Quantifying the incompleteness of the physics model in seismic inversion

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ABSTRACT

At each stage of seismic processing we adopt an underlying physical or mathematical model, and rely on its degree of incompleteness (relative to what is actually experienced by a seismic wave) being either negligible, or irrelevant to the processing task in question. For some tasks we can proceed with remarkably simplistic approximations, but in seismic inversion, our choices are few: any parameter or process affecting the seismic wave has to be included. The geophysical literature reflects the broad range of possible responses to this issue: suppressing elastic conversions in order to use acoustic physics; selecting analysis domains which boost P-wave influences over S-wave, etc. A smaller number of researchers have grappled directly with incompleteness, through probabilistic and numerical schemes. We may be able to come to grips with this problem more deterministically, if we learn to view complex physical models as being perturbations of simple physical models. This will be an important part of a meaningful attempt to apply full waveform inversion / IMMI to multicomponent land data.

INTRODUCTION

A problem we are likely permanently stuck with in seismic inversion is that we don't have an equation that exactly expresses seismic wave propagation through a real geological volume, we just have approximations. Approximations that have had at least moderate success and staying power in our field (e.g., scalar, acoustic, elastic-isotropic, viscoelastic, elastic-anistropic) are those which have found environments where the incompleteness of the physics model is insignificant.

Note that *insignificant* might mean "the errors due to the incompleteness of the physics model are small", but it also might mean "the neglected physics is important generally, but has no bearing the processing algorithm we are currently using". What is, or is not, an adequate physics model depends on whether the task at hand is this or that type of deconvolution, deghosting, demultipling, imaging, inversion, etc. For instance, scalar physics involving V_P but not density is often satisfactory for imaging the locations of reflectors. But for properly imaging *reflectivity* at depth, density, V_S and Q may all be necessary, since they dictate transmission losses through the overburden.

In full waveform inversion, there would seem to be few shortcuts: if a certain parameter influences wave propagation, even if we have no interest in knowing it, then it must be included in the inverse framework, even if only in a "garbage variable" through which its associated data variations are explained. Of course, we don't *know* what the full set of parameters are for any given geological model and/or production process we would like to investigate with seismic data. One is reminded of the "known-unknowns" political discussions of a few years ago*.

The geophysical inversion literature contains many instances in which *physics incompleteness* plays an important role. In full waveform inversion, a range of approaches, for instance in which elastic codas are suppressed in order to use acoustic physics (Brenders and Pratt, 2007), and in which certain domains suppress later-time events whose origins tend to require more complex models (Shin and Cha, 2009) are reported, as are, more recently, attempts to include as much elastic wave physics as possible (Jun et al., 2014). In particular, Jun et al. attribute to Operto et al. (2013) the statement that "when applying an acoustic FWI to real onshore data, incomplete modelling increases nonlinearity". While no doubt true, this may be too broad a characterization to allow for progress†. The purpose of this paper is to begin framing a clearer characterization of the unavoidable incompleteness of our seismic wave propagation model. Hansen et al. (2014) make very significant progress accounting for imperfect physics in a GPR experiment, using a probabilistic formulation. What we are after is something more deterministic, and accessible to analysis as well as numerics.

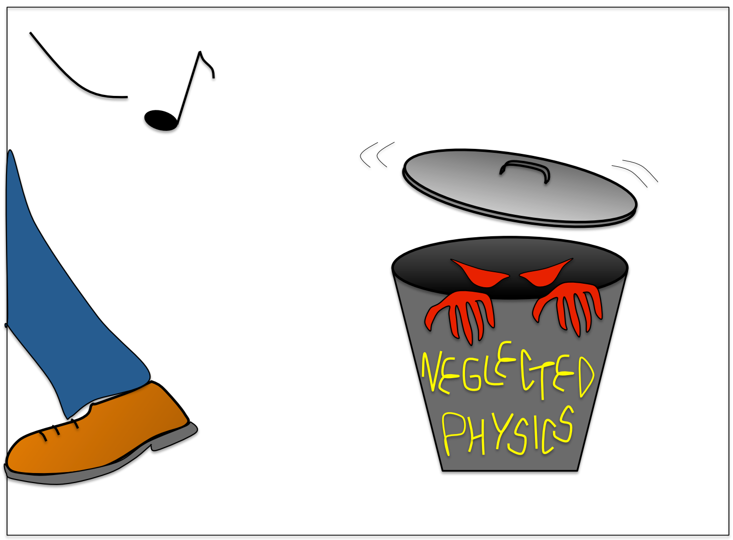

FIG. 1. Just don't forget the little fellow's in there.

Incompleteness of the physics model technically falls within the domain of appraisal and/or resolution analysis (Miller and Routh, 2007; Snieder, 1998; Oldenburg and Li, 1999), in the category of modelling errors (Hansen et al., 2014). But, here we will work with it as if it is distinct from other ways a "true" model is badly constrained by a geophysical experiment.

**http://en.wikipedia.org/wiki/There_are_known_knowns*

†Suppose one were to do AVO inversion on an event that had undergone strong attenuation in the overburden, but which had had its amplitudes preprocessed using elastic assumptions only. We could *refer* to the resulting difficulty and error as being the result of increased nonlinearity, but we wouldn't be accomplishing much by doing so. It is difficult to imagine that for instance iterating the elastic AVO inversion in this circumstance would lead us to, or even towards, the right answer.

A GENERAL PERTURBATION MODEL

The idea as we see it currently is that a forward modelling equation like the wave equation is expressed as an operator equation

$$\mathcal{L}(\mathbf{r},\omega|\mathbf{m})P(\mathbf{r},\omega|\mathbf{m}) = 0, \tag{1}$$

that depends on space and time (or in this case frequency), and also on a general model vector $\mathbf{m}$. The vector $\mathbf{m}$ is a function of space— we view it as containing all of the information we could want to determine in a certain geological volume—but it has a second dimensionality, which is the number and type of parameter we solve for. A realization of $\mathbf{m}$ could therefore be of length M×N, where there are M model parameters and N volume elements. Or, we could imagine equally well that there is a separate M length vector assigned to each of the N volume elements. The key point is that one $\mathbf{m}$ can be distinguished from another by having different numerical values for a certain parameter, but also in the number, and type, of parameters enumerated inside it.

An example is the one-parameter scalar operator

$$\mathcal{L}(\mathbf{r},\omega|c) = \nabla^2 + \frac{\omega^2}{c^2(\mathbf{r})}; \tag{2}$$

another is the two-parameter acoustic operator

$$\mathcal{L}(\mathbf{r},\omega|\kappa,\rho) = \nabla\cdot\frac{1}{\rho(\mathbf{r})}\nabla + \frac{\omega^2}{\kappa(\mathbf{r})}, \tag{3}$$

and the two-parameter viscoacoustic operator

$$\mathcal{L}(\mathbf{r},\omega|c,Q) = \nabla^2 + \frac{\omega^2}{c^2(\mathbf{r})}\left[1 + \frac{i}{2Q(\mathbf{r})} - \frac{1}{\pi Q(\mathbf{r})}\log\left(\frac{\omega}{\omega_0}\right)\right]^2. \tag{4}$$

This can go on, to include elastic, viscoelastic, anisotropic, etc. media. Let us imagine two geophysicists, a right one and a wrong one. The right one has chosen a physical model and operator, say $\mathcal{L}(\mathbf{r},\omega|\mathbf{m})$, that adequately describes the geophysical experiment, and the wrong one has chosen one, say $\mathcal{L}(\mathbf{r},\omega|\mathbf{m}')$, which has either spurious parameters or the wrong number of them. The right geophysicist has equations like the ones we have described above, and the wrong one has others, namely

$$\mathcal{L}(\mathbf{r},\omega|\mathbf{m}')P(\mathbf{r},\omega|\mathbf{m}') = 0, \tag{5}$$

where perhaps an acoustic model

$$\mathcal{L}(\mathbf{r},\omega|\kappa',\rho') = \nabla\cdot\frac{1}{\rho'(\mathbf{r})}\nabla + \frac{\omega^2}{\kappa'(\mathbf{r})}, \tag{6}$$

rather than the more appropriate three-parameter viscoacoustic model is assumed. The perturbation picture of incomplete physical modelling calls for a perturbation operator

$$\mathcal{V}(\mathbf{r},\omega|\mathbf{m},\mathbf{m}') = \mathcal{L}(\mathbf{r},\omega|\mathbf{m}) - \mathcal{L}(\mathbf{r},\omega|\mathbf{m}'), \tag{7}$$

to be defined to characterize the difference between the medium of the right geophysicist and the wrong geophysicist. Now, neither the right geophysicist nor the wrong geophysicist yet know the correct values of the parameters in the subsurface. So, there are two ways for a general perturbation of this kind to take on nonzero values. If the two operators are in agreement on the type of model to solve for, but the numerical values of these parameters differ, $\mathcal{V} \neq 0$. However, if the numerical values of shared model parameters between the two types of model are equal, but there are extra parameters, also $\mathcal{V} \neq 0$.

A perturbation operator of this kind can be included in a sensitivity calculation, in principle. The Green's operator or resolvent being

$$\mathcal{G} = \mathcal{L}^{-1}, \tag{8}$$

the perturbation in the field associated with any of the kinds of

$$\mathcal{V} \neq 0$$

above is

$$\delta P = \mathcal{G}\mathcal{V}P. \tag{9}$$

The question is, within this model, how, precisely and in detail, does the influence of model-type errors co-mingle with standard errors? Where by *standard errors* we mean discrepancies between correctly-chosen medium parameter estimates and their true values.

The coupling of known-unknowns with unknown-unknowns in amplitude inversion

To begin to grapple with this issue, we consider a difference quantity Δ, defined as the difference between two reflection coefficients. The difference in general is

$$\begin{aligned} &\Delta(\theta, \omega, ...|\{\text{parameter set 1}\}, \{\text{parameter set 2}\}) \\ &\qquad = R(\theta, \omega, ...|\{\text{parameter set 2}\}) - R(\theta, \omega, ...|\{\text{parameter set 1}\}). \end{aligned} \tag{10}$$

For example, parameter set 1 could be two-parameter acoustic, and parameter set 2 could be three-parameter viscoacoustic:

$$\begin{aligned} &R(\theta|\{\kappa_1, \rho_1\}), \\ &R(\theta, \omega|\{\kappa_2, \rho_2, Q_2\}), \end{aligned} \tag{11}$$

for a sensitivity of

$$\Delta(\theta, \omega|\{\kappa_1, \rho_1\}, \{\kappa_2, \rho_2, Q_2\}) = R(\theta, \omega|\{\kappa_2, \rho_2, Q_2\}) - R(\theta, \omega|\{\kappa_1, \rho_1\}). \tag{12}$$

Expanding in series about jumps across the boundary in each parameter, we have

$$\Delta(\theta, \omega|\{\kappa_1, \rho_1\}, \{\kappa_2, \rho_2, Q_2\}) = \Delta^{(1)} + \Delta^{(2)} + ..., \tag{13}$$

where

$$\Delta^{(1)} = \Gamma_\kappa \left[\left(\frac{\Delta\kappa}{\kappa}\right)_2 - \left(\frac{\Delta\kappa}{\kappa}\right)_1 \right] + \Gamma_\rho \left[\left(\frac{\Delta\rho}{\rho}\right)_2 - \left(\frac{\Delta\rho}{\rho}\right)_1 \right] + \Gamma_Q \left(\frac{\Delta Q}{Q}\right)_2, \tag{14}$$

at first order, with coefficients Γ_x, and

$$\begin{aligned}\Delta^{(2)} =&\Gamma_{\kappa\kappa}\left[\left(\frac{\Delta\kappa}{\kappa}\right)_2^2-\left(\frac{\Delta\kappa}{\kappa}\right)_1^2\right]+\Gamma_{\kappa\rho}\left[\left(\frac{\Delta\kappa}{\kappa}\right)_2\left(\frac{\Delta\rho}{\rho}\right)_2-\left(\frac{\Delta\kappa}{\kappa}\right)_1\left(\frac{\Delta\rho}{\rho}\right)_1\right]\\&+\Gamma_{\rho\rho}\left[\left(\frac{\Delta\rho}{\rho}\right)_2^2-\left(\frac{\Delta\rho}{\rho}\right)_1^2\right]+\Gamma_{\kappa Q}\left(\frac{\Delta\kappa}{\kappa}\right)_2\left(\frac{\Delta Q}{Q}\right)_2\\&+\Gamma_{\rho Q}\left(\frac{\Delta\rho}{\rho}\right)_2\left(\frac{\Delta Q}{Q}\right)_2+\Gamma_{QQ}\left(\frac{\Delta Q}{Q}\right)_2^2,\end{aligned}\tag{15}$$

with coefficients Γ_{xy}, etc. At first order, equation (13), we see a separation of the sensitivity into two independent sets of contributions. The first two terms represent jumps in the the reflectivity due to our imperfect reconstruction of correctly-included parameters — "known unknowns". Whereas in the third term we have a linear contribution from the "unknown unknown" — the parameter Q whose presence we are ignorant of in this example. At second order, an even more interesting phenomenon appears. We have second order contributions of the "unknown unknowns" (the Γ_{QQ} term), and "known unknowns" (e.g., the $\Gamma_{\kappa\kappa}$ term). However, we have cross-terms also, in which known- and unknown-unknowns couple to determine the sensitivity. For instance, the $\Gamma_{\rho Q}$ term.

CONCLUSIONS

How exactly to use a formalism like this to analyze and manage incomplete physics is not clear. A nominal approach is to analyze residuals in a given inversion methodology as compared to the expected sensitivities in a certain modelling domain, and infer "the unmodelled sensitivities" as being given by "one minus the modelled sensitivities". Another possibility is that one develops a library of model types and the interrelated Δ operators between them, and match the behaviour of our inversion as it proceeds. This is speculative; we will proceed in the coming following the more cautious but compelling motivation that, at the very least, a quantitative accounting of the missing physics in our inversion schemes is needed.

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