REFLECTIVITY MODELING BY FINITE DIFFERENCE

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Basic Equations – the Acoustic Wave Problem

- **Standard reflectivity method (e.g. Müller, 1985) uses summation over the lateral wavenumber and plane- wave domain propagator matrices for vertical propagation.**
- **The finite difference method (Mikhailenko, 1970- present) uses the same summation over the lateral wavenumber but finite difference in *z* and *t*.**
- **A major effect of this is the run time is independent of the number of "layers"**
- **Also, the finite difference code cannot easily suppress the surface effects.**

Basic Equations – the Acoustic Wave Problem

$$(1).\quad \rho\alpha^2\nabla^2\psi(r,z,t)-\rho\frac{\partial^2\psi}{\partial t^2}-b\frac{\partial\psi}{\partial t}=\frac{\delta(r)\delta(z-z_s)}{2\pi r}f(t)$$

Finite Hankel Transform and Inverse

$$\psi(k_j,z,t)=\int_0^a \psi(r,z,t)J_1(k_j r)rdr$$

$$\psi(r,z,t)=\frac{2}{a^2}\sum_{j=0}^{\infty}\frac{\psi(k_j,z,t)J_1(k_j r)}{\left(J_0(k_j r)\right)^2}$$

$J_1(k_j a)=0 \quad \leftarrow \quad$ trancendental equation

infinite number of roots

$$(2).\quad \rho\alpha^2 \frac{\partial^2\psi(r,z,t)}{\partial z^2} - \rho\alpha^2 k_j^2 \psi(r,z,t) - \rho \frac{\partial^2\psi(r,z,t)}{\partial t^2} - b\frac{\partial\psi(r,z,t)}{\partial t} = \frac{\delta(z-z_S)}{4\pi} f(t)$$

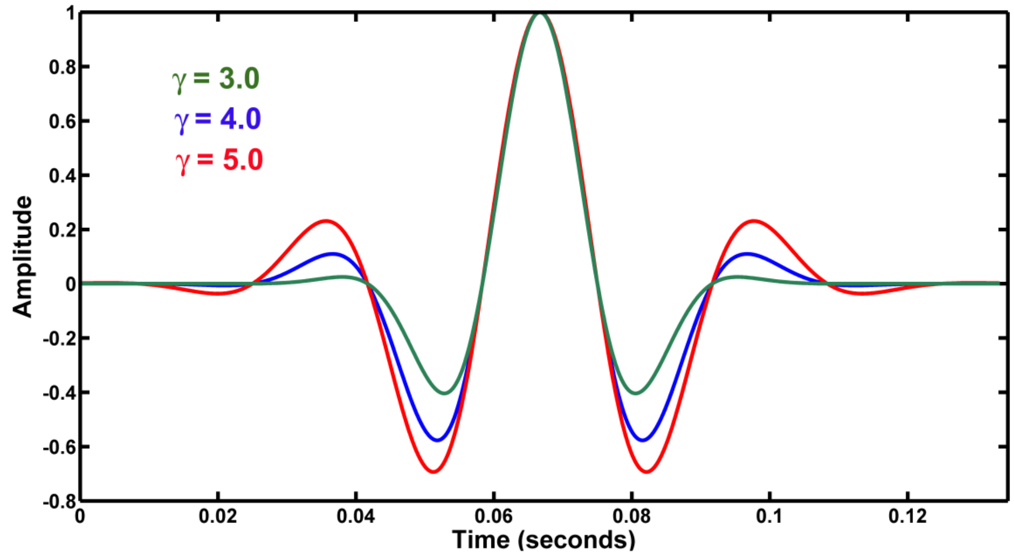
f(t) **– band limited source wavelet allows for a finite truncation of the infinite inverse series.**

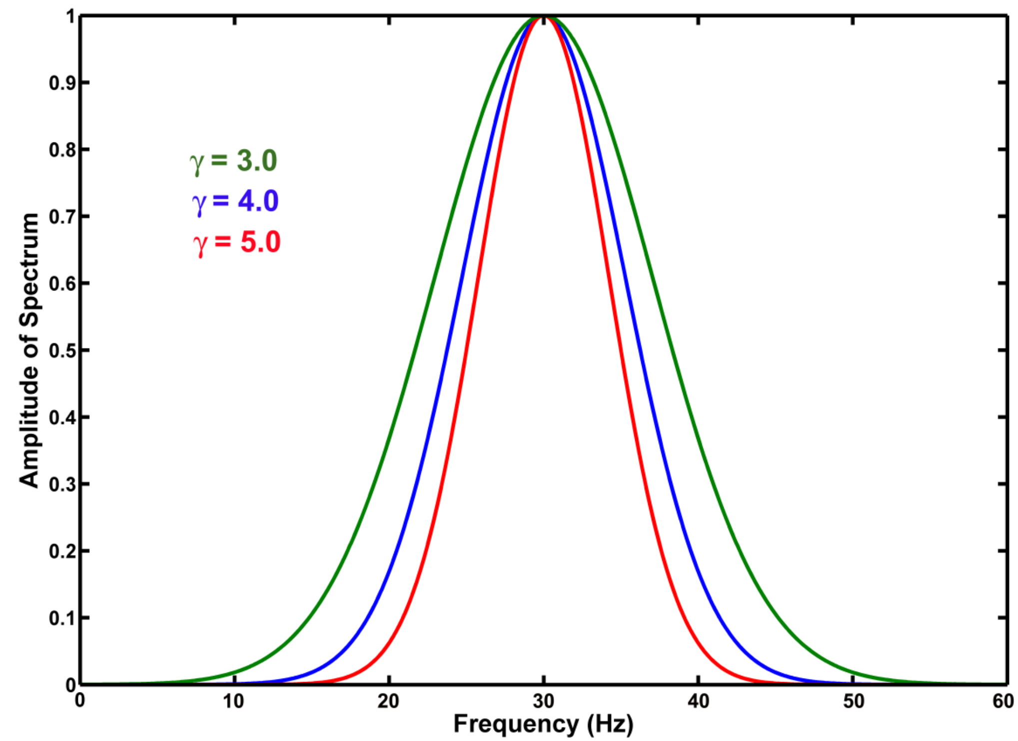
b **is an attenuation factor proportional to** Q^{-1}

Required that some form of finite difference analogue be constructed for equation (2).
O(Δz^2, Δt^2) is sufficiently accurate.

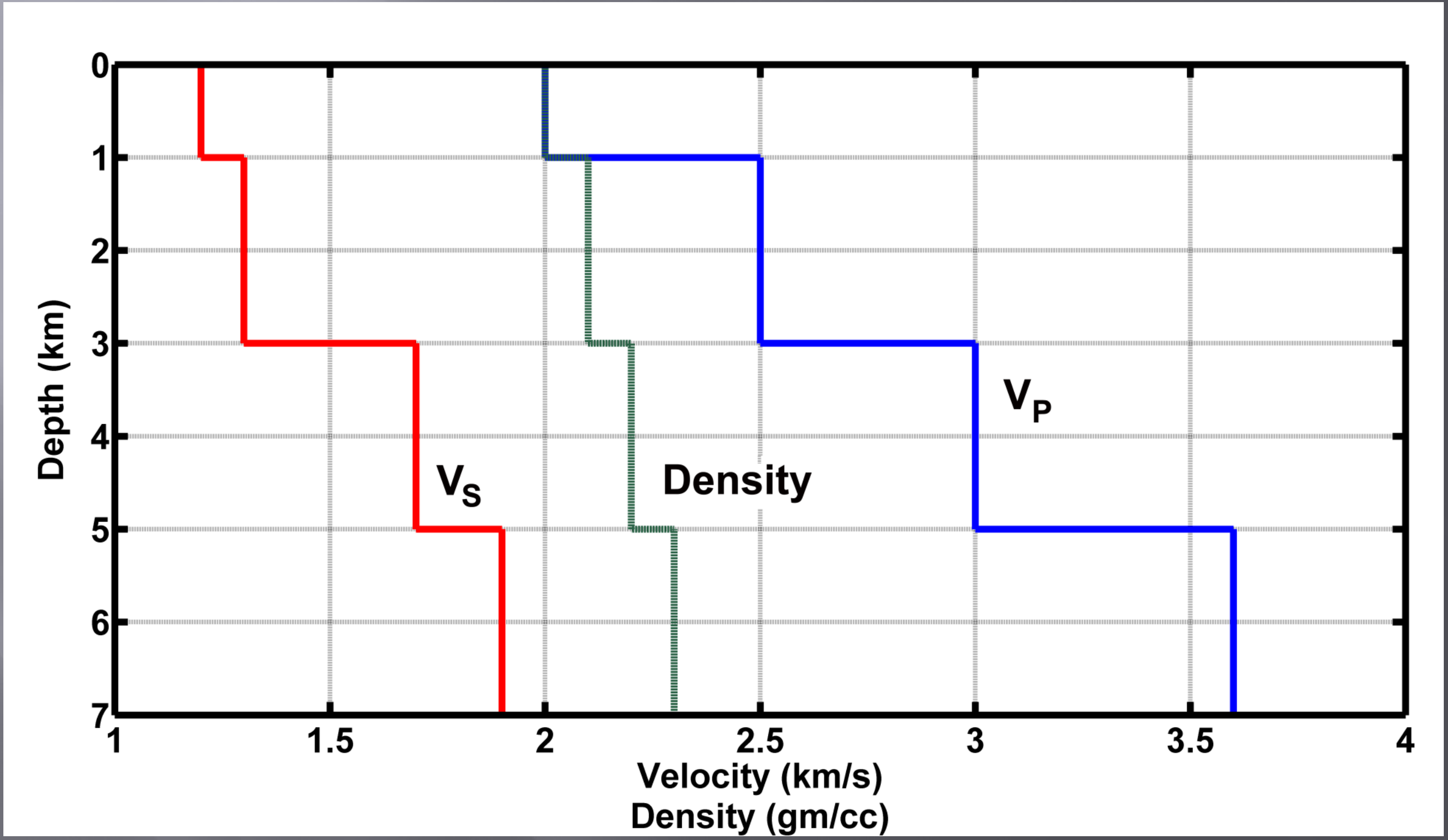
Gabor Wavelet and Spectrum 30Hz

$$f(t) = \cos \omega_0 t \, \exp\left[-(\omega_0 t/\gamma)\right]$$

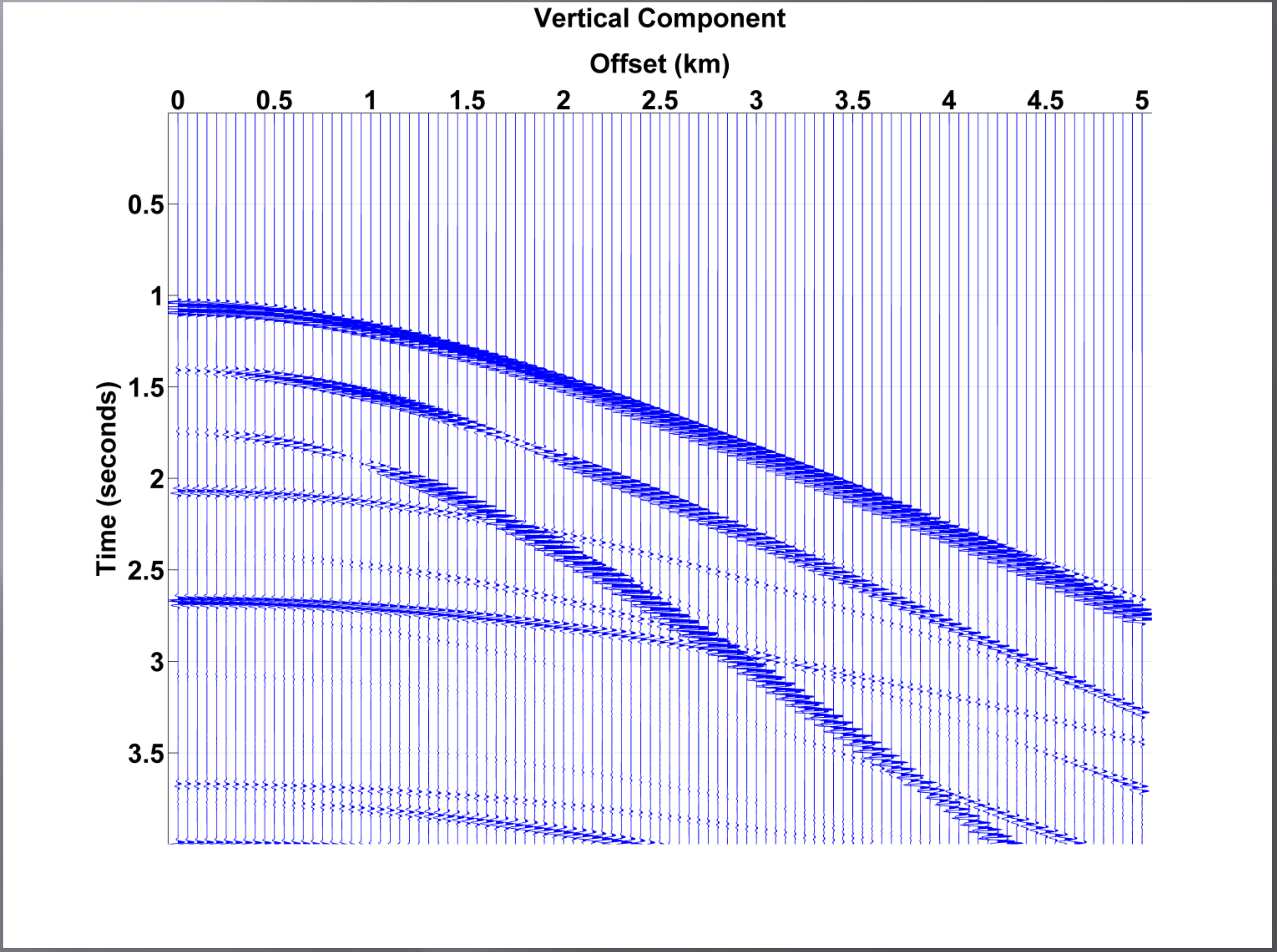




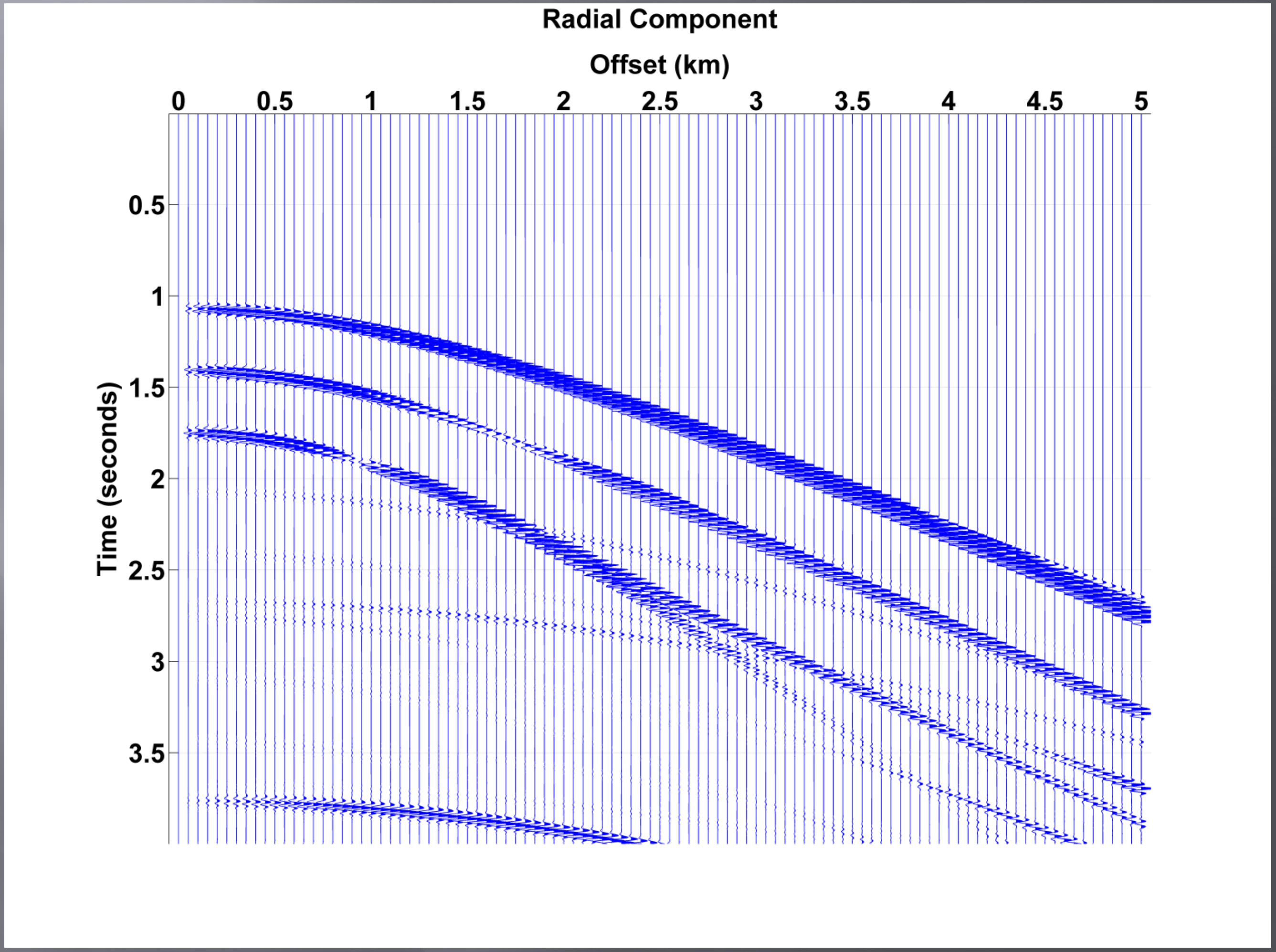
Velocity (Density) – Depth Model



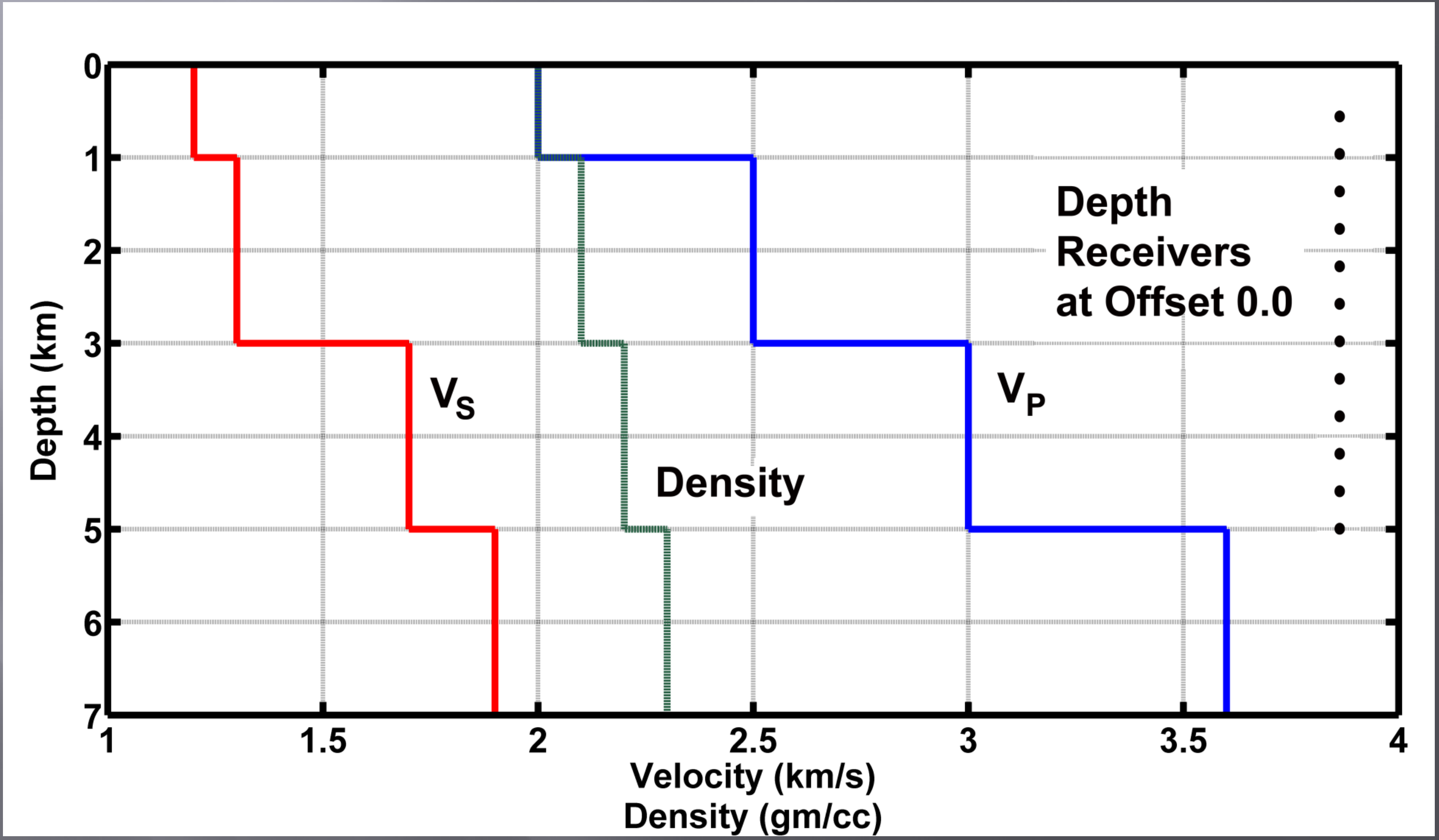
Isotropic Vertical Component



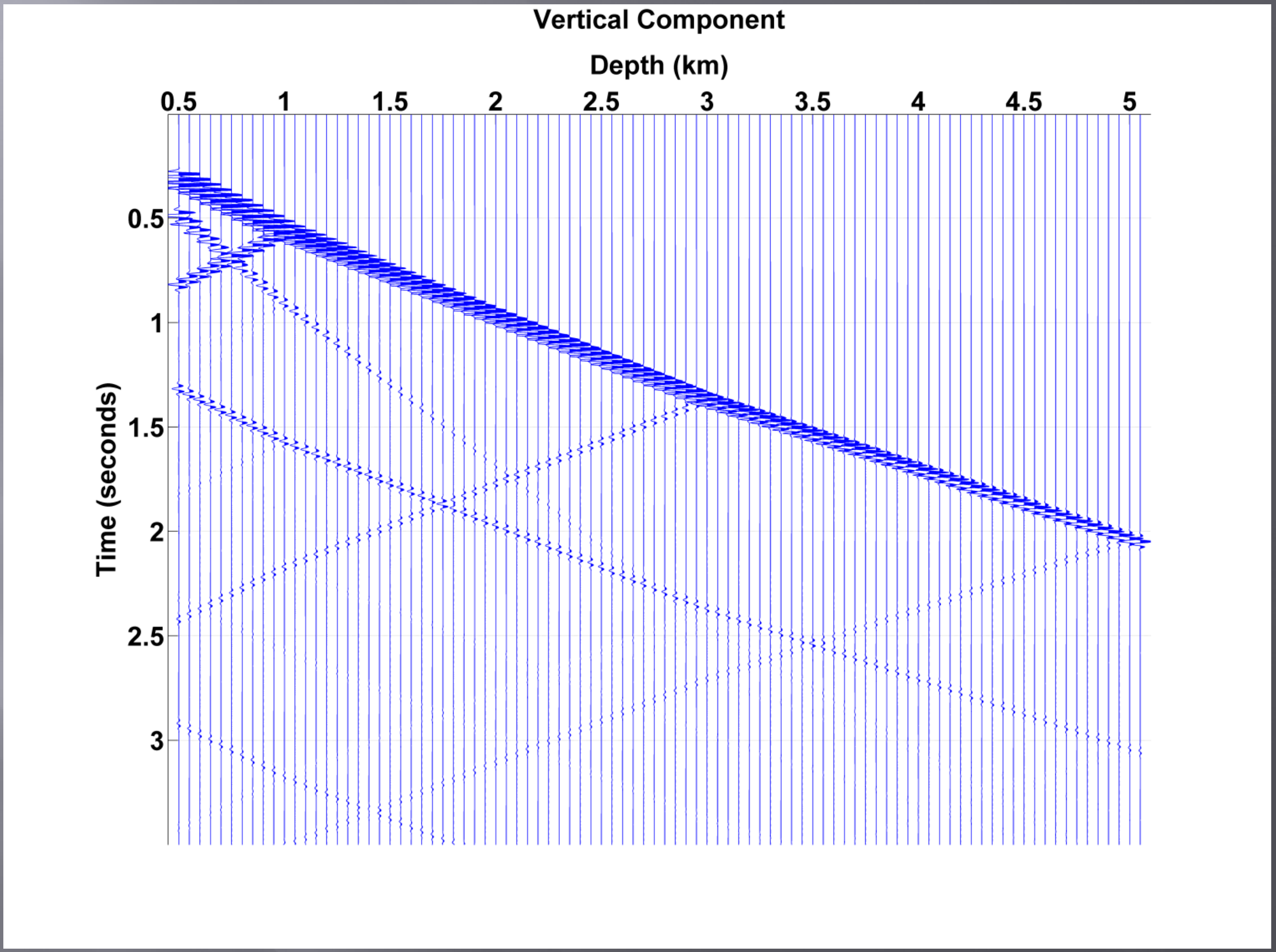
Isotropic Radial Component



Velocity (Density) – Depth Model Zero-offset VSP

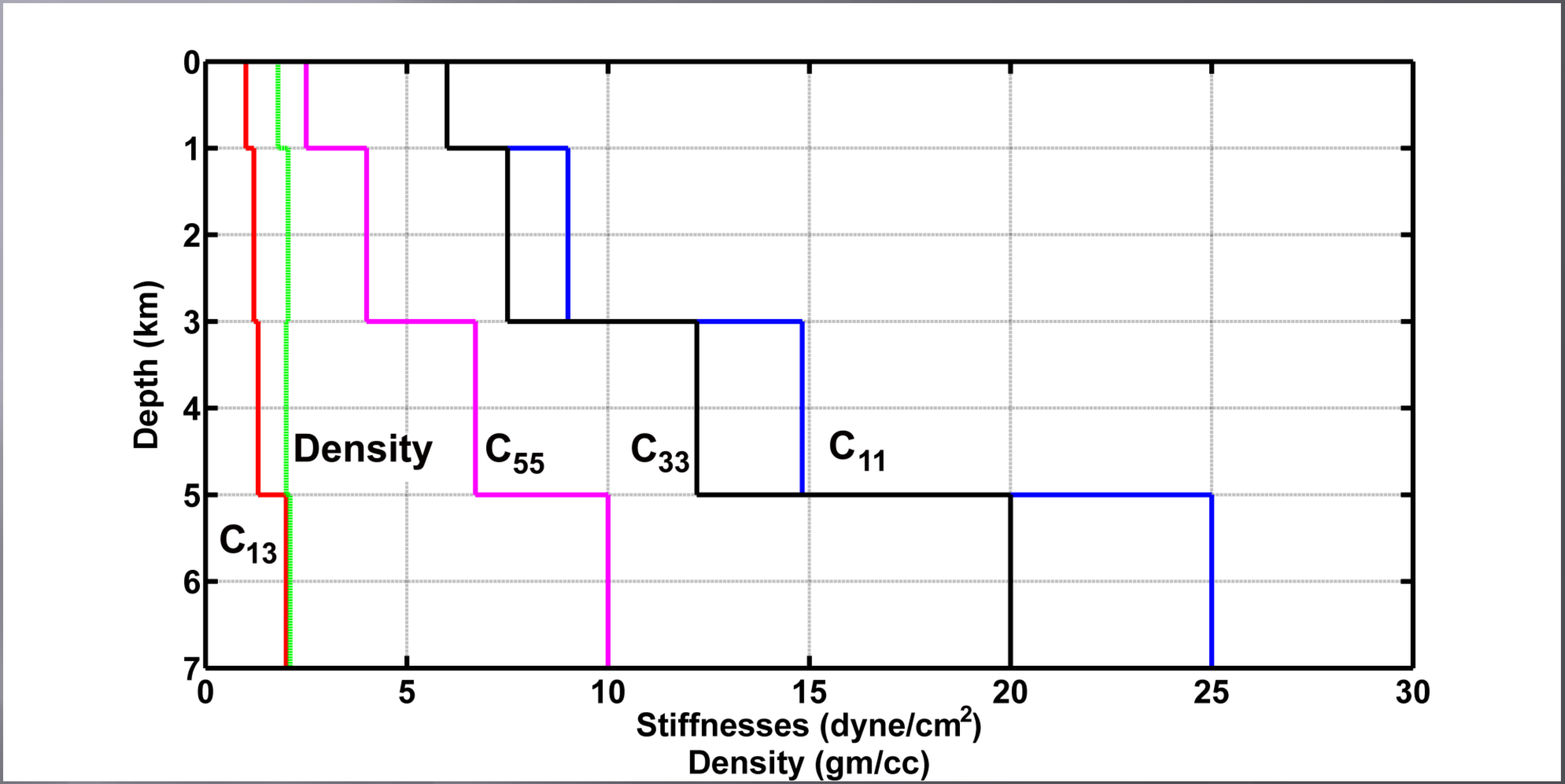


Vertical Component (VSP)



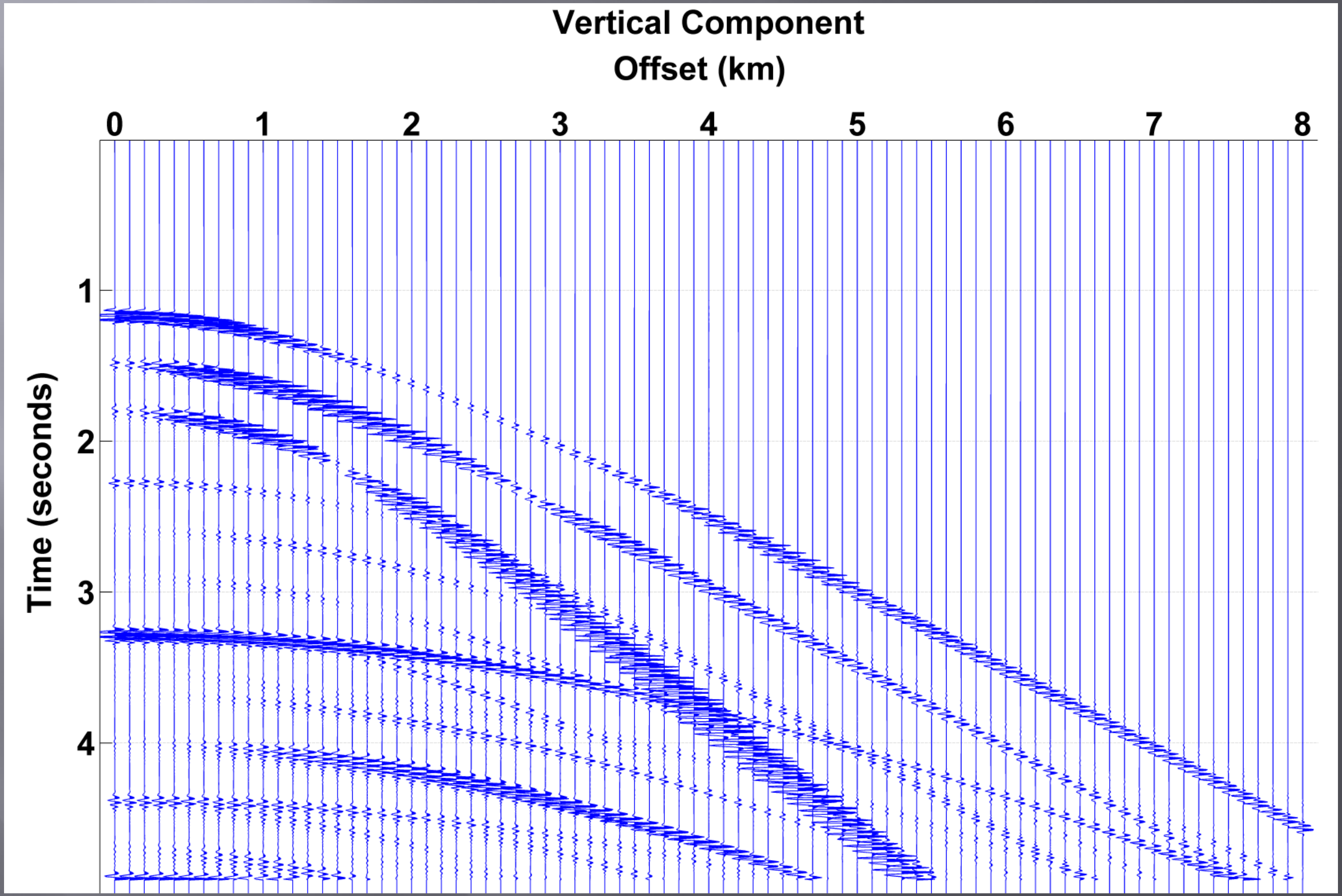
Source and receivers in bore hole.

TI Depth Model

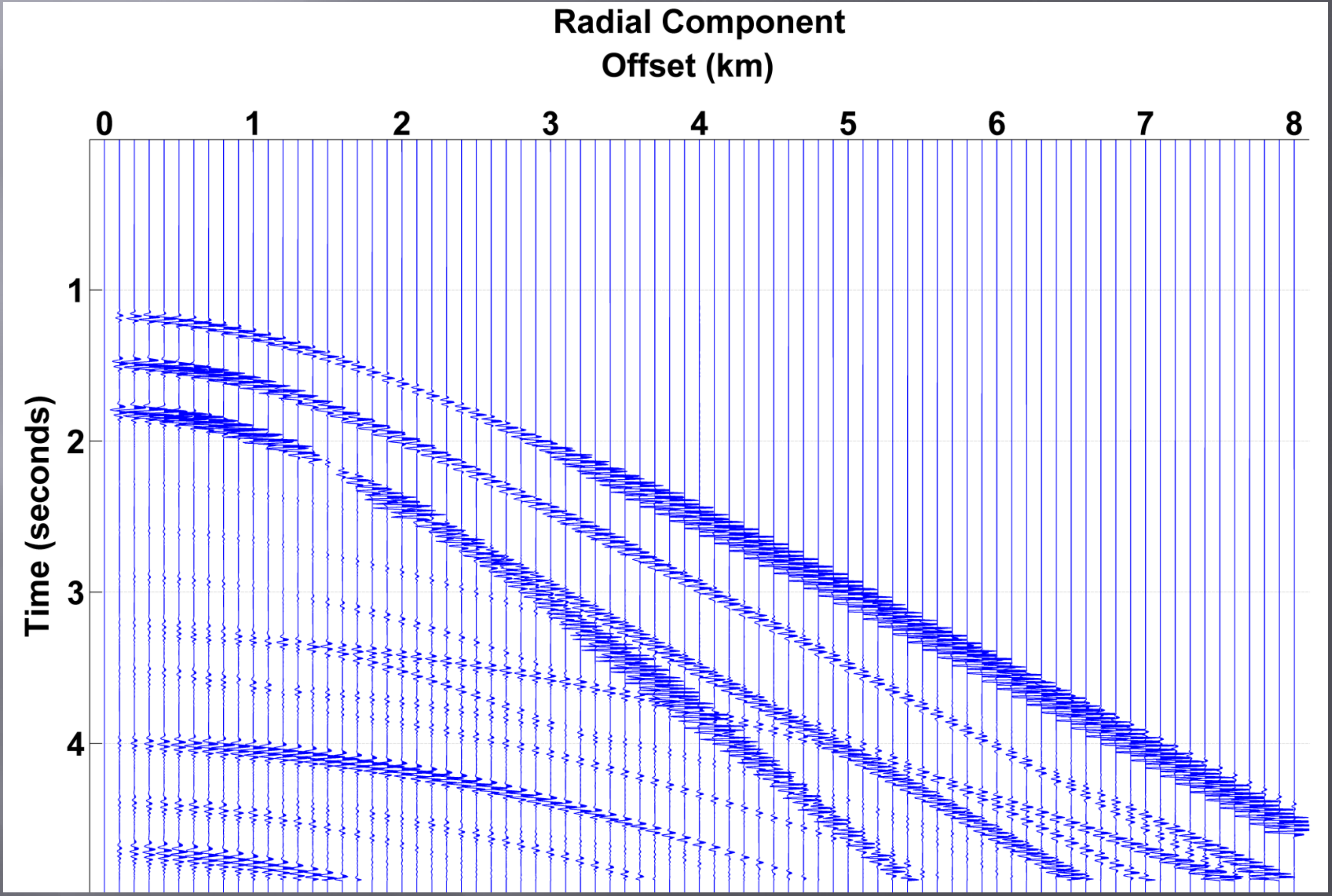


(Function Exists for α_0, β_0, ε, $\delta \rightarrow C_{ij}$)

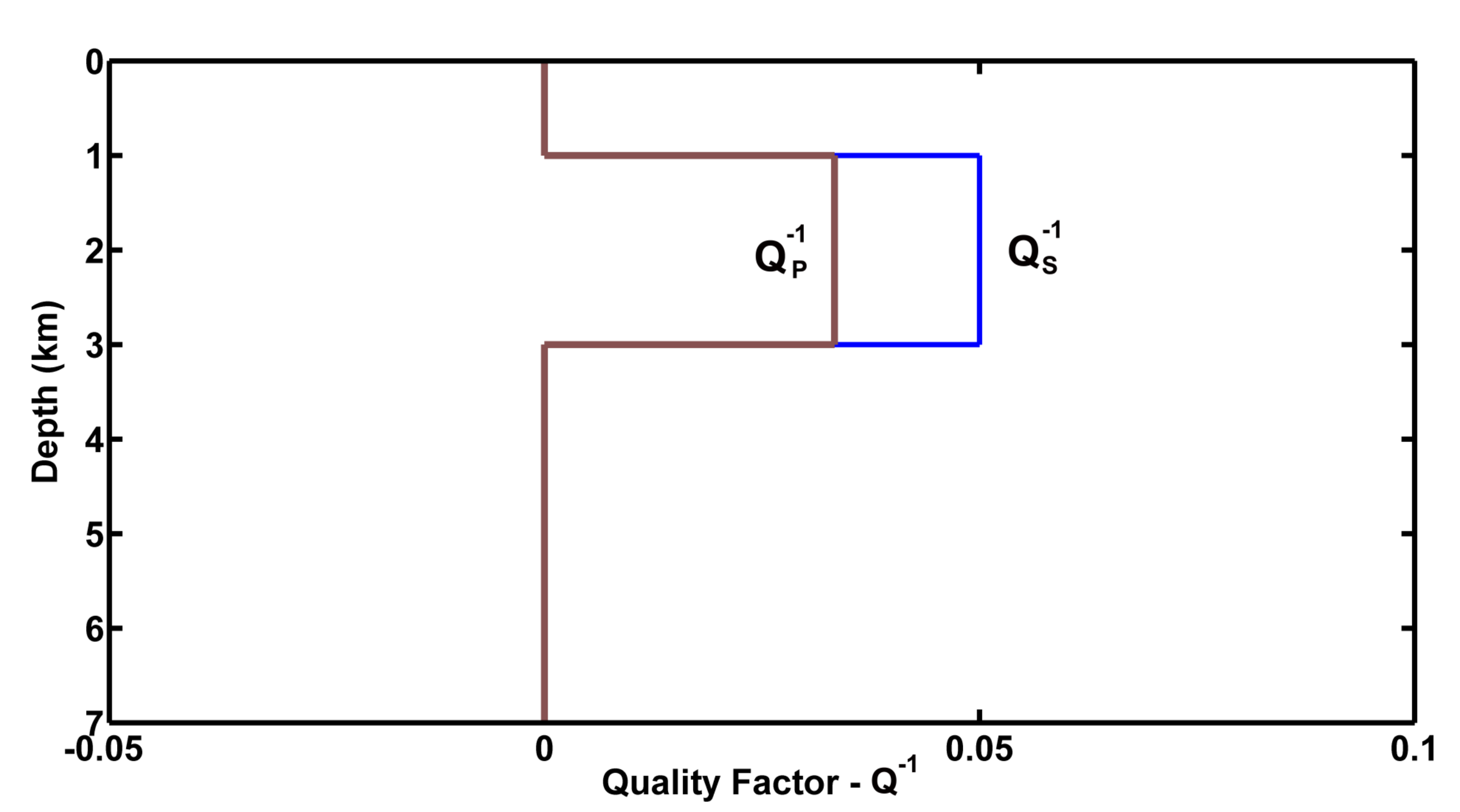
TI Vertical Component



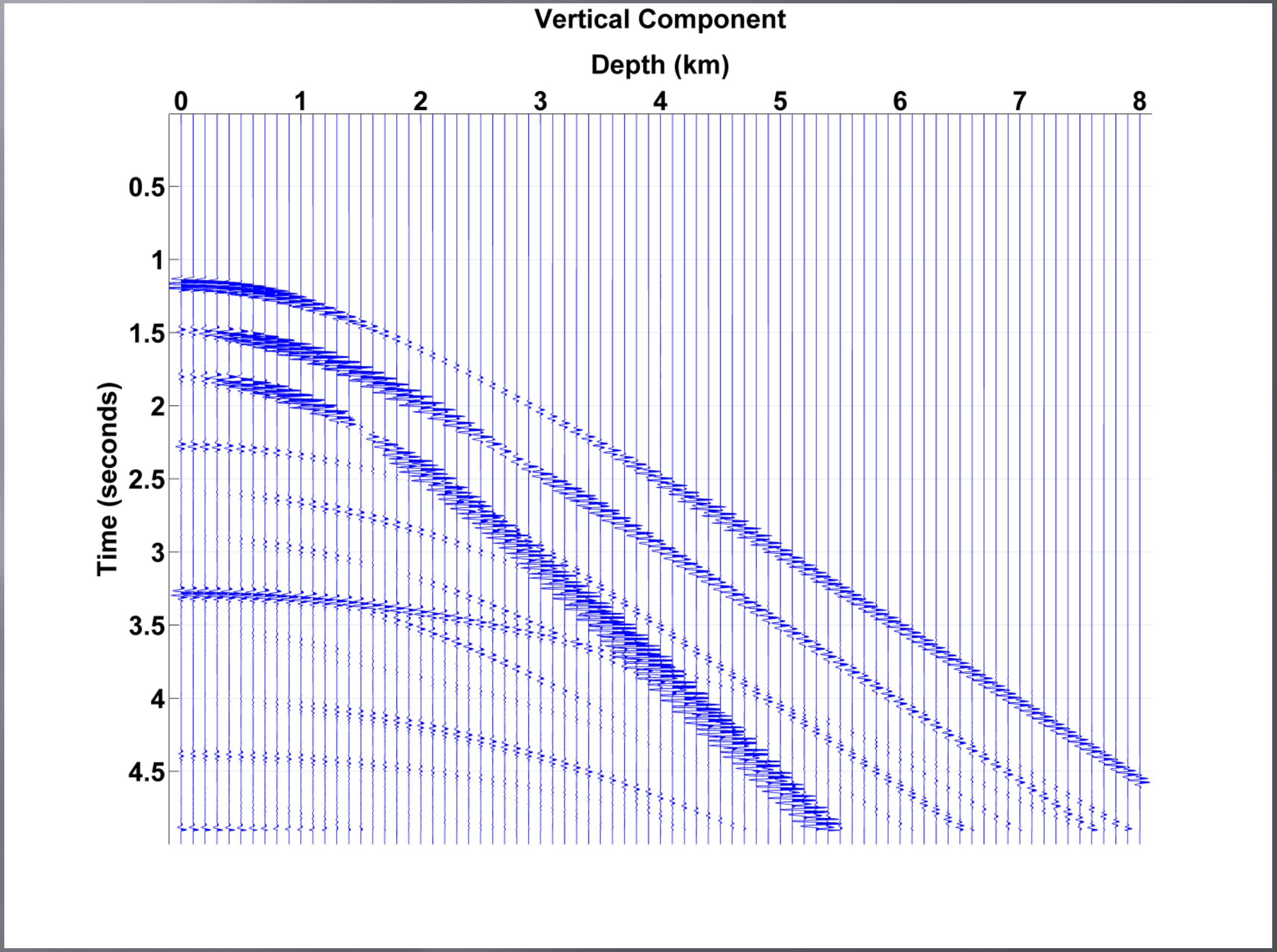
TI Radial Component



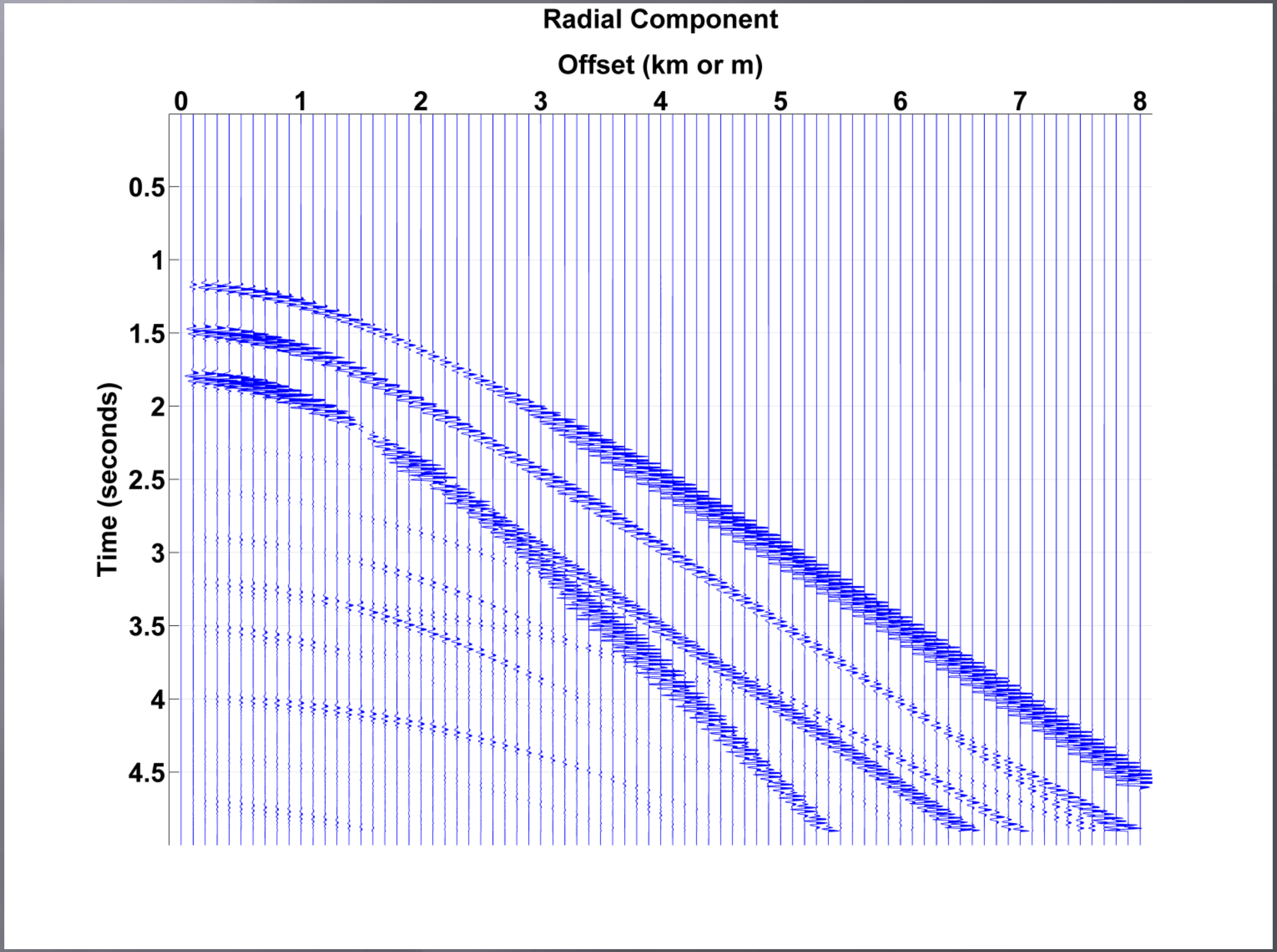
1/Q versus Depth



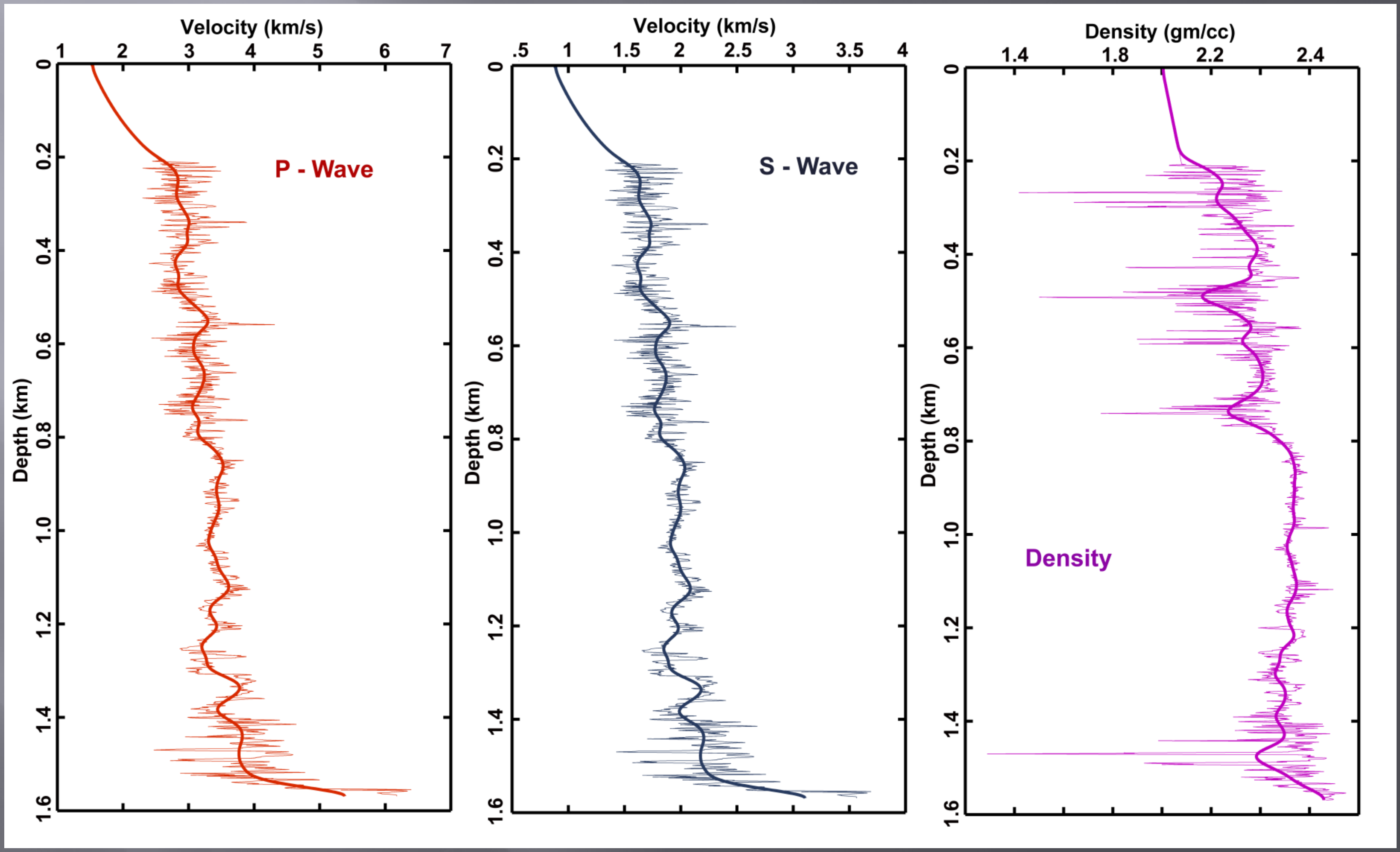
TI Vertical Component (with Q)



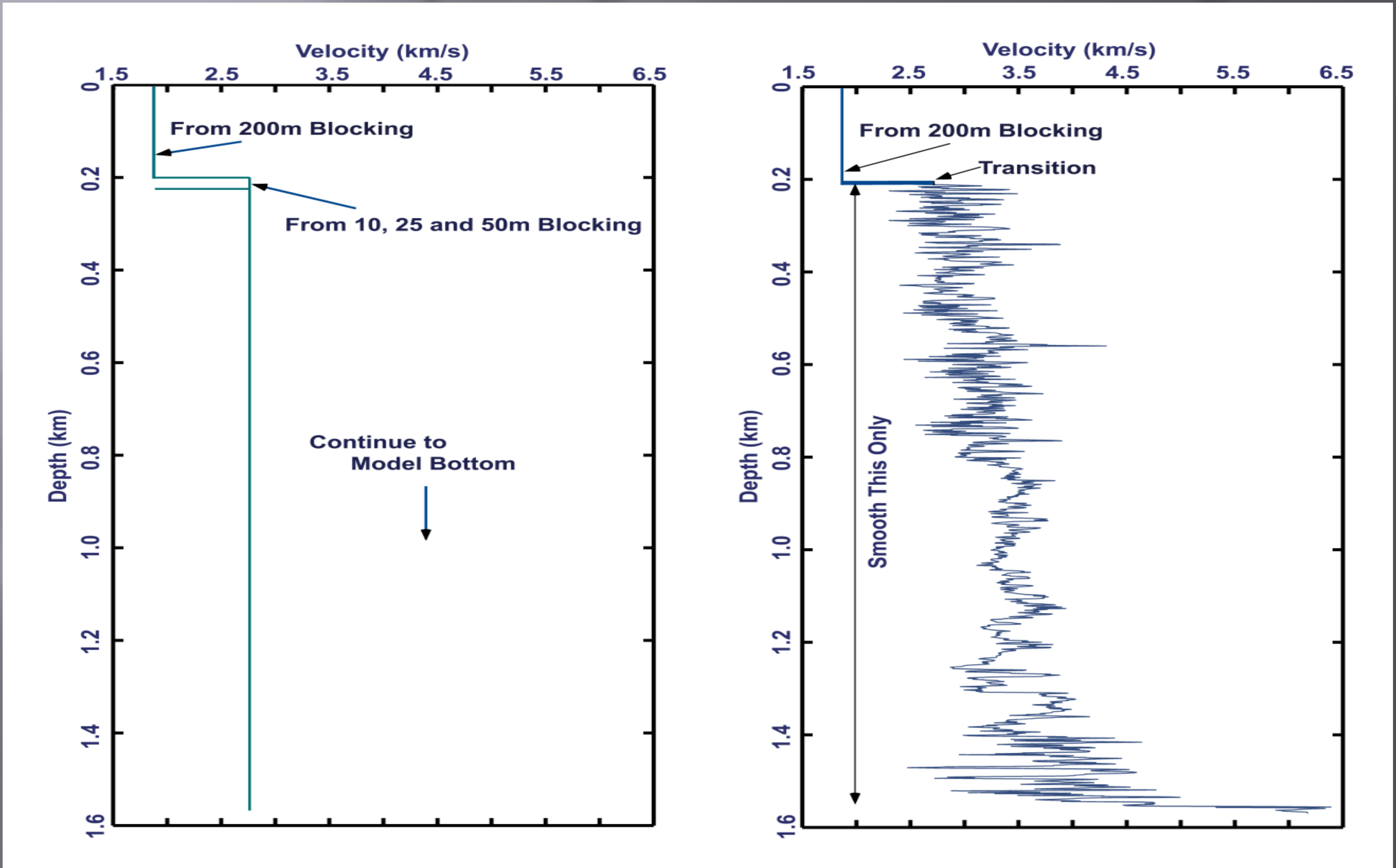
TI Radial Component (with Q)



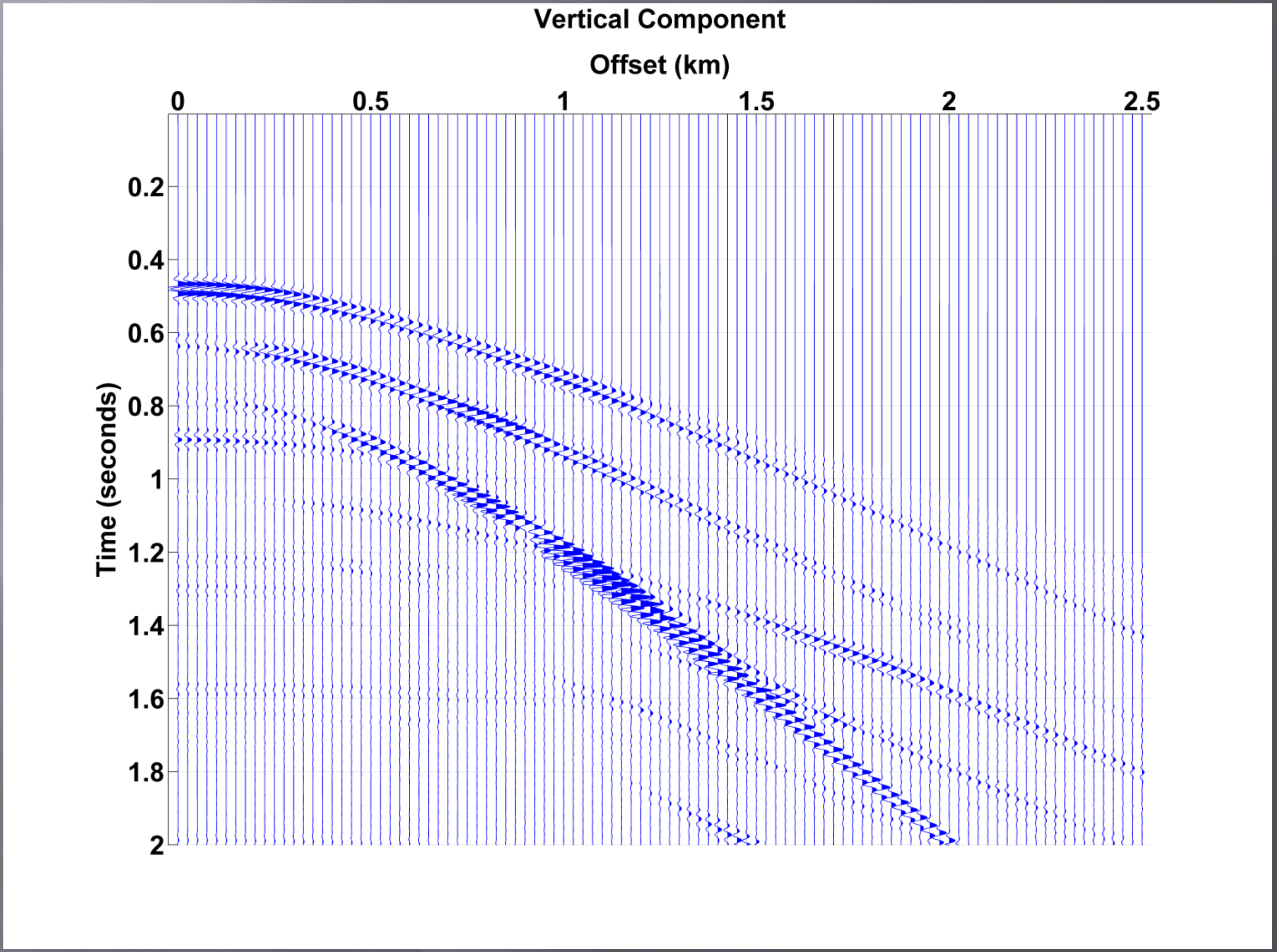
Smoothed Logs



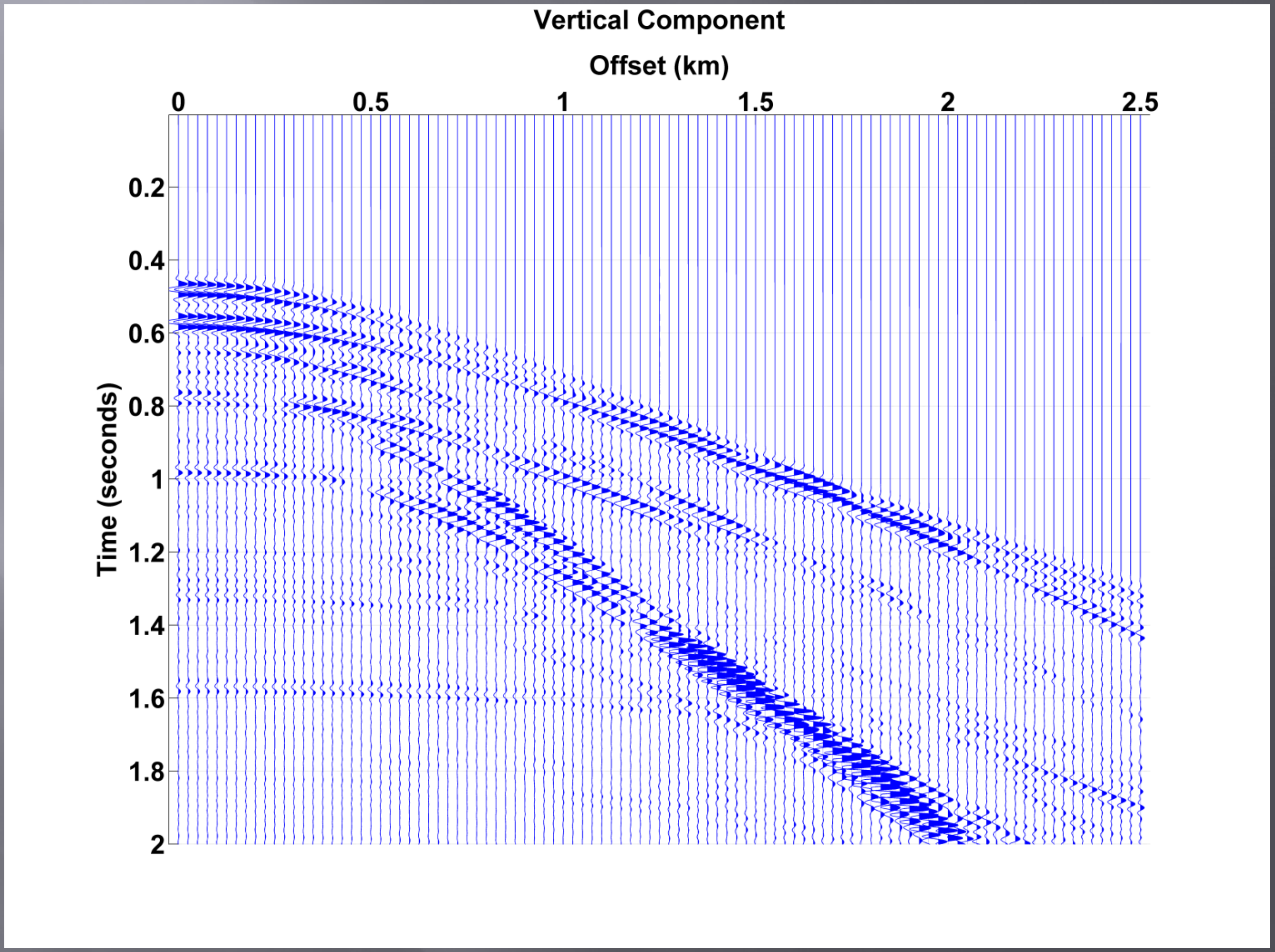
Modify Logs for Smoothing



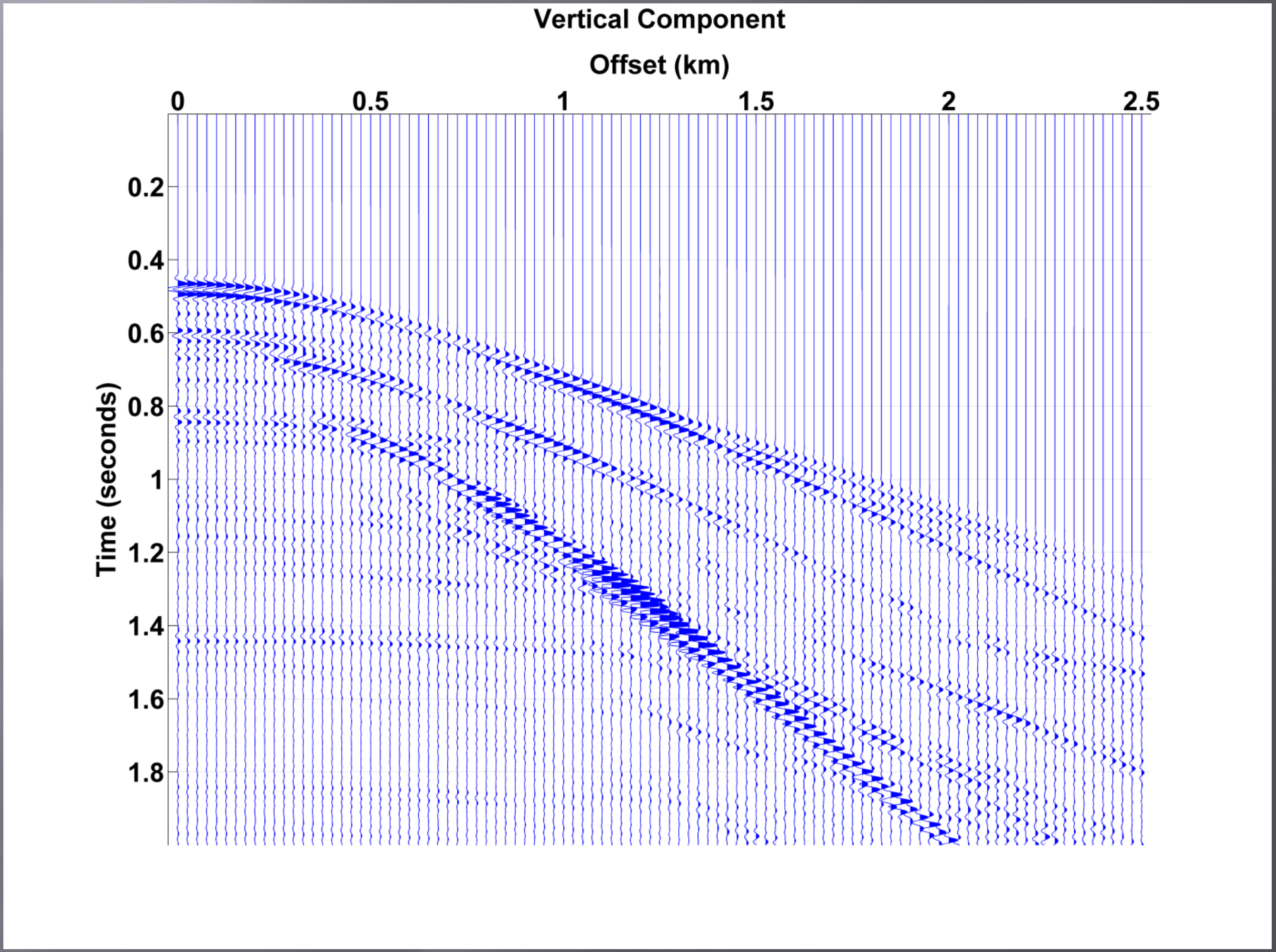
Vertical – Smoothing High



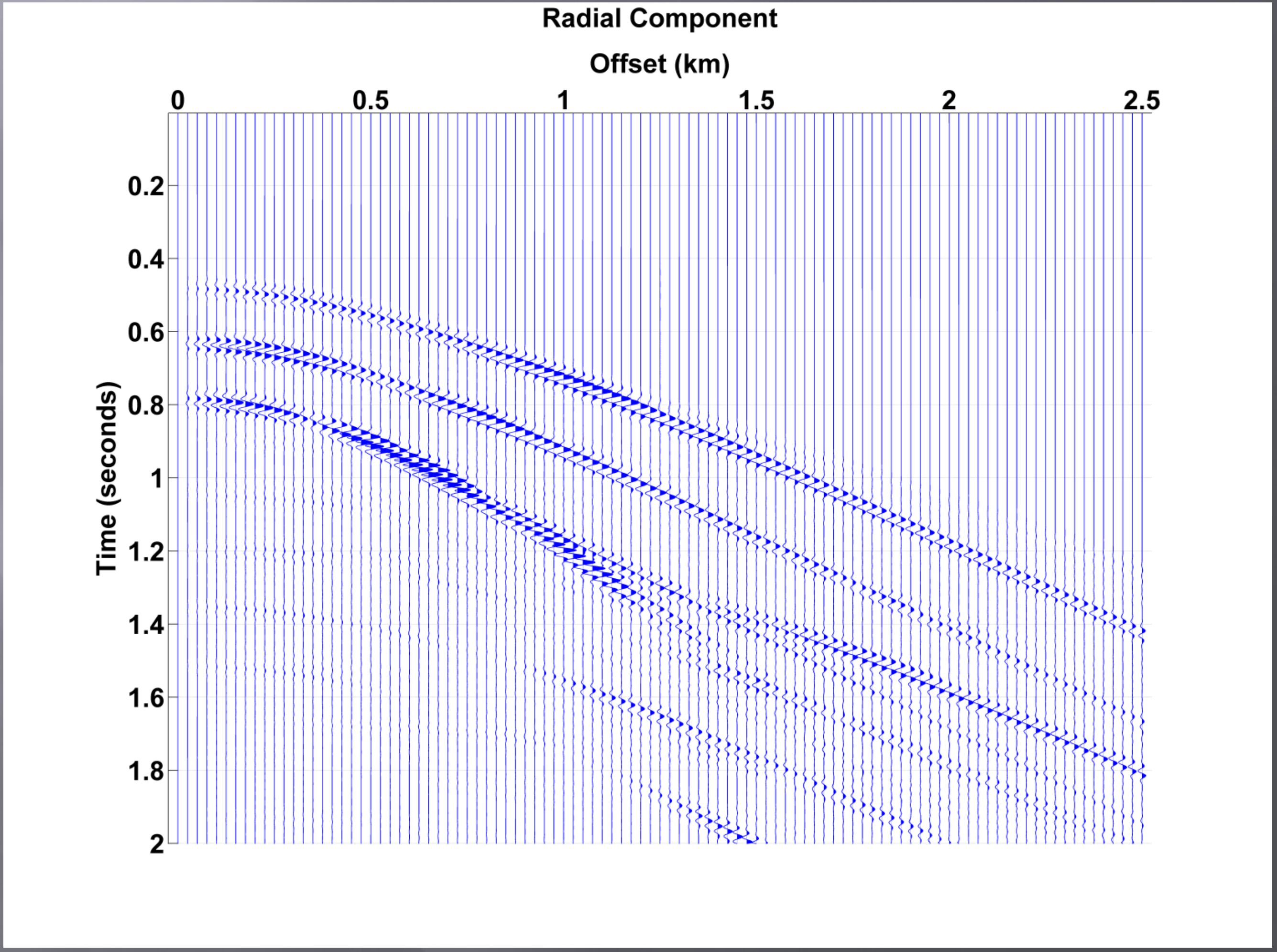
Vertical – Smoothing Moderate



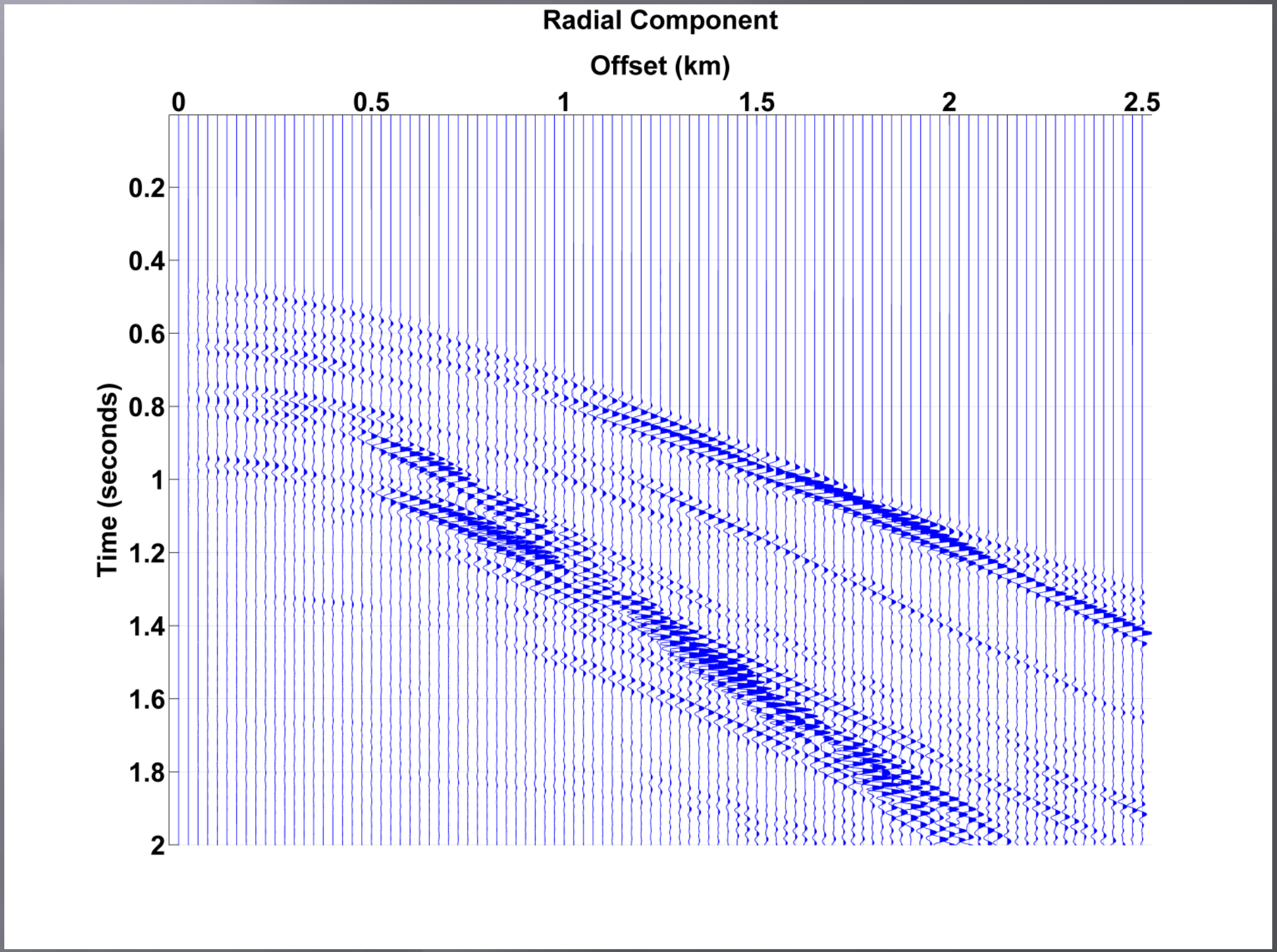
Vertical – Smoothing Low



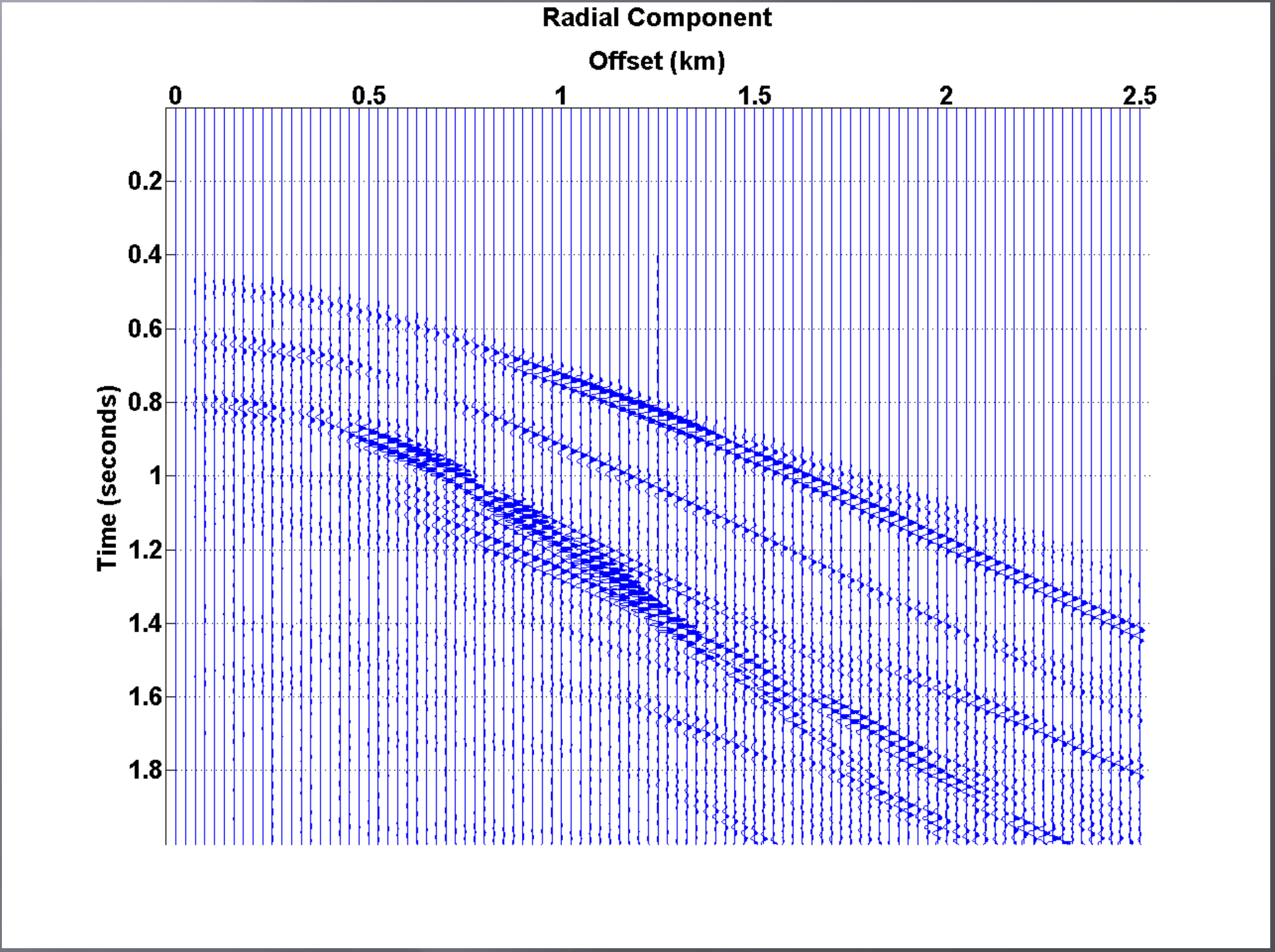
Horizontal – Smoothing High



Horizontal – Smoothing Moderate



Horizontal – Smoothing Low



Conclusions

- Same cost in CPU time as the true Reflectivity Method.
- Advantage of having the elastic parameters varying arbitrarily with depth.
- Orthorhombic and monoclinic anisotropy codes are in development.
- The upgrade to lateral variation of elastic parameters has been investigated (Mikhailenko et al.).
- The generalization to true 3D has been developed by others (Mikhailenko et al.).

Acknowledgements

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