Numerical analysis of scattering in a viscoelastic medium

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ABSTRACT

Recently we have developed a theoretical picture of viscoelastic scattering applicable to seismic waves propagating in arbitrary multidimensional geological volumes. The purpose of this paper is to begin to integrate this theoretical analysis with numerical analysis. That combination will permit very general versions of attenuation/Q related analysis, processing, and inversion in multicomponent seismic to be formulated. Here we used the code developed by Martin and Komatitsch (2009) to simulate the reflections caused by general viscoelastic contrasts designed to be comparable to the results from the Born approximation. We apply the code to a viscoelastic geological model involving a contrast between two layers with different elastic and anelastic properties. We show that the anelastic contrasts generate reflection amplitudes which quantitatively are in agreement with those derived theoretically by Moradi and Innanen (2013).

INTRODUCTION

The goal of seismic inversion is estimation of physical properties of subsurface earth from recorded data. In an inverse problem recorded wavefield is known whereas the subsurface properties in which the wave field propagates are unknown. In order to solve the inverse problem, it is essential to understand the forward problem in which we model the the observed data from physical characteristics of subsurface.

The scattering of seismic waves in a heterogenous medium in the context of the Born approximation has been investigated by many authors (Beylkin and Burridge, 1990; Stolt and Weglein, 2012). Stolt and Weglein (2012) introduced a formal theory for the description of the multidimensional scattering of seismic waves based on an isotropic-elastic model. We have elsewhere identified as a research priority the adaptation of this approach to incorporate other, more complete pictures of seismic wave propagation. We have progressed one of these, the extension to include anelasticity and/or viscoelasticity (Flugge, 1967), which brings to the wave model the capacity to transform elastic energy into heat. Anelasticity is generally held to be a key contributor to seismic attenuation, or "seismic Q", which has received several decades worth of careful attention in the literature (e.g., Aki and Richards, 2002; Futterman, 1967).

Wave propagation in linear viscoelastic media has been extensively studied numerically (Carcione et al., 1988b; Carcione, 1993; Carcione et al., 1988a). Borcherdt (2009) has presented a complete theory for seismic waves propagating in layered anelastic media, assuming a viscoelastic model to hold. Borcherdt in particular predicts a range of transverse inhomogeneous wave types unique to viscoelastic media (Type I and II S waves), and develops rules for conversion of one type to another during interactions with planar boundaries.

Motivated by the need to derive and characterize increasingly sophisticated seismic data analysis and inversion methods incorporating wave dissipation, the problem of scattering of homogeneous and inhomogeneous waves from perturbations in five viscoelastic parameters (density, P- and S-wave velocities, and P- and S-wave quality factors), formulated in the context of the Born approximation (Moradi and Innanen, 2013). In this report, we validate those formulation using existing numerical forward modeling schemes (Carcione, 1993).

The paper is organized as follows. In section 1 we review the physical models for viscoelastic medium based on the dash-pot spring systems and introduced the constitutive equation between stress and strain. In section 2, the wave equation for viscoelastic medium based on the memory variables are described. In section 3, we briefly describe the scattering potential components in Born approximation. In section 4, we simulate the wave propagation in a two layer medium with elastic and anelastic properteis. Finally in section 5 we summarized the results and clarify the future directions for this research.

REVIEW OF COMMON VISCOELASTIC MODELS

For a linear elastic medium the stress and strain has linear relationship. If the stress is removed, a linear elastic medium instantaneously returns to its original shape. Mathematically it is said that elastic medium behaviour is time-independent. In contrast, a viscoelastic medium has a time-dependent behaviour, when stress is loaded and unloaded. Such a medium has both viscosity and elasticity. For viscoelastic medium, stress not only is a function of strain but also time variation of strain (Flugge, 1967; Borcherdt, 2009).

To mimic the viscoelastic behaviour of medium, various combinations of springs and dashpots are used. Springs display the elastic properties and dashpots simulate the viscous characteristics. The simplest analogous model can by obtained by connecting springs and dashpots in parallel or in series. The first one called Kelvin-Voigt model and the second one Maxwell model. In Kelvin-Voigt, since the spring and dashpot are in parallel, the displacement is the same throughout the system but different stresses are experienced. A Maxwell model in which the spring and dashpot are in series, the stress is the same throughout the system but different displacements are experienced.

Springs represents the elastic properties of the medium and dashpots simulates the fluid behavior which are assumed to deform continuously. In the Maxwell model when stress applied to the system spring deformation is finite but continuous so long as the stress is maintained. Due to this the Maxwell model is said simulate a viscoelastic fluid. In contrast, in Kelvin-Voigt model when stress is applied to the model, since the dashpot is in parallel to the spring, the dashpot deformes as long as spring keep deforming. In other words, the dashpot can not deform continuously. As a result Kelvin-Voigt model behaves as a viscoelastic solid medium. Neither Kelvin-Voigt model nor Maxwell model represent a real viscoelastic model, however in combination with additional springs in series or in parallel can explain most properties of a viscoelastic medium. One example is referred to as the standard linear model.

Let us consider a one-dimensional viscoelastic model. In this case the relation between stress (σ) and strain(e) is given by a convolution (Borcherdt, 2009)

$$\sigma(t) = r * \dot{\varepsilon} = \int_{-\infty}^{\infty} r(t-\tau) \left[\frac{d\varepsilon}{dt}\right]_{t=\tau} d\tau, \tag{1}$$

where r is relaxation function. Equation (1) implies that the stress at any time t is determined by the entire history of the strain until time t. The inverse of this equation which relates the strain to stress also can be written as a convolution

$$\varepsilon(t) = c * \sigma_{,t} = \int_{-\infty}^{\infty} c(t-\tau) \left[\frac{d\sigma}{dt}\right]_{t=\tau} d\tau, \tag{2}$$

where c is the creep function and,t denotes the derivative respect to time. The complex modulus M is next defined as

$$M(\omega) = i\omega R(\omega), \tag{3}$$

where $R(\omega)$ is Fourier transform of the relaxation function. The fractional energy loss expressed in terms of the ratio of the imaginary and real parts of the complex modulus (Borcherdt, 2009) is then used to define the reciprocal of the Q factor

$$Q^{-1} = \frac{\Im M}{\Re M} = \frac{M_I}{M_R}. \tag{4}$$

The relaxation and creep functions for standard linear model are given by (Flugge, 1967)

$$r(t) = M_r \left[1 - \left(1 - \frac{\tau_\varepsilon}{\tau_\sigma}\right) e^{-t/\tau_\sigma}\right] H(t), \tag{5}$$

$$c(t) = \frac{1}{M_r} \left[1 - \left(1 - \frac{\tau_\varepsilon}{\tau_\sigma}\right) e^{-t/\tau_\sigma}\right] H(t). \tag{6}$$

Here $H(t)$ is step function and τ_ε and τ_σ stand for relaxation times for strain and stress respectively

$$\tau_\sigma = \frac{\eta}{k_1 + k_2}, \qquad \tau_\varepsilon = \frac{\eta}{k_2}. \tag{7}$$

In addition M_r refers to the relaxed elastic modulus

$$M_r = \frac{k_1 k_2}{k_1 + k_2}, \tag{8}$$

where k is a constant relating stress and strain in Hooke's law for a spring and η is the viscosity of the dashpot component. It can be seen that elastic limit is recovered by setting $\tau_\sigma = \tau_\varepsilon$. By applying the Fourier transform to (5) and inserting in (3) we obtain the complex modulus

$$M(\omega) = M_r \frac{1 + i\omega\tau_\varepsilon}{1 + i\omega\tau_\sigma}. \tag{9}$$

Unrelaxed or high-frequency modulus is an instantaneous elastic response of the viscoelastic material which is given by

$$M_u = \lim_{\omega\to\infty} M(\omega) = M_r \frac{\tau_\varepsilon}{\tau_\sigma}. \tag{10}$$

On the other side, low-frequency or relaxed modulus is a long term equilibrium response

$$M_r = \lim_{\omega\to 0} M(\omega). \tag{11}$$

Finally using the definition of quality factor in (4) we arrive at

$$Q^{-1} = \frac{\omega(\tau_\varepsilon - \tau_\sigma)}{1 + \omega^2 \tau_\varepsilon \tau_\sigma}. \tag{12}$$

We can see that for $\tau_\sigma = \tau_\varepsilon$ the attenuation factor goes to zero. The above analysis can be generalized to the l-mechanism system. In this case complex modulus takes the following form (Flugge, 1967)

$$M(\omega) = M_r \left(1 - L + \sum_{l=1}^{L} \frac{1 + i\omega\tau_{\varepsilon l}}{1 + i\omega\tau_{\sigma l}}\right). \tag{13}$$

Similar to the one-mechanism case, the unrelaxed modulus is

$$M_u = M_r + \sum_{l=1}^{L} M_l, \tag{14}$$

where

$$M_l = M_r \left(\frac{\tau_{\varepsilon l}}{\tau_{\sigma l}} - 1\right). \tag{15}$$

In this case the quality factor is given by

$$Q(\omega) = \frac{1 - L + \sum_{l=1}^{L} \frac{1+\omega^2\tau_{\varepsilon l}\tau_{\sigma l}}{1+\omega^2\tau_{\sigma l}^2}}{\sum_{l=1}^{L} \frac{\omega(\tau_{\varepsilon l}-\tau_{\sigma l})}{1+\omega^2\tau_{\varepsilon l}\tau_{\sigma l}}}. \tag{16}$$

There is a method to extract the desired Q-constant model for given values of relaxation times called τ-model (Blanch et al., 1995). To describe this method first we define a dimensionless parameter τ

$$\tau = \frac{\tau_{\varepsilon l}}{\tau_{\sigma l}} - 1. \tag{17}$$

For real materials $\tau \ll 1$, so we can approximate (16) as

$$Q^{-1}(\omega, \tau_{\sigma l}, \tau) \approx \sum_{l=1}^{L} \frac{\omega\tau_{\sigma l}\tau}{1 + \omega^2\tau_{\sigma l}^2}. \tag{18}$$

Using the least-squares inversion, the optimization variables $\tau_{\sigma l}$ and τ are determined. The following function is minimized numerically in a least-squares sense

$$J(\tau_{\sigma l}, \tau) = \int_{\omega_a}^{\omega_b} \left[Q^{-1}(\omega, \tau_{\sigma l}, \tau) - Q_0^{-1}\right]^2 d\omega. \tag{19}$$

In Figure 1, we plot the quality factor for P- and S-waves versus frequency for a two-mechanism model. We observe that in the range of $30 < f < 100$, quality factor is nearly constant. In can be shown that a larger number of relaxation mechanisms gives better constant-Q approximations, especially for higher frequencies.

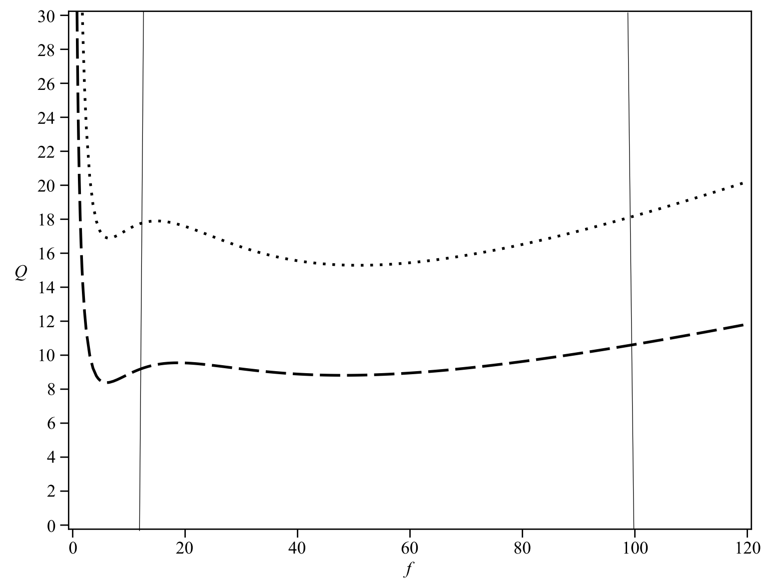

FIG. 1. Quality factor Q for different numbers of relaxation mechanisms in the frequency band from 0 to 120 Hz. The frequency band from 30 to 100 Hz, for which Q is constructed to be approximately constant, is separated by vertical lines. Dot line is for Q_p and dash line for Q_s

EQUATION OF MOTION

In one dimension, say in x-direction, the particle velocity is given by

$$\dot{u}_{,x} = v_{,x} = \dot{\varepsilon}. \tag{20}$$

where u is displacement in x-direction, v is the particle velocity and ', x' denotes the partial derivative respect to x. Now, the equation of motion for 1-D viscoelastic medium is given by

$$\rho\dot{u} = \sigma_{,x}, \tag{21}$$

$$\dot{\sigma} = \dot{r} * v_{,x}. \tag{22}$$

To eliminate the convolution term in Eq. (22), memory variables are defined (Carcione, 1993). Differentiating equation (1) with respect to t

$$\dot{\sigma}(t) = \left(\left\{M_r + \sum_{l=1}^{L} M_l e^{-t/\tau_{\sigma l}}\right\}\delta(t) + \sum_{l=1}^{L}\frac{M_l}{\tau_{\sigma l}}e^{-t/\tau_{\sigma l}}H(t)\right) * v_{,x}. \tag{23}$$

By definition of the L-memory variable we have

$$m_l = \left[\frac{M_l}{\tau_{\sigma l}}e^{-t/\tau_{\sigma l}}H(t)\right] * v_{,x} \tag{24}$$

In which case equation Eq.(23) reduces to

$$\dot{\sigma}(t) = M_u v_{,x} + \sum_{l=1}^{L} m_l. \tag{25}$$

The only convolutional term that left is in equation (24). To remove that we take the time derivative and obtain

$$\dot{m}_l = \left[\frac{M_l}{\tau_{\sigma l}} e^{-t/\tau_{\sigma l}} \delta(t) - \frac{1}{\tau_{\sigma l}} \left\{ \frac{M_l}{\tau_{\sigma l}} e^{-t/\tau_{\sigma l}} \right\} H(t) \right] * v_{,x}, \tag{26}$$

finding that the memory variables satisfy in a first order differential equation

$$\dot{m}_l = \frac{m_l}{\tau_{\sigma l}} + \frac{M_l}{\tau_{\sigma l}} v_{,x}. \tag{27}$$

Equations (21),(25), and (27) comprise a set of $2+L$ equations, referred to 1-D viscoelastic wave propagation in a medium with L sets of standard linear solids.

VISCOELASTIC SCATTERING AMPLITUDE

Allowing for inhomogeneity, there are three types of waves that propagate in a viscoelastic medium. P and SI waves with elliptical motion in the plane defined by propagation and attenuation directions, and SII with linear polarization perpendicular to that plane. If the two half-space medium are very similar we can define the linearized reflectivity functions in terms of changes in density, velocities and quality factors. The fractional perturbation in property τ is defined as

$$A_\tau = \frac{\Delta \tau}{\bar{\tau}}, \tag{28}$$

where $\tau = \rho, \alpha, \beta, Q_p, Q_s$ and

$$\Delta \tau = \tau_2 - \tau_1, \tag{29}$$

and

$$\bar{\rho} = \frac{\tau_2 + \tau_1}{2}. \tag{30}$$

Frequency independent scattering potential for scattering of P-wave to P-wave is given by (Moradi and Innanen, 2013)

$$\begin{aligned} {}^P_P\mathbb{V}_{visco} = &\left({}^P_P\mathbb{V}^\alpha_e\right) A_\alpha + \left({}^P_P\mathbb{V}^\rho_e + i {}^P_P\mathbb{V}^\rho_{ane}\right) A_\rho + \left({}^P_P\mathbb{V}^\beta_e + i {}^P_P\mathbb{V}^\beta_{ane}\right) A_\beta \\ &+ i \left({}^P_P\mathbb{V}^{Q_{hs}}_{ane}\right) A_{Q_s} + i \left({}^P_P\mathbb{V}^{Q_p}_{ane}\right) A_{Q_p}. \end{aligned} \tag{31}$$

We can see that, is a complex function. The term corresponds to the P-wave velocity is real and terms related to S-wave and density are complex. In addition contributions for perturbation in quality factors of P- and S-wave are pure imaginary. Scattering potentials for P to SI and SI to SI are give by

$${}^P_{SI}\mathbb{V}_{visco} = \left({}^P_{SI}\mathbb{V}^\rho_e + i {}^P_{SI}\mathbb{V}^\rho_{ane}\right) A_\rho + \left({}^P_{SI}\mathbb{V}^\beta_e + i {}^P_{SI}\mathbb{V}^\beta_{ane}\right) A_\beta + i \left({}^P_{SI}\mathbb{V}^{Q_{hs}}_{ane}\right) A_{Q_s}, \tag{32}$$

$${}^{SI}_{SI}\mathbb{V}_{visco} = \left({}^{SI}_{SI}\mathbb{V}^\rho_e + i {}^{SI}_{SI}\mathbb{V}^\rho_{ane}\right) A_\rho + \left({}^{SI}_{SI}\mathbb{V}^\beta_e + i {}^{SI}_{SI}\mathbb{V}^\beta_{ane}\right) A_\beta + i \left({}^{SI}_{SI}\mathbb{V}^{Q_{hs}}_{ane}\right) A_{Q_s}. \tag{33}$$

It can be seen from (32) and (33) that only relative differences in density, S-wave velocity and it's quality factor influence the scattered waves. On the other hand fractional perturbations in P-wave velocity and it's quality factor has no contributions in these cases. In the next section we numerically examine the effects of changing in elastic and anelastic properties of medium on the scattered wave.

NUMERICAL IMPLEMENTATION

In two dimensions we have $8 + 7L$ dynamic variables; three stress values $\sigma_{xx}, \sigma_{xy}, \sigma_{yy}$, and corresponding $3L$ memory variables $m_{xxl}; m_{yyl}; m_{xyl}$, $4L$ relaxation times, $\tau^p_{\varepsilon l}; \tau^p_{\sigma l}; \tau^s_{\varepsilon l}; \tau^s_{\sigma l}$, two components of particle velocity $v_x; v_y$ and three material parameters $\mu; \pi; \rho$. Where π is the relaxation modulus corresponding to P-wave analogues to $\lambda + 2\mu$ in the elastic case where λ and μ are Lame parameters. Stress, memory variables and velocity are the wave variables, and relaxation times and material parameters define the and make-up of the viscoelastic medium.

Figure 2 is a simple two layer model that uses 900×200 grid with spacing $D_x = D_y = 5m$. We put the layer boundary at the depth of 400m to set up the contrasts in elastic and anelastic properties of medium. In addition, we buried the source in depth 50m by injecting a vertical displacement wavelet with a central frequency of 45Hz. We expect not only to see P-to-P modes, but to record the SI-to-P and SI-to-SI modes also of the surface effects.

Contributions of perturbations in elasic and anelastic properties to the scattered waves are numerically examined as follows. First the viscoelastic code is run with the perturbations in density, velocities and quality factors on a homogeneous background model in place. Second, we isolate the upgoing scattered wave field by re-running the code without the perturbations and subtracting the resulting direct wave field. The results are displayed in Figures 3 to 7.

Figures 2, 3 and 4 illustrate the reflections caused by $\Delta\rho$, $\Delta\alpha$ and $\Delta\beta$. Similar to the elastic medium changing in the P-wave velocity produced only PP scattered wave, which is expected as we have one term in P-to-P scattering potential (Eq.31). However, perturbation in density and S-velocity generate all modes (Figures 4).

According to Eq.(16), in order to simulate the contrast in Q, we changed the corresponding relaxation times of stress and strain for P- and S-waves, and left all other properties constant. Figure 6 displays the scattering of P-to-P wave for contrast in Q_p. As seen, there is one reflection which corresponds to only one perturbation term in Q_p in scattering potential. Figure 7 shows the influence of perturbation in Qs on scattered waves. Demonstrate the agreement with a plot. Perturbation in Q_p only influence the P-to-P mode according to Eq. (31). However contribution of perturbation in Q_s exists in all modes in Eq.(31) to (33).

SUMMARY AND FUTURE DIRECTION

In summary, the numerical analysis of scattering in viscoelastic medium in the context of Born approximation is investigated. Scattering potential in the presence of anelasticity has been studied in (Moradi and Innanen, 2013). There are two important features, first the perturbation in the quality factors of P- and S-waves has contribution in scattering potential. Second, scattering potential is a complex function in which the real part is elastic scattering potential and imaginary part corresponds to anelasticity in medium. In this report numerically we examine the first feature. We show that perturbation in quality factor for P-wave between two layers generate only P-to-P reflection. The latter result is concordance with the viscoelastic scattering potential, which indicate that there is one term due to perturbation in Q_p.

The consistency of our theoretical/scattering treatment with the numerical results obtained by an independent modeling code (based on the framework of Carcione) is a significant step towards the development of several processing and inversion applications for data with nonneglible P and S wave attenuation. These in include standard Q estimation techniques, but also viscoelastic extensions of land seismic reflection full waveform inversion.

Additionally, we can investigate the second feature of scattering amplitude, which is related to complex terms induced by anelasticity. From equation (31) to (33) we can see that scattering potential elements corresponds to perturbation in dencity and S-velocity have imaginary parts. So comparing to the elastic case we expect the changes not only in amplitude of scattered wave but also in the phase behaviour.

ACKNOWLEDGMENTS

We thank the sponsors of CREWES for their support. We also gratefully acknowledge support from NSERC (Natural Science and Engineering Research Council of Canada) through the grant CRDPJ 379744-08.

APPENDIXE: 3-D VISCOELASTIC MEDIUM

The constitutive equation for a 2-D(or 3D) linear isotropic homogeneous viscoelastic medium is given by

$$\sigma_{ij} = \dot{\Lambda} * \delta_{ij}\varepsilon_{kk} + 2\dot{M} * \varepsilon_{ij}, \tag{34}$$

Time derivative of the strain tensor can be written as

$$\dot{\varepsilon}_{ij} = \frac{1}{2}(\partial_i v_j + \partial_j v_i). \tag{35}$$

Where v is particle velocity. For a standard linear model of viscoelastic medium we can define

$$\Pi = \Lambda + 2M = \pi\Gamma^p(t)H(t), \tag{36}$$

and

$$M = \mu\Gamma^s(t)H(t), \tag{37}$$

where $\tau^p_{\varepsilon l}$ is relaxation time of strain for P-wave, $\tau^s_{\varepsilon l}$ is relaxation time of strain for S-wave and $\tau_{\sigma l}$ is relaxation time of stress for both P- and S-wave. In addition we define

$$\Gamma^k(t) = 1 - \sum_{l=1}^{L}\left(1 - \frac{\tau^k_{\varepsilon l}}{\tau_{\sigma l}}\right)e^{-t/\tau_{\sigma l}}, \qquad k = p, s \tag{38}$$

After some algebra we arrive at

$$\dot{\sigma}_{ij} = (\pi\Gamma^p_0 - 2\mu\Gamma^s_0)\,\partial_k v_k + 2\mu\Gamma^s_0\partial_i v_j + \sum_{l=1}^{L} m_{ijl}, \qquad i = j \tag{39}$$

and

$$\dot{\sigma}_{ij} = \mu\Gamma^s_0(\partial_i v_j + \partial_j v_i) + \sum_{l=1}^{L} r_{ijl}, \qquad i \neq j \tag{40}$$

Where $\Gamma_0 = \Gamma(t = 0)$ and m_{ijl} is a memory tensor for mechanism-l, which satisfies in the following differential equation

$$\dot{m}_{ijl} = -\frac{1}{\tau_{\sigma l}}\left[m_{ijl} + \left\{\pi\left(\frac{\tau^p_{\varepsilon l}}{\tau_{\sigma l}}\right) - 2\mu\left(\frac{\tau^s_{\varepsilon l}}{\tau_{\sigma l}}\right)\right\}\partial_k v_k + 2\mu\left(\frac{\tau^s_{\varepsilon l}}{\tau_{\sigma l}}\right)\partial_i v_j\right], \quad i = j \tag{41}$$

for diagonal terms, and

$$\dot{m}_{ijl} = -\frac{1}{\tau_{\sigma l}}\left[r_{ijl} + \left(\frac{\tau^s_{\varepsilon l}}{\tau_{\sigma l}}\right)(\partial_i v_j + \partial_j v_i)\right], \quad i \neq j \tag{42}$$

The linearized equation for wave propagation in absence of body forces is given by

$$\rho\ddot{u}_i = \sigma_{ij,j} \quad i = x, y. \tag{43}$$

Where $, j$ is spacial derivative, ρ is density, u denotes the displacement and σ refers to the stress.

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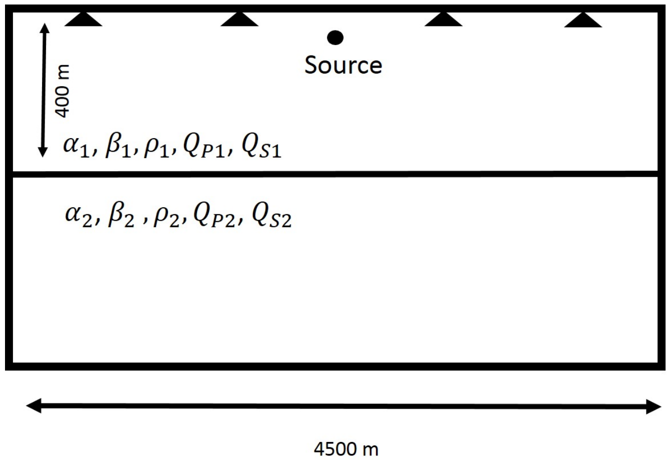

FIG. 2. Model description of two layer viscoelastic medium.

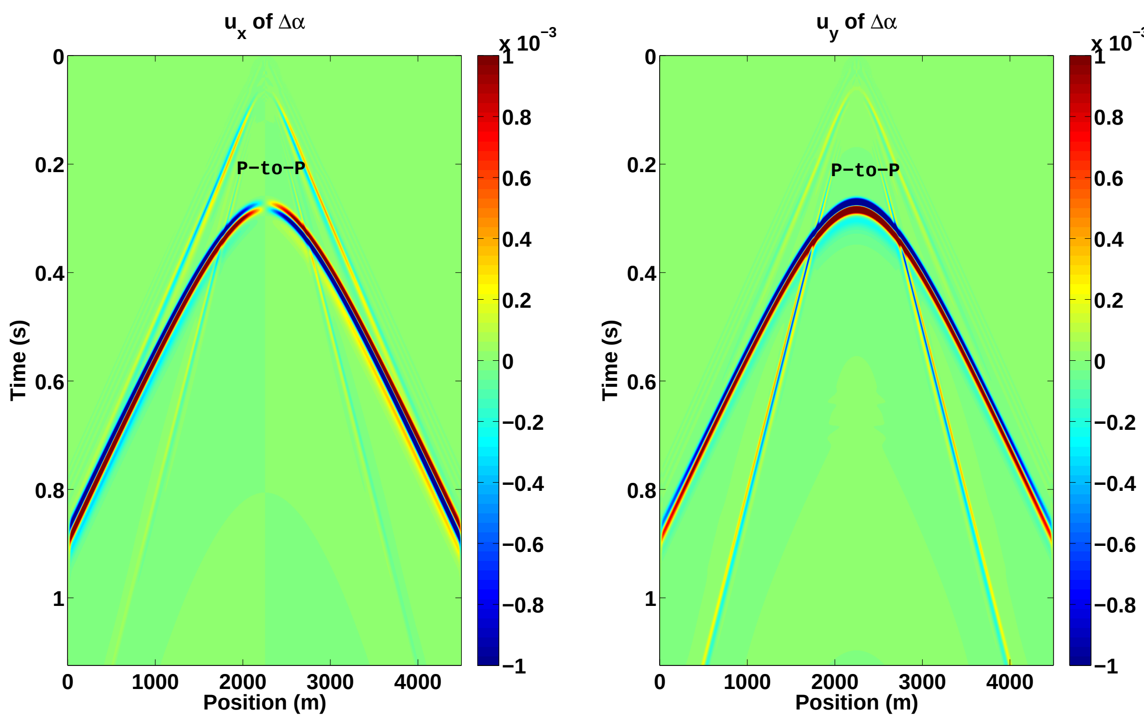

FIG. 3. Simulated seismic data corresponding to the contrast in P-wave velocity α. The left figure is the x-component of displacement and right is the y-component of displacement.

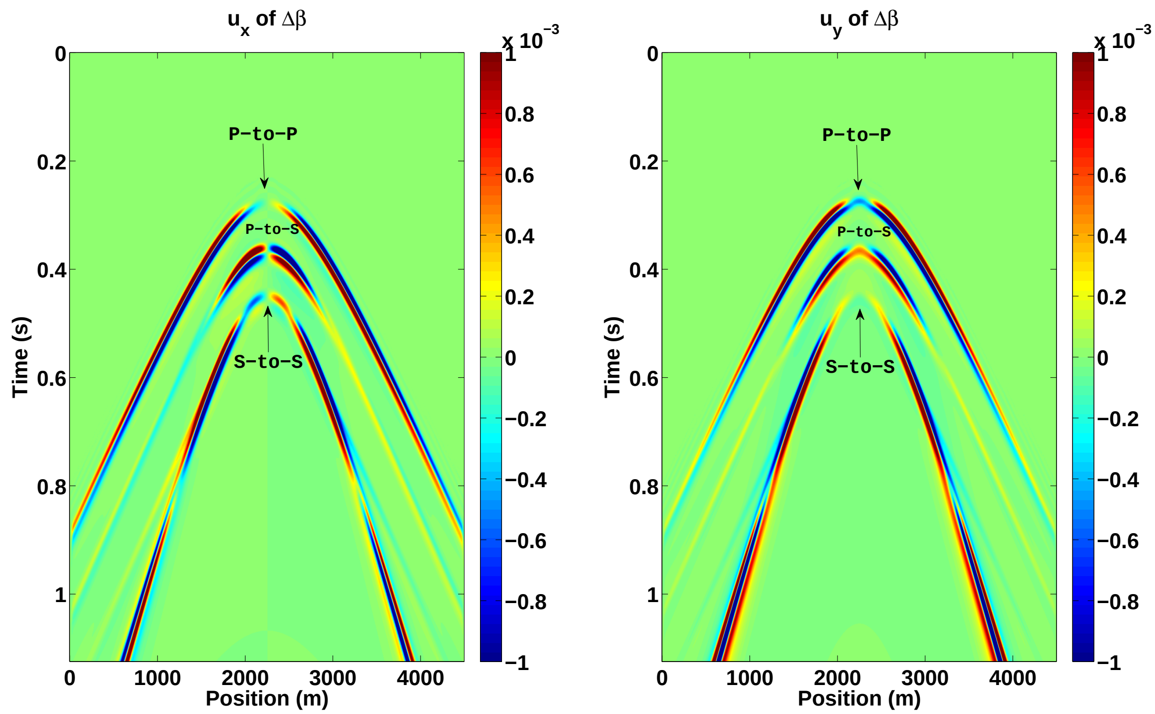

FIG. 4. Simulated seismic data corresponding to the contrast in S-wave velocity β. The left figure is the x-component of displacement and right is the y-component of displacement.

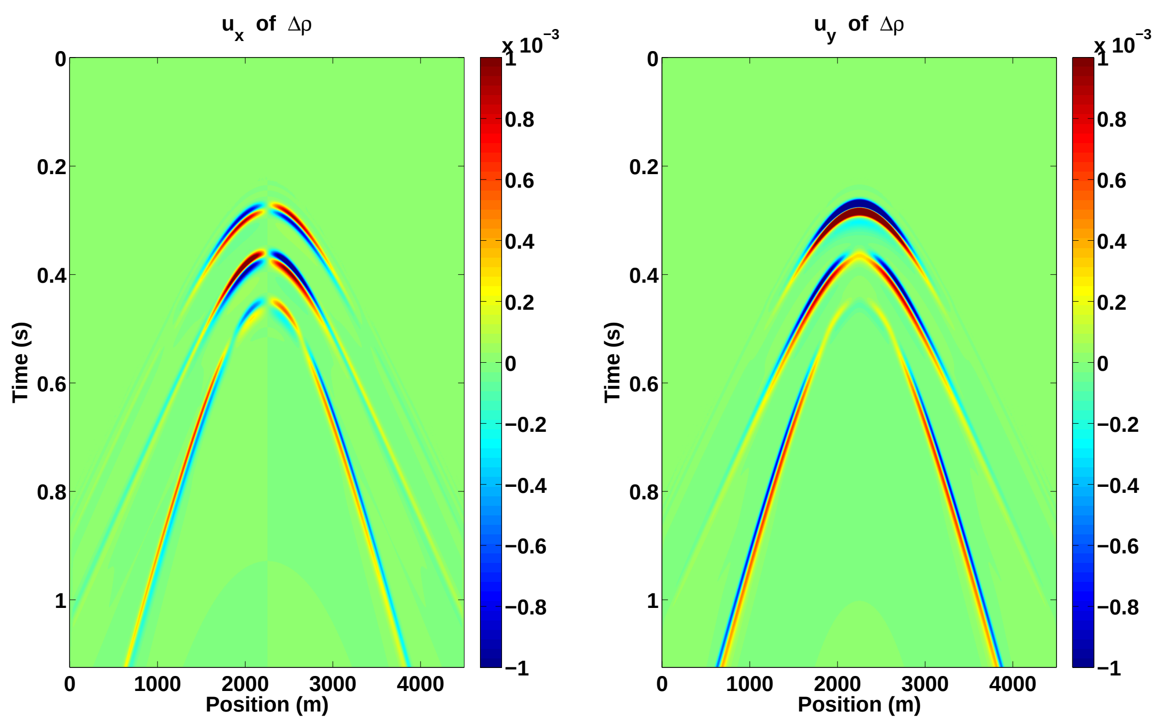

FIG. 5. Simulated seismic data corresponding to the contrast in density ρ.The left figure is the x-component of displacement and right is the y-component of displacement.

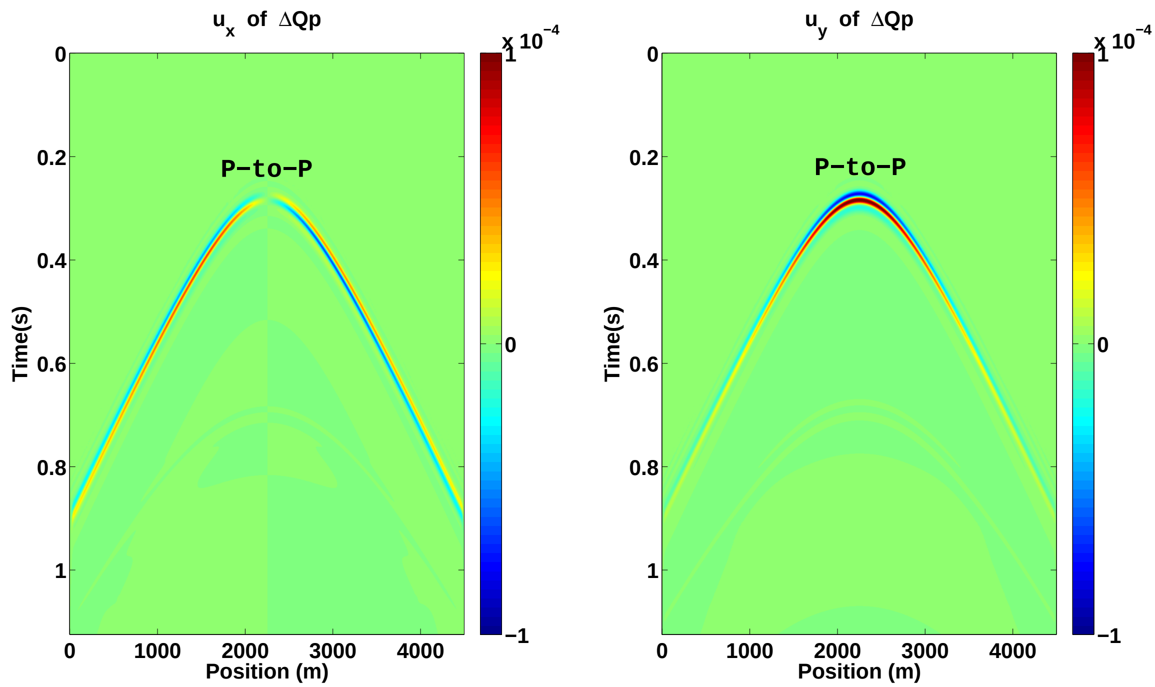

FIG. 6. Simulated seismic data corresponding to the contrast in quality factor for P-wave velocity Q_p. The left figure is the x-component of displacement and right is the y-component of displacement.

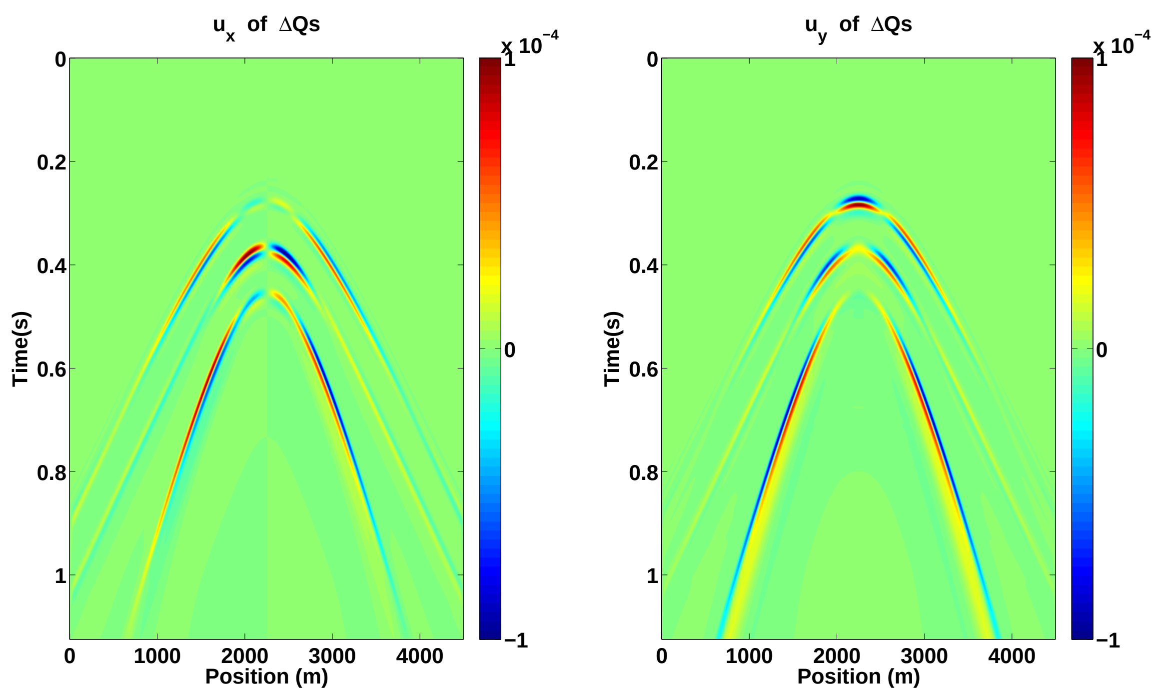

FIG. 7. Simulated seismic data corresponding to the contrast in quality factor for S-wave velocity Q_s. The left figure is the x-component of displacement and right is the y-component of displacement.