# FWI and the “Noise” Quandary

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## ABSTRACT

Full waveform inversion (FWI) has been termed the most general inversion method in which we attempt to model every digital sample in the seismic trace by adjusting model parameters so that the discrepancies between data and model response are minimized – often in a least squares sense. FWI attempts to model every data sample, even though the data values are signal contaminated with “noise”. This begs the question. What is noise? This paper attempts to define “noise” and how it might affect FWI.

## INTRODUCTION

In full waveform inversion, the goal is to produce a model whose response provides a good fit to all the digital amplitude values in a seismic trace. In mathematical terms, the goal is to minimize some objective function measuring differences between the observed wavefield, $u(y_{rs},t)$ and the modeled wavefield, $g(y_{rs},t)$, where $y_{rs}$ denotes the location of the receiver, r, for a given source point, s, and t denotes the time sample. Shin (1988) and Pica et al. (1990) used the least squares objective function to do FWI. That is, the following norm, S, is minimized.

$$S = \sum_r \sum_t \left\| u \ (y_r,t) - g(y_r,t) \right\|^2 \tag{1}$$

Necessary criteria for minimization of S require that $\frac{\partial S}{\partial \theta_j} = 0$ for all of the model parameters $\theta_j$ As shown by Lines and Treitel (1984), this minimization leads to a series of ill-posed linearized equations of the form:

$$\mathbf{Ax} = \mathbf{b} \tag{2}$$

Here A is the n by p Jacobian matrix, for n data values and p parameter values, whose elements are given by $A_{ij} = \frac{\partial f_i}{\partial \theta_j}$; x is the parameter change vector with values $x_j = \theta_j - \theta_j^0$ , and $\mathbf{b} = \mathbf{u} - \mathbf{g}_0$ is the discrepancy vector between the data vector,**u**, and the vector containing the initial model response values, $\mathbf{g}_0$ . This least squares form of full waveform inversion can be successful under the following conditions.

1. The form of the wave equation used to compute the modeled wavefield is appropriate for the recorded data.

2. We have the appropriate number of parameters to minimize S. ,

3. We have an immense amount of computer power to minimize S for a realistic number of traces and digitized times.
4. The observed seismic data are dominated by signals caused by changes in the Earth's interior and are not corrupted by noise.

With the advent of parallel processing and ever increasing computation power, conditions 1-3 are being met and worthwhile results have been shown. Condition 4 relates more to the signal-to-noise ratio in our data which may have a deleterious effect on full waveform inversion. **In full waveform inversion, all input data are treated as signal. Both coherent and incoherent noise will pose a problem for waveform inversion, as pointed out by Shin et al. (2007).**

In this report, we will examine effects on FWI of two different types of "noise". The first will test the effects due to noise that is random and unrelated to the reflectivity properties of the Earth. These types of noise could be due to wind, ocean waves, instrument noise and will be considered to be uncorrelated to seismic reflection properties of the Earth.

The second type of noise is defined as "unwanted signal" and will not be primary reflection signal. According to Sheriff (1991), this would include seismic recordings we will to suppress. In this report, we will examine multiples and how these arrivals (normally classified as unwanted) can be used by FWI to compute reflection coefficients. Examples of this inversion will be taken from Lines (1996) and O'Brien (1997).

## METHODOLOGY AND RESULTS

A test of FWI performance on data contaminated by random noise has been given by Lines et al. (2013). In this paper, the inversion for seismic-Q was performed on model data with decreasing signal-to-noise ratios. S/N ratios of infinity (no noise), 5 and 2.5 were tested. These seismic model data are shown in Figure 1.

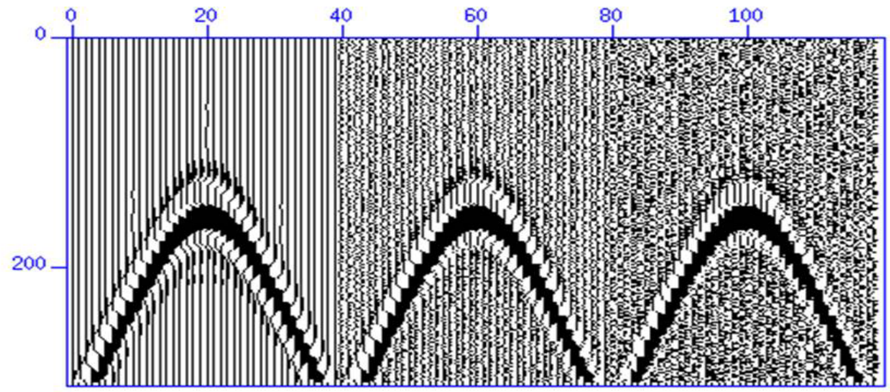


FIG. 1 Seismic model data (left) with various signal-to-noise ratios of infinity, 5.0 and 2.5 were inverted to estimate Q value of 2 pi (6.28).

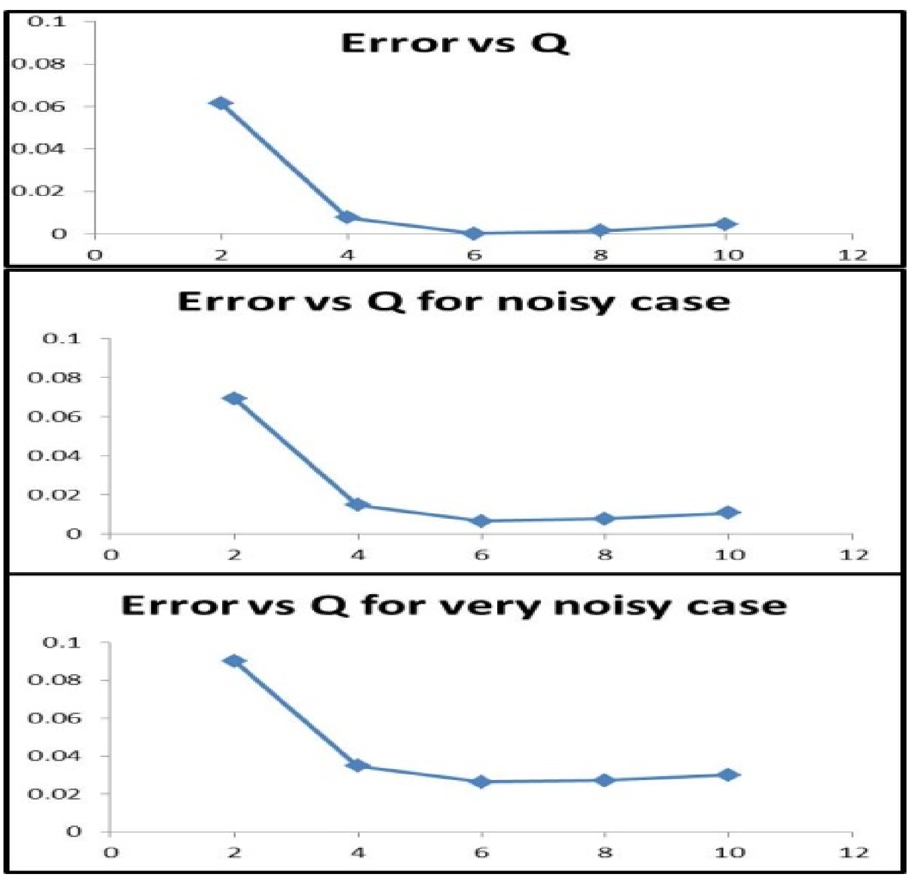


FIG. 2 While the error of fit worsened with increasing noise, all inversions converged close to the correct value of 6.28 within 3 iterations.

As expected the error of fit, given by equation (1) increased as the noise levels increased. The error of fit is shown in Figure 2 for the three cases. However, although the error worsened with each case, the convergence to the correct value of Q (which was $2\pi$) occurred after 3 iterations in all three cases. While the error levels in Figure 2 increased with noise, the position of the local minimum stayed in almost the same location despite increasing random noise levels. Hence, **for modest random noise levels, FWI appears to be robust.** A mathematical explanation can be found, if we examine the least squares solution to equation (2).

Since A in equation (2) is in general rectangular, we compute a least-squares solution as given by:

$$\mathbf{A}^T\mathbf{A}\mathbf{x} = \mathbf{A}^T\mathbf{b} \tag{3}$$

Now if we consider the noisy data case we replace b by b+n and equation (9) becomes:

$$\mathbf{A}^T\mathbf{A}\mathbf{x} = \mathbf{A}^T\mathbf{b} + \mathbf{A}^T\mathbf{n} \tag{4}$$

However, if the noise is random and zero mean, the second term on the right hand side, $\mathbf{A}^T\mathbf{n}$, will tend to zero especially if $\mathbf{A}$ is smoothly varying. This second term is essentially a crosscorrelation of a random sequence with one that is smoothly varying and its value will generally be small compared to the first term. This and the behavior of the error function have meant that the full waveform inversion is robust, at least for modest noise levels. Therefore equation (4) will closely resemble equation (3), which is valid for the noiseless case.

For the case of multiples, we can attempt to model the multiples in our data by adjusting the reflection coefficients. In other words, the multiples can be considered as signal. Whereas normal seismic processing attempts to suppress multiple energy before invertint the primary reflections to derive information about reflection coefficients and subsequently the acoustical impedance, we could try to invert the primary and multiple arrivals to estimate the reflection coefficients. The latter approach was applied to multiples in model data by Lines (1996) and applied to marine multiples from offshore Newfoundland by O'Brien (1997). In these cases, multiples were considered as signal.

Figure 3 from Lines (1996) shows an FWI result obtained by model fitting of primaries and multiples by adjustment of reflection coefficients. The actual reflection coefficients (traces 1-20) are compared with the estimated reflection coefficients (traces 21-40) after 5 iterations. The starting estimates for the reflection coefficients were all set at values 0.1 at positions where the reflection coefficients above a threshold value existed. In practice, this activation of reflection coefficient parameter values could be done through use of sparse-spike inversion or by thresholding of reflection coefficients from sonic and density logs. After a few iterations, the model response fits the data containing primaries plus multiples to within the estimated level while producing reliable reflection coefficient values.

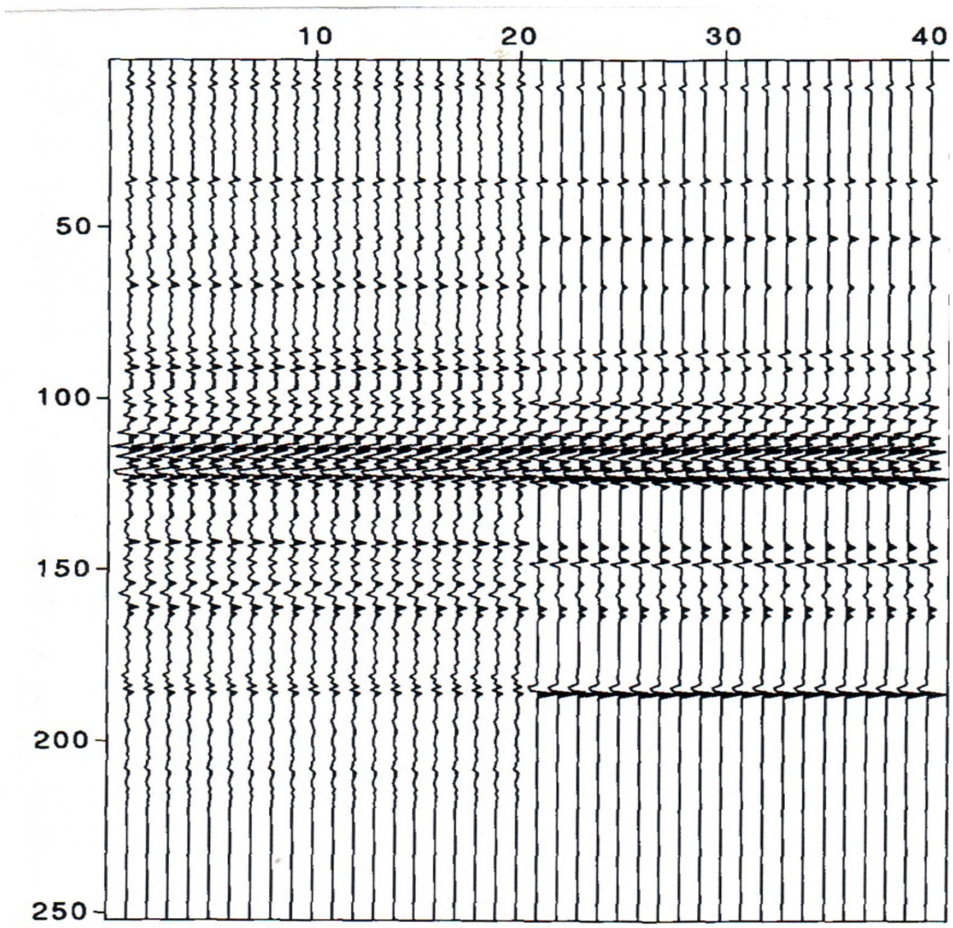


FIG. 3 The actual reflection coefficients for a model example (traces 1-20 in plot) are compared to the estimated reflection coefficients (traces 21-40) that are obtained from the inversion after 5 iterations (from Lines, 1996).

In this example the normally "undesired signal", the multiples are used with primaries in the inversion to produce reliable reflection coefficients. This FWI application uses the multiples as signal rather than noise.

## CONCLUSIONS

One of the main concerns with FWI or model-based inversion of amplitudes has been that the method treats all data as signal, even if the data are contaminated with noise. While the author originally believed that this could be the nemesis of FWI, it appears that FWI (from these tests at least) may be robust for both random noise and for the noise as undesired signal. In the case of data containing random noise, FWI with least squares appears to be robust for cases of moderate noise since the noise is uncorrelated with the Jacobian matrix. In the case of strong undesired signal ("organized noise") such as multiples, FWI can produce reliable model parameters by modeling the multiples. In either case, FWI appears to do better than the author anticipated while producing useful results. More tests on noisy data are required.

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