Determining elastic constants of an orthorhombic material by physical seismic modeling

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CREWES Project

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Objective

Determination of elastic constants
for an orthorhombic material
using group velocity measurements
via physical seismic modeling

Outline

- Elastic constant tensor: c_{ijkl}

$$(\sigma_{ij} = c_{ijkl} e_{kl})$$

- C_{ij}: 6×6 matrix (Voigt's notation)
- General anisotropic medium: 21 independent elastic constants
- Isotropic medium: 2 independent elastic constants (λ, μ)
- A_{ij}: density normalized elastic constants

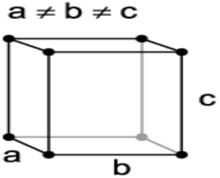
$$A_{ij} = C_{ij} / \rho$$

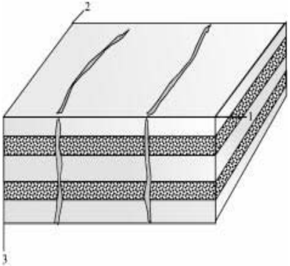
Orthorhombic symmetry

- Three distinctive directions
- 9 independent elastic constants

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & & & \\ & A_{22} & A_{23} & & 0 & \\ & & A_{33} & & & \\ & & & A_{44} & & \\ & & & & A_{55} & \\ & & & & & A_{66} \end{bmatrix}$$

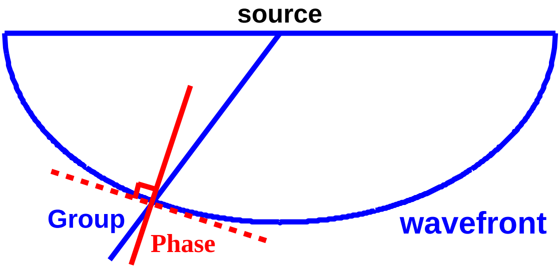
- Vertical fractures and horizontal layering
- VTI, HTI: a degenerate case of orthorhombic





[Schoenberg and Helbig, 1997]

- Anisotropic homogeneous medium



- Phase velocity: normal to the wavefront
- Group velocity: raypath direction
- Phase velocity = group velocity (principal directions)

Phase velocity relation to elastic constants

- Christoffel equation (solves for phase velocity and polarization vector)

$$\begin{bmatrix} G_{11}-\rho v^2 & G_{12} & G_{13} \\ G_{21} & G_{22}-\rho v^2 & G_{23} \\ G_{31} & G_{32} & G_{33}-\rho v^2 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$G_{ik} = c_{ijkl} n_j n_l$$

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$$G_{ik} = c_{ijkl} n_j n_l$$

- Linear equation for P-wave phase velocity

$$\begin{aligned} {v_P}^2(\vec{n}) \cong\ & A_{11} n_1^4 + A_{22} n_2^4 + A_{33} n_3^4 + 2(A_{12} + 2A_{66}) n_1^2 n_2^2 + \\ & 2(A_{13} + 2A_{55}) n_1^2 n_3^2 + 2(A_{23} + 2A_{44}) n_2^2 n_3^2 \end{aligned}$$

[Backus, 1965, Daley and Krebes, 2006]

- Linear equation for SH-wave, x-z plane

$$v_{SH}^2(\vec{n}) = A_{66} n_1^2 + A_{44} n_3^2$$

A_{ij} from phase velocity

$$A_{ij} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & & & \\ & A_{22} & A_{23} & & 0 & \\ & & A_{33} & & & \\ & & & A_{44} & & \\ & & & & A_{55} & \\ & & & & & A_{66} \end{bmatrix}$$

$A_{11} = V_{11}^2$ (V_P: propagation in x-axis)

$A_{22} = V_{22}^2$ (V_P: propagation in y-axis)

$A_{33} = V_{33}^2$ (V_P: propagation in z-axis)

A_{ij} from phase velocity

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$A_{44} = V_{23}^2$ (V_S: propagation in z-axis polarization in y-axis)

$A_{55} = V_{13}^2$ (V_S: propagation in z-axis polarization in x-axis)

$A_{66} = V_{12}^2$ (V_S: propagation in x-axis polarization in y-axis)

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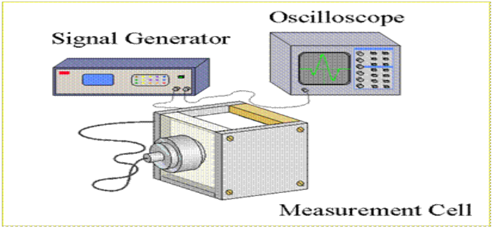
$A_{66} = V_{12}^2$ (V_S: propagation in x-axis polarization in y-axis)

A_{23}, A_{13}, A_{12} : from P-wave velocity in $\pm 45^\circ$

1. Basic theory
2. **Elastic constants estimation**
   - Previous work
   - Group velocity relation to elastic constants
3. Physical seismic modeling
   - P- and S-wave group velocity measurements
4. Elastic constants estimations
5. Conclusions
6. Future work

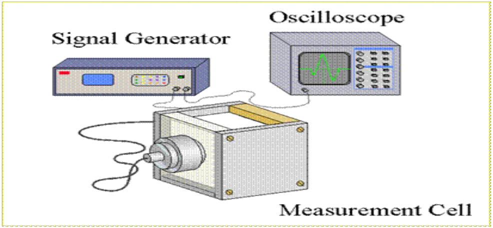
Previous work

- Calculation of A_{ij} from ultrasonic phase-velocity measurements [Jones and Wang, 1981, Lo et al., 1986, Vernik and Nur, 1992, Every and Sachse, 1992, Dellinger and Vernik, 1994] · · ·



Previous work

- Calculation of A_{ij} from ultrasonic phase-velocity measurements [Jones and Wang, 1981, Lo et al., 1986, Vernik and Nur, 1992, Every and Sachse, 1992, Dellinger and Vernik, 1994] · · ·



- Laboratory phase velocity measurements are problematic
  - Plane wave assumption required
  - Transducer size should be comparable to the source-receiver distance, [Auld, 1973, Dellinger and Vernik, 1994]
  - otherwise group velocity is measured
  - Group velocity $\neq$ phase velocity

Group velocity relation to elastic constants [Daley and Krebes, 2006]

- Phase velocity ($\vec{n}$: phase direction)
  $v_P^2(\vec{n}) \cong A_{11} n_1^2 + A_{22} n_2^2 + A_{33} n_3^2 + E_{12} n_1^2 n_2^2 + E_{13} n_1^2 n_3^2 + E_{23} n_2^2 n_3^2$
- Anellipsoidal deviation terms
  $E_{23} = 2(A_{23} + 2A_{44}) - (A_{22} + A_{33})$
  $E_{13} = 2(A_{13} + 2A_{55}) - (A_{11} + A_{33})$
  $E_{12} = 2(A_{12} + 2A_{66}) - (A_{11} + A_{22})$
- Group velocity ($\vec{N}$: group direction)

$$\frac{1}{V_P^2(\vec{N})} \cong \frac{N_1^2}{A_{11}} + \frac{N_2^2}{A_{22}} + \frac{N_3^2}{A_{33}} - \frac{E_{12}}{N_1^2 N_2^2} A_{11} A_{22} - \frac{E_{13}}{N_1^2 N_3^2} A_{11} A_{33} - \frac{E_{23}}{N_2^2 N_3^2} A_{22} A_{33}$$

Group velocity relation to elastic constants [Daley and Krebes, 2006]

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$$BE_{23} + FE_{13} + LE_{12} = D(\theta)$$

$$\begin{pmatrix} B_1 & F_1 & L_1 \\ \vdots & \vdots & \vdots \\ B_n & F_n & L_n \end{pmatrix} \begin{pmatrix} E_{23} \\ E_{13} \\ E_{12} \end{pmatrix} = \begin{pmatrix} D(\theta_1) \\ \vdots \\ D(\theta_n) \end{pmatrix}$$

Our plan

- Diagonal elastic constants: $A_{ii} : i = 1 : 6$
  Direct measurements of P- and S group velocity in principal axes
- Off diagonal elastic constants: $A_{ij} : i \neq j$
  Least-squares inversion of several P group velocity for (E_{23}, E_{13}, E_{12})

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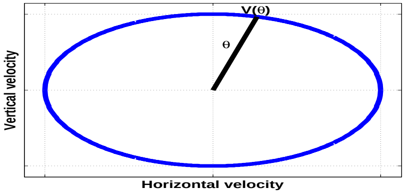
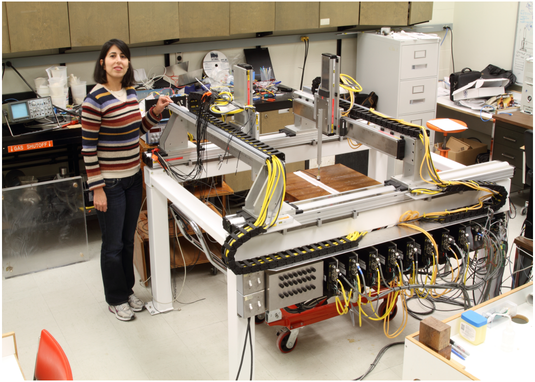
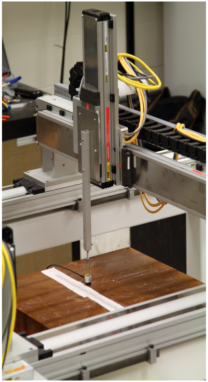
Elliptical anisotropy: ellipsoidal wavefront ($E_{23} = E_{13} = E_{12} = 0$)
Group velocity surface = Wavefront at unit time
Polar graph of group velocity vs. group angle



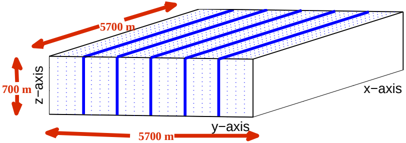
CREWES physical modeling facility

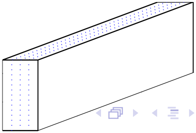




Scale: distance (1:10000), frequency (10000:1)

- Material: Phenolic LE, orthorhombic symmetry
- Model: a HTI layer

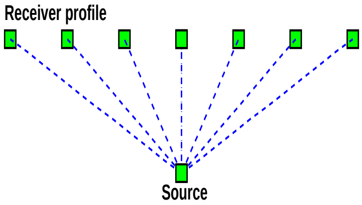




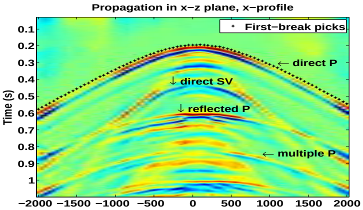
How is the P-wave group velocity measured?

Receiver: P-transducer sensitive to normal displacement

Receiver profile



Vertical-component data



$$\text{Group velocity} = \frac{\text{Source-receiver distance}}{\text{Direct arrival traveltime}}$$

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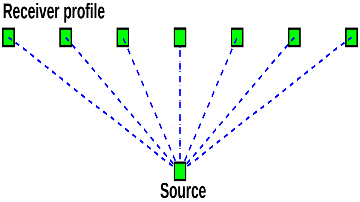

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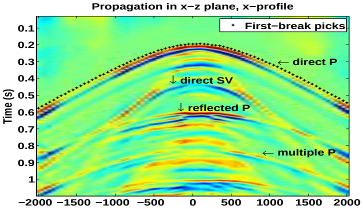

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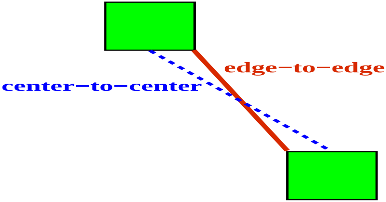
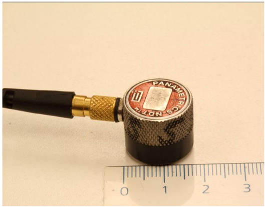
Edge-to-edge correction



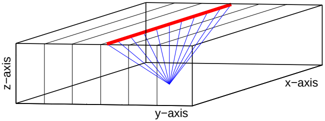
Transducer size: 13*mm*



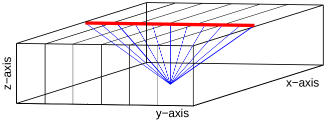
Transmission profiles

- To measure the group velocity in different directions

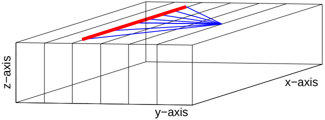
x-z plane



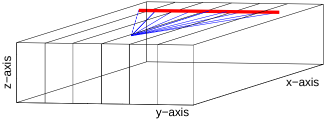
y-z plane



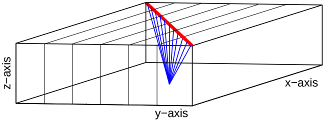
x-y plane (x-profile)



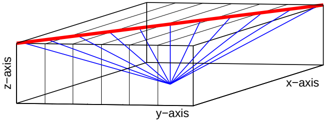
x-y plane (y-profile)



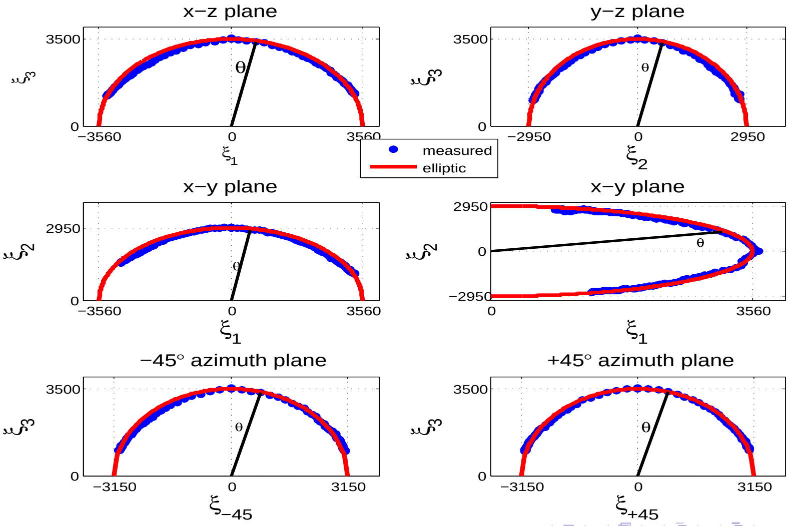
azimuth $-45°$ plane



azimuth $+45°$ plane

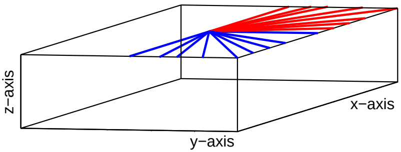


Group velocity surface (Wavefront at unit time)

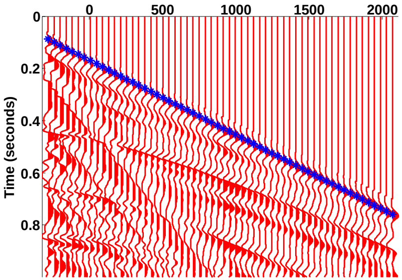


Reflection azimuth data

Azimuth profiles

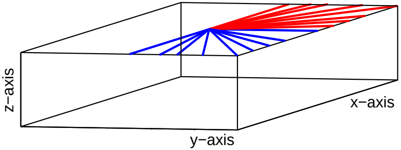


$+14^\circ$ azimuth data



Reflection azimuth data

Azimuth profiles



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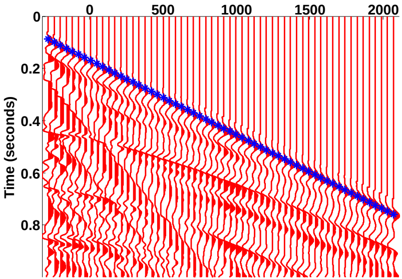
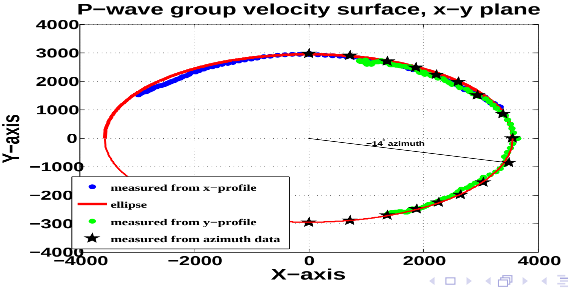
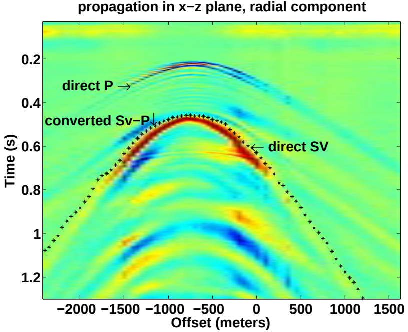


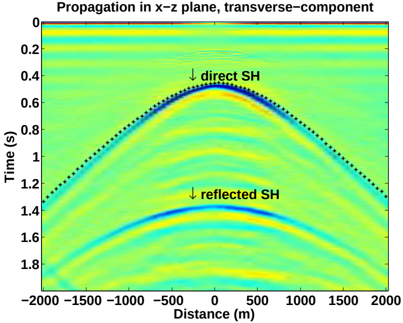


- Receiver: S-transducer sensitive to shear displacement

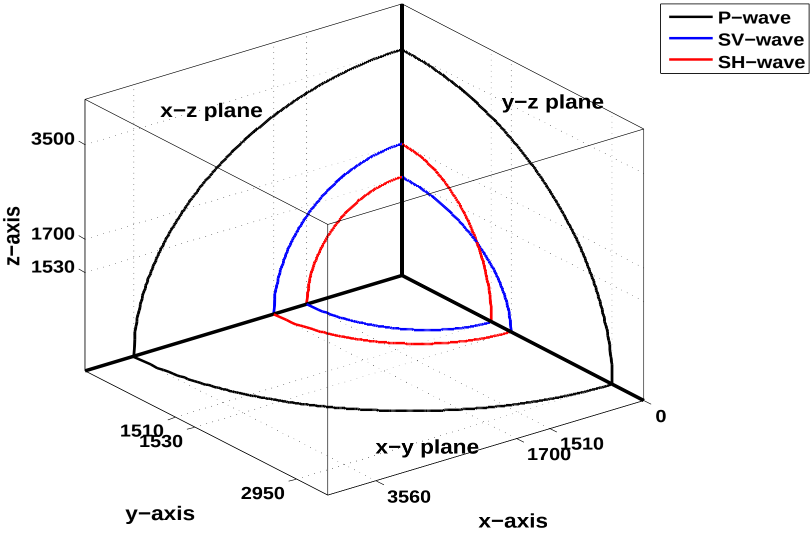
SV-wave group velocity
Radial component



SH-wave group velocity
Transverse component



P- and S-wave Group velocity surfaces



Elastic constants for phenolic model

Table: Density normalized elastic constants, units of $(km/s)^2$

12.67 ± 0.006	6.13 ± 0.003	6.68 ± 0.003	0	0	0
	8.70 ± 0.006	5.79 ± 0.003	0	0	0
		12.25 ± 0.006	0	0	0
			2.34 ± 0.001	0	0
				2.89 ± 0.001	0
					2.28 ± 0.001

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		12.25 ± 0.006	0	0	0
			2.34 ± 0.001	0	0
				2.89 ± 0.001	0
					2.28 ± 0.001

The relationship between elastic constants to the generic Thomsen parameters [Rüger, 2001].

Table: Calculated Thomsen anisotropy parameters.

	ϵ	γ	δ
x-z plane	0.0173	−0.0130	0.0175
y-z plane	−0.1448	−0.1055	−0.1318
x-y plane	−0.1567	0.1173	−0.1417

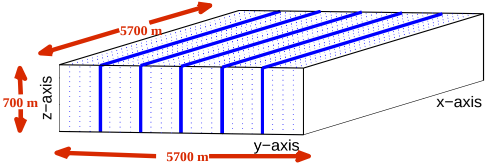
Conclusions

- Orthorhombic elastic constants from group velocity using [Daley and Krebes, 2006].
- Method eliminates the difficulties of measuring the phase velocity.
- The relation between the group velocity and elastic constants, can be used in laboratory experiments as a robust method in determining all A_{ij}.
- Some of these experiments maybe applicable in bore-hole environment.
- Phenolic model shows elliptical anisotropy with close values for the ϵ and δ parameters.

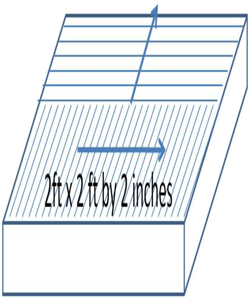
Some of the acquired seismic data are available in our poster

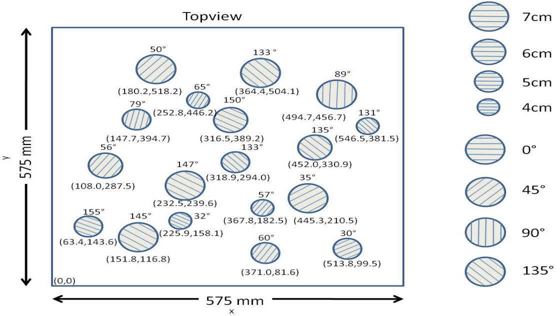
Future work

AVAZ inversion for anisotropy parameters



Future models





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