1.5D Internal multiple prediction in the plane wave domain

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ABSTRACT

The inverse scattering series internal multiple attenuation algorithm was developed by Weglein and collaborators in the 1990s, based on the fact that internal multiples could be constructed from subevents satisfying the lower-higher-lower relationship in the pseudo-depth domain. Innanen (2012) implemented a 1.5D version of the algorithm in MATLAB and complemented the 1D version of the algorithm studied by Hernandez and Innanen (2012). We present a 1.5D MATLAB implementation of this algorithm in plane wave domain, as initially formulated by Coates and Weglein (1996) and analyzed by Nita and Weglein (2009). Compared to the application in the wavenumber-pseudo depth domain, we generate improved numerical accuracy and reduced Fourier artifacts from internal multiple prediction in the plane wave domain. Furthermore, the procedure for generating the input data is greatly simplified. We believe the plane wave domain is a promising way of posing full 2D versions of the algorithm.

INTRODUCTION

Multiple identification and removal is essential and plays an important role in seismic data processing because most imaging algorithms (some based on the Born approximation) deal correctly only with primary reflections. Therefore, the quality and completeness of multiple attenuation will directly affect the accuracy of seismic imaging and subsequent interpretation. The inverse scattering series internal multiple attenuation algorithm leverages the fact that an internal multiple can be predicted from a combination of amplitudes and arrival times of primary reflection subevents which satisfy a certain lower-higher-lower relationship in pseudo-depth. More importantly, during the processing the primaries will remain intact, and there is no seismic information from subsurface required: the algorithm is fully data-driven.

In this paper, we review the basic features of the inverse scattering series internal multiple attenuation algorithm, which was introduced to the geophysical industry in the 1990s (Araújo et al., 1994; Weglein et al., 1997, 1998, 2003). And the connection between pseudo-depth and intercept time, which transfers the 1.5D inverse scattering series internal multiple attenuation algorithm from the wavenumber-pseudo depth domain to the plane-wave domain, was presented by Nita and Weglein (2009). The purpose of this paper is to demonstrate the application of the internal multiple attenuation algorithm in the plane wave domain to 1.5D data using a MATLAB implementation. Our plan forward is to explore the field application of this plane wave domain algorithm, making use of the fact that in tau-p no resampling procedure is required, and implement a full 2D version as well.

We begin with a review of the algorithm both in the wavenumber-pseudo depth domain and the plane wave domain, then describe how to generate the inputs and how these are implemented in the plane wave domain using MATLAB codes. The implementation of the algorithm in the plane wave domain is then tested on a simple synthetic data set.

THEORY

Transforming the input data to the pseudo-depth domain

The procedure for generating the input data to the inverse scattering series internal multiple attenuation algorithm (Weglein et al., 1997) was reviewed in the context of the MATLAB implementation by Innanen (2012) and further analyzed by Pan and Innanen (2013). We begin with a data set measured over intervals in lateral source location x_s, lateral receiver location x_g, and two way travel time t. First, the Fourier transform are applied to the data over all three of these coordinates:

$$d(x_g, x_s, t) \rightarrow D(k_g, k_s, \omega). \tag{1}$$

And a change of variables is made from ω to k_z by resampling:

$$D(k_g, k_s, \omega) \rightarrow D(k_g, k_s, k_z). \tag{2}$$

where $k_z = q_g + q_s$ and

$$q_g = \frac{\omega}{c_0}\sqrt{1 - \frac{{k_g}^2 {c_0}^2}{\omega^2}}, \quad q_s = \frac{\omega}{c_0}\sqrt{1 - \frac{{k_s}^2 {c_0}^2}{\omega^2}}. \tag{3}$$

Then data are scaled by $-2iq_s$, forming

$$B_1\left(k_g, k_s, k_z\right) = (-2iq_s)D(k_g, k_s, k_z), \tag{4}$$

Finally, the quantity input $b_1(k_g, k_s, z)$ in the wavenumber-pseudo depth domain is generated by an inverse Fourier transform over k_z:

$$B_1\left(k_g, k_s, k_z\right) \rightarrow b_1(k_g, k_s, z)\ . \tag{5}$$

Internal multiple prediction algorithm in 2D (wavenumber-pseudo depth domain)

The prediction algorithm (Weglein et al., 1997) in its full 2D form is then, making use of the input generated in the previous section,

$$\begin{aligned} &b_{3_{IM}}\left(k_g, k_s, \omega\right) \\ &= \frac{1}{(2\pi)^2}\iint_{-\infty}^{+\infty} dk_1 e^{-iq_1(\varepsilon_g - \varepsilon_s)} dk_2 e^{-iq_2(\varepsilon_g - \varepsilon_s)} \int_{-\infty}^{+\infty} dz e^{i(q_g + q_1)z} b_1\left(k_g, k_1, z\right) \\ &\times \int_{-\infty}^{z-\epsilon} dz' e^{-i(q_1+q_2)z'} b_1(k_1, k_2, z') \int_{z'+\epsilon}^{+\infty} dz'' e^{i(q_2+q_s)z''} b_1(k_2, k_s, z'') \end{aligned} \tag{6}$$

where

$$q_X = \frac{\omega}{c_0}\sqrt{1 - \frac{{k_X}^2 {c_0}^2}{\omega^2}}; \qquad k_z = q_g + q_s; \tag{7}$$

q_X , are vertical wave numbers associated with the carious lateral wave numbers and the reference velocity. z, z' and z'' are the pseudo-depth which satisfy the lower-higher-lower relationship. The left-hand side of equation (6) is inverse Fourier transformed over all three Fourier variables, and the result is added to the original data to attenuate the multiples, normally with an adaptive component to account for small phase and amplitude mismatches between the prediction and the actual multiples.

Internal multiple prediction in 1.5D (wavenumber-pseudo depth domain)

However, if the data have offset but the Earth is nearly layered, a 1.5D version of the algorithm can be considered, in which

$$k_g = k_s, \tag{8}$$

Then the 1.5D algorithm can be expressed as

$$b_{3_{IM}}(k_g,\ \omega) = \int_{-\infty}^{+\infty} dz e^{ik_z z} b_1(k_g,\ z) \int_{-\infty}^{z-\epsilon} dz' e^{-ik_z z'} b_1(k_g, z') \\ \times \int_{z'+\epsilon}^{+\infty} dz'' e^{ik_z z''} b_1(k_g, z'') \tag{9}$$

where $k_g = 2q_g$. Compared to the 2D algorithm, the computation cost has been dramatically reduced, with the equivalent of a single 1D prediction for every output k_g .

The connection between pseudo-depth and intercept time

Notice that for each planewave component of fixed k_g, k_s and ω we have (Nita and Weglein, 2009)

$$\omega\tau_a = k_z^{actual} z_a^{actual}, \tag{10}$$

where k_z^{actual} is the actual, velocity dependent, vertical wavenumber and z_a^{actual} is the actual depth of the turning point of the planewave. Since the velocity of the actual medium is assumed to be unknown, this relation is written in terms of the reference velocity as (Nita and Weglein, 2009)

$$\omega\tau_a = k_z z_a\ , \tag{11}$$

where k_z is the vertical wavenumber of the planewave in the reference medium and can be calculated by $k_z = q_g + q_s$, here, q_g, q_s are expressed as in equation (3).

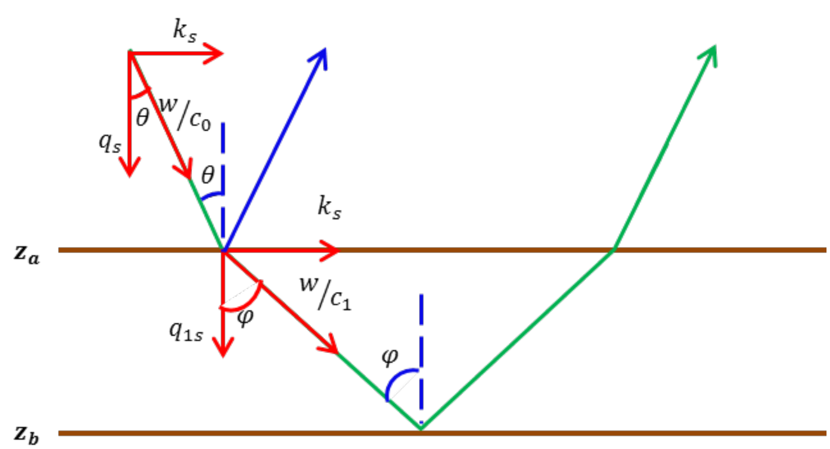

FIG. 1. Geometrical representation of the two primaries (Nita and Weglein, 2009).

Geometrical representations of primaries are shown in Figure 1. Let us take the downgoing wave as an example to explain the relationship between pseudo-depth and intercept time. For the first primary, the blue event shown in Figure 1, the vertical wavenumber of downgoing wave can be written as

$$q_s = \frac{\omega}{c_0} cos\theta \qquad (12)$$

The two way intercept time τ_a of first primary can be thought of as a sum of the vertical travel time of downgoing wave τ_a^d and the vertical travel time of upgoing wave τ_a^u: $\tau_a = \tau_a^d + \tau_a^u$. Here, the vertical travel time of downgoing wave τ_a^d can be expressed as

$$\tau_a^d = \frac{z_a}{c_0/cos\theta} = \frac{z_a cos\theta}{c_0} \qquad (13)$$

Then we have

$$q_s z_a = \frac{\omega cos\theta}{c_0} * z_a = \frac{z_a cos\theta}{c_0} * \omega = \omega\tau_a^d \qquad (14)$$

Similarly to the vertical wavenumber and the vertical travel time of upgoing wave,

$$q_g z_a = \omega\tau_a^u \qquad (15)$$

Summing the last equation (14) and (15) we find the relationship among wavenumber, intercept time and pseudo depth (compare with Eq. (11))

$$q_s z_a + q_g z_a = \omega\tau_a^d + \omega\tau_a^u \qquad (16)$$

which implies

$$k_z z_a = \omega\tau_a \qquad (17)$$

where k_z is the vertical wavenumber of the planewave in the reference medium and can be calculated by $k_z = q_g + q_s$. z_a is the pseudo-depth of the first primary we used in internal multiple prediction algorithm and is equal to the actual depth for the first primary. τ_a is the intercept time of the first primary and computed by $\tau_a = \tau_a^d + \tau_a^u$.

For the second primary, the green event shown in Figure 1, we can write the relationship among the vertical wavenumber, intercept time, and depth in second layer,

$$\gamma_1(z_b - z_a) = \omega(\tau_b - \tau_a) \qquad (18)$$

where $\gamma_1 = q_{1s} + q_{1g}$ is the vertical wavenumber in second layer, τ_b is the total vertical travel time of the second primary.

Summing Eq. (17) and Eq. (18), the relation between wavenumber and intercept time can be written as

$$k_z z_a + \gamma_1(z_b - z_a) = \omega\tau_a + \omega(\tau_b - \tau_a) = \omega\tau_b \qquad (19)$$

Since the velocity of the second layer is unknown, we can write $\omega\tau_b$ in terms of c_0 only as follows (Eq. (11))

$$k_z {z_b}' = \omega\tau_b \tag{20}$$

where ${z_b}'$ is a pseudo-depth of the second primary in terms of c_0 which can be calculated by the vertical travel time τ_b and the vertical speed of the first medium. This pseudo-depth is exactly the one we involved in the inverse scattering series internal multiple prediction algorithm.

Through equations (17) and (20), we find that the pseudo-depth of each event used in the internal multiple prediction algorithm, is determined by its intercept time and the constant reference velocity. The pseudo-depths of those events have the same monotonicity property condition and can be calculated by

$$z' = \frac{c_0\tau}{2} \tag{21}$$

where z' is the pseudo-depth of the event, τ is the corresponded two-way intercept time.

Internal multiple prediction in 1.5D (horizontal slowness-pseudo depth domain)

The inverse scattering series internal multiples attenuation algorithm was first introduced in wavenumber-pseudo depth domain (Araújo et al., 1994; Weglein et al., 1997). It was demonstrated that the algorithm still works even though we change the variable from wavenumber to horizontal slowness. The 1.5 D algorithm in the horizontal slowness and pseudo-depth domain was first introduced by Coates and Weglein (1996), and further discussed and analyzed by Nita and Weglein (2009), can be expressed as

$$\begin{aligned} b_{3_{IM}}(p_g,\, \omega) = \int_{-\infty}^{+\infty} dz e^{i2q_g z} b_1(p_g,\, z) \int_{-\infty}^{z-\epsilon} dz' e^{-i2q_g z'} b_1(p_g, z') \\ \times \int_{z'+\epsilon}^{+\infty} dz'' e^{i2q_g z''} b_1(p_g, z'') \end{aligned} \tag{22}$$

where $q_X = \frac{\omega}{c_0}\sqrt{1 - {p_X}^2 {c_0}^2}$, are vertical wave numbers associated with a given frequency, lateral wave number and the reference velocity. The quantities - z, z' and z'' are the pseudo-depths of events which on account of the integral limits are constrained to satisfy the lower-higher-lower relationship.

In this paper, we described how to construct the input data in horizontal slowness-pseudo depth domain according to the steps for generating the input data in wavenumber-pseudo depth domain. After that, the algorithm is applied to 1.5D synthetic data in horizontal slowness-pseudo depth using MATLAB code. An inverse Fourier transform and an inverse $\tau - p$ transform have to be carried out after processing with the algorithm complete. We begin with the data set measured over intervals in lateral source location x_s, lateral receiver location x_g, and two way travel time t.

First, a $\tau - p$ transform is applied over all three of these coordinates:

$$d(x_g, x_s, t) \to D(p_g, p_s, \tau). \tag{23}$$

Second, the data are Fourier transformed from τ to ω,

$$D(p_g, p_s, \tau) \to D_1(p_g, p_s, \omega). \tag{24}$$

Third, a change of variables is made by resampling from ω to k_z (where $k_z = q_s + q_g$),

$$D_1(p_g, p_s, \omega) \to D_1(p_g, p_s, k_z). \tag{25}$$

Fourth, the data are scaled by$-2iq_s$, forming

$$B_1(p_g, p_s, k_z) = (-2iq_s)D_1(p_g, p_s, k_z), \tag{26}$$

Finally, the quantity input $b_1(p_g, p_s, z)$ in the horizontal slowness-pseudo depth domain is generated by an inverse Fourier transform over k_z:

$$B_1(p_g, p_s, k_z) \to b_1(p_g, p_s, z)\ . \tag{27}$$

Internal multiple prediction in 1.5D (plane wave domain)

Coates and Weglein (1996) proposed a plane wave version of the original algorithm (Araújo et al., 1994; Weglein et al., 1994). In it, the internal multiple attenuation term may be written explicitly in the source and receiver slowness domain as

$$b_{3_{IM}}(p_g, \omega) = \int_{-\infty}^{+\infty} d\tau e^{i\omega\tau} b_1(p_g,\ \tau) \int_{-\infty}^{\tau-\epsilon} d\tau' e^{-i\omega\tau'} b_1(p_g, \tau') \times \int_{\tau'+\epsilon}^{+\infty} d\tau'' e^{i\omega\tau''} b_1(p_g, \tau'') \tag{28}$$

where p_g=p_s are the source and receiver horizontal slownessnes (which are equal for 1.5D problems) respectively. The time variables τ, τ' and τ'' are intercept time of three events satisfied lower-higher-lower relationship.

In this paper, we show how to construct the input to the algorithm and MATLAB codes are which apply internal multiple prediction algorithm to 1.5D synthetic data in plane wave domain. An inverse Fourier transform and a $\tau - p$ transform will be applied after the application the algorithm. We still begin with the data set $d(x_g, x_s, t)$ which is $\tau - p$ transformed over all three coordinates:

$$d(x_g, x_s, t) \to D(p_g, p_s, \tau). \tag{29}$$

Then, a Fourier transform are applied from τ to ω,

$$D(p_g, p_s, \tau) \to D_1(p_g, p_s, \omega). \tag{30}$$

Third, the data are scaled by $-2iq_s$,

$$B_1(p_g, p_s, \omega) = (-2iq_s)D_1(p_g, p_s, \omega). \tag{31}$$

Finally, an inverse Fourier transform are used to calculate the input data $b_1(p_g, p_s, z)$,

$$B_1(p_g, p_s, \omega) \rightarrow b_1(p_g, p_s, \tau). \tag{32}$$

Slant stack or $\tau - p$ transform

Radon transform is a mathematical technique and a very useful data processing tool due to its ability to decompose a seismic matrix to events of the constant horizontal slowness p and provide an increased separation between different seismic waves. It is also known as the $\tau - p$ transform or the slant-stack technique. A conventional linear $\tau - p$ transform (Zhou and Greenhalgh, 1994) can be defined as follows:

$$D(p, \tau) = \int_{-\infty}^{+\infty} d(x, \tau + px) dx. \tag{33}$$

where p is the horizontal slowness and $d(x, \tau + px)$ can be considered as a time shift of $d(x, \tau)$.

As we known, a static shift is equivalent to a linear phase shift. Then equation (33) for frequency domain can be written as

$$\widehat{D}(p, \omega) = \int_{-\infty}^{+\infty} \hat{d}(x, \omega) e^{i\omega px} dx \tag{34}$$

where $\hat{d}(x, \omega)$ and $\widehat{D}(p, \omega)$ are the Fourier transforms of $d(x, t)$ and $D(x, \tau)$, respectively.

In this paper, *tptran.m* and *itptran.m* from CREWES tool box are used for $\tau - p$ transform and inverse $\tau - p$ transform, respectively.

SYTHETIC EXAMPLE

Construction of the input data

In order to compare with the algorithm in wavenumber-pseudo depth domain, we make use of the simple two-interface model used by Innanen (2012) to test the internal multiple prediction in Figure 2a.

In Figure 2b a single shot record of data is illustrated. The raw seismic data is created using the CREWES acoustic finite difference function *afd_shotrec.m*, with all four boundaries set as “absorbing” to suppress the creation of free-surface multiples (Innanen, 2012). The zero offset travel times of two primaries are indicated in yellow and two internal multiples zero offset travel times are indicated in red. Our objective is to use the primaries as sub events to predict these two internal multiples at all offsets.

In Figure 3a the input data is illustrated for the horizontal slowness-pseudo depth algorithm using the procedure discussed as Eq. (23)-(27). For the plane wave domain algorithm, the input data are generated using the steps stated as Eq. (29)-(32). Both of them are applied to test the algorithm in the horizontal slowness-pseudo depth domain and the plane wave domain, respectively.

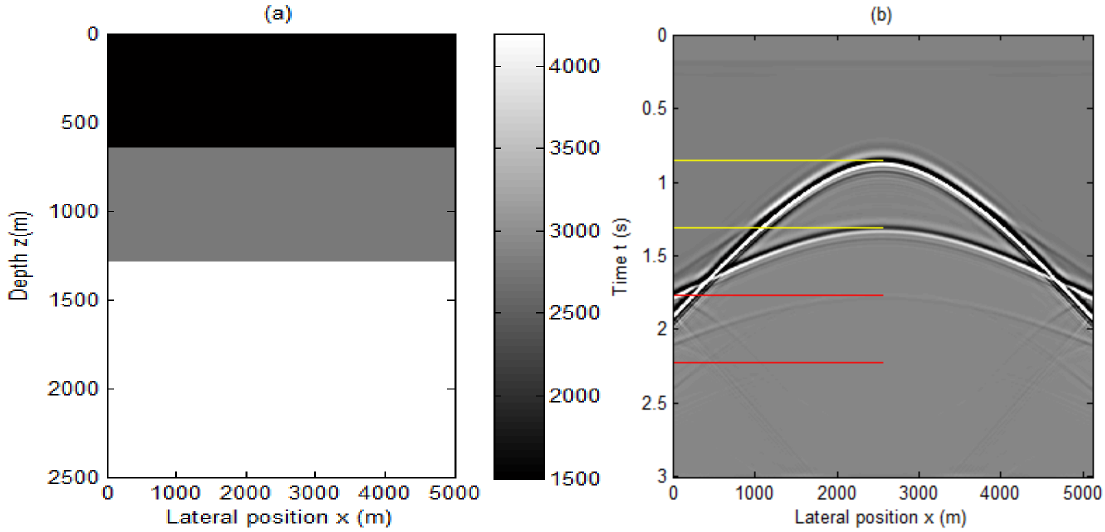

FIG. 2. Synthetic model and data acquired to test 1.5D internal multiple prediction algorithm. (a) Layered two interfaces model to generate the synthetic data. (b) Synthetic data calculated using synthetic model in Figure 2a. The CREWES code *afd_shotrec.m* was applied to create the raw synthetic data. Primary zero offset travel times are indicated in red and multiple zero offset travel times are indicated in yellow.

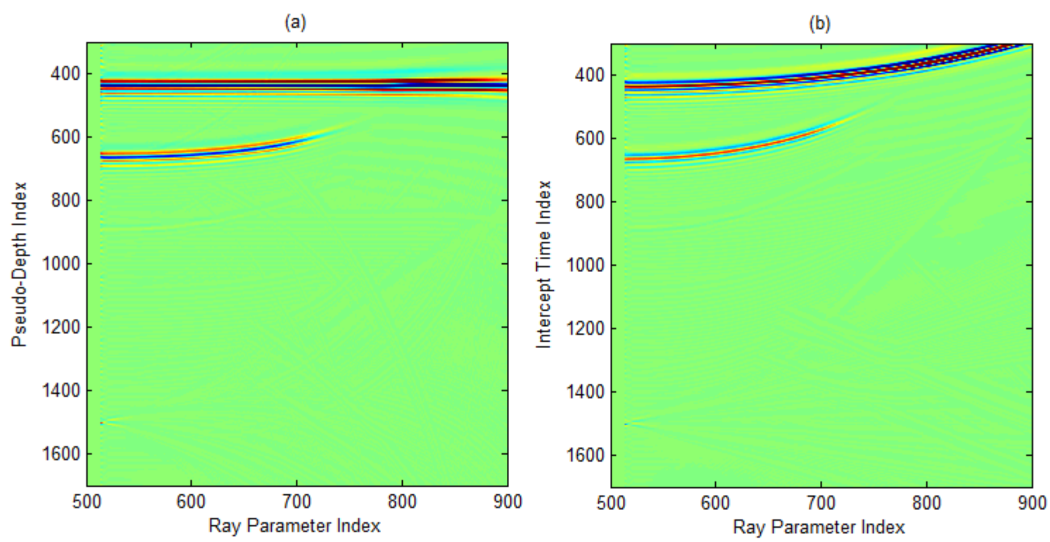

FIG. 3. The algorithm Input data is generated using procedure we discussed above. (a) The input data is generated using procedure discussed in horizontal slowness- pseudo depth domain corresponding to Eq. (23)-(27). (b) The input data is generated using procedure discussed in plane wave domain corresponding to Eq. (29)-(32).

In Figure 4, we stack the input data over horizontal slowness we created in Figure 3 and compare them to the raw zero offset seismic data. Figure 4a shows the original zero offset seismic data collected from the raw seismic data. Figure 4b and Figure 4c indicate the input data, drawn in Figure 3a and Figure 3b, are stacked over horizontal slowness respectively.

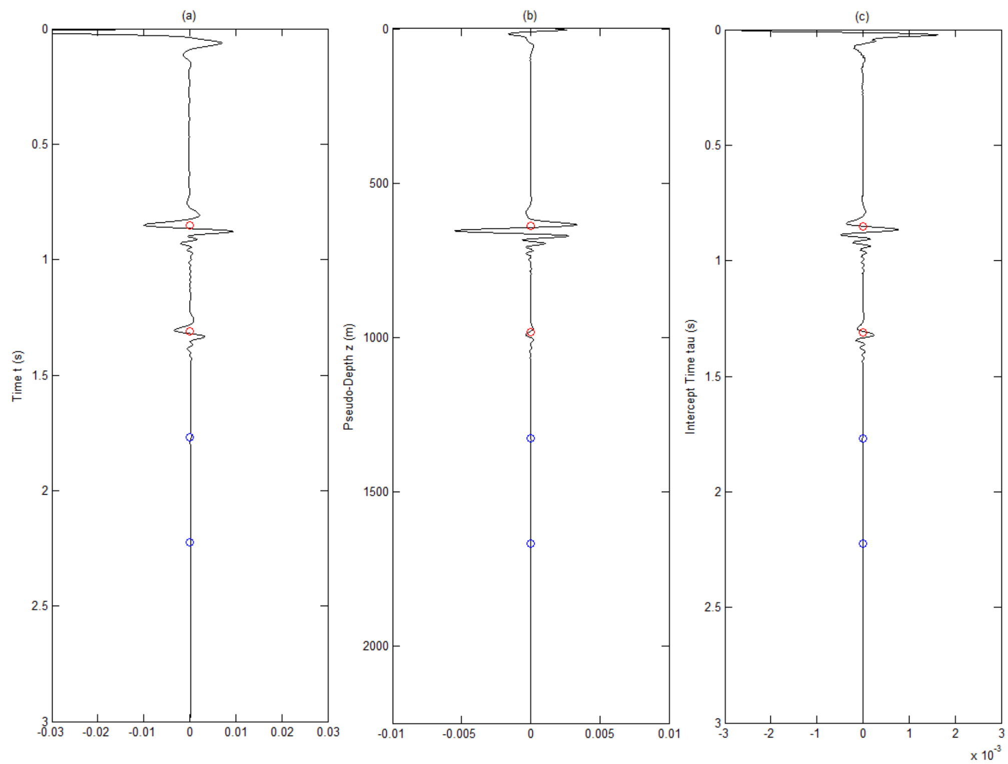

FIG. 4. Raw recorded data and the input data at zero-offset. (a) The raw data we created with the synthetic model at zero-offset. (b) The stacked input data over horizontal slowness in horizontal slowness-pseudo depth domain. (c) The stacked input data over horizontal slowness in plane wave domain.

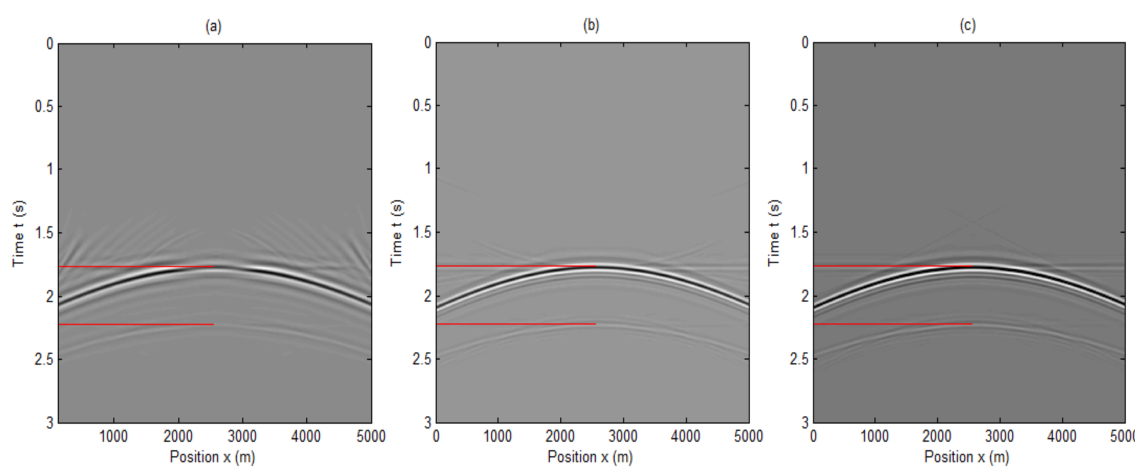

FIG. 5. The internal multiple predictions are generated using the inverse scattering series internal multiple prediction algorithm in different domains. (a) The result is predicted using the algorithm (Eq. 9) in wavenumber-pseudo depth domain (Innanen, 2012). (b) The prediction is generated using the algorithm (Eq. 22) in horizontal slowness- pseudo depth domain. (c) The outcome is created using the algorithm (Eq.28) in plane wave domain.

Predicting internal multiples

We finally input the constructed b_1 matrix into the algorithm. The results, after a matched processing, are displayed in Figure 5. In Figure 5a the prediction, after a 2D inverse Fourier transform, is displayed with a range of visible artifacts and the zeros offset travel time are indicated in red. For horizontal slowness-pseudo depth domain the final results, which can be calculated by an inverse Fourier transform and a followed $\tau - p$ transform, are shown in Figure 5b. It indicates that better predictions can be achieved in horizontal slowness-pseudo depth domain rather than wavenumber-pseudo depth domain. In Figure 5c the internal multiple prediction is displayed with better focusing and accuracy, less artifacts.

CONCLUSIONS

We implement a 1.5D version of the inverse scattering series internal multiple attenuation algorithm in the plane wave domain. This algorithm does not need any subsurface information and is suitable for the situation of primaries and internal multiples mixed together. Free-surface multiples have to be removed before the application of the method. Compared to the wavenumber-pseudo depth algorithm, the computation cost of the algorithm in plane wave domain has been dramatically reduced. A better focusing prediction can be achieved with the internal multiple attenuation pane wave algorithm. Another advantage of the internal multiples prediction algorithm in plane wave domain is that free-surface multiples suppression with a predictive deconvolution often works better in plane wave domain. Therefore, the study of 1.5D inverse scattering series internal multiple attenuation algorithm in plane wave domain strongly suggestive that 2D and ultimately 3D implementations in plane wave domain will bring significant technological value to the critical matter of internal /interbed multiple identification and suppression.

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