Internal multiple prediction in the continuous wavelet transform maxima domain

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ABSTRACT

We test possible applications of seismic processing in the continuous wavelet transform maxima (CWTM) domain, which is the prediction of internal or interbed multiples. Both the use of the algorithm in the CWTM domain itself and the use of CWTM preprocessed data in the nominal (time or pseudodepth) domain are considered. In this initial study it is established that the outcome of the use of the prediction algorithm in the CWTM domain is close to that produced in the nominal domain, opening this domain up to examination, in particular with respect to scale dependent integration limit parameters. It is also established using a particularly noisy poststack land data trace that aggressive denoising in the CWTM domain, while potentially harmful in action on traces whose amplitude information will be interpreted, could be very effective if used on the data ingredients of the prediction operator.

INTRODUCTION

In this paper we will examine one possible use of the Mallat-Zhong framework for processing of wavelet transform modulus maxima as developed in this year's report by Innanen (2013). As described in a variety of venues (which are reviewed by Hernandez and Innanen, 2013), noise on land data traces is a serious impediment to successful internal multiple prediction as developed by Araújo (1994) and Weglein et al. (1997), in which the data themselves are used to produce the prediction operator. We have demonstrated that with an aggressive soft-thresholding scheme, based on the continuity of parts of the signal across scale, a dramatic reduction in random noise can be obtained on a field trace. The trace was extracted from the NEBC data set studied and discussed in detail by Hernandez and Innanen (2013) and Zuleta (2012). The problem was, that in so denoising, the signal in the trace, while "kept", is altered in phase and amplitude, and the damage to the signal is likely too high for what remains, however impressive, to be used.

However, the aggressively denoised trace, even with damaged primaries, could be used as part of the internal multiple prediction operator without this same concern being raised. The prediction itself requires much further processing in any case prior to subtraction, so that fact that amplitudes have been altered in the CWTM processing is much less of an issue. Further, we take an interest in the action of the prediction operation, not on CWTM-denoised data, but on *while it is in the* WTM domain. In this paper we report on some initial testing of both these ideas.

PREDICTION IN THE WTM DOMAIN

In the first section, we analyze with some synthetic examples the ability to predict internal multiples by applying the prediction algorithm in the CWTM domain itself. This is by no means guaranteed to be a "legitimate" procedure, for although the forward and inverse CWT are linear transforms, the internal multiple prediction algorithm is not—the predicted output is the product of three signals b_1:

$$b_{3\text{IM}}(k_z) = \int_{-\infty}^{\infty} dz' e^{ik_z z'} b_1(z') \int_{-\infty}^{z'-\epsilon} dz'' e^{-ik_z z''} b_1(z'') \int_{z''+\epsilon}^{\infty} dz''' e^{ik_z z'''} b_1(z'''). \quad (1)$$

Nevertheless the effect of this may be small, and the opportunities for using much smaller, and/or scale-varying ϵ values (Weglein and Matson, 1998), may make it worth the effort.

Analysis of internal multiple prediction in the WTM domain

In Figure 1a is a simple trace associated with a 1D, two interface medium. Two primaries are visible, as is one multiple at around 0.7s. In Figure 1b, the multiple is modelled alone.

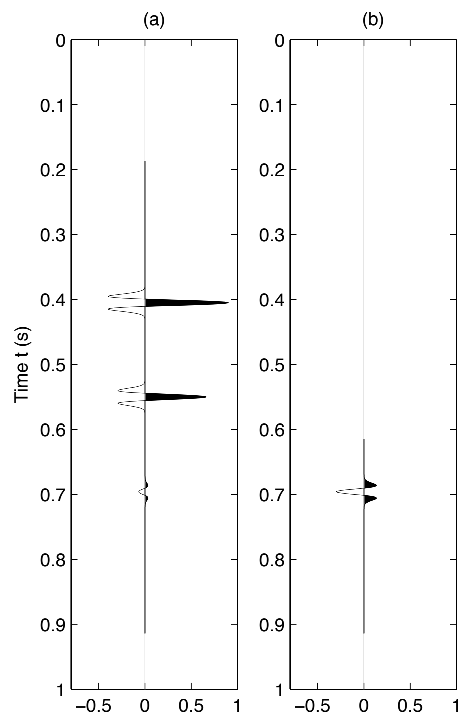

FIG. 1. Synthetic trace. (a) Two primaries and one multiple. (b) The multiple plotted in isolation.

To begin the study, we create two sets of CWTMs; first, those derived directly from the modelled multiple alone (i.e., Figure 1b). Mathematically,

$$\tilde{d}_1(t, \sigma) = \text{MW}d(t). \quad (2)$$

These maxima will be our reference. Second, we produce the CWTMs from the full trace (i.e., Figure 1a), and carry out the prediction process scale by scale. Mathematically, we construct $b_1(z)$ in the pseudo-depth domain from the input trace $d(t)$ (Weglein and Matson, 1998), and then transform it into the CWTM domain:

$$\tilde{b}_1(z, \sigma) = \text{MW}b_1(z). \quad (3)$$

We will then carry out the prediction in equation (1) using this as input. The prediction is run scale by scale:

$$\begin{aligned}\tilde{B}_{3\mathrm{IM}}(k_z,\sigma) = &\int_{-\infty}^{\infty} dz' e^{ik_z z'} \mathrm{MW} b_1(z',\sigma) \\ &\times \int_{-\infty}^{z'-\epsilon} dz'' e^{-ik_z z''} \mathrm{MW} b_1(z'',\sigma) \\ &\times \int_{z''+\epsilon}^{\infty} dz''' e^{ik_z z'''} \mathrm{MW} b_1(z''',\sigma).\end{aligned} \tag{4}$$

The point of caution is that this is not guaranteed to be equivalent to the CWTM domain version of the prediction alone:

$$\tilde{b}_{3\mathrm{IM}}(z,\sigma) = \mathrm{MW} \int dk_z e^{ik_z z} \tilde{b}_{3\mathrm{IM}}(k_z,\sigma), \tag{5}$$

that is, $\tilde{B}_{3\mathrm{IM}}(z,\sigma) \neq \tilde{b}_{3\mathrm{IM}}(z,\sigma)$.

In Figures 2a–c, three scales of the result of the prediction of the CWTM are plotted in black, zoomed in on the multiple. For reference the CWTMs of the modelled multiple are plotted in blue, upshifted. There is reasonable agreement, but the prediction has generated a larger array of maxima. This is likely a consequence of the triple product of the data amongst themselves in the CWTM domain.

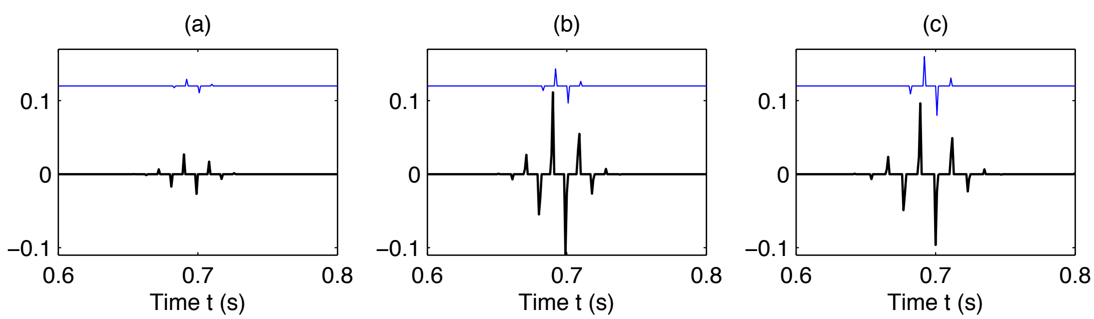

FIG. 2. Multiple predictions in the continuous wavelet transform maximum (CWTM) domain. In black, the multiple prediction output; in blue, the CWTMs of the modelled multiple for refernce. (a)-(c) Scales 2^j, with $j = 2, 3, 4$.

However, the extra spikes to the left and right of the arrival time are small in amplitude, and hence we expect that the process of soft thresholding may be adequate to finalize the prediction and arrive at a meaningful output. The process is illustrated in Figure 3. In the top row we repeat the input—in black the raw prediction maxima, and in blue the idealized maxima. In the bottom row are the prediction scales after thresholding, in which the amplitudes below a certain size are set to zero. The remaining amplitudes are not equal to the idealized maxima, but a simple thresholding procedure has led to a similar number and arrival time.

Finally in Figure 4 we illustrate the results of the prediction and thresholding, reconstructed (as per Innanen, 2013) and displayed in the time domain. In Figure 4a the original

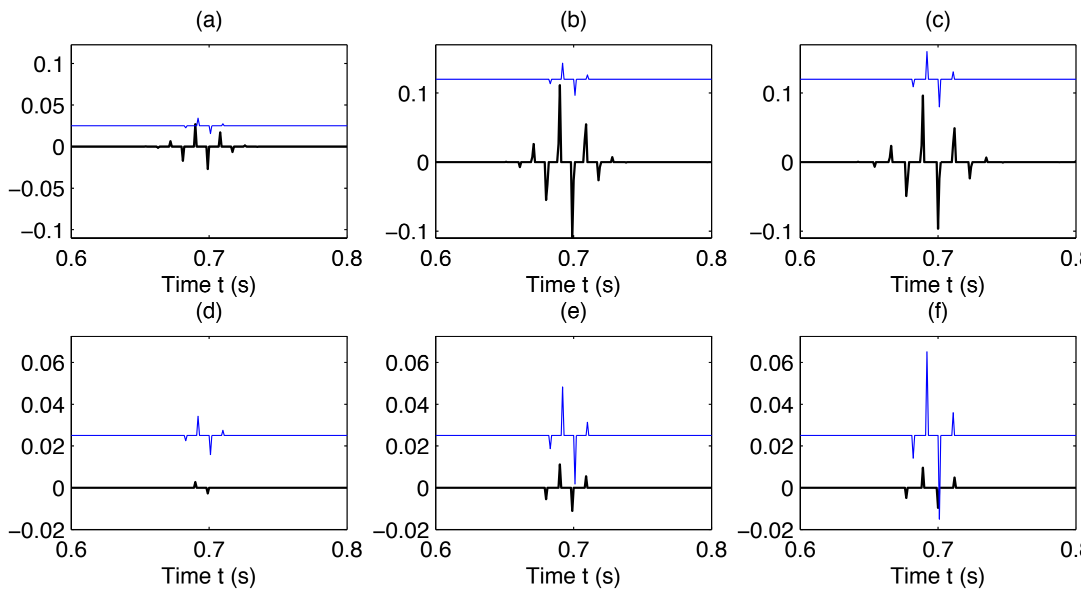

FIG. 3. (a)-(c) Input raw predictions in the CWTM domain for scales 2^j, with $j = 2, 3, 4$. Black are the predictions; blue are the idealized CWTMs. (d)-(f) After thresholding.

trace is plotted; in Figure 4c the modelled multiple is plotted, and in Figure 4b, in the centre, the reconstructed multiple prediction, generated in the CWTM domain, is plotted. There are some phase differences between the modelled multiple and the prediction, but such phase effects occur in standard internal multiple prediction also to roughly the same degree. We conclude that multiple prediction can plausibly occur in the CWTM domain.

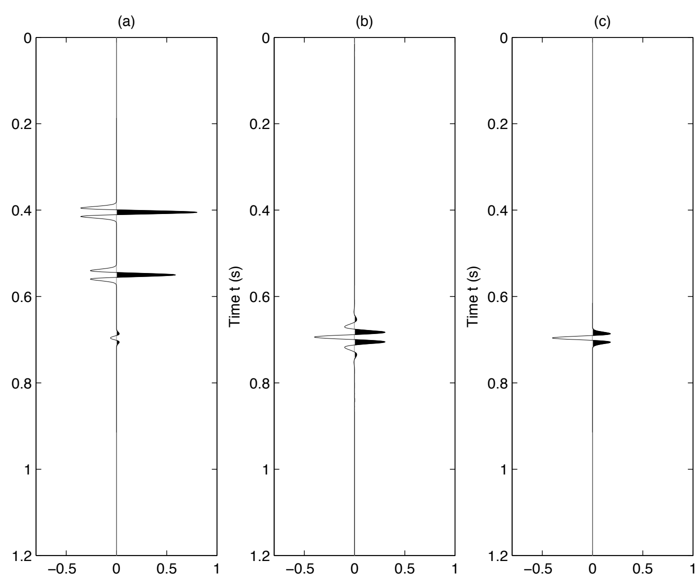

FIG. 4. (a) Input trace; (b) multiple as predicted in the CWTM domain; (c) modelled multiple.

AGGRESSIVE THRESHOLDING OF WTM INPUT

The second initial test of continuous wavelet transform maxima multiple processing we would like to make is the use of the aggressively denoised data in the construction of the prediction operator, namely the construction of the inputs $b_1(z)$. Note that now the internal multiple prediction itself has nothing to do with the CWTM transform and reconstruction—the idea is to investigate the effect of pre-processing prediction input with strong denoising.

Hernandez and Innanen (2013) used the NEBC land data set, known to have internal multiple issues, to investigate the utility of the 1D prediction algorithm. That data set contains some traces with significant noise, as reviewed in that paper. In Figure 5a, a representative poststack trace is displayed. Especially without the benefit of a section to highlight spatial coherency, this is not an easy trace to identify multiples in. Nevertheless, there is nothing stopping us from applying the internal multiple prediction algorithm to it: the result is plotted in Figure 5b. Unfortunately, as tends to be the case with such noisy data, the output too is noisy and difficult to interpret.

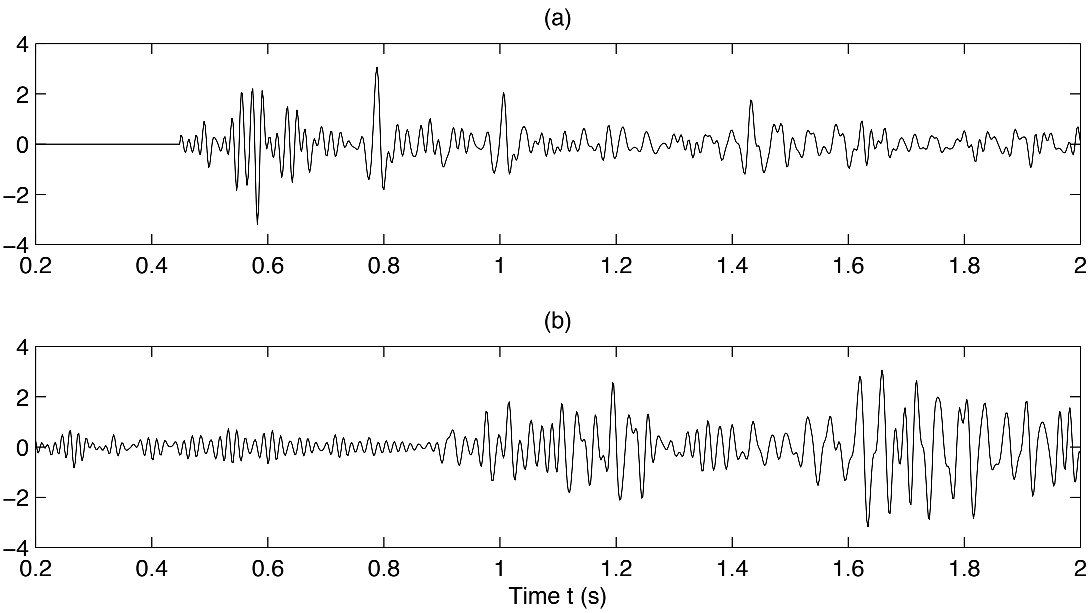

FIG. 5. (a) Raw noisy trace extracted from NEBC poststack section. (b) Prediction computed using the raw trace in the prediction operator.

Because the NEBC data were supported by well log data, through blocking Hernandez and Innanen (2013) was able to generate synthetic traces with the primaries and multiples associated with the major geological markers. If we subject the synthetic traces to the internal multiple prediction algorithm, and compare the result with the prediction carried out on the noisy trace (Figure 6a vs. b), we essentially confirm that at the expected multiple arrival times there is little to see. In the prediction study, the continuity of the traces with CMP number in the poststack section, and the presence of some traces less noisy than others, led to good overall results. Picking a specifically noisy trace from that data set, as we have done here, leads to much more compromised outcomes.

We next predict the same internal multiples using the aggressively denoised trace in the design of the prediction operator. Comparing the result (which is in Figure 7a) against the raw noisy prediction (which is in Figure 7b), there are clear differences. In particular, individual events are identifiable in the prediction from the denoised operator, whereas it was not really possible to say that about the raw output.

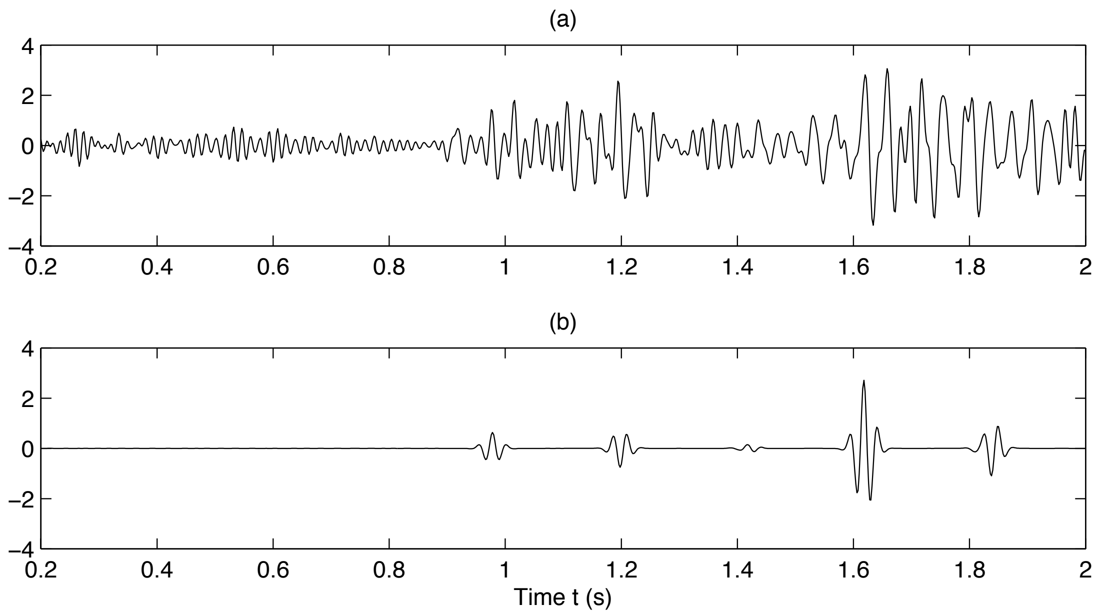

FIG. 6. (a) Prediction using raw trace in the prediction operator. (b) Prediction using the synthetic trace derived from blocked well log data in the prediction operator.

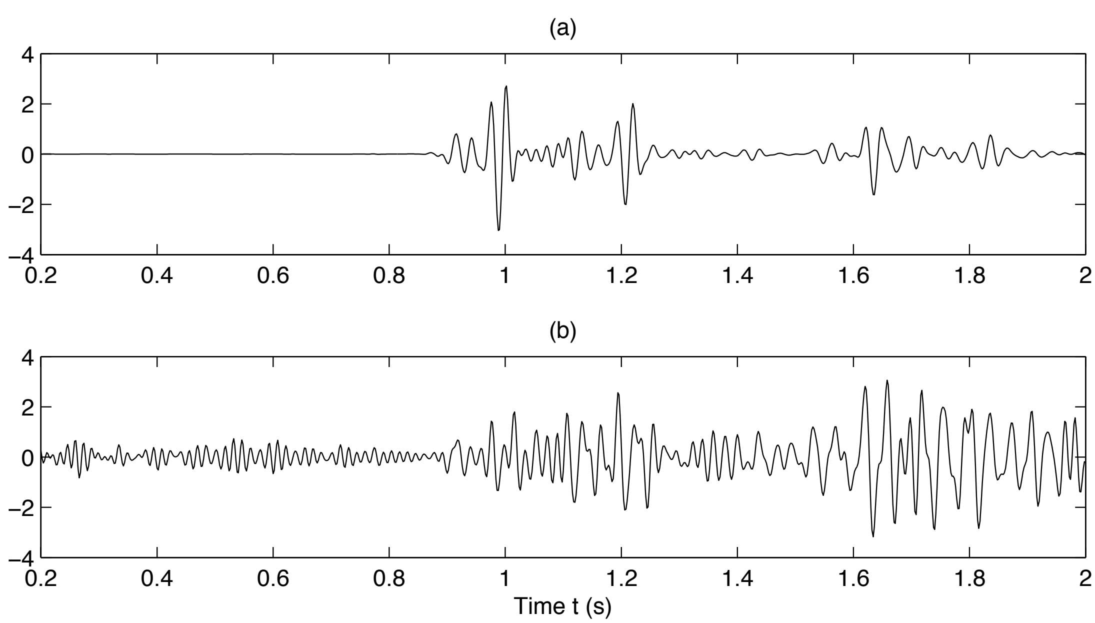

FIG. 7. (a) Prediction using the CWTM-denoised NEBC trace in the prediction operator. (b) Prediction using raw trace in the prediction operator.

The key test is to compare the predicted events in the denoised operator against the expected arrivals generated in the synthetic predictions from well data. The result is again plotted in Figure 7a, against the synthetic predictions Figure 7b. There is a clear correlation between regions of large predicted multiple energy and the expected arrival times.

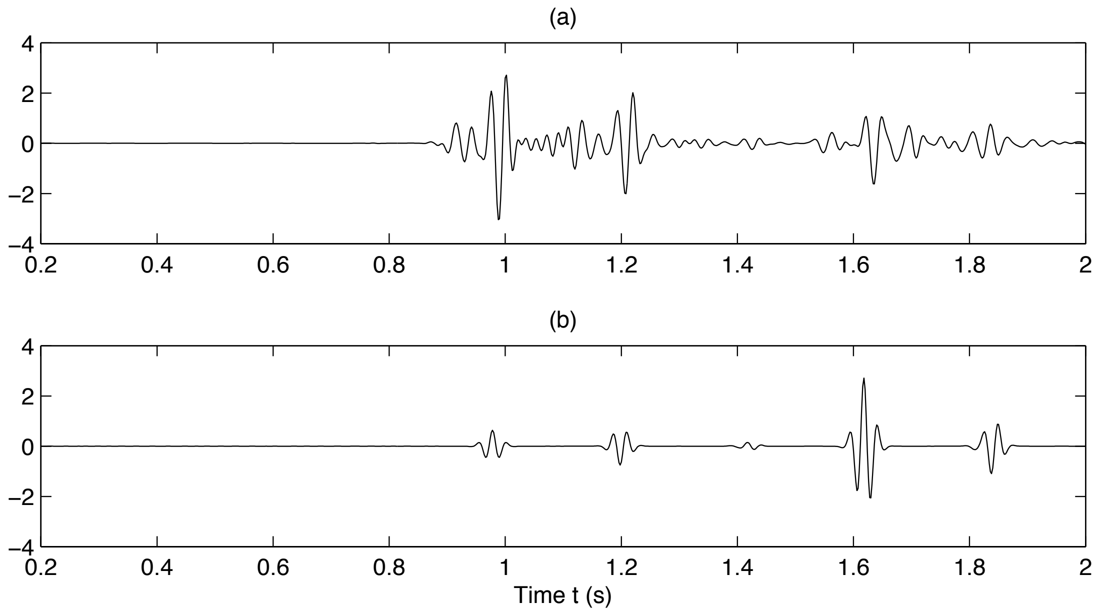

FIG. 8. (a) Prediction using the CWTM-denoised NEBC trace in the prediction operator. (b) Prediction using the synthetic trace derived from blocked well log data in the prediction operator.

CONCLUSIONS

We test application of seismic processing in the continuous wavelet transform maxima (CWTM) domain on the prediction of internal or interbed multiples. We establish that the use of the prediction algorithm in the CWTM domain is "sufficiently close" to that produced in the nominal domain to make the CWTM domain a viable one in which to predict; investigation into the scale and threshold dependent choice of algorithm parameters is therefore possible going forward. It is also established using noisy poststack traces that aggressive denoising in the CWTM domain is very effective when used in the construction of the prediction operator.

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