The relationship between dry rock bulk modulus and porosity – an empirical study

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Introduction

- There are various approaches to computing K_{dry} as a function of porosity, and thus inferring changes in fluid content as a function of porosity.
- One common approach is the pore space stiffness method, and a second approach uses critical porosity.
- Mavko and Mukerji (1995) propose a useful template for plotting the various porosity functions.
- Using a dataset collected by De Hua Han, and the Mavko-Mukerji template, we will evaluate the suitability of each method.
- Using the Han dataset, we will also derive an empirical relationship between pore space stiffness and pressure.

Pore space stiffness

- To model dry rock bulk modulus at different porosities, we can use pore space stiffness, the inverse of the dry rock space compressibility at a constant pore pressure, written:

$$\frac{1}{K_{dry}} = \frac{1}{K_m} + \frac{\phi}{K_\phi}, \quad \text{where:}$$

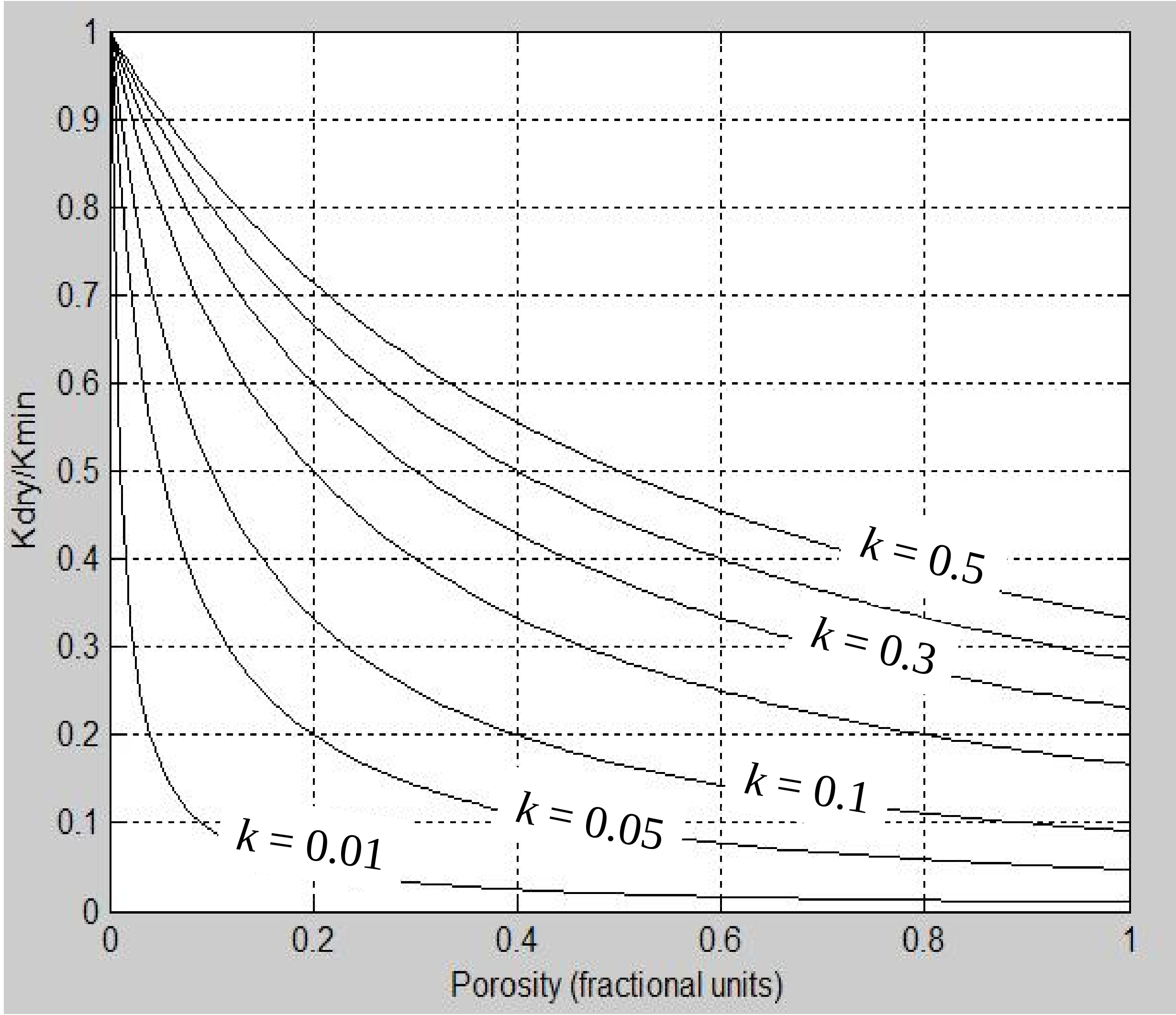
K_{dry} = dry rock bulk modulus, K_m = mineral bulk modulus,

K_ϕ = pore space stiffness, and ϕ = porosity.

- If we calculate K_{dry} directly, and divide through by K_m, this equation can be re-written as:

$$\frac{K_{dry}}{K_m} = \frac{1}{1 + \dfrac{\phi}{k}}, \quad \text{where: } k = \frac{K_\phi}{K_m}.$$

Constant pore space stiffness



Note that a family of constant k curves can be drawn on a plot of K_{dry}/K_m versus porosity, allowing us to estimate K_ϕ trends from rock physics measurements.

Voigt and Reuss bounds

- Mavko and Mukerji,(1995), discuss other models such as the Voigt (high bound) and Reuss (low bound) averages, given by:

$$K_{dry}^{Voigt} = K_m(1-\phi) + K_{air}\phi \approx K_m(1-\phi) \Rightarrow \frac{K_{dry}^{Voigt}}{K_m} = 1-\phi$$

$$K_{dry}^{Ruess} = \left[\frac{1-\phi}{K_m} + \frac{\phi}{K_{air}}\right]^{-1} \approx 0$$

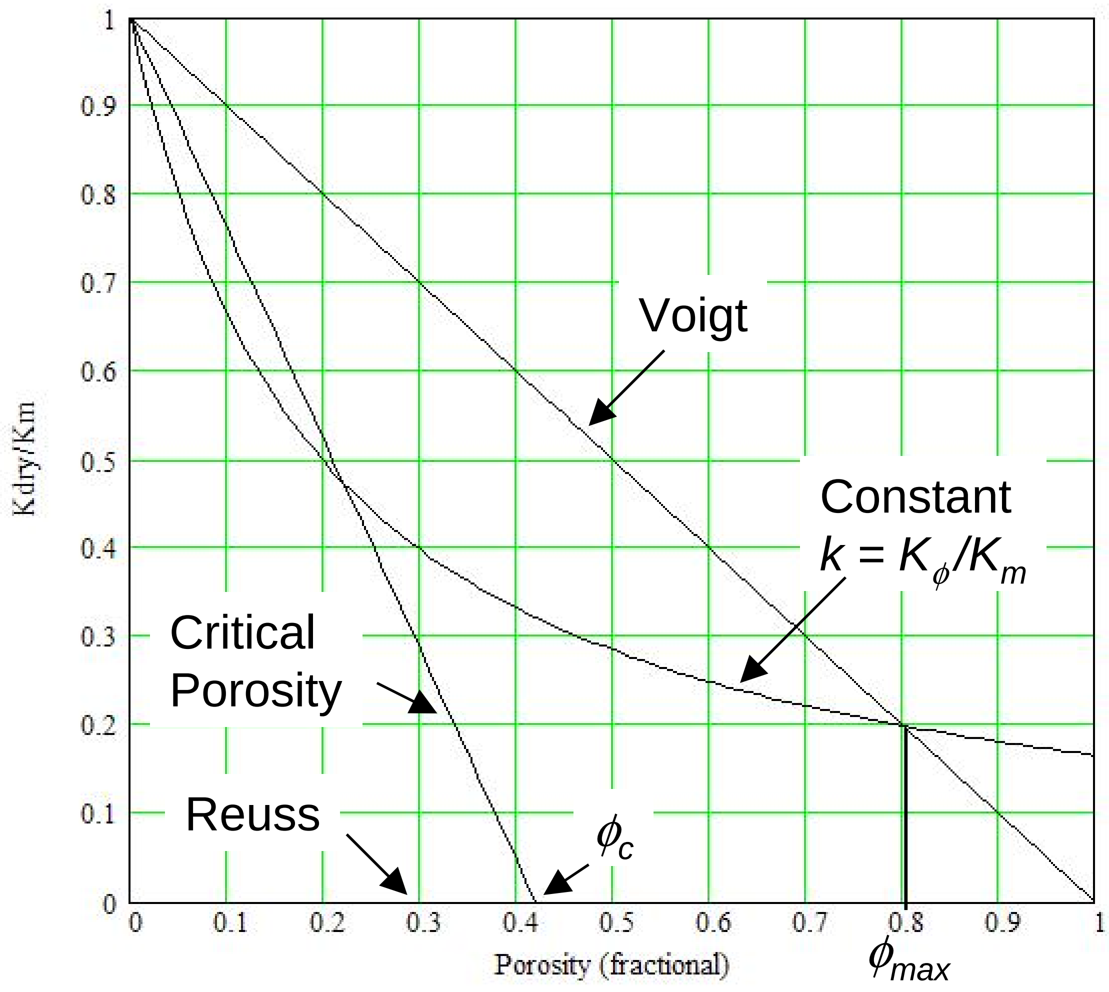
- The critical porosity model (Nur, 1992) is given by:

$$\frac{K_{dry}}{K_m} = 1 - \frac{\phi}{\phi_c}\left(1 - \frac{K_{dry}^{Voigt}}{K_m}\right) \approx 1 - \frac{\phi}{\phi_c}, \text{ where } \phi_c = \text{critical porosity.}$$

- ϕ_c separates load-bearing sediments ($\phi < \phi_c$) from suspensions ($\phi > \phi_c$) and is like a scaled Voigt model.

The Mavko-Mukerji Template

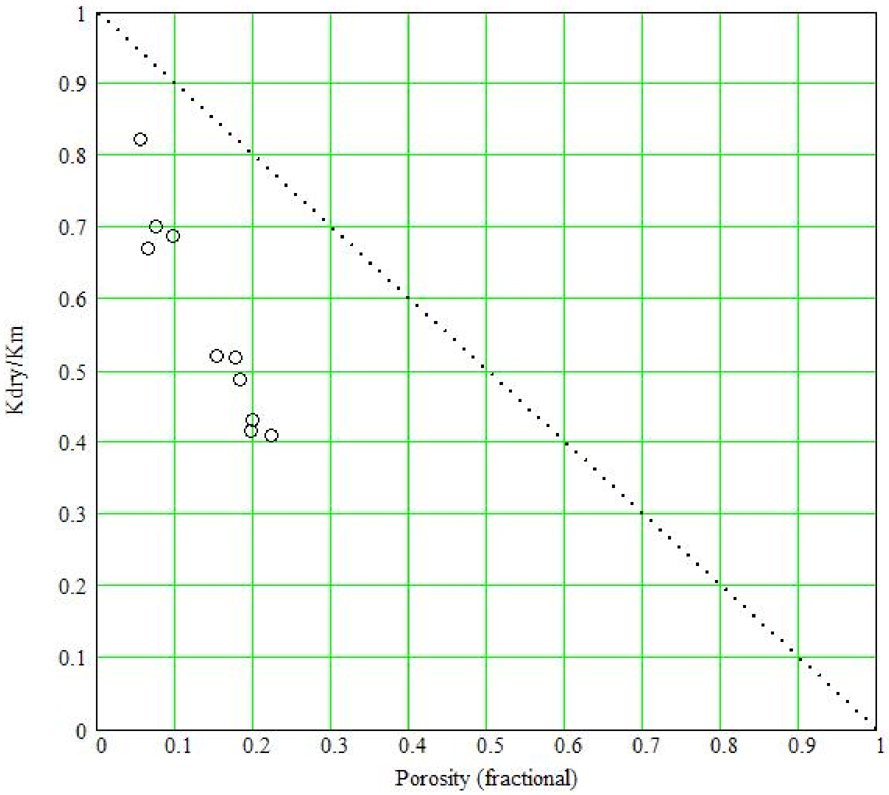
Mavko and Mukerji devised a template which plotted K_{dry}/K_m against porosity, and with which the various models could be compared. Here is their template with the different curves annotated.



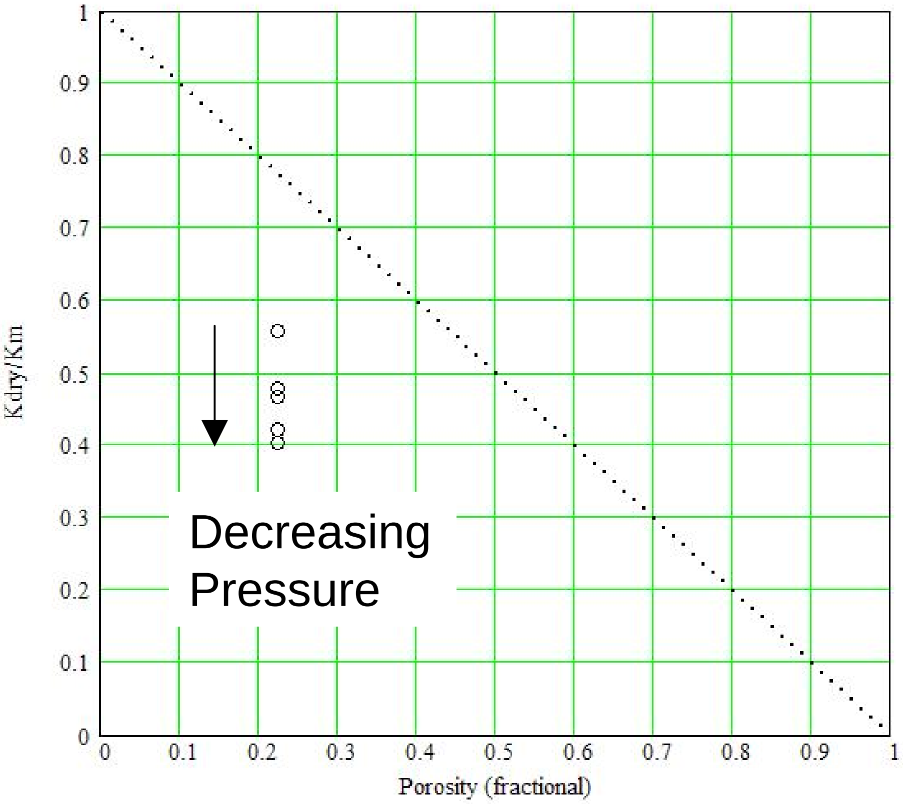
Han’s Dataset

- To test the various models, Mavko and Mukerji (1995) used a dataset collected by De Hua Han for his 1986 Ph.D. thesis. This dataset was graciously provided to us by Dr. Han of the University of Houston.
- Han’s dataset consisted of a number of sandstones of various porosities and clay content, measured at different pressures and saturations.
- Han was able to derive empirical formulae for *P* and *S*-wave velocities versus porosity and clay content (Han et al., 1986).
- From Han’s measurements, Mavko and Mukerji (1995) used the 10 clean sandstones at 40 MPa and a single clean sandstone at 5, 10, 20, 30, and 40 MPa.

Data points used by Mavko and Mukerji



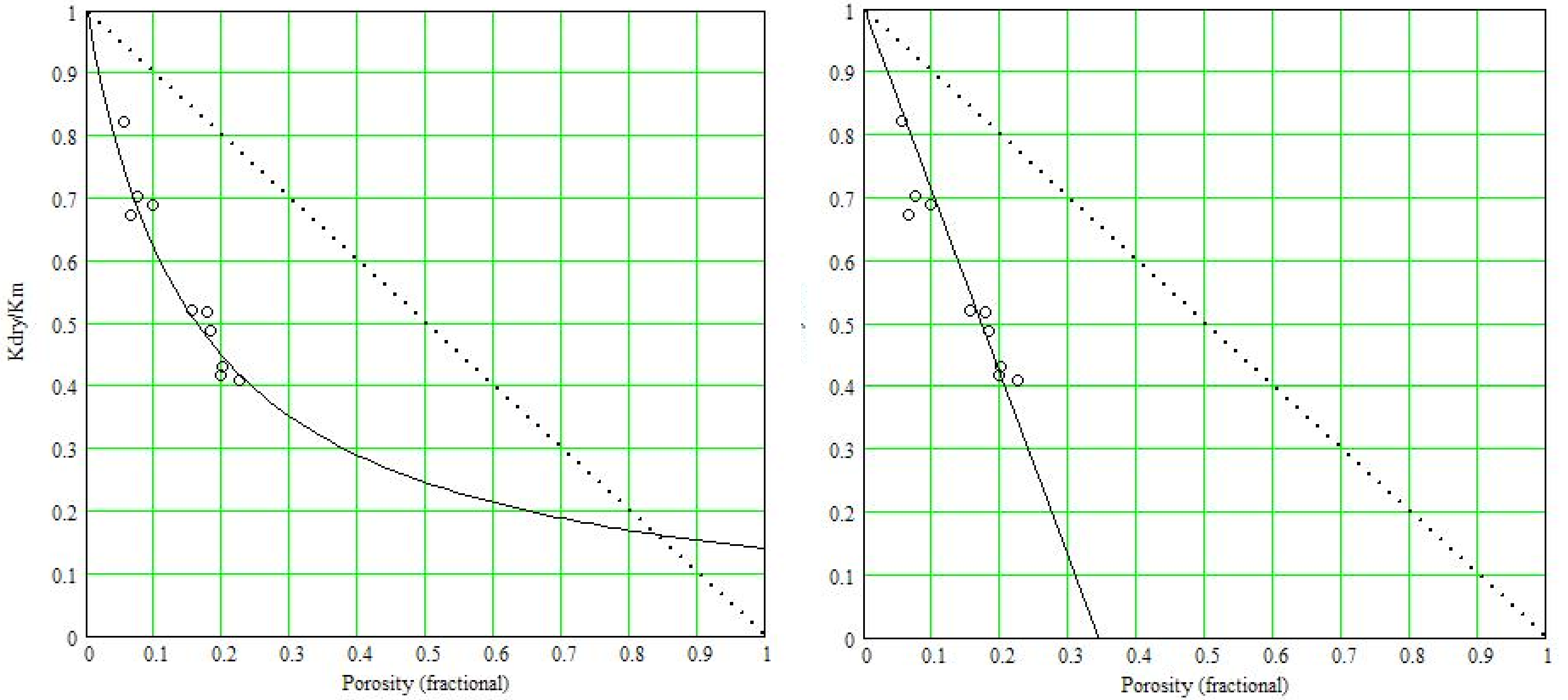
(a)



(b)

Figure (a) shows the ten clean dry sandstones at a constant pressure of 40 MPa. Figure (b) shows a single clean dry sandstone at pressures of 5, 10, 20, 30, and 40 Mpa. The dotted line is the Voigt limit.

Best fits for constant pressure



(a) (b)

Figure (a) shows the best pore space stiffness fit ($K_\phi/K_m = 0.162$ with RMS error = 0.039), and (b) shows the best critical porosity fit ($\phi_c = 34.3\%$ with RMS error = 0.058). Based on RMS error, the K_ϕ fit is the best.

Modeling K_{dry} versus porosity

- To model K_{dry} at different porosities using pore space stiffness, K_ϕ can be calculated for the in-situ porosity using the original equation:

$$\frac{\phi_{in-situ}}{K_\phi} = \frac{1}{K_{dry}} - \frac{1}{K_m} \Rightarrow K_\phi = \phi_{in-situ}\left[\frac{1}{K_{dry}} - \frac{1}{K_m}\right]^{-1}$$

- Once we have estimated the in-situ value for the pore space stiffness, we can calculate a value for K_{dry} at a new porosity at the same pressure using:

$$K_{dry\_new} = \left[\frac{\phi_{new}}{K_\phi} + \frac{1}{K_m}\right]^{-1}$$

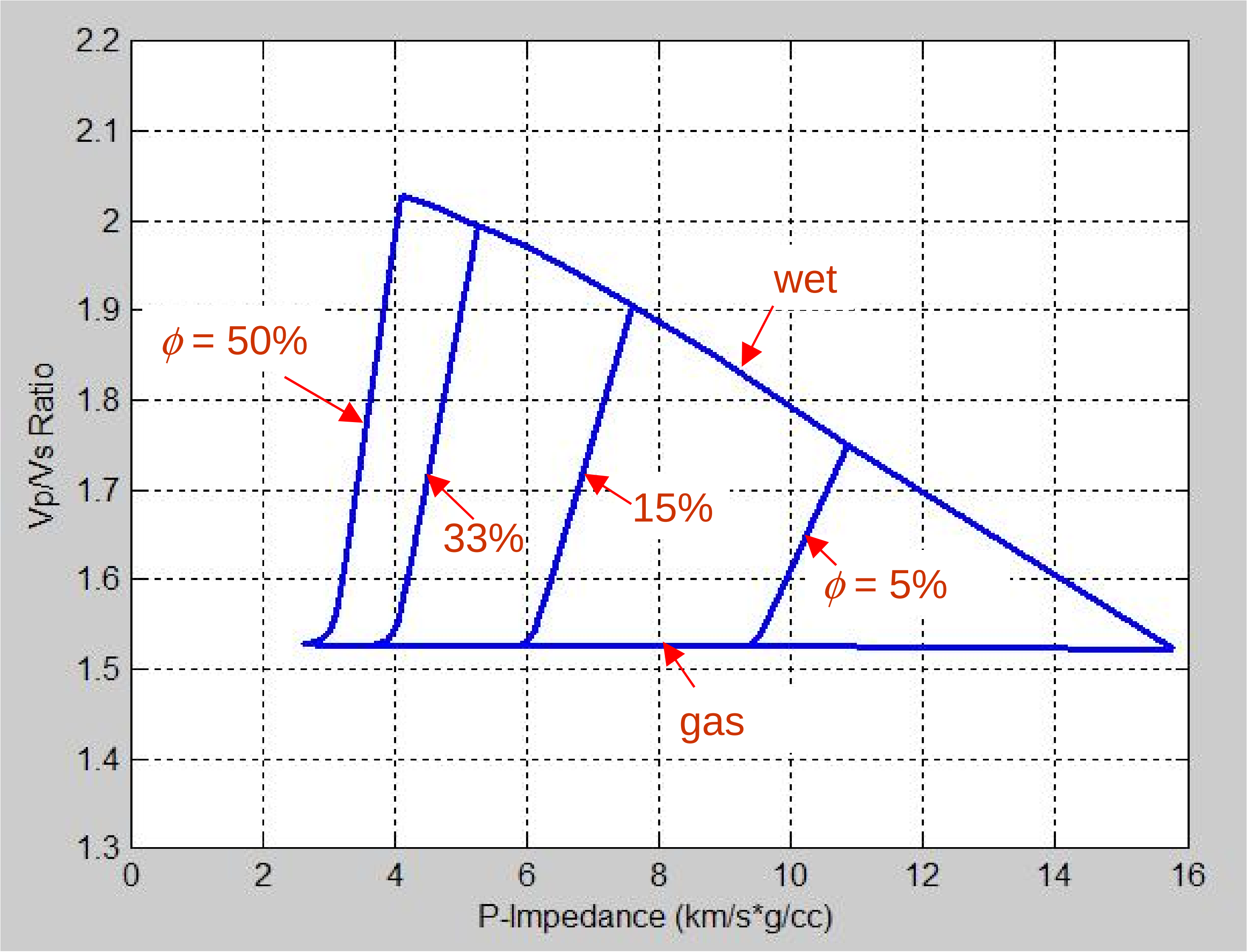
Modeling μ versus porosity

- Murphy et al (1993) measured K_{dry} and μ for clean quartz sandstones, and found the ratio of K_{dry}/μ is constant for varying porosity. We can therefore compute the new value of μ from:

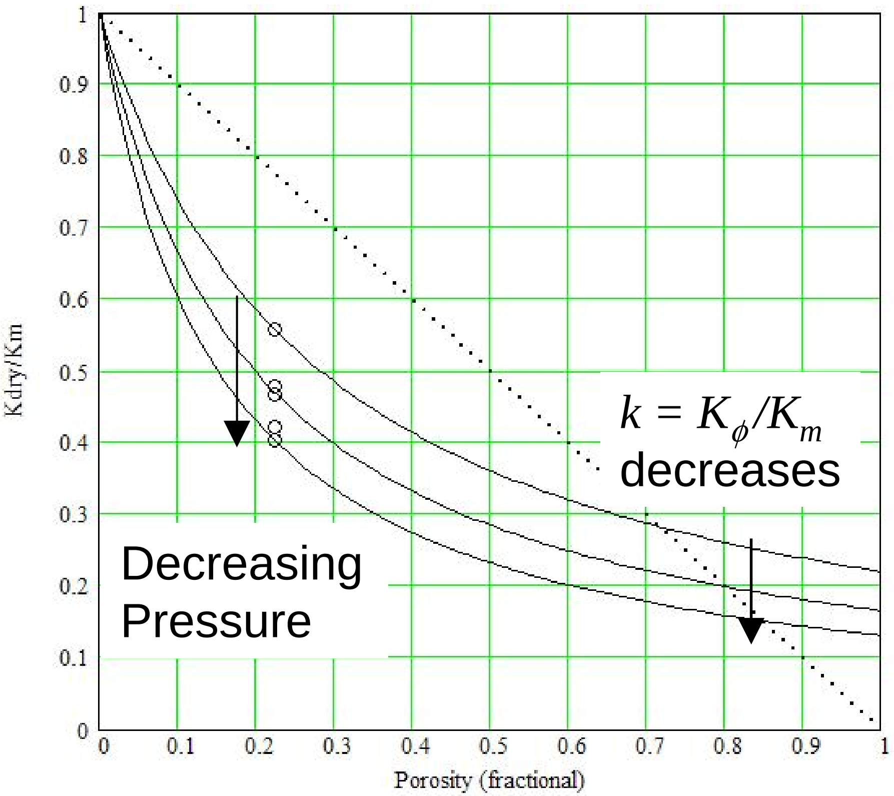
$$\mu_{new} = \mu_{in-situ} \frac{K_{dry\_new}}{K_{dry\_in-situ}}$$

- The figure on the next slide shows a cross-plot of V_P/V_S ratio versus P-impedance as a function of porosity and water saturation in a gas-charged sand using the following three assumptions:
  - Biot-Gassmann is used to model fluid changes
  - The pore space stiffness is used to model K_{dry} vs porosity change.
  - The above equation is used to model μ vs porosity change.

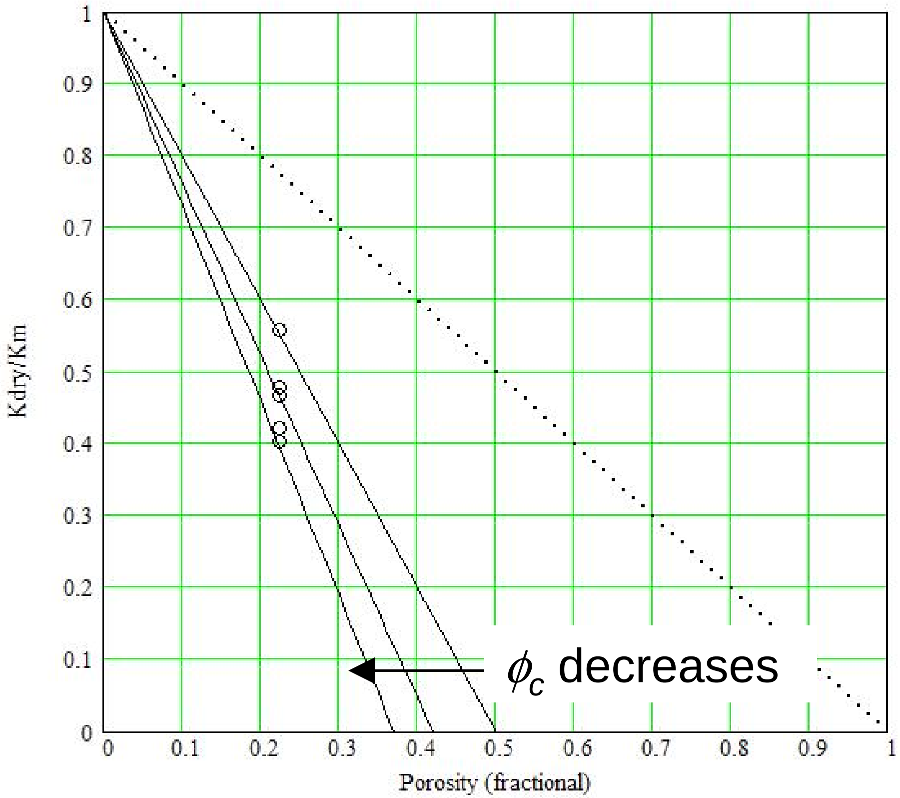
V_P/V_S Ratio vs P-Impedance



Variable pressure



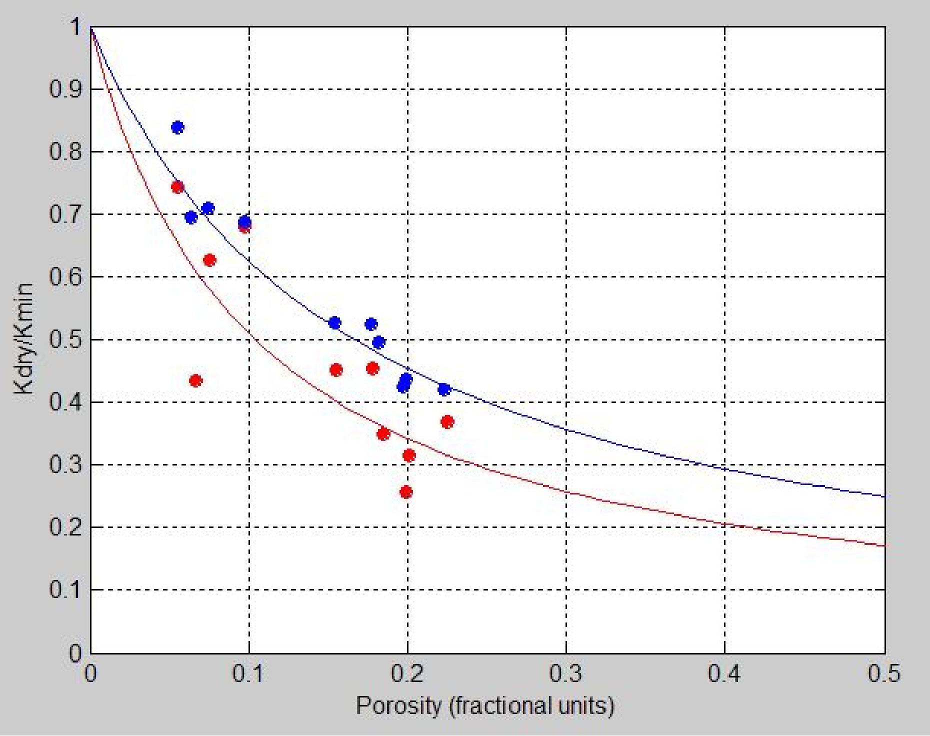
(a)



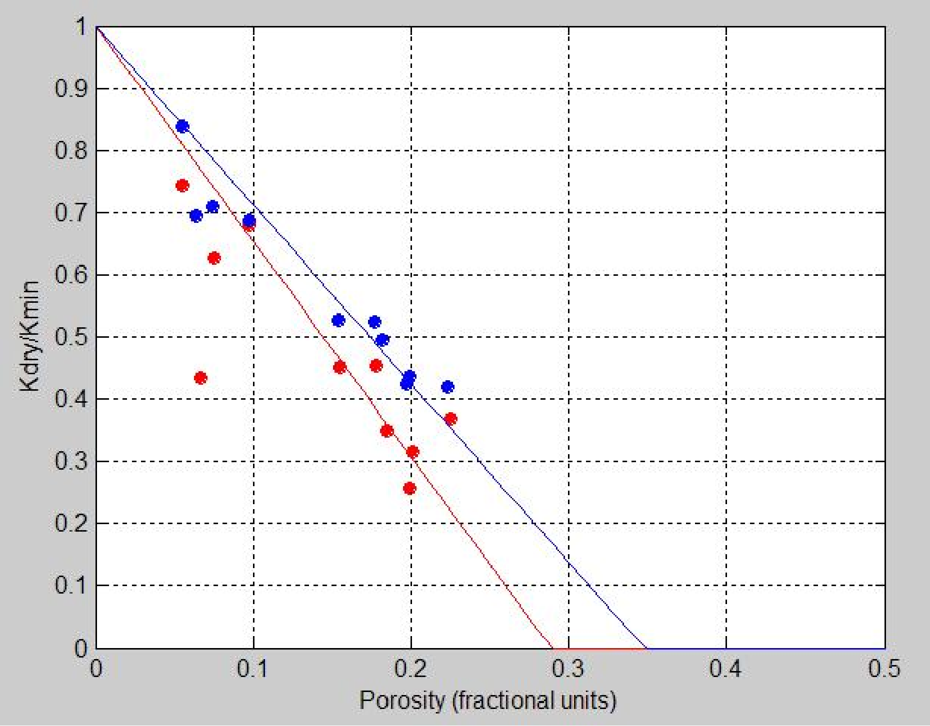
(b)

Next, consider the variable pressure case. Figure (a) shows that $k = K_\phi / K_m$ decreases as pressure decreases. Figure (b) shows that ϕ_c also decreases with decreasing pressure.

Variable pressure



(a)



(b)

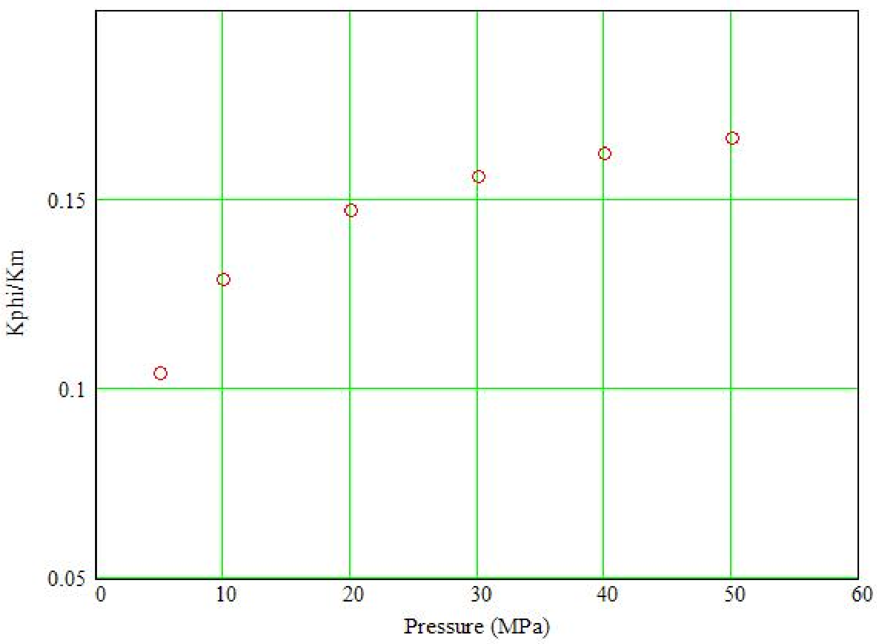
Han measured pressures of 5, 10, 20, 30, 40 and 50 MPa. Figure (a) shows the optimum pore stiffness fits for 50 MPa (blue) and 5 MPa (red). Figure (b) shows the equivalent ϕ_c fits.

Variable pressure

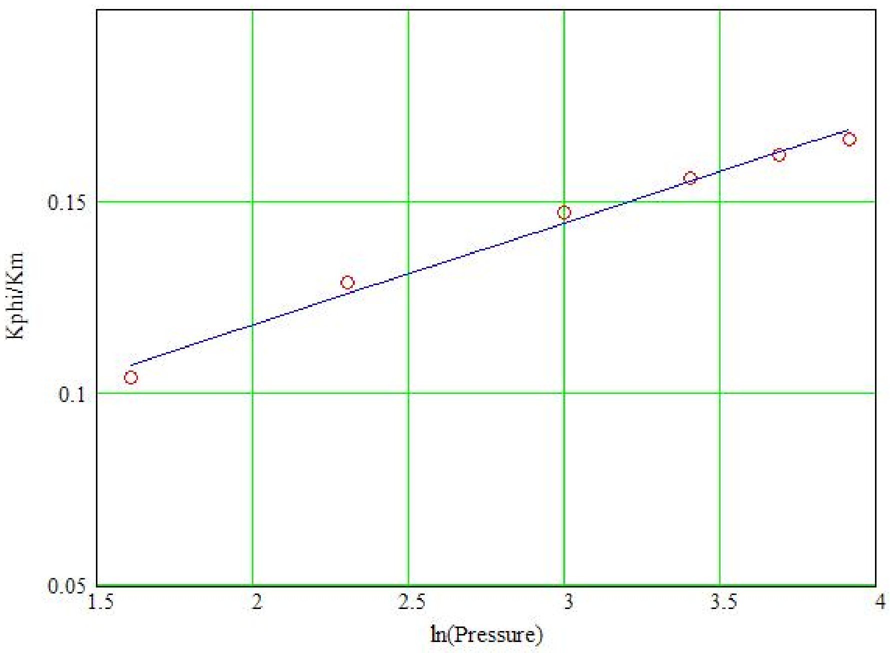
P(MPa)	ϕ_c	RMSE	K_ϕ/K_m	RMSE
5	0.289	0.126	0.104	0.094
10	0.311	0.107	0.129	0.076
20	0.329	0.079	0.147	0.055
30	0.338	0.069	0.156	0.044
40	0.343	0.058	0.162	0.039
50	0.348	0.053	0.166	0.038

This table shows the best fits and RMS errors for both models at all pressures. Note that K_ϕ and ϕ_c increase with increasing pressure and the error is smallest in all cases for K_ϕ.

Variable pressure



(a)



(b)

Here are the plots of K_ϕ/K_m vs (a) pressure and (b) natural log of pressure. Note that the relationship in (b) shows a good linear fit.

Variable pressure

- From the previous plot, the best least-squares fit is:

$$\frac{K_\phi}{K_m} = 0.065 + 0.027\ln(P)$$

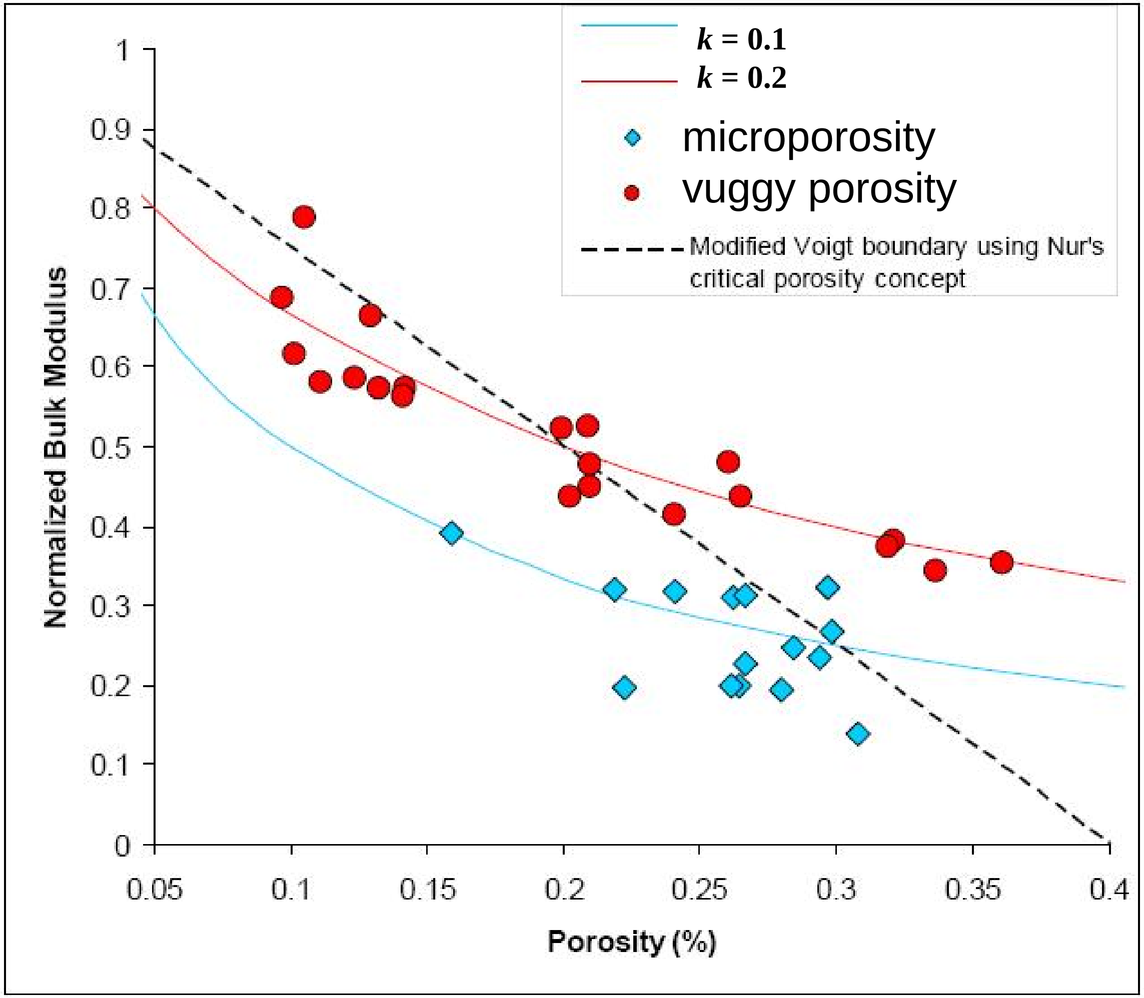
- From this equation, we can derive the relationship between change in pore space stiffness and pressure:

$$\frac{dK_\phi}{dP} \approx \frac{\Delta K_\phi}{\Delta P} = \frac{0.027K_m}{P} \Rightarrow \Delta K_\phi = 0.027K_m\frac{\Delta P}{P}$$

- This equation allows us derive constant K_ϕ curves at different pressures than the in-situ pressure, and hence predict a depth variable K_{dry} versus porosity relationship.

Carbonate example

Finally, Baechle et al. (2006) modeled carbonates using pore space stiffness. They show that dolomites with microporosity (blue points on right) fit the curve $k = 0.2$ and dolomites with vuggy porosity (red) fit the curve $k = 0.1$. The dashed line is the critical porosity fit.



Baechle et al. (2006)

Conclusions

- We have reviewed the two different approaches (pore space stiffness and critical porosity) to model porosity changes in fluid-saturated rocks using the dry rock bulk modulus.
- When tested using Han’s measurements, the pore space stiffness method gave the best fit to the data.
- We then showed a model incorporating the pore space stiffness method with Biot-Gassmann fluid changes.
- We next derived an equation that allowed us to calculate pore space stiffness as a function of pressure.
- Finally, we looked at a carbonate example from the literature using the critical porosity method.
- Limitations of this study were that only clean sandstones were modeled and that the pore space stiffness method is appropriate only at porosities much less than critical porosity.

References

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