Estimation of shear velocity from P-P and P-S seismic data

Thais A. Guirigay and John C. Bancroft

ABSTRACT

Prestack migration of converted wave data requires accurate acoustic and shear velocities V_p and V_s. New methods to estimate the shear velocities using a single converted wave velocity V_c are shown in this paper.

The estimated shear velocity V_s is used with V_p in a full prestack migration using the equivalent offset method to form complete prestack migration gathers. This process is referred as converted wave equivalent offset migration (C-EOM). The quality of the method is demonstrated with a one real dataset.

INTRODUCTION

One of the major problems of converted wave processing is the scale of the axis that defines the velocities. This scale may be in time or depth, and the time scale could be in P-wave times, S-wave times or, for converted wave data, C-wave times. This intend of this paper is demonstrate the practical methods for estimation of converted wave velocities V_c and then the estimation of shear velocities for be used in the full Equivalent offset migration. Real data set from Hussar area in Southeastern of Alberta illustrate the progress of the method.

Velocity consideration

There are numerous properties that use the term velocity which are related to the actual velocity of the medium. These are referred to as velocity types, with the main four "velocities" referred to as: interval, average, root-mean-squared, and stacking velocities.

The interval velocities are defined over a time interval usually associated with velocity model building. When that time interval is the sampling interval, they are equivalent to the instantaneous velocities. These velocities may be derived from well log where the scale is originally in depth, but may be converted to a vertical two-way time t_0 when it is converted to a synthetic seismogram for comparison to real seismic data.

The average velocity is the ratio of the distance along a certain path to the time to traverse this path. Vertical two-way traveltimes t_0 are related to vertical depths z_0 with the average velocity V_{ave}

$$V_{ave}\,(t_0\,, or\; z_0) = \;\frac{2}{t_0} z_0, \tag{1}$$

$$z_0 = \; t_0 V_{ave}\,(t_0\,)/2, \tag{2}$$

$$t_0 = \;\frac{2\,z_0}{V_{ave}\,(z_0)}, \tag{3}$$

where the average velocity may be defined in time or space, depending on the direction of the conversion. It is defined from the instantaneous velocity $V_{Int}\,(n)$ and interval time t_n defined in the n^{th} layer by

$$V_{ave}(\,t_{0-n}) = \;\frac{\sum_{i=1}^{n}\;\; V_{Int}\,(n)\;t_n}{\sum_{i=1}^{n} t_n}, \tag{4}$$

When using the root-mean square (RMS) velocity V_{rms}, the scale is in vertical time. These velocities are also computed from the interval velocities using:

$$V_{rms}(t_{0-n}\,) = \;\sqrt{\frac{\sum_{i=1}^{n}\, V_{Int}^2 t_n}{\sum_{i=1}^{n} t_n}}\,. \tag{5}$$

Equations (4) and (5) can be modified to compute the interval velocity from average or RMS velocities, allowing one type of velocity to be converted into another, i. e., Vave $\Leftrightarrow$ Vint $\Leftrightarrow$ Vrms. With one mode, velocities can be expressed in time or, if necessary, in depth.

A typical moveout equation for horizontal data defines the offset traveltime *t* as

$$t = \;\sqrt{t_0^2 + \frac{4\,h^2}{V_{rms}^2}}\,, \tag{6}$$

where h is half offset or distance between the source and receiver. It is convenient to remove the time from the square-root using the depth z_o with

$$t = \;\sqrt{\frac{z_0^2}{V_{ave}^2} + \frac{4\,h^2}{V_{rms}^2}}\,, \tag{7}$$

or

$$t = \;\frac{1}{V_{rms}^2}\sqrt{z_0^2\,\frac{V_{rms}^2}{V_{ave}^2} + 4\,h^2}\,. \tag{8}$$

For convenience, we may use a pseudo depth $\hat{z}_0$ defined by

$$\hat{z}_0 = \; z_0\,\frac{V_{rms}(\,\hat{z}\,)}{V_{ave}(\,\hat{z}\,)}\,, \tag{9}$$

simplifying the traveltime computation to be

$$t = \frac{1}{V_{rms}} \sqrt{\ \hat{z}_0{}^2 + 4\,h^2}\ . \tag{10}$$

Introducing a new value for γ based on RMS velocities

The ratio between P-wave and S-wave velocities as should be written as a function of depth using interval velocities

$$\gamma\,(z) = \frac{V_{p-Int}}{V_{s-Int}}. \tag{11}$$

We can have RMS velocities for P-waves, S-waves and also for C-waves, and hence could write a relationship γ between the corresponding RMS velocities. However, the times of the corresponding velocities are different, i. e.,

$$\hat{\gamma}_{rms}\,(z\ or\ ?\,) = \frac{V_{p-rms\,(t_p)}}{V_{s-rms}(t_s)}, \tag{12}$$

where the times t_p and t_s are at the same depth.

Ideally we should continue to use depth as the common parameters to compare different modes of propagation; however it is convenient to use one common time scale. Here, we are going to use P-wave time t_p, as the P-wave velocities are usually defined first, and are more reliable. Then, scale the S and P-data to align events on the same display.

This requires converting the S-wave velocity from t_s time to t_p time, i, e.,

$$\hat{\gamma}_{rms}\ (t_p) = \frac{V_{p-rms\,(t_p)}}{V_{s-rms-p}(t_p)}. \tag{13}$$

The following section will consider practical methods for estimated V_c by using a small range of x and allows h to range from zero to maximum values of h_{max} according to the geometry shown in Figure 1.

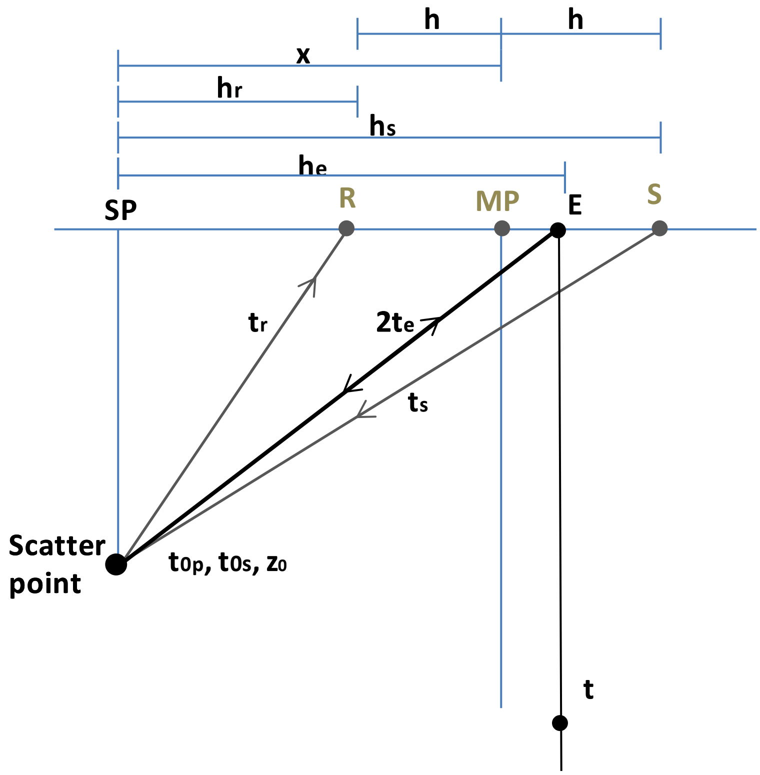

FIG 1. The raypaths and traveltime for a scatter or conversion point.

Estimating an initial value for V_c.

One method of computing an initial velocity for V_c is to scale V_p with an assumed value for $\hat{\gamma}$. This requires adjusting of the velocity V_p, and shifting the time t_{0p} to a larger time of t_{0c}, i.e.,

$$V_{c-rms}(t_{0c}) = \frac{2}{(1+\hat{\gamma})} V_{p-rms}(t_{0p}), \tag{14}$$

where

$$t_{0c} = \frac{1+\gamma}{2} t_{0p} \ . \tag{15}$$

Tests could be run with different constant values of $\hat{\gamma}$ to establish more accurate values of $V_{c\text{-}rms}$ that vary with time t_0.

Another method for estimating V_c is to use the equivalent offset method with short offsets. Consider the DSR equation expressed in the midpoint location x and half offset h and according to the geometry shown in Figure 1.

$$t_c = \sqrt{\frac{t_{0p}^2}{4} + \left(\frac{x+h}{V_{p-rms}(t_{0p})}\right)^2} + \sqrt{\frac{t_{0s}^2}{4} + \left(\frac{x-h}{V_{s-rms}(t_{0s})}\right)^2}\ . \qquad (16)$$

Using a pseudo depth, the double square root (DSR) becomes

$$t_{c-zo}(x.h, \hat{z}_0) = \frac{1}{V_{p-rms}(\hat{z}_0^2)}\ \sqrt{\hat{z}_0^2 + (x+h)^2} + \frac{1}{V_{s-rms}(\hat{z}_0^2)}\ \sqrt{\hat{z}_0^2 + (x-h)^2}\ . \qquad (17)$$

When x is small relative to h, we can assume

$$|x+h| \approx |x-h| \approx |h|, \qquad (18)$$

and we combine the two square-roots, and convert the S velocity to a P velocity giving

$$t_{c,x\ll h} = \frac{1+\gamma_{rms}}{V_{p-rms}(\hat{z}o)}\ \sqrt{\hat{z}_0^2 + h^2}\ , \qquad (19)$$

where $\hat{\gamma}_{rms}$ is the ratio

$$\hat{\gamma}_{rms}\ (\hat{z}) = \frac{V_{p-rms}(\hat{z})}{V_{s-rms}(\hat{z})}. \qquad (20)$$

Equation (19) can be written in terms of a RMS converted wave velocity V_c

$$t_{c,x\ll h} \approx \frac{2}{V_c(\hat{z}o)}\ \sqrt{\hat{z}_0^2 + h^2}\ , \qquad (21)$$

defined as,

$$V_{c-rms}(\hat{z}_0) = \frac{2}{(1+\hat{\gamma}_{rms})} V_{p-rms}(\hat{z}_0)\ . \qquad (22)$$

Equation (22) tells us that we can approximate an initial equivalent offset h_e with an estimate of V_{c-rms} to form gathers with short displacements x. There will be no energy at zero offset, but if a gather can be formed with a short displacement x, then a simple velocity analysis will provide a more accurate converted wave velocity V_c . This velocity may also be used for moveout correction but more importantly can be used for an initial estimation of V_s, which can then be used to form complete CSP gathers.

Extending to all offsets

Given the P-wave velocity and a good estimate of the S-wave velocity, the source and receiver traveltimes can be computed for an Equivalent Offset Migration (EOM) that encompasses all offsets.

Equation (16) may be used to compute a converted wave traveltime and is repeated with the actual times of the velocity,

$$t_c(t_{0p}, x, h) = \frac{1}{V_{p-rms}(t_{0p})}\ \sqrt{\hat{z}_0^2 + (x+h)^2} + \frac{1}{V_{s-rms}(t_{0s})}\ \sqrt{\hat{z}_0^2 + (x-h)^2}\ , \tag{23}$$

that is equated to an equivalent offset for a collocated source and receiver,

$$t_c(t_{0p}, h_e) = \frac{1}{V_{p-rms}(t_{0p})}\ \sqrt{\hat{z}_0^2 + {h_e}^2} + \frac{1}{V_{s-rms}(t_{0s})}\ \sqrt{\hat{z}_0^2 + {h_e}^2}\ , \tag{24}$$

or

$$t_c(t_{0p}, h_e) = \left[\frac{1}{V_{p-rms}(t_{0p})} + \frac{1}{V_{s-rms}(t_{0s})}\right]\ \sqrt{\hat{z}_0^2 + {h_e}^2}\ , \tag{25}$$

or

$$t_c(t_{0p}, h_e) = \frac{2}{V_{c-rms}(t_{0p})}\ \sqrt{\hat{z}_0^2 + h_e}\ . \tag{26}$$

This equation tells us that we can compute a converted traveltime using equation (23), and assign it an equivalent offset h_e using equation (26). A prestack migration gather can be formed using $V_{p\text{-}rms}$ and $V_{s\text{-}rms}$, and then been processed like conventional data using $V_{c\text{-}rms}$. This process is referred to as converted wave equivalent offset migration (C-EOM).

Note however, that the times of the velocities in equation (23) are different and need to be aligned. This is discussed in the following section where we match the traveltime for P, S and C wave data.

Matching the traveltime for P-, and C-wave data.

The objective is to map the traveltimes between various velocities for the different modes. In the case where we want to map P velocities to C velocities to match an initial guess of V_c, we start with $V_{p\text{-}rms}$ (t_{0p}). Then, scale the amplitude and times to an estimated converted wave velocity $V_{c\text{-}rms}$ (t_{0c}) to t_{op} times $V_{c\text{-}rms\text{-}p}$ (t_{0p}).

Real data will be used in the following sections to illustrate the progress of the methods. The input velocities that were picked from the real data were smoothed for easier viewing.

Please note that some migration methods use interval velocities, but a Kirchhoff migration requires the velocities to be in an RMS form. The velocities may be converted from one form to another. The γ function is usually defined for layers in depth with defined interval velocities. A corresponding γ was defined for RMS velocities.

Method 1

This method is straight forward and starts with the RMS P velocities, ($V_{p\text{-}rms}$) and converts then to interval P velocity ($V_{p\text{-}Int}$), then ($V_{c\text{-}Int}$) then to the RMS C velocities ($V_{c\text{-}rms}$).

1. Convert $V_{p\text{-}rms}\,(t_{0\text{-}p})$ to interval velocities $V_{p\text{-}Int}(t_{0\text{-}p})$

$$V_{p-Int}(n) = \sqrt{\frac{t_n V_{p-rms}^2(n) - t_{n-1} V_{p-rms}^2(n-1)}{t_n - t_{n-1}}}\,. \tag{27}$$

2. Use the interval velocities to map the times to depth $t_{0\text{-}p} \Rightarrow z_0$.

$$z_n = \frac{1}{2} V_{p-Int}\,(n) * (t_n - t_{n-1}). \tag{28}$$

3. Scale the amplitude of $V_{c\text{-}Int}$ to $V_{p\text{-}Int}$ at z (same as $t_{0\text{-}p}$) using γ, (as illustrated in Figure 2b where $V_{p\text{-}Int}$ (z) is in blue, and $V_{c\text{-}Int}$ (Gz) is in green),

$$V_{c-Int}\left(z, or\ t_{0p}\right) = \frac{2}{(1+\gamma)} V_{p-Int}(z, or\ t_{0p})\,. \tag{29}$$

4. Use $V_{c\text{-}Int}$ *(z)* and the corresponding depth increments, compute the C times at each depth.

$$V_{c-Int}\left(\, t_{0c\ at\ z_n}\right) = \ V_{c-Int}(z_n)\,. \tag{30}$$

5. Resample $V_{c\text{-}Int}$ from irregular times (at depth) to equal time increments

$$V_{c-Int}\left(\, t_{0p\ at\ z_n}\right) \Rightarrow \ V_{c-Int}(n\delta t)\,. \tag{31}$$

6. Convert the interval C velocity ($V_{c\text{-}Int}$) to RMS C velocities ($V_{c\text{-}rms}$),

$$V_{c-rms}(\,n) = \sqrt{\frac{\sum_{i=1}^{n} t_i\, V_{c-Int}^2(i)}{\sum_{i=1}^{n} t_i}}, \tag{32}$$

RMS, interval, and average P velocities ($V_{p\text{-}rms}\,(t_{0\text{-}p})$, $V_{p\text{-}Int}(t_{0\text{-}p})$ and $V_{p\text{-}ave}(t_{0\text{-}p})$ are illustrated in Figure 2: a) in time, and b) in depth.

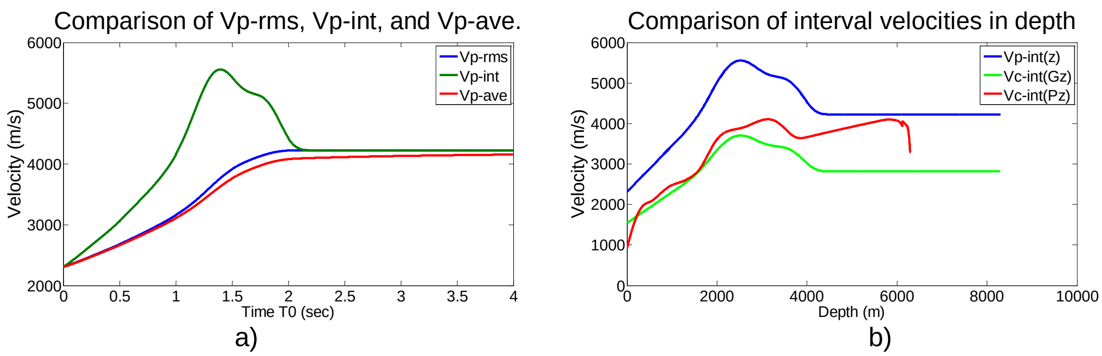

FIG 2. Plots of initial estimates of the converted wave velocity V_c with a) the RMS, interval, and average P velocities, and b) the interval P and C velocities in depth from the gamma function in green and from picked velocities in red.

Method 2

This method uses a simple approximation, is much simpler than method 1, but has a similar accuracy. Using the corresponding average velocity we get the depth z_0 from

$$2\,z_0 = t_{0c}\,V_{c-ave}(z) = t_{0p}\,V_{p-ave}(z), \tag{33}$$

and assuming the ratio of average P and C velocities to be similar to the ratio of P and C RMS velocities

$$t_{0c} = t_{0p}\frac{V_{p-ave}(z)}{V_{c-ave}(z)} \approx t_{0p}\frac{V_{p-rms}(z)}{V_{c-rms}(z)}, \tag{34}$$

we relate the time t_{0p} and t_{0c} with γ, using equation (22), i. e.,

$$t_{0c} \approx t_{0p}\frac{1+\hat{\gamma}}{2}. \tag{35}$$

Equation (35) allows us to simply map the P-wave times to converted wave times without the need to convert to interval velocities. We can get the C velocity values by converting $V_{p\text{-}rms}\,(t_o)$ on approximate depth using the RMS velocities $V_{p\text{-}rms}\,(z)$ using:

$$V_{c-rms}\,(z) = \frac{2}{1+\gamma_{Int}\,(z)}\,V_{p-rms}(z). \tag{36}$$

The estimated $V_{c\text{-}rms}$ velocities in approximate depth are then converted back into time to complete mapping equation:

$$V_{c-rms-p}\left(t_{0p}\right) = \frac{2}{1+\gamma_{Int}\,(t_{0p})}\,V_{p-rms}\left(t_{0p}\right). \tag{37}$$

If we are given the P-wave velocities, and a chosen specific value for γ_{Int}, we can scale the P velocities to C velocities, and then map the P times to the C times. In summary the processing steps are:

1. Create an initial array of $\hat{V}_{c-rms}\,(n)$ by scaling the amplitude of $V_{p-rms}(n)$ using

$$\hat{V}_{c-rms}\,(n) = \frac{2}{1+\hat{\gamma}}\,V_{p-rms}(n). \tag{38}$$

2. Resample $\hat{V}_{c-rms}$ (m) to V_{c-rms} (n) using equal increments of m, where

$$m = \frac{1+\hat{\gamma}}{2}\,n. \tag{39}$$

The second method is illustrated in Figure 3a where values from the fast method or method 2 are plotted in yellow colour ($V_{c\text{-}rms2}$). Note that the maximum time of the fast method is less than that of the exact method and only extend to 4 sec. The error between the two methods is shown in Figure 3b and is less than 0.1%.

Once we have an initial estimate of V_c, then we can create limited converted wave CSP (LCCSP) gathers at a few locations to get an improved estimate of V_c from a semblance analysis of the gathers. These improved or picked V_c velocities were converted to interval velocities and then depth and are displayed as the red curve in Figure 2b as $V_{c\text{-}int}(Pz)$.

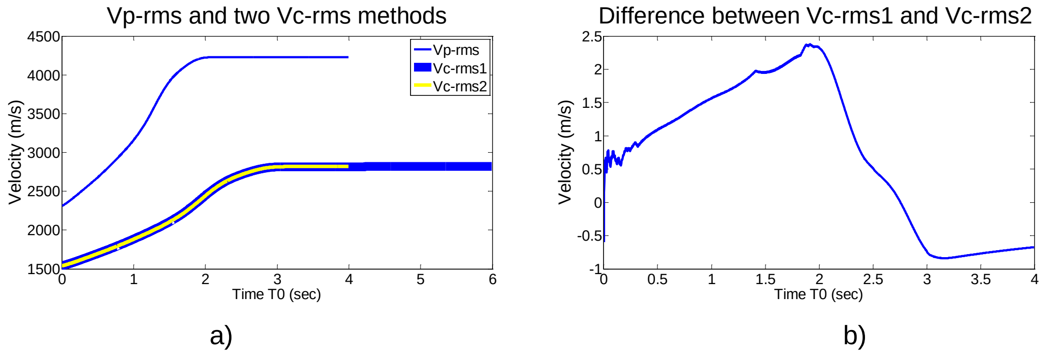

FIG 3. Plots of comparison converted wave velocity V_c with two methods. a) the P RMS velocities and the two methods of computing the C RMS velocities in time, and b) the error or difference between the two methods of computing C RMS velocity V_c.

The simplicity and accuracy of Method 2 makes it the preferred method of choice when converting P velocities to C velocities.

In Figure 4, we see the defined P velocities in green and the C velocities in red for one value of gamma. Note the additional time required for the C velocities, indicated by the extended red box.

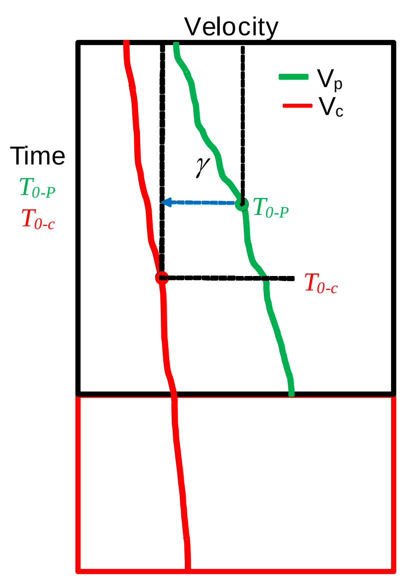

FIG 4. Mapping P and C velocities in P and C times respectively.

Figure 5 shows a comparison between original Vp-rms velocity in blue, Vc-rms(G) computed from Vp-rms and gamma equal to two in green and the more accurate Vc-rms(P) in red.

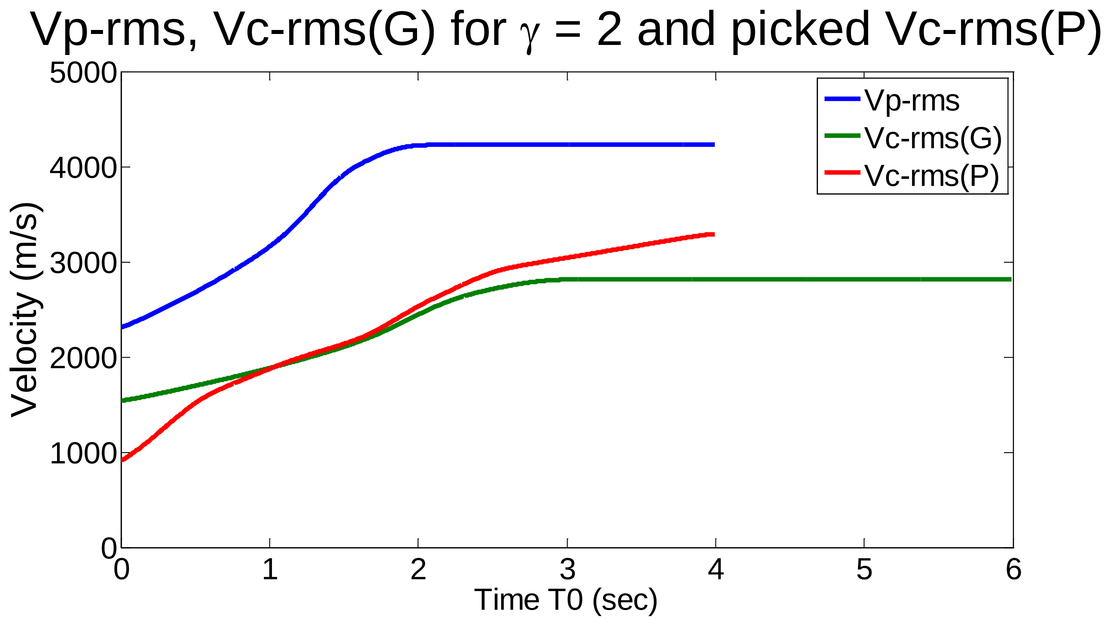

FIG 5. Mapping P and C RMS velocities.

Mapping converted wave time to P-wave times

Previously we saw the process of mapping P times to C times which was straight forward when we knew the P-wave velocities and a defined gamma function.

The reverse process of mapping C velocities and data to P-wave times enable converted wave data to be mapped to corresponding P data. This process is more complex. Again, we assume the ratios of the average and RMS velocities are similar, and compute a vector of pseudo-depths for both P time $\hat{z}_p\left(t_{0p}\right)$ and C time $\hat{z}_c\left(t_{0c}\right)$ at the sample time intervals.

We start with a depth at a given P time t_{0p}, and then search for a corresponding depth in the C array to define a corresponding C time t_{0c}.

$$\hat{z}_p\left(t_{0p}\right) = \hat{z}_c\left(t_{0c}\right). \tag{40}$$

The C velocities at t_{0c} are them mapped to a new array at the P times t_{0p}.

$$V_{c-rms}(\,t_{0c}) \Rightarrow \; V_{c-rms-p}(t_{0p})\,. \tag{41}$$

Estimating the S velocities

The previous section of estimating the C velocities is now used to estimate the S velocities for the full prestack migration that uses the DSR equation (17).

C velocities are useful for estimating a γ function and S-wave velocities. However, C velocities are not useful for forming CSP gather for a complete prestack migration of P-S data. After the LCCSP gathers have been formed, a new and more accurate estimate of V_c can be picked from the LCCSP gathers for moveout correction to complete the prestack migration.

The full prestack migration requires P and S velocities that use the DSR equation (17), defined at the same depths.

Depth arrays are computed from the P and C velocities respectively. The depths of the C velocities are matched to the depths of the P velocities. At a defined depth, P and C velocities are used to compute the S velocities, which are then mapped to the corresponding time of the P velocity. Now, when the DSR equation is used to compute a traveltime, the t_0 time can be used for both the P and S velocities. When the S velocities are mapped to $t_{o\text{-}p}$ times, and using the same pseudo depths $\hat{z_0}$ in each of the square roots, equation (16) becomes

$$t_c(\hat{z}_0^2) = \frac{1}{V_{p-rms}(\hat{t}_{0p}^2)}\ \sqrt{\hat{z}_0^2 + \ (x+h)^2} + \frac{1}{V_{s-rms-p}(t_{0p}^2)}\ \sqrt{\hat{z}_0^2 + (x-h)^2}\,. \quad (42)$$

This equation is then used to compute the times and equivalent offset for forming the CSP gathers. After these gathers are formed a third estimated of V_c is obtained from velocity analysis to apply moveout correction, amplitude scaling, and stacking to complete the prestack migration.

Method 1 of Vs

The interval S velocities $V_{s\text{-}Int}$ are computed similar to the V_c process using $V_{p\text{-}Int}$ and $V_{c\text{-}Int}$ from equation (11) at the same depth $\hat{z_0}$ from

$$V_{s-Int}(\hat{z}_0) = \frac{V_{p-Int} * V_{c-Int}}{2V_{p-Int} - V_{c-Int}}\,. \quad (43)$$

The interval velocities $V_{p\text{-}Int}$, $V_{s\text{-}Int}$, and $V_{c\text{-}Int}$ in depth are shown in Figure 6.

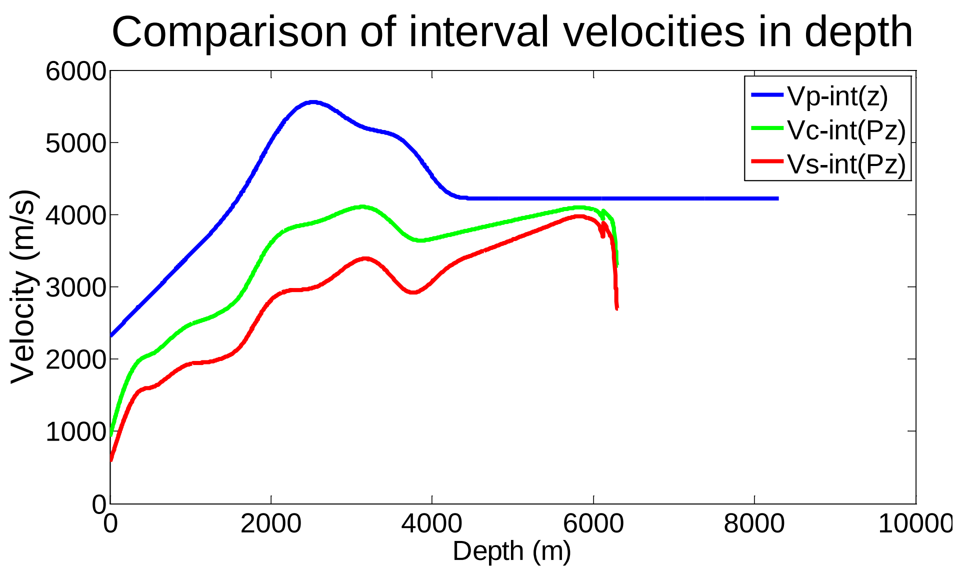

FIG 6. Plots of initial estimates of the converted wave velocity V_s with the Interval P, C and S velocities

The interval S velocities $V_{s\text{-}Int}$ are then converted to RMS velocities, $V_{s\text{-}rms}$, and mapped to P times at the corresponding depth.

Fast methods of Vs

This process can be performed more efficiently similar with the assumption that the velocity ratio of V_{rms}/V_{ave} and V_{rms}/V_{Int} are similar for both the P and S velocities. Pseudo depth arrays of $\hat{z}_0$ for both $V_{p\text{-}rms}$ and $V_{c\text{-}rms}$ are created and corresponding depths used to map $V_{c\text{-}rms}$ to the same time of $V_{p\text{-}rms}$ as $\breve{V}_{c-rms}$ when

$$\hat{z}_{Vs-rms}(t_s) = \hat{z}_{Vp-rms}(t_p) \ , \tag{44}$$

then

$$\breve{V}_{c-rms-p}(t_p) = V_{c-rms}(t_c) \ , \tag{45}$$

and we estimate $V_{s\text{-}rms}$ from

$$V_{s-rms-p}\left(t_p\right) = \frac{V_{p-rms}(t_p) * \breve{V}_{c-rms-p}(t_p)}{2V_{p-rms}(t_p) - \breve{V}_{c-rms-p}(t_p)} \, . \tag{46}$$

Figure 7a shows this RMS Velocity $V_{s\text{-}rms}$ using the method 1, $V_{s\text{-}rms\ exact}$ and method 2, $V_{s\text{-}rms\ fast}$. The difference in two methods is approximately 3%, for times less than 100 ms, and less than 1% for times greater than 200 ms and is illustrated in Figure 7b.

The improved estimate of V_c is used for as initial estimate of V_s for the formation of the unlimited CCSP gathers. The V_p and V_s velocities in t_{op} times can then be used to compute the RMS velocity value for γ.

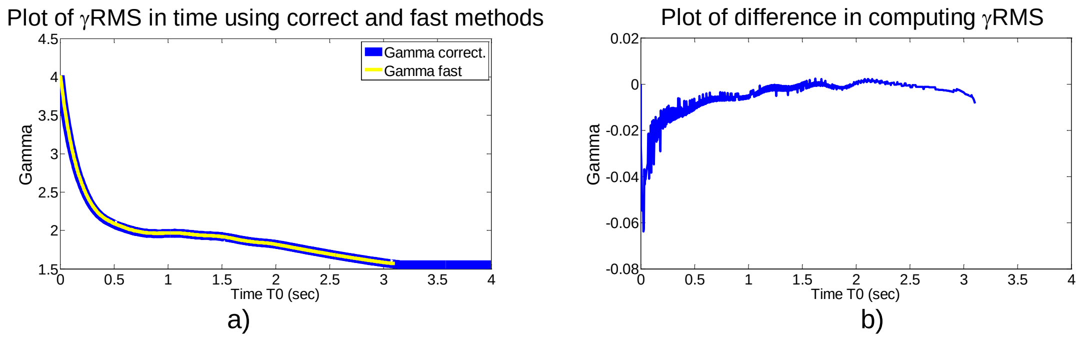

FIG 7. Plots of initial estimates of the converted wave velocity V_s with a) the RMS S velocity using two methods of computing the RMS velocities in time and b) the difference between both methods.

COMMENTS AND CONCLUSIONS

The processing of converted wave data requires a reasonably accurate estimate of converted wave velocities, V_c that is required to form Common Conversion Scatterpoint (CCSP) gathers as part of the Equivalent Offset Migration of converted waves. This velocity was first estimated from RMS velocities V_p and an initial constant value for the V_p/V_s ratio γ. This velocity was used first to form limited converted CSP (LCCSP) gathers that then provided an improved and second estimate of V_c. The resulting C velocities are then used to compute an estimated of the shear wave velocities V_s that can be used in the DSR equation for the full Equivalent offset prestack migration. A third estimate of V_c is obtained from velocity analysis of the full CSP gathers for moveout correction to completed the prestack migration.

ACKNOWLEDGMENTS

We thank to the all CREWES sponsors and staff for their support.

REFERENCES

Guirigay, T. A., and Bancroft, J. C., 2010, Converted wave processing in the EOM domain: CREWES Research Report, 22, 23.1-24

Guirigay, T., 2012, Estimation of Shear wave velocities from P-P and P-S seismic data using Equivalent Offset Migration: M.S. thesis, University of Calgary, Alberta, Canada, 99 p