Amplitude calibration of a fast S-transform with application to anelastic AVF inversion

CREWES

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Outline

1. Fast S-transform
- Review
- Amplitude Calibration
- Fidelity thresholds as a function of proximity of events
2. Anelastic AVF Inversion (Amplitude variations with frequency)
- AVF signature of anelastic reflectivity and its inversion
- AVF in the fast S-transform domain and its inversion
- Accuracy
- Future work

The S-transform and seismic data processing

- A seismic signal is often non-stationary, meaning its frequency content changes with time (Margrave 1998)
- The S-transform is a technique of time-frequency decomposition, enabling non-stationary processing of seismic data
  - ➢A basis for seismic interpolation (Naghizadeh and Innanen 2010)
  - ➢A tool for the estimation of local spectra

S-transform

- For a time signal f(t), The S-transform [Stockwell et al. (1996)] is defined as

$$S(\tau,\omega)=\int_{-\infty}^{\infty} f(t)\,\underbrace{\frac{|\omega|}{\sqrt{2\pi}}e^{\frac{-(\tau-t)^2\omega^2}{2}}}_{\text{Gaussian window}}\,\underbrace{e^{-i2\pi\omega\cdot t}}_{\text{Fourier kernel}}\,dt$$

- The Gaussian window provides (1) time localization and (2) progressive resolution.
- High computational cost has limited its use.
- FGFT, proposed by Brown et al. (2010) and implemented by Naghizadeh and Innanen (2010) is a fast, non-redundant method

S-transform from α-domain

- It is possible to calculate the S-transform from Brown et al.(2010) refer to as the α-domain

$$\alpha(\omega',\omega) = G(\omega'+\omega)\cdot W(\omega',\omega)$$

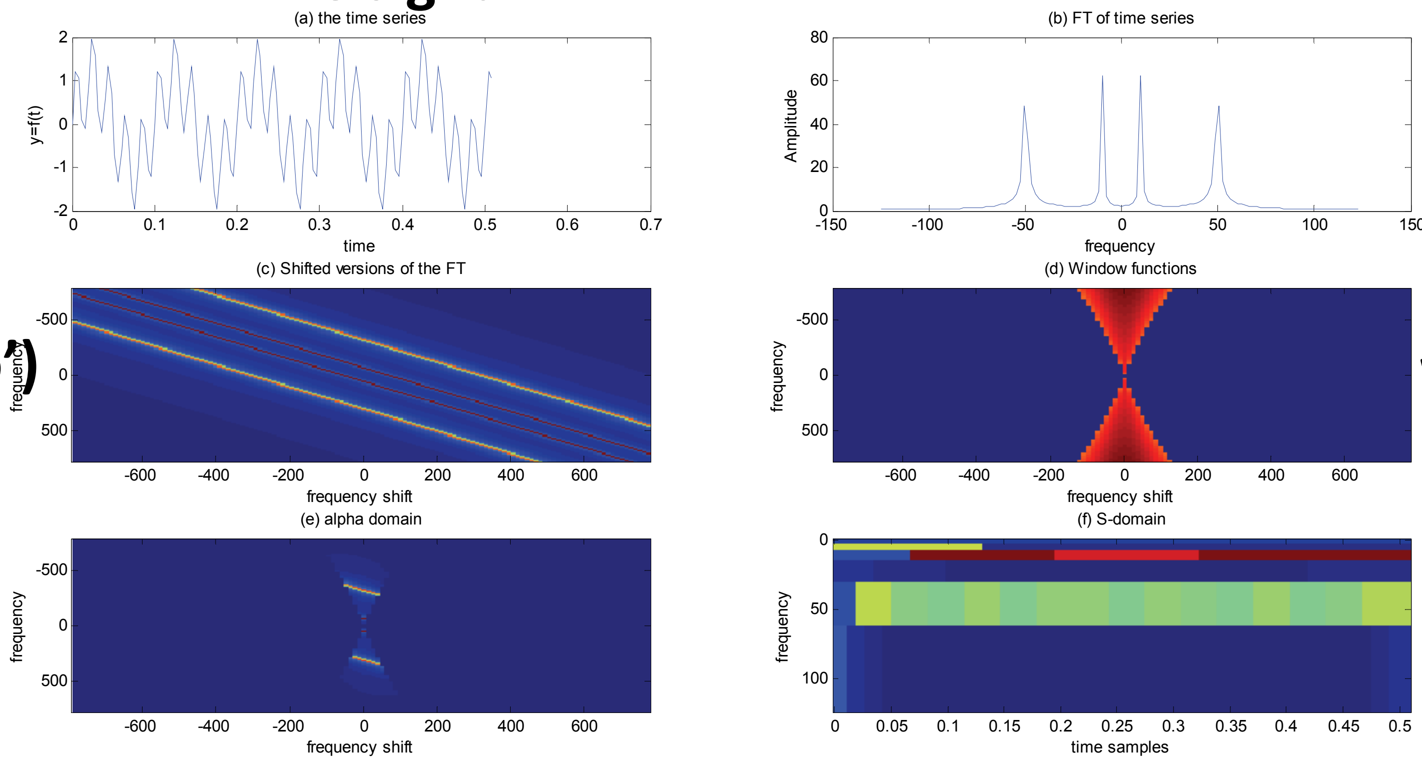
- Where G(ω+ ω’) is the FT of the signal shifted by ώ.
- W(ω’, ω) is the FT of the Gaussian window functions.
- S-domain is obtained by taking FT of α-domain along the ω’ axis. Given in Brown et al. (2010)

$$S(\tau,\omega) = \int_{-\infty}^{\infty} \alpha(\omega',\omega)\cdot e^{i2\pi\tau\omega'} d\omega'$$

Example: sum of two harmonics

Time signal

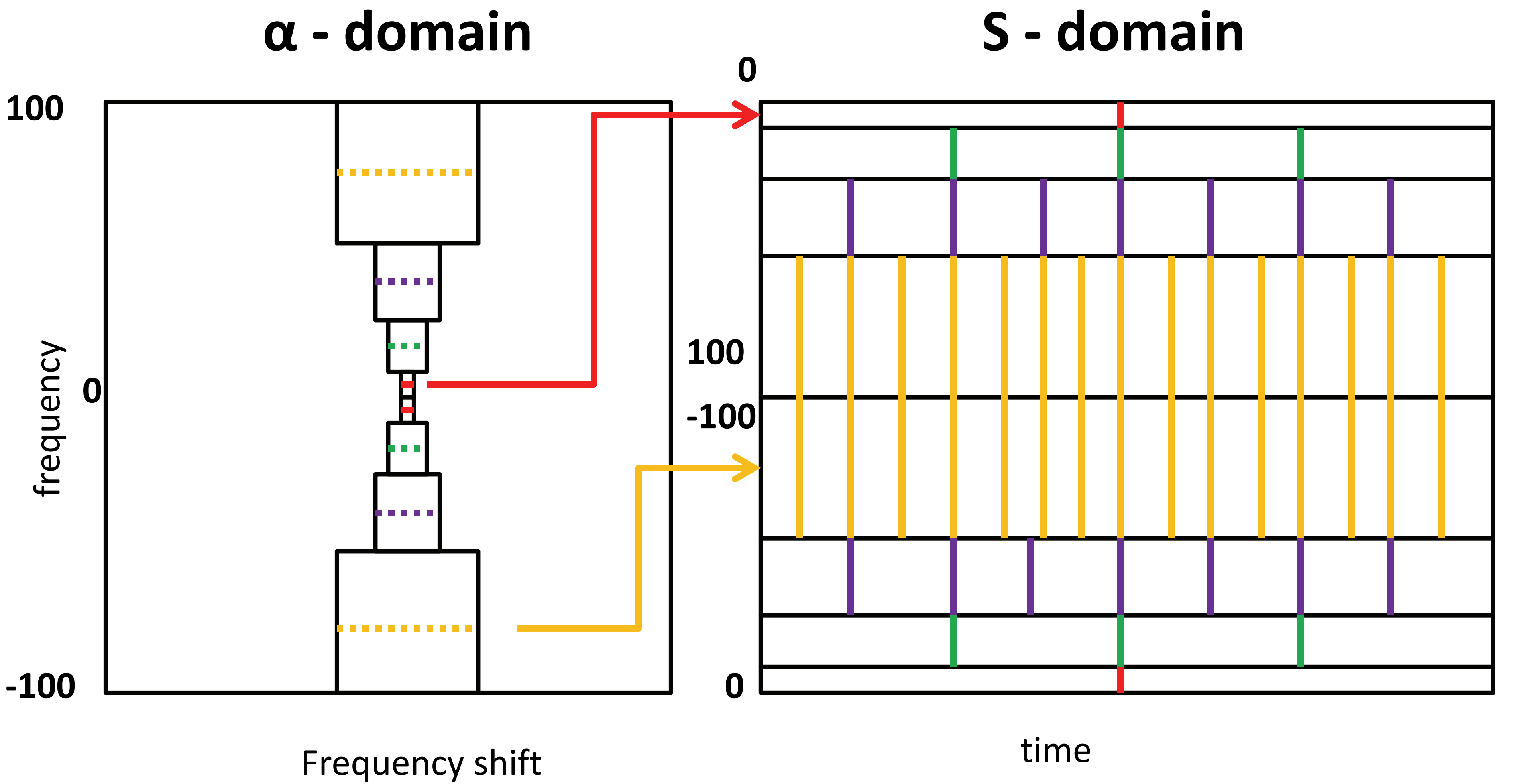
Fourier Transform



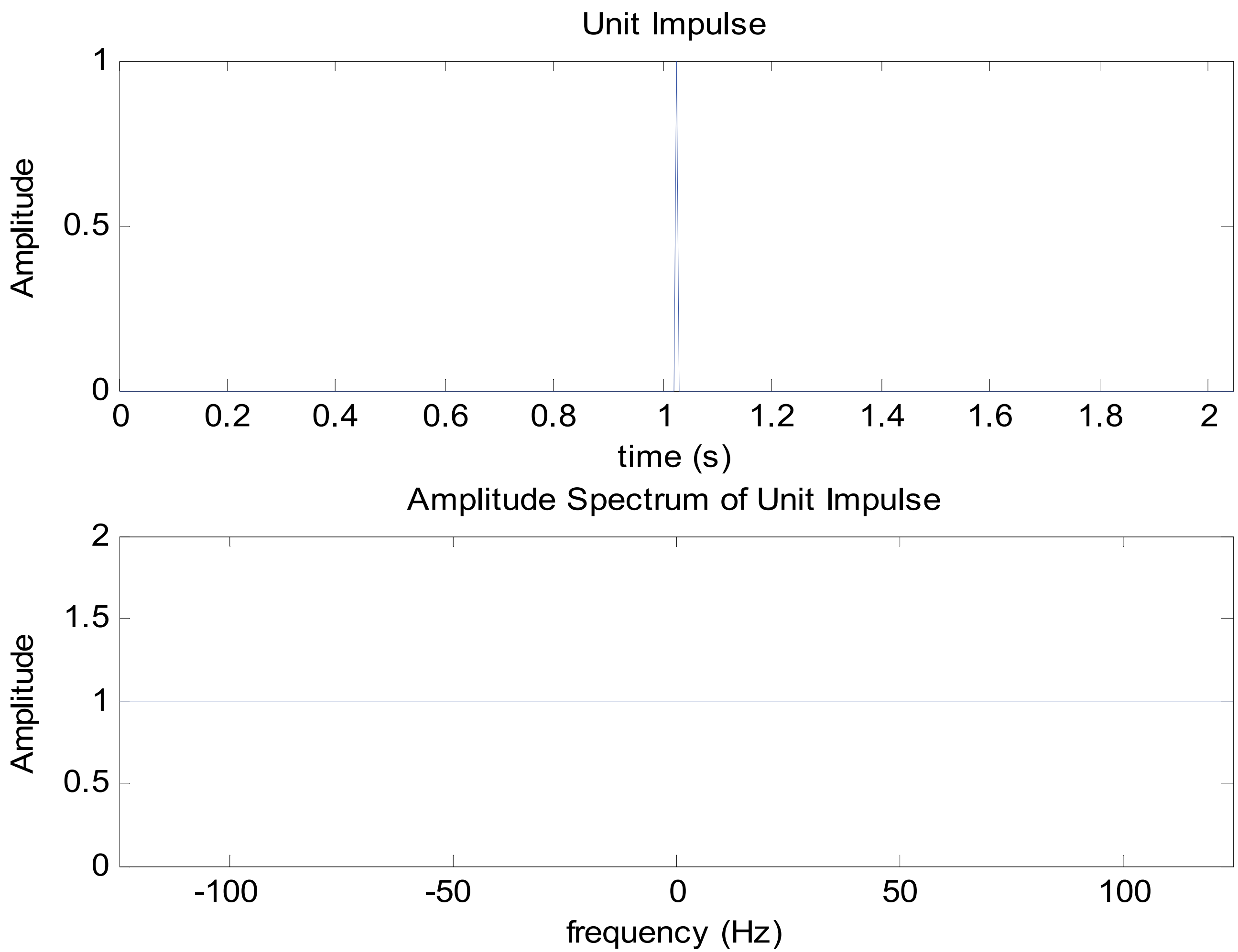
α-domain

S-domain

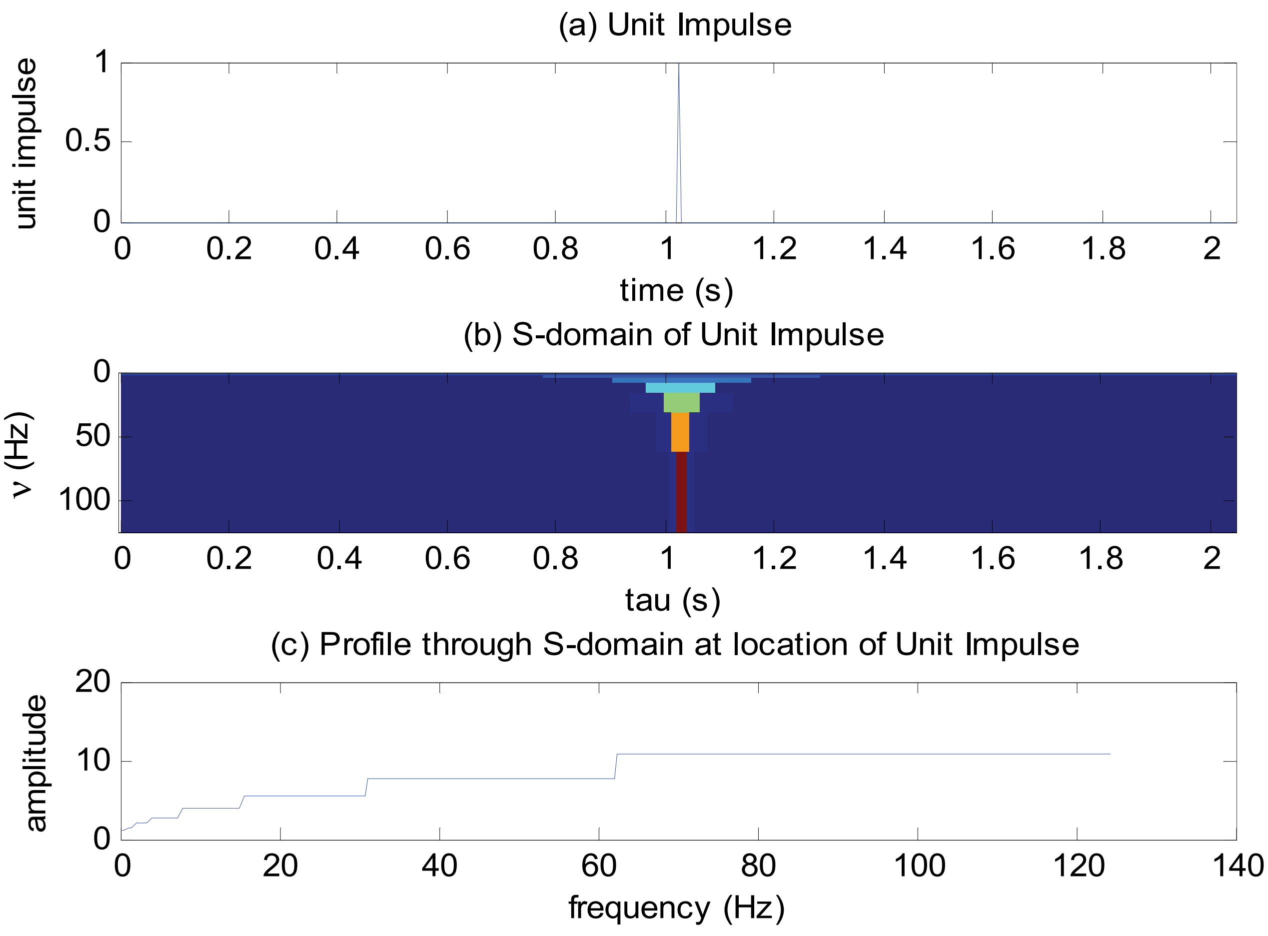
To make fast, segment in the α-domain



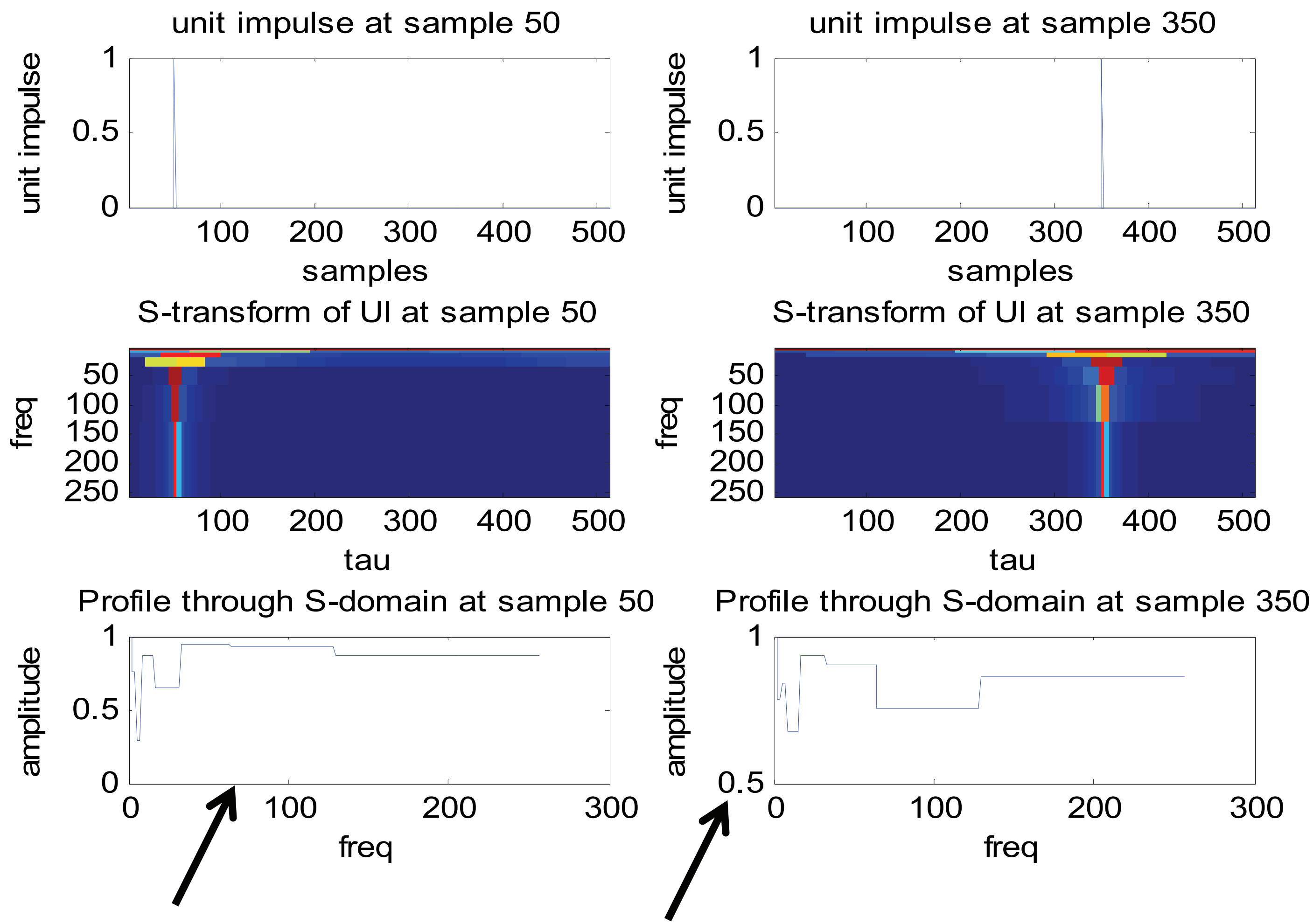
Unit Impulse Response of Fourier transform



What about the Unit Impulse Response of the FST

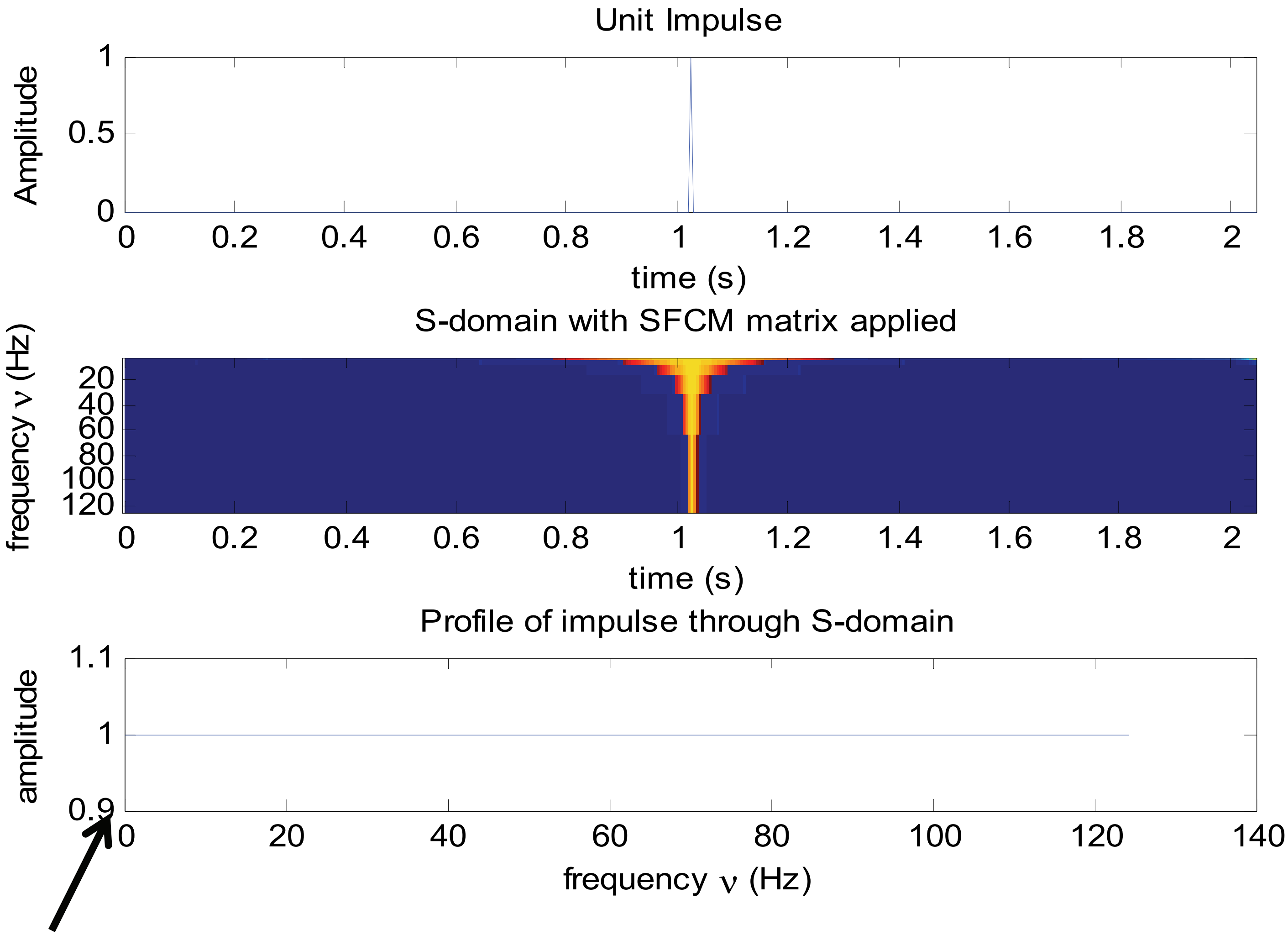


Dependence upon position of unit impulse in time



Spectrum dependent upon location in time

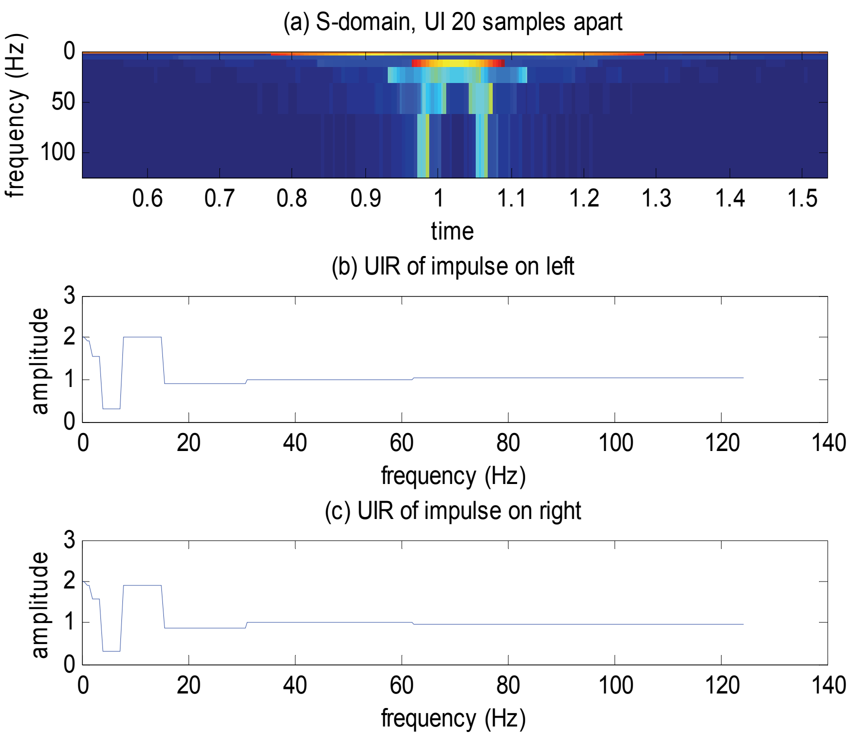
The calibrated fast S-transform (FST)



Nice flat spectrum

Spectrum fidelity thresholds based on proximity of events

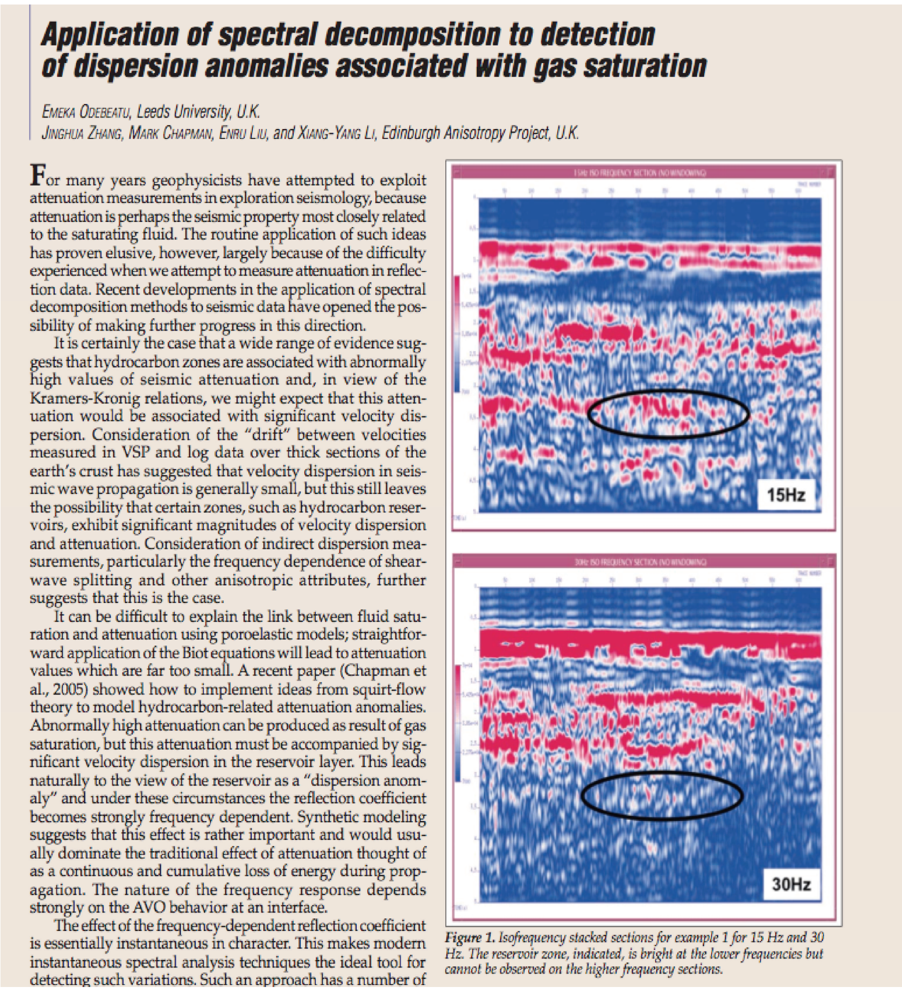
- Due to low resolution at low frequency nature of the FST the spectra of individual events will interfere as a function of proximity
- Defined thresholds or trust regions as a function of proximity to other events (see report for details)



AVF

- Frequency dependent reflections in seismic field data have been associated with highly attenuative targets (Odebeatu et al. 2006)
- Geologically, this may occur for a gas saturated reservoir
- AVF (Amplitude variations with frequency) inversion presents an avenue of determining subsurface rock properties/reservoir characterization

Anelastic Reflectivity



Application of spectral decomposition to detection of dispersion anomalies associated with gas saturation

Emeka Odebeatu, Leeds University, U.K.
Jinghua Zhang, Mark Chapman, Enru Liu, and Xiang-Yang Li, Edinburgh Anisotropy Project, U.K.

For many years geophysicists have attempted to exploit attenuation measurements in exploration seismology, because attenuation is perhaps the seismic property most closely related to the saturating fluid. The routine application of such ideas has proven elusive, however, largely because of the difficulty experienced when we attempt to measure attenuation in reflection data. Recent developments in the application of spectral decomposition methods to seismic data have opened the possibility of making further progress in this direction.

It is certainly the case that a wide range of evidence suggests that hydrocarbon zones are associated with abnormally high values of seismic attenuation and, in view of the Kramers-Kronig relations, we might expect that this attenuation would be associated with significant velocity dispersion. Consideration of the "drift" between velocities measured in VSP and log data over thick sections of the earth's crust has suggested that velocity dispersion in seismic wave propagation is generally small, but this still leaves the possibility that certain zones, such as hydrocarbon reservoirs, exhibit significant magnitudes of velocity dispersion and attenuation. Consideration of indirect dispersion measurements, particularly the frequency dependence of shear-wave splitting and other anisotropic attributes, further suggests that this is the case.

It can be difficult to explain the link between fluid saturation and attenuation using poroelastic models; straightforward application of the Biot equations will lead to attenuation values which are far too small. A recent paper (Chapman et al., 2005) showed how to implement ideas from squirt-flow theory to model hydrocarbon-related attenuation anomalies. Abnormally high attenuation can be produced as result of gas saturation, but this attenuation must be accompanied by significant velocity dispersion in the reservoir layer. This leads naturally to the view of the reservoir as a "dispersion anomaly" and under these circumstances the reflection coefficient becomes strongly frequency dependent. Synthetic modeling suggests that this effect is rather important and would usually dominate the traditional effect of attenuation thought of as a continuous and cumulative loss of energy during propagation. The nature of the frequency response depends strongly on the AVO behavior at an interface.

The effect of the frequency-dependent reflection coefficient is essentially instantaneous in character. This makes modern instantaneous spectral analysis techniques the ideal tool for detecting such variations. Such an approach has a number of

Figure 1. Isofrequency stacked sections for example 1 for 15 Hz and 30 Hz. The reservoir zone, indicated, is bright at the lower frequencies but cannot be observed on the higher frequency sections.

- Frequency dependent reflection coefficient associated with a gas saturated target (Odebeatu et al. 2006)
- Our goal is to develop the means to extract target information from this type of variability

AVF Inversion

- AVF inversion presents an untapped avenue of determining subsurface rock properties/reservoir characterization
- Theory for determining target Q, Qp or Qs given frequency dependent reflection coefficient as input

Anelastic Reflection coefficients

- Reflection Coefficient(R) in terms of vertical wavenumber

Normal incidence ⟶

$$R = \frac{k_{z_i} - k_{z_{i+1}}}{k_{z_i} + k_{z_{i+1}}} \qquad (1)$$

Use expression for wavenumber from Aki and Richards

$$k = \frac{\omega}{c}\left(1 + \frac{F(\omega)}{Q}\right) \qquad (2)$$

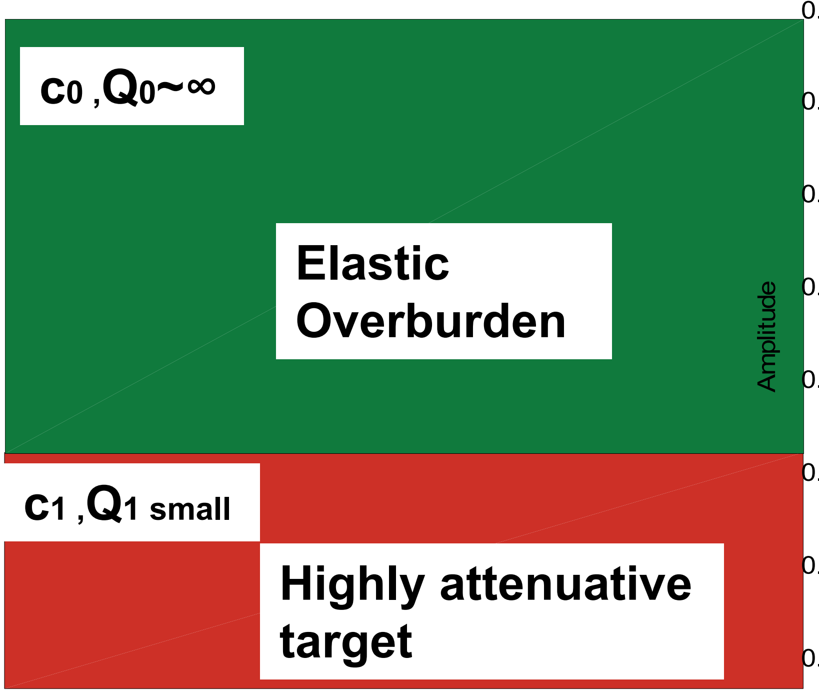
where

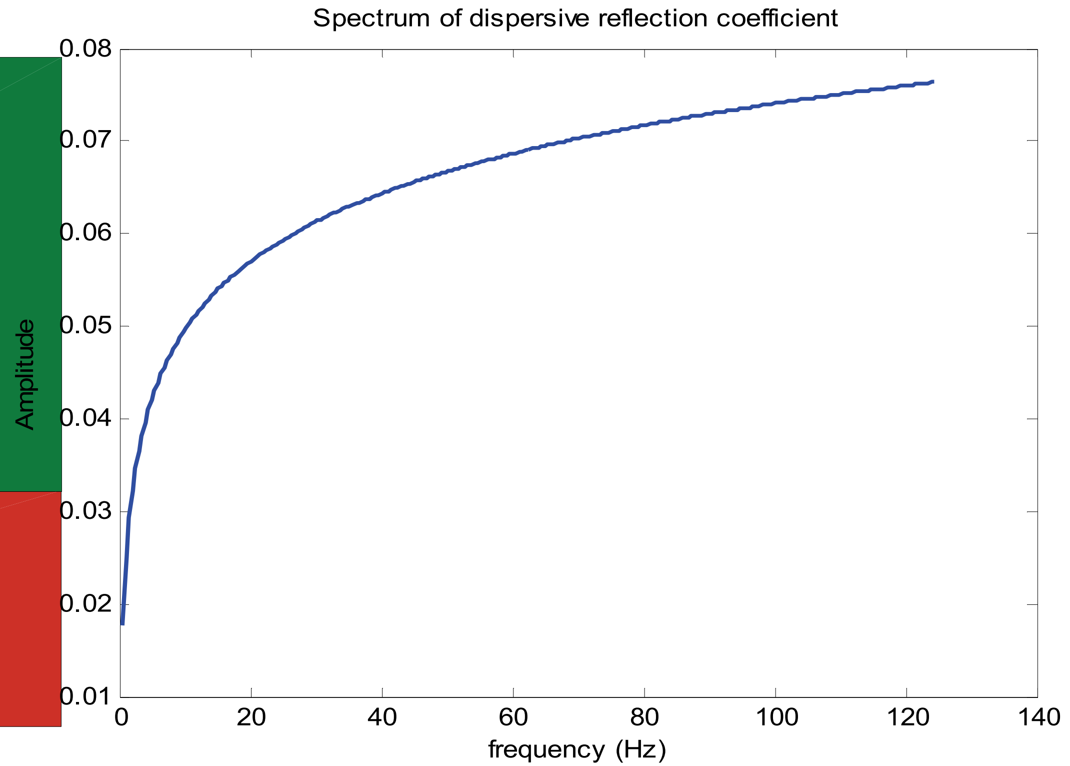
$$F_{(\omega)} = \frac{i}{2} - \frac{1}{\pi}\log(\omega / \omega_r)$$

- Substitute (2) into (1) To obtain expression for anelastic Reflection coefficients.

$$R(\omega) = \frac{\frac{1}{c_i}\left(1 + \frac{F(\omega)}{Q_i}\right) - \frac{1}{c_{i+1}}\left(1 + \frac{F(\omega)}{Q_{i+1}}\right)}{\frac{1}{c_i}\left(1 + \frac{F(\omega)}{Q_i}\right) - \frac{1}{c_{i+1}}\left(1 + \frac{F(\omega)}{Q_{i+1}}\right)} \qquad (3)$$

Anelastic Reflection Coefficients





- Equation (3) becomes

$$R(\omega) = \frac{\dfrac{1}{c_i} - \dfrac{1}{c_{i+1}}\left(1 + \dfrac{F(\omega)}{Q_{i+1}}\right)}{\dfrac{1}{c_i} - \dfrac{1}{c_{i+1}}\left(1 + \dfrac{F(\omega)}{Q_{i+1}}\right)} \qquad (4)$$

The Forward AVF Problem

- Innanen et al. (2008) cast the problem in terms of perturbations:

$$a_c = 1 - \frac{{c_0}^2}{{c_1}^2} \quad a_Q = \frac{1}{Q_1}$$

- Expand and linearize equation (4) to obtain

$$R(\omega) = -\frac{1}{2} a_Q F(\omega) + \frac{1}{4} a_c \tag{5}$$

- This represents the forward problem

AVF inversion

- Equation (5) $R(\omega) = -\frac{1}{2} a_Q F(\omega) + \frac{1}{4} a_c$

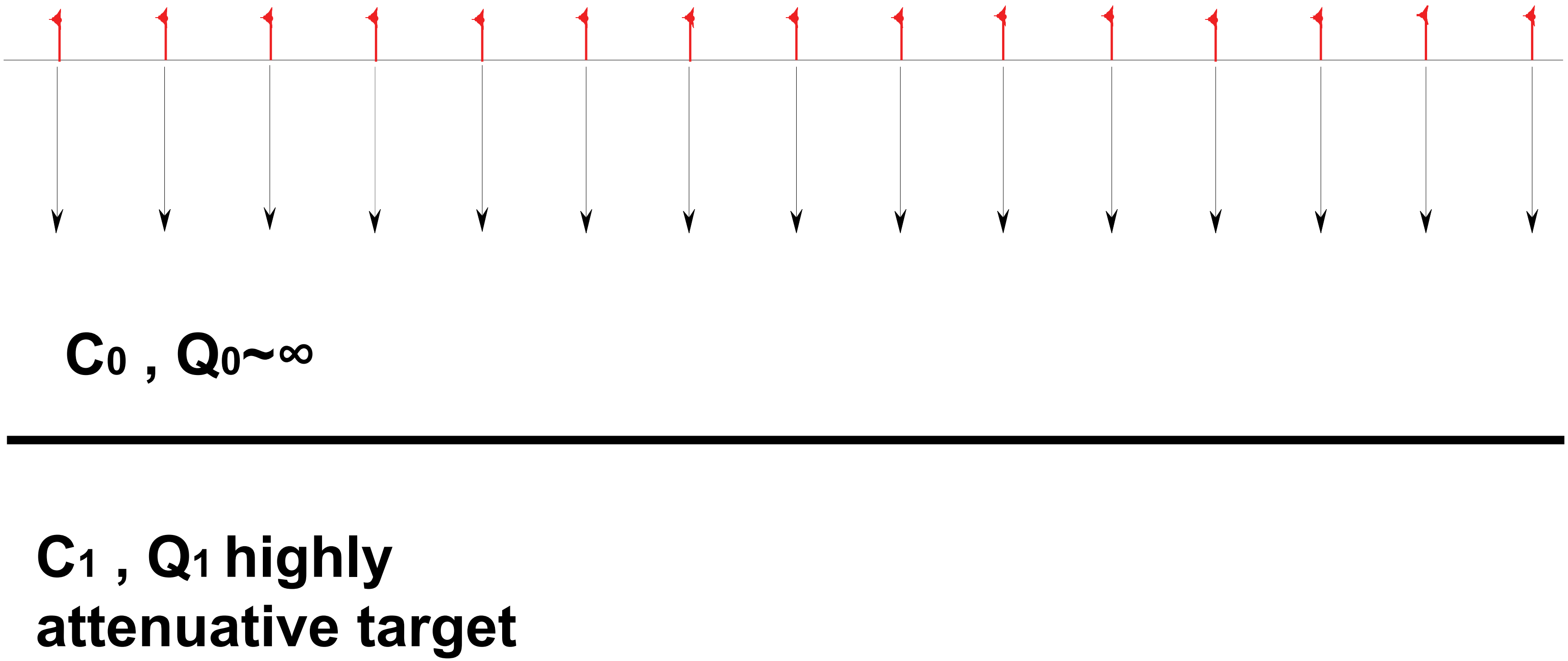
- We are interested in the inverse problem of determining a_Q from estimates of $R(\omega)$.
- Given $R(\omega)$ at two exact values of frequency, ω_1 and ω_2, a_Q is estimated linearly as

$$a_Q = -2\left(\frac{R(\omega_2) - R(\omega_1)}{F(\omega_2) - F(\omega_1)}\right) \qquad (6)$$

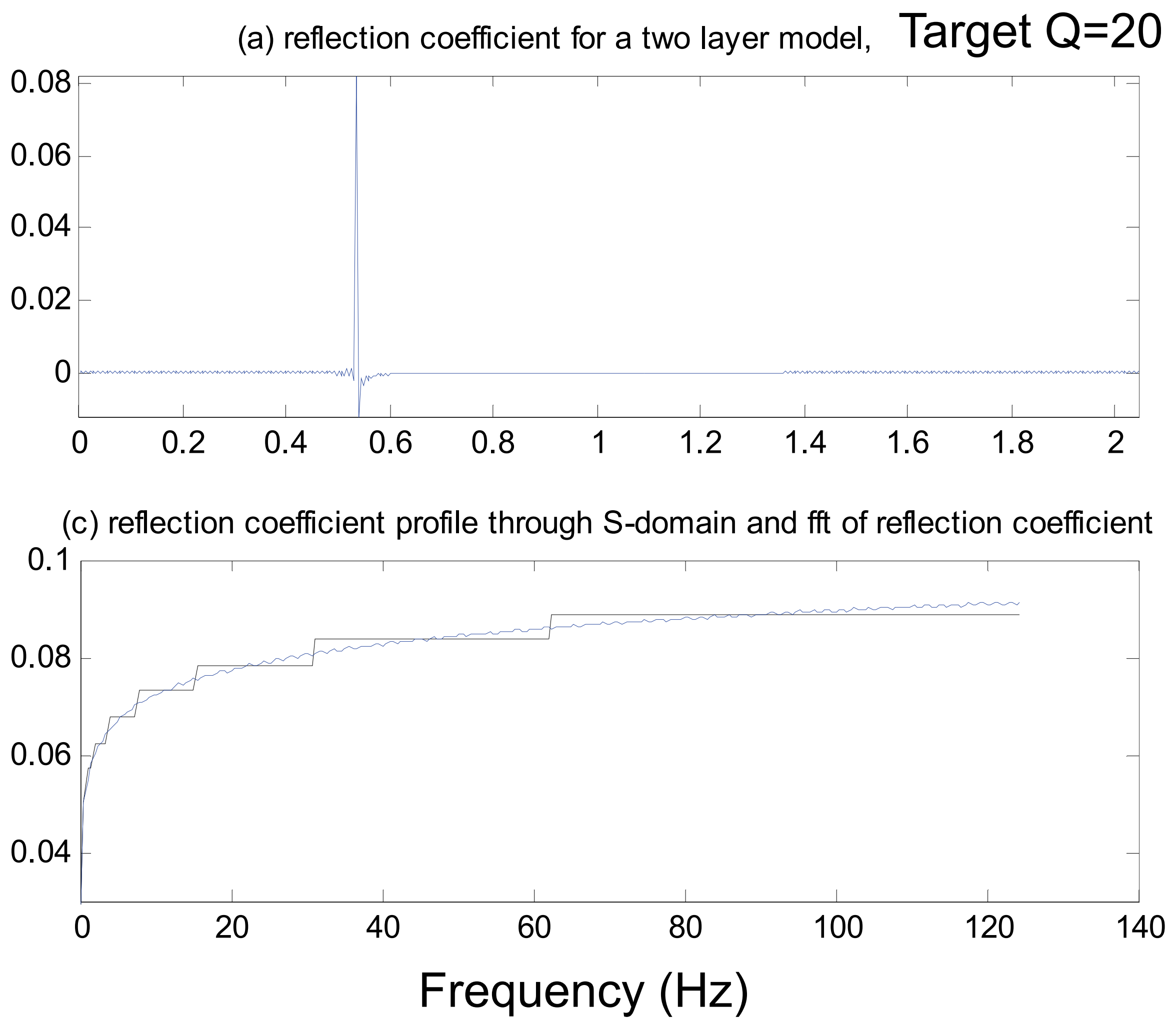
Realistic input for AVF Inversion

- A time-frequency decomposition such as the FST generates average amplitudes over fixed frequency bands
- Alter inverse theory to accommodate input from FST
  - Use forward modeling codes to generate traces with anelastic reflection coefficients
  - Implement the FST to estimate R(ω). Call this estimate $\widetilde{R}(\omega)$
  - Compare $\widetilde{R}(\omega)$ with R(ω)
  - Reformulate AVF inversion to take $\widetilde{R}(\omega)$ as input

Forward modeling anelastic reflection coefficients



Spectrum from FST compared with FFT



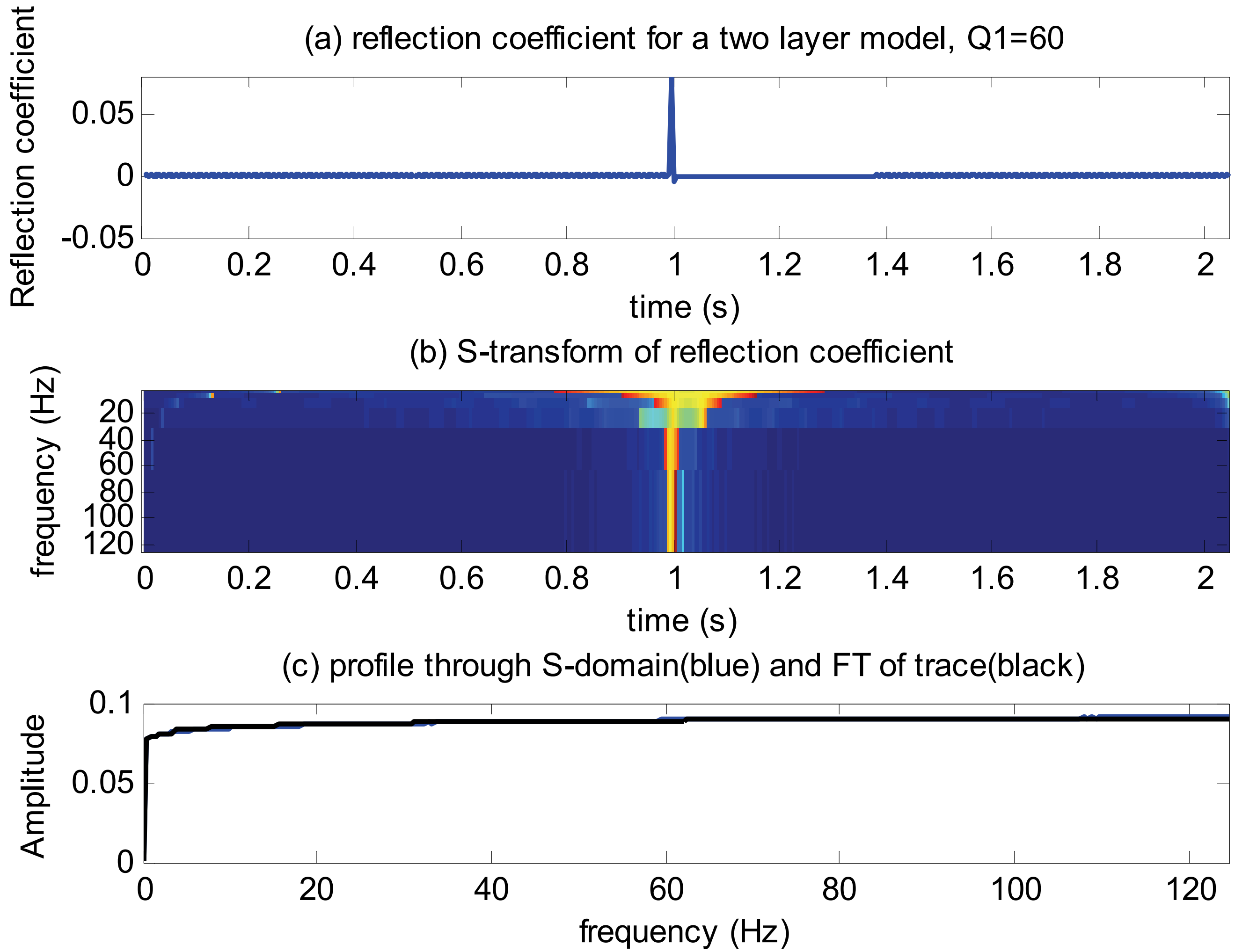
Analytic spectrum

Spectrum estimated by FST

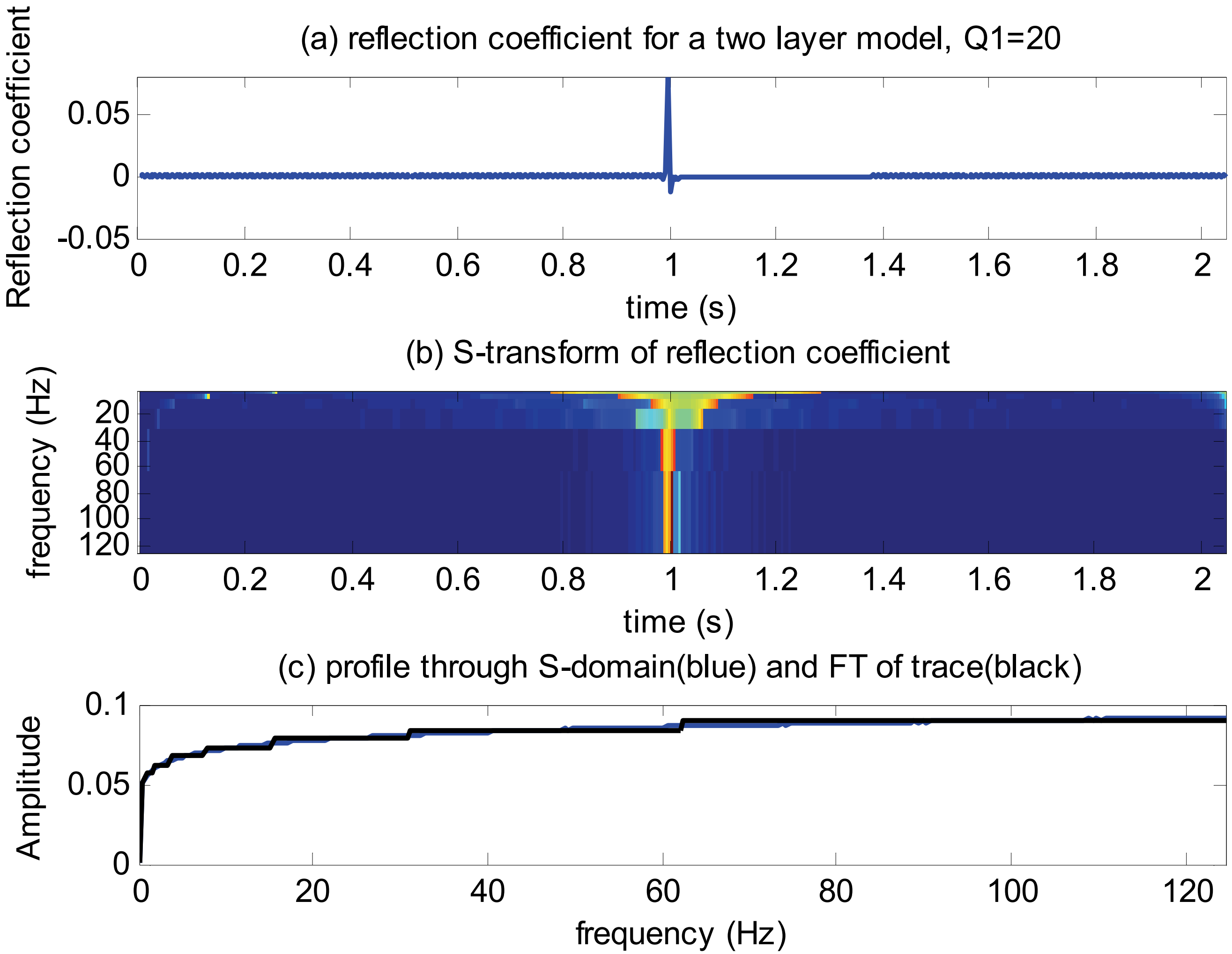
The FST provides a very good approximation to the average

Comparison of FST spectrum to FFT

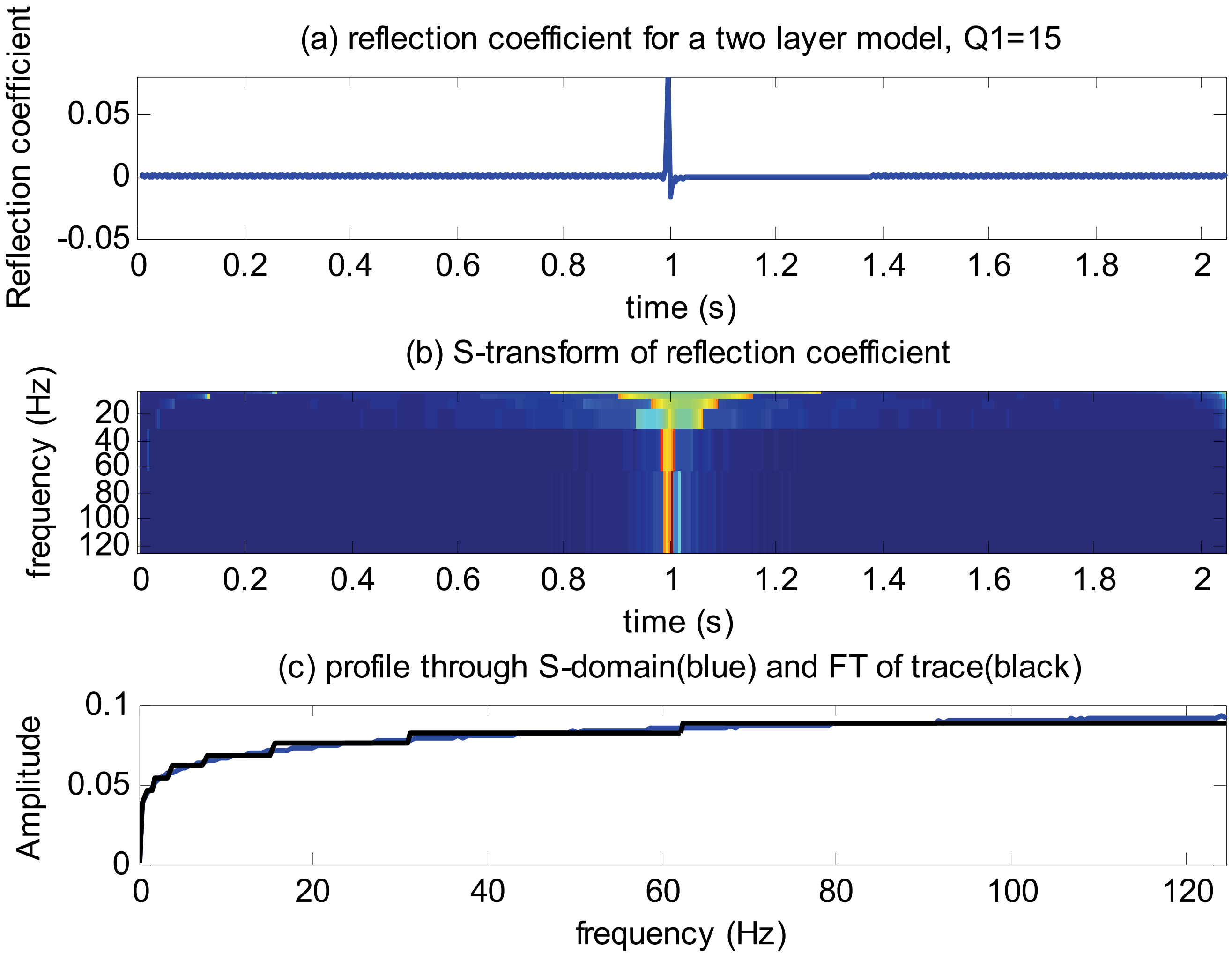
Target Q=60



Target Q=20

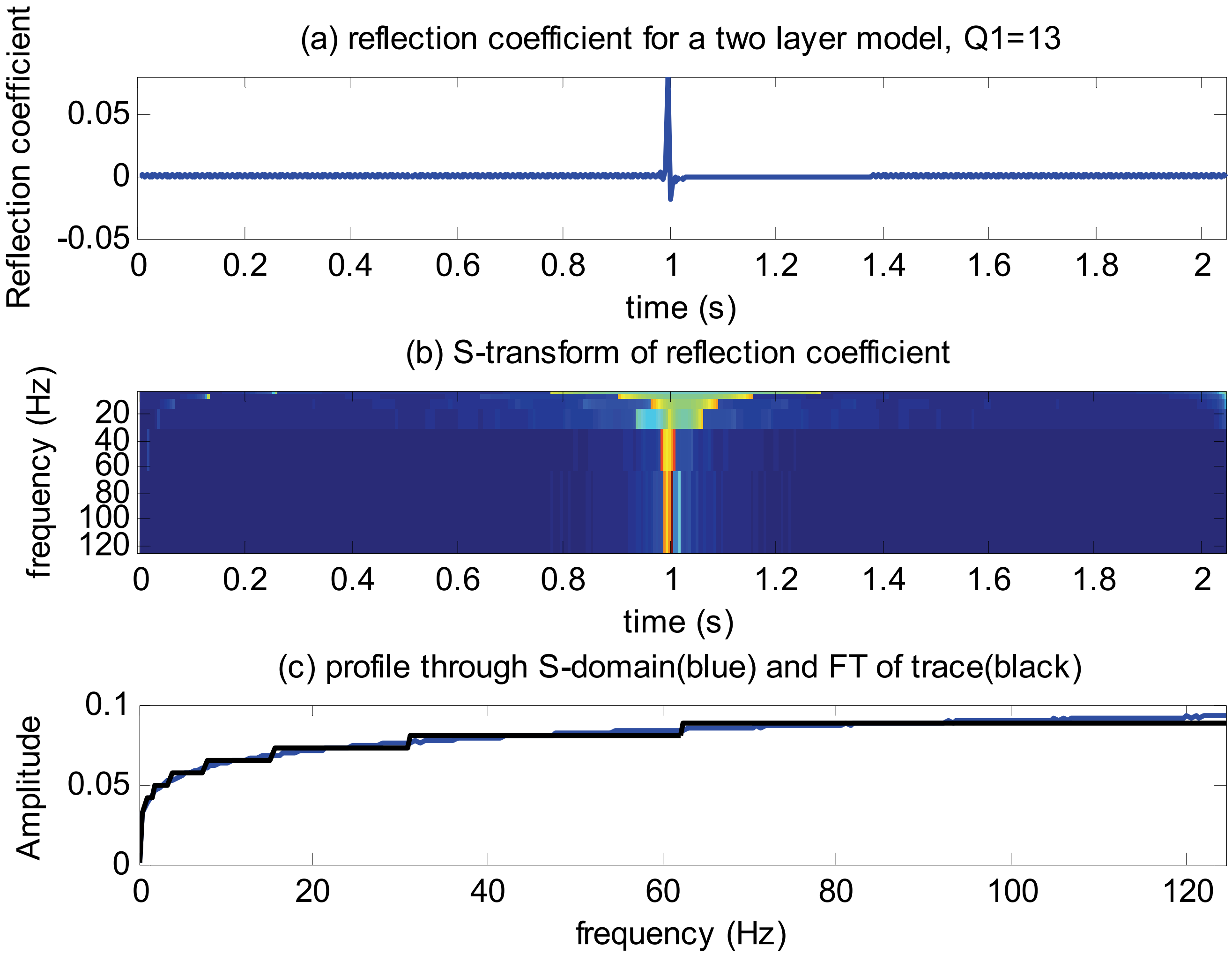

(a) reflection coefficient for a two layer model, Q1=20
Reflection coefficient
0.05
0
-0.05
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time (s)
(b) S-transform of reflection coefficient
frequency (Hz)
20
40
60
80
100
120
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time (s)
(c) profile through S-domain(blue) and FT of trace(black)
Amplitude
0.1
0.05
0
0
20
40
60
80
100
120
frequency (Hz)

Target Q=15



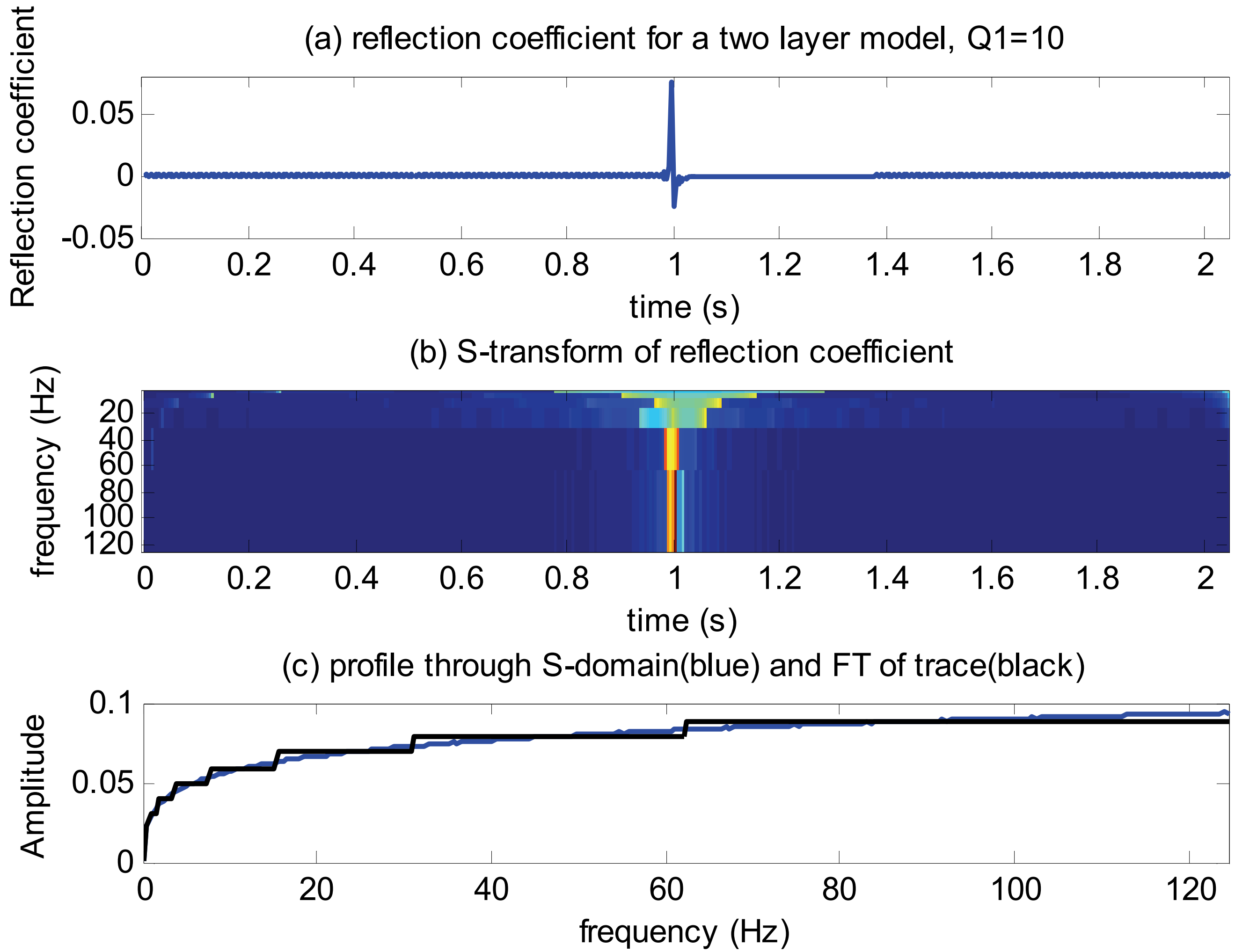
(a) reflection coefficient for a two layer model, Q1=15
Reflection coefficient
0.05
0
-0.05
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time (s)
(b) S-transform of reflection coefficient
frequency (Hz)
20
40
60
80
100
120
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time (s)
(c) profile through S-domain(blue) and FT of trace(black)
Amplitude
0.1
0.05
0
0
20
40
60
80
100
120
frequency (Hz)

Target Q=13



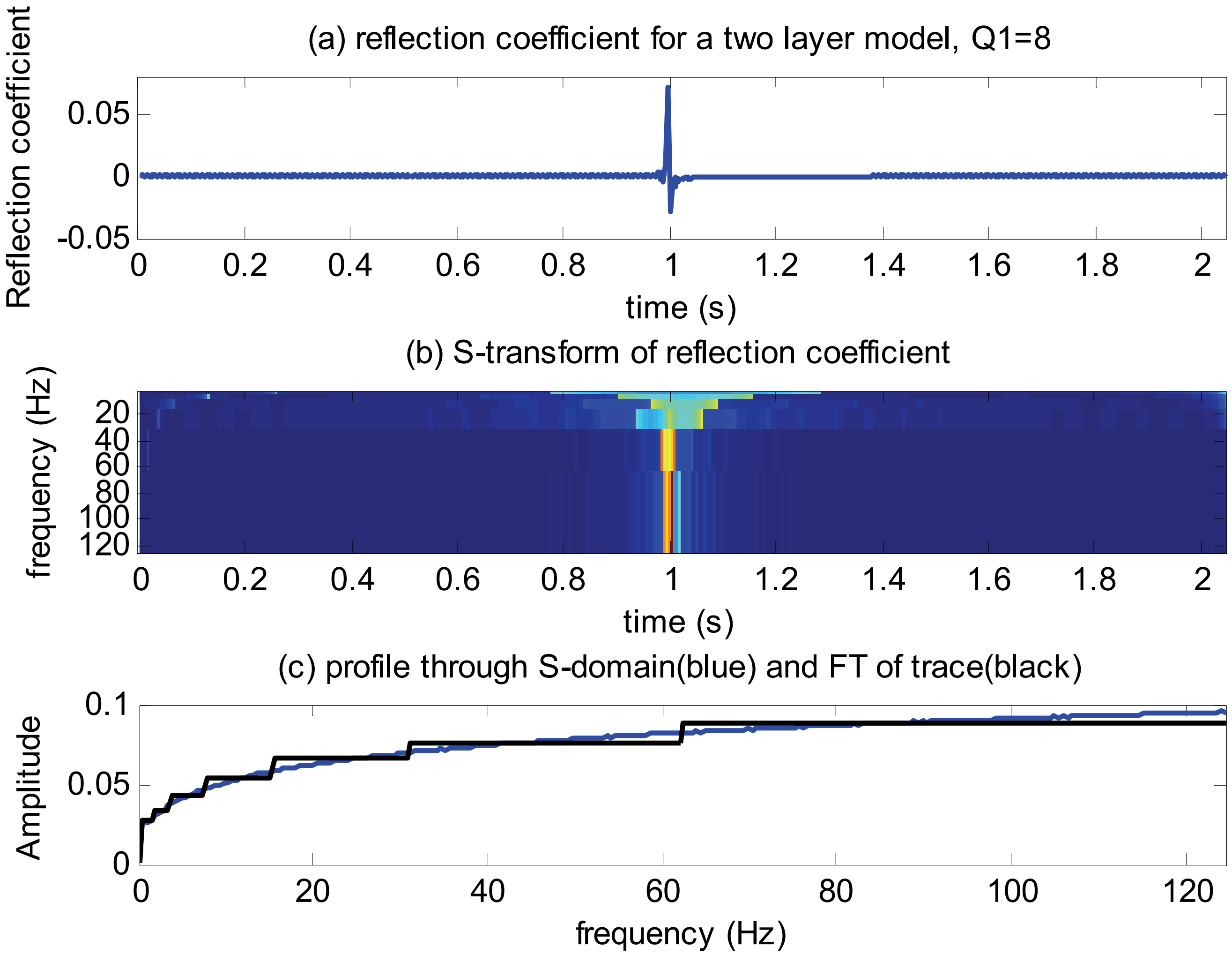
(a) reflection coefficient for a two layer model, Q1=13
Reflection coefficient
0.05
0
-0.05
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time (s)
(b) S-transform of reflection coefficient
frequency (Hz)
20
40
60
80
100
120
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time (s)
(c) profile through S-domain(blue) and FT of trace(black)
Amplitude
0.1
0.05
0
0
20
40
60
80
100
120
frequency (Hz)

Target Q=10



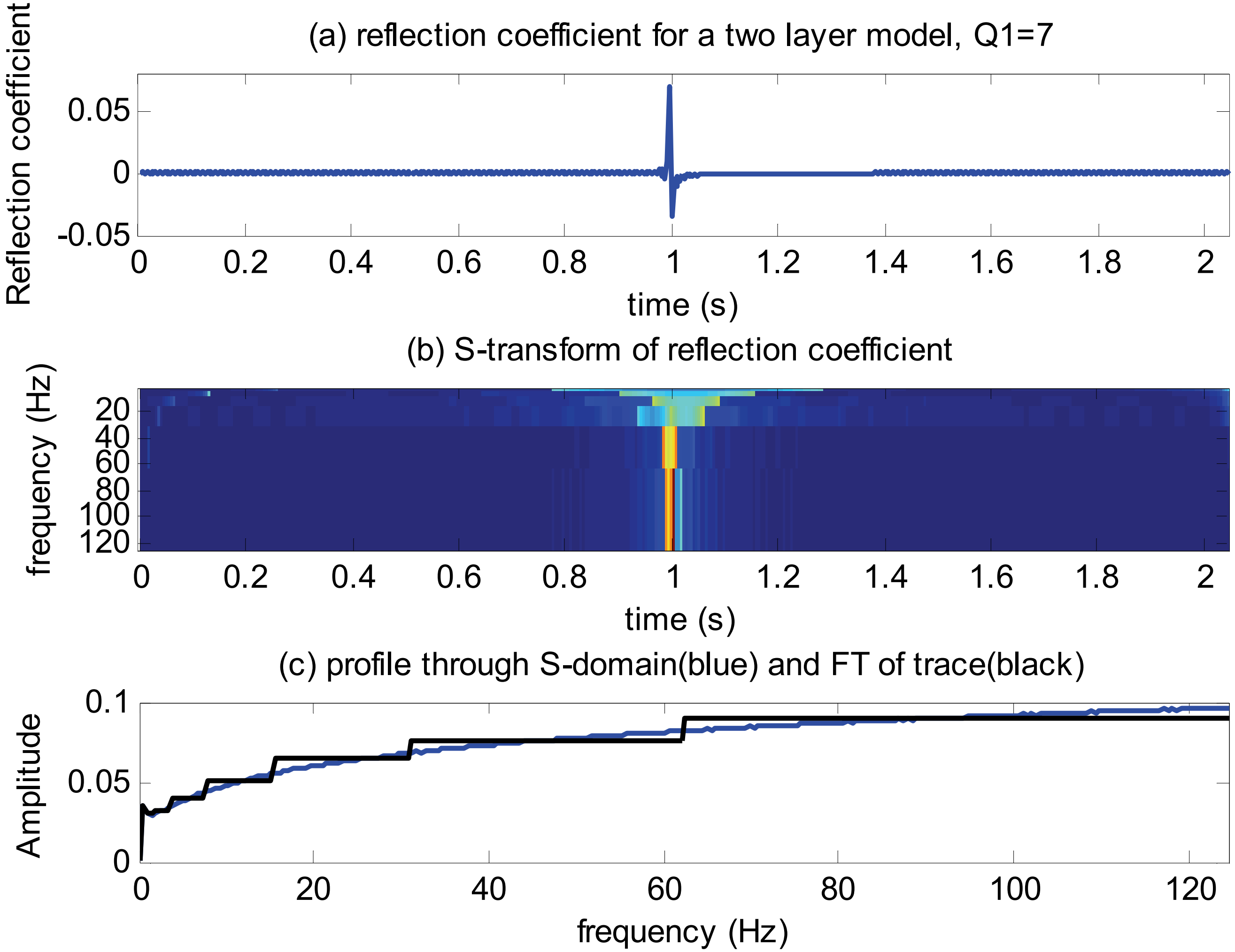
(a) reflection coefficient for a two layer model, Q1=10
Reflection coefficient
0.05
0
-0.05
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time (s)
(b) S-transform of reflection coefficient
frequency (Hz)
20
40
60
80
100
120
time (s)
(c) profile through S-domain(blue) and FT of trace(black)
Amplitude
0.1
0.05
0
20
40
60
80
100
120
frequency (Hz)

Target Q=8



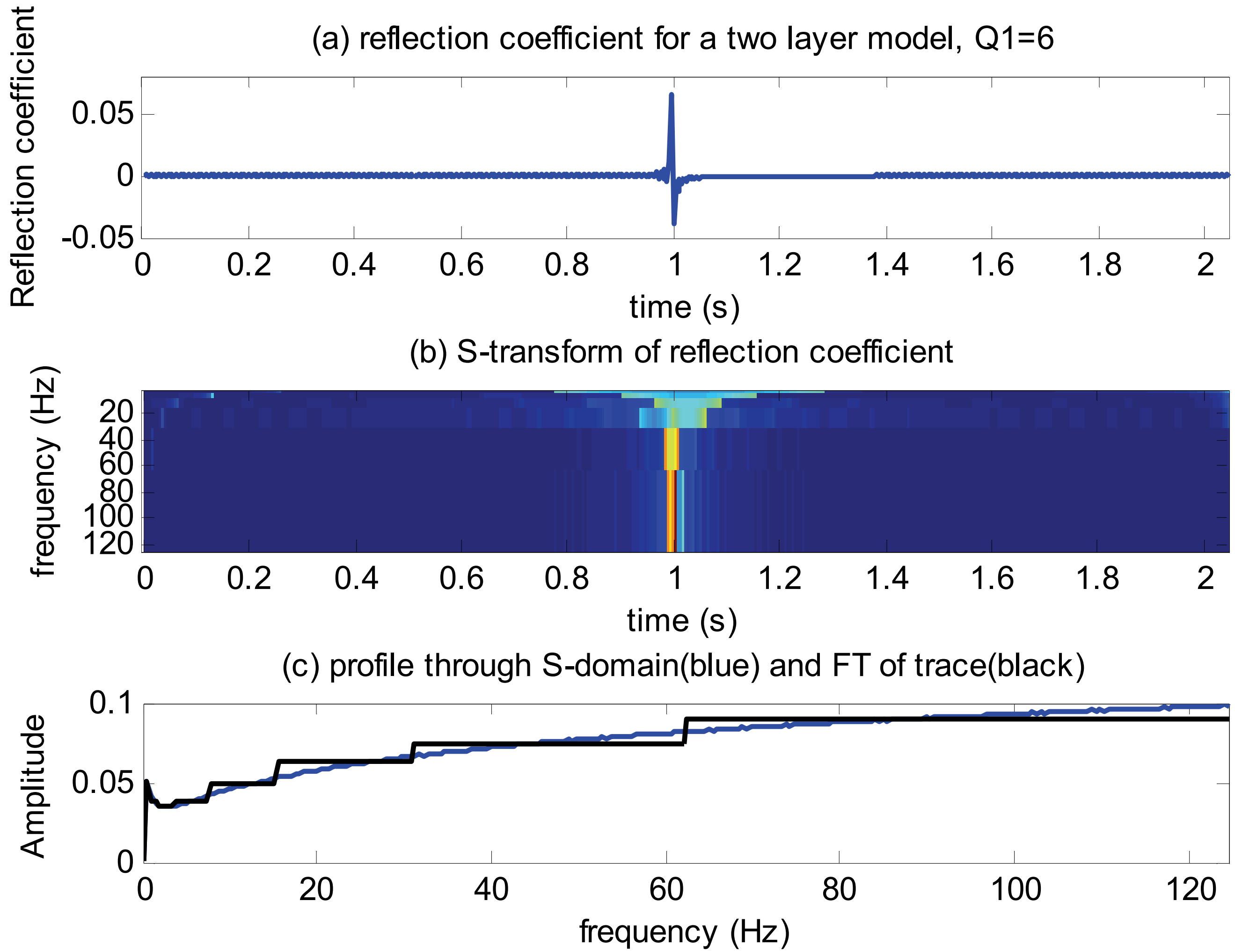
(a) reflection coefficient for a two layer model, Q1=8
Reflection coefficient
0.05
0
-0.05
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time (s)
(b) S-transform of reflection coefficient
frequency (Hz)
20
40
60
80
100
120
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time (s)
(c) profile through S-domain(blue) and FT of trace(black)
Amplitude
0.1
0.05
0
0
20
40
60
80
100
120
frequency (Hz)

Target Q=7

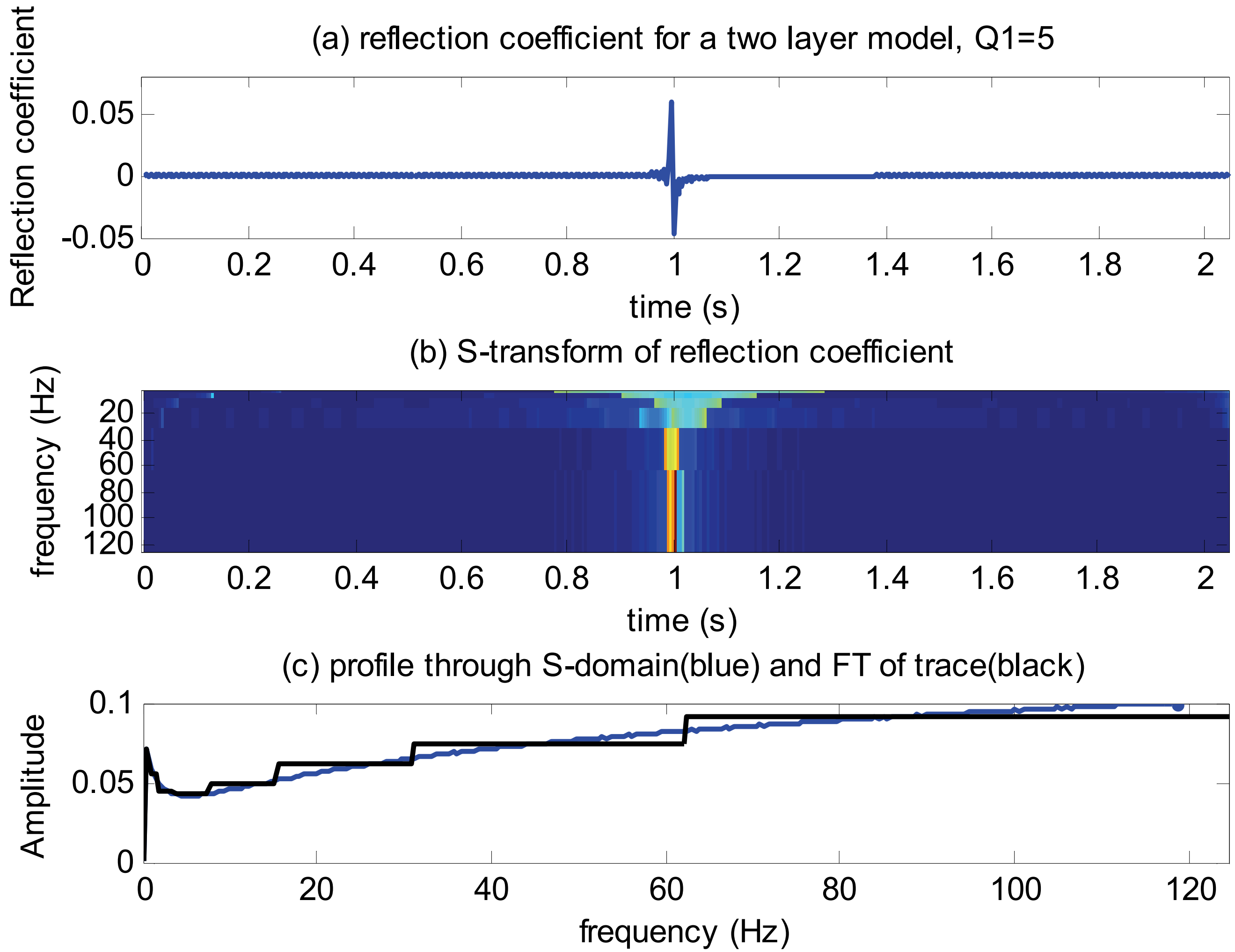


(a) reflection coefficient for a two layer model, Q1=7
Reflection coefficient
0.05
0
-0.05
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time (s)
(b) S-transform of reflection coefficient
frequency (Hz)
20
40
60
80
100
120
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time (s)
(c) profile through S-domain(blue) and FT of trace(black)
Amplitude
0.1
0.05
0
0
20
40
60
80
100
120
frequency (Hz)

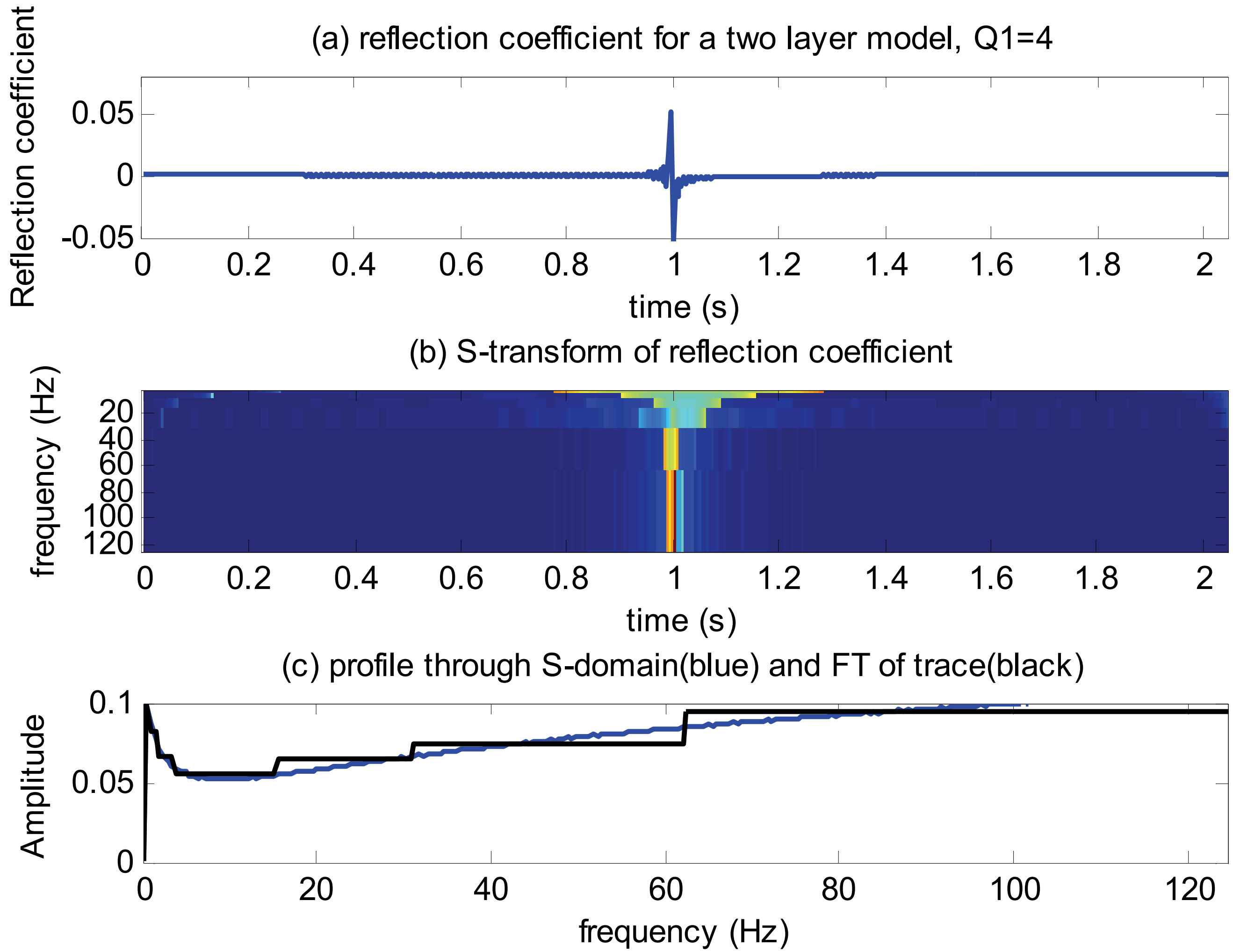
Target Q=6


(a) reflection coefficient for a two layer model, Q1=6
Reflection coefficient
0.05
0
-0.05
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time (s)
(b) S-transform of reflection coefficient
frequency (Hz)
20
40
60
80
100
120
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time (s)
(c) profile through S-domain(blue) and FT of trace(black)
Amplitude
0.1
0.05
0
0
20
40
60
80
100
120
frequency (Hz)

Target Q=5

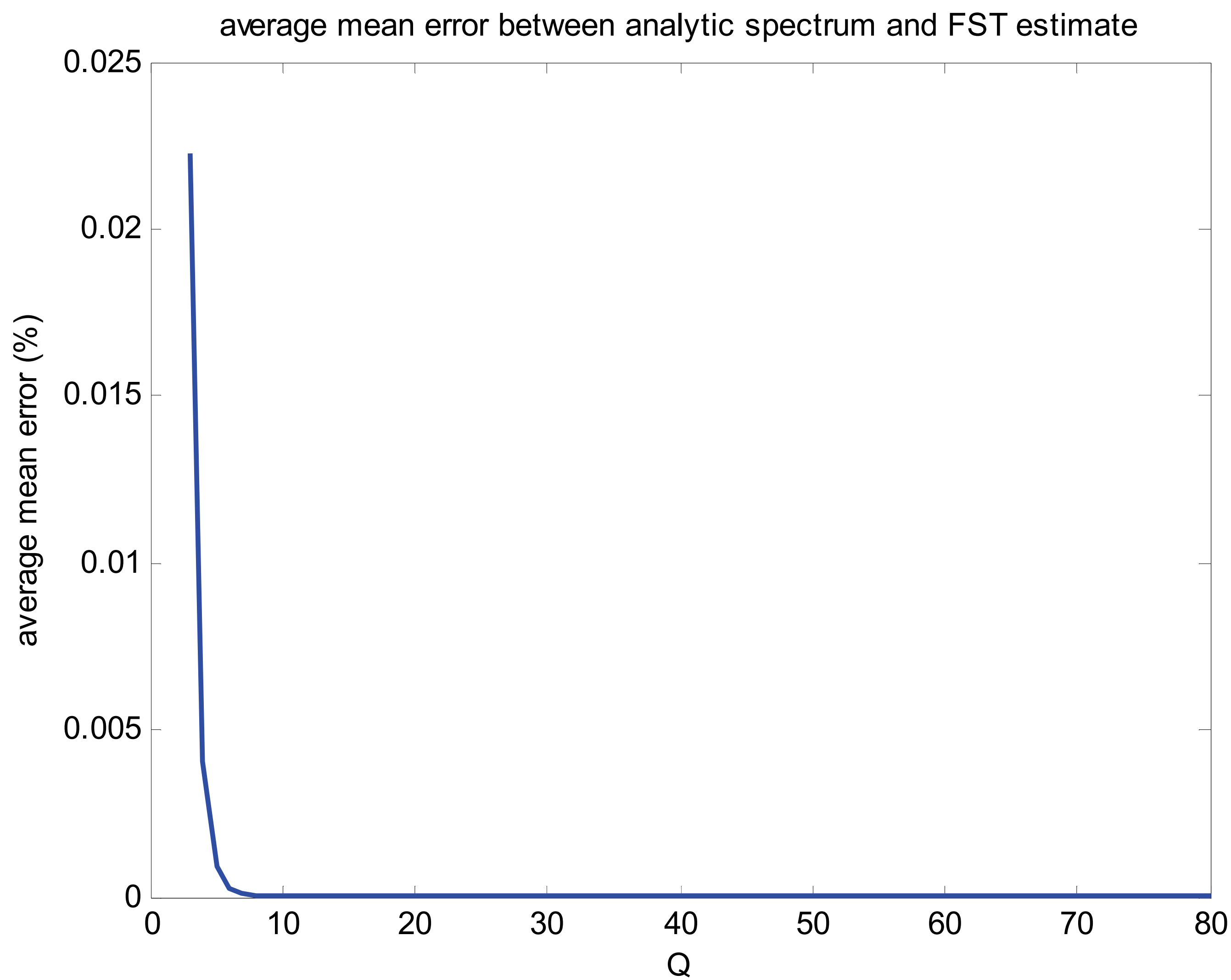

(a) reflection coefficient for a two layer model, Q1=5
Reflection coefficient
0.05
0
-0.05
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time (s)
(b) S-transform of reflection coefficient
frequency (Hz)
20
40
60
80
100
120
time (s)
(c) profile through S-domain(blue) and FT of trace(black)
Amplitude
0.1
0.05
0
20
40
60
80
100
120
frequency (Hz)

Target
Q=4



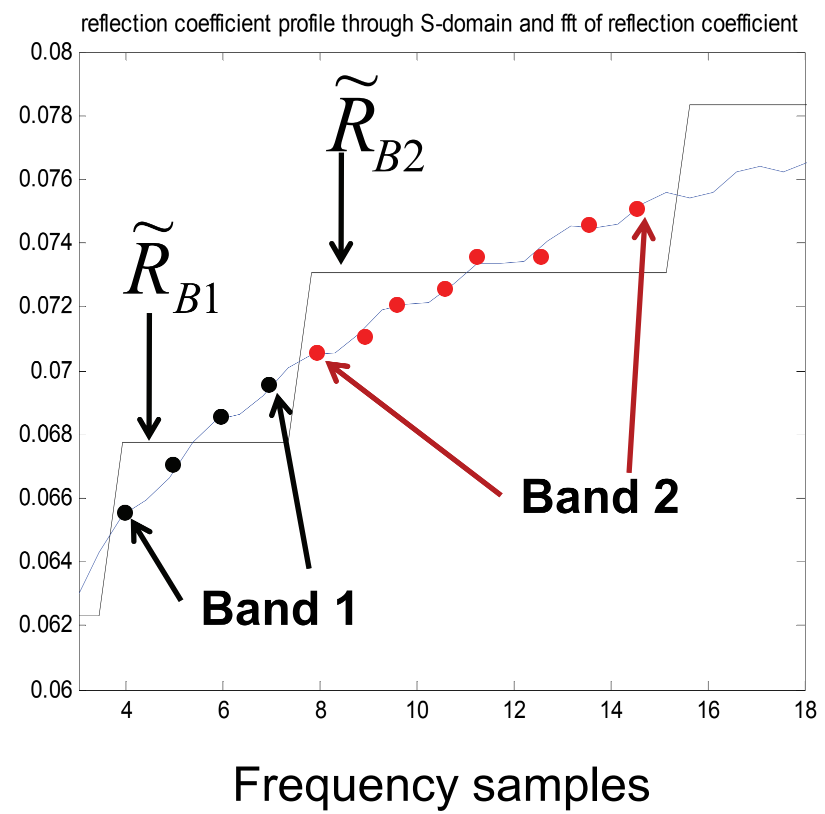
(a) reflection coefficient for a two layer model, Q1=4
Reflection coefficient
0.05
0
-0.05
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
time (s)
(b) S-transform of reflection coefficient
frequency (Hz)
20
40
60
80
100
120
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
time (s)
(c) profile through S-domain(blue) and FT of trace(black)
Amplitude
0.1
0.05
0
0 20 40 60 80 100 120
frequency (Hz)

Error between FST and analytic



Reformulating inversion to take $\tilde{R}(\omega)$ as input

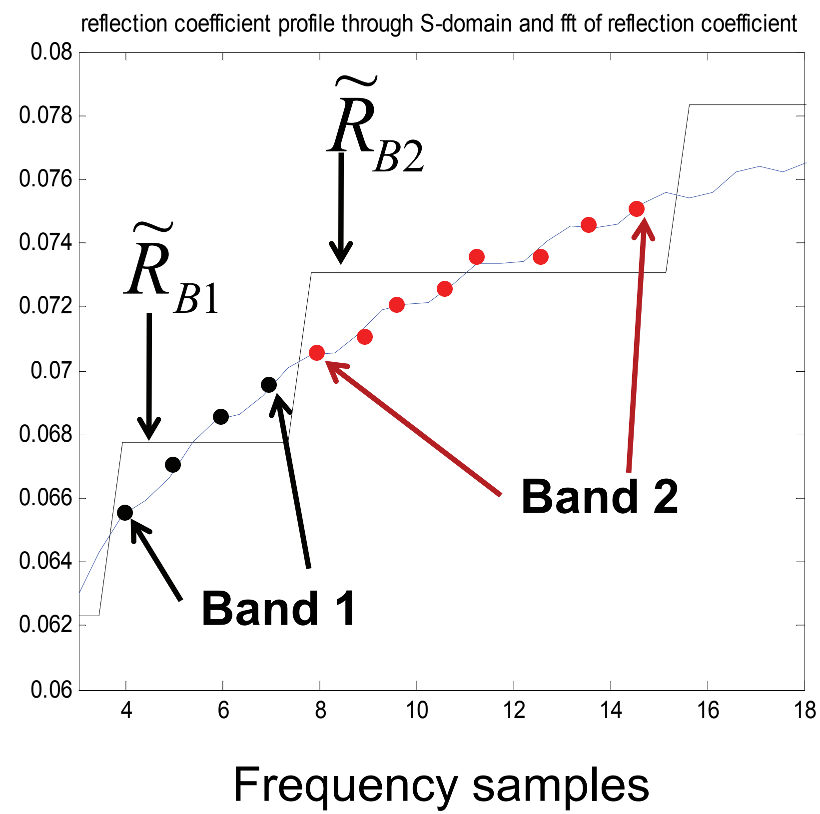
- The $\tilde{R}(\omega)$ values are an average of the true spectrum over the frequency band of each tile.
- Therefore, $\tilde{R}_{B1}$ and $\tilde{R}_{B2}$ are the mean amplitudes of their respective frequency bands
- Also, define $\tilde{F}_{B1}$ and $\tilde{F}_{B2}$ as the sum of the known F(ω) function over these frequency bands



Reformulating inversion to take $\widetilde{R}(\omega)$ as input

- let m_1 and m_2 be the number of samples in Band 1 and Band 2
- Z is a normalization parameter
- Then the equation for AVF inversion becomes

$$a_Q = -2\frac{(m_1\widetilde{R}_{B1} - Zm_2\widetilde{R}_{B2})}{\widetilde{F}_{B1} - Z\widetilde{F}_{B2}}$$

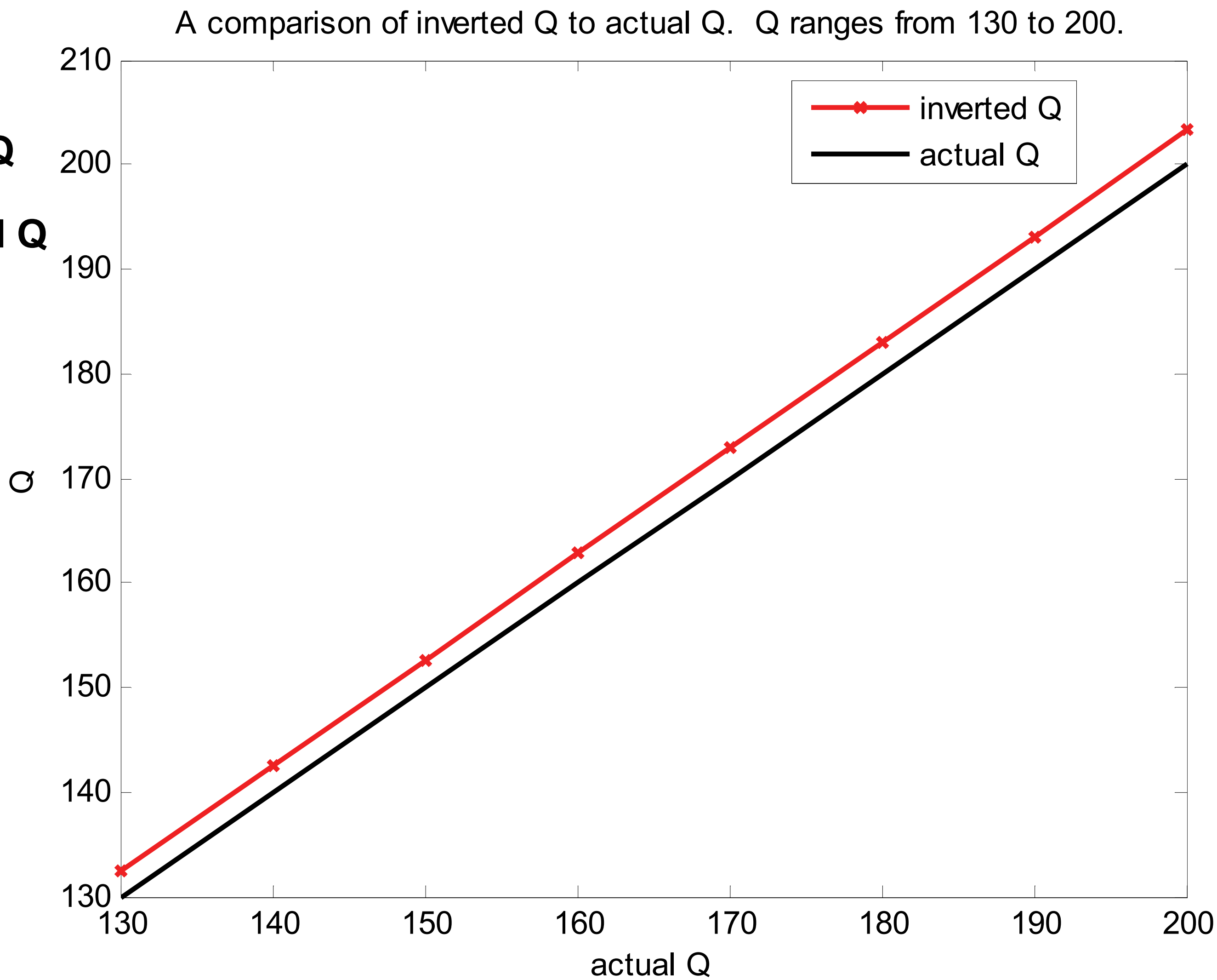


See report for details

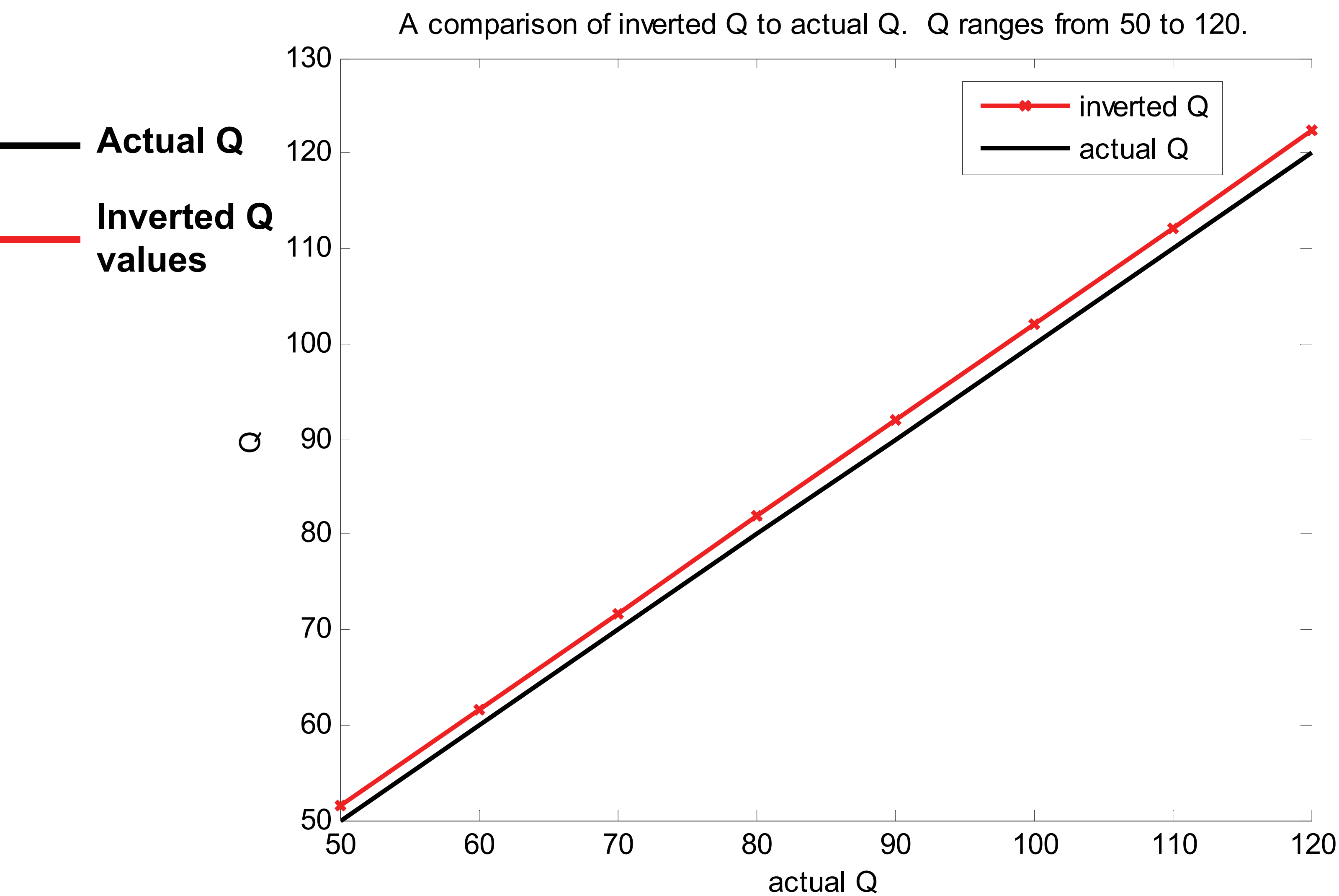
Initial Results
Q ranges from 130 to 200

Actual Q

Inverted Q values



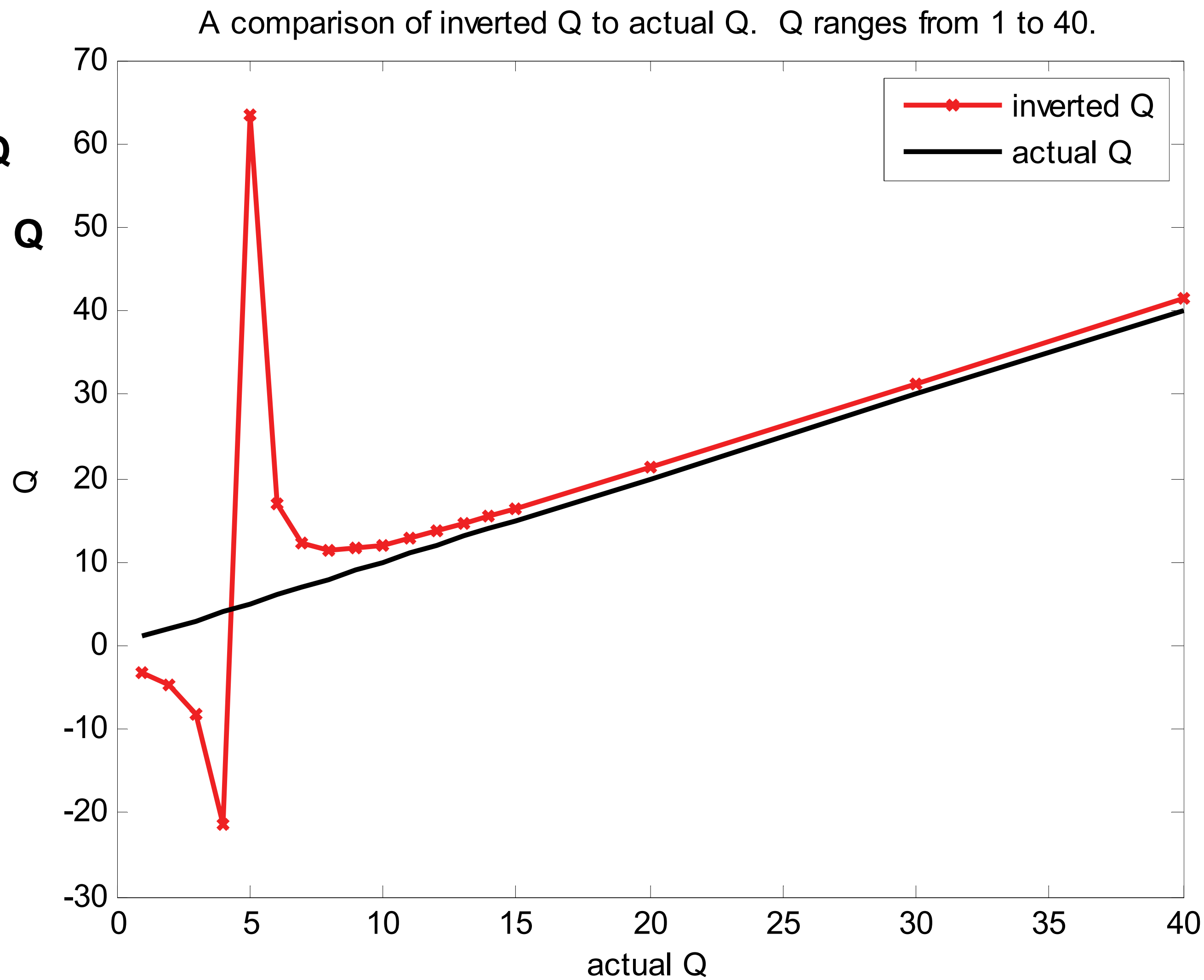
Initial Results
Q ranges from 50 to 120



Initial Results
Q ranges from 1 to 40

Actual Q

Inverted Q values



Conclusions

- The calibrated FST produces close approximation of spectrum
- The AVF inversion theory was reformulated to use FST estimates of reflectivity as input
- The inverted values of Q are in close agreement with the actual values until Q drops below 8. In part, this is due to linearization error.

Future Work

- Non-normal incidence
  - Aperture limitations
  - Consider problem in space domain
- Implement non-linear corrections
- Anelastic formulations
- Wavelet, Noise
- Fidelity thresholds of spectrum based on proximity of events

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- CREWES sponsors
- Thanks for listening