A review of internal multiple prediction

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ABSTRACT

Multiple events can be mistaken for primary reflections, and may distort primary events and obscure the task of interpretation (Hernandez, and Innanen, 2011). So, to eliminate these effects, internal multiple prediction becomes a necessity in the industry. In this paper, we determine the definitions of primaries, multiples, and the most important concept in this research, internal multiples. Inverse scattering series will be introduced here. Then we review the basic principles of 1D and 2D internal multiple prediction algorithm, which were introduced to geophysics literature in the 1990s (Araújo et al., 1994; Weglein et al., 1997, 2003), and demonstrate 1D algorithm's use to 1D synthetic data using a MATLAB implementation. Also the basic idea of a lower-higher-lower relationship will be discussed. Then the role and importance of the parameter ϵ are emphasized and the effects of badly chosen epsilon values are shown. The 1D internal multiple algorithm has been tested with good results on band-limited synthetic data. Analytical and numerical examples will be used to exemplify the usefulness of 1D internal multiple prediction algorithm.

INTRODUCTION

For the exploration of oil and gas reservoirs, multiples can be one of the main issues in applying the seismic method. The key characteristic of the inverse scattering series based method is that they do not require any a priori information from the subsurface as they are fully data-driven. Furthermore, the primary reflections remain untouched. They will compute internal multiples from all possible generators. The output of the algorithm is a data set that contains the predicted internal multiples (Hernandez, and Innanen, 2012).

Every event in the seismic record can be thought of as a group of subevents. This algorithm predicts an internal multiple from interpreted subevents by performing a convolution and a cross-correlation of the data. One of the most important characteristic of this algorithm is that it selects all the subevents that suit the lower-higher-lower ideal (Weglein et. al., 1998). The parameter ϵ limits the selection or searching of the subevents and is related to the source wavelet. The inverse scattering attenuation method has three basics assumptions in order to work properly: knowledge of the source wavelet within the seismic frequency band, the input data must be rid of free-surface multiples, and accomplish the lower-higher-lower relationship in pseudo-depth (Hernandez, and Innanen, 2012).

METHODOLOGY

Definitions

First of all, we will discuss primaries, free-surface multiples and internal multiples which can help us to better understand how the algorithm works. Primaries are events which have experienced one upward reflection and no downward reflections during their history. Any events that are reflected from the free surface is a free-surface multiple. Free-surface multiples (FSMs) are themselves classified by their order. Events which experience at least one downward reflection in the subsurface, and never interact with the free-surface, are called interbed or internal multiples (IMs). Internal multiples are also classified by their order (see Figure 1).

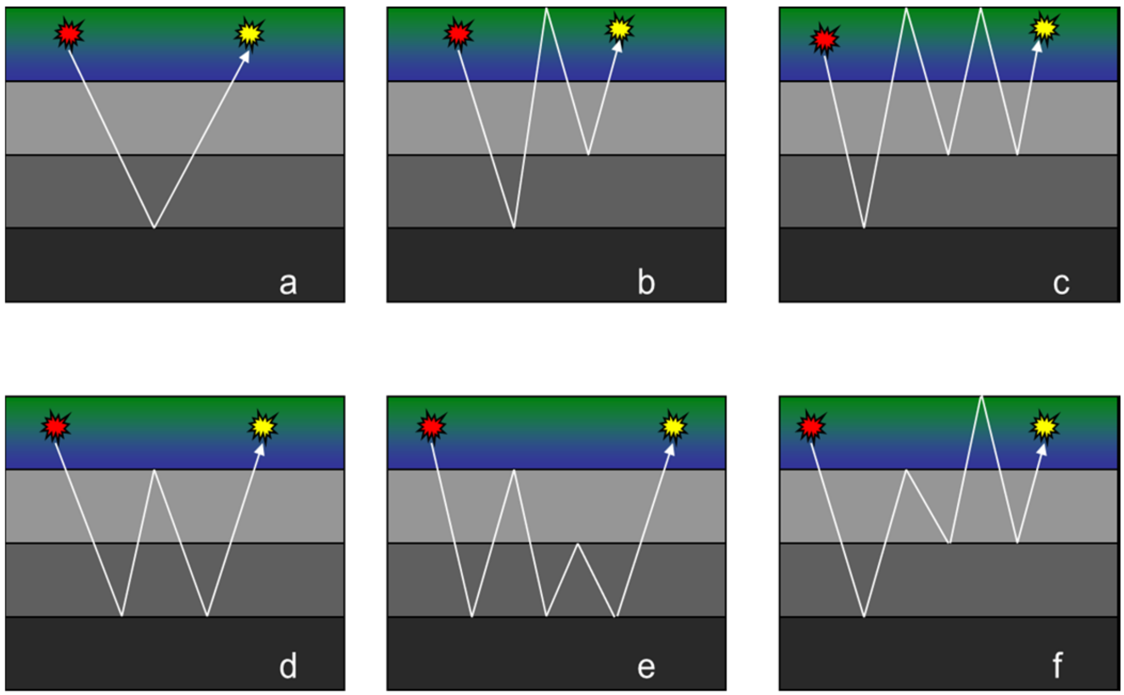

FIG. 1. Examples of primary and multiples (after Weglein and Dragoset, 2005). The blue-green area represents the water layer. The red and yellow stars indicate the positions of seismic source and receiver, respectively. The white lines are raypaths of the events being defined. (a) A primary event; (b) A first-order free-surface multiple; (c) A second-order free-surface multiple; (d) A first-order internal multiple; (e) A second-order internal multiple; (f) A free-surface multiple.

Inverse scattering series

The inverse scattering series is a multidimensional inversion method that can determine subsurface physical properties using measured data, D, and a reference medium Green's function, G_0. The methodology in this paper was introduced by Araújo et al. (1994); Weglein et al. (1997, 2003). Let us define a perturbation operator V, which is the difference between the actual medium, G and reference medium, G_0. An relationship among V, G and G_0 is called the scattering equation,

$$G = G_0 + G_0 V G, \tag{1}$$

which is an operator identity that relates the reference and actual wavefield propagation to the difference between the reference and actual medium, V. The scattered field ψ_s can be defined as

$$\psi_s = G - G_0. \tag{2}$$

Taking equation 1 into 2 we can get an iterative solution for ψ_s in terms of V and G_0,

$$\psi_s = G_0 V G_0 + G_0 V G_0 V G_0 + \cdots = (\psi_s)_1 + (\psi_s)_2 + \cdots, \tag{3}$$

where $(\psi_s)_n$ is an nth-order function of V. The goal of inversion is to solve V, which can be expressed as:

$$V = V_1 + V_2 + V_3 + \cdots, \tag{4}$$

where V_n is an nth-order function of the measured data, D. Substituting equation 4 into equation 3 and expanding V in terms of the measured data, we can get the inverse scattering series:

$$D = G_0 V_1 G_0 \tag{5}$$

$$G_0 V_2 G_0 = -G_0 V_1 G_0 V_1 G_0 \tag{6}$$

$$\begin{aligned} G_0 V_3 G_0 = &-G_0 V_1 G_0 V_1 G_0 V_1 G_0 \\ &-G_0 V_1 G_0 V_2 G_0 - G_0 V_2 G_0 V_1 G_0 \end{aligned} \tag{7}$$

$$\vdots$$

Note that the procedure for solving V does not require any a priori subsurface information. So the inverse scattering series allows removal of internal multiples when a priori information is unavailable.

2D internal multiple prediction algorithm

Now we will focus on the algorithm of internal multiple prediction. A portion of the third term in equation 7, $G_0 V_1 G_0 V_1 G_0 V_1 G_0$, contains the leading order contribution to the generation of the first order internal multiples (Weglein et al., 1997). This portion of the third term is then

$$\begin{aligned} b_{3IM}(k_g, k_s, q_g + q_s) = &\left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_1 dk_2 \\ &\times \int_{-\infty}^{\infty} dz e^{i(q_g+q_1)z} b_1(k_g, k_1, z) \\ &\times \int_{-\infty}^{z-\epsilon} dz' e^{-i(q_1+q_2)z'} b_1(k_1, k_2, z') \\ &\times \int_{z'+\epsilon}^{\infty} dz'' e^{i(q_2+q_s)z''} b_1(k_2, k_s, z''), \end{aligned} \tag{8}$$

and the

$$q_x = \frac{\omega}{c_0}\sqrt{1 - \frac{k_x^2 c_0^2}{\omega^2}}\,, \tag{9}$$

are vertical wave numbers in terms of the various lateral wave numbers and the reference velocity c_0. b_1 is the input to the prediction algorithm, which is defined in terms of the original pre-stack data with surface multiples eliminated. The procedure for getting the input was given in Innanen (2012) as we begin with a data set measured over intervals in lateral source location x_s, lateral receiver location x_g, and time t. The data can be Fourier transformed to the frequency domain:

$$d(x_g, x_s, t) \rightarrow D(k_g, k_s, \omega), \tag{10}$$

and then change from ω to k_z:

$$D(k_g, k_s, \omega) \to D(k_g, k_s, k_z), \tag{11}$$

where $k_z = q_g + q_s$. The data are scaled by $-i2q_s$, we can get

$$b_1\left(k_g, k_s, k_z\right) = (-i2q_s)D(k_g, k_s, k_z). \tag{12}$$

Note that the obliquity factor, $-i2q_s$, is used to transform an incidence wave into a plane wave in the Fourier domain (Weglein et al., 2003). Finally let's inverse Fourier transform b_1 over k_z, appearing in the pseudo-depth domain as

$$b_1\left(k_g, k_s, k_z\right) \to b_1(k_g, k_s, z). \tag{13}$$

The quantity $b_1(k_g, k_s, z)$ is the input to the prediction algorithm.

1D internal multiple prediction algorithm

Now we will reduce this multidimensional prediction algorithm to 1D, using the replacement used by Hernandez and Innanen (2012)

$$k_g = k_s = 0, \tag{14}$$

then we can obtain the prediction algorithm in 1D normal incidence case,

$$b_{3IM}(k_z) = \int_{-\infty}^{\infty} dz e^{ik_z z} b_1(z) \int_{-\infty}^{z-\epsilon} dz' e^{-ik_z z'} b_1(z') \int_{z'+\epsilon}^{\infty} dz'' e^{ik_z z''} b_1(z''), \tag{15}$$

where $k_z = 2\omega/c_0$ is the vertical wavenumber, which is the conjugate of the pseudo-depth ($z = c_0 t/2$).

Lower-higher-lower relationship

Figure 2 is a construction of the travel times of an internal multiple. The red primary has travel time t_1, the green primary has travel time t_2, and the dashed line primary has travel time t_3. The travel time of the internal multiple equals $t_1 + t_2 - t_3$.

In Figure 2, sums and differences of travel times produce internal multiple travel time, but not every combination of sums and differences do. In fact, most of the combination of sums and differences of travel time triplets correspond to no event at all. We have two combinations of sums and differences as seen in Figure 3. For internal multiples, we desire the first kind of combination, while the second case corresponds to spurious events, artifacts.

A notion emerges from the above graphs, is it possible for us to restrict the sums and differences such the artifacts are suppressed? It can be seen that, in Figure 3a, the travel time being subtracted is smaller than the travel times being added. Artifacts come from subtraction of travel times which are larger than those being summed. We expressed equation 15 in pseudo-depth, rather than time, disallowing subtraction of larger travel times will take the form of disallowing subtraction of lower events. As this event is incorporated through correlation, the trace whose events are to be subtracted is the middle one, $b_1(z')$. What we need to do now is to disallow contributions from the middle integral coming from events in that trace which are lower than those of two traces $b_1(z)$, and $b_1(z'')$. Because we use explicit integration over pseudo-depth, so it is convenient to restrict the combinations of events to reflect this lower-higher-lower relationship. We need to ensure three events that satisfy $z'' > z'$ and $z > z'$. Therefore we restrict the z'' integration so that it begins at z'. By the same principle, we restrict the z' integration such that it ends at z, disallowing contributions from any z' value greater than z (Innanen, 2011). The parameter ϵ is included in equation 15 to ensure that $z'' > z'$ and $z > z'$. Note that for band-limited data, this parameter is different for every data set and is related to the width of the wavelet.

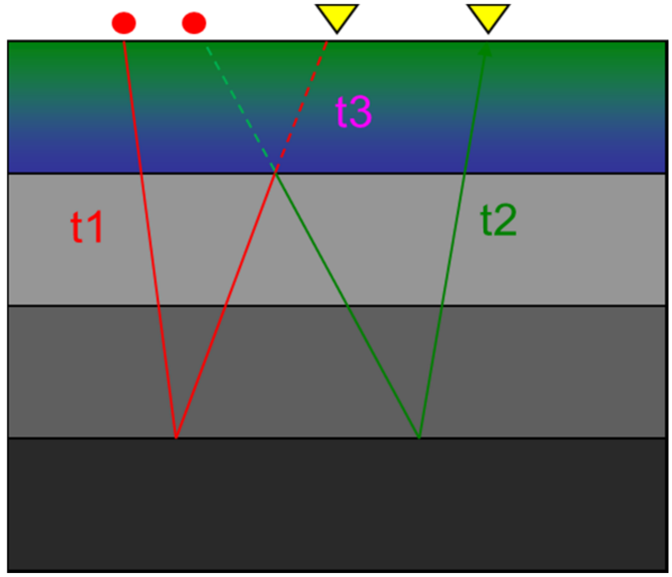

FIG. 2. Construction of the travel times of an internal multiple.

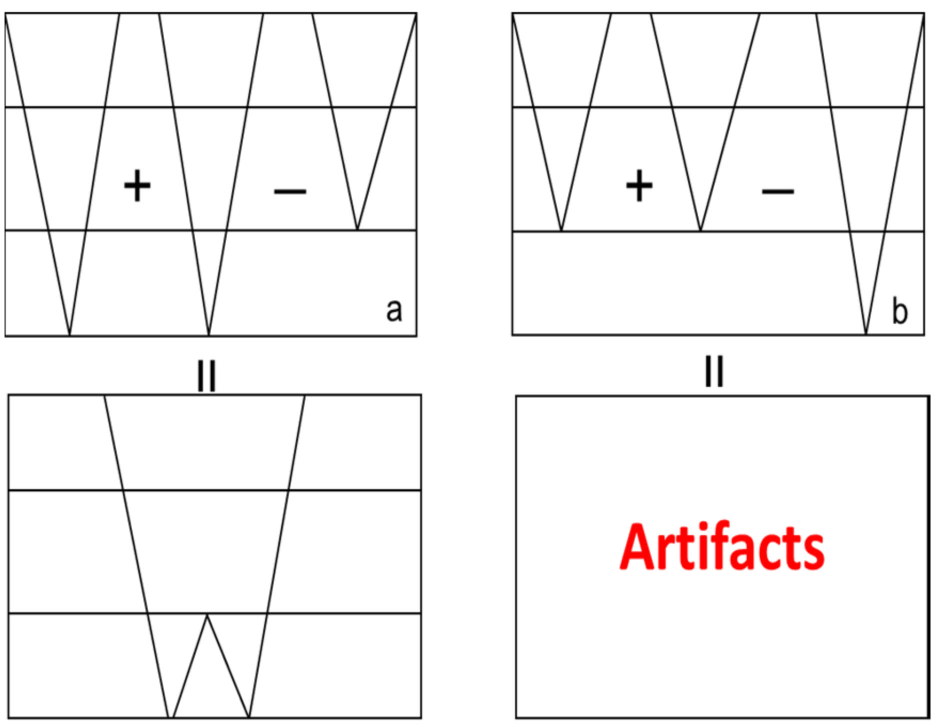

FIG. 3. Two combinations of sums and differences.

ANALYTIC EXAMPLE

In this section, we will show how the 1D internal multiple attenuation algorithm works using the concept of subevents. This technique does not require subsurface information to achieve the suppression of the internal multiples. We will use the model in Figure 4, in which a multiple is generated and it can be seen as a convolution and a correlation of the three subevents.

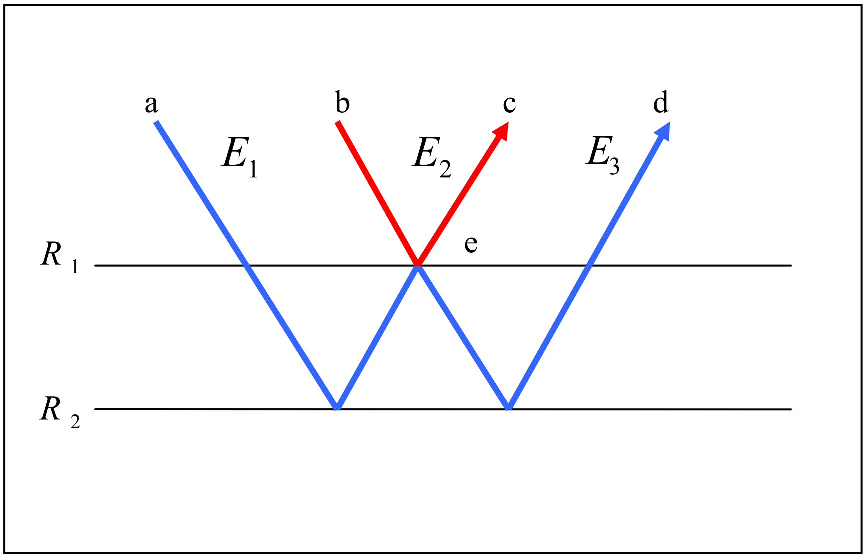

FIG. 4. Construction of an internal multiple using subevent interpretation. The first subevent is a primary reflection that travels from point 'a', reflected from the second reflector, and is received at point 'c'. The second subevent is a primary that propagates from point 'b', reflected from the first interface 'e', and received at point 'c'. The third one is a primary from point 'b' to 'd', reflected from the second interface.

The first subevent in Figure 4 is

$$E_1(\omega) = T_{01}R_2T_{10}e^{i\omega t_2} \,, \tag{16}$$

which is a primary reflection that propagates from a source at 'a', reflected from the second reflector, and is measured at 'c'. While the second subevent is a primary that propagates from 'b' to 'c', reflected from the first reflector which is 'e',

$$E_2(\omega) = R_1e^{i\omega t_1} \,. \tag{17}$$

The third one propagates from 'b', reflected from the second interface, and is measured at 'd',

$$E_3(\omega) = T_{01}R_2T_{10}e^{i\omega t_2} \,. \tag{18}$$

The internal multiple attenuation algorithm predicts an internal multiple from these subevents by performing a convolution and a correlation. Now let us substitute them into equation 15, then we can get

$$IM_{est}(k_z) = \int_{-\infty}^{\infty} dz e^{ik_z z} E_1\,(z) \int_{-\infty}^{z-\epsilon} dz' e^{-ik_z z'} E_2\,(z') \int_{z'+\epsilon}^{\infty} dz'' e^{ik_z z''} E_3\,(z'') \,, \tag{19}$$

where IM_{est} is the estimated internal multiple. Since the three subevents are discrete localized events and satisfy the condition $z'' > z'$ and $z > z'$, so the integration limits can be extended to $\pm\infty$. Hence we can write down

$$IM_{est}(k_z) = \int_{-\infty}^{\infty} dz e^{ik_z z} E_1(z) \int_{-\infty}^{\infty} dz' e^{-ik_z z'} E_2(z') \int_{-\infty}^{\infty} dz''\, e^{ik_z z''}(z'')$$
$$= E_1(k_z) E_2(-k_z) E_3(k_z). \tag{20}$$

Applying a Fourier transform to equation 20, this can be written in the frequency domain as,

$$IM_{est}(\omega) = E_1(\omega) E_2(-\omega) E_3(\omega)\ . \tag{21}$$

From which we can see that this equation describe the cross-correlation of subevent 1 with subevent 2 followed by a convolution with subevent 3. Substituting the above three subevents into equation 21 gives us

$$IM_{est}(\omega) = R_1 R_2^2 T_{10}^2 T_{01}^2 e^{i\omega(2t_2 - t_1)}\ . \tag{22}$$

While the actual internal multiple in the frequency domain should be

$$IM_{act}(\omega) = T_{01} R_2 (-R_1) R_2 T_{10} e^{i\omega(2t_2 - t_1)}\ . \tag{23}$$

Comparing equation 22 and 23, we can tell that the predicted amplitude is always less than the true internal multiple. The difference between the actual and the predicted multiple is that the actual multiple does not experience a transmission at the downward reflection point 'e', whereas the internal multiple algorithm models the multiple from subevents that have experienced a transmission loss at point 'e'. The error is a factor known as the attenuation factor of the predicted internal multiple, which is $T_{01}T_{10}$. In a previous work (Weglein and Matson, 1998) pointed out that this error could be due to the leading order term in the internal multiple attenuation series does not properly take transmission effects into account. Though there is error in the prediction algorithm, for typical earth velocities this error is very small and the predicted multiples give a satisfactory result (Hernandez and Innanen, 2012).

Also, we should notice that for time prediction it takes the time of the first event, plus the time of the third event minus the time of the second. This subtraction can be seen in the negative phase of the second depth integral in equation 19. This process gives the correct arrival time and phase (Ramirez and Weglein, 2005).

NUMERICAL EXAMPLES

For numerical examples, we need to generate a synthetic model. From this model three primaries and associated two first-order internal multiples are generated and plotted in Figure 5. The IM prediction algorithm is implemented and run on the input data, the results are plotted in Figure 6. Table 1 illustrates the detail information of this synthetic model.

Notable here is the parameter ϵ value, several values of epsilon are tested with the optimal value shown to be 7. For smaller epsilon values, artifacts will be seen at the arrival times of primaries in the output data. While larger epsilon values could damage important information present in the output data. If the overestimation of the value is large enough, the output data will not show any events at all.

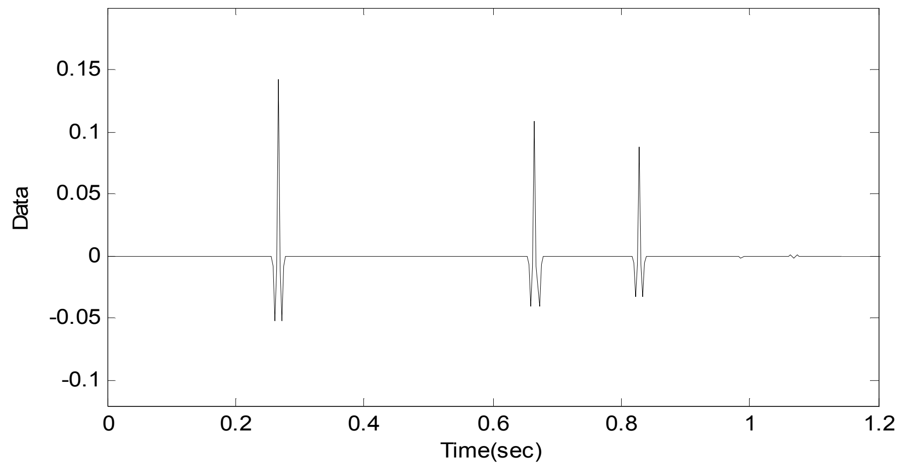

FIG. 5. A numerical example of internal multiple prediction. A zero offset trace with three primaries and two first-order internal multiples. The arrival times for the two internal multiples are $t_1 = 0.98s$ and $t_2 = 1.07s$.

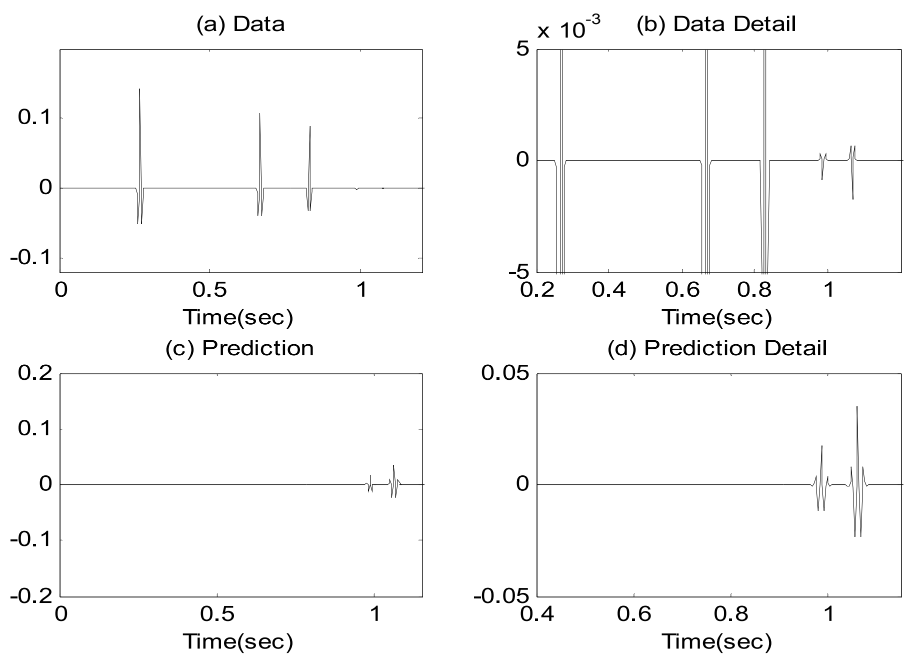

FIG. 6. Applying the 1D internal multiple attenuation algorithm to the synthetic model. (a) Input data; (b) Input data with focus on internal multiples; (c) Prediction output; (d) Prediction output with focus on internal multiples.

PARAMETER	**VALUE**
Sample number	*512*

Interval sample time	*3ms*
Velocity and depth of the first interface	*2000m/s at 200m*
Velocity and depth of the second interface	*2500m/s at 600m*
Velocity and depth of the third interface	*3000m/s at 800m*
Wave speed of the source/receiver medium	*1500m/s*
Type of wavelet	*Ricker*
Epsilon	*7*
Wavelet central frequency	*80Hz*

Table 1. Parameters for synthetic model

CONLUSTIONS

We have employed the 1D algorithm from the inverse scattering series theory for the prediction of internal multiples. Analytical examples were used to exemplify that no velocity information from the subsurface is required using the inverse scattering series theory. Its performance was demonstrated with a band-limited synthetic data. Based on the results, we can conclude that output prediction depends strongly on the epsilon value. For smaller epsilon values, artifacts will be seen at the arrival times of primaries in the output data. While larger epsilon values could damage important information present in the output data. If the overestimation of the value is large enough, the output data will not show any events at all.

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REFERENCES

Araújo, F. V., Weglein, A. B., Carvalho, P. M., and Stolt, R. H., 1994, Inverse scattering series for multiple attenuation: an example with surface and internal multiples: SEG Expanded Abstracts, 13, 1039–1041.

Hernandez, M., and Innanen, K. A., 2012, Application of internal multiple prediction: from synthetic to lab to land data: CREWES Annual Report, 24.

Hernandez, M., Innanen, K. A., and Wong, J., 2011, Internal Multiple Attenuation based on inverse scattering: Theoretical review and implementation in synthetic data: CREWES Annual Report, 23.

Hernandez, M., Innanen, K. A., and Wong, J., 2011, Internal Multiple Attenuation based on inverse scattering: Implementation in physical model seismic data: CREWES Annual Report, 23.

Innanen, K. A., 2012, 1.5D internal multiple prediction in MATLAB: CREWES Annual Report, 24.

Innanen, K. A., 2011, Multiples: GOPH657 Course Notes.

Ramirez, A. C., and Weglein, A. B., 2005, An inverse scattering internal multiple elimination method: Beyond attenuation, a new algorithm and initial tests, in SEG/ Houston 2005 Annual Meeting, 2115-2118.

Weglein, A. B., Araújo, F. V., Carvalho, P. M., Stolt, R. H., Matson, K. H., Coates, R. T., Corrigan, D., Foster, D. J., Shaw, S. A., and Zhang, H., 2003, Inverse scattering series and seismic exploration: Inverse Problems , No. 19, R27–R83.

Weglein, A. B., Gasparotto, F. A., Carvalho, P. M., and Stolt, R. H., 1997, An inverse-scattering series method for attenuating multiples in seismic reflection data: Geophysics, 62, No. 6, 1975–1989.

Weglein, A. B., and Matson, K. H., 1998, Inverse scattering internal multiple attenuation: An analytic example and subevent interpretation, in SPIE Conference on Mathematical Methods in Geophysical Imaging, 108–117.