An analysis of time-lapse phase shifts using perturbation theory

Shahin Jabbari and Kristopher A. Innanen

ABSTRACT

Scattering or perturbation theory has been widely used in many applications in seismology, including time-lapse problems. One of the main challenges in using scattering theory to predict the model for the difference data in a time-lapse problem is, the reference medium, the baseline survey, being a medium as complicated as the perturbed medium, the monitoring survey. We produce the linear and higher order terms in the forward scattering series for the difference data for the phase-shift changes between the baseline and monitor surveys in a reservoir. The baseline surveying is taken to be a homogenous single layer for simplicity, but can be extended for a more complicated medium in future research. Green's function for characterizing the wavefield in this reference medium has a term describing the wave reflected from the interface at the time of the baseline survey. This leads to extra terms in the first and higher order approximations of the difference data when compared with a standard scattering problem. These extra terms are a function of the reflectivity of the single interface in the baseline survey and the perturbation due to production in the reservoir. Our perturbation theory for nonlinear time-lapse seismic inversion, which is the future steps in this research, will accommodate multidimensional and multi parameter problems which will lead to more complete and general versions.

INTRODUCTION

Reservoir properties change over time due to production or the employment of enhanced oil recovery techniques (EOR). Up-to-date information on a reservoir, generally interpreted with the help of programs specific to the purpose is used to optimize the management of a reservoir and to extend the useful life of an oilfield. A time lapse survey introduces an important contribution to the production of hydrocarbons. In the time-lapse monitoring process, a baseline survey is acquired prior to production of a reservoir. This is followed seismic surveys (monitor surveys) over a particular interval of time when geological/geophyical characteristics of a reservoir may change. Comparison of repeated seismic surveys over months, years, or decades add the dimension of calendar time to the seismic data (Greaves and Fulp, 1987; Lumley, 2001; Arts et al., 2004). Seismic trace can differ in amplitude, frequency, polarity, or the location of the interfaces from the baseline survey relative to the monitor survey. The time-lapse difference data between the baseline and monitor surveys indicate the change in the amplitude and travel time of the seismic trace. Time-lapse seismic images display the results which are not predicted by reservoir modeling and emphasize the effect of the production rather than lithological variation. Major oil companies now use time lapse seismic in reservoir management (Johnston, 2013; Waal and Calvert, 2003; Tura, 2003).

Perturbation or scattering theory can be used as a powerful theoretical approach to model and invert seismic data, including 4D time-lapse data (Weglein et al., 2003; Zhang, 2006; Innanen, 2008). The main idea in scattering theory is to compute a wavefield in an inhomogeneous medium by using a wavefield in a reference medium, perturbed with a function which is related non-linearly to local earth properties. Zhang and Weglein's innovation was to describe the difference data in 4D time-lapse by setting the baseline survey to be the reference or background medium, and the monitoring survey to be the perturbed medium. The difference data are then the scattered wavefield data. Innanen et al. (2013) have pointed out that this analysis requires a representation of difference data as an expansion in terms of both the time-lapse perturbation and the baseline medium properties in order to be self consistent (Innanen et al., 2013). When properties of a reservoir change during the production, the elastic properties of a reservoir change and this will induce a phase-shift in the time-lapse data. A model-based inversion method has been developed by Thore and Hubans (2012) to estimate the changes in the elastic properties of a reservoir due to production. They suggested a formula relating the difference between the baseline trace and the monitor trace to the changes in the P-velocity and the density (Thore and Hubans, 2012; Williamson et al., 2007).

Thore and Hubans (2012) and Williamson (2007) make use of a model based formula which takes into account time-shifts of events of interest which occur because of time-lapse changes in the P-wave velocity of the overburden. This formula, which is not derived but simply intuited, is a highly nonlinear, since the shift of a $\Delta V_P/V_P$ interface is given by the sum of $\Delta V_P/V_P$ itself from the surface to the interface.

We suspect that our perturbation theory for linear and/or nonlinear time-lapse seismic inversion, in addition to being useful for posing time-lapse AVO analysis, is a natural framework for deriving such nonlinear shift formulas. Since the theory is fully self-consistent and accommodates multidimensional and multi parameter problems, it may go beyond putting such formulas on a firm mathematical foundation, and in fact lead to more complete and general versions.

In this paper we begin the process of analyzing phase issues in nonlinear time-lapse perturbation theory. We will not in this paper arrive at processing or inversion formulas, but merely continue to study the basics of the modelling equations. Our aim is to report next year on the ability of the framework to provide practical algorithms.

THEORY

A forward problem is designed to characterize the wavefield emanating from a source and propagating through an earth model by representing the wavefield in terms of a reference wavefield with a known velocity model and a perturbation in the medium (Matson, 1996). We will consider two seismic experiments involved in a time-lapse survey, the baseline survey, followed by a monitoring survey. The acoustic medium is one-dimensional, varying in depth only, with a normal incident plane source. We begin with a one dimensional constant density acoustic wave equation for the baseline wavefield which is the reference wavefield:

$$L_0 G_0 = \delta(z - z_s), \tag{1}$$

where L_0 represents the differential operator that describes wave propagation in the reference medium, G_0, and for simplicity, the source is a pulse which is presented by a delta function at $z = z_s$. L defines a differential operator for describing the wave propagation in the monitoring survey, G:

$$LG = \delta(z), \tag{2}$$

The Lippmann-Schwinger equation plays a pivotal role in the scattering theory and basically shows that a wavefield in an inhomogeneous medium is the sum of the wavefield in a reference medium and an integral that represents the scattered field due to the perturbation. Based on the Lippmann-Schwinger equation, G_0 and G are related as:

$$G - G_0 = G_0(L_0 - L)G. \tag{3}$$

If we define the perturbation as $V = L_0 - L$, iterating the Lippmann-Schwinger equation back into itself generates:

$$G = G_0 + G_0VG_0 + G_0VG_0VG_0 + ...,$$

Then, defining the scattered wavefield or time-lapse difference data as $\psi = G - G_0$, we have:

$$\psi = \sum_{n=1}^{\infty} G_0(VG_0)^n. \tag{4}$$

Now, let's consider a 1-D constant density acoustic wave equation governing the propagation of a signal from the source at z_s to the receiver at z_g in the reference medium:

$$\left[\frac{\partial^2}{\partial z^2} - \left(\frac{1}{c_0^2}\right)\left(\frac{\partial^2}{\partial t^2}\right)\right] G_0(z_g, z_s, k) = \delta(z_g - z_s). \tag{5}$$

where ω is the angular velocity, C_0 is the homogenous reference velocity, $k = \frac{\omega}{c_0}$, and $V = k^2\alpha$. Here,

$$L_0 = \left[\frac{\partial^2}{\partial z^2} - \left(\frac{1}{c_0^2(z)}\right)\left(\frac{\partial^2}{\partial t^2}\right)\right].$$

The velocity c(z) can be characterized by a constant reference velocity c_0 and a perturbation $\alpha(z)$ so that

$$\frac{1}{c^2(z)} = \left(\frac{1}{c_0^2}\right)\left[1 - \alpha(z)\right],$$

or

$$\alpha(z) = 1 - \frac{c_0^2}{c^2(z)}. \tag{6}$$

The wave equation for the perturbed medium, thus is:

$$\begin{aligned} &\left[\frac{\partial^2}{\partial z^2} - \left(\frac{1}{c^2(z)}\right)\left(\frac{\partial^2}{\partial t^2}\right)\right] G(z_g, z_s, k) = \delta(z_g - z_s), \\ &L = \left[\frac{\partial^2}{\partial z^2} - \left(\frac{1}{c_0^2(z)}\right)\left(\frac{\partial^2}{\partial t^2}\right)\right]. \end{aligned} \tag{7}$$

Substituting equation (6) into equation (7) and using Green's function as a reference wavefield, an integral form corresponding to equation (7) is (Weglein, 1985):

$$\psi(z_g, z_s; k) = \int_{-\infty}^{\infty} G_0(z_g, z'; k) k^2 \alpha(z') G(z', z_s; k) dz'. \tag{8}$$

Iterating $G(z, z_s; k)$ back into $G_0(z, z_s; k)$ as in equation (4), we can form the scattering series for the difference data:

$$\begin{aligned}\psi(z_g, z_s; k) =& \int_{-\infty}^{\infty} G_0(z_g, z'; k) k^2 \alpha(z') G_0(z', z_s; k) dz' \\ &+ \int_{-\infty}^{\infty} G_0(z_g, z'; k) k^2 \alpha(z') dz' \int_{-\infty}^{\infty} G_0(z', z''; k) k^2 \alpha(z'') G_0(z'', z_s; k) dz'' \\ &+ \int_{-\infty}^{\infty} G_0(z_g, z'; k) k^2 \alpha(z') dz' \int_{-\infty}^{\infty} G_0(z', z''; k) k^2 \alpha(z'') \\ &\int_{-\infty}^{\infty} G_0(z'', z'''; k) k^2 \alpha(z''') G_0(z''', z_s; k) dz''' + ... \\ &= \psi_1 + \psi_2 + \psi_3 + ...\end{aligned} \tag{9}$$

A structural perturbed time-lapse problem

In this study, we define the medium at the time of the baseline survey as a single interface in a constant density acoustic medium whose depth and rock properties change before the time of the monitoring survey (Figure 1). The Green's function for the reference medium, the baseline survey, satisfies equation (6) and describes a direct wave from the source to the receiver plus a reflected wave at the interface propagating to the receiver:

$$G_0(z_g, z_g, \omega) = \frac{e^{ik|z_g - z_s|}}{i2k} + R_I \frac{e^{ik(z_I - z_g)} e^{ik(z_I - z_s)}}{i2k}. \tag{10}$$

where $k = \frac{\omega}{c_0}$, $R_I = (c_I - c_0)/(c_I + c_0)$. The first term in this equation represents the direct wavefield propagating from the source to the receiver, and the second term is the reflection from the interface at location z_I (Figure 1). Using zero offset for convenience, $z_s = z_g = 0$:

$$G_0(0, 0, \omega) = \frac{1}{i2k} + R_I \frac{e^{i2kz_I}}{i2k}, \tag{11}$$

We form scattering series in terms of the perturbation and the Green's function as in equation (10), and categorize them based on their order in the perturbation, α. The first order in difference data is:

$$\psi_1(0, 0; k) = \int_{-\infty}^{\infty} G_0(0, z'; k) k^2 \alpha(z') G_0(z', 0; k) dz'. \tag{12}$$

where

$$\begin{aligned}G_0(0, z'; k) =& \frac{e^{ikz'}}{i2k} + R_I \frac{e^{i2kz_I} e^{-ikz'}}{i2k}, \\ G_0(z', 0; k) =& \frac{e^{ikz'}}{i2k} + R_I \frac{e^{i2kz_I} e^{-ikz'}}{i2k}.\end{aligned} \tag{13}$$

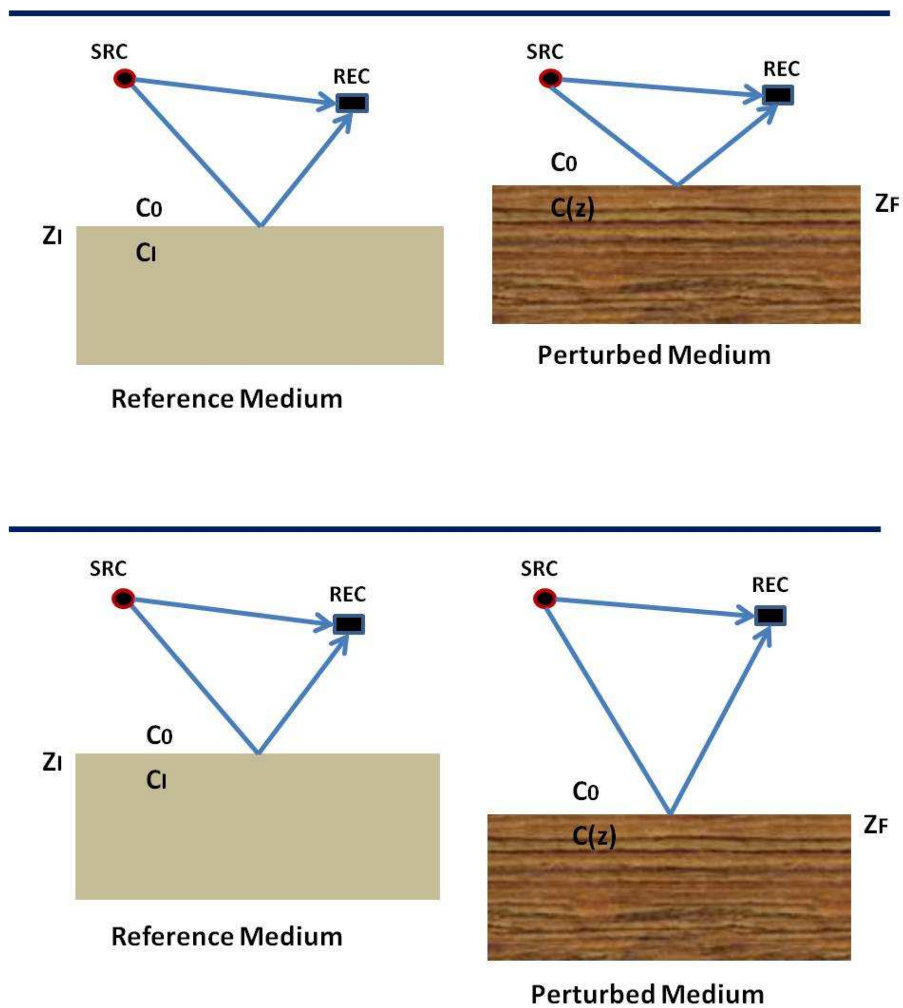

FIG. 1. Illustration of a structural perturbed time-lapse problem

Substituting equation (15) into equation (16) and solving the integral, and assuming small contrast at the interface in the baseline such that $R^2 \ll 1$ we have (for more details refer to Innanen (2008)):

$$\psi_1(0,0;k) = -\frac{1}{4}\int_{-\infty}^{\infty} e^{i2kz'}\alpha(z')dz' - \frac{2R}{4}e^{i2kz_I}\int_{-\infty}^{\infty}\alpha(z')dz'. \tag{14}$$

Next, we compute the second order term in the difference data, ψ_2:

$$\psi_2(0,0;k) = +\int_{-\infty}^{\infty} G_0(0,z';k)k^2\alpha(z')dz'\int_{-\infty}^{\infty} G_0(z',z'';k)k^2\alpha(z'')G_0(z'',0;k)dz''. \tag{15}$$

Substituting the appropriate Green's function and assuming $R^2 \ll 1$, ψ_2 is computed as:

$$\begin{aligned}\psi_2(0,0;k) =& -\frac{1}{4}\int_{-\infty}^{\infty} e^{i2kz'}\left(\frac{1}{2}\right)\left[\alpha^2(z') + \frac{d}{dz}\alpha(z')\int_{-\infty}^{z'}\alpha(z'')dz''\right]dz' \\ & -\frac{R}{2}e^{i2kz_I}\Bigg((-ik)\int_{-\infty}^{\infty}\alpha(z')dz'\int_{-\infty}^{z'}\alpha(z'')dz'' - \frac{1}{2}\int_{-\infty}^{\infty}dz' \\ & \left[\alpha^2(z') + \frac{d}{dz}\alpha(z')e^{i2kz'}\int_{-\infty}^{z'}\alpha(z'')e^{-i2kz''}dz''\right]\Bigg).\end{aligned} \tag{16}$$

The third order contribution to ψ, or ψ_3, in the integral form in terms of Green's functions and third order perturbation is:

$$\begin{aligned}\psi_3(0,0;k) =& \int_{-\infty}^{\infty} G_0(0,z';k)k^2\alpha(z')dz'\int_{-\infty}^{\infty} G_0(z',z'';k)k^2\alpha(z'')dz''. \\ & \int_{-\infty}^{\infty} G_0(z'',z''';k)k^2\alpha(z''')G_0(z''',0;k)dz'''\end{aligned} \tag{17}$$

ψ_3 is computed following the same process by substituting the appropriate Green's functions. As multiples show up in the third and higher order approximation , the result after

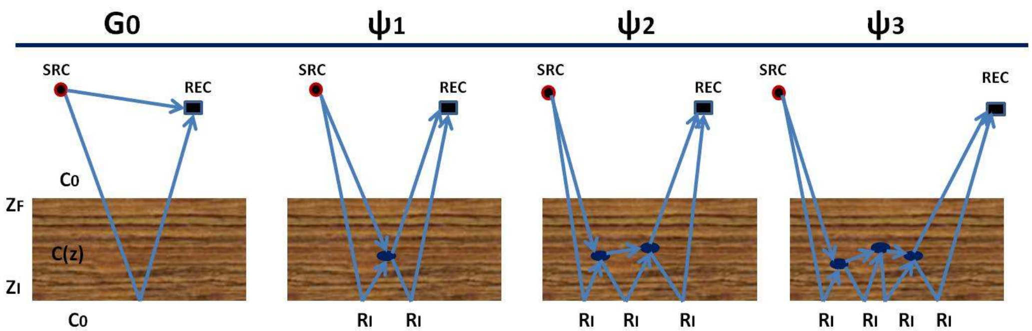

FIG. 2. Illustration of the geometry of the time-lapse difference field for the Green's function, ψ_1 (first order term), ψ_2 (second order term), and ψ_3 (third order term), in the scattering series for time-lapse difference data.

removing the multiples is:

$$
\begin{aligned}
\psi_3(0,0;k) = &-\left(\frac{1}{32}\right)\int_{-\infty}^{\infty} e^{i2kz'}\frac{d^2}{dz^2}\left[\alpha(z')\left(\int_{-\infty}^{z'}\alpha(z'')dz''\right)^2\right]dz' \\
&-\frac{R}{2}e^{i2kz_I}(ik)^2\left(\frac{1}{2}\right)\Bigg[\left(\int_{-\infty}^{\infty}\alpha(z')dz'\right)^2\int_{-\infty}^{z'}\alpha(z'')dz'' \\
&-\frac{1}{2ik}\int_{-\infty}^{\infty}\alpha(z')dz'\int_{-\infty}^{\infty}\alpha(z'')^2dz'' \\
&+\frac{1}{(2ik)^2}\int_{-\infty}^{\infty}\alpha(z')dz'\int_{-\infty}^{\infty}\alpha(z'')\frac{d}{dz}\alpha(z'')dz'' \\
&+\frac{1}{(2ik)^2}\int_{-\infty}^{\infty}\alpha(z')dz'\int_{-\infty}^{\infty}e^{-i2kz''}\alpha(z'')dz''\int_{z''}^{\infty}e^{i2kz''}\frac{d^2}{dz^2}\alpha(z'')dz''\Bigg] \\
&-\frac{R}{2}e^{i2kz_I}\frac{(ik)^2}{4}\Bigg[-\frac{1}{2ik}\int_{-\infty}^{\infty}\alpha(z')dz'\int_{-\infty}^{z'}\alpha^2(z'')dz'' \\
&+\frac{1}{(2ik)^2}\int_{-\infty}^{\infty}\alpha(z')dz'\int_{-\infty}^{z'}\alpha(z'')\frac{d}{dz}\alpha(z'')dz'' \\
&+\frac{1}{(2ik)^2}\int_{-\infty}^{\infty}\alpha(z')dz'\int_{-\infty}^{z'}e^{-i2kz''}\alpha(z'')dz''\int_{z''}^{\infty}e^{i2kz''}\frac{d^2}{dz^2}\alpha(z'')dz''\Bigg]
\end{aligned}
\qquad (18)
$$

To represent the contribution of the first, second, and third order terms to the time-lapse difference data, an assumption of the upward migration of the interface between cap rock and the reservoir has been made and the geometry is illustrated in Figure 2. In each ψ_n every path following the arrows from the source to the receiver represents a possible perturbation in the difference data.

DISCUSSION

Employing scattering theory in geophysics problem like time-lapse is worthwhile but presents specific challenges. Setting the baseline survey as a reference wavefield encounters two particular difficulties. The reference medium is as complicated as the perturbed medium, and therefore assigning an smooth reference medium to simplify the problem is not an option. Another concern is due to the reflected data in the baseline survey which are absent in the reference medium for a standard scattering method. The wavefield describing the reflected wave due to the interface of the single layer in the baseline survey as the reference medium, introduces new extra terms in the first and higher order approximations for the difference data which are the function of the combination effects of the perturbation in the monitor survey and the reflectivity at the interface in the baseline survey.

The future work for this project will be developing a general form for the inversion of the time-lapse phase shift. This will provide us a tool to investigate the effect of the perturbation or the change in the elastic parameters on the difference data, as well as the effect of the extra reflected wave term due to the interface of the cap rock and the reservoir at the time of the baseline survey.

Furthermore, at this stage of the study, the baseline is considered as a single and homogenous layer for the simplicity. The general form of the inversion for time-lapse phase shift should describe the real world in which a baseline survey is described based on a non-homogenous multiple layers.

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