Trace Interpolation and Elevation Statics by Conjugate Gradients

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Motivation

Ferguson 2006 - Regularization and datuming of seismic data by weighted, damped least squares.

- set up trace regularization and datuming as an inverse problem
- handle vertical and lateral velocity variation
- solve using phase-shift migration as a forward model

Phase-Shift Operator

Shifts a monochromatic wavefield in depth by multiplication in the Fourier domain

$$\varphi_\omega(k_x, z_1) = \alpha(z_1 - z_0, k_x, \omega) \cdot \varphi_\omega(k_x, z_0)$$

Phase-Shift Operator

Shifts a monochromatic wavefield in depth by multiplication in the Fourier domain

$$
\begin{aligned}
\varphi_\omega(k_x, z_1) &= \alpha(z_1 - z_0, k_x, \omega) \cdot \varphi_\omega(k_x, z_0) \\
\alpha(z, k_x, \omega) &= e^{izk_z}
\end{aligned}
$$

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\alpha(z, k_x, \omega) &= e^{izk_z} \\
k_z^2 &= \left(\frac{\omega}{v}\right)^2 - {k_x}^2
\end{aligned}
$$

Phase-Shift Migration

Gazdag 1978

- Models the propagation of the upward traveling wavefield in a homogeneous medium
- Fourier transform in x, followed by multiplication by α, then an inverse Fourier transform

$$\varphi_0 = P_{0 \leftarrow 200m} \varphi_{200m}$$

Depth Stepping

We accommodate velocity variation in depth by splitting P into a number of constant velocity depth steps

$$P_{0\leftarrow 4} P_{4\leftarrow 8} \ldots P_{196\leftarrow 200} \varphi_{200} = \varphi_0$$

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Depth Stepping

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$$P_{4\leftarrow 8}\varphi_8 \quad = \quad \varphi_4$$

Depth Stepping

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$$P\varphi_m = \varphi_d$$

Trace Padding

Finally, we model irregular spatial sampling using a weighting matrix W_d, which assigns a weight of 1 to live traces, and a weight of 0 to noisy or padded traces, and assume normally distributed error ϵ related to ambient noise and measurement error.

$$W_d P \varphi_m = \varphi_d + \epsilon$$

The Misfit Equation

To perform the inversion, we minimize a combination of the prediction error and solution roughness

$$M(\varphi) = \|P\varphi - \varphi_d\|_{W_d}^2 + \varepsilon\|\varphi - \varphi_m\|_{W_m}^2$$

The Misfit Equation

This is equivalent to solving the least-squares equation:

$$P^A W_d \varphi_d = \left[P^A W_d P + \varepsilon W_m \right] \varphi_m$$

Conjugate Gradients

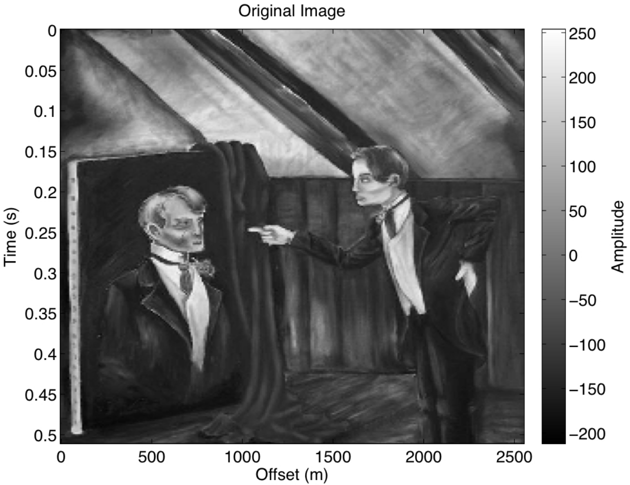
Benefits

- Runtime can be measured as a function of the forward operator
- Assuming perfect arithmetic, returns exact solution
- $\sqrt{N}$ steps for well-conditioned systems

Conjugate Gradients

Drawbacks

- Requires positive definite input
- Our system is not well-conditioned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Offset (m)
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Amplitude
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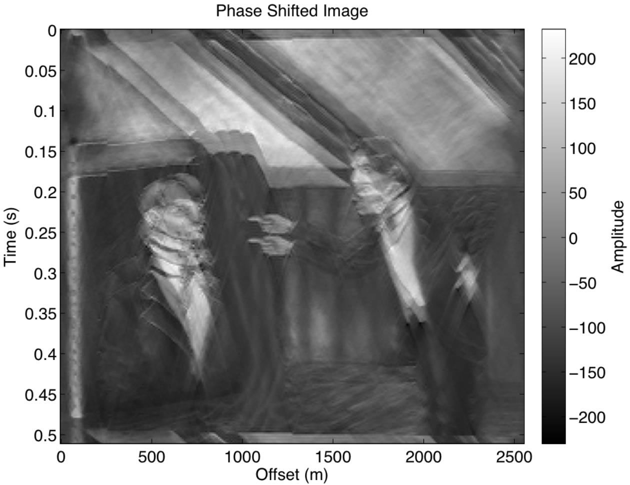
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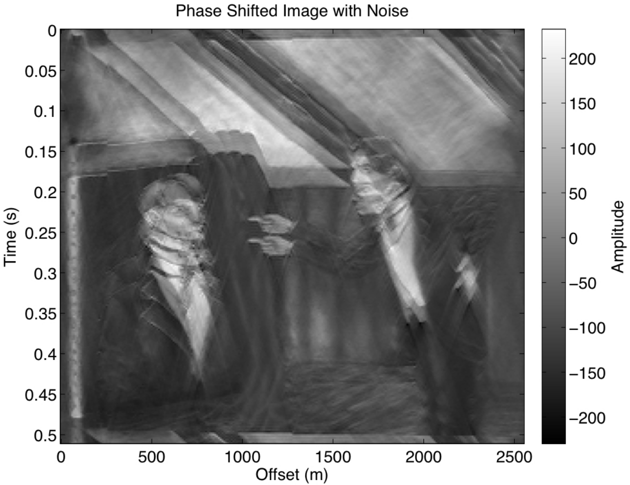
Phase Shifted Image with Noise
Time (s)
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Offset (m)
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Amplitude
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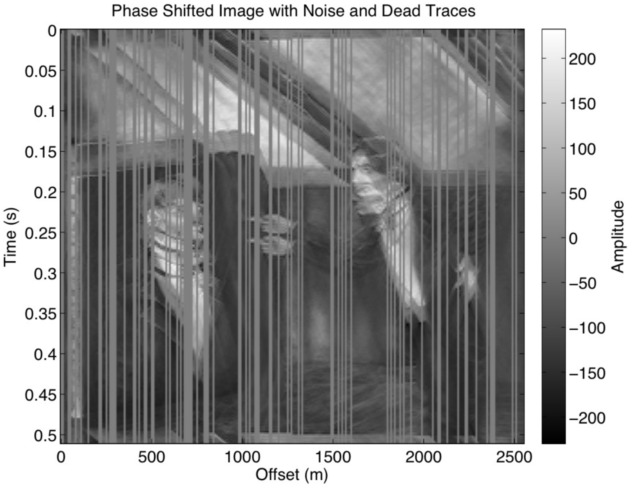
Phase Shifted Image with Noise and Dead Traces
Time (s)
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0.4
0.45
0.5
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Offset (m)
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−150
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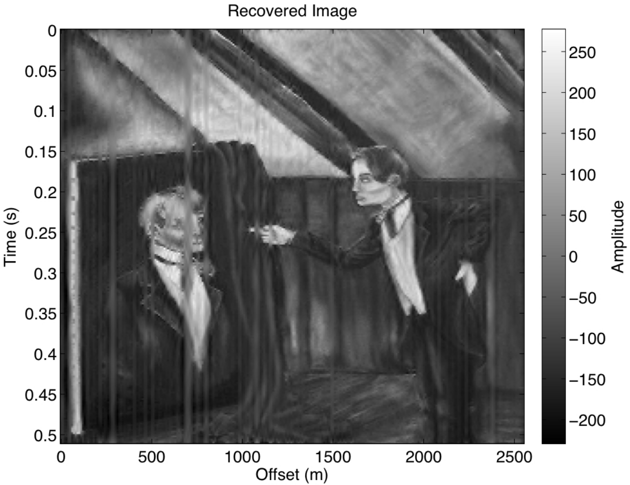
Amplitude


Recovered Image
Time (s)
0
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0.5
Offset (m)
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Amplitude
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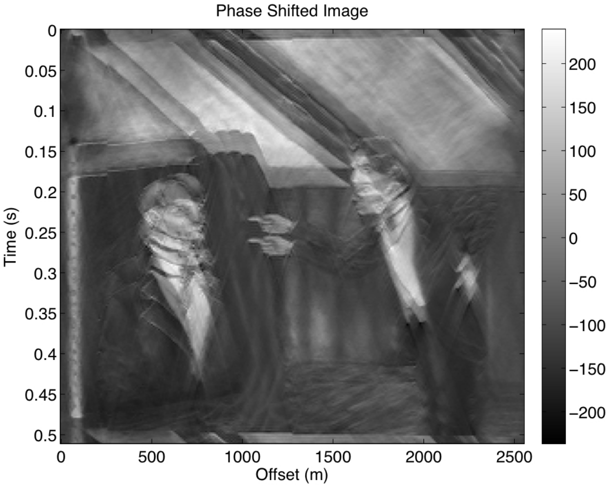
Best Case

- Known velocity model
- No trace decimation
- $\sqrt{N}$ iterations


Phase Shifted Image
Time (s)
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Offset (m)
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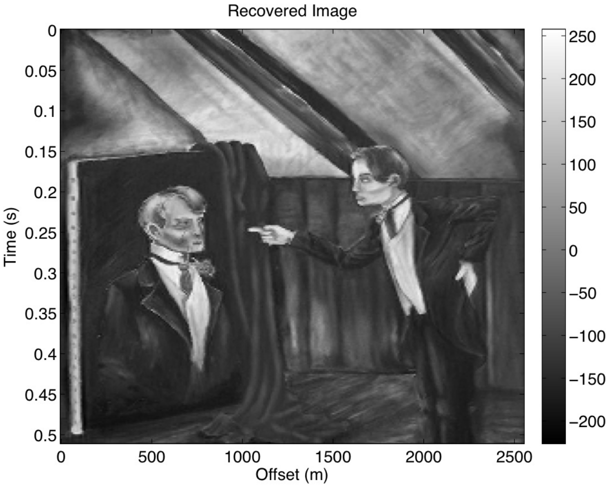
Recovered Image
Time (s)
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Offset (m)
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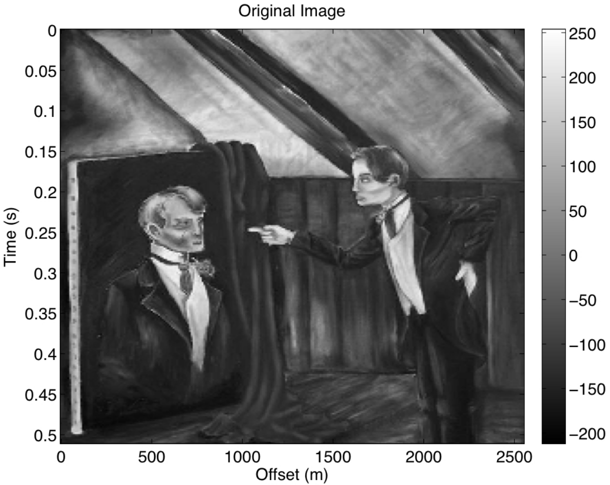
Original Image
Time (s)
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Offset (m)
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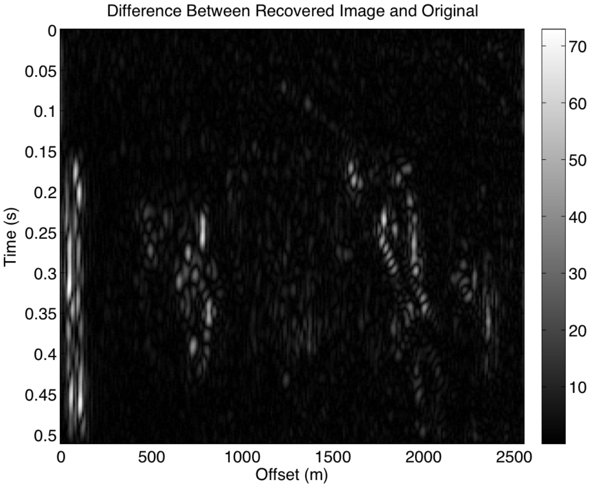
Difference Between Recovered Image and Original
Time (s)
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Offset (m)
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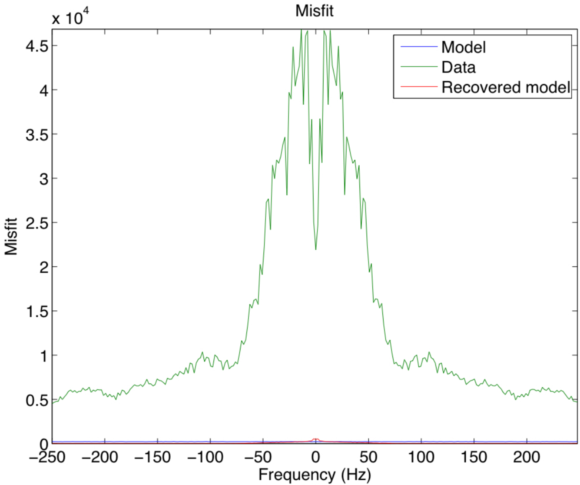
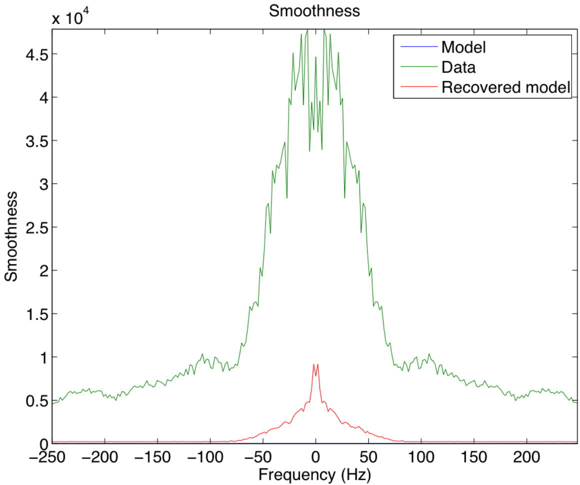

Misfit
x 10^4
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Misfit
−250
−200
−150
−100
−50
0
50
100
150
200
Frequency (Hz)
Model
Data
Recovered model

Trace Interpolation and Elevation Statics by Conjugate Gradients

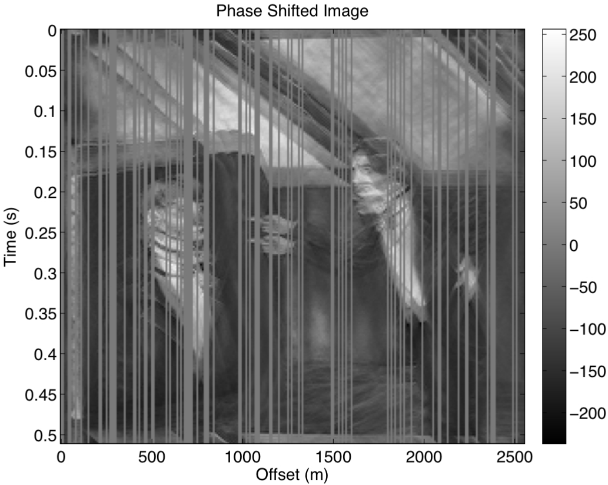
Marcus Wilson

Outline
Motivation
Forward Model
The Misfit Equation
Conjugate Gradients
Example
Results
Future Work



Worst Case

- Roughly known velocity model
- 30% trace decimation
- $\sqrt{N}$ iterations


Phase Shifted Image
Time (s)
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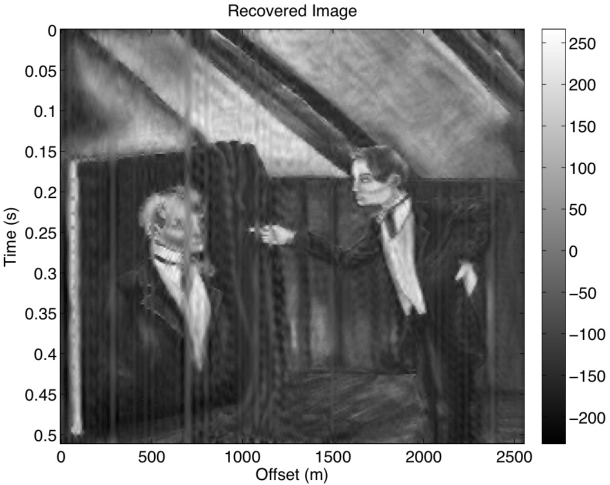
Recovered Image
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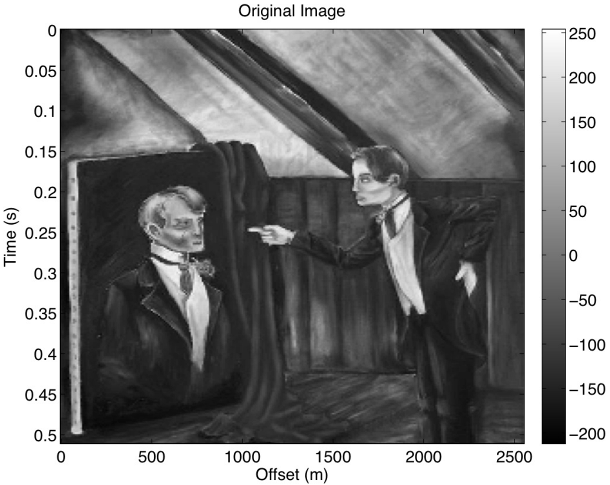
Original Image
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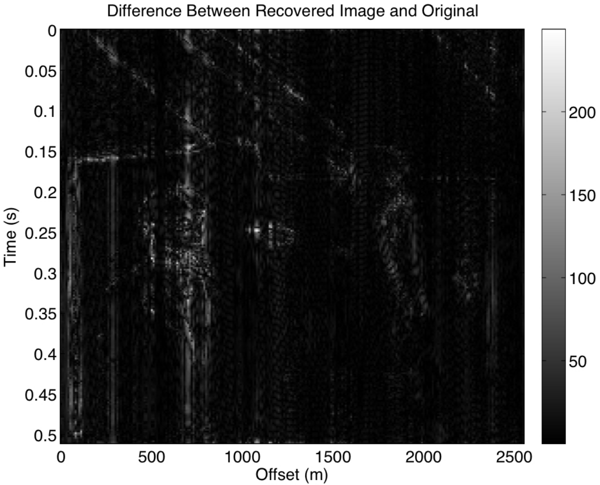
Difference Between Recovered Image and Original
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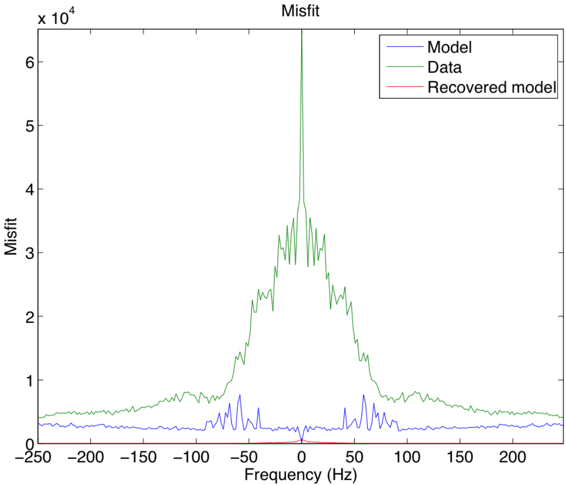
Misfit
x 10^4
Model
Data
Recovered model
Misfit
Frequency (Hz)
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6
−250
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−150
−100
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100
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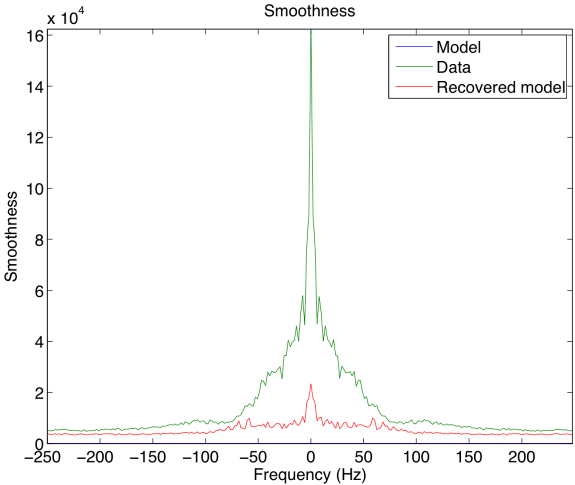
Smoothness
x 10^4
16
14
12
10
8
6
4
2
0
Smoothness
Model
Data
Recovered model
−250
−200
−150
−100
−50
0
50
100
150
200
Frequency (Hz)

Future Work

- Develop a framework to run CG on operators
- Compare runtime of series expansion, matrix-based implementation with operator-based implementation
- Consider separate treatment of wavelike and evanescent regions
- Investigate preconditioning operators

Acknowledgements

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