Improvements to Spherical-Wave AVO Modeling

Chuck Ursenbach, Arnim Haase and Jon Downton

CREWES Sponsors Meeting

November 30, 2006

Research Reports

- Ursenbach, Haase and Downton, “Improved modeling of spherical-wave AVO”
- Ursenbach and Haase, “AVO modeling of monochromatic spherical waves: comparison to band-limited waves”

Outline

- Review: An efficient approach for modeling spherical-wave reflectivity
- Improvement 1: more compact and general expressions → shorter run time, higher *n* values
- Comparison to monochromatic results
- Improvement 2: fitting parameters to those of common wavelets
- Demonstration of updated Explorer

Efficient modeling of spherical-wave AVO

Standard Approach

•Obtain displacement spectrum for a single ω by numerical integration of weighted plane-wave reflection coefficients

•Repeat for other ω as required

•Multiply displacement and wavelet spectra and apply $FT^{-1} \rightarrow$ yields time trace

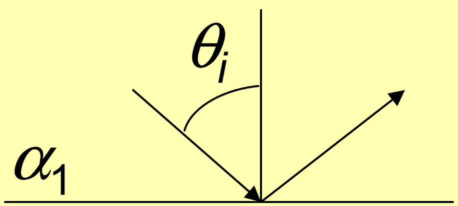
•Extract reflection coefficient from maximum in time trace envelope

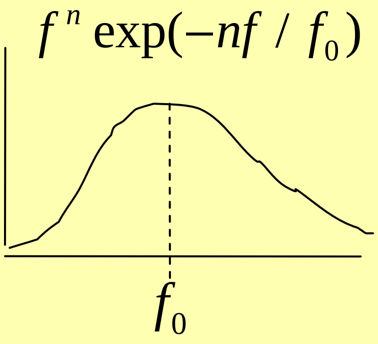
Efficient Approach

•Multiply Rayleigh wavelet spectrum and weighting function and integrate *analytically* over frequency

•Set time equal to ray theoretical arrival time

•Perform a single numerical integration to obtain reflection coefficient





Improvement #1

- Simplified expression

$$R_{\mathrm{PP}}^{\mathrm{sph}}(\theta_i) = \left[\int_0^1 - \int_{i0}^{i\infty}\right] R_{\mathrm{PP}}(\theta) W_n(\theta, \theta_i, S) d(\cos\theta).$$

$$\begin{aligned}
W_n &= -\frac{(nS)^{n+2}}{\overline{\tau}^{n+4}} \frac{B P_n(\overline{T}/\overline{\tau}) + C P_{n+1}(\overline{T}/\overline{\tau})}{1 + iSn/(n+1)}, \\
B &= (n+1)(i + nS)\overline{\tau}, \\
C &= -n^2(1+n)S^2 - inS[2(n+1) + \cos\theta\cos\theta_i] \\
&\quad + n(\sin^2\theta + \sin^2\theta_i) + 3(1 - \cos\theta\cos\theta_i) - 2(\cos\theta - \cos\theta_i)^2, \\
\overline{\tau} &= \sqrt{\overline{T}^2 + \sin^2\theta\sin^2\theta_i}, \\
&= \sqrt{(nS)^2 + 2inS(1 - \cos\theta\cos\theta_i) + (\cos\theta - \cos\theta_i)^2}, \\
\overline{T} &= nS + i(1 - \cos\theta\cos\theta_i).
\end{aligned}$$

- Shorter runtime
- Can use arbitrary values of *n*

Outline

- Review: An efficient approach for modeling spherical-wave reflectivity
- Improvement 1: more compact and general expressions → shorter run time, higher *n* values
- Comparison to monochromatic results
- Improvement 2: fitting parameters to those of common wavelets
- Demonstration of updated Explorer

Monochromatic wavelet

The spectrum of the displacement along the ray path:

$$u_{\|}(\omega) = Ai\omega\exp(-i\omega t)\int_0^{\infty}\frac{p}{\xi}R_{\mathrm{PP}}(p)\underbrace{\left[-\omega p J_1(\omega pr)\sin\theta_i + i\omega\xi J_0(\omega pr)\cos\theta_i\right]\exp[i\omega\xi(z+h)]}_{\text{weighting function}}\,dp.$$

Normalize by

$$u_{\|}^{R_{\mathrm{PP}}=1}(\omega) = A\left(-\frac{1}{R^2}+\frac{i\omega}{R\alpha_1}\right)\exp\left[-i\omega\left(t-\frac{R}{\alpha_1}\right)\right].$$

near-field

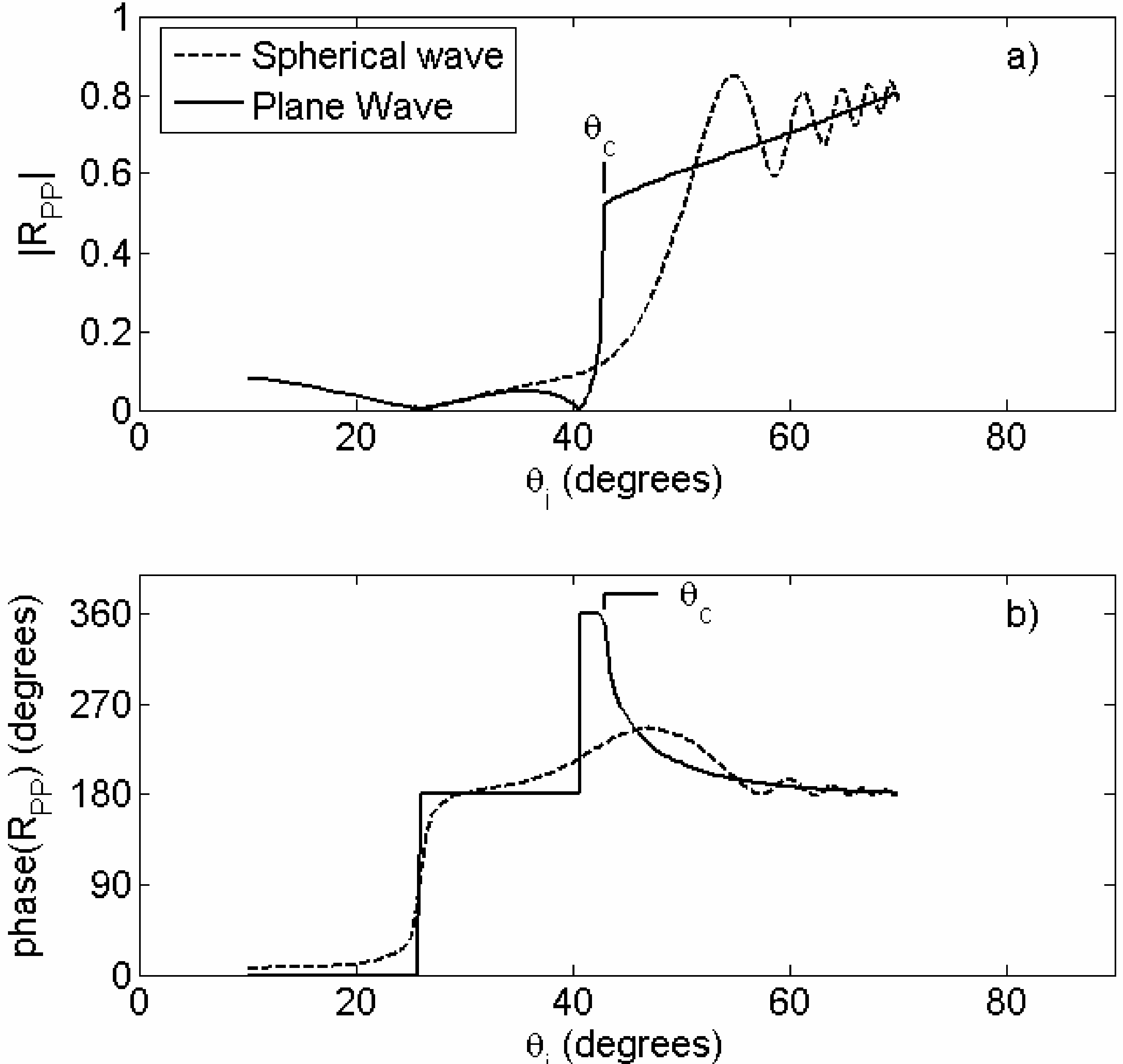
far-field

Final result:

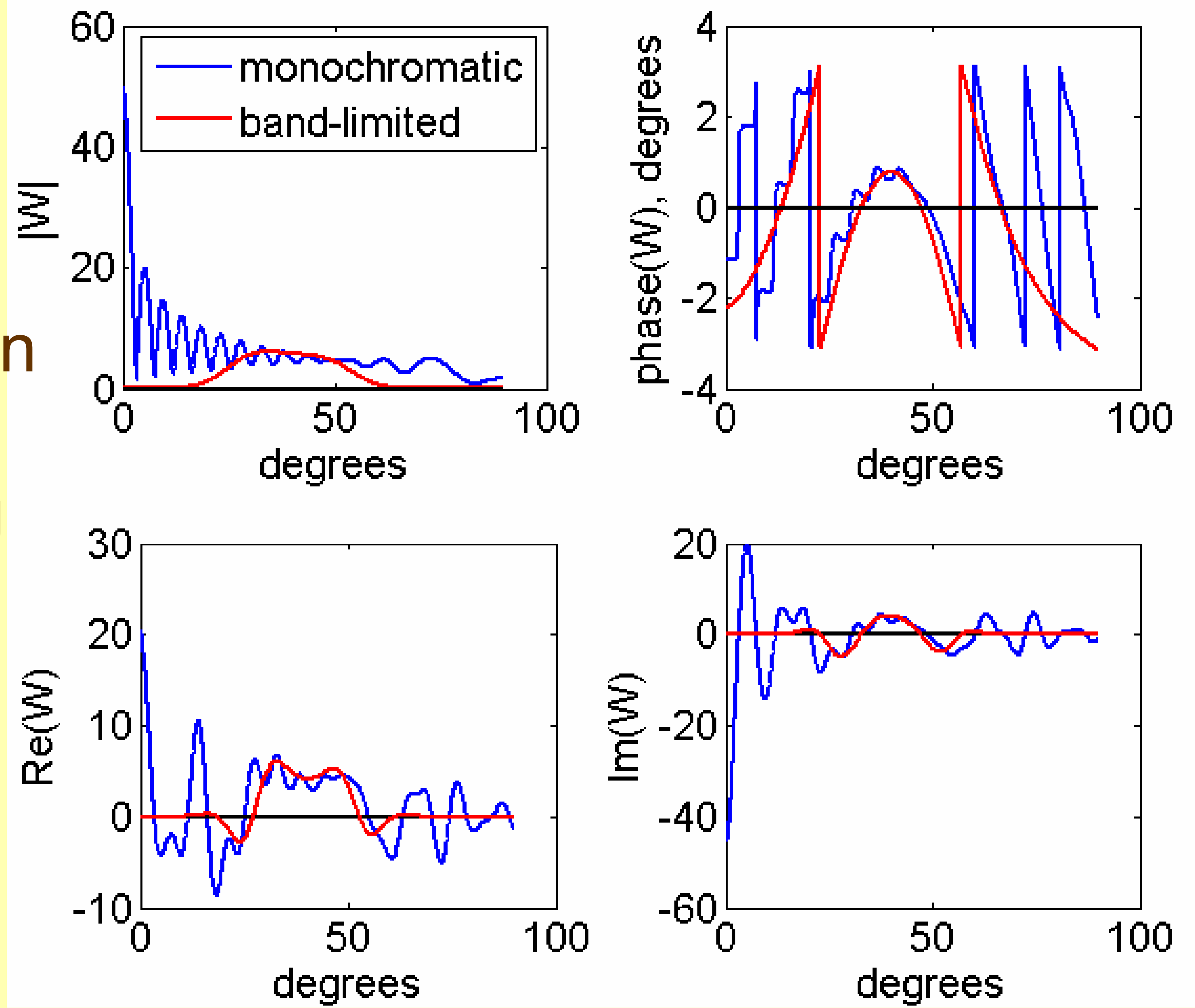
$$R_{\mathrm{PP}}^{\mathrm{spherical}}(\theta_i) = \int_{\Gamma} W(S,\theta,\theta_i)R_{\mathrm{PP}}(\theta)\,d(\cos\theta), \qquad S = \frac{\alpha_1}{R\omega},$$

$$W(S,\theta,\theta_i) = \frac{\left[-J_1(\sin\theta\sin\theta_i/S)\sin\theta\sin\theta_i + iJ_0(\sin\theta\sin\theta_i/S)\cos\theta\cos\theta_i\right]}{S(1-iS)\exp\left[i(1-\cos\theta\cos\theta_i)/S\right]}.$$

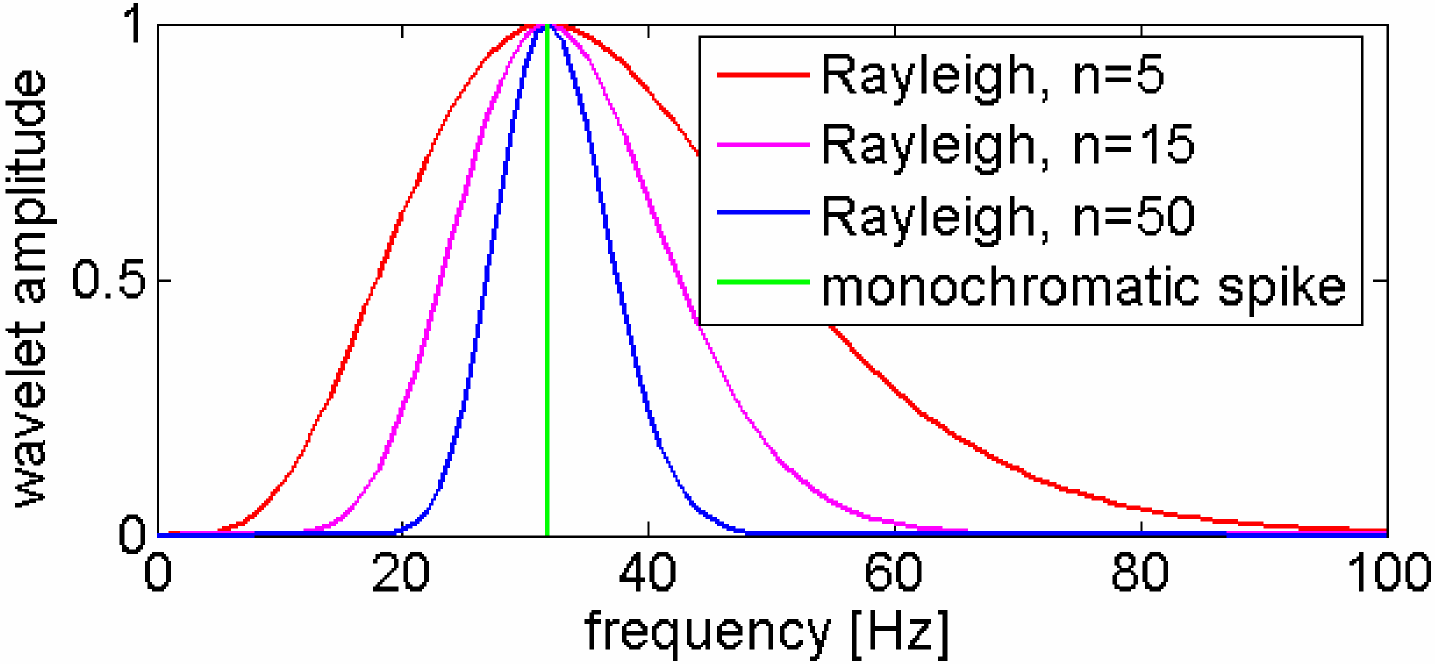
Reflectivity Curves for a Mono-chromatic Wavelet

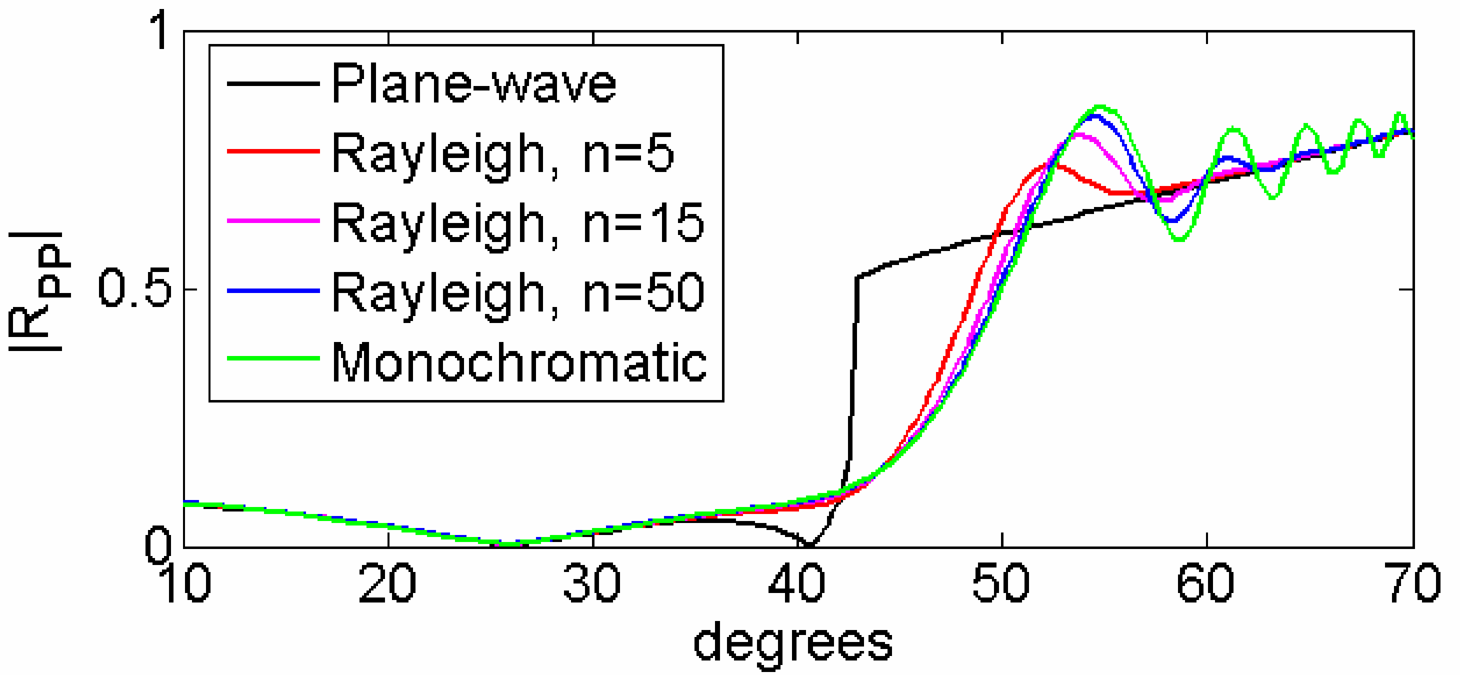


Comparison of Weighting Functions

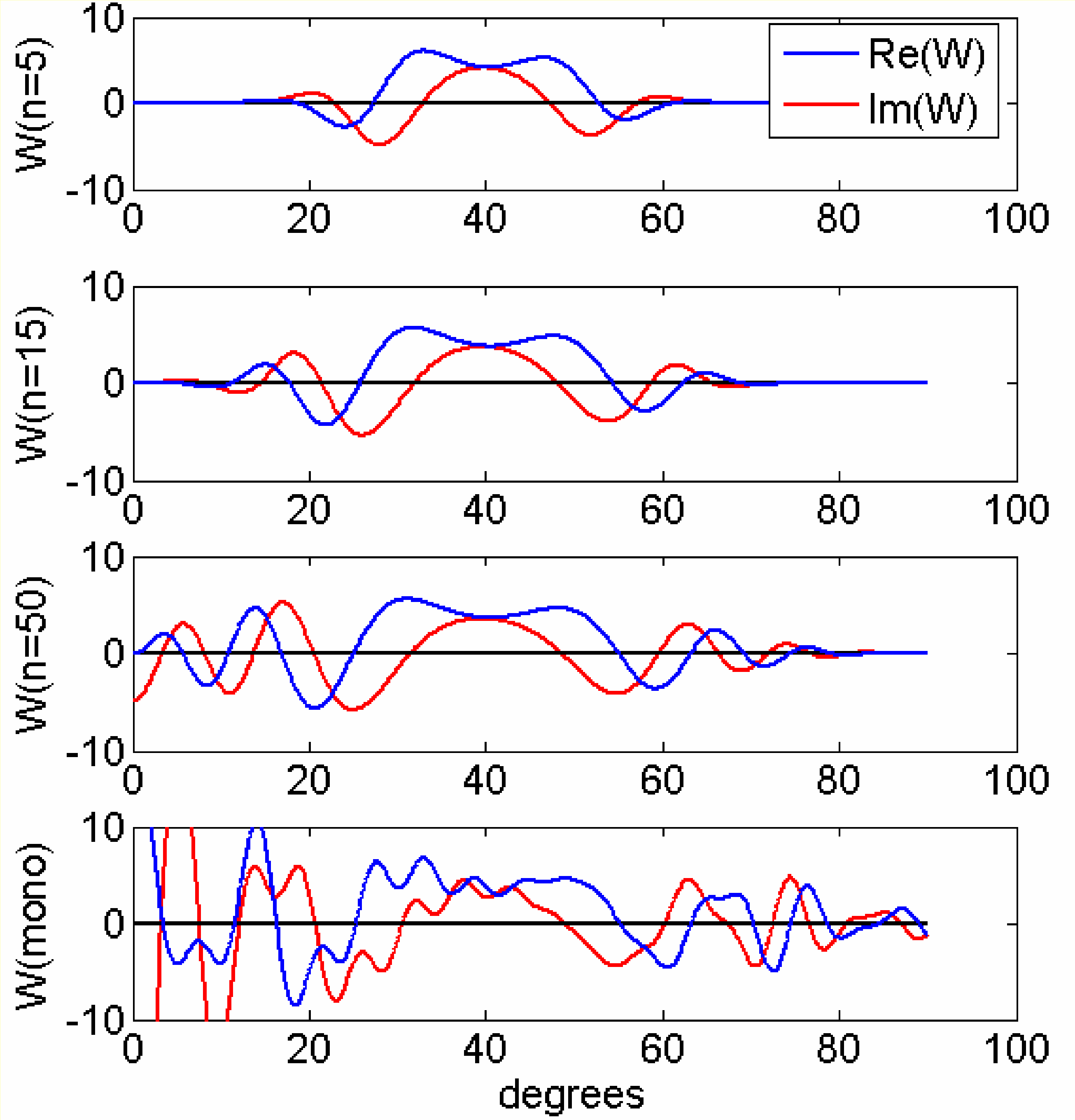


High *n* Reflectivity Curves for Rayleigh Wavelets





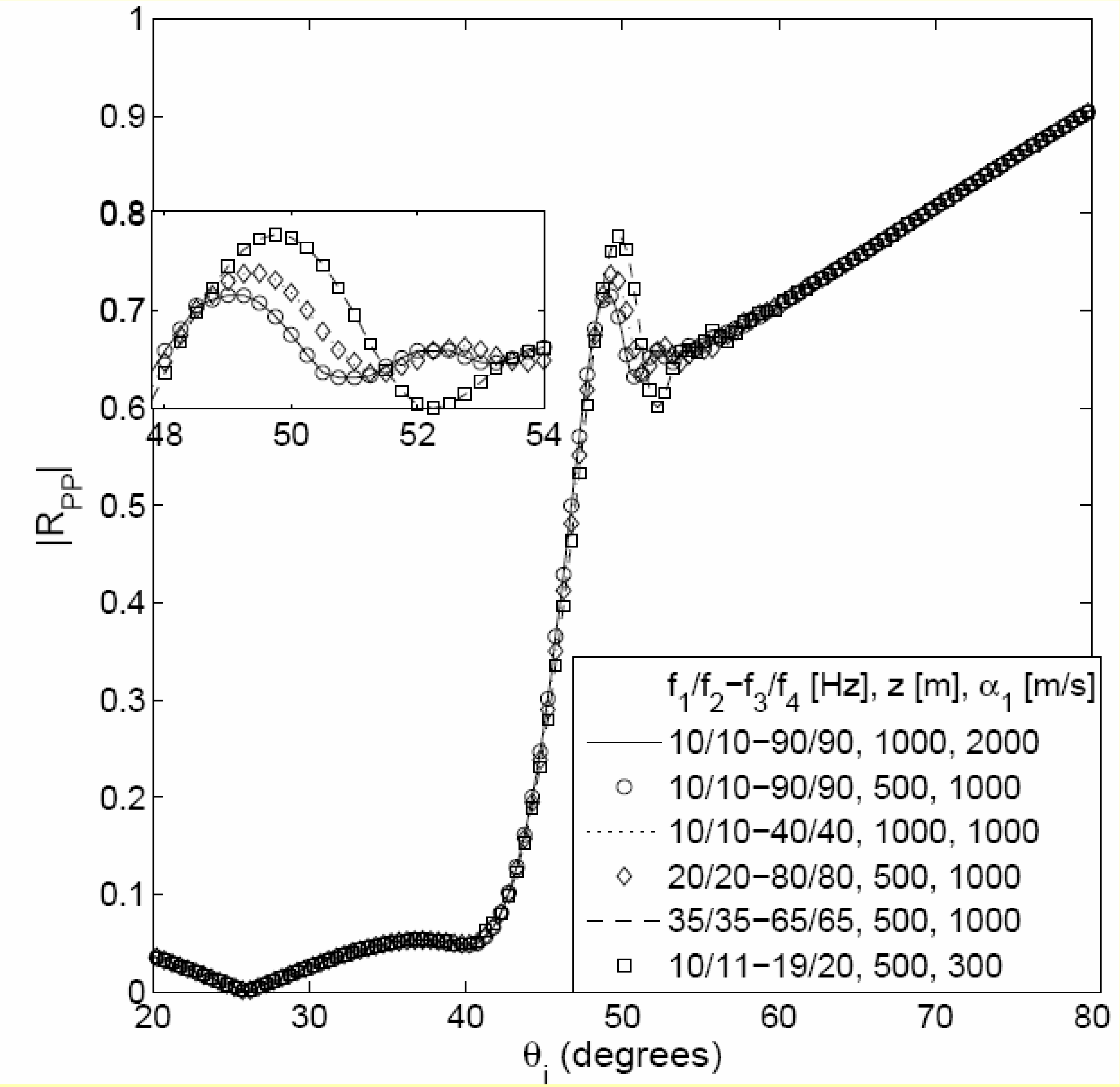
High *n* Weighting Functions



Outline

- Review: An efficient approach for modeling spherical-wave reflectivity
- Improvement 1: more compact and general expressions → shorter run time, higher *n* values
- Comparison to monochromatic results
- Improvement 2: fitting parameters to those of common wavelets
- Demonstration of updated Explorer

Fundamental Ormsby parameters



$$S_{\text{ave}} = \frac{\alpha_1}{(2Z)\omega_{\text{ave}}} \approx .0032$$

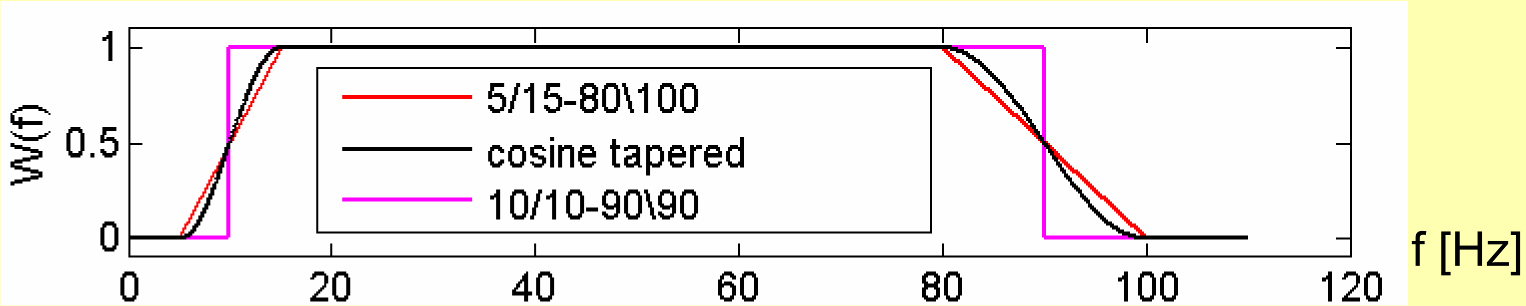
$$B_r = \frac{f_1 + f_2}{f_3 + f_4}$$

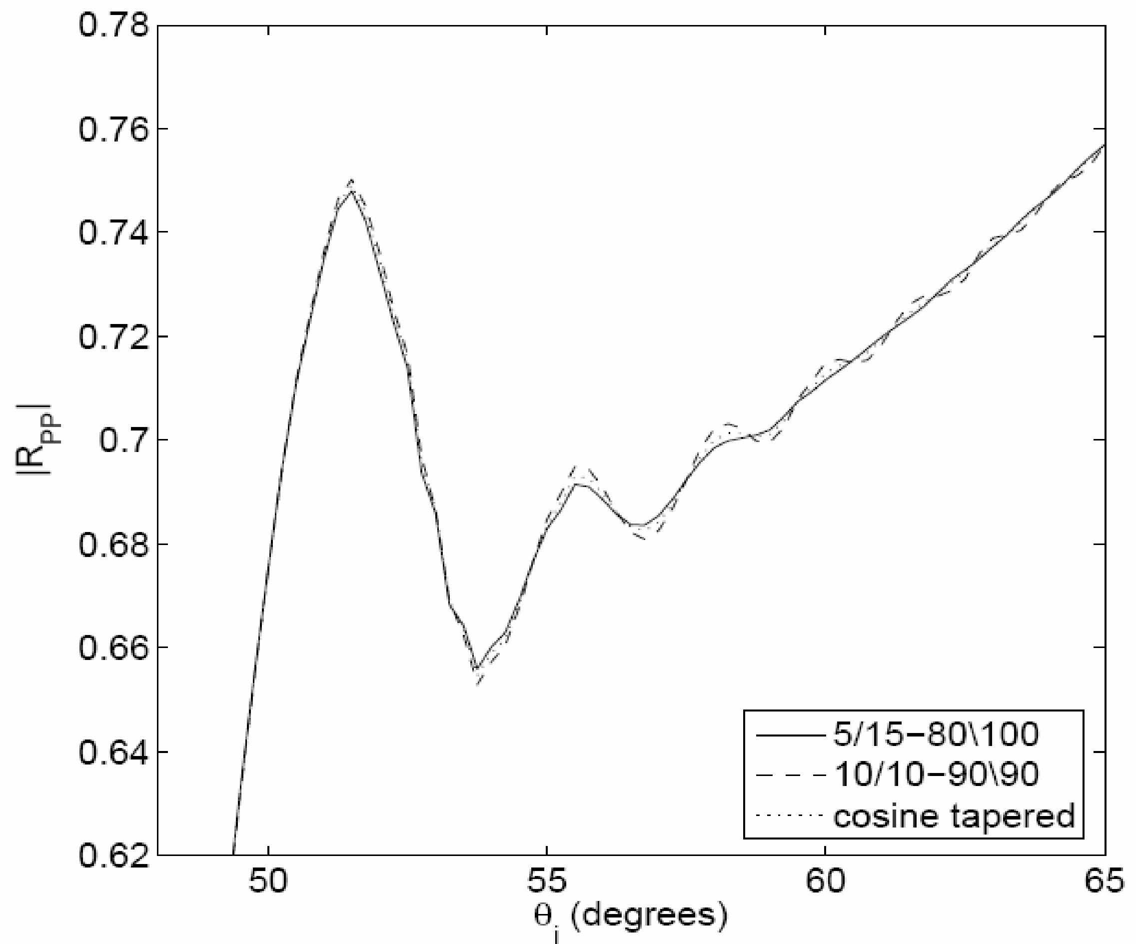
$$B_r \approx 0.11$$

$$B_r \approx 0.25$$

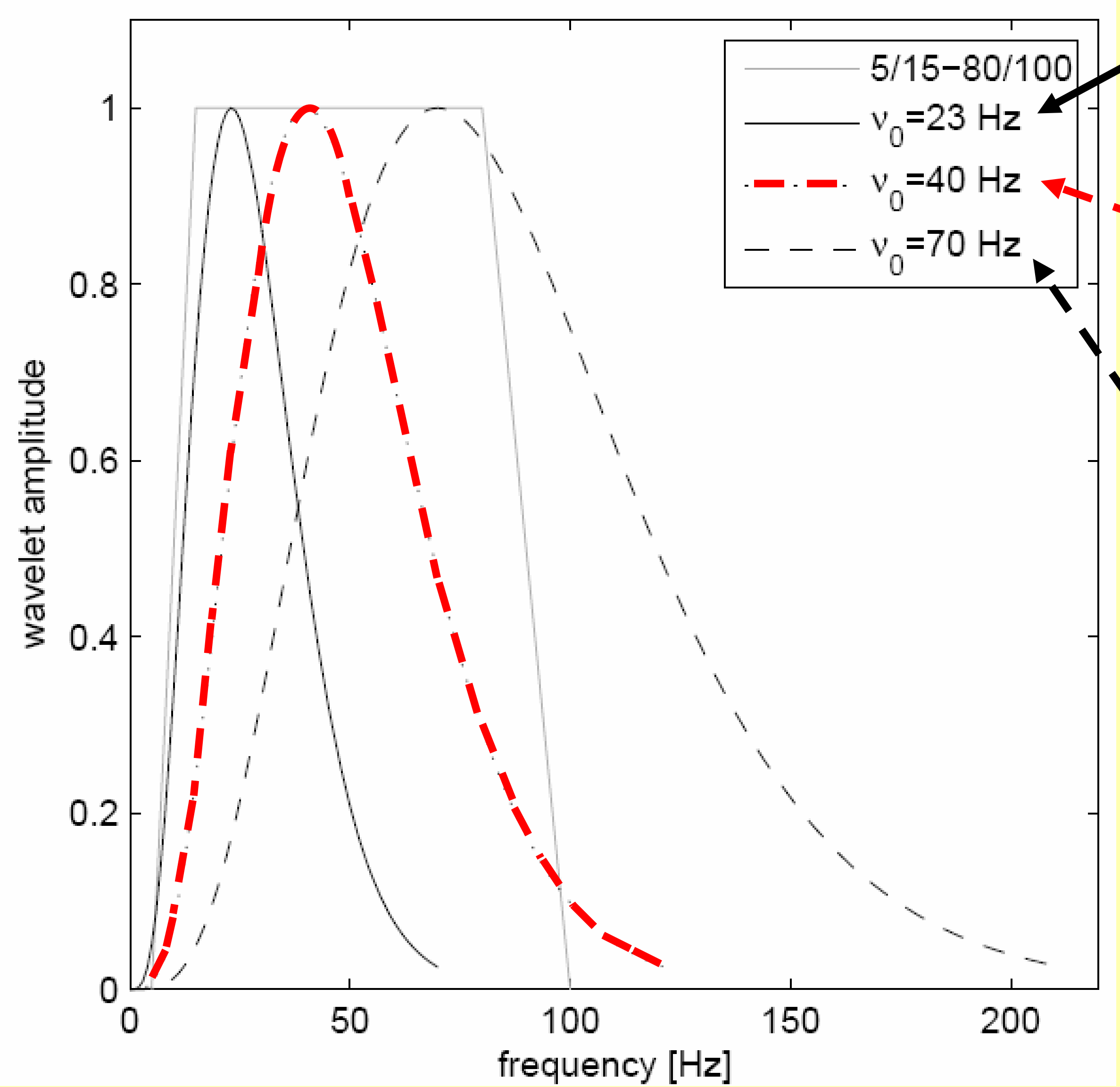
$$B_r \approx 0.54$$

Influence of tapers





Determining S

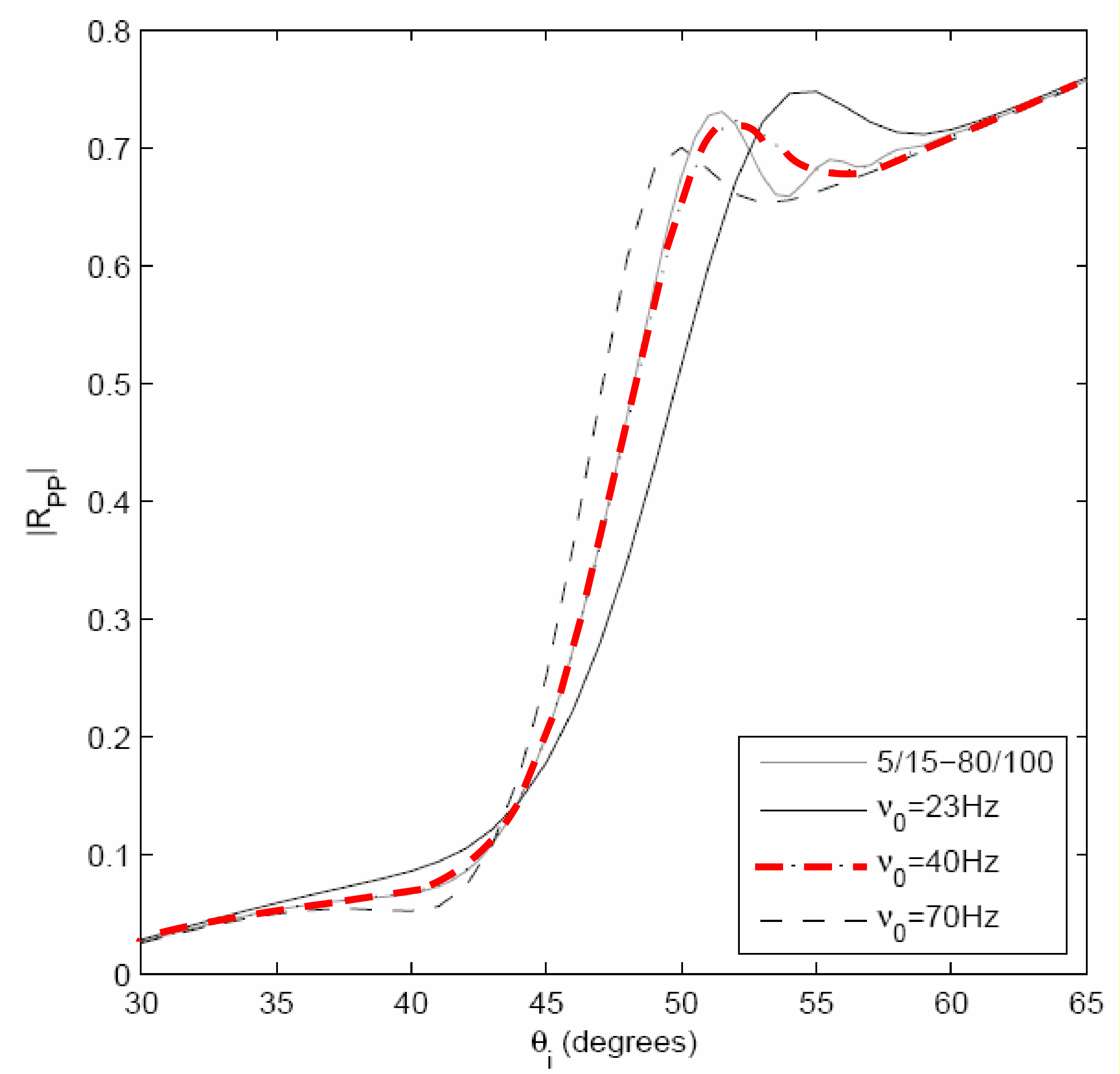


matches lower band edge

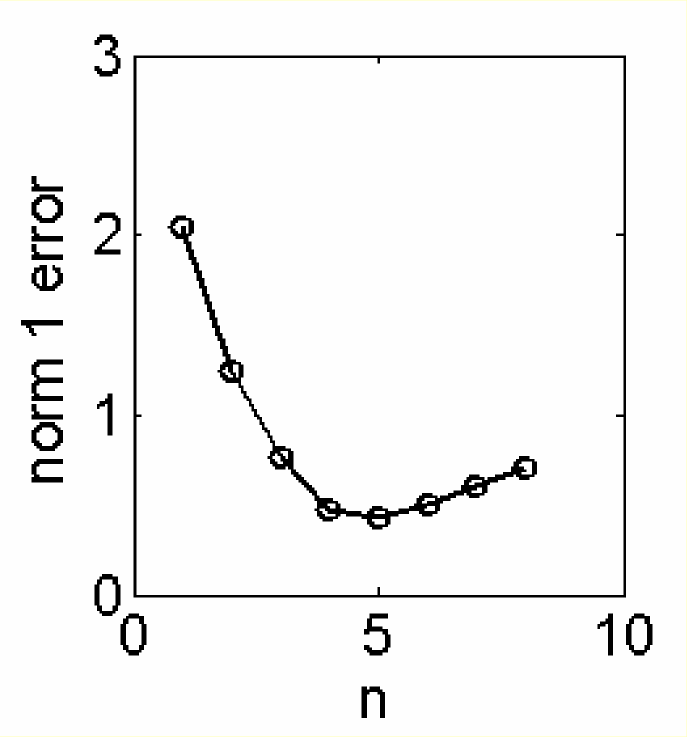
matches avg. frequency

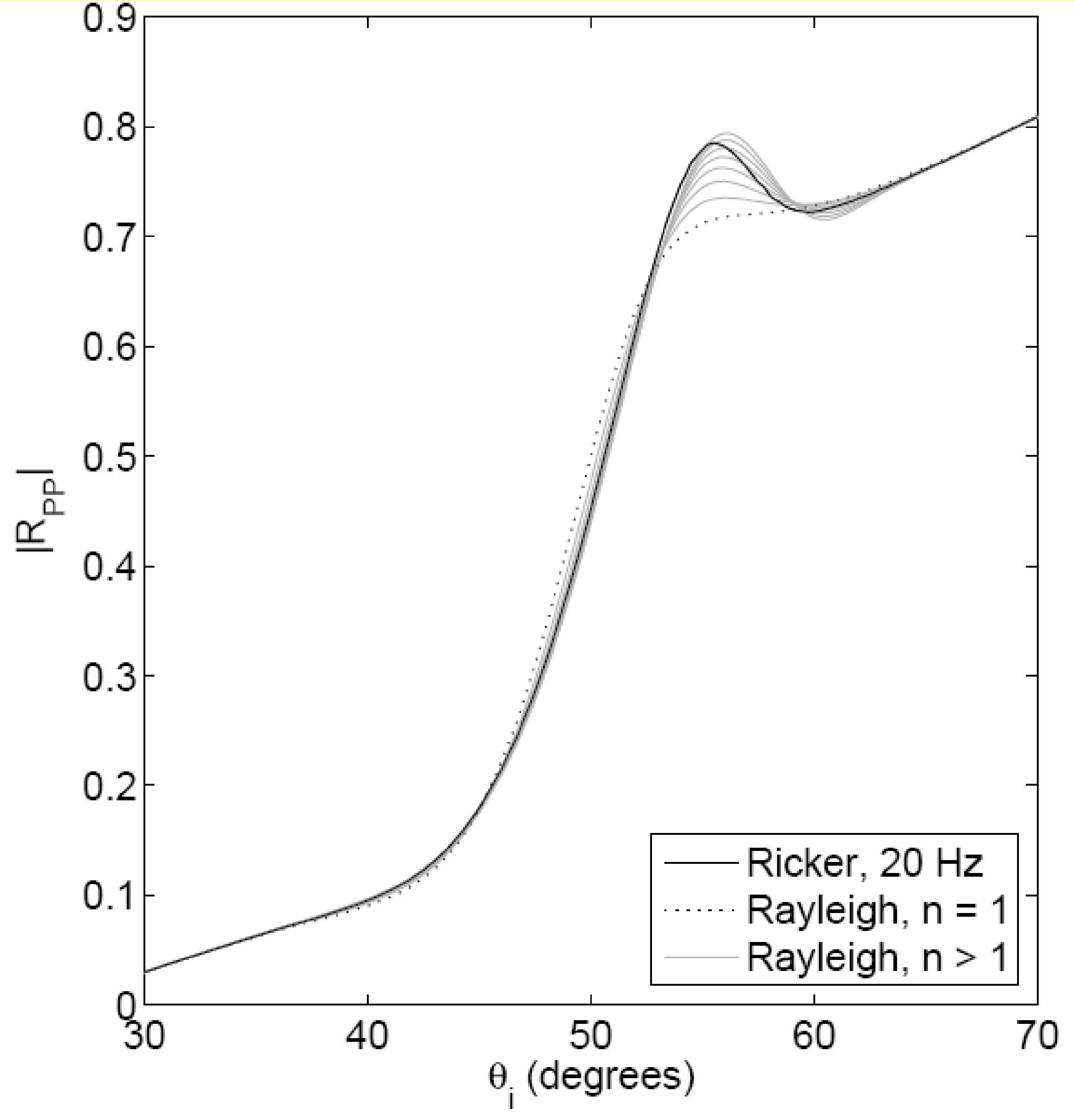
matches upper band edge and bandwidth

Determining *S*



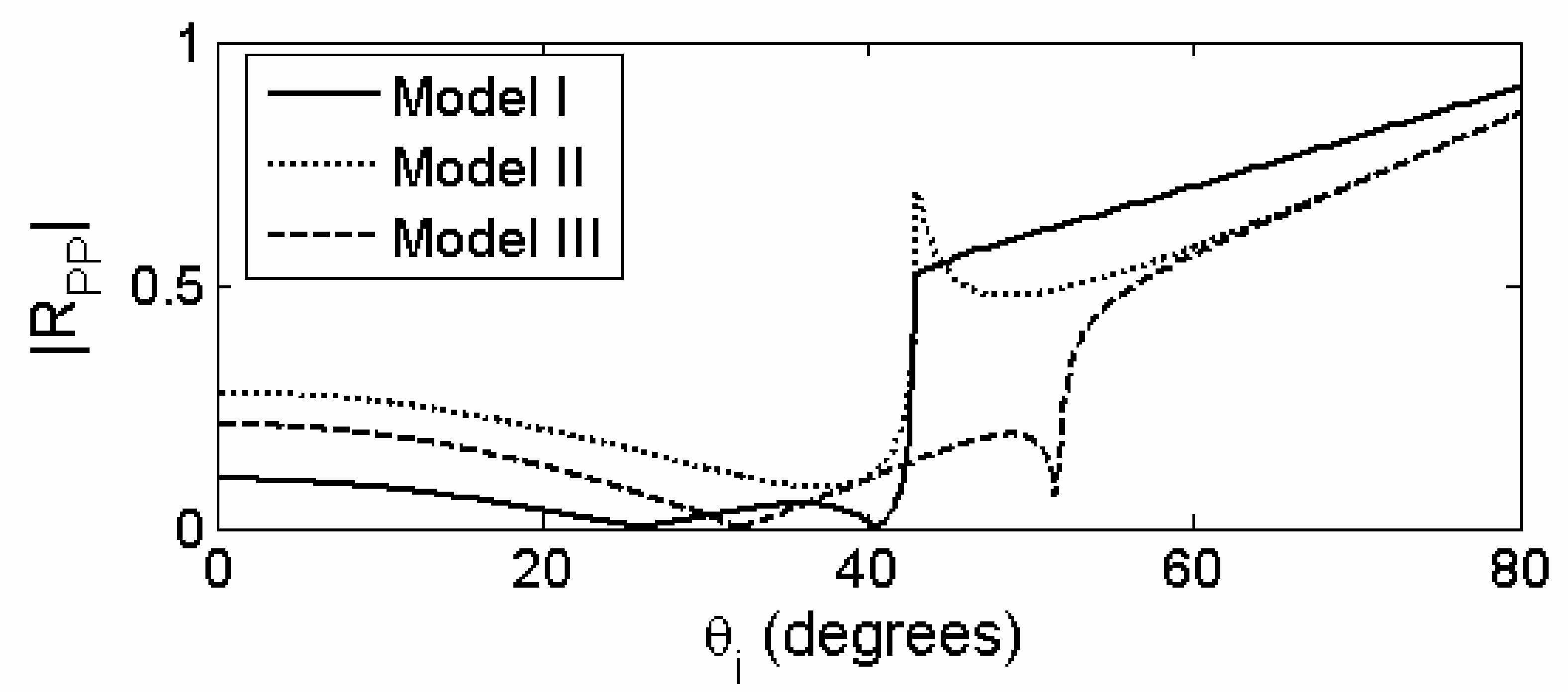
Determining *n*





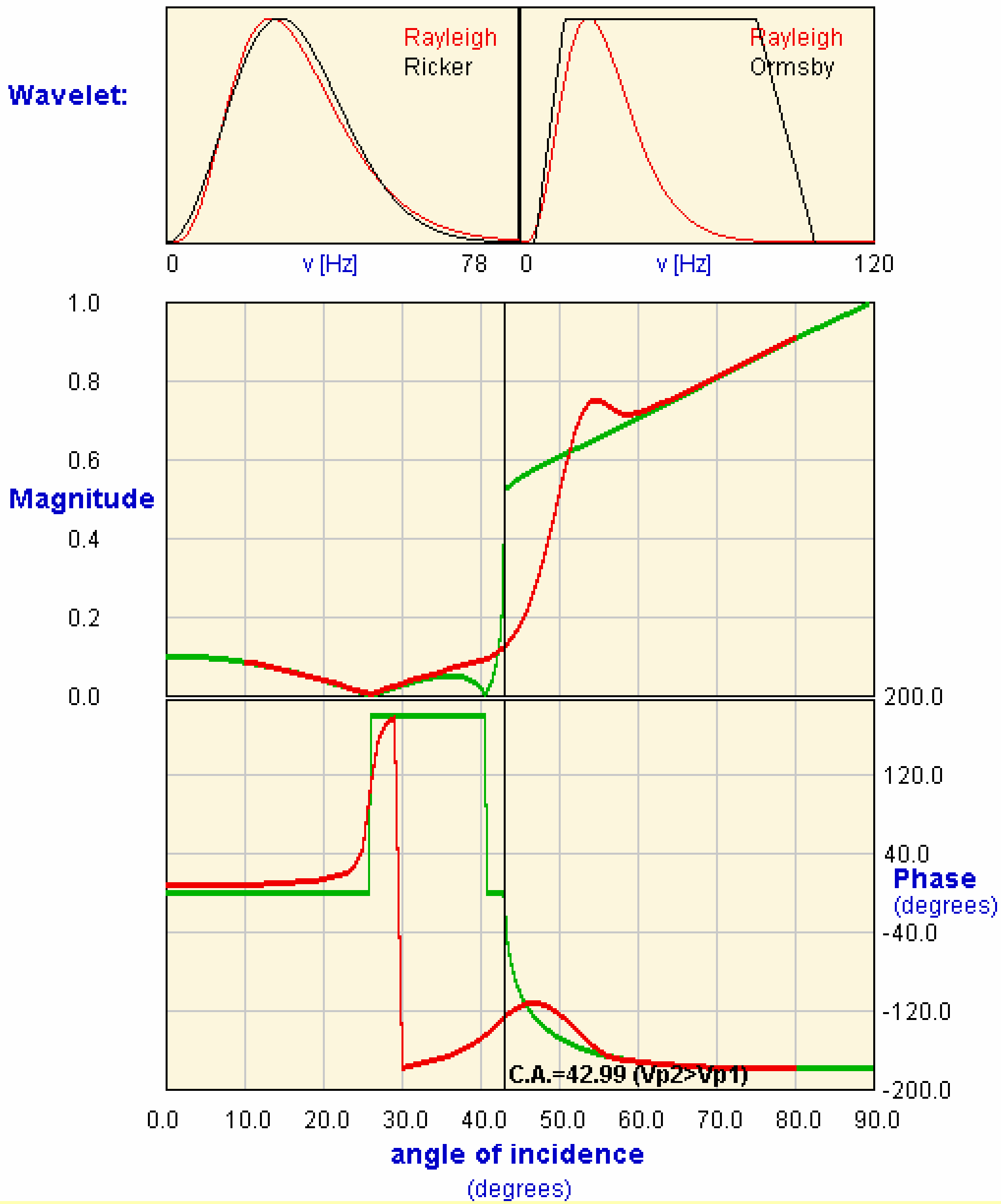
Determining *n*

Wavelet			Optimal n value for Rayleigh Wavelet		
Name	B_r	$\bar{S}_z$	Model I	Model II	Model III
Ricker	-	.01	5	5	5
		.1	5	5	5
		1	5	5	≥ 8
Ormsby	1/9	.01	3	4	3
		.1	3	3	3
		.5	3	3	4
Ormsby	1/4	.01	7	≥ 8	7
		.1	6	6	6
		.5	6	6	≥ 8



CREWES Spherical Zoeppritz Explorer

www.crewes.org



Sphericity: α1 / (2 Z f0) = 0.086 Z [m]: 500.0

◉ Rayleigh f0 [Hz]: 23.121 n: 4

○ Ricker f0-Ricker) 25.0

○ Ormsby 5.0 | 15.0 | 80.0 | 100.0

Ave.freq.: Rayleigh 28.901, Ricker 28.209, Ormsby 50.156

Upper layer density (ρ1): 2400.0 kg/m³

Upper layer Vp (α1): 2000.0 m/s

Upper layer Vs (β1): 880.0 m/s

Lower layer density (ρ2): 2000.0 kg/m³

Lower layer Vp (α2): 2933.0 m/s

Lower layer Vs (β2): 1882.0 m/s

☑ Spherical Zoeppritz ☐ Spherical Aki-Richards

☑ Zoeppritz ☐ Aki-Richards

Angle limits (integers, 0 to 90): 0 | 90

Magnitude limits: 0.0 | 1.0

Phase limits (integers): -200 | 200

Click here to recalculate graph

Units: ◉ m/s and kg/m³ ○ ft/s and g/cm³

Conclusions

- The Spherical Zoeppritz Explorer is now based on simpler and more general expressions
- It now runs more quickly, and allows one to choose values of $n>5$
- As before, the Rayleigh wavelet parameters may be input directly
- A new feature is that the Rayleigh wavelet parameters may alternatively be calculated to represent a Ricker or Ormsby wavelet whose parameters are input instead