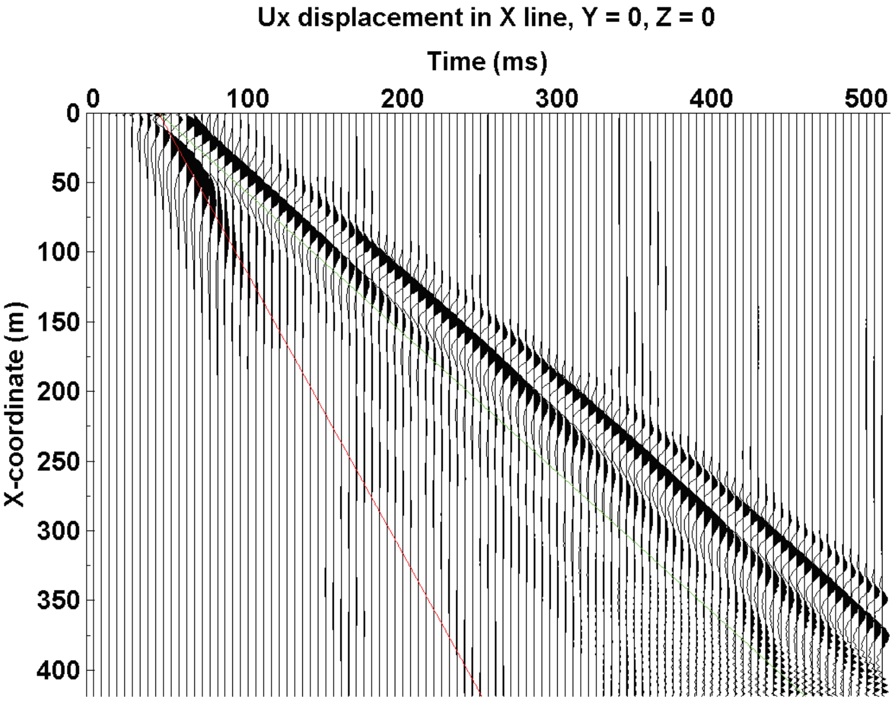

FIG. 4. The Rayleigh wave generated from a Ricker wavelet recorded on a free surface of 1000 m/sec shear velocity but only 7 metres thick. The shear velocity underneath is 1500 m/sec and the Rayleigh wave penetrates deep enough to be affected. This shingling effect is dispersion.

Figure 4 shows the traces initiated by the same wavelet in Figure 2, but here the surface layer of 1000 m/sec is only 7 metres thick, and below it is material with a shear wave velocity of 1500 m/sec. The shingling effect is caused by the interaction between the parts of the wave travelling in the two media. It is very difficult to invert from this display to the layering which causes it.

Some of the difficulties of interpreting the ground roll which is generated from a broad band sweep can be reduced by using a series of narrow band sweeps. An example of a single frequency sweep used in a model is shown in Figure 5, and the recording made from it is shown in Figure 6. The earth model here is the same one that is described for Figure 4.

Figure 6 still shows some of the shingling of Figure 4, but a valid Rayleigh wave velocity may be picked. The green line shows 1000 m/sec and the event velocity is somewhat faster than that. Since a classic Rayleigh wave here must be slower than 1000 m/sec, it is apparent that the wave is being affected by the deeper material. If we are fortunate the real and finite-difference worlds will act in the same way, and the model will allow an accurate assessment of the real.

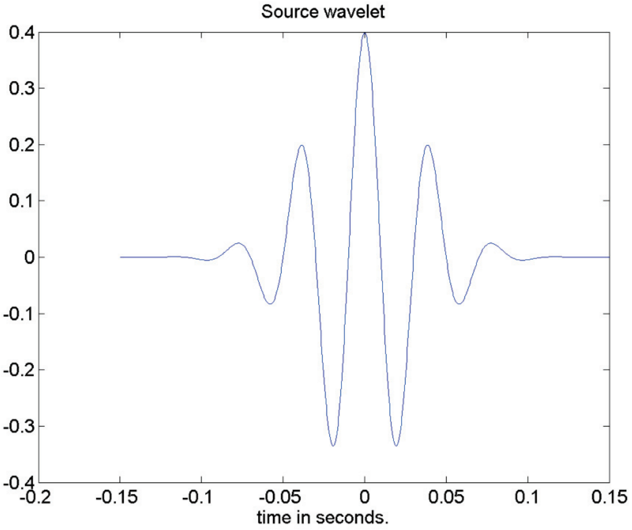

FIG. 5. The near constant frequency wavelet proposed as a vibrator sweep. The wavelet is a 25 Hz cosine tapered with a Gaussian. The long end tapers are required to minimize finite-difference artifacts.

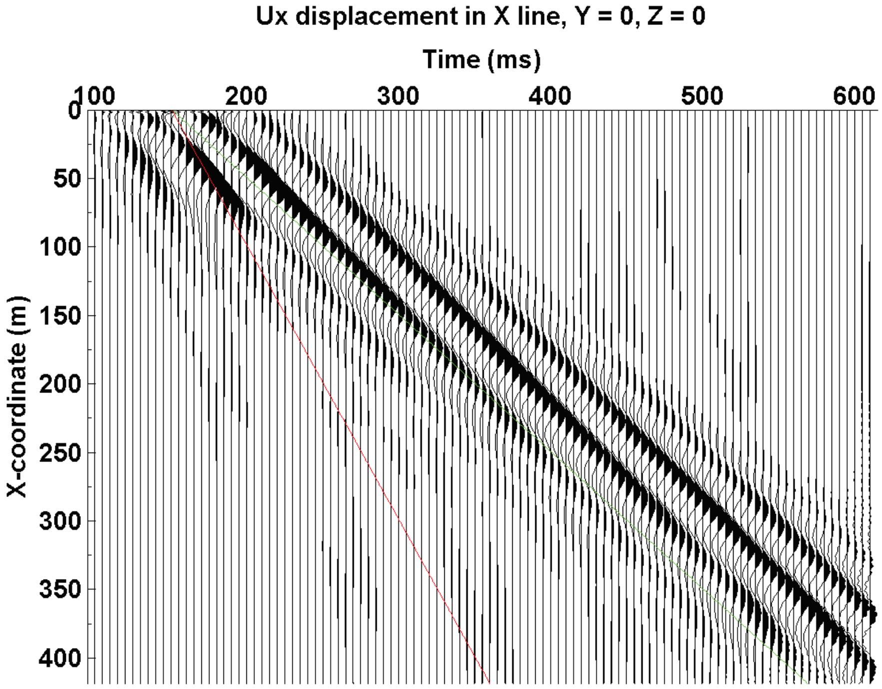

FIG. 6. A finite-difference model recording made from the source in Figure 5 on the free surface of a layer 7 m thick and with a shear wave velocity of 1000 m/sec, exactly like the model for Figure 4. There is still some shingling as in Figure 4, but a definite reasonable Rayleigh wave velocity may be obtained.

The object then would be to run a series of models and sweeps, and compare them to real data collected with similar sweeps. As an example, some finite-difference models and sweeps were run, the apparent velocities were picked by hand, and the results were gathered in Figure 7, ready for real data comparison.

If the real data velocities for the same sweep are compatible with the model data, then an interpretation can be made. If not, the geologic model assumptions must be modified in the direction in which the real data suggest until the real and model results are compatible. This does not guarantee a perfect interpretation but it is more likely to be close.

As an example, the real layers are most likely to have a velocity which increases quite smoothly from top to bottom, because of the likely greater compaction with depth. The ranges of these velocities are very difficult to predict, and probably vary in different areas.

A further type of complication which is almost certain to appear is changes in the elastic parameters within the range of the real survey. Some of these may not be subtle at all, and examples are shown in Figures 8 and 9. Figure 8 shows an abrupt change in layer thickness from 10 to 5 metres, and Figure 9 shows a more gradual change of the same type. Interpretations of this type should be possible with data collected in the proposed way.

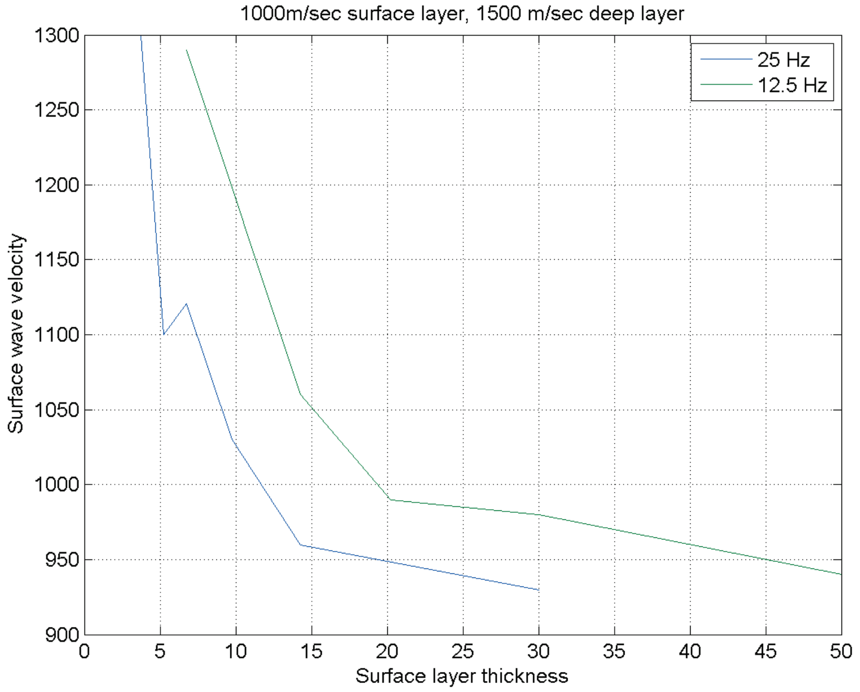

FIG. 7. The relationship between surface layers of several thicknesses and two sweeps that were used to illuminate them in finite-difference models. The velocities of the Rayleigh waves were picked manually from plots similar to Figure 6. The lower frequencies are better able to detect a thicker layer.

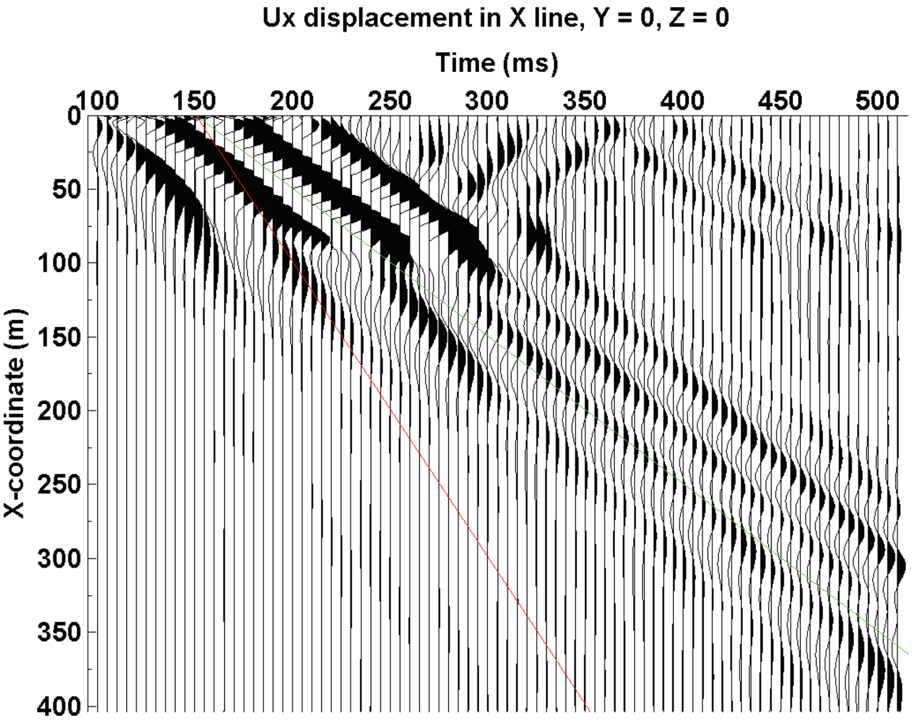

FIG. 8. A sudden decrease in layer thickness from 10 to 5 metres between X-coordinate 80 m and 90 m causes very obvious effects to the reflected and transmitted energy.

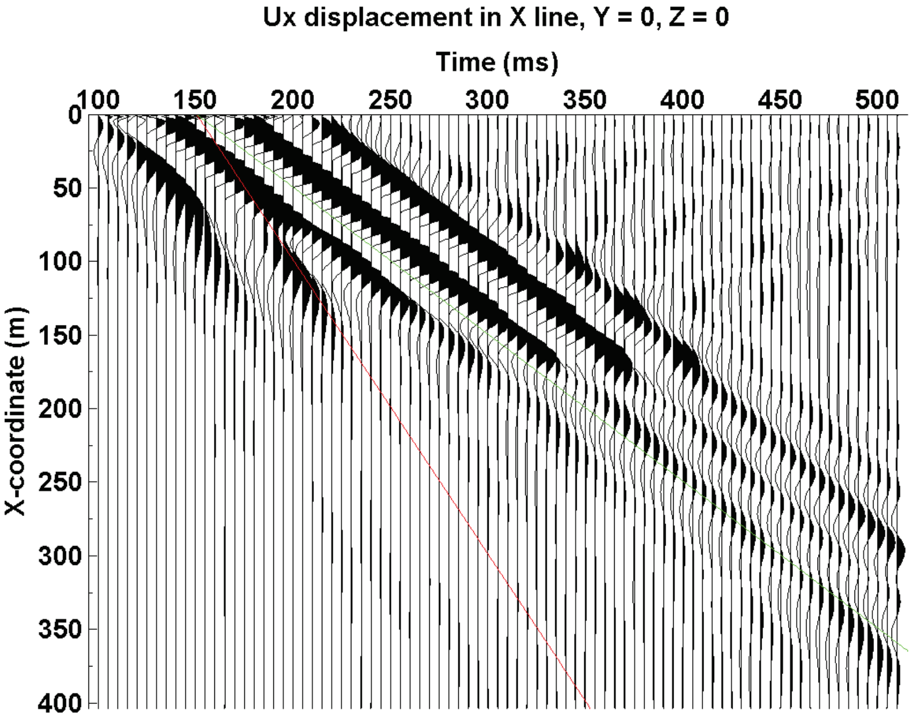

FIG. 9. A gradual decrease in layer thickness from 10 to 5 metres between X-coordinate 80 m and 190 m still causes quite obvious effects.

REAL DATA

One of the advantages of using models to represent real data is that the models can sometimes provide a degree of complexity that may be valid, even if unexpected. This was the case with the following data set.

An example of ground-roll from a recent Alberta CREWES vibrator project at Hussar is shown in Figure 10. It may be seen that below the first breaks in time, and from zero to 1000 metres in offset, is a continuous display of noise sufficient to mask even the strongest reflection signals

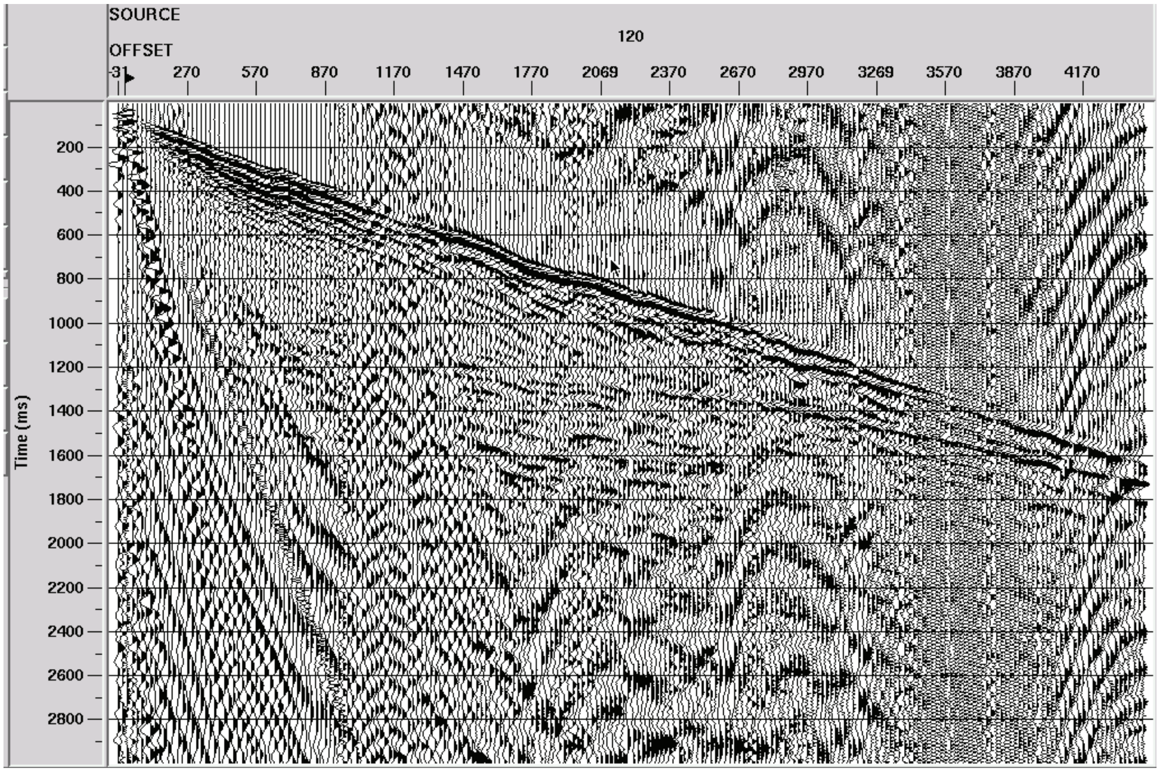

FIG. 10. A seismic vibroseis record from the open Alberta plains (the CREWES Hussar project). The shot generated surface noise dominates any signal after the first breaks up to at least 1000 metres.

Inspection of Figure 10 showed a possibly simple ground-roll event originating at the source and angled down at about 170 m/sec. It was decided to experiment using a broad band wavelet (from Figure 2) on a surface layer of 170 m/sec shear wave velocity, a pressure wave velocity of double that, and an arbitrary thickness of 10 metres. The results are displayed as a final snapshot in Figure 11 and a plot of the acquired surface data in conventional trace format in Figure 12.

The trace data are encouraging in principle, even though they are not yet entirely convincing in detail. The most interesting features are the events which peel off the main ground-roll event with a slope nearly equivalent to the pressure wave velocity, and which then die out. Similar events appear on the real data.

Inspection of Figure 12 shows that the high energy package which propagates along the surface consists of Rayleigh waves (which tend to have displacements in vertical columns), and shear waves trapped in the layer with wavefronts at an angle to the vertical. The velocity of the subsidiary events seems to be that of a shear wave propagating at an angle. Also, the Rayleigh and shear waves seem to be tied together, possibly with the Rayleigh wave reinforcing the shear wave.

It would be possible to tie the model and data together more closely by adjusting some of the parameters. For example, it was found that the subsidiary event spacing was directly dependent on the layer thickness.

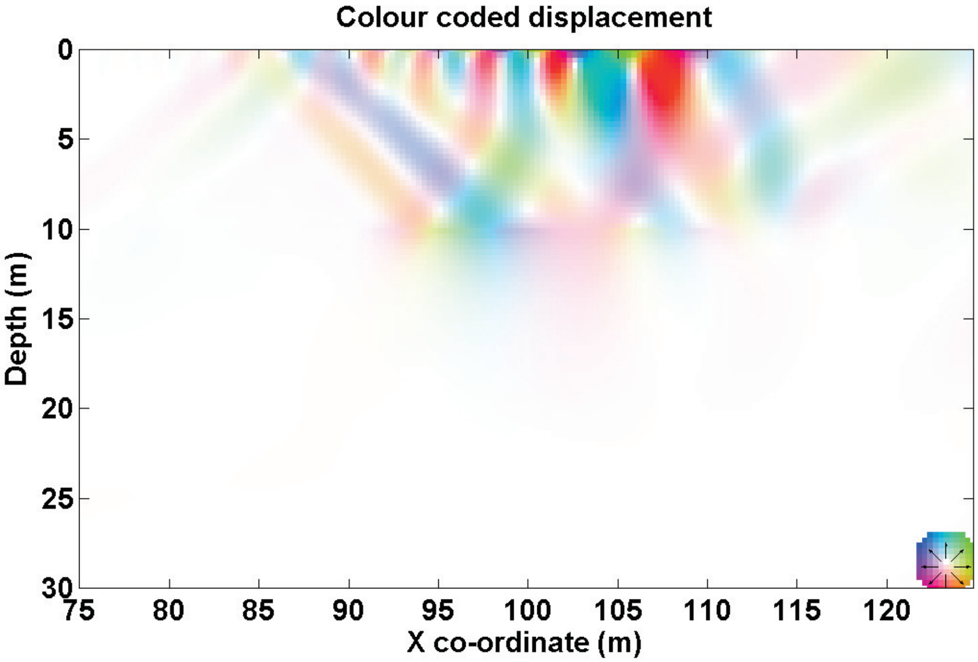

FIG. 11. Broad band displacement within a 10 metre surface layer designed to model some of the surface noise on the seismic record of Figure 10. This shows an interacting combination of Rayleigh waves (vertical wavefronts) and shear waves (the wavefronts at 45 degrees).

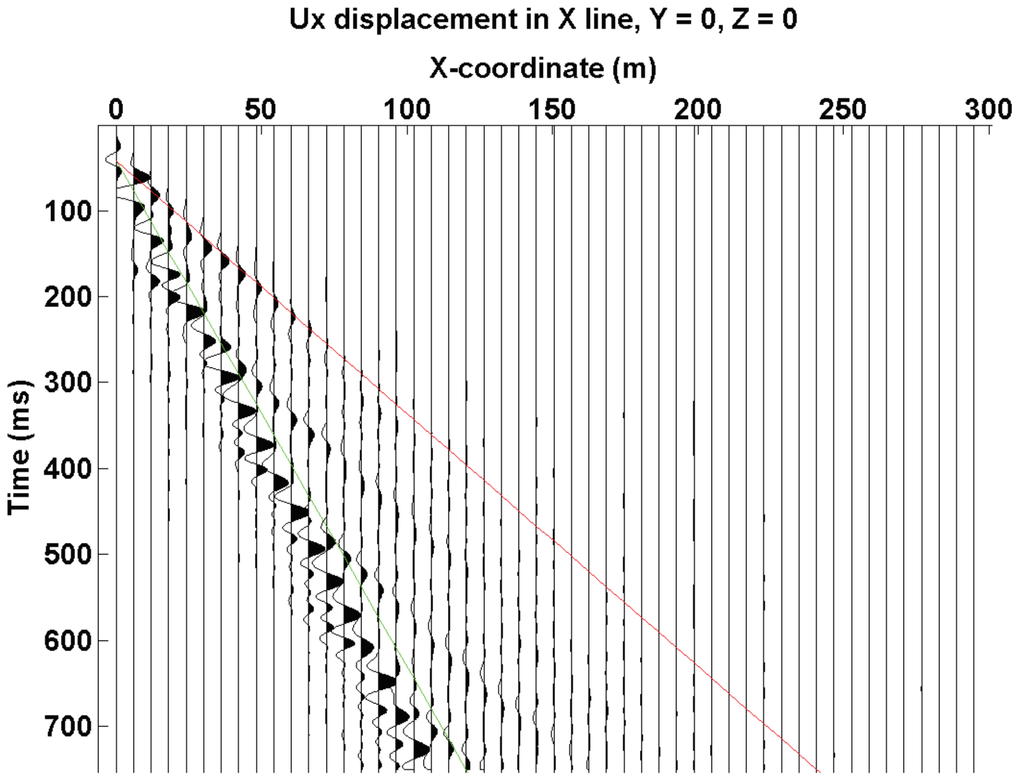

FIG. 12. The conventional trace display from the model of Figure 11. The ground-roll appears at the left, but the interaction between it and the trapped shear wave seems to generate short lived pressure wave refractions at intervals.

CONCLUSIONS

There are hints that staggered-grid finite-difference models with simple energy source initiations are good simulations of the near surface earth. Specialized testing would be worth doing to confirm this.

Specialized vibrator sweeps would be very useful in the study of near surface seismic noise. Their use should either reinforce or reveal gaps in our understanding of vibrator energy, near surface conditions, and the finite-difference model. At best this approach could lead to methods that reduce or correct for near surface noise, or even lead to new methods of surveying in the near surface.

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