Particle model of walkaway VSP data

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ABSTRACT

Earlier treatment of zero-offset VSP data arrival times in terms of perfectly inelastic particle interactions is extended to include walkaway modes. The space-time tracks of particles moving under the influence of a uniform force field accelerate in such a way that under a particular coordinate transformation they reproduce the moveout in the VSP gather. In zero offset data and walkaway modes NMO correction is realized through transformations from the laboratory reference frame of the notional collisions to rest frames of individual particles before and after colliding. Galilean transformations alter the linear NMO patterns of zero-offset data, whereas transformations to noninertial reference frames are needed for walkaway modes.

INTRODUCTION

A seismic event in a data set traces out a remarkably coherent curve in the geophone location/time plane. The curves bear a strong resemblance to the tracks formed by particles as they drift in space, in a classical kinematical view. In fact we may find it tempting to try to impose on a seismic event (and its coherence in space and time) the interpretation of a track or path being followed by a particle in space as time elapses. In this paper we find that the predictive capability of such a picture is far from negligible, and it provides us with the ability to see certain seismic data processing tasks (e.g. NMO correction) in a new light.

This paper extends previous analysis from zero-offset VSP environments (Innanen, 2010) to walkaway environments. The presence of lateral source offset obviously changes the character of VSP events significantly, requiring a switch from linear to curved trajectories. However, the basic idea of inelastic collisions occurring (say) between two carts on a frictionless air-track still holds, with a slight modification, as we shall see. Beyond that, we apply the particle model to practical aspects of VSP data handling: in particular, NMO corrections (e.g., Zhang et al., 1994) are here seen to correspond to coordinate transformations between the laboratory frame and the rest frames of the various particles. For the zero offset case, Galilean transformations "flatten" VSP events; when the source is laterally offset, non-Galilean transformations which retain classical interpretations are possible, with ultimately transformations with some relativistic properties necessary.

VSP experimental configuration

The configuration we consider in this paper is the walkaway VSP (WVSP) experiment (Figure 1). The variable x_s will be used to represent the lateral distance of the source to the well, and z_g will represent the depth of the geophone down the well. Ray paths lengthen as the hypotenuses of right triangles as we move down the well, and bend conserving horizontal slowness as they pass through interfaces and/or regions of smooth change in velocity. Our interest in the current paper is primarily in the travel time curvature caused by the offset x_s, which leads to more pronounced hyperbolic character in the measured events (Figures 1a-c).

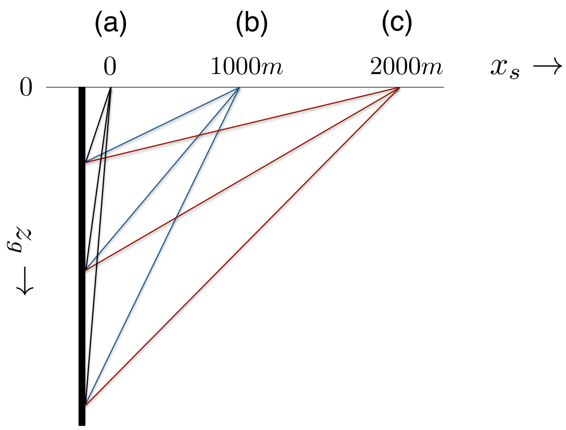

FIG. 1. Walkaway VSP configuration. (a-c) Increasing offset leads to a more pronounced hyperbolic character in all measured events, including the direct arrival.

ZERO OFFSET VSP DATA

Previous discussions on this subject have focused mostly on zero offset VSP data, a special case of WVSP with $x_s = 0$, wherein measured events exhibit a linear moveout. In Figure 2a is an example zero offset shot record*, with depth increasing from left to right. The direct and reflected events, which are distinct at $z_g > z_1$, meet at the depth z_1 of the reflecting interface. The transmitted event is measured at depths below z_1. In Figure 2b the travel times of the three events, t_1 for the direct arrival, t_R for the reflected arrival, and t_T for the transmitted arrival, are labelled.

Particle/collision picture of zero offset VSP data

The particle model of zero offset datasets—such as that illustrated in Figure 2—grows out of the observation we made in the introduction, namely that an *event* is a curve in the time and in space plane of the data record, and that several of these curves generally co-exist and intersect in regions of the volume of time and space of the data. It turns out that we can treat these curves as if they were the time histories of particles drifting around in space, accelerating or drifting with uniform velocity, and sometimes colliding and sticking together.

To do so we have to first reverse the roles played by time and space. Consider a single trace in the VSP experiment in Figure 2a. For the sake of definiteness, take the trace at 1500m depth. In that trace there are two events, the direct arrival and the reflected arrival. Consider only the direct arrival, found at roughly 0.5s. We define the "position" of a particle in our model to be the value of the time coordinate at which the seismic event is found. Thus, in this trace we would say there is a particle at "position" $t = 0.5s$. And, we

*All synthetic data in this paper were calculated with the fourth-order finite difference acoustic solver *afd_shotrec.m* in the CREWES matlab toolbox.

define the value z_g along depth axis of the well to be the current "time". So, the particle passed "position" t =0.5s at "time" z_g =1500m.

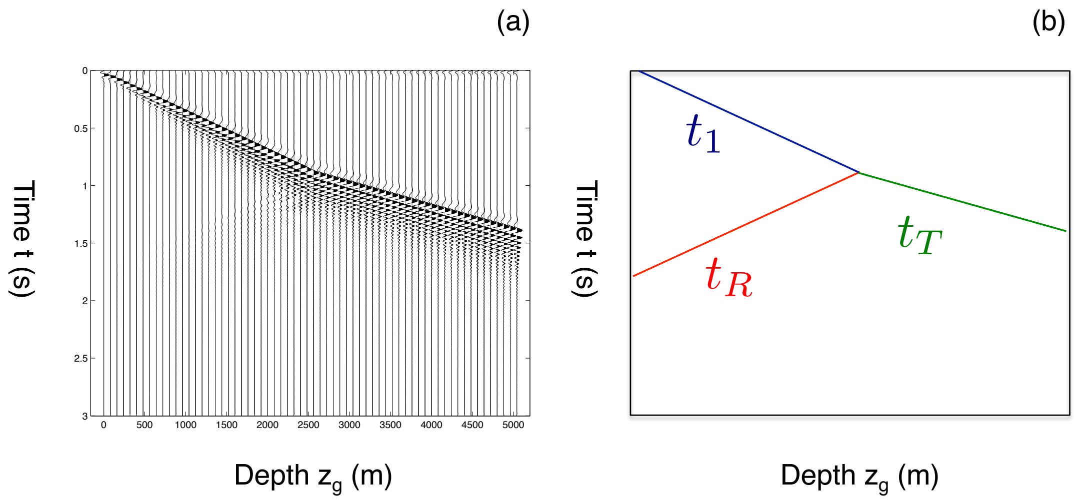

FIG. 2. Synthetic zero offset VSP experiment for a single interface model with velocity c_0 above z_1 and velocity c_1 below. (a) Synthetic shot record; (b) arrival times of three events interpreted in z_g and t.

Tracking a particle in kinematics we determine the position as a function of time $x(t)$ in one dimension, or $[x(t), y(t), z(t)]$ in three dimensions. Hence in our model to track each "particle" we will need to determine their respective functions $t(z_g)$. These are simply the travel time curves of the events. In a zero-offset case, for a reflector at depth z_1, the particle associated with the direct arrival has a trajectory given by

$$\begin{aligned} t_1(z_g) &= \frac{1}{c_0} z_g + t_1^0, \quad z_g < z_1, \\ t_1^0 &= 0, \end{aligned} \tag{1}$$

whereas the reflected arrival has

$$\begin{aligned} t_R(z_g) &= -\frac{1}{c_0} z_g + t_R^0, \quad z_g < z_1, \\ t_R^0 &= \frac{2z_1}{c_0}, \end{aligned} \tag{2}$$

and the transmitted arrival has

$$\begin{aligned} t_T(z_g) &= \frac{1}{c_1} z_g + t_T^0, \quad z_g > z_1, \\ t_T^0 &= z_1 \left(\frac{1}{c_0} - \frac{1}{c_1} \right). \end{aligned} \tag{3}$$

These are all linear travel time curves, reflecting the fact that the source-geophone separation and the ray paths grow in proportion as we move down the well.

Wave boundary conditions as momentum conserving inelastic collisions

Consider a trace in the VSP experiment illustrated in Figure 2a, or better yet, its representation in Figure 2b, in which a trace is obtained by drawing a vertical line anywhere through the box-shaped domain of z_g and t. The events occur at points where this line intersects the red, blue or green travel time curve.

The process we would like to examine is the evolution in time (whose coordinate we have defined to be z_g) generated by the uniform motion of this vertical line and its intersections with t_1, t_R and t_T from left to right. It is what we would see if we made a movie out of the seismic traces in Figure 2a, with each trace being a frame of the film, and the frames were played sequentially starting from low values of z_g and going to high.

Consider what the direct, reflected and transmitted events would look like in such a movie. As the vertical line sweeps from $z_g = 0$ to z_1, the "particles" represented by the reflected and direct arrivals drift towards each other. At "time" $z_g = z_1$ the two collide. Beyond that a single particle continues, drifting at a slightly altered rate. The impression is of a perfectly inelastic collision.

Let these particles be assigned masses proportional to the amplitudes of their respective arrivals. That is, the three tracks in Figure 2b are assumed to represent particles with masses m_1, m_R and m_T where

$$m_1 = 1, \quad m_R = R, \quad m_T = T. \tag{4}$$

The rates at which they drift towards each other, which we will call the velocities v, are determined by the slopes of the travel time curves in Figure 2b. Since we have defined z_g as a time coordinate and t as a space coordinate, the velocities are in fact the slownesses of the wave fronts. The three particles therefore have drift velocities

$$v_1 = (1/c_0), \quad v_R = -(1/c_0), \quad v_T = (1/c_1). \tag{5}$$

These velocities are a consequence of the decision to treat z_g as a time coordinate and t as a space coordinate, but the masses were arbitrarily chosen. The motivation for this choice lies in the observation that at z_1 a perfectly inelastic collision appears to take place. The masses and velocities in equations (4)–(5) make it possible to write down expressions for the total mass and total momenta of the system before and after the collision. Conservation of these quantities implies

$$\begin{aligned} m_1 + m_R = m_T \quad &\rightarrow \quad 1 + R = T \\ m_1 v_1 + m_R v_R = m_T v_T \quad &\rightarrow \quad (1/c_0) - (R/c_0) = (T/c_1). \end{aligned} \tag{6}$$

Thus, in our scheme the scalar boundary conditions by which R and T are normally determined for plane waves are replaced by mass and momentum conservation rules before and after the collision.

Moveout corrections as Galilean transformations

Continuing in the zero-offset VSP experiment format, we can pursue the idea of colliding particles by considering coordinate transformations. The particle motions examined so far have been in the equivalent of a *laboratory frame*, but to the extent these VSP events mimic particle experiments, other frames may be suitable also. Since we have set z_g as a time coordinate and t as a space coordinate, a Galilean transform between, for instance, the laboratory frame and the rest frame of the particle of mass m_1 is

$$\begin{aligned} t' &= t - \frac{1}{c_0} z_g, \\ z'_g &= z_g. \end{aligned} \tag{7}$$

Applying this transform to the kinematic expressions for the three VSP particles, we have for the particle of mass m_1

$$\begin{aligned} t'_1 &= t_1 - \frac{1}{c_0} z_g \\ &= \frac{1}{c_0} z_g + t_1^0 - \frac{1}{c_0} z_g \\ &= t_1^0, \end{aligned} \tag{8}$$

which sits still in this frame at location t_1^0, and for the other two particles

$$\begin{aligned} t'_R &= -\frac{2}{c_0} z_g + t_R^0 \\ t'_T &= \left(\frac{1}{c_0} - \frac{1}{c_1} \right) z_g + t_T^0. \end{aligned} \tag{9}$$

The VSP data and particle tracks in the original laboratory frame of reference are illustrated in Figures 3a and c, and are illustrated transformed to the m_1 rest frame in Figures 3b and d. Riding along on the particle m_1, the particle m_R appears to approach at twice the lab frame rate, and the post collision particle can be seen to now drift backwards (since in this example $c_1 < c_0$).

Note then that the mathematics and processing associated with VSP data moveout correction for a particular event, appears in the particle view as a classical or Galilean transformation to an inertial reference frame in which the associated particle is at rest.

Invariance of collision rules under general linear moveout corrections

We might think to ask if the VSP moveout corrections we make alter the viability of the interpretation in terms of particle collisions. The answer is no. To see this, consider a general laboratory frame triplet of incident-reflected-transmitted events:

$$\begin{aligned} t_1(z_g) &= p_0 z_g \\ t_R(z_g) &= -p_0 z_g + t_R^0 \\ t_T(z_g) &= p_1 z_g + t_T^0, \end{aligned} \tag{10}$$

subject to a general Galilean transformation (i.e., any linear moveout correction):

$$\begin{aligned} t' &= t - p' z_g, \quad z'_g = z_g \\ t &= t' + p' z_g, \quad z_g = z'_g. \end{aligned} \tag{11}$$

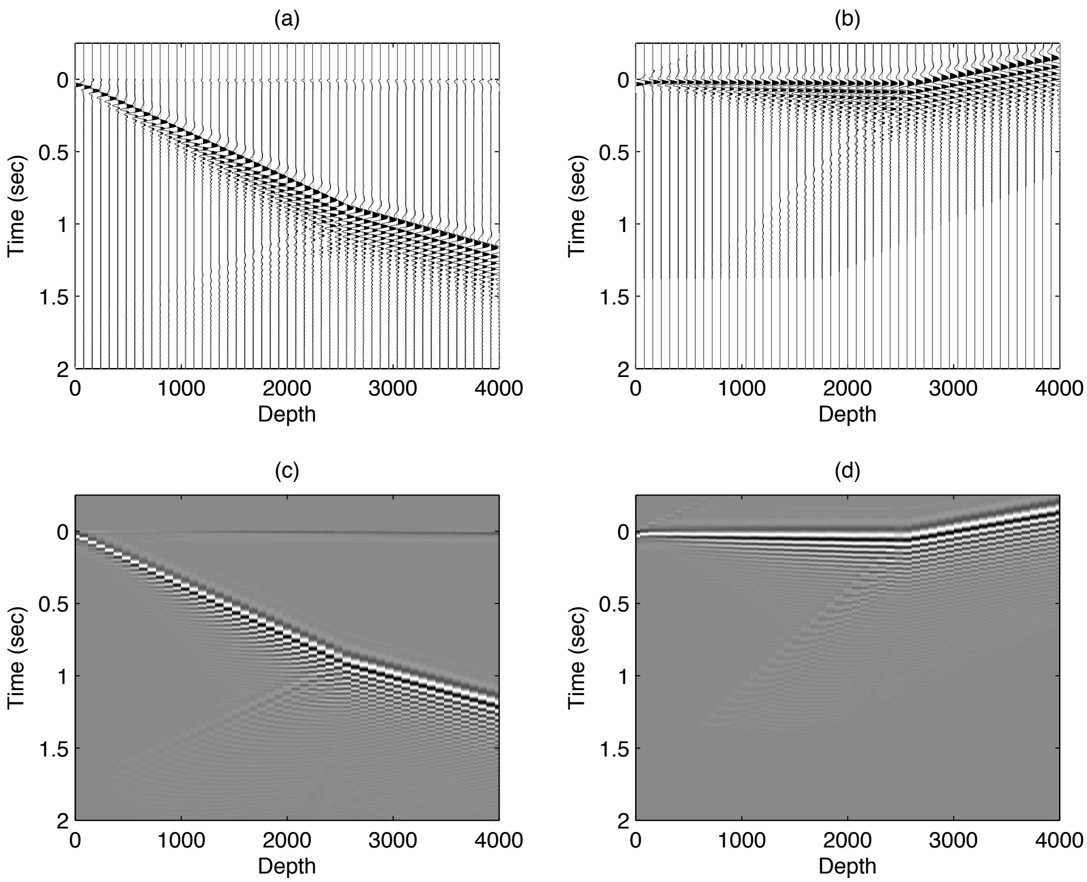

FIG. 3. Galilean transform example. (a)-(c) Original zero offset VSP data set, in the equivalent of the laboratory frame; (b)-(d) after a Galilean transformation into the rest frame of the particle m_1 associated with the direct arrival.

The mass and momentum rules for the laboratory frame are

$$\begin{aligned} m_1 v_1 + m_R v_R = m_T v_T \quad &\rightarrow \quad 1 \times p_0 - R p_0 = T p_1 \\ m_1 + m_R = m_T \quad &\rightarrow \quad 1 + R = T. \end{aligned} \tag{12}$$

Applying the transformation we find the new relations between the time z_g and space t coordinates of the three particles:

$$\begin{aligned} t_1'(z_g) + p' z_g &= p_0 z_g \\ t_R'(z_g) + p' z_g &= -p_0 z_g + t_R^0 \\ t_T'(z_g) + p' z_g &= p_1 z_g + t_T^0, \end{aligned} \tag{13}$$

or, re-arranging,

$$\begin{aligned} t_1'(z_g) &= (p_0 - p') z_g \\ t_R'(z_g) &= -(p_0 + p') z_g + t_R^0 \\ t_T'(z_g) &= (p_1 - p') z_g + t_T^0. \end{aligned} \tag{14}$$

Reading the drift velocities from these equations, we have then for the momenta before and after the z_1 collision in the new coordinates

$$\begin{aligned} & m_1 v_1' + m_R v_R' = 1 \times (p_0 - p') + R \times (-p_0 - p') = -p'(1 + R) + p_0 - p_0 R \\ & m_T v_T' = T \times (p_0 - p') = -p' T + p_1 T, \end{aligned} \tag{15}$$

which proves that if equation (12) holds, i.e., if mass and momentum are conserved in one frame $m_1 v_1' + m_R v_R'$ must be equal to $m_T v_T'$ in all frames attainable through the Galilean transform. Thus, any linear moveout correction conserves the basic collision rules.

PARTICLES COLLIDING CLASSICALLY IN A UNIFORM FORCE FIELD

Our main purpose in this report is to extend these zero-offset rules, discussed in previous reports, to walkaway VSP data. Before we treat VSP data with the source offset from the well, we review the new aspects of classical particle kinematics we will make use of. In particular, kinematics in the presence of simple external force fields.

Suppose an object or particle of unit mass $m_1 = 1$ is moving along a coordinate axis x, and is at all times subject to a uniform force field such that it experiences an acceleration $a < 0$ in the direction of positive x. This might correspond, for instance, to a cart on an air track which has been tilted slightly upward in the positive x direction (see Figures 4a–c). Just to be definite, suppose that the particle is sent upward from near the bottom of the track, such that it passes a point $x = L_1$ with velocity v_1 at time $t = 0$. Its position at subsequent times is from Newtonian kinematics

$$x(t) = L_1 + v_1 t + \frac{1}{2} a t^2 = x_1(t). \tag{16}$$

We label this $x_1(t)$. Meantime, suppose a second object of mass m_R is sent upwards from near the bottom of the track a moment later, chasing, as it were, the first object. Again for

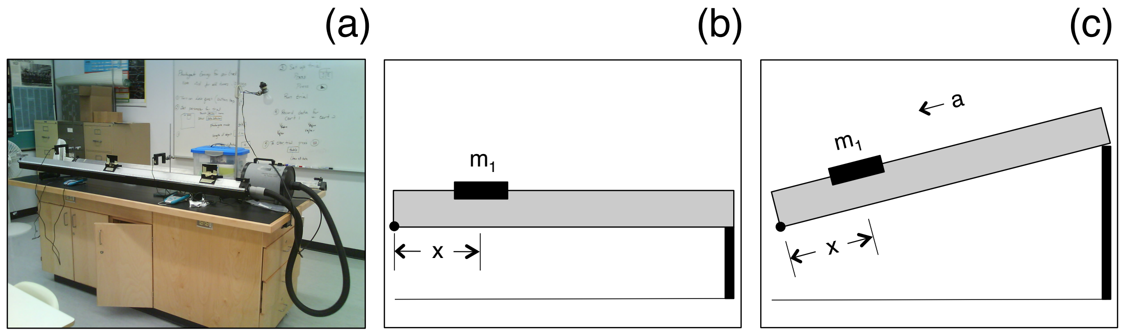

FIG. 4. (a) A cart on an air track is a useful physical model with which to illustrate classically colliding particles (photo credit *B. Chauhan http://en.wikipedia.org/wiki/File:Airtrack.JPG*). (b) Schematic diagram of (a). (c) To allow the particles/carts to experience a uniform acceleration, we imagine tilting the track slightly upward in the positive x direction.

definiteness, let us suppose it is so configured that it passes the point L_R with velocity v_R at time $t = 0$. Its position then is given by

$$x_R(t) = L_R + v_R t + \frac{1}{2}at^2. \tag{17}$$

If L_1, v_1, L_R and v_R are chosen correctly, the two-particle history illustrated in Figures 5a-c can be arranged. That is, as m_R moves upward, m_1 reaches a turning point and begins to accelerate back down the track (Figure 5a), and after an interval the two objects collide. We shall prescribe this collision to be *perfectly inelastic*, meaning the carts have velcro on them (or something) and stick together after colliding (Figure 5b). The upward moving m_R slows the downward moving m_1 down briefly, but the two now proceed downhill together as a single object with mass $m_T = m_1 + m_R$ (Figure 5c).

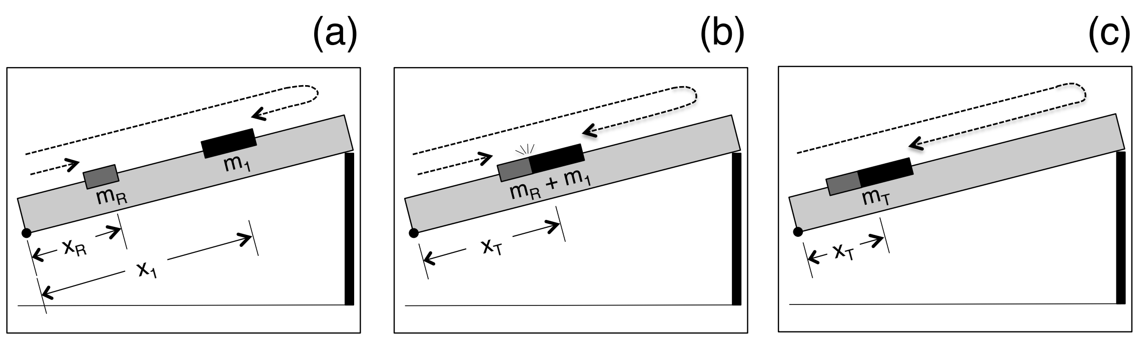

FIG. 5. Time history of a collision with all particles accelerating uniformly. (a) m_1 slides up the track first, followed by m_R; m_1 slows, stops and reverses course, accelerating downward as m_R comes up to meet it. (b) The two particles collide and stick together in a perfectly inelastic collision. (c) The two proceed downward as one mass m_T, having been delayed slightly by the upward hit from m_R.

The kinematic formulas in equations (16) and (17), along with the conservation of mass and momentum, can be used to determine an expression for the trajectory of the new particle mass m_T. First, the time of the collision t_c is the time at which $x_R = x_1$, which is

$$t_c = \frac{L_R - L_1}{v_1 - v_R}. \tag{18}$$

The location along the track where the collision occurs is found by substituting the time of the collision into either equation (16) or (17):

$$x_c = L_1 + v_1 t_c + \frac{1}{2} a t_c^2. \tag{19}$$

At the moment of the collision, mass m_1 and mass m_R are traveling with velocities

$$\left.\frac{dx_1}{dt}\right|_{t_c} = v_1 + at_c, \quad \left.\frac{dx_R}{dt}\right|_{t_c} = v_R + at_c \tag{20}$$

respectively, which means the total momentum of the two particles the instant before the collision was

$$p_{\text{before}} = m_1 \left(v_1 + at_c\right) + m_R \left(v_R + at_c\right). \tag{21}$$

This being equal to the post-collision momentum, $p_{\text{before}} = p_{\text{after}}$, we may now determine the velocity of the composite object immediately after the collision:

$$v_T^c = \frac{p_{\text{after}}}{m_T}. \tag{22}$$

The particle trajectory $x_T(t)$ will be of the form

$$x_T(t) = L_T + v_T t + \frac{1}{2} a t^2. \tag{23}$$

The velocity v_T^c can be extrapolated back to $t = 0$:

$$v_T = v_T^c - at_c, \tag{24}$$

and, after this, one point on the curve in equation (23), namely (x_c, t_c), can be used to determine L_T:

$$L_T = x_c - v_T t_c - \frac{1}{2} a t_c^2. \tag{25}$$

The entire trajectory of the two colliding objects is now determined from the initial conditions L_1, L_R, v_1 and v_R:

$$\begin{aligned} &x_1(t) = L_1 + v_1 t + \frac{1}{2} a t^2, \quad x_R(t) = L_R + v_R t + \frac{1}{2} a t^2, \quad t < t_c \\ &x_T(t) = L_T + v_T t + \frac{1}{2} a t^2, \quad t > t_c. \end{aligned} \tag{26}$$

An example set of trajectories is plotted in Figure 6, with the position of the two particles on the vertical axis and the time on the horizontal axis. A "movie" of the interaction can be produced by sweeping a vertical line from left to right and observing the evolution of the intersections of the vertical line with each particle's path.

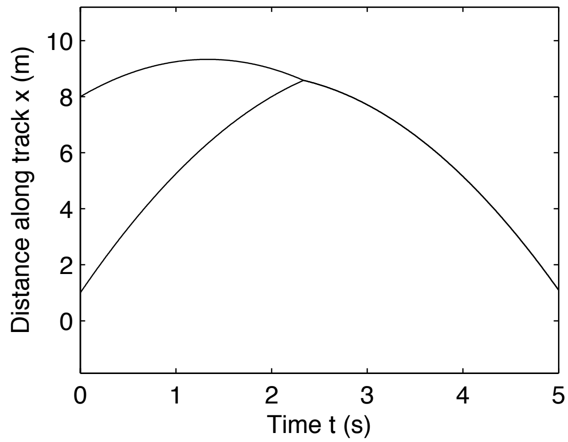

FIG. 6. Classical space-time diagram of the three particles colliding while all experiencing a uniform acceleration. Any particular "now" is focused on by drawing a vertical line somewhere in the distance/time domain; the positions of the particles are given by points of intersection of the tracks in the diagram with this vertical line. The physical process is visualized by having the vertical line sweep from left to right. The collision occurs just before 2.5s.

Change of variables $(x, t) \rightarrow (t, z_g)$

Finally, anticipating our use of this model to discuss a walkaway VSP data set, let us make a change of variables, the position variable x going to a time variable t, and the time variable t going to a depth variable z_g. The velocities v will be replaced with the symbols p for later convenience. We thus have the trajectories of the three particles in the form

$$\begin{aligned} &t_1(z_g) = L_1 + p_1 z_g + \frac{1}{2} a z_g^2, \quad t_R(z_g) = L_R + p_R z_g + \frac{1}{2} a z_g^2, \quad z_g < z_1 \\ &t_T(z_g) = L_T + p_T z_g + \frac{1}{2} a z_g^2, \quad z_g > z_1. \end{aligned} \tag{27}$$

Another example of the trajectories of particles before and after a perfectly inelastic collision is illustrated in Figure 7, this time with our pathological variables z_g for time and t for position. In this example (not in the expressions above) we have added a slight wrinkle, which is to change the acceleration after the collision to a different value. As if we had tilted the air track up by putting it onto some blocks, and the force of the collision knocked out one of the blocks, changing the amount of gravity felt.

Transformation to the m_1 rest frame

We can also arrange for a transformation to the rest frame of an accelerating particle. For instance, the transformation

$$\begin{aligned} t' &= t - \frac{1}{2} a z_g^2, \\ z_g' &= z_g, \end{aligned} \tag{28}$$

since $p_1 = 0$, generates for the example in Figure 7,

$$\begin{aligned} &t_1'(z_g) = L_1, \quad t_R'(z_g) = L_R + p_R z_g', \quad z_g' < z_1 \\ &t_T'(z_g) = L_T + p_T z_g' + \frac{1}{2}(a - a_1) z_g^2, \quad z_g' > z_1, \end{aligned} \tag{29}$$

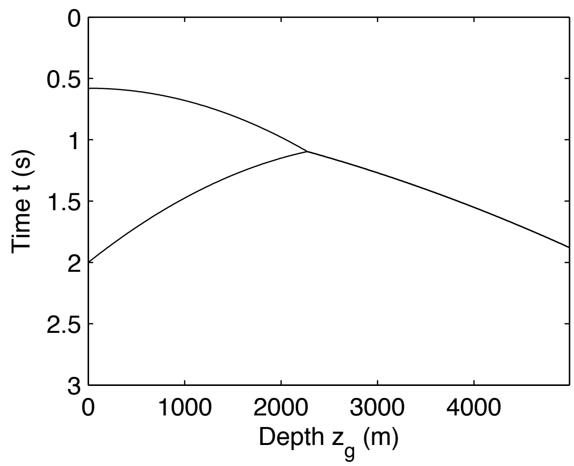

FIG. 7. Particle collision tracks in the new variables, with z_g being the new time coordinate, and travel time being the position coordinate.

where we have correctly included the change in the acceleration after the collision in that example. Now, evidently, m_1 is motionless, and m_R moves with a uniform velocity. m_T, because it experiences a different acceleration, does not transform to a state of uniform velocity in the accelerating reference frame. The particle tracks in the laboratory frame and the m_1 rest frame are illustrated in Figures 8a and b.

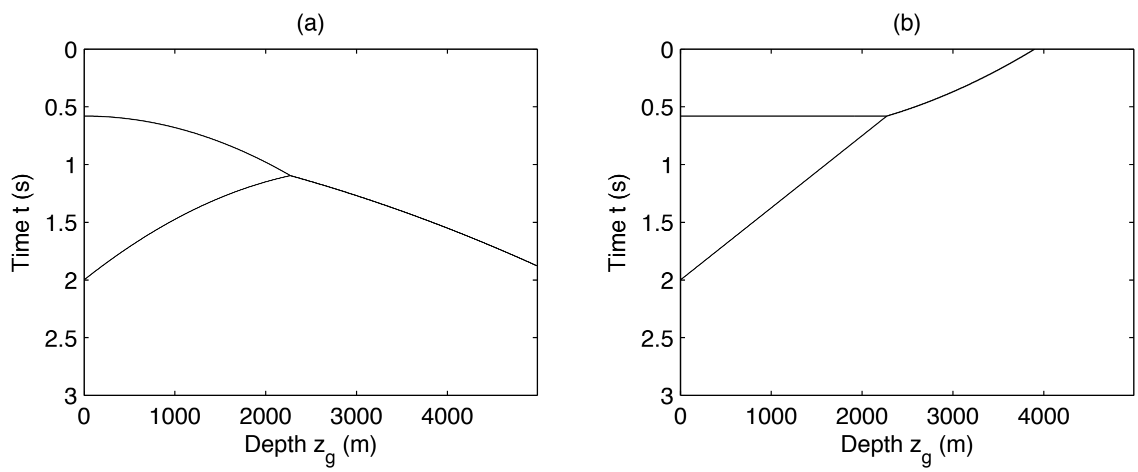

FIG. 8. Carts undergoing uniform acceleration. (a) Laboratory frame. (b) m_1 rest frame.

WALKAWAY VSP DATA

The travel time curves characteristic of a single interface in a WVSP experiment are sketched in Figure 9a-c. Two we may write down exact expressions for t_1 and t_R, and the third must be determined with additional calculations. We have

$$\begin{aligned} t_1(z_g) &= \frac{1}{c_0}\left(z_g^2 + x_s^2\right)^{1/2}, \quad t_R(z_g) = \frac{1}{c_0}\left[(2z_1 - z_g)^2 + x_s^2\right]^{1/2}, \quad z_g < z_1 \\ t_T(z_g) &= \frac{1}{c_0}\left(\frac{z_1}{\cos\theta}\right) + \frac{1}{c_1}\left(\frac{z_g - z_1}{\cos\theta_1}\right), \quad z_g > z_1, \end{aligned} \tag{30}$$

where $\theta_1 = \sin^{-1}[(c_0/c_1)\sin\theta]$, and where θ can be found for a given output z_g value with a simple shooting method.

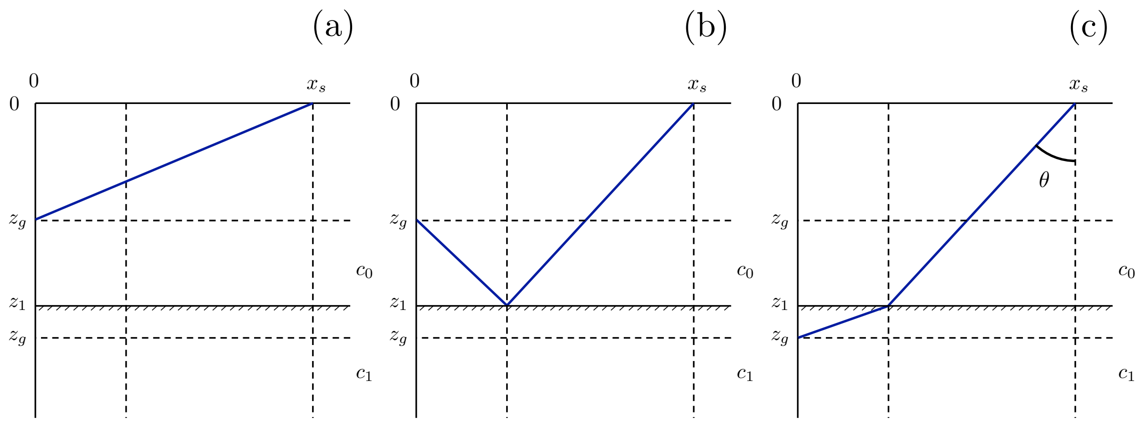

FIG. 9. Raypaths for the interaction of waves in a WVSP experiment with one interface and the well. (a) Direct arrival; (b) reflected arrival; (c) transmitted arrival. Traveltimes along these rays are given in equations (30).

We can likewise easily generate a WVSP shot record making use of finite difference acoustic modelling. By varying the lateral offset of the source (compare Figures 10–11) we can gain a qualitative sense of the effect of lateral offset – the hyperbolic character of the arrivals becomes more pronounced.

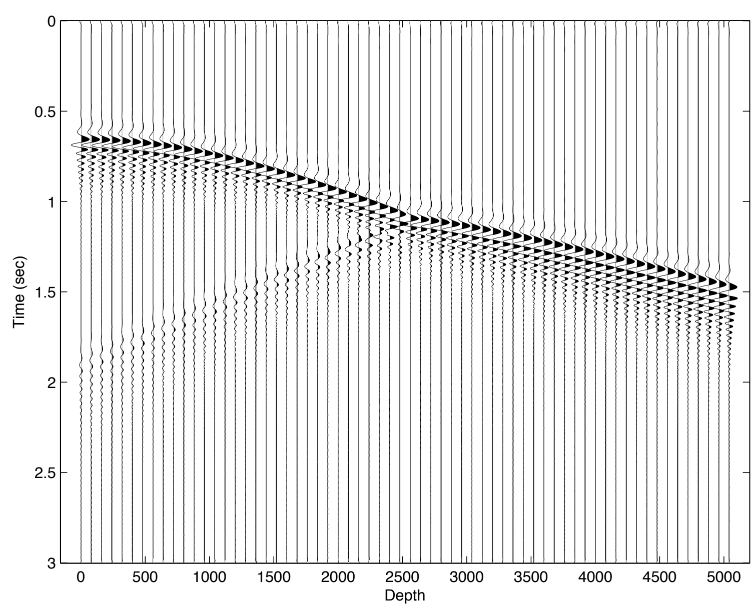

FIG. 10. Walkaway VSP data set with x_s at 1/5 the maximum depth of the well.

Qualitative relationship between lateral offset and acceleration in a uniform field

We began the analysis of zero-offset VSP data in particle terms by noticing the similarities between the shot record and an accounting of the space and time history of particles drifting with uniform velocity (e.g., Figure 2a-b). Let us now do something similar for

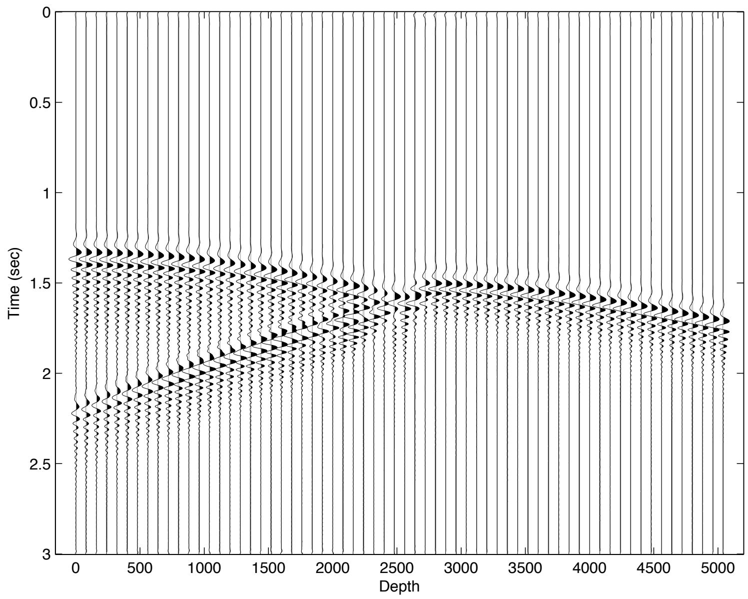

FIG. 11. Walkaway VSP data set with x_s at 2/5 the maximum depth of the well.

walkaway VSP data by comparing a shot record (and particle tracks for the particles colliding under uniform acceleration. We do this in Figure 12a (data) and Figure 12b (particle tracks). The WVSP particle model leverages this similarity.

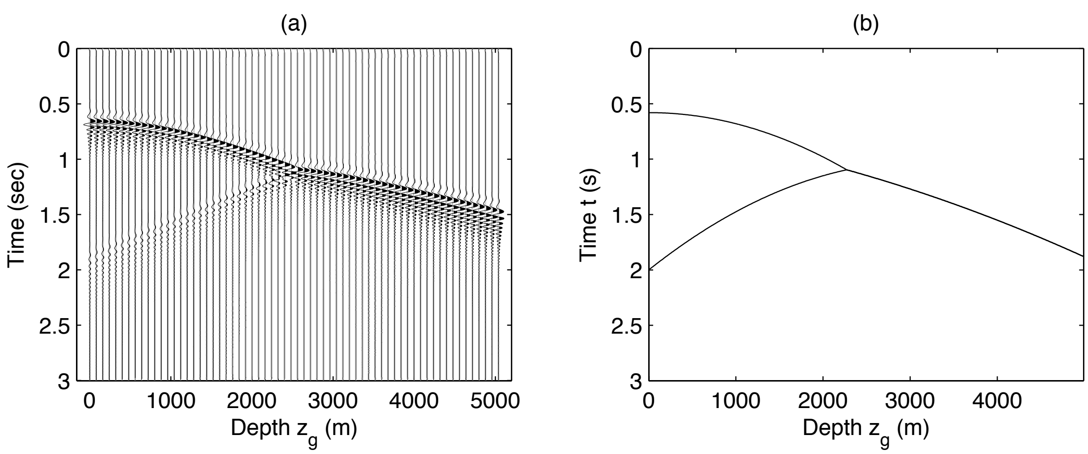

FIG. 12. Comparison of (a) a WVSP shot record and (b) particle tracks associated with a perfectly inelastic collision, in the coordinate system t, z_g. The similarity is leveraged to extend the zero-offset particle model.

Discrepancy far from the source

The qualitative relationship illustrated in Figures 12a-b is compelling, and for near offsets we can make this relationship predictive in the same way we have done for the zero offset case. However, a problem as offsets grow is evident: particles which experience uniform accelerations have, in classical kinematics, parabolic trajectories, that is they (in principle) can accelerate to arbitrarily high velocities. Whereas, event moveout is hyperbolic; a matching particle trajectory must have a *speed limit*, one which coincides with the asymptotic behaviour of the hyperbolic event.

CONCLUSIONS

Fortunately, in relativistic kinematics we have ready access to a theory for particle motion which comes equipped with a speed limit. Of course, in relativity c is a true speed limit, whereas we may need many, one to associate with each layer or homogeneous volume we consider. To concluded and summarize, the space-time tracks of particles moving under the influence of a uniform force field accelerate in such a way that under a particular coordinate transformation they reproduce the moveout in the VSP gather. In zero offset data and walkaway modes NMO correction is realized through transformations from the laboratory reference frame of the notional collisions to rest frames of individual particles before and after colliding. Galilean transformations alter the linear NMO patterns of zero-offset data, whereas transformations to noninertial reference frames are needed for walkaway modes.

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