A collision theory of seismic waves applied to elastic VSP data

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ABSTRACT

In previous CREWES reports we have been assembling a theory for seismic wave propagation which is based on particles with well-defined momenta and masses colliding inelastically. Here we discuss some early arguments towards applying this collision picture to elastic VSP data, such that it describes elastic processes, such as conversions. We choose a candidate reflection in the Husky Cold Lake 3C walkaway VSP data set, as discussed by Hall et al. in the 2012 CREWES Report, which has clear evidence of P-P interactions and P-S conversions, and report on the beginning of our attempts to validate the collision model.

INTRODUCTION

We have discussed the possibility of using particle dynamical quantities to describe seismic waves. In 1D scalar reflection and transmission problems (Innanen, 2010), the conservation of event “masses” and “momenta” before and after a reflection (as defined with time and depth exchanged) can be applied directly to the wave amplitudes to get consistent results. By thinking of the wave amplitudes as being descriptive of a large number of identical particles whose velocities obey uniform, Poisson, or Pareto-Levy statistics, bandlimited behaviour, wavelets, attenuation and dispersion can be included without complication (Innanen, 2011). Multidimensional problems and wave obliquity along the observation axis add no fundamental problems either (Innanen, 2012). What does require some additional concept development is the influence of multiple parameters varying across reflecting boundaries, and the use of, e.g., displacement as opposed to potential reflection and transmission coefficients.

In this report, we introduce two new aspects: (1) the observation of particle-like behaviour in field VSP data, and (2) the discussion of the particle interactions in terms of elastic wave theory. Both are incompletely developed, but neither can be sensibly discussed without the other.

We will consider the case of an incident P-wave in elastic media. The boundary conditions in any given problem can be used to relate the transmission coefficient(s) to incidence wave quantities and the reflection coefficient(s). From the point of view of the particle model, these relationships define the mass and momentum conservation rules obeyed by seismic “particles” which interact inelastically. We begin by re-writing the Zoeppritz equations in this manner, producing the isotropic elastic versions of the mass and momentum rules. We tie this in with a description of the sets of expected particles and behaviours when elastic reflection and conversion are both permitted. Finally, we discuss a field VSP data set (Hall et al., 2012) in the context of colliding elastic particles.

SOME USEFUL ELASTIC AMPLITUDE RELATIONS

In the scalar case (Innanen, 2010) we decided that of the two equations deriving from the scalar boundary conditions, one,

$$T = 1 + R, \tag{1}$$

was interpretable in the collision framework as a statement of conservation of mass. "After" the collision, i.e., at VSP depths greater than the depth of the interface, the aggregate particle has mass T, the sum of the masses 1 and R of the two colliding particles "before" the collision. The other equation of the two,

$$\left(\frac{1}{c_1}\right) T = \left(\frac{1}{c_0}\right) 1 + \left(-\frac{1}{c_0}\right) R, \tag{2}$$

or alternatively

$$T = \left(\frac{c_1}{c_0}\right) 1 - \left(\frac{c_1}{c_0}\right) R, \tag{3}$$

since it involves velocities*, in products with the newly minted mass quantities, are naturally interpretable as momenta.

The elastic problem is significantly more complicated, but it is possible to put the elastic boundary condition equations into roughly similar form. Assuming a reflection/transmission configuration as illustrated in Figure 1, we may start with the half of the Zoeppritz equations appropriate for an incident P-wave (Aki and Richards, 2002).

Beginning with the top two rows of the 4×4 matrix of Keys (1989), we arrive at two equations which generalize the scalar case in equation (1). One is a solution for the transmitted P amplitude:

$$T_{\text{PP}} = \frac{\cos(\theta - \phi_1)}{\cos(\theta_1 - \phi_1)} - \frac{\cos(\theta + \phi_1)}{\cos(\theta_1 - \phi_1)} R_{\text{PP}} + \frac{\cos(\phi + \phi_1)}{\cos(\theta_1 - \phi_1)} R_{\text{PS}}, \tag{4}$$

and the other for the transmitted conversion (S) amplitude:

$$T_{\text{PS}} = \frac{\sin(\theta - \theta_1)}{\cos(\theta_1 - \phi_1)} + \frac{\sin(\theta + \theta_1)}{\cos(\theta_1 - \phi_1)} R_{\text{PP}} + \frac{\cos(\phi + \theta_1)}{\cos(\theta_1 - \phi_1)} R_{\text{PS}}. \tag{5}$$

Then using the bottom two rows of the 4×4 Zoeppritz matrix we further generalize the scalar momentum equation (3). This is a slightly more complicated set of relationships, but it can be written compactly in terms of two 2×2 matrices:

$$\begin{bmatrix} T_{\text{PP}} \\ T_{\text{PS}} \end{bmatrix} = -\left(\frac{V_{S_0}}{V_{S_1}}\right)\left(\frac{\rho_0}{\rho_1}\right) \mathbf{M}^{-1}\mathbf{N}, \tag{6}$$

*We agreed, as one of the rules of the game, to exchange depth and time, which means collision model velocities appear as reciprocals of normal velocities, i.e., slownesses.

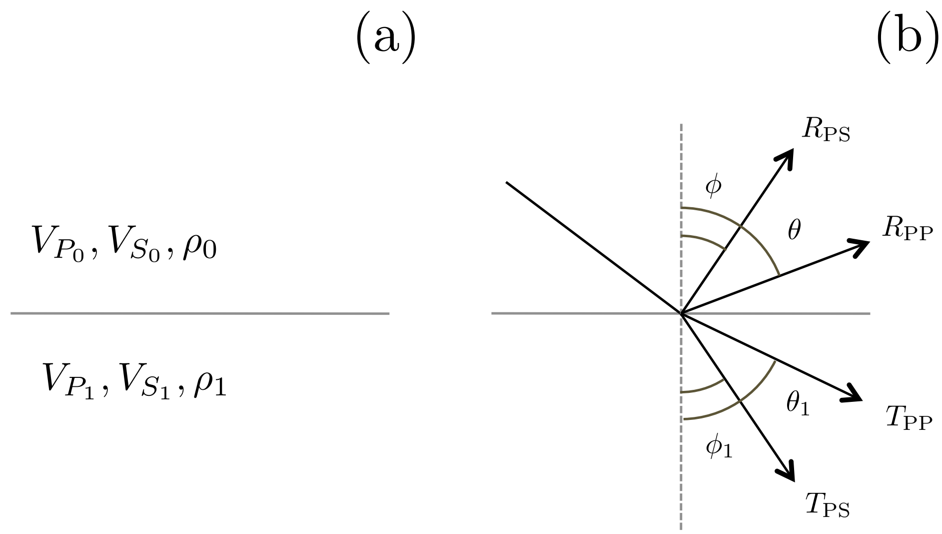

FIG. 1. The elastic particle model for an incident P-wave. (a) A horizontal planar boundary separates an upper half space with elastic properties V_{P_0}, V_{S_0} and ρ_0 from a lower half space with properties V_{P_0}, V_{S_0} and ρ_0. (b) An incident P wave excites reflected P and S waves and transmitted P and S waves.

where

$$\mathbf{M} = \begin{bmatrix} \sin\phi_1 \cos\theta_1 & (1/2 - \sin^2\phi_1) \\ \gamma_1(1/2 - \sin^2\phi_1) & -\sin\phi_1\cos\phi_1 \end{bmatrix} \tag{7}$$

and

$$\mathbf{N} = \begin{bmatrix} \sin\phi\cos\theta(1 - R_{\mathrm{PP}}) & -(1/2 - \sin^2\phi)R_{\mathrm{PS}} \\ \gamma_0(1/2 - \sin^2\phi)(1 + R_{\mathrm{PP}}) & -\sin\phi\cos\phi R_{\mathrm{PS}} \end{bmatrix}. \tag{8}$$

Although we will use them in the collision model, these results are not tied to such a model at all—they are true statements about the transmission and reflection coefficients in the Zoeppritz equations.

THE ELASTIC COLLISION PICTURE FOR AN INCIDENT P-WAVE

The collision model involves visualizing wave processes occurring as if time and depth (or whatever preferred space axis one cares to choose) were interchanged. That is, whereas we normally experience phenomena over an extended interval in space evolving "frame by frame" with time, the collision model requires us to experience the wave over an extended interval in time, evolving frame by frame with depth. In Figure 2a, we sketch the time and depth history of a plane P wave incident on a single elastic boundary (if the wave is obliquely incident, we interpret the plot as the projection of the event onto the depth axis). Depth is the vertical coordinate, and time is the horizontal coordinate. We can visualize a VSP experiment being carried out by drawing a vertical line at the left side of Figure 2a, calling that point $t = 0$, and then allowing it to sweep at a uniform rate to the right. At any given moment during the experiment, the vertical line intersects all wave events at the points in depth they are to be found.

For the first few moments, the vertical “now-line” (which separates the past and the future in the diagram), as it moves from left to right, intersects only the incident wave (solid black line). The incident wave propagates downward—that is, it intersects the now-line at points of increasing depth on the vertical axis. At the instant the wave hits the interface, its energy is partitioned into four components: reflected PP (blue solid), reflected PS (red), transmitted PP (blue dashed) and PS (red dashed). Thereafter these events propagate up or down at their prescribed velocities.

In the collision model, where time and depth are interchanged, we instead visualize the process unfold by drawing a horizontal line at the top of the diagram, and allowing it to sweep downwards at a uniform rate. In this model, therefore, a “moment” is defined as a fixed point in depth, not time. The intersection of the new now-line with any of the wave paths again shows where the event is at that moment.

Figures 2a–d illustrate four such depth moments, labelling them t_1 through t_4. Watching where the events are as we step from t_1 to t_4, the collision picture emerges. At early moments during the collision process, three particles move towards a point of eventual collision. One moves from left to right, and two move from right to left. The latter two are at different locations, but they also drift with different velocities. The three locations and velocities of the three particles are just such that at the moment t_3 corresponding to the depth of the interface, they collide. An inelastic interaction occurs, whose details are such that, after it is over, two particles emerge, both propagating downwards.

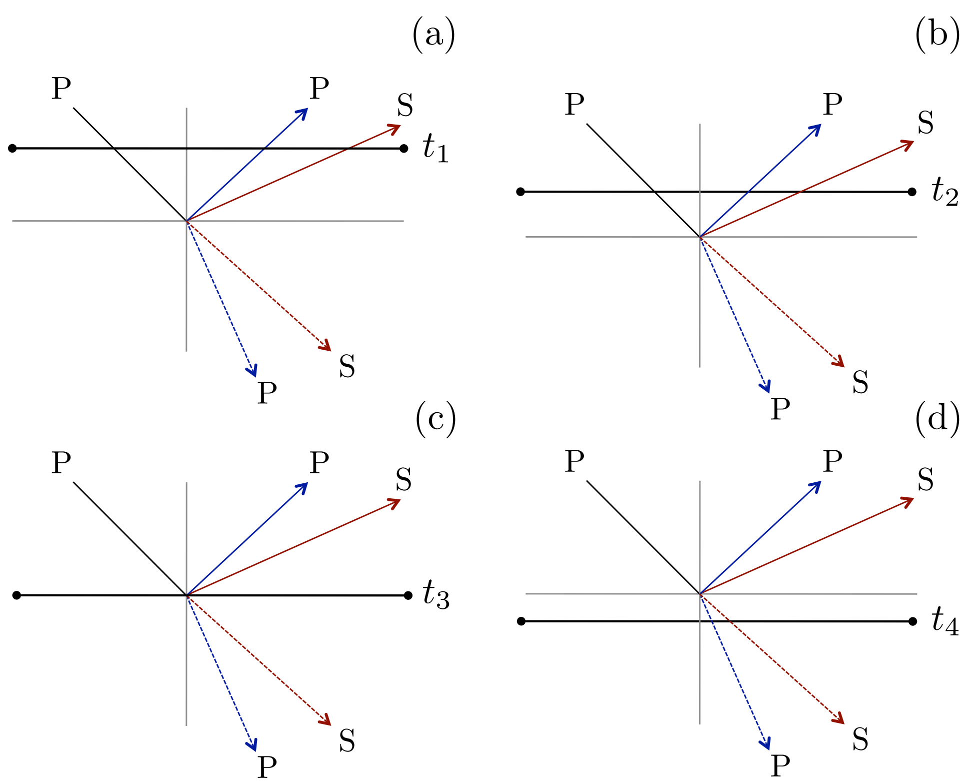

FIG. 2. Four “moments” during the seismic particle collision process. (a)–(d) Three particles approach one another, two at equal and opposite speeds, the other slower. They collide at t_3, and thereafter two particles propagate away, both in the same direction but at different speeds.

The way the masses and momenta are shared after the interaction in terms of their values before the interaction is given by equations (4) and (5) (mass relations), and equations (6) (momentum relations).

GATHERS FROM THE HUSKY 2011 3C WALKAWAY VSP DATA SET

In addition to pursuing further the momentum and mass transfer between particles further, we have also sought to track this behaviour in field VSP data. The Husky 2011 walkaway 3C data set (Hall et al., 2012) contains some distinct reflections and conversions, and so we will use this data set to take some initial steps towards characterizing field data in terms of these interacting particles.

In Figure 3, the vertical (a) and radial (b) components of the data set are illustrated for the near offset vibe shot. The data are filtered with Ormsby parameters $f_1 = 60.0$Hz, $f_2 = 85.0$Hz, $f_3 = 290.0$Hz, and $f_4 = 310.0$Hz.

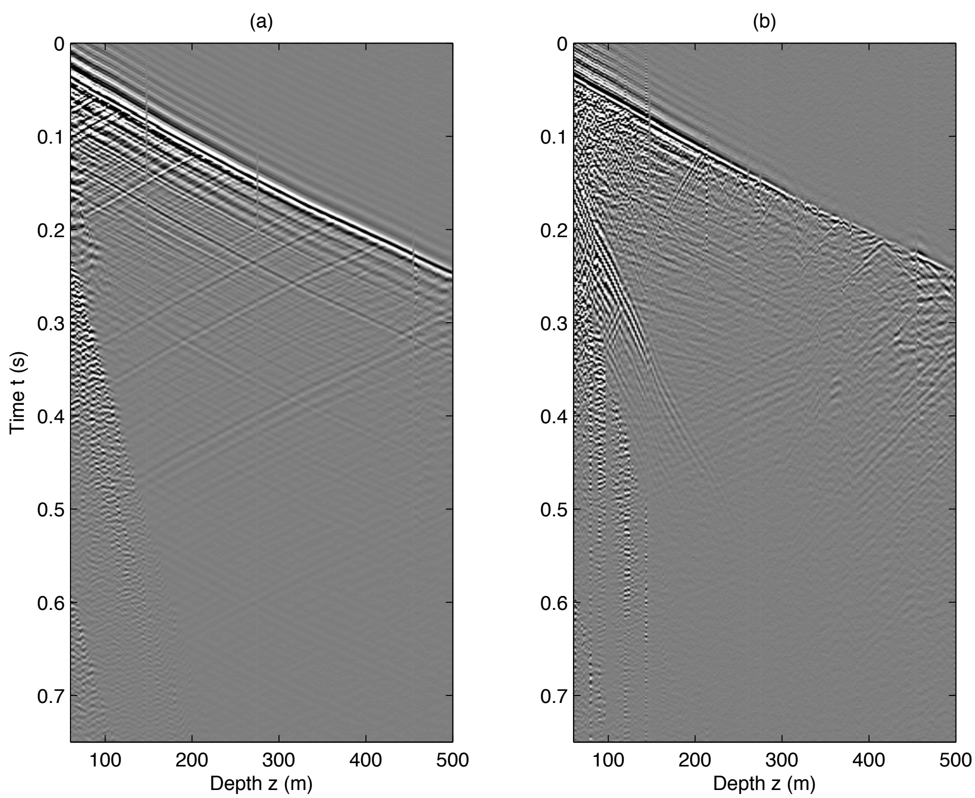

FIG. 3. Husky 3C walkaway VSP data set, near offset, (a) vertical component, (b) radial component. Filtered with Ormsby parameters $f_1 = 60.0$Hz, $f_2 = 85.0$Hz, $f_3 = 290.0$Hz, and $f_4 = 310.0$Hz.

Our plan will be to attempt to identify a candidate interaction in this data set, which is appropriate as a test case. This is not necessarily a simple task, in spite of the ease with which events are identifiable. The collision model requires us to view the VSP data trace by trace, and trace by trace reflected and converted events can be difficult to distinguish by eye.

A PARTICLE COLLISION POINT

A reflector at roughly 200m depth appears to fit the bill nicely, as it excites a PP reflection (visible in the vertical gather) and a PS reflection (visible in the radial gather). In Figure 4a the vertical gather is plotted with dashed cross-hairs indicating the reflection point.

Esimating from the slopes of the direct, transmitted and reflected waves the P-wave velocity is close to 1900m/s above 200m, and 2050m/s below. Using these values in Figure 4a we draw in the events. The direct wave is plotted in yellow, and the reflected and transmitted P waves are plotted in red. Similarly, estimating from the slopes an average S wave velocity roughly 550m/s, we draw in the S events in Figure 4b in blue.

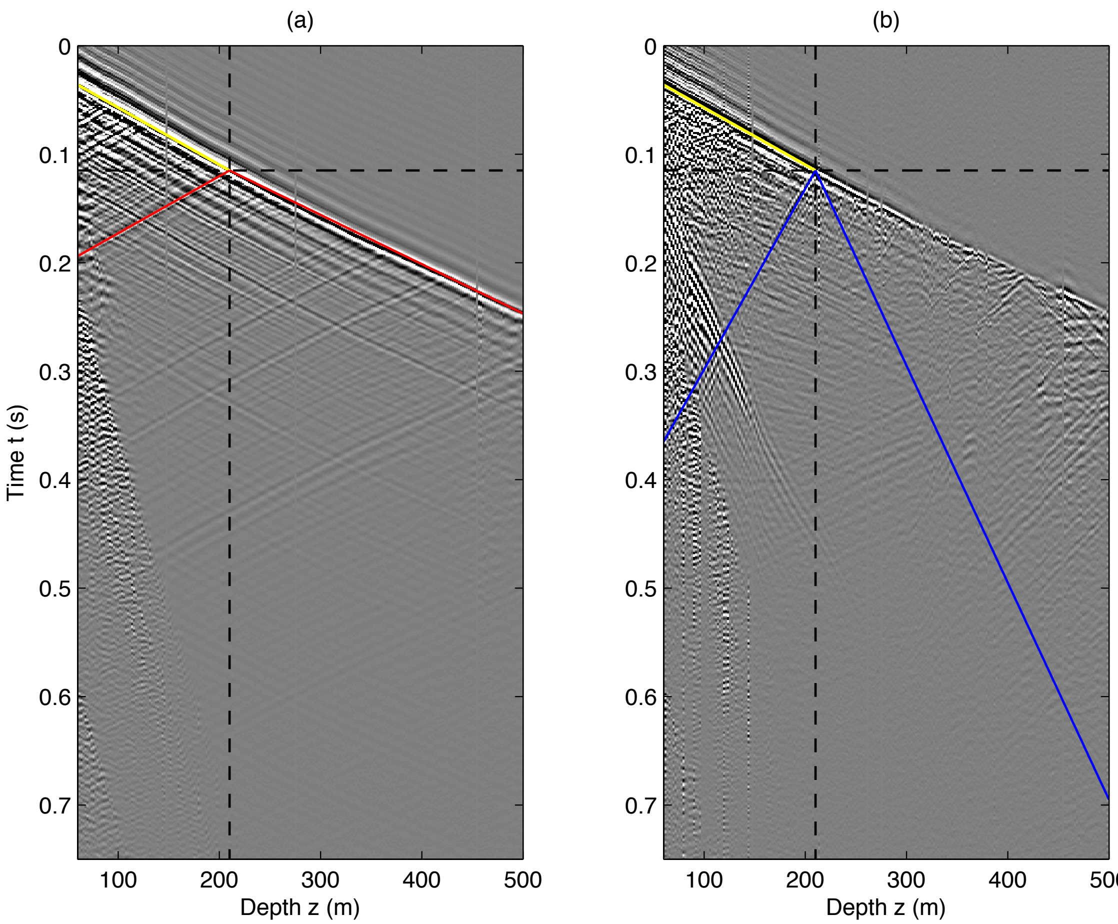

FIG. 4. (a) Vertical component gather with time-depth paths; incident wave in yellow, PP reflected and transmitted waves in red. (b) Radial component gather in with time-depth paths; incident wave in yellow; PS reflected and transmitted waves in blue.

Tracking the incident P and reflected P waves in the vertical component

Let us get a preliminary sense of what will be necessary by tracking the PP and PS events as if the collision processes were occurring now. A "movie frame" is constructed by extracting a vertical slice from data like that in Figure 4a or b — namely a trace. We repeat this process with the vertical slice location advancing from left to right.

In Figure 5a we begin this process on the PP reflection by analyzing the vertical component gather, for which the PP energy is the strongest. On the right hand panel we reproduce the data vertical component data. The vertical slice location is illustrated with a vertical dashed line. In Figure 5b we repeat the illustrations absent the data themselves. Where the vertical slice (i.e., the "moment" at which we observe the data) crosses the yellow and red lines, we expect to find the incident and reflected events destined to collide. Two additional dashed lines, these ones horizontal, highlight these points.

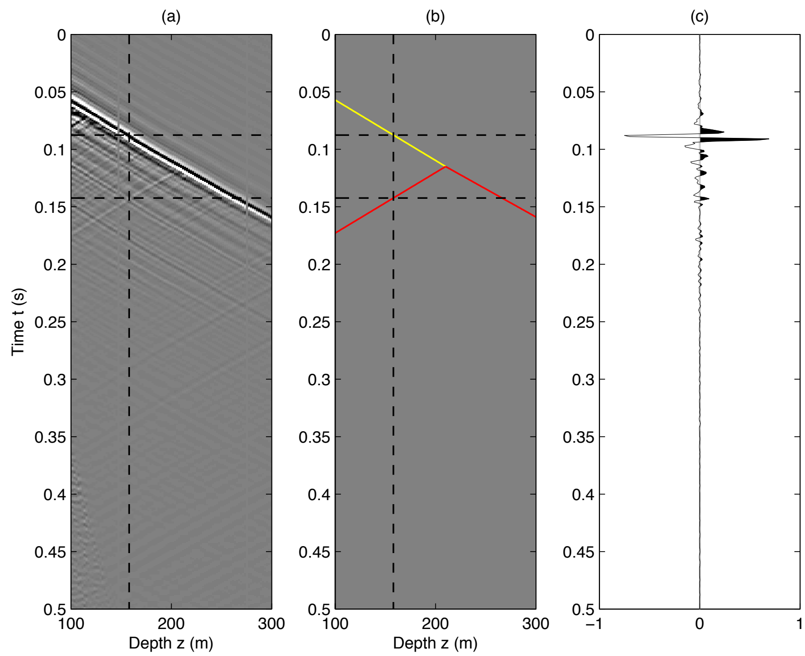

FIG. 5. (a) Vertical component data gather with the first "moment" indicated with a vertical dashed line. The intersections of the PP reflection and the incident P wave with the vertical line. (b) Interpreted events in a blank data matrix. (c) VSP trace corresponding to the current depth "moment".

The two horizontal dashed lines line up with the points in the trace in Figure 5c where the two events are found. There is of course no doubt as to where on the trace the incident wave is found. The reflected event is visible but significantly lower in amplitude.

In Figures 6–7 we repeat the same plots, with the vertical slice advancing to increasing depths. On the traces the reflected event and the incident wave drift towards one another as expected—though with the reflected event occasionally vanishing in the noise.

Tracking the incident P and reflected S waves in the radial component

In Figures 8–10 we repeat the procedure for the PS event, using the radial component gather wherein we expect the conversion to be more strongly visible. It is, but in the traces alone (i.e., without the visual aid of spatial coherence in a gather), the converted event is quite low in amplitude.

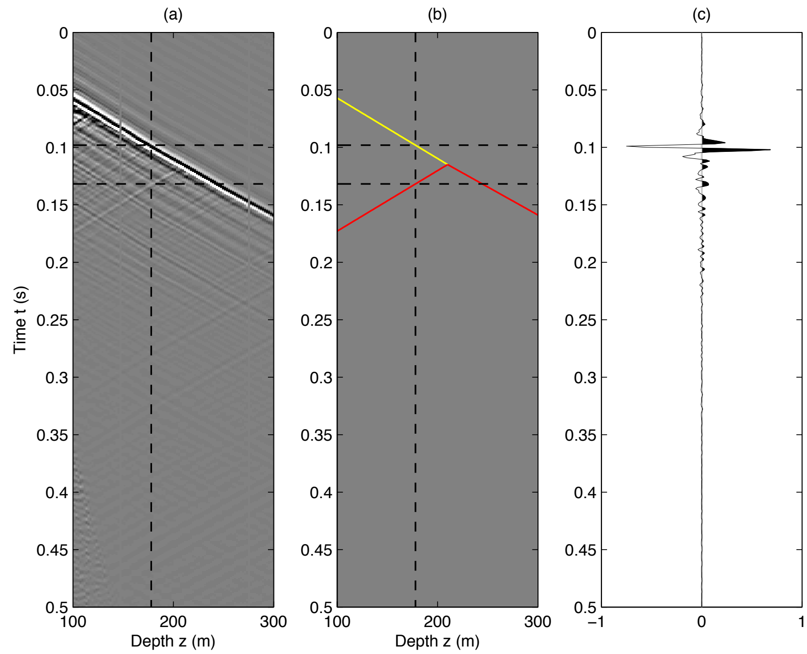

FIG. 6. (a) Vertical component data gather with a subsequent moment indicated with the vertical dashed line. The intersections of the PP reflection and the incident P wave with the vertical line. (b) Interpreted events in a blank data matrix. (c) VSP trace corresponding to the current depth “moment”.

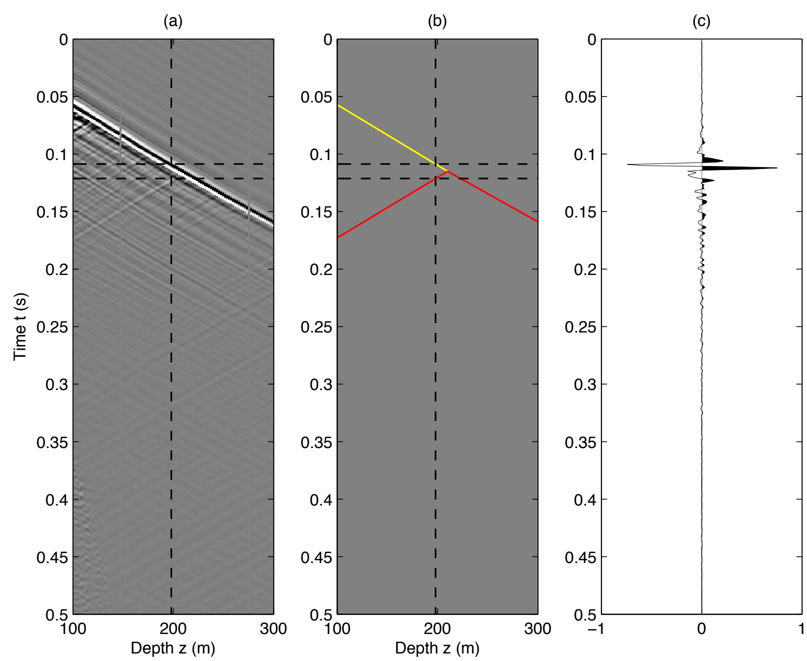

FIG. 7. (a) Vertical component data gather with a subsequent moment indicated with the vertical dashed line. The intersections of the PP reflection and the incident P wave with the vertical line. (b) Interpreted events in a blank data matrix. (c) VSP trace corresponding to the current depth “moment”.

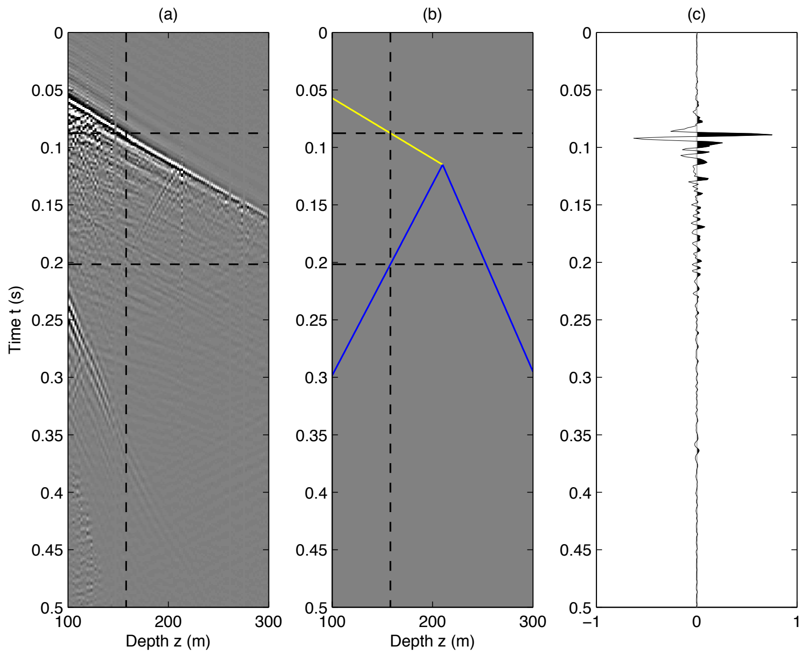

FIG. 8. (a) Radial component data gather with the first “moment”, or vertical slice, indicated with the vertical dashed line. The intersections of the PS reflection and the incident P wave with the vertical line. (b) Interpreted events in a blank data matrix. (c) VSP trace corresponding to the current depth “moment”.

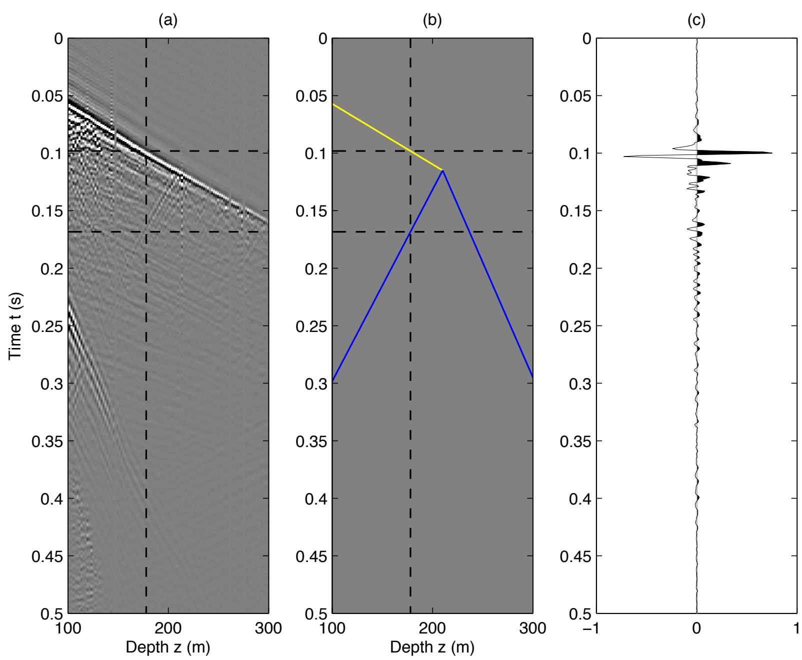

FIG. 9. (a) Radial component data gather with a subsequent moment’ indicated with the vertical dashed line. The intersections of the PS reflection and the incident P wave with the vertical line. (b) Interpreted events in a blank data matrix. (c) VSP trace corresponding to the current depth “moment”.

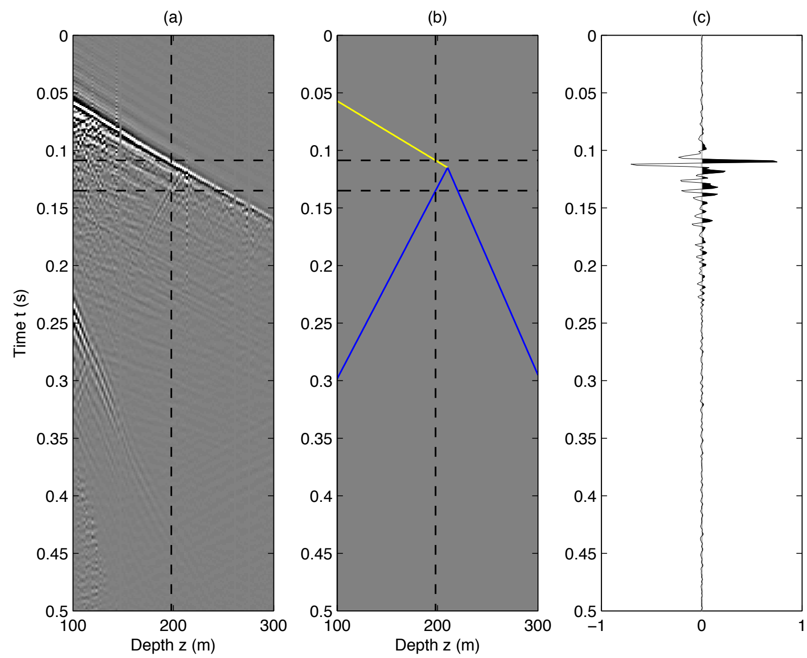

FIG. 10. (a) Radial component data gather with a subsequent moment' indicated with the vertical dashed line. The intersections of the PS reflection and the incident P wave with the vertical line. (b) Interpreted events in a blank data matrix. (c) VSP trace corresponding to the current depth "moment".

AMPLITUDES BEFORE AND AFTER THE POINT OF COLLISION

If we extract the traces at these "moments", and string them together like a movie, the particle processes are visible in the field VSP data. In Figures 11 and 12 we illustrate some selected frames from the movie as extracted from the vertical component data set. Frames similarly extracted from the radial component gather are illustrated in Figure 13. Where possible and/or necessary the amplitudes of the approaching events are indicated with horizontal dashed lines, and the event times themselves with vertical dashed lines.

CONCLUSIONS

In previous CREWES reports we have been assembling a theory for seismic wave propagation which is based on particles with well-defined momenta and masses colliding inelastically. Here we have discussed applying this collision picture to elastic VSP data, such that it describes elastic processes, such as conversions. Interpreting a field VSP data set in terms of the picture allows us to point to some outstanding issues. The model, like the models used to derive the Zoeppritz equations, is in terms of total motion. To ascertain that the collision model is consistent with field data we will need to incorporate the projection of the total motion onto three components, most likely the post-processing vertical-radial-transverse set.

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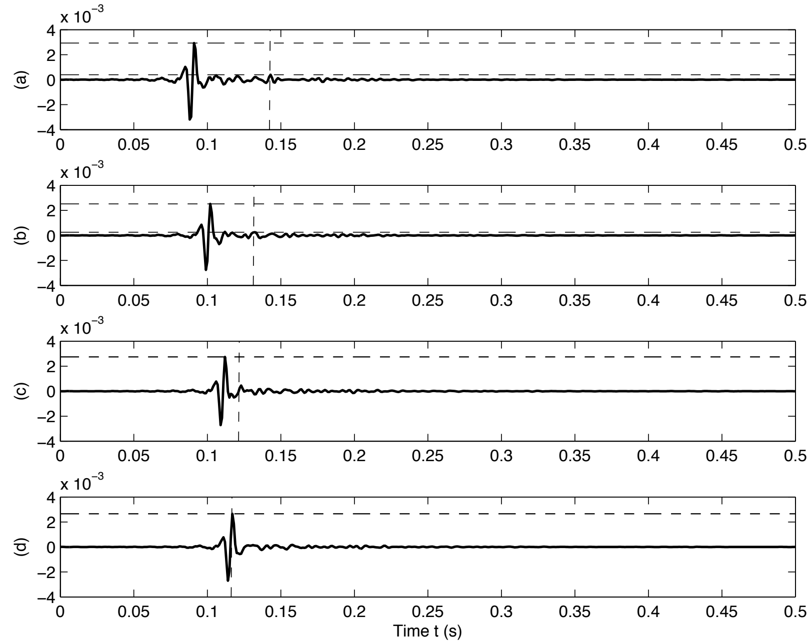

FIG. 11. (a)–(d) Frames of the collision process extracted from the vertical component VSP data set, for depth “moments” leading up to the point of collision. The reflected event is indicated with dashed lines.

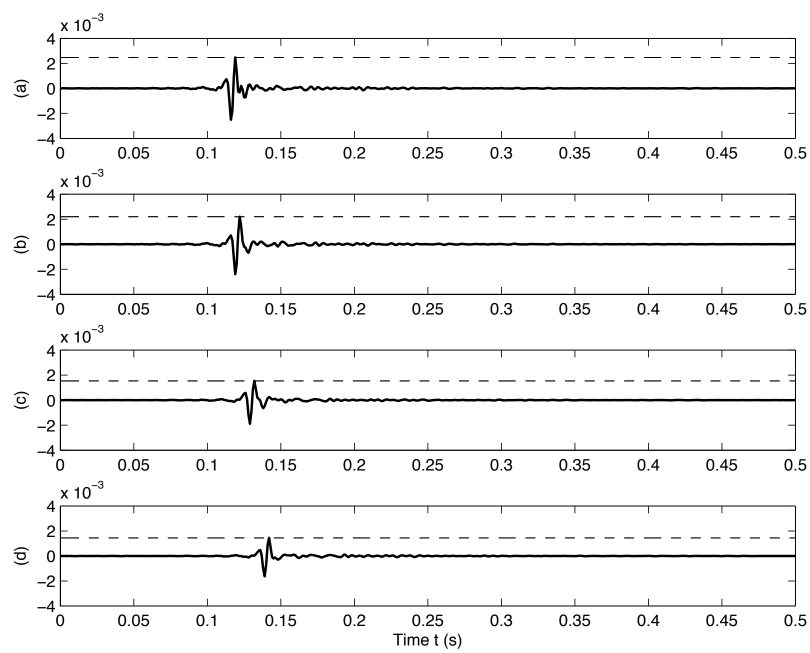

FIG. 12. (a)–(d) Frames of the collision process extracted from the vertical component VSP data set, for depth “moments” after the point of collision. The amplitude of the transmitted event is indicated with dashed lines.

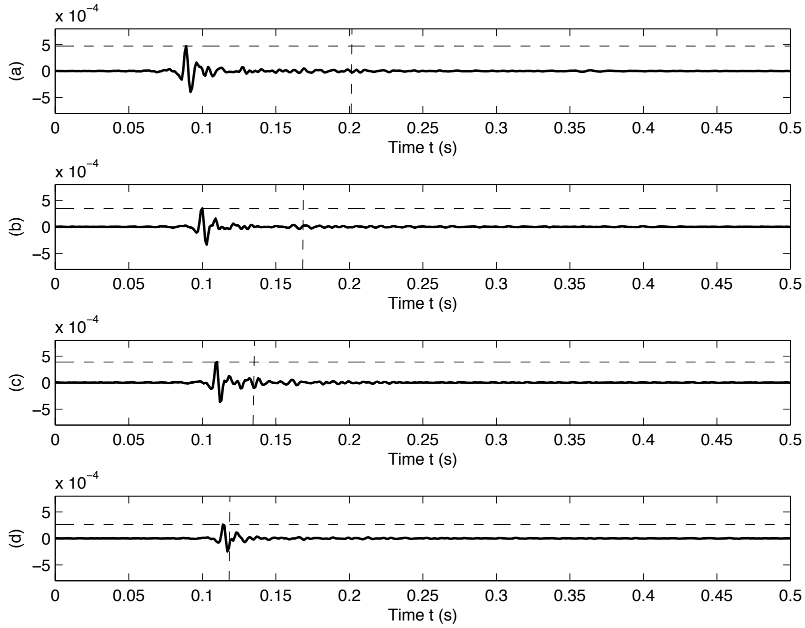

FIG. 13. (a)–(d) Frames of the collision process extracted from the radial component VSP data set, for depth "moments" leading up to the point of collision. The converted event is indicated with dashed lines.

ticipate in the 2011 VSP experiment and to publish results involving the data.

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