A pot of porridge and a tutorial
or
there's a shark in my transform

John C. Bancroft

CREWES/University of Calgary

Acknowledgements

- CREWES sponsors
- NSERC
- Mark Ng of GEO-X

Synopsis

- *Imaging and inversion*
  - Continued fraction expansion *Bancroft*
  - Practical implementations of equivalent offset processing *Bancroft*
  - Tutorial on downward extrapolation operators *Bancroft*
  - 2-D wave equation modeling and migration by a new finite difference scheme based on the Galerkin method *Du*
  - Modeling and migration by a new finite difference scheme based on the Galerkin method for irregular grids *Du*
  - Poststack and prestack depth migrations using Hale's extrapolator *Al-Saleh*
  - Stability and accuracy analysis of the space-frequency domain wavefield extrapolators *Liu*

Synopsis

- Anisotropy
  - Anisotropic velocity modeling *Elapavuluri*
  - Seismic modeling in structurally complex anisotropic media *(2nd author)* VTI media *Elapavuluri*
  - Characteristics of P-, SV- and SH-wave propagation in a weakly anisotropic medium *Kelter*
  - Estimation of Thomsen's anisotropy parameters in layered VTI media *Xiao*
- Noise Reduction
  - Multiple Attenuation by Semblance Weighted Radon Transform *Cao*
- Computing
  - Multigrid deconvolution of seismic data *Millar*
  - Solving surface consistent statics with multigrid *Millar*

Multigrid inversion *John Millar*

$$\mathbf{W}\mathbf{r} = \mathbf{s}$$

- **W** and **s** known
- Reduce to smallest size
- Antialias filter each reduction
- Save

W	**s**
31×31	31
15×15	15
7×7	7
3×3	3
1	1

Average value

Multigrid inversion *John Millar*

Gauss-Seidel $\mathbf{r} \approx \widehat{\mathbf{W}}^{-1}\mathbf{s}$

- Estimate $\mathbf{r}_1$
- Interpolate $\mathbf{r}_3$
- Improved estimate of $\mathbf{r}_3$
- Interpolate $\mathbf{r}_7$
- Improved estimate of $\mathbf{r}_7$
- Interpolate $\mathbf{r}_{15}$
- Improved estimate of $\mathbf{r}_{15}$
- Interpolate $\mathbf{r}_{31}$
- Improved estimate of $\mathbf{r}_{31}$

$\mathbf{r}_n$	$\mathbf{W}$	$\mathbf{s}$	$\mathbf{r}_{n+1}$
1	1	1	1
3	3×3	3	3
7	7×7	7	7
15	15×15	15	15
31	31×31	31	31

Multigrid inversion *John Millar*

Gauss-Seidel $\mathbf{r} \approx \widehat{\mathbf{W}}^{-1}\mathbf{s}$

- Estimate $\mathbf{r}_1$
- Interpolate $\mathbf{r}_3$
- Improved estimate of $\mathbf{r}_3$
- Interpolate $\mathbf{r}_7$
- Improved estimate of $\mathbf{r}_7$
- Interpolate $\mathbf{r}_{15}$
- Improved estimate of $\mathbf{r}_{15}$
- Interpolate $\mathbf{r}_{31}$
- Improved estimate of $\mathbf{r}_{31}$

$\mathbf{r}_n$	$\mathbf{W}$	$\mathbf{s}$	$\mathbf{r}_{n+1}$
1	1	1	1
3	3×3	3	3
7	7×7	7	7
15	15×15	15	15
31	31×31	31	31

Multigrid inversion *John Millar*

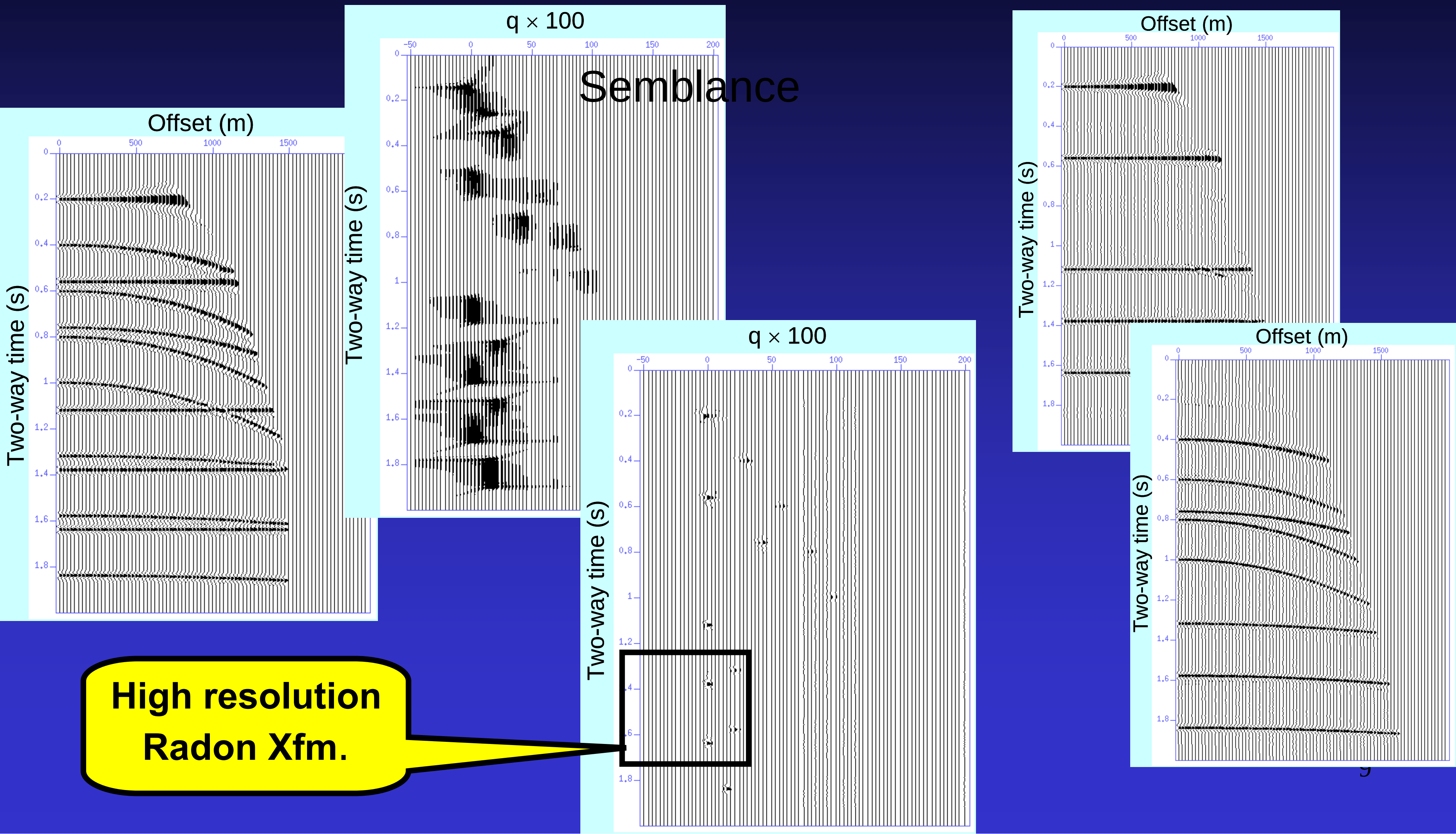
Gauss-Seidel $\mathbf{r}$ » $\widehat{\mathbf{W}}^{-1}\mathbf{s}$

$\mathbf{r}_n$	$\mathbf{W}$	$\mathbf{s}$	$\mathbf{r}_{n+1}$
1	1	1	1
3	3×3	3	3
7	7×7	7	7
15	15×15	15	15
31	31×31	31	31

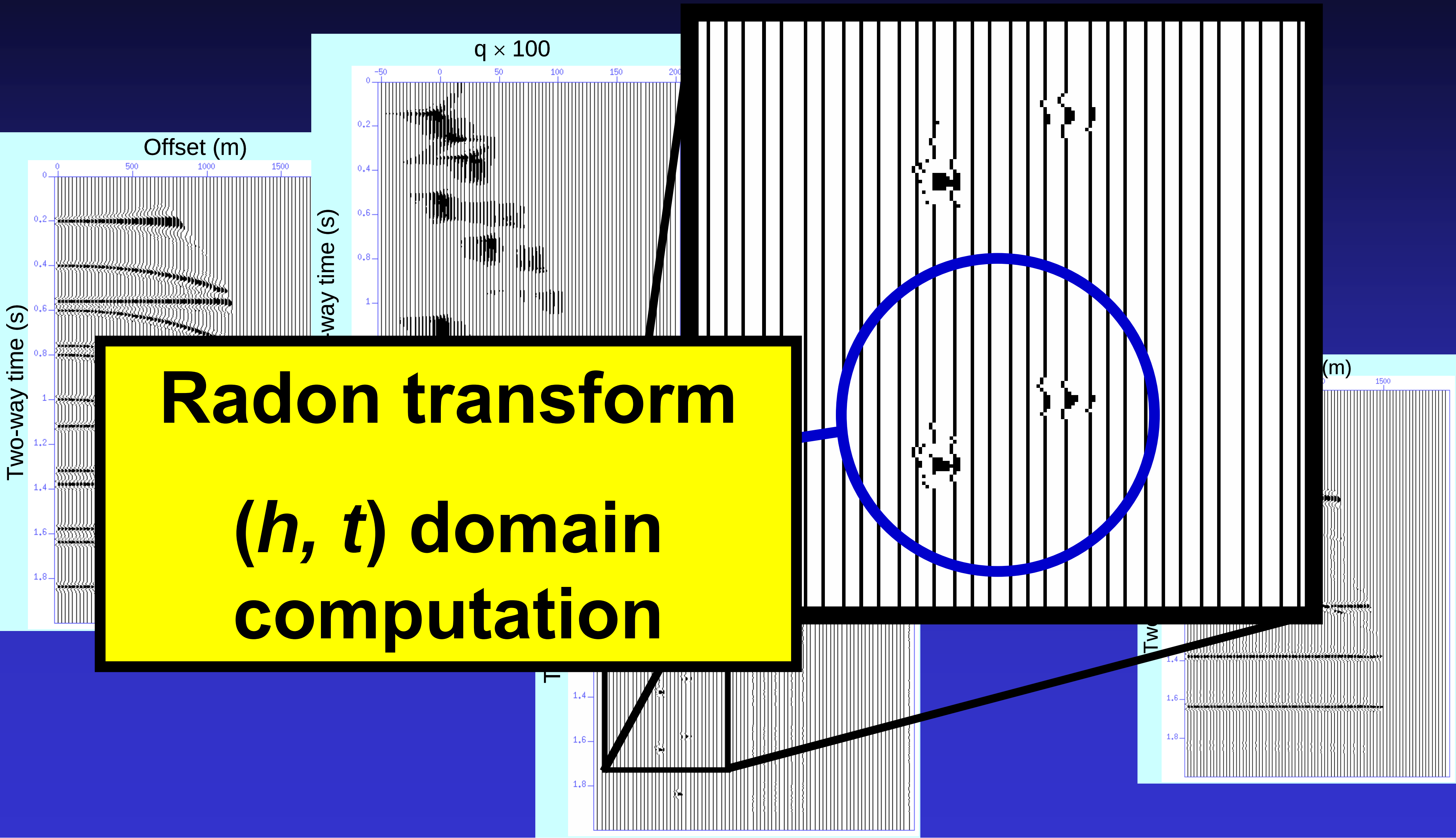
- Estimate $\mathbf{r}_1$
- Interpolate $\mathbf{r}_3$
- Improved estimate of $\mathbf{r}_3$
- Interpolate $\mathbf{r}_7$
- Improved estimate of $\mathbf{r}_7$
- Interpolate $\mathbf{r}_{15}$
- Improved estimate of $\mathbf{r}_{15}$
- Interp
- Impr of $\mathbf{r}_{31}$

Final estimation

Multiple attenuation *Zhihong (Nancy) Cao*



Multiple attenuation *Zhihong (Nancy) Cao*

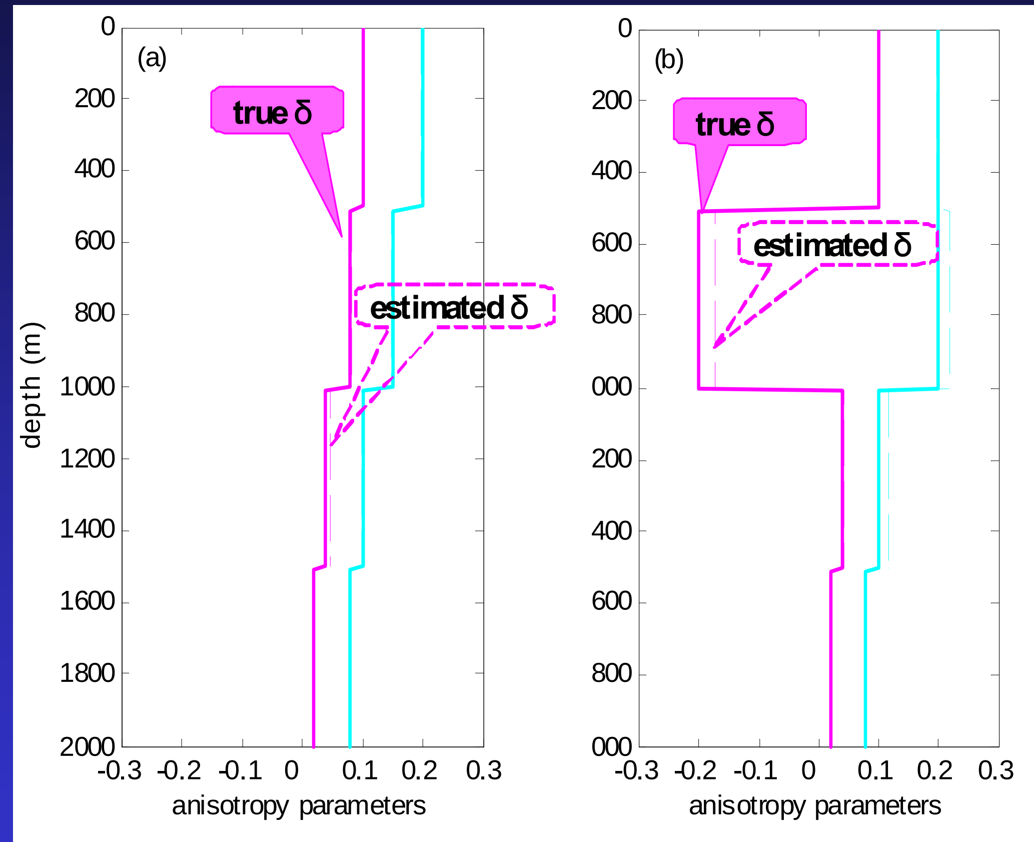


Anisotropy *Mary Xiao*

$$t^2(x) = t_0^2 + \frac{x^2}{V_{\mathrm{NMO}}^2}$$

$$t^2(x) = t_0^2 + A_2 x^2 + \frac{A_4 x^4}{1 + A x^2}$$

$$t(x, \tau_s) = \tau_s + \sqrt{\tau_x^2 + \frac{x^2}{SV_{\mathrm{NMO}}^2}}$$



Anisotropy *Pavan Elapavuluri*

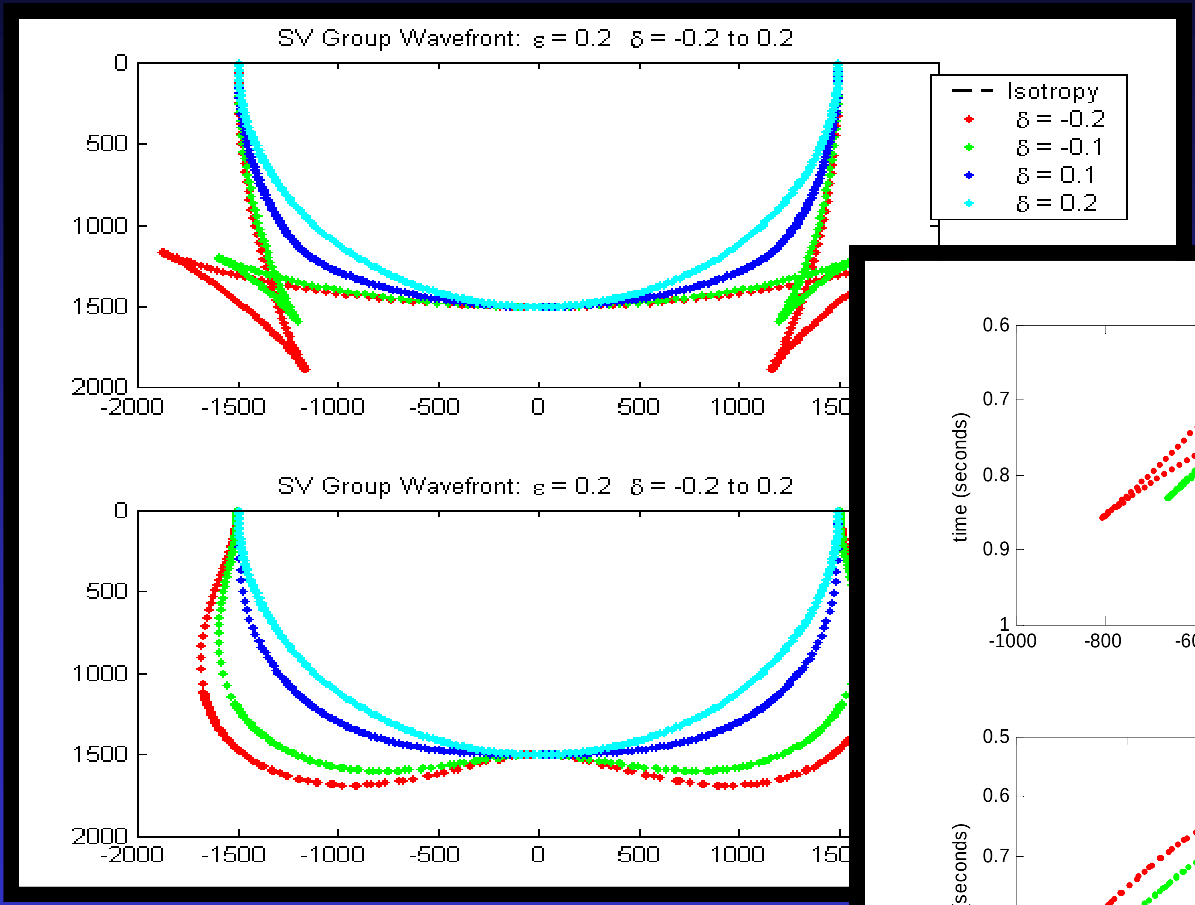
$$\sqrt{s}+\sqrt{r}=\sqrt{\ldots h_e^2}$$

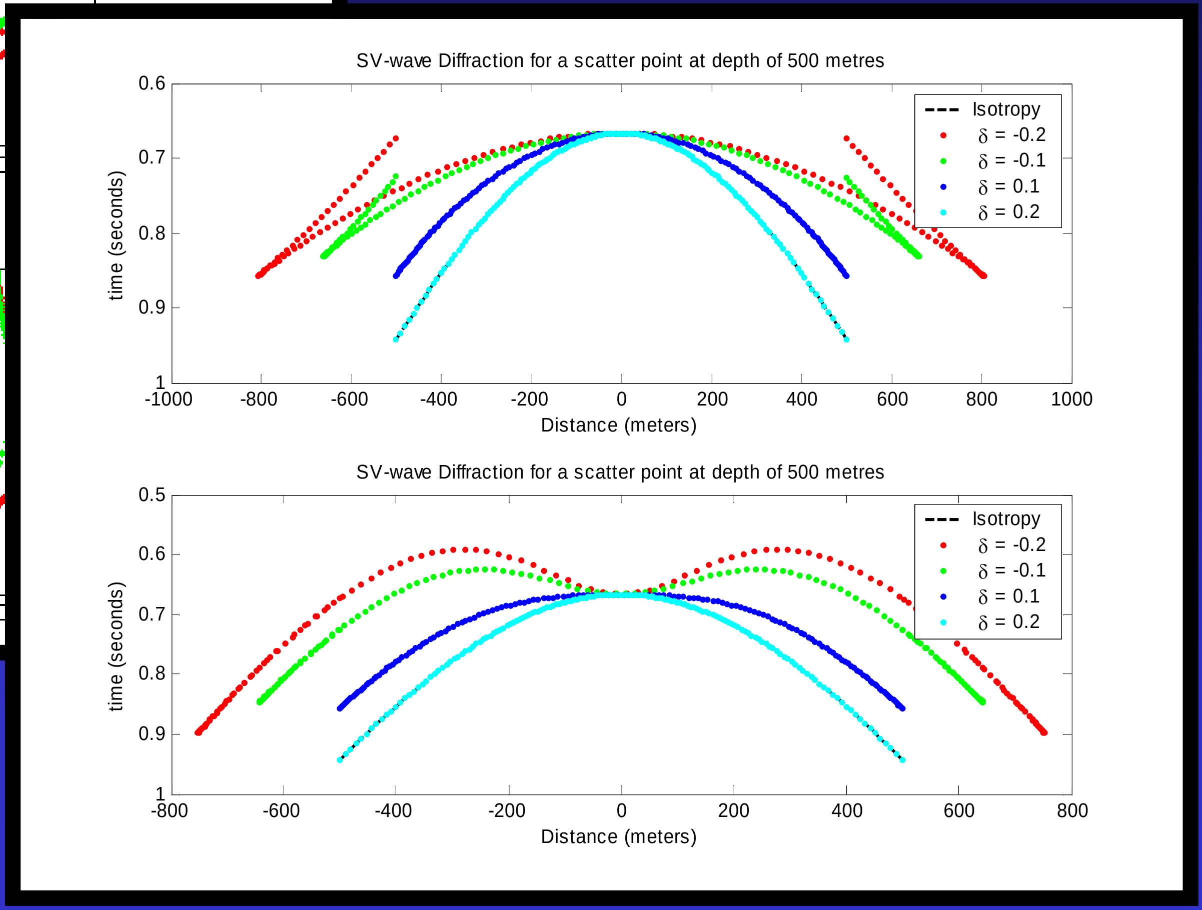
Aniso.

Aniso.

- Conventional EO uses hyperbolic eqn.
- Now use shifted hyperbola to include anisotropy effects
- Objective: improve velocity model

Anisotropy *Amber Kelter*

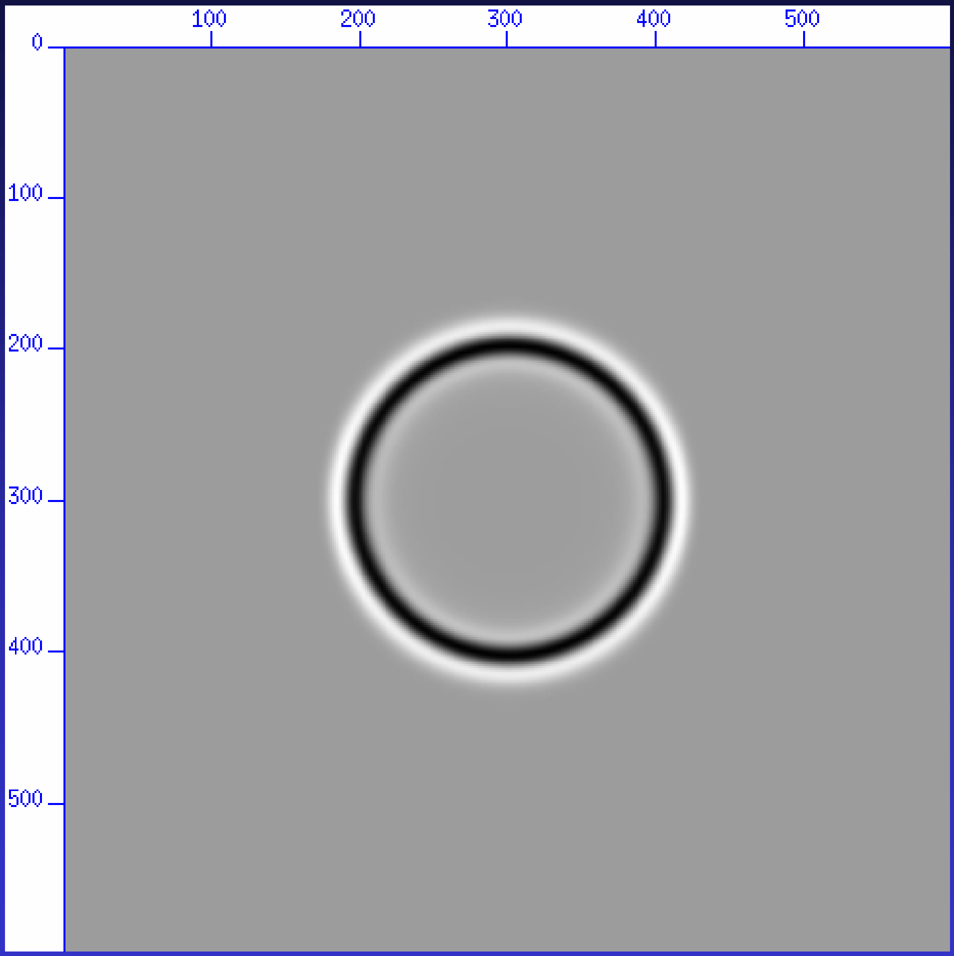




Finite element / difference *Xiang Du*

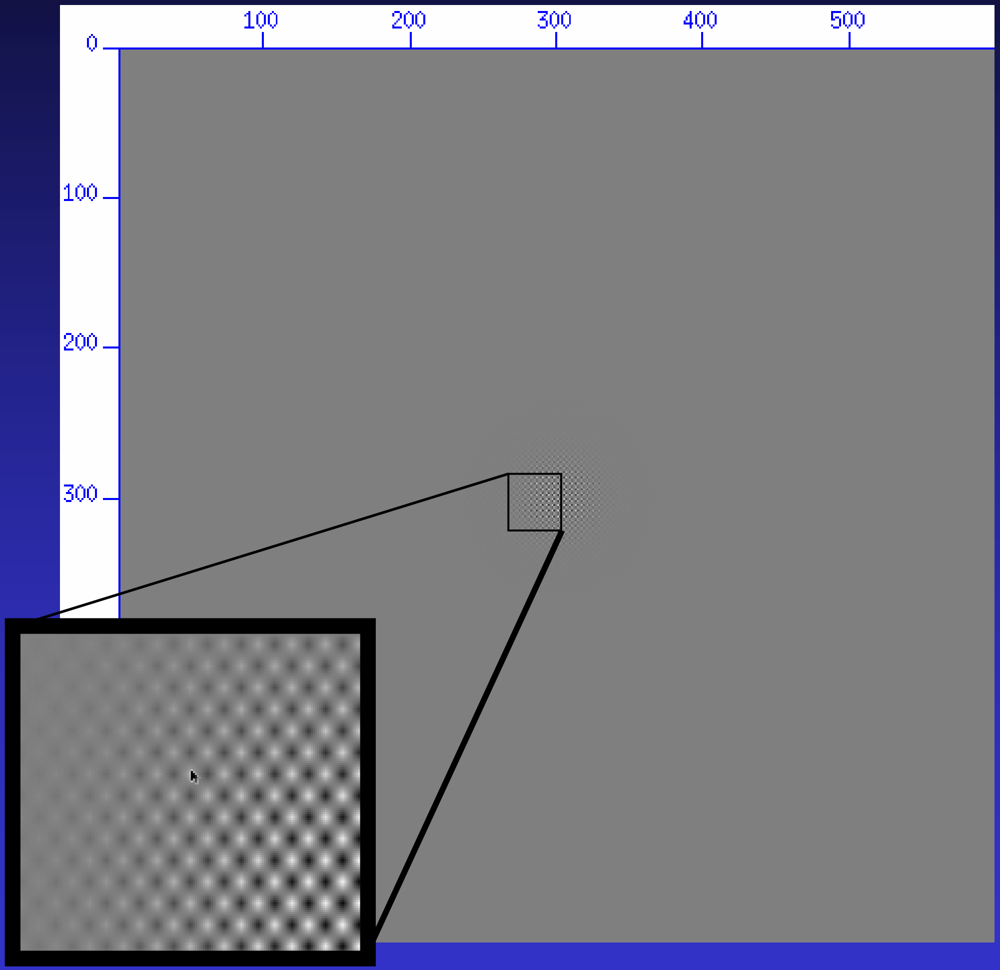
- Combine finite-element with finite difference
- Full wave equation
- Greater stability
- Larger step size
- Variable depth increment
- Modelling and Migration

FE – FD



Stable response

FD

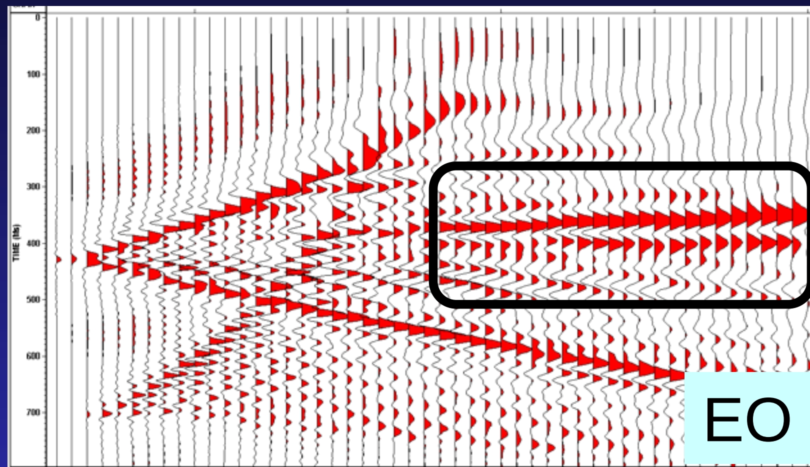


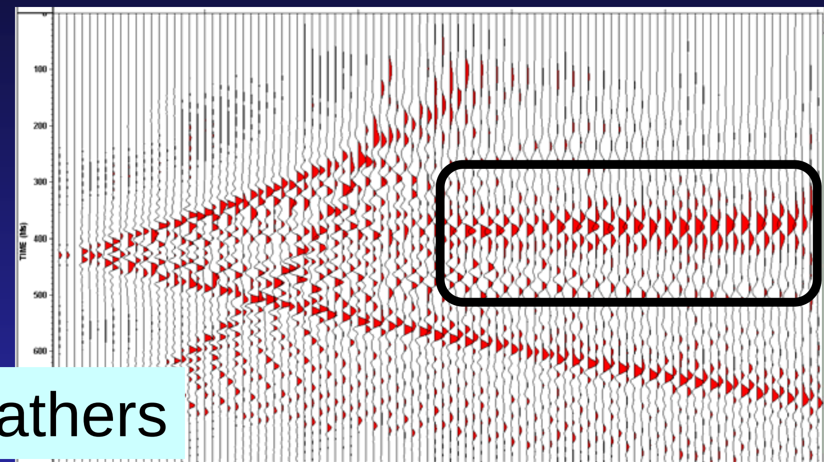
Unstable response

EOM

Bancroft

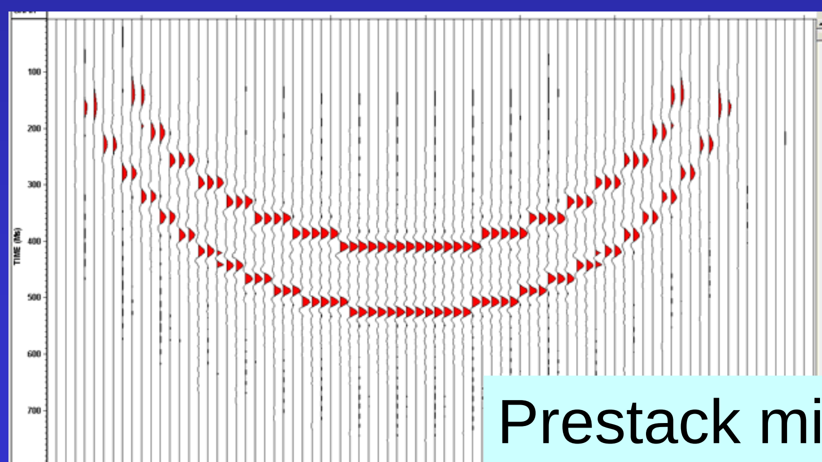
Bin interval

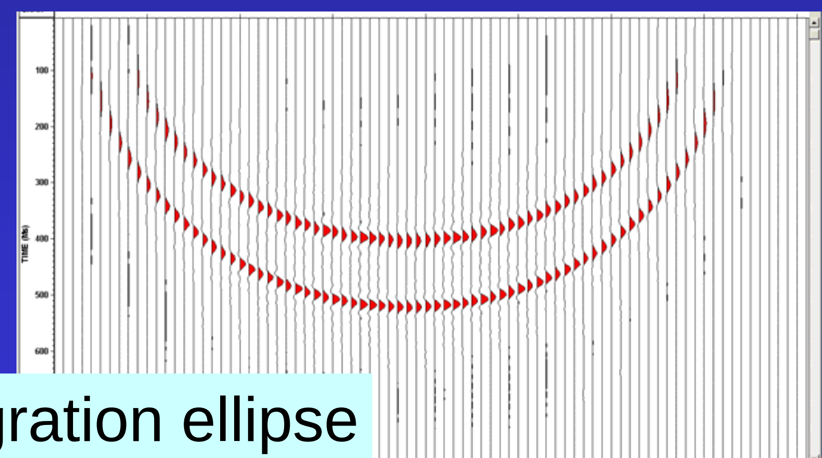




EO gathers

Bin interpolation





Prestack migration ellipse

Continued fraction expansion

$$g = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}}}$$

$$g = [1 : 1, 1, 1, 1, 1, \ldots]$$

Continued fraction expansion

$$g = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}}}$$

$$g = [1:1,1,1,1,1,\ldots]$$

$$\sqrt{2} = [1:2,2,2,2,2,2,2,2,2,2,2,\ldots] = [1:2^{*}]$$

Continued fraction expansion

$$g = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}}}$$

$$g = \left[1:1,1,1,1,1,\ldots\right]$$

$$\sqrt{2} = \left[1:2,2,2,2,2,2,2,2,2,2,2,\ldots\right] = \left[1:2^{*}\right]$$

$$\pi \approx \frac{3}{1}, \quad \frac{22}{7}, \quad \frac{333}{106}, \quad \frac{355}{113}, \quad \frac{103993}{33102}, \quad \frac{104384}{33215}, \quad \frac{208341}{66317}, \quad \text{and} \quad \frac{312689}{99532}$$

Continued fraction expansion

$$g = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}}}$$

$$g = [1:1,1,1,1,1,\ldots]$$

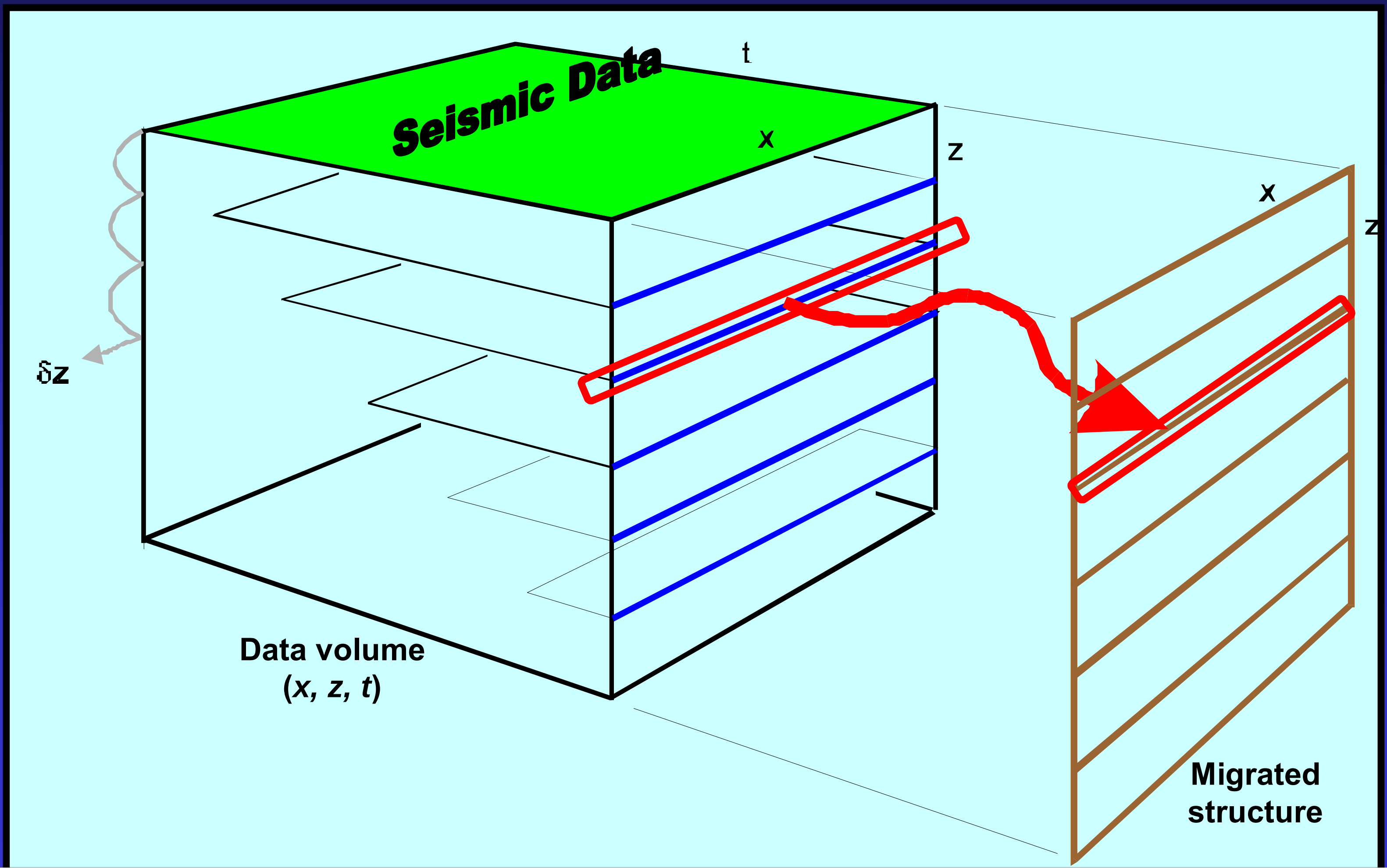
$$f(x) = \sqrt{1 - x^2} = a_0 + \cfrac{x^2}{a_1 + \cfrac{x^2}{a_2 + \cfrac{x^2}{a_3 + \cfrac{}{\ldots}}}}$$

$$\sqrt{2} = [1:2,2,2,2,2,2,2,2,2,2,2,\ldots] = \left[1:2^*\right]$$

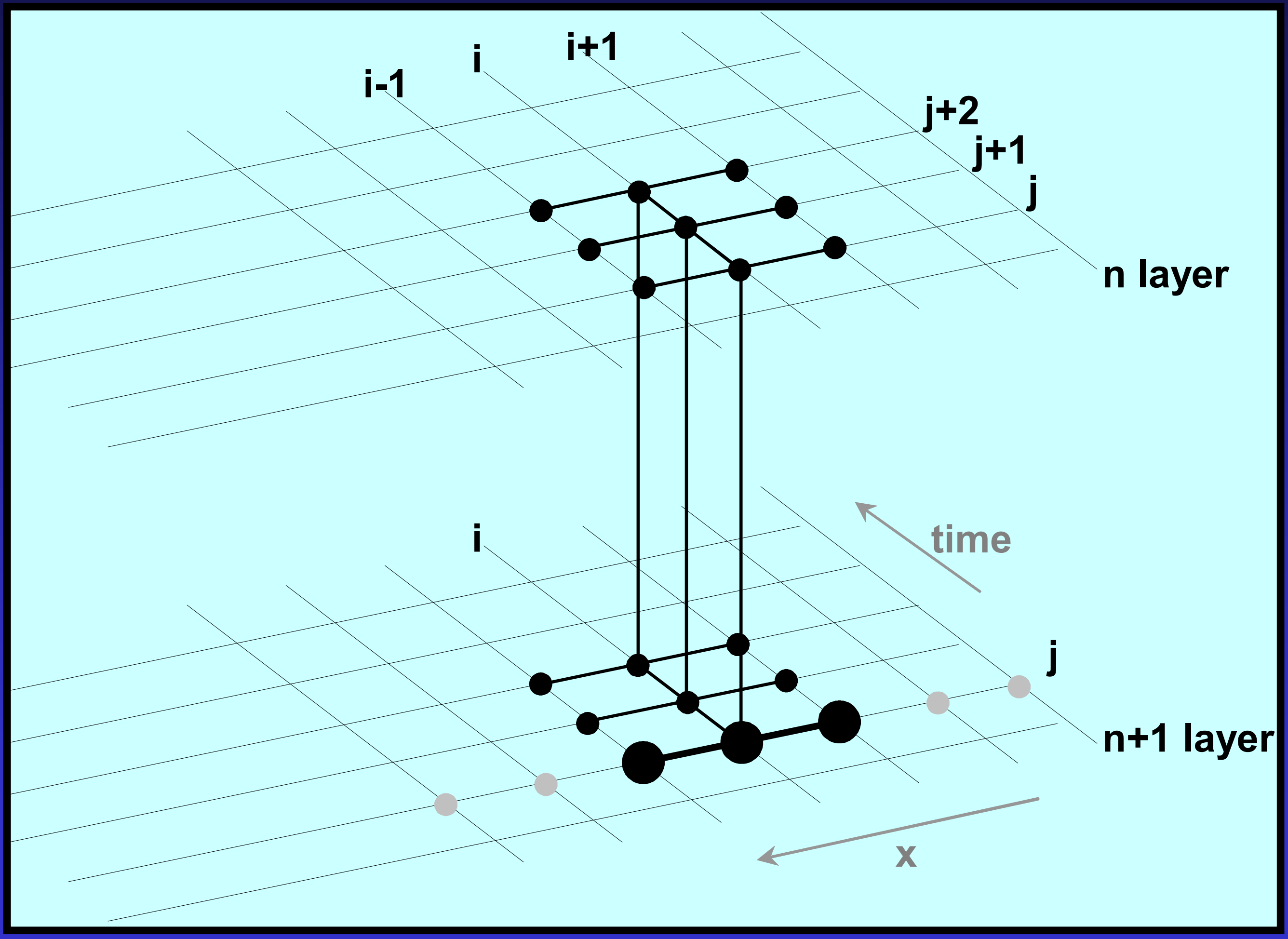
$$\pi \approx \frac{3}{1}, \quad \frac{22}{7}, \quad \frac{333}{106}, \quad \frac{355}{113}, \quad \frac{103993}{33102}, \quad \frac{104384}{33215}, \quad \frac{208341}{66317}, \quad \text{and} \quad \frac{312689}{99532}$$

Downward continuation
Downward extrapolation
Downward marching …

Bancroft

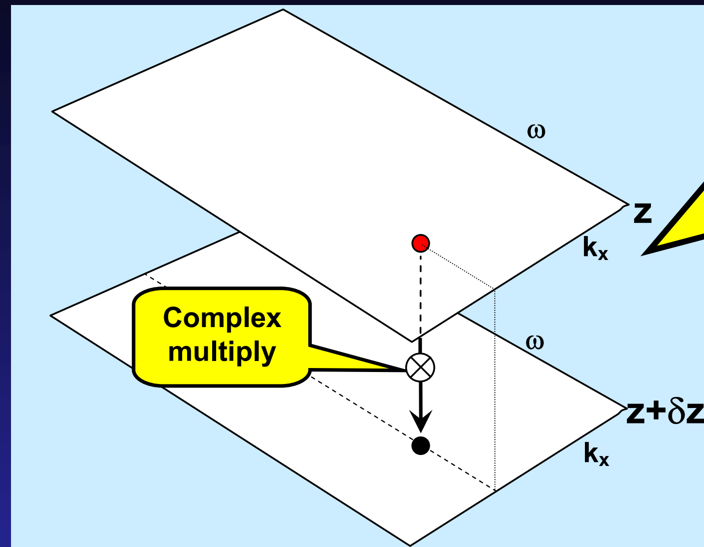


Finite difference



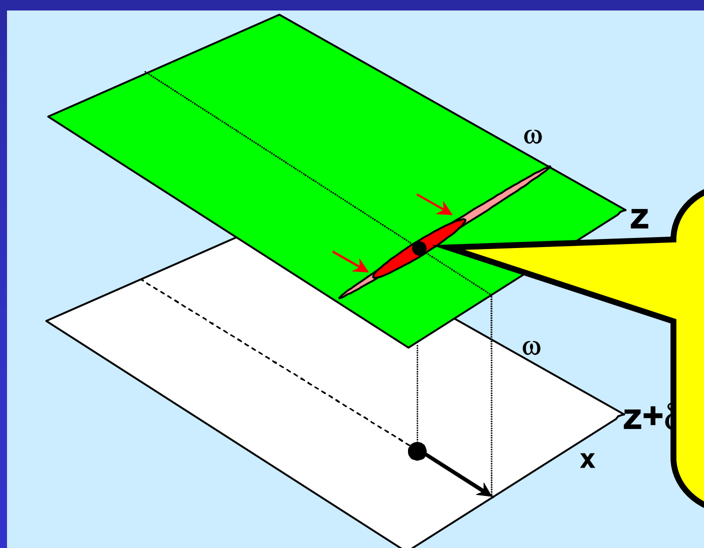
Phase-shift

(k_x, ω)

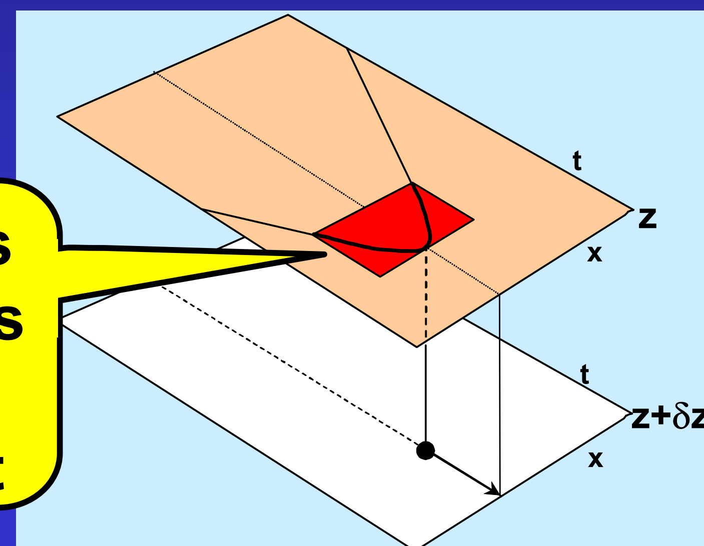


No x dependence
…
Assume constant velocity at this layer

(x, ω)



(x, t)

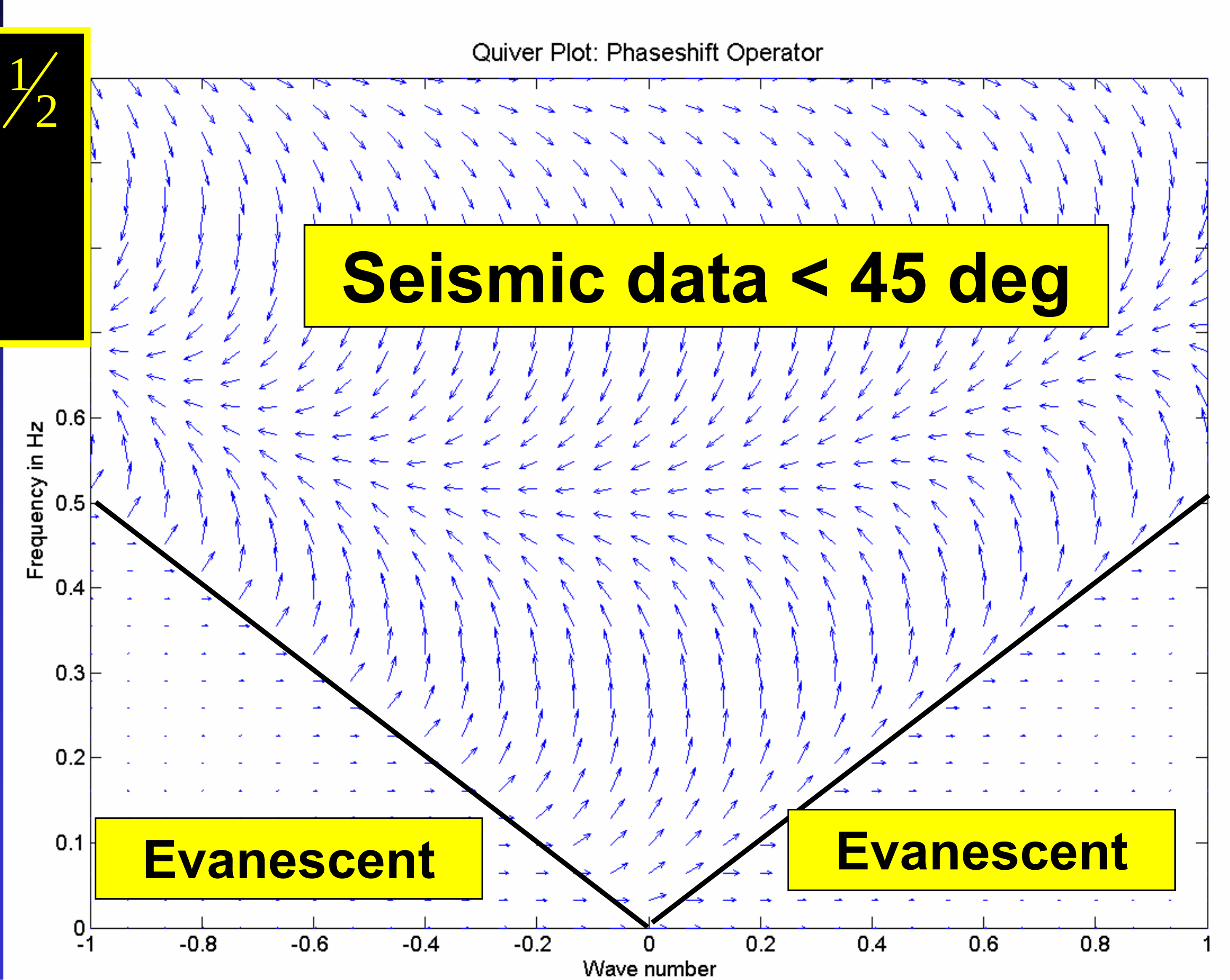


Assumes velocity is locally constant

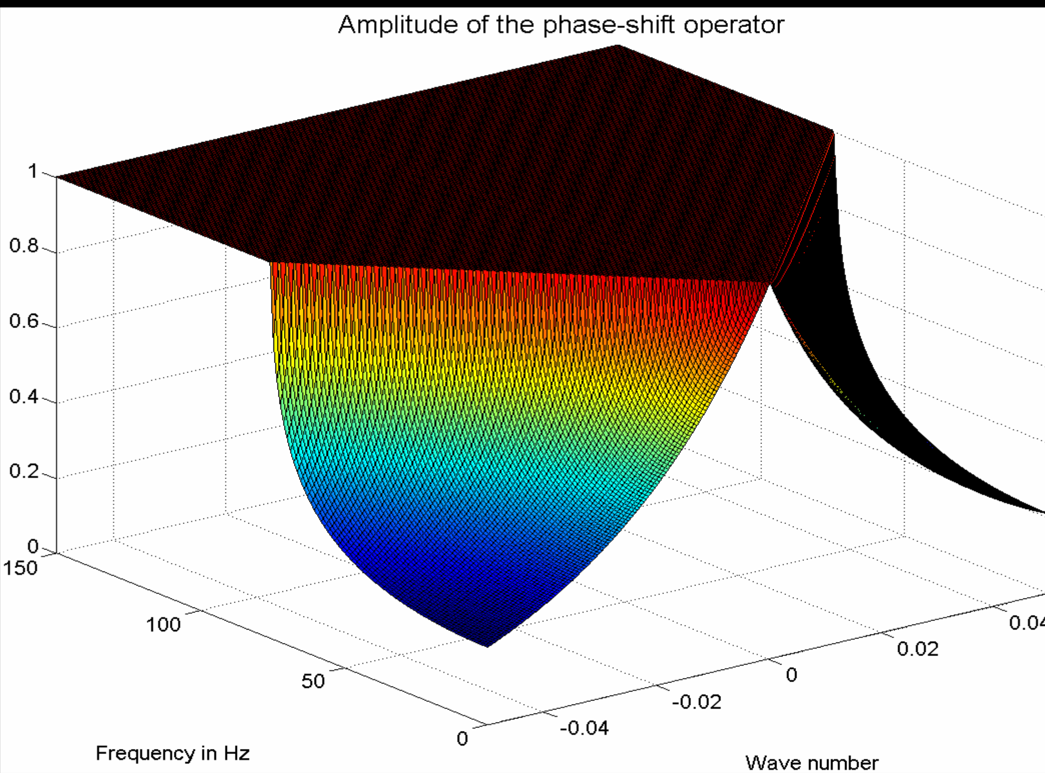
Phase-shift operator (k_x, z) space

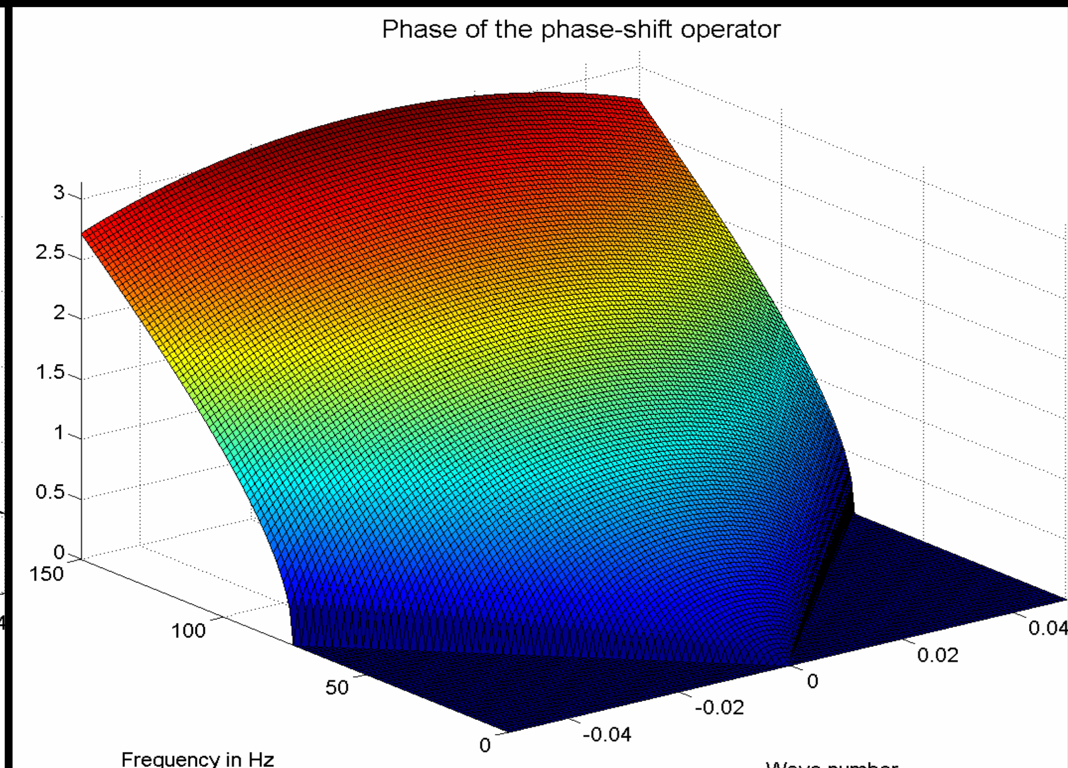
$$e^{j\delta z\left(\frac{\omega^2}{v^2}-k_x^2\right)^{1/2}}$$

Quiver plot

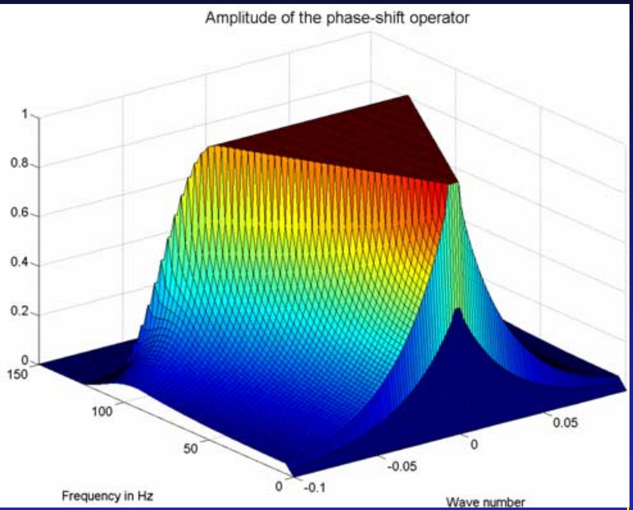


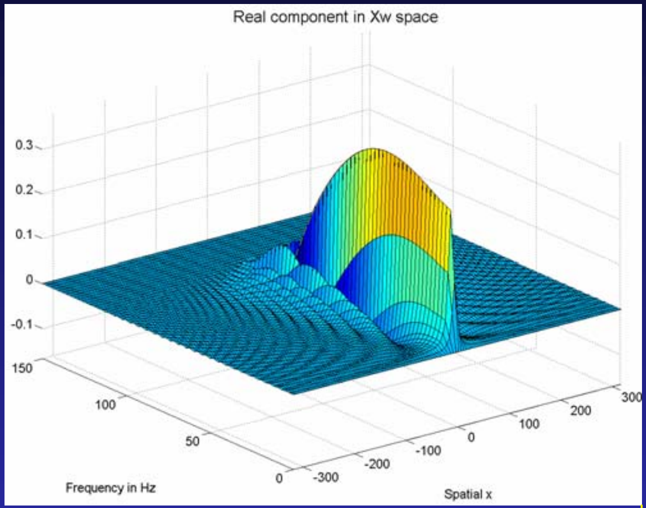
Phase-shift operator (k_x, z) space

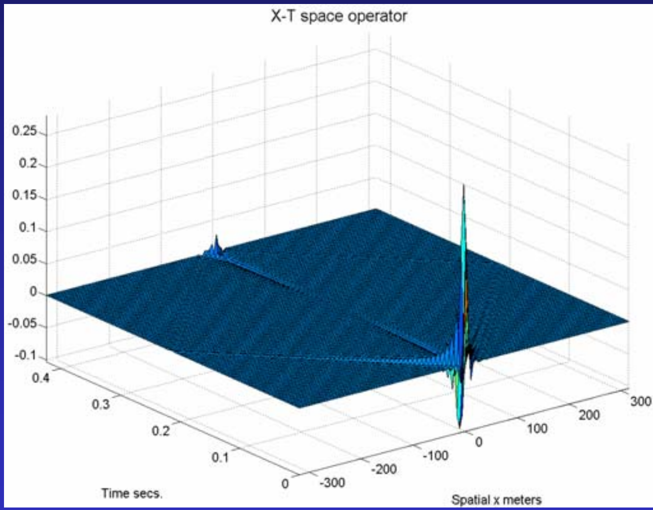




Phase-shift operator (k_x, z), (x, z), (x, t)



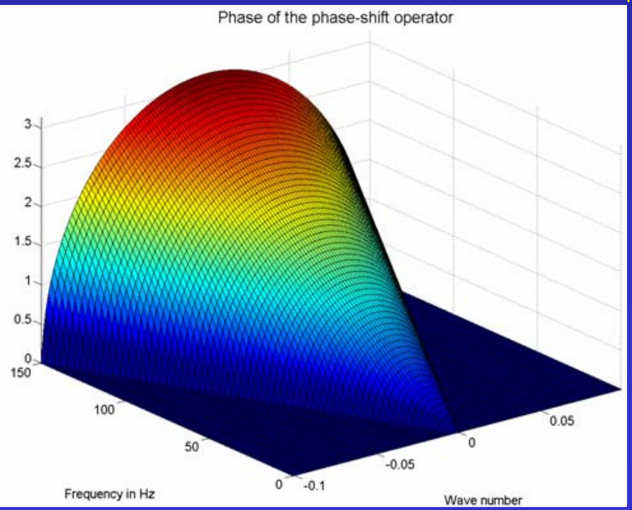


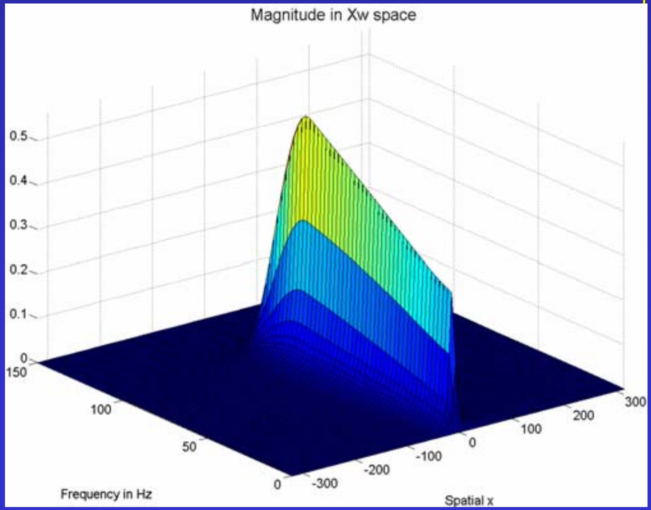


(k_x, z)

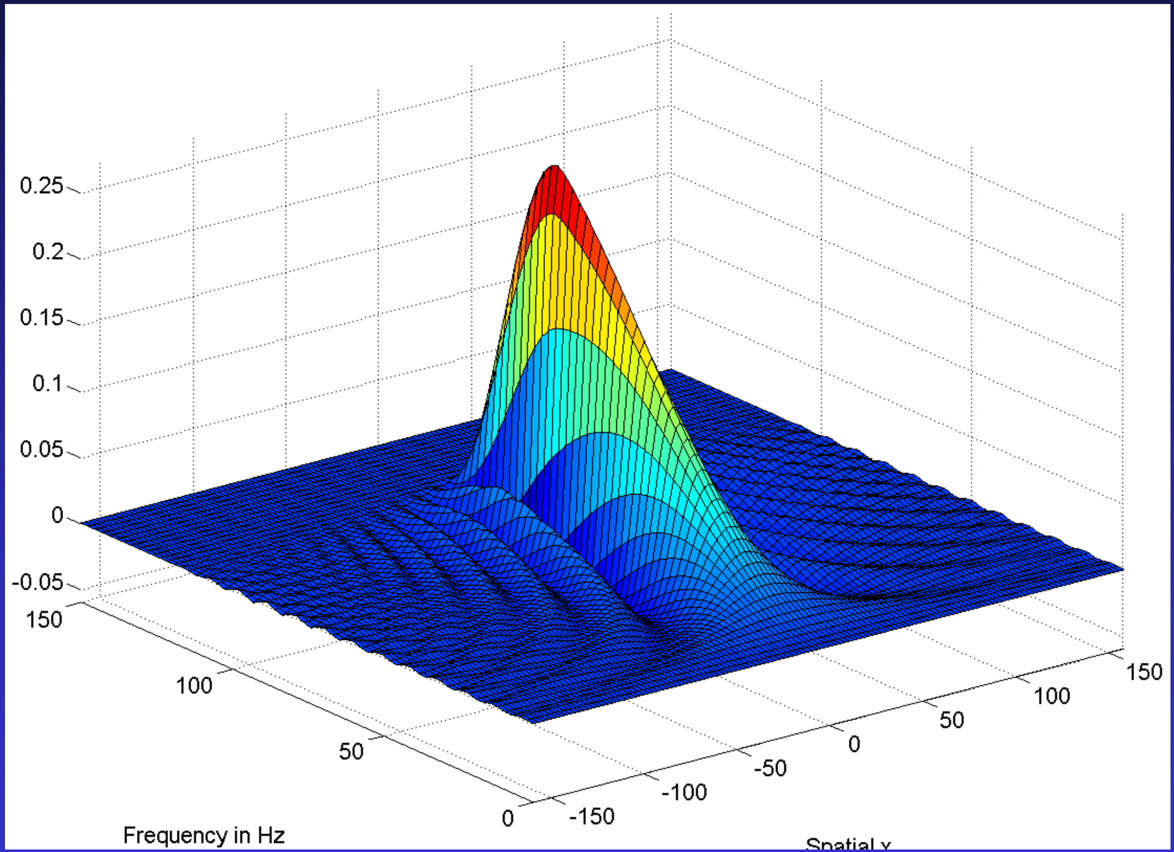
(x, z)

(x, t)



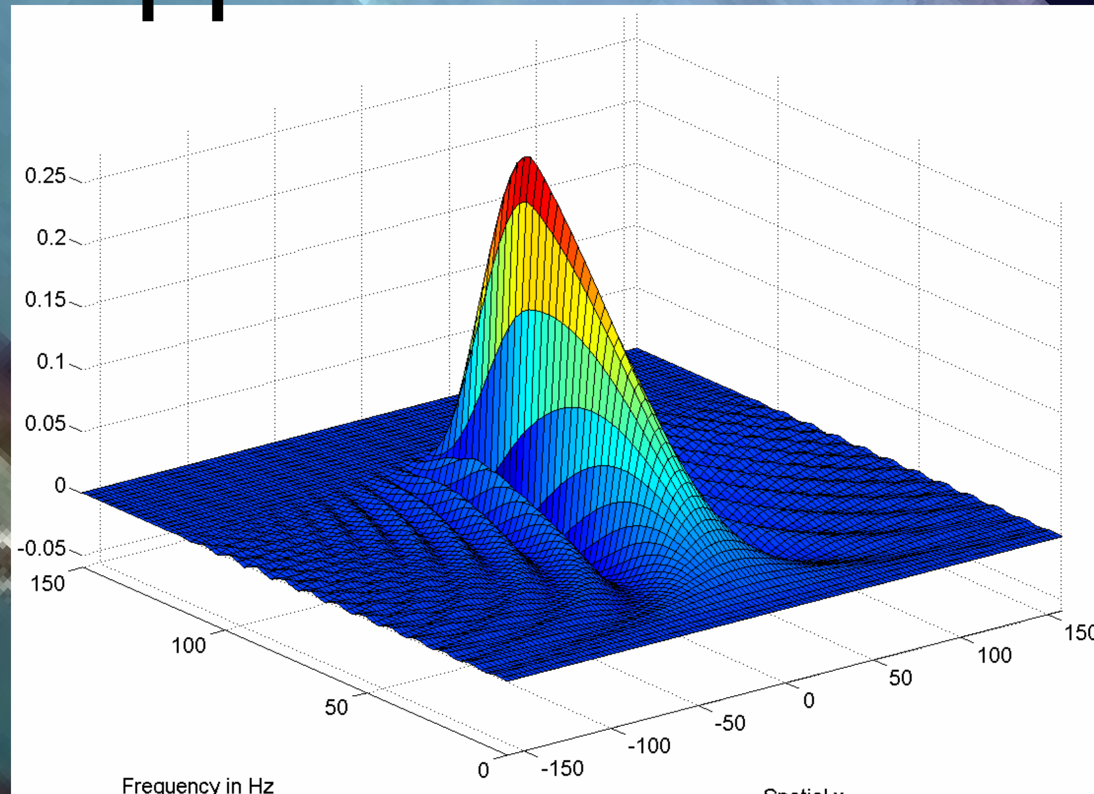


Magnitude of the phase-shift op. (x, z) space

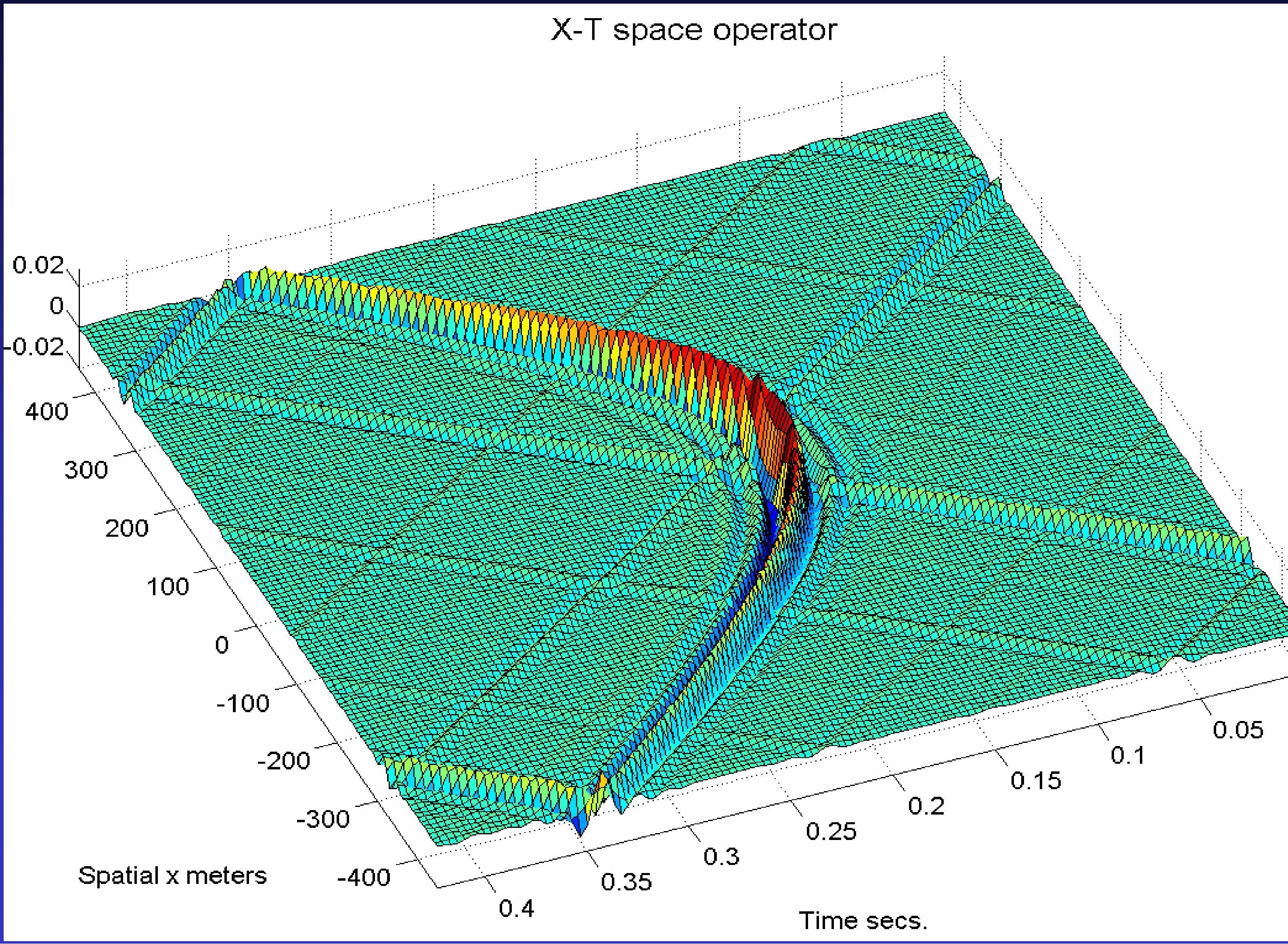


(x, ω)

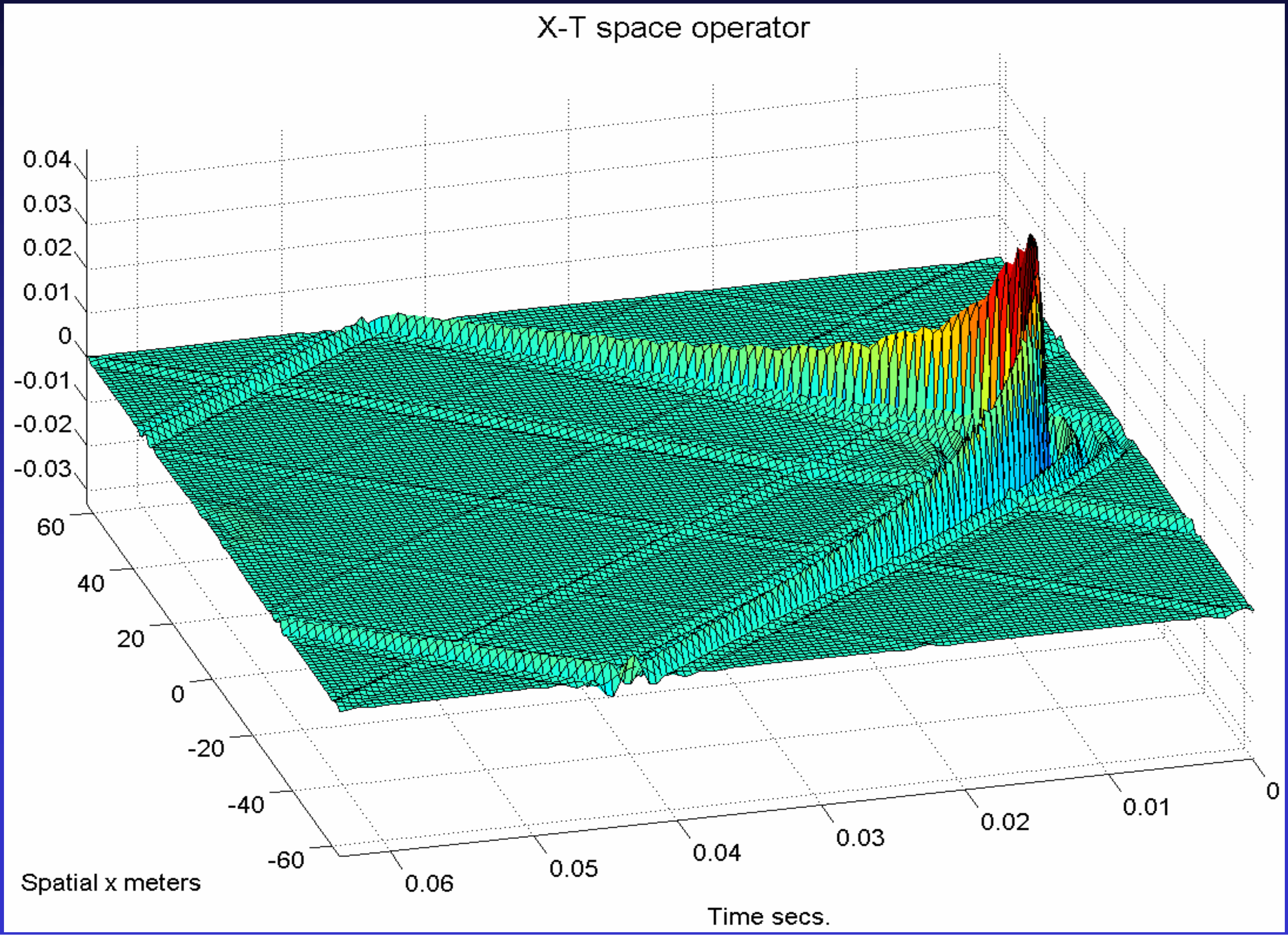
The red-tipped shark



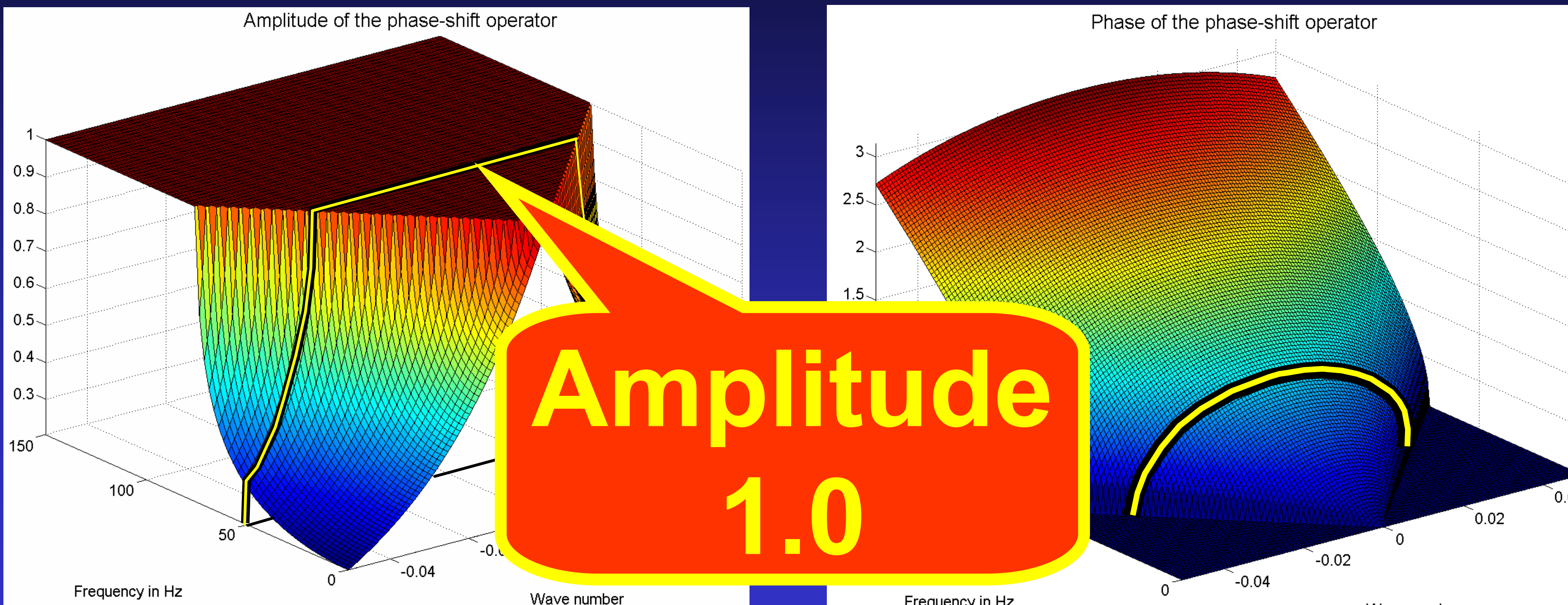
Phase-shift op. (*x*, *t*) space gz = <u>250m</u>



Phase-shift op. (*x*, *t*) space δz = <u>10m</u>

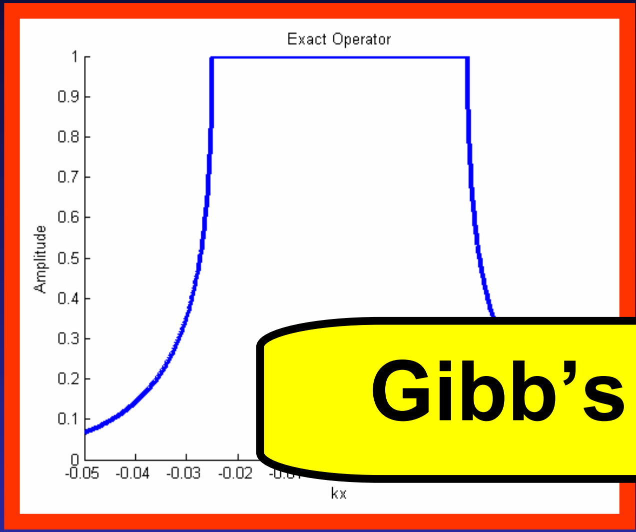


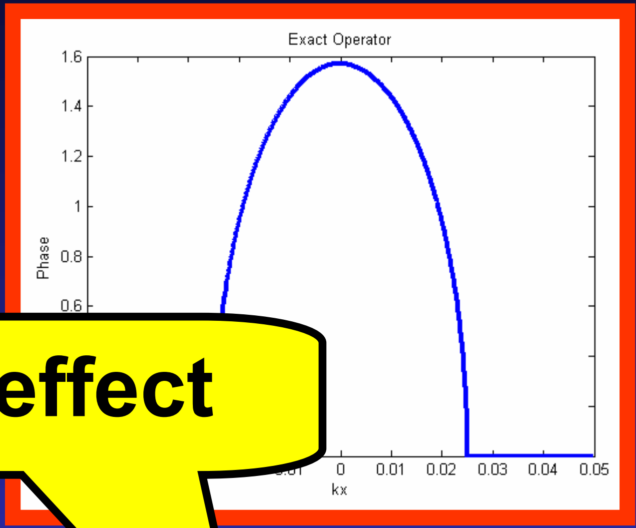
Phase-shift op. (k_x, z) space gz = 10m



Constant frequency operator

Phase-shift op. (k_x, z) space   z constant



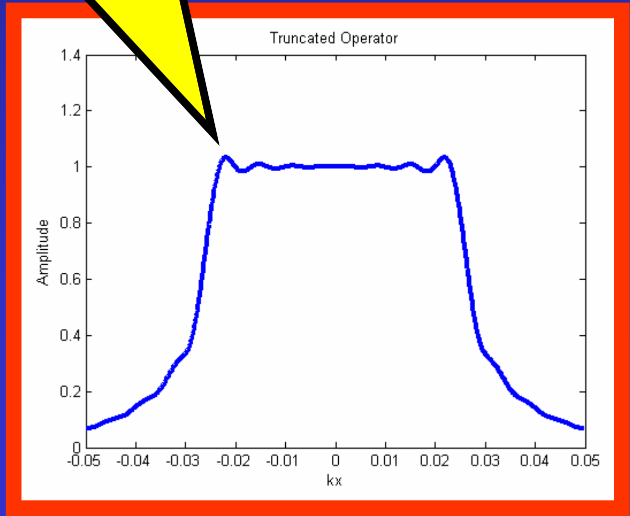


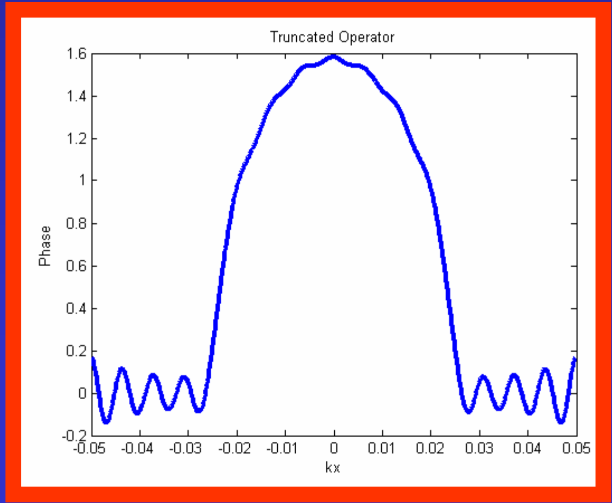
Desired

Gibb's effect

Get with

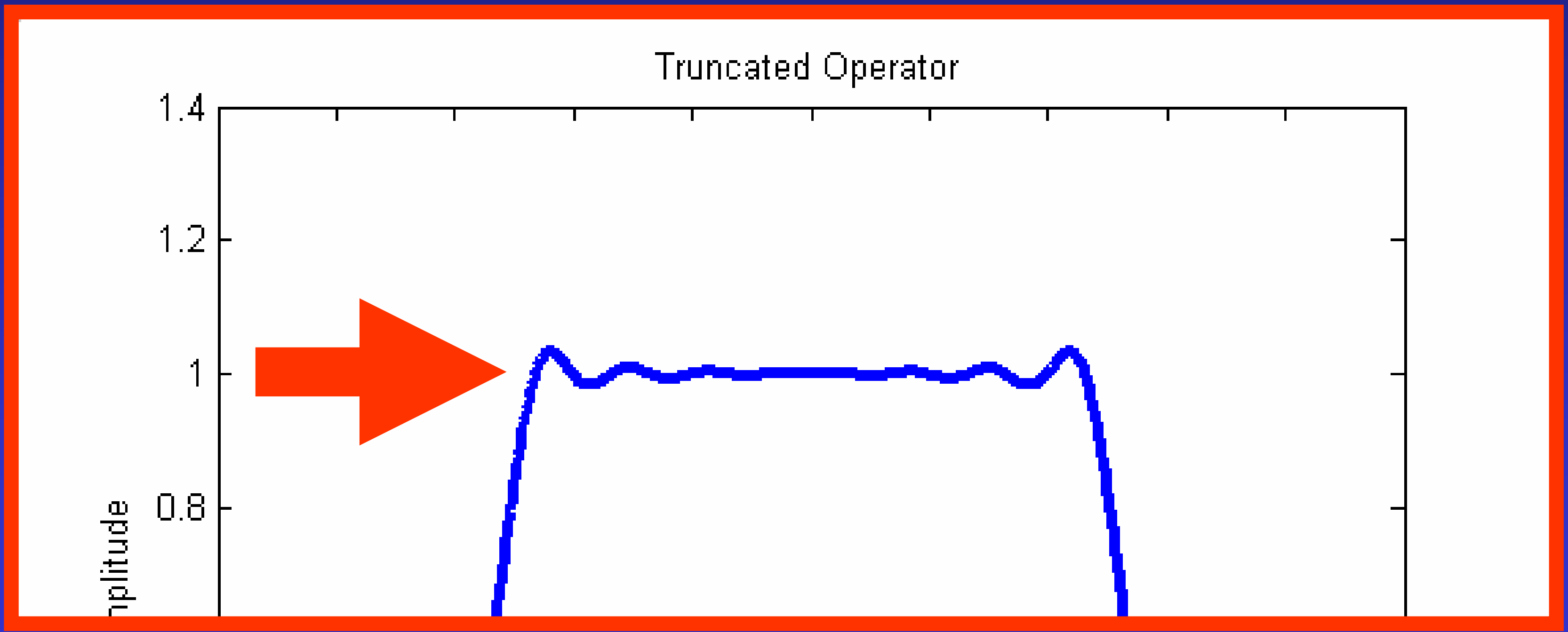
(x, z) operator





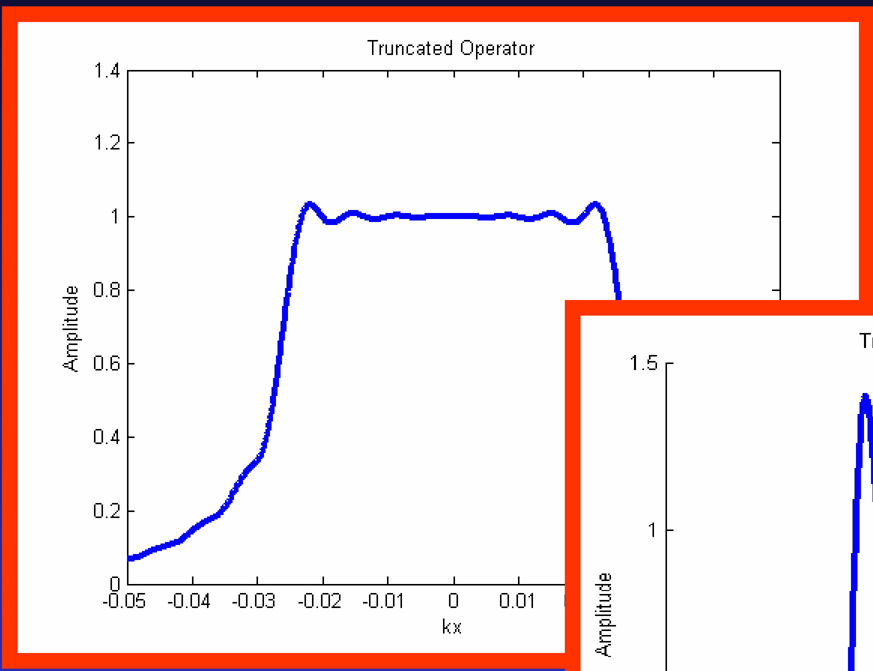
Phase-shift op.

- Applied (convolved) many times in **(*x*, z)**
- Multiplied many times in **(k_x, z)**
- Amplitudes greater than one will “blow up”

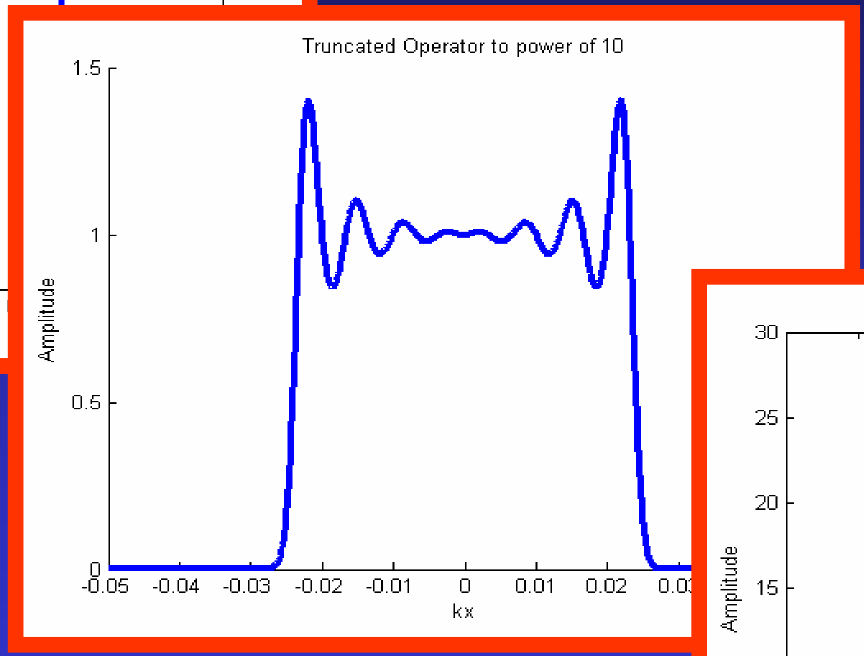


Phase-shift op.

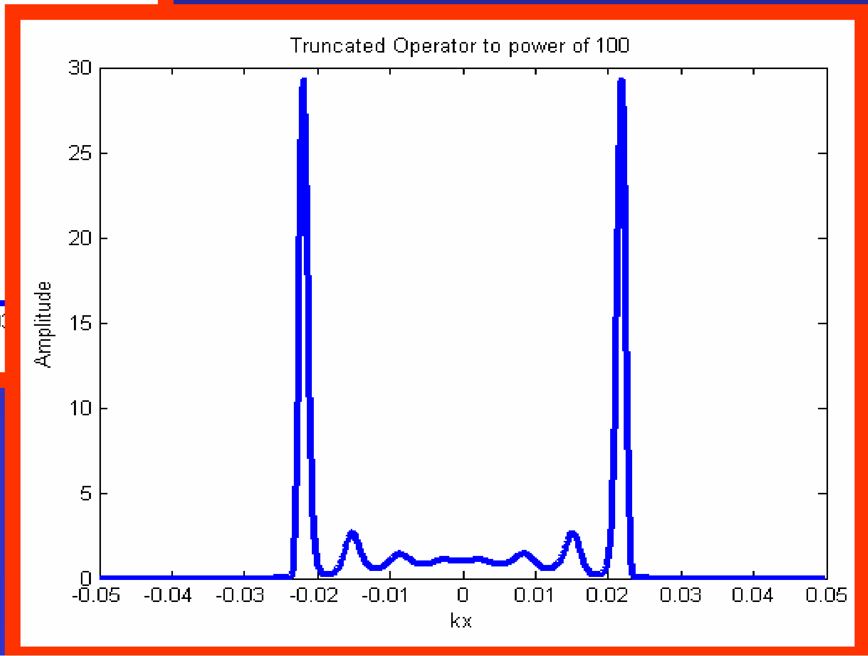
We need > ^500



^1

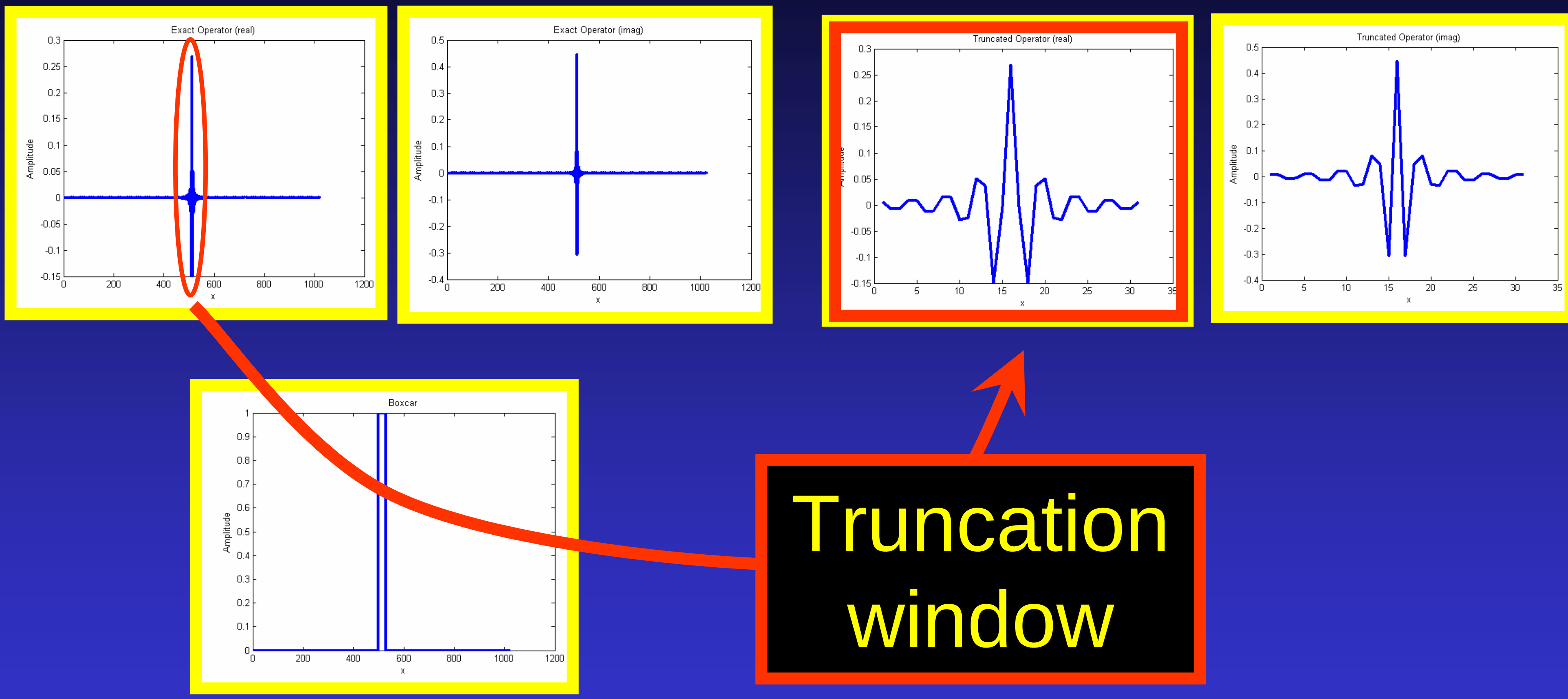


^10

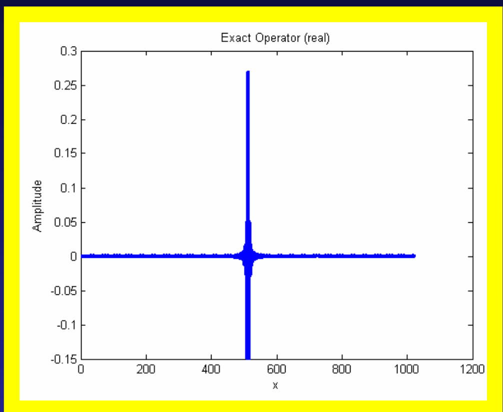


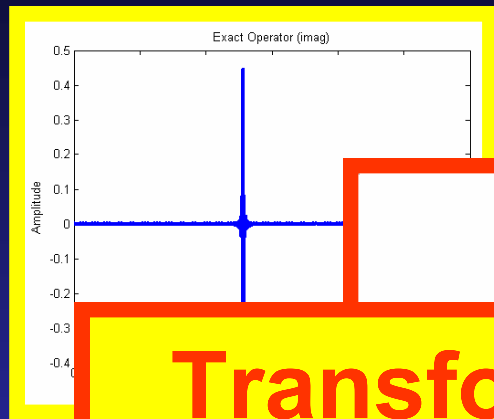
^100

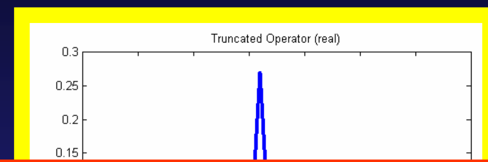
Phase-shift op.   (x, z) space

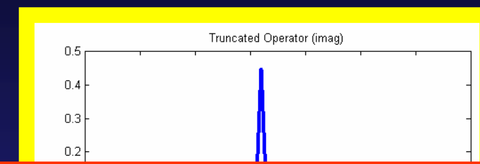


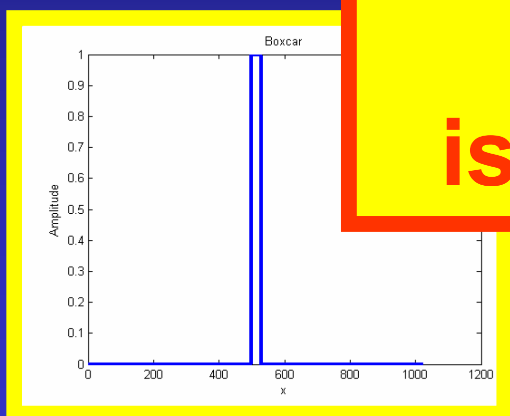
Phase-shift op.

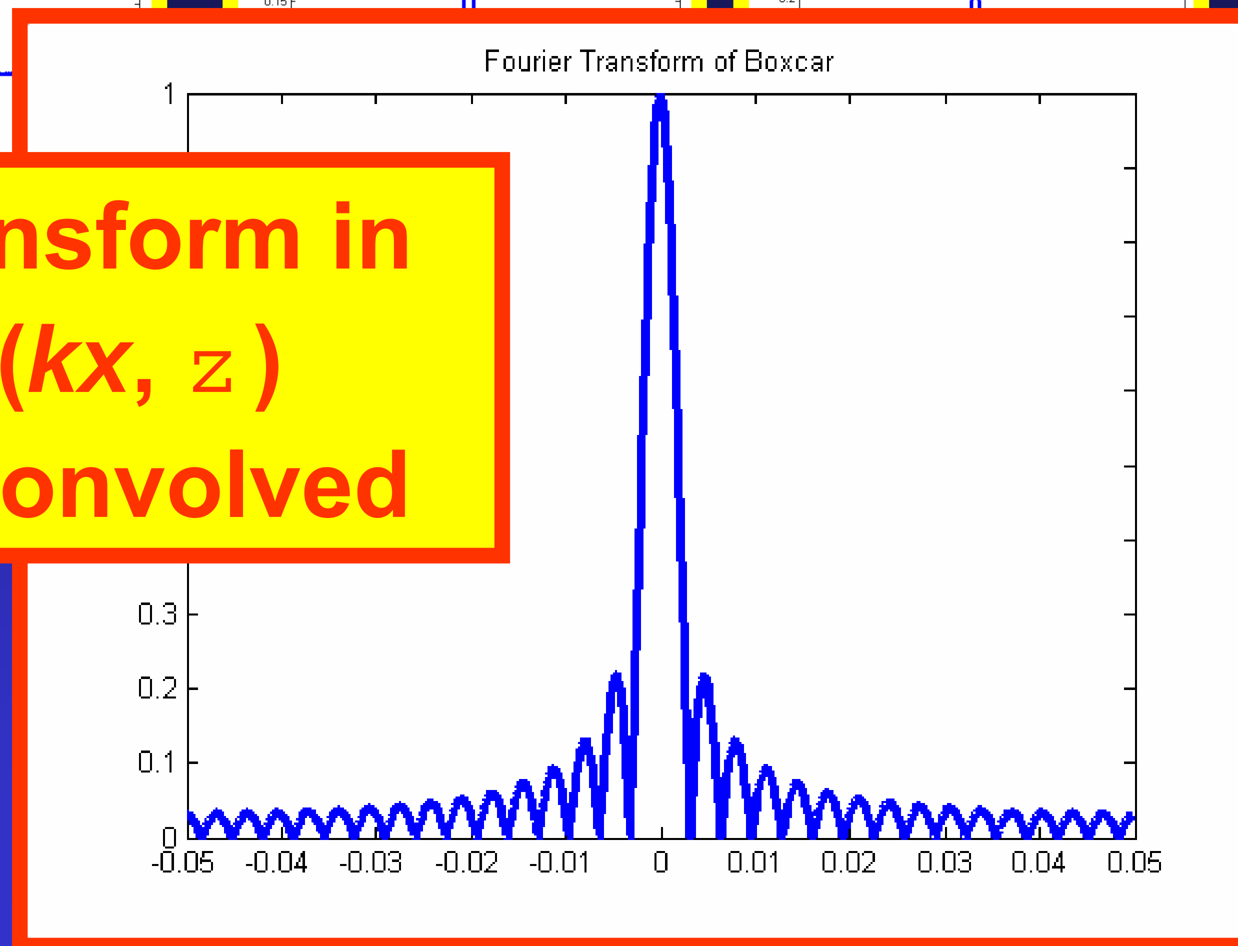






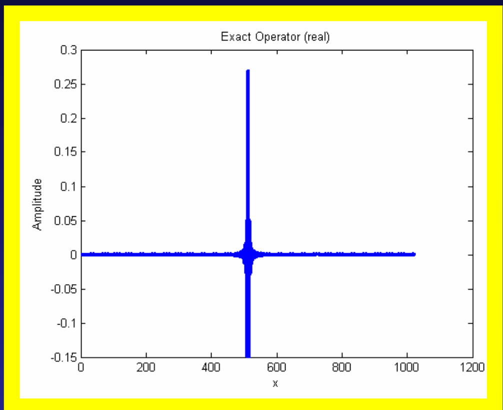


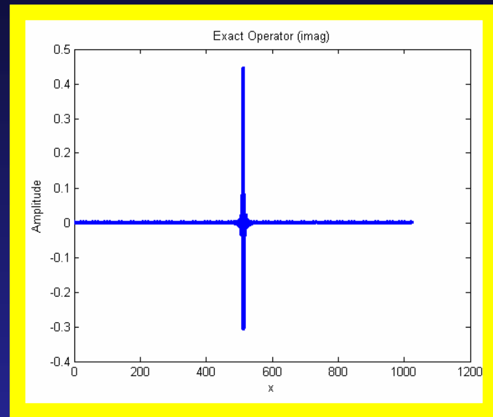


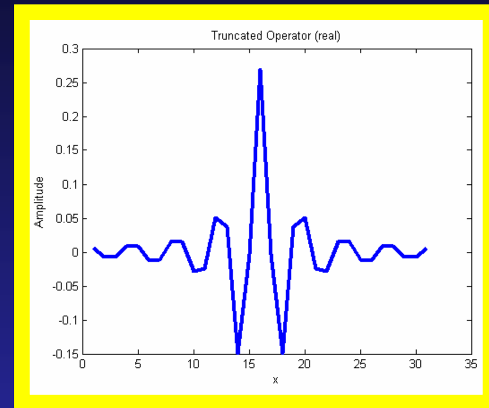


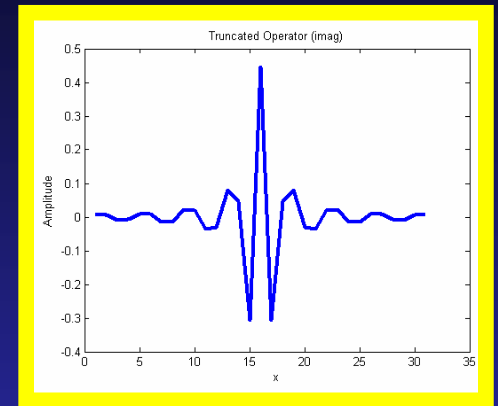
Transform in (kx, z) is convolved

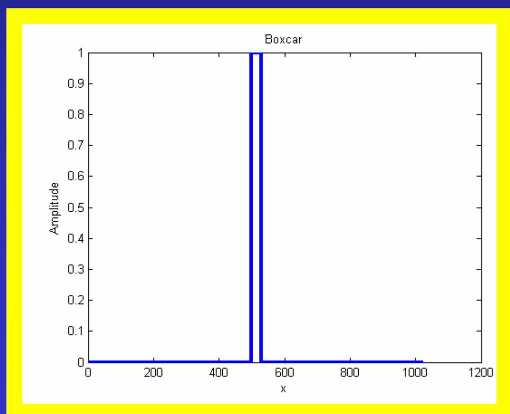
Phase-shift op. (k_x, z) space

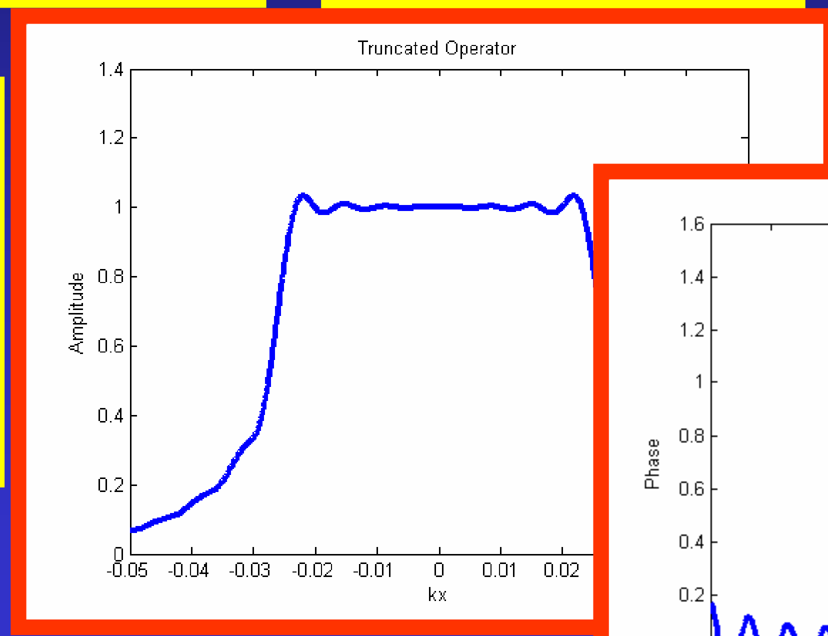


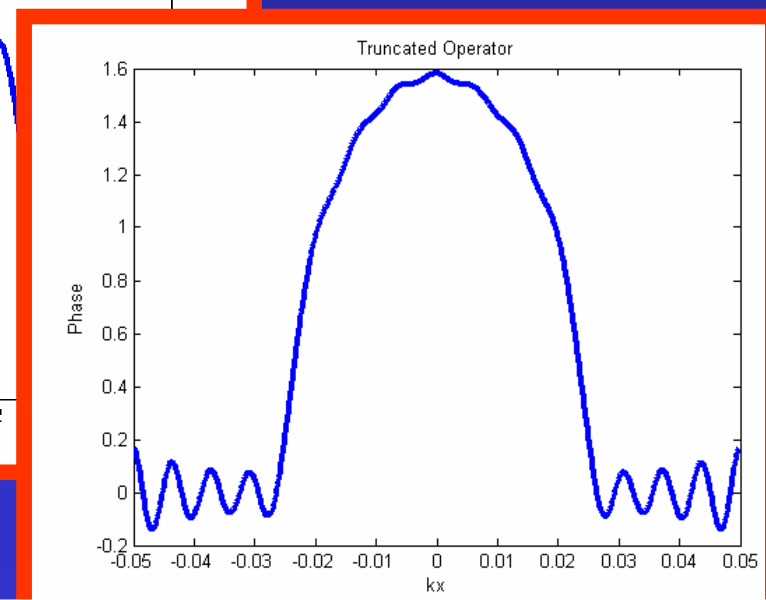




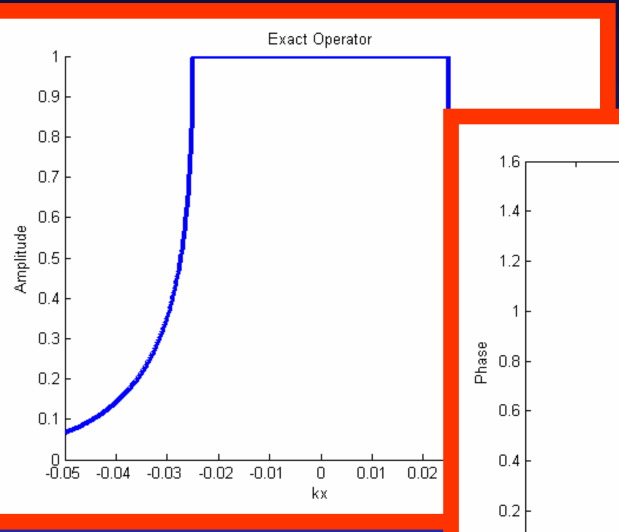


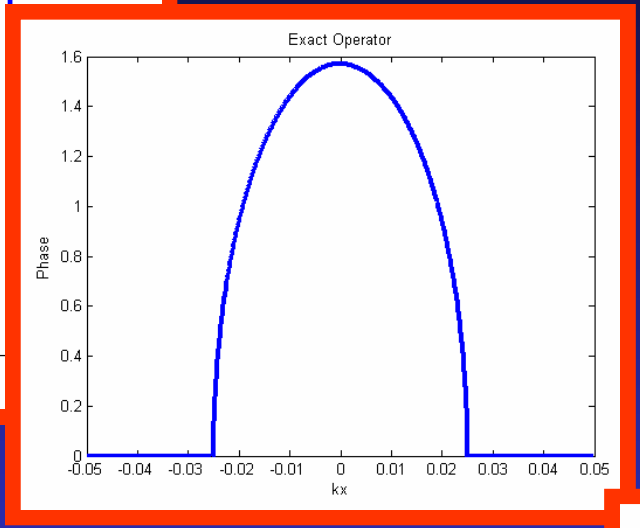


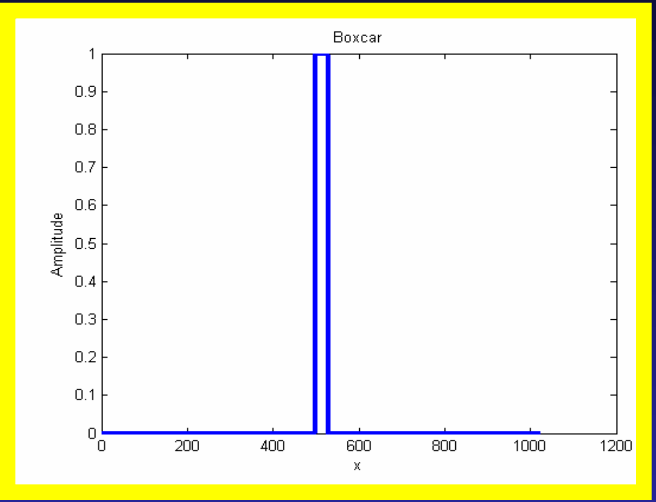


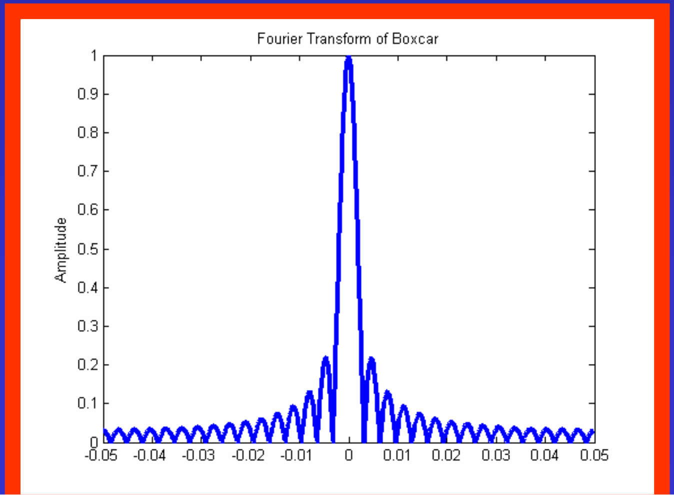


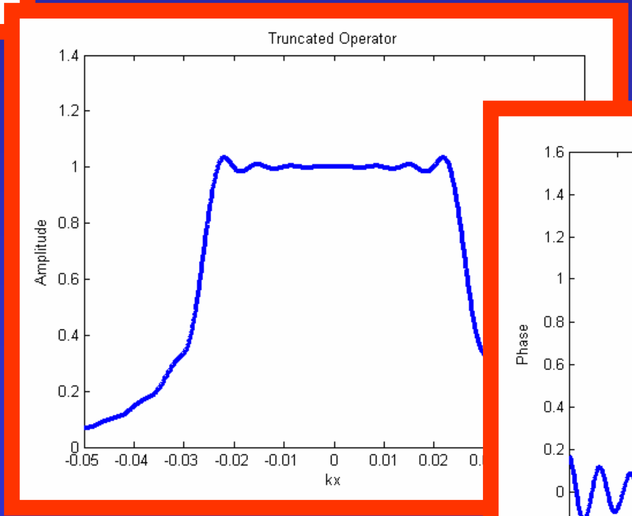
Phase-shift op. (*kx*, *z*) space Desired

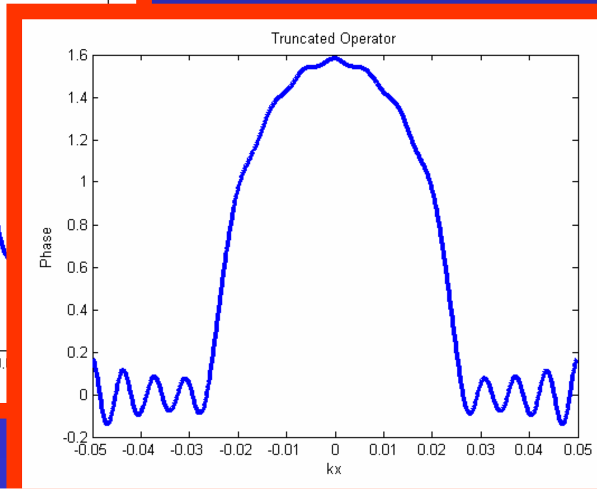








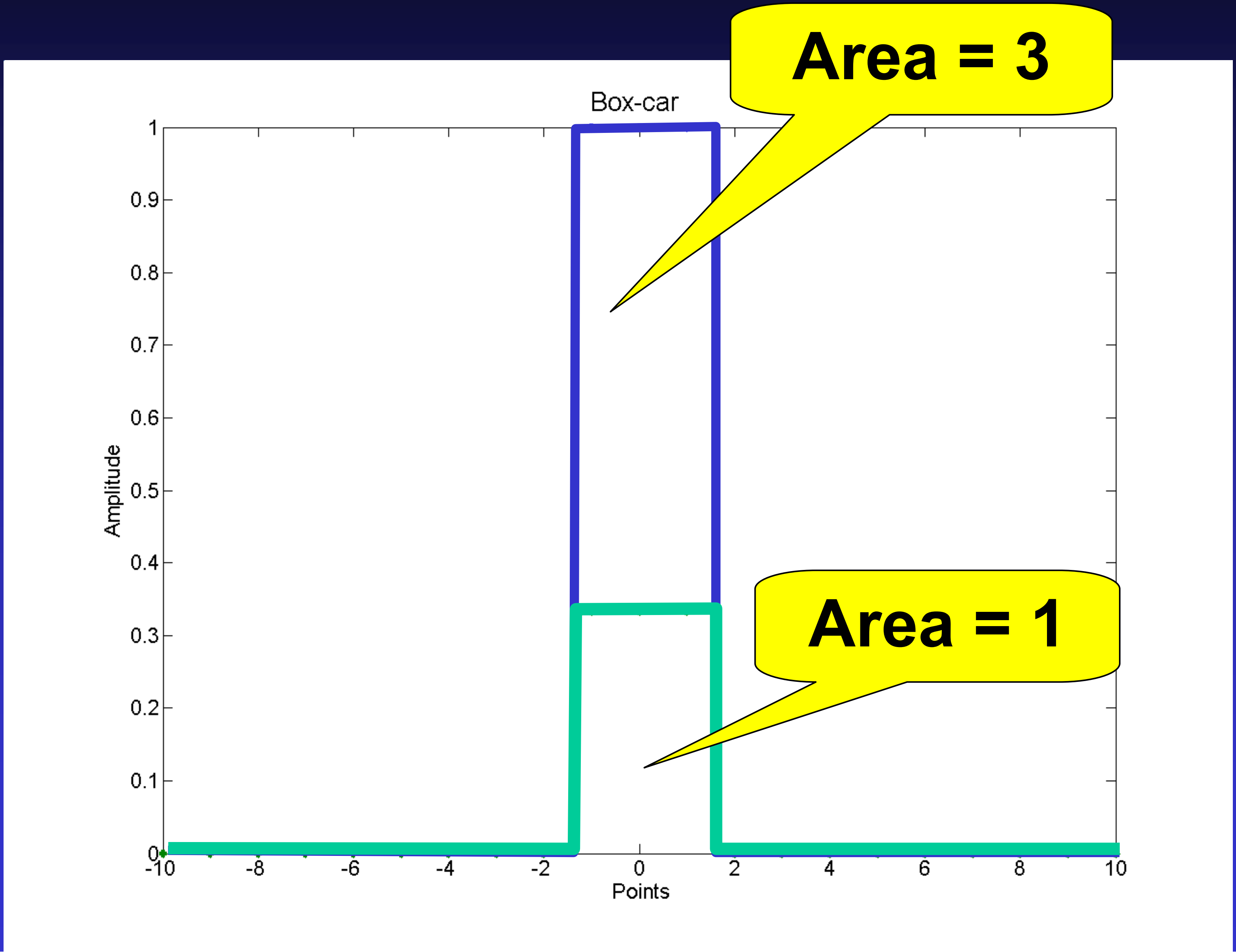




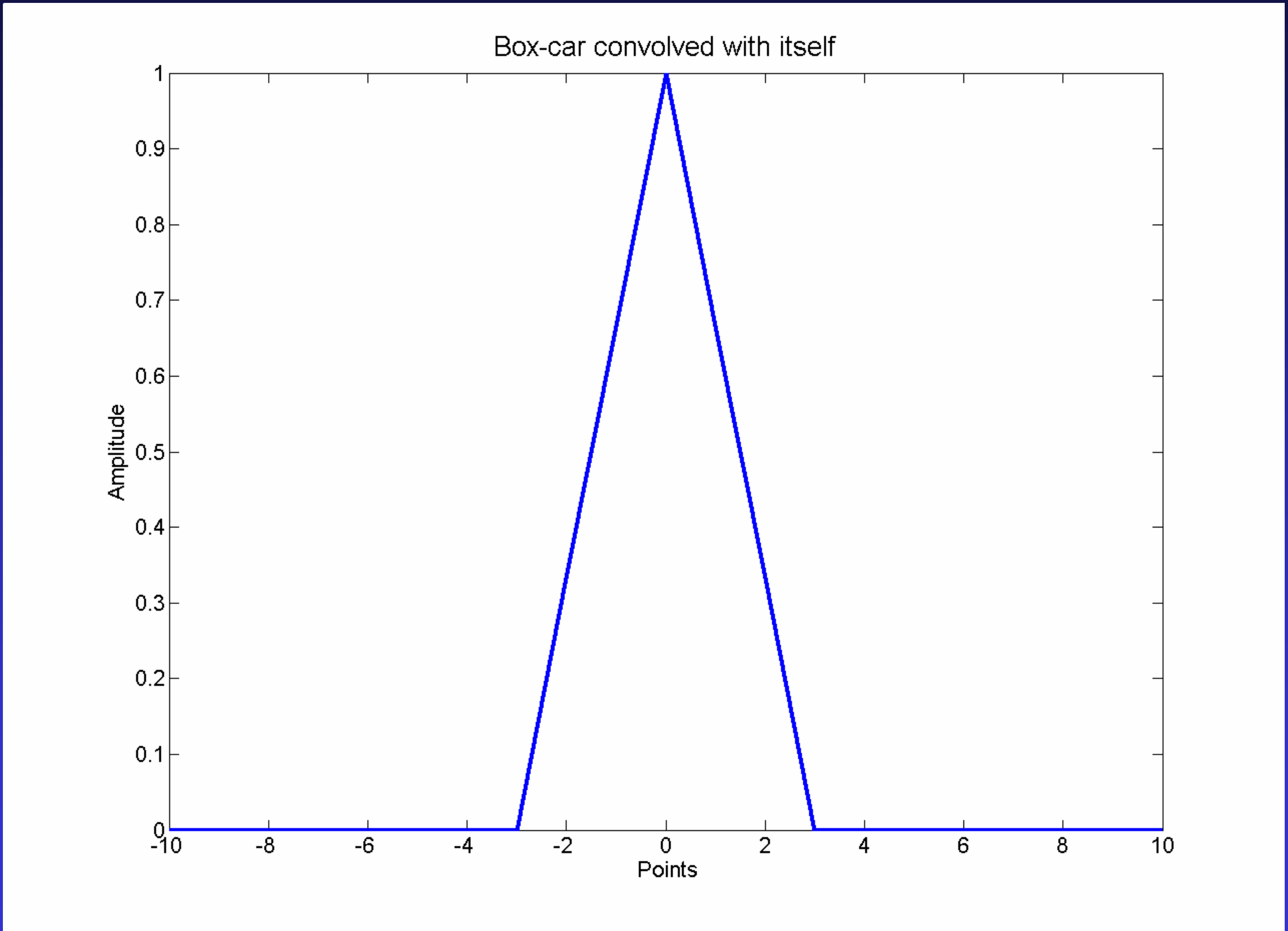
Boxcar

- Any time we truncate, we convolve with sinx/x in the "other" domain.
- Poor choice
- Can use a better shaped window
- Consider convolving a boxcar with itself (cascading)

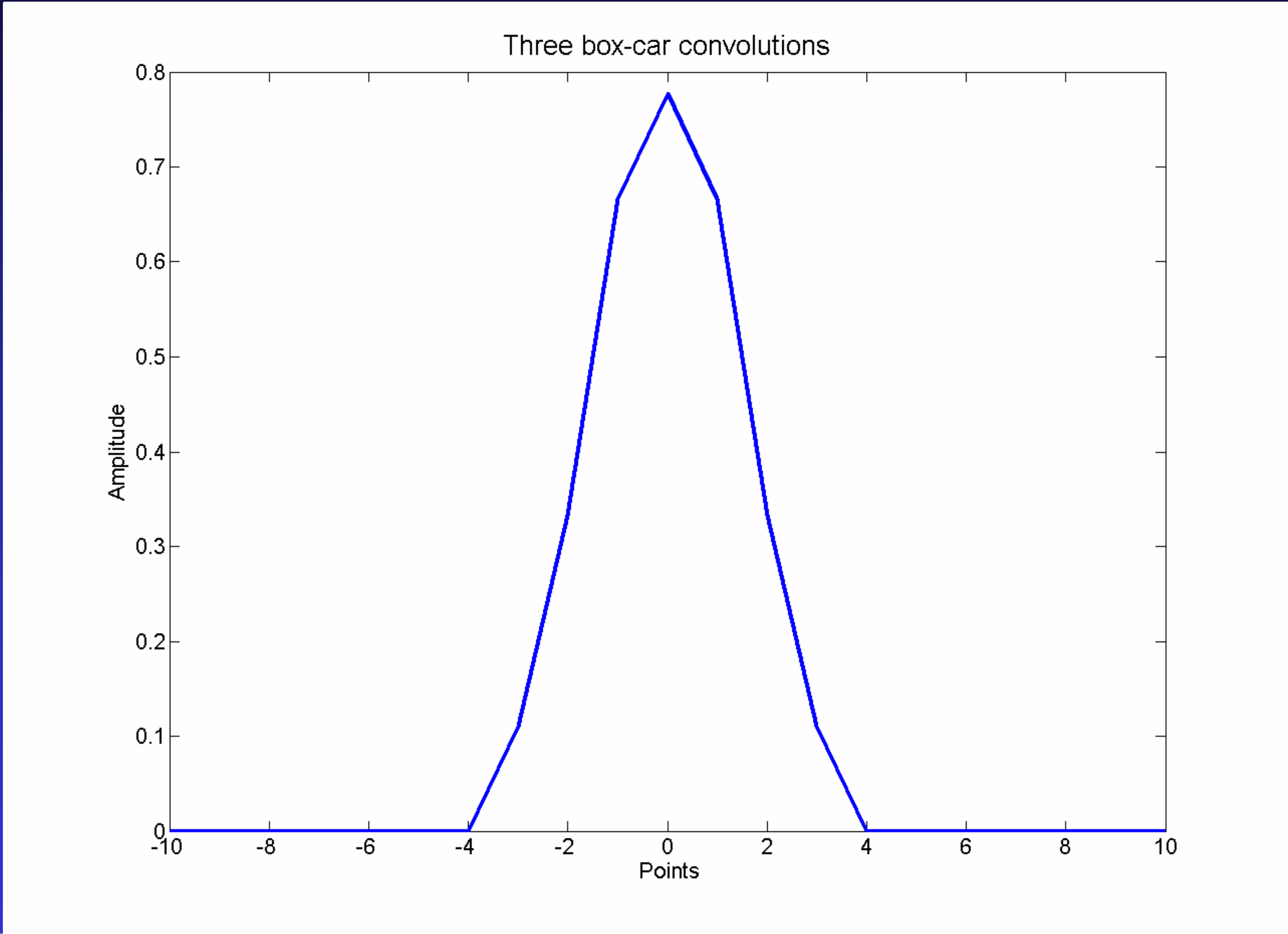
3 point boxcar



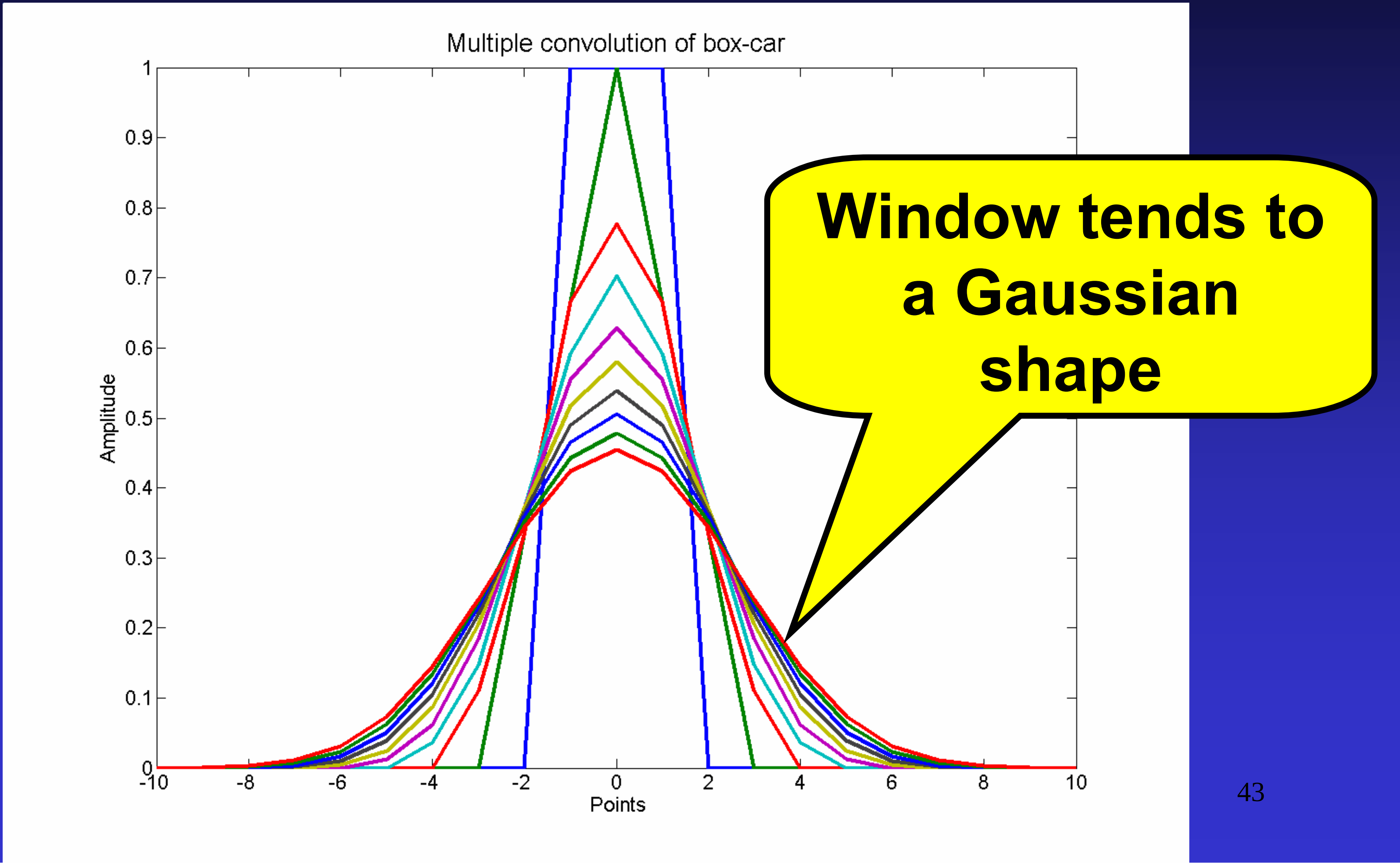
Boxcar convolved with itself



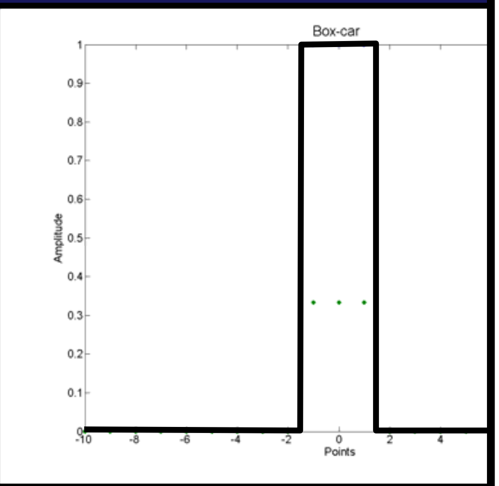
Cascading boxcar 3 times

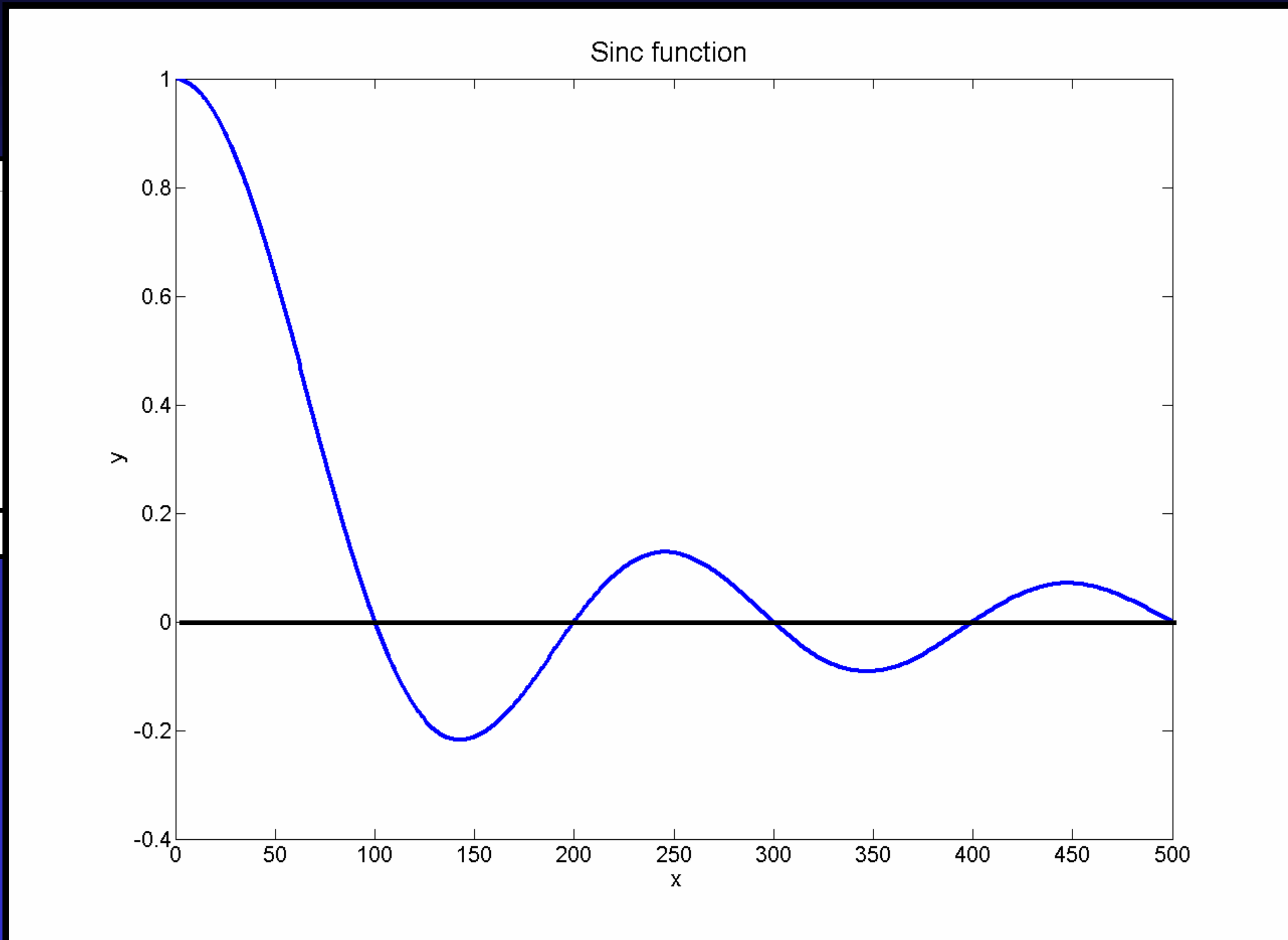


Cascading boxcar 1 to 10 times

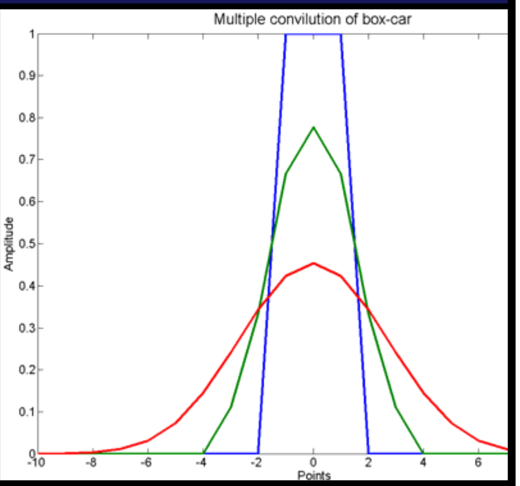


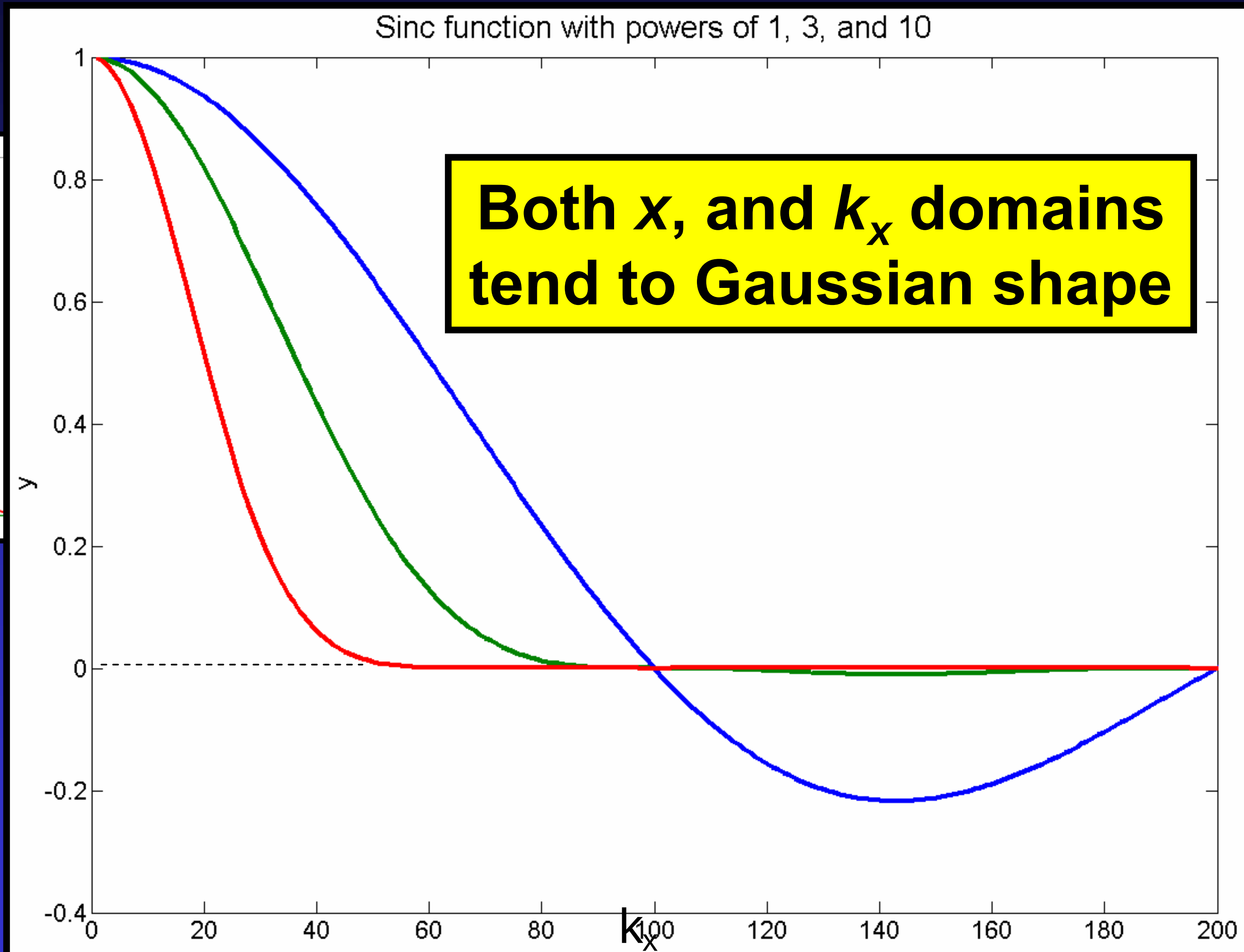
Boxcar in x, and sinc function in k_x



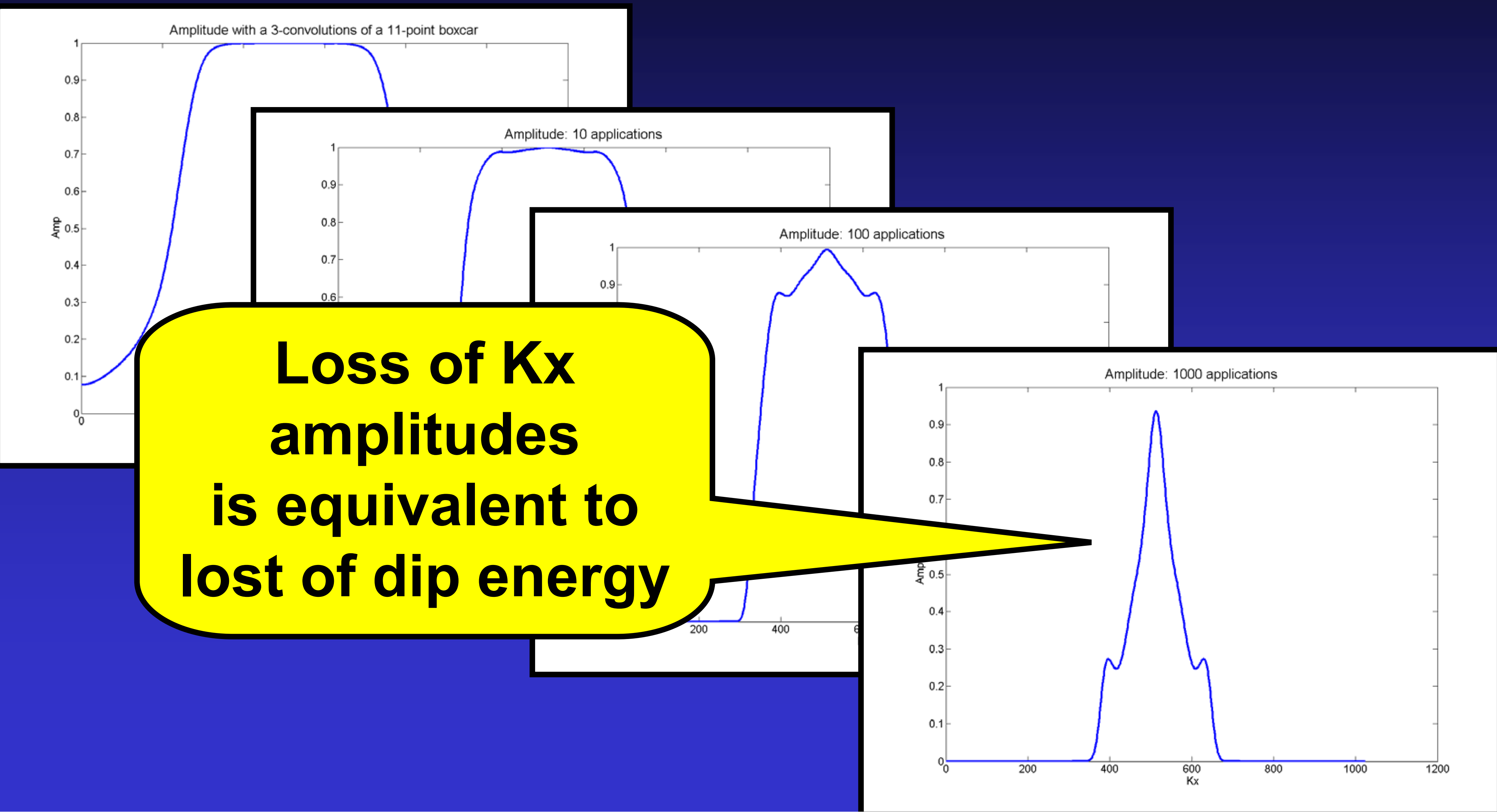


Cascading 1, 3, and 10





Application of 113 to amplitude of phase-shift operator, 1, 10, 100, 1000



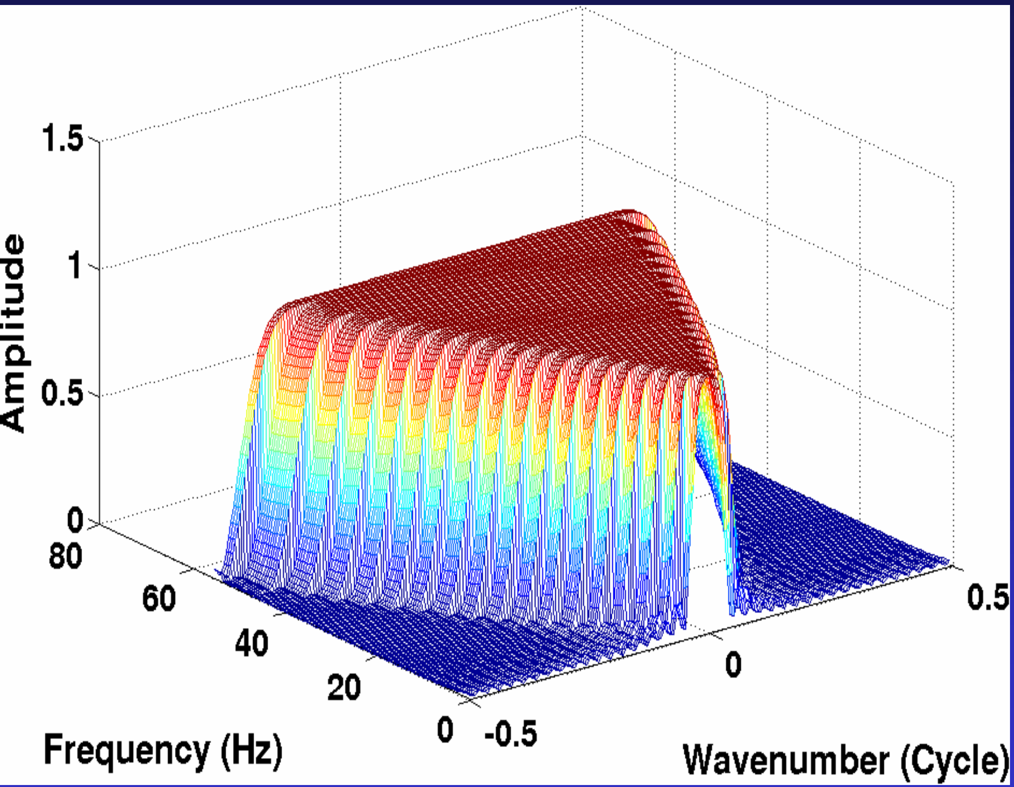
Comments on z X PS operator

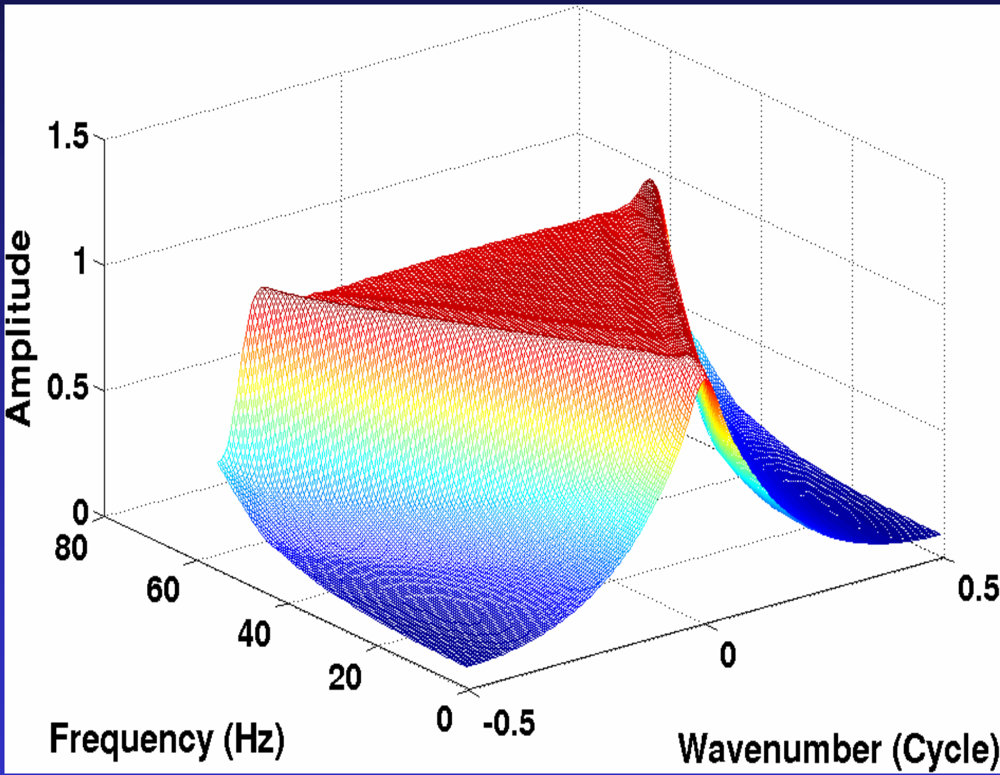
1. Any truncation in (x, z) space causes sinc ripple
2. Ripple reduced by improved window shape
3. Larger window in x space improves shape of PS op.
4. Larger window does not reduce amplitude of ripple
5. Operator at same frequency changes with velocity reducing cascading effect
6. Cascaded operator attenuates higher dips
7. But…lower part of section has reduced dips

Design goals for z X PS operator

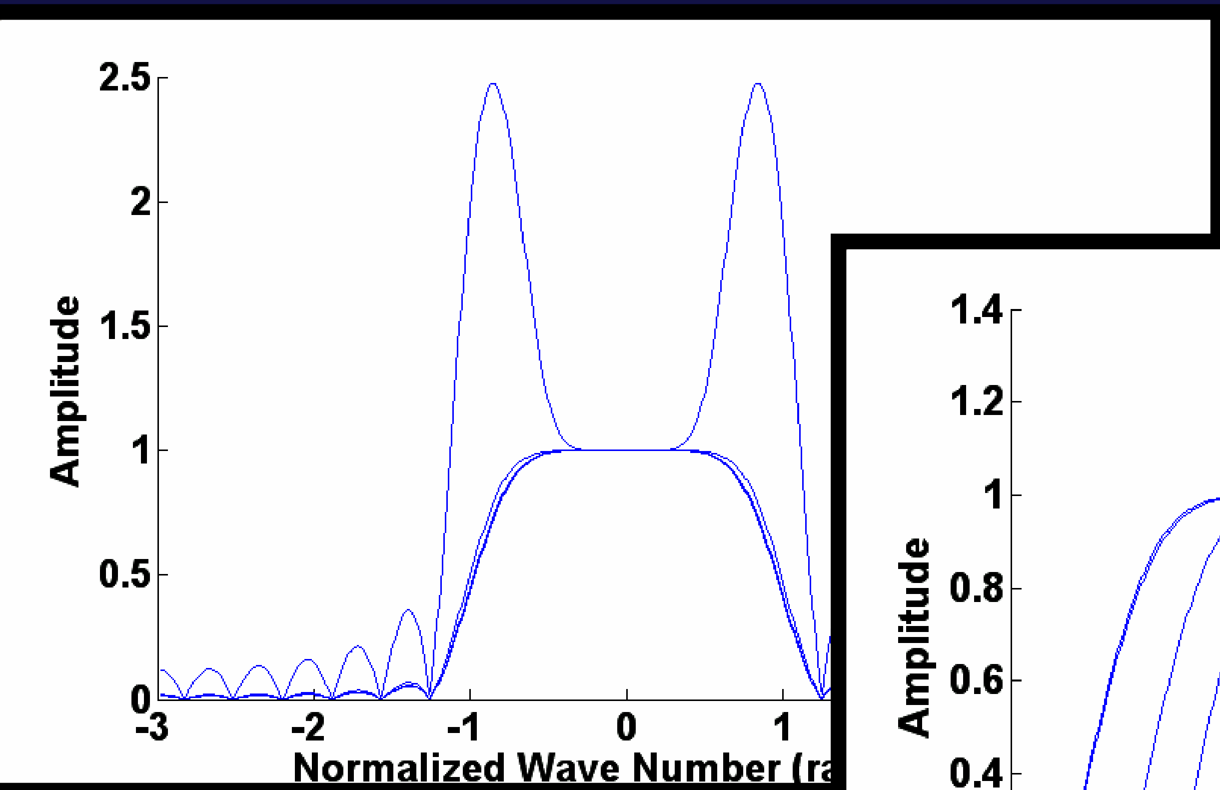
1. Shorter operator produces faster runtimes
2. Size of operator for acceptable attenuation of higher dips
3. Smoothness of operator related to stability in (k_x, z) space

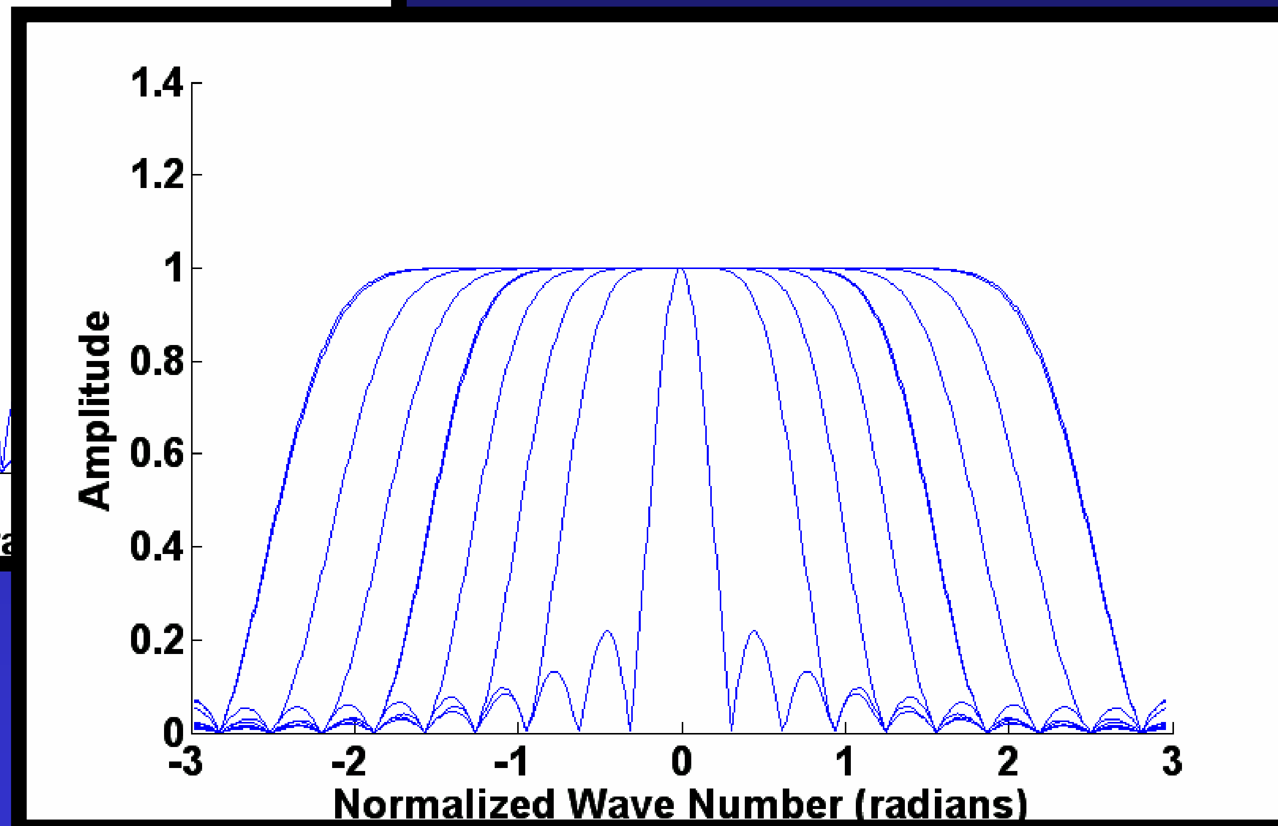
Downward contin. Extrapolators *Kun Liu*





Hale's extrapolators *Saleh Al-Saleh*






The End