Time-lapse AVO inversion: Application to synthetic data

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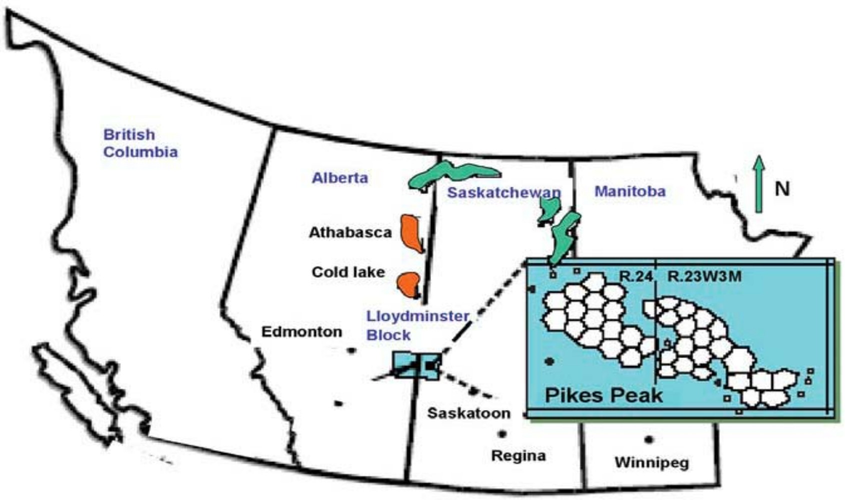
Outline

- Survey area & motivations
- Time-lapse model building
- Time-lapse AVO inversion
- Conclusions and future work
- Acknowledgments

Survey area & motivations



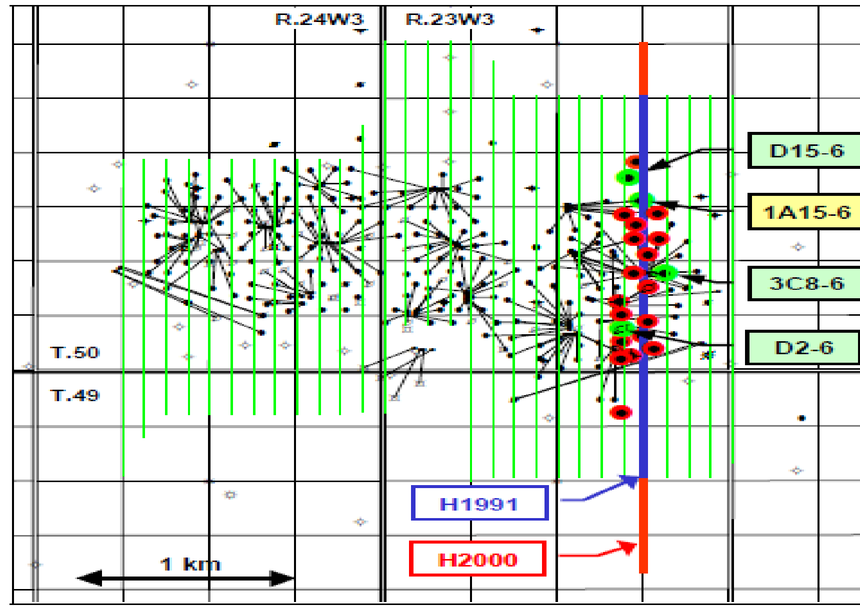


Fig. 1: Left: Location map of the Pikes Peak oil field.
Right: Pike Peak time – lapse survey lines (*after Watson, 2004*)

- **Pikes Peak survey area -** 40Km east of Lloydminster, AB-SK
- **Heavy oil reservoir - Wasica Formation -**located on E-W structural high within an incised valley-fill channel

- **CSS –** reduce viscosity - increase mobility of oil

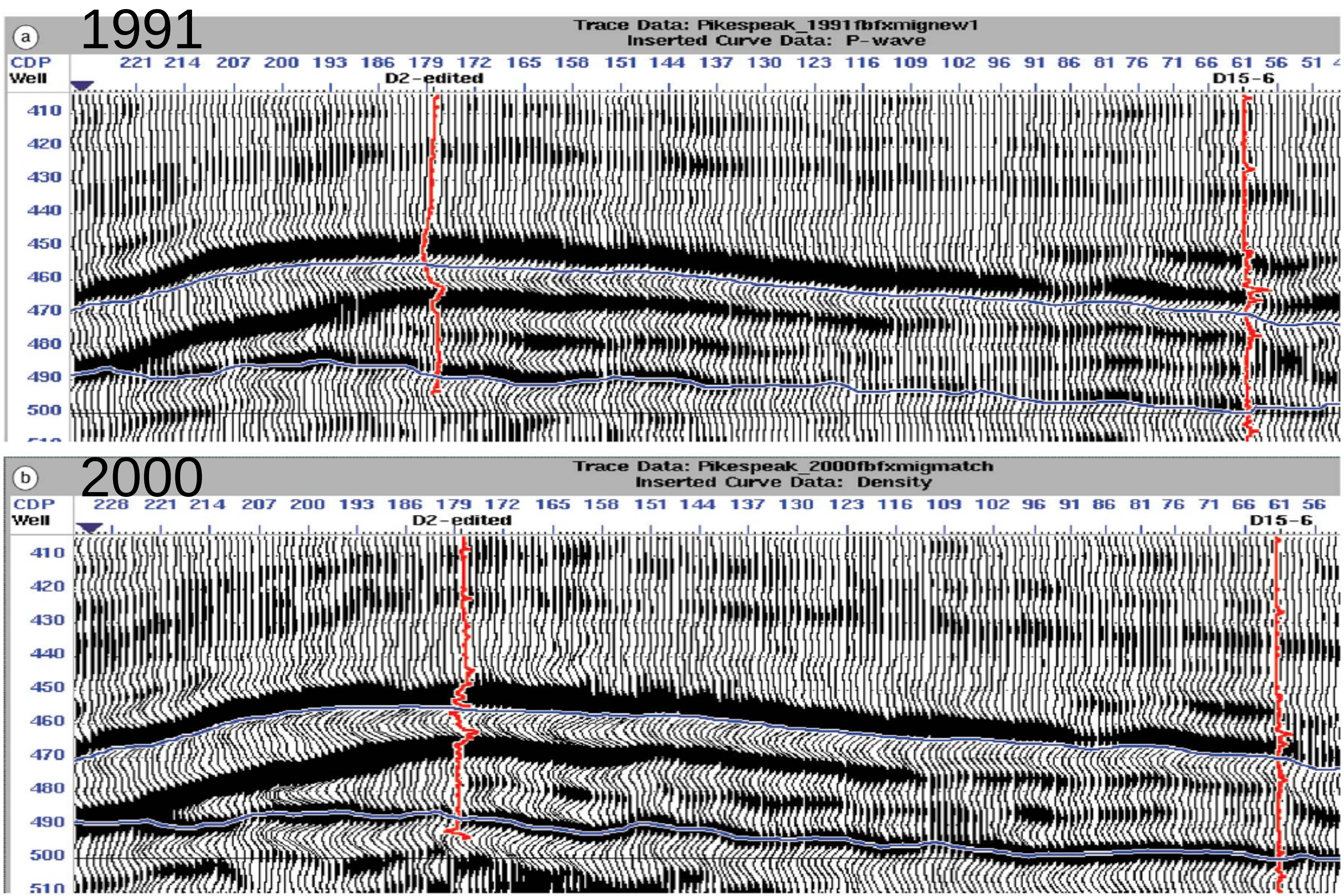


Fig. 2: Time-lapse of P-P migrated sections of (a) 1991 and (b) 2000 (*after Zou et. al, 2005*).

➢Production-induced amplitude changes can be seen in the lower part of the reservoir

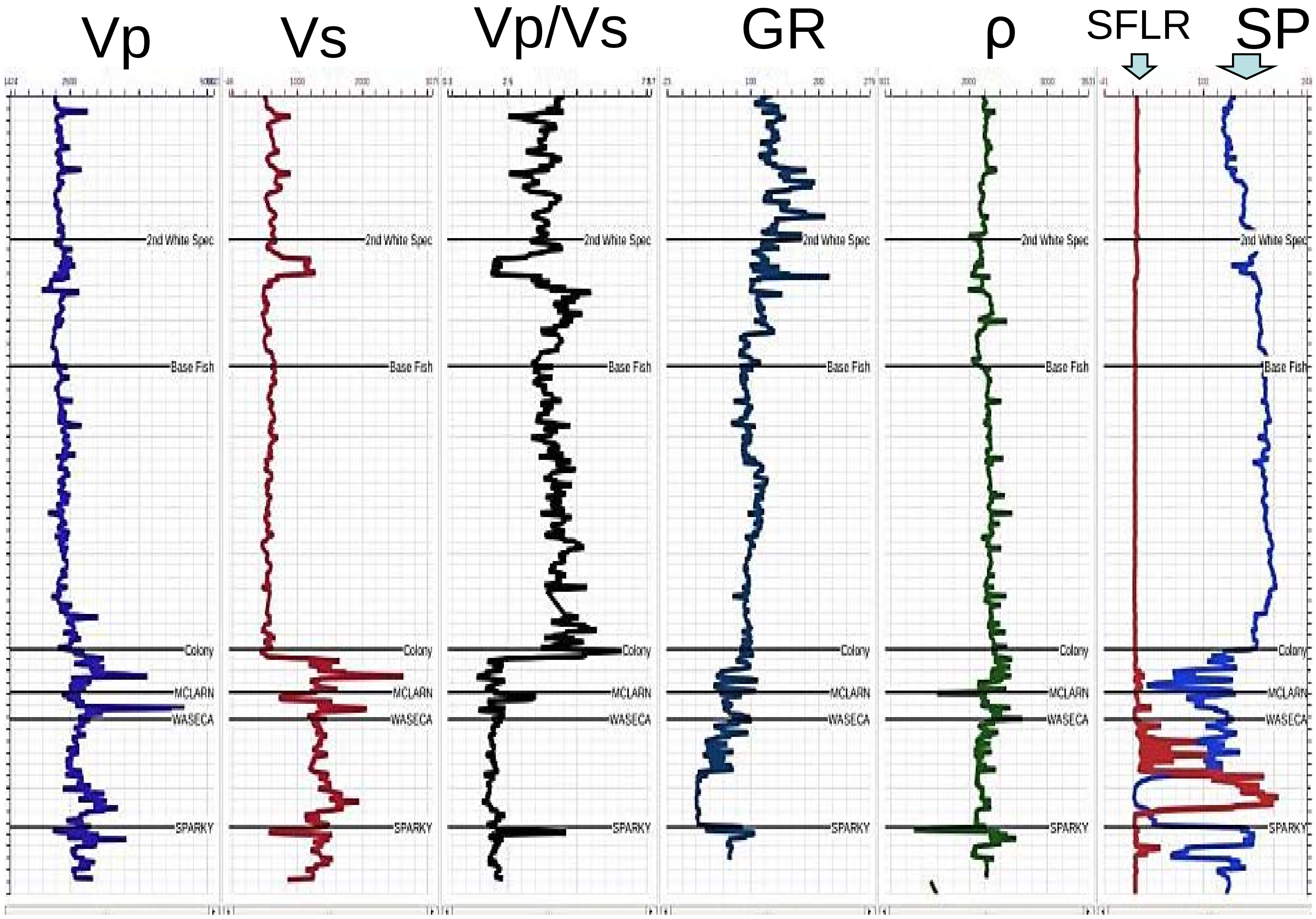
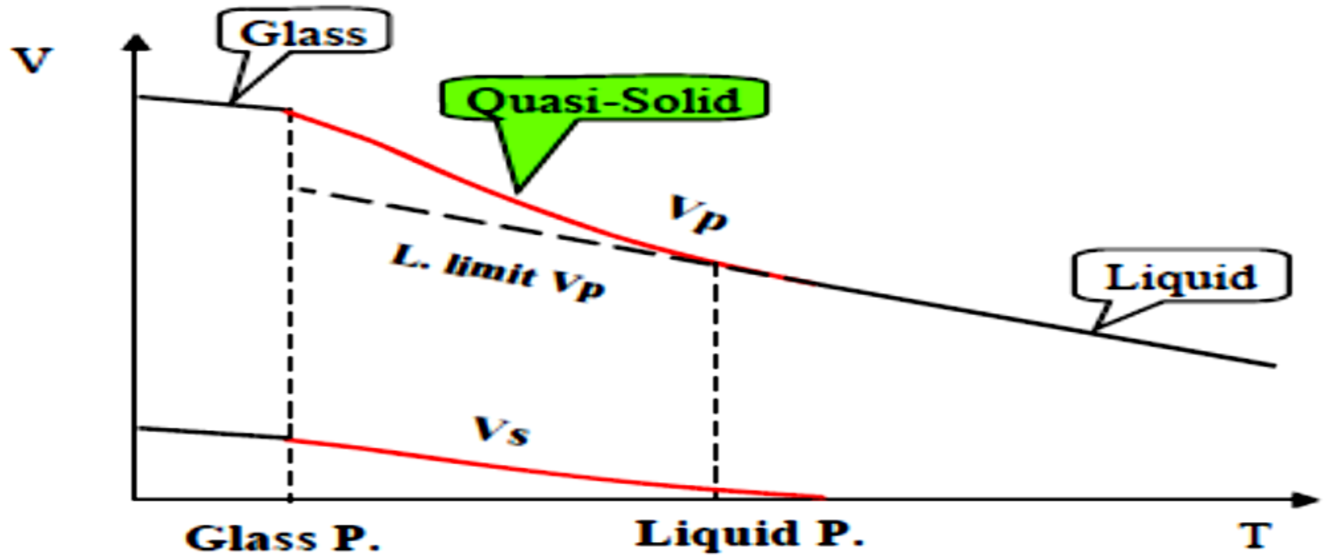


Fig.3. Well logs for Well 15A-6, Pikes Peak.



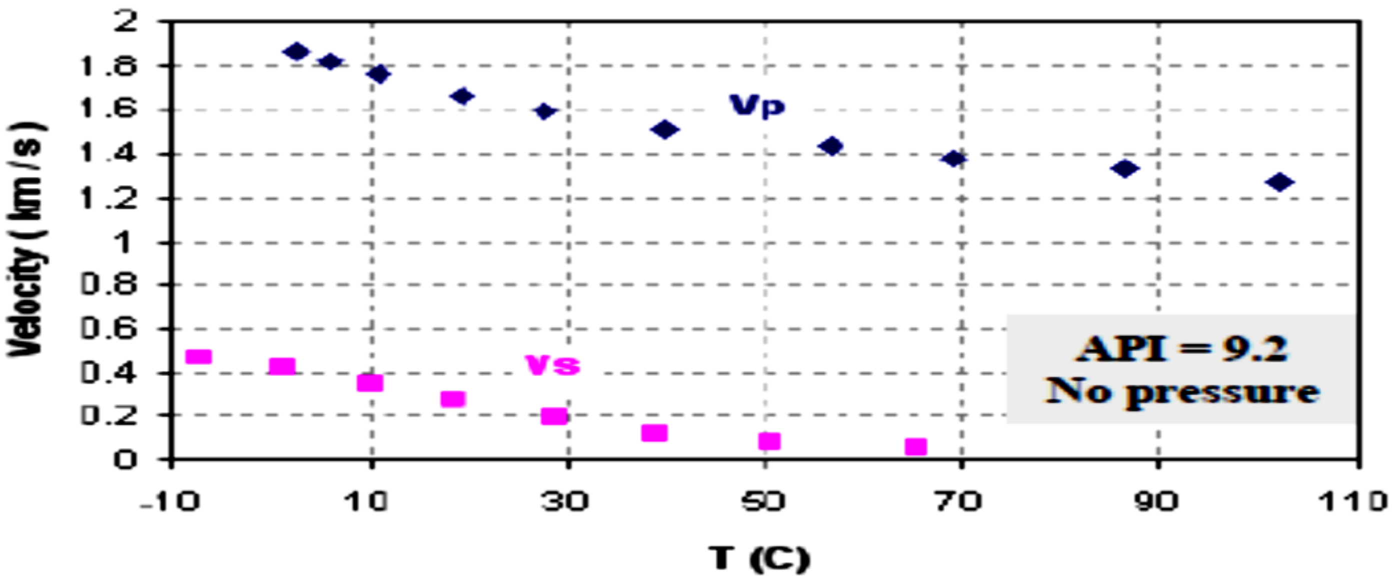


Fig.4. Properties of Heavy oil sand (*after Han et. al., 2006*)

Building time-lapse model

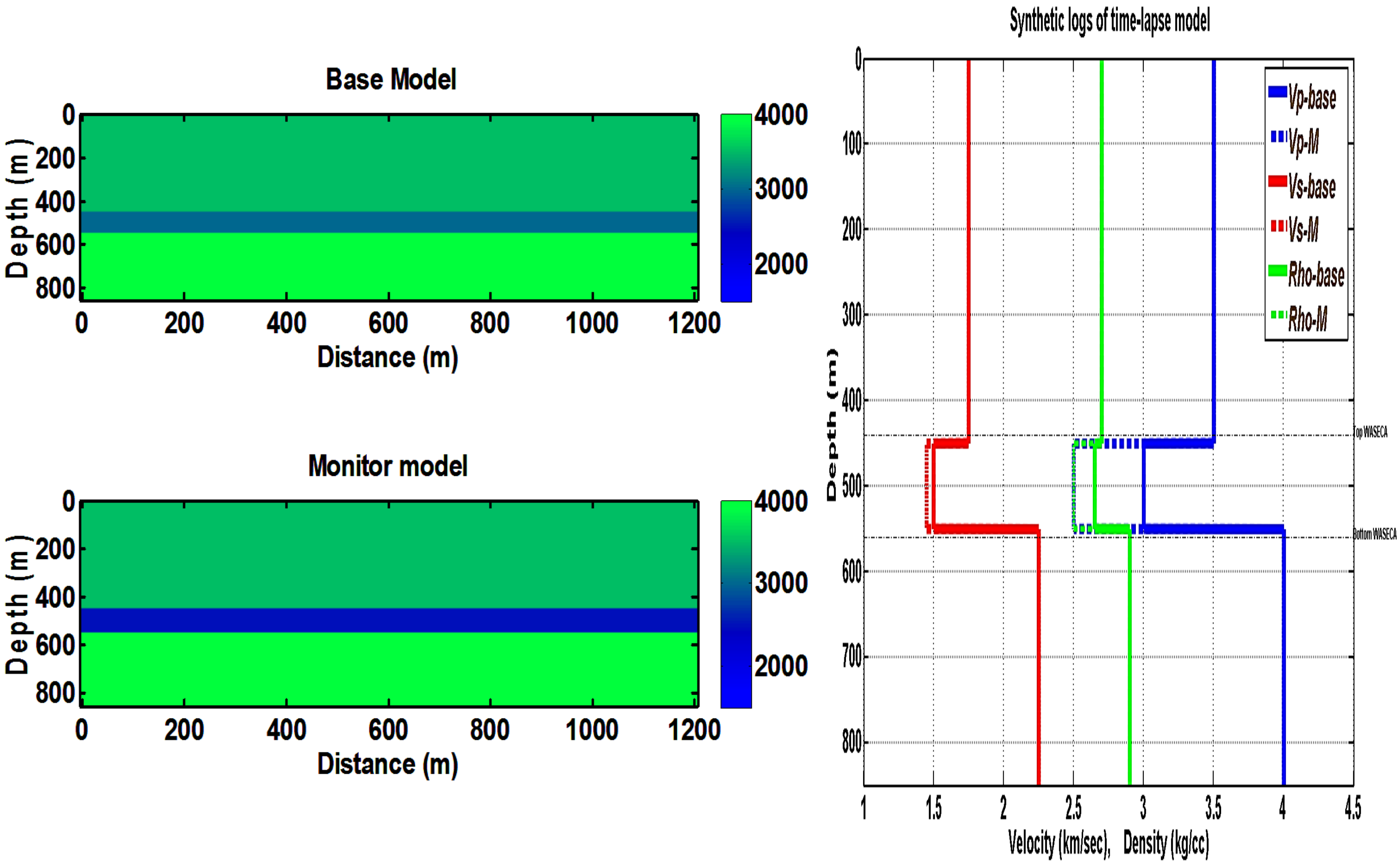
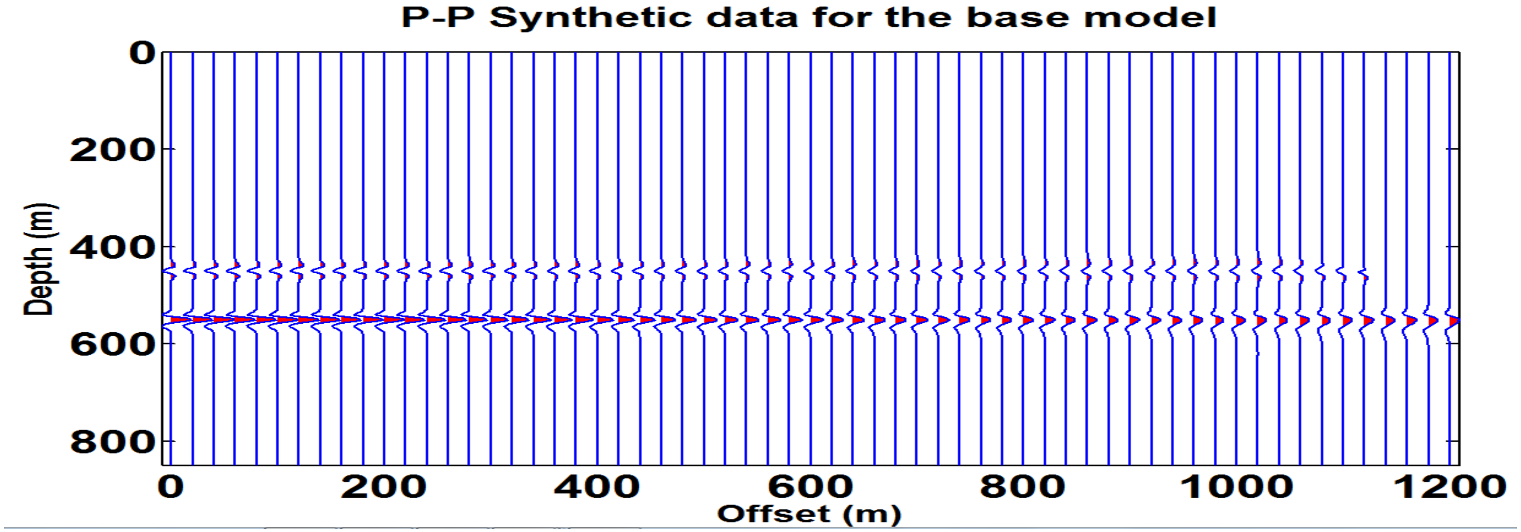


Fig.5. Left: Time-lapse model. Right: Synthetic logs



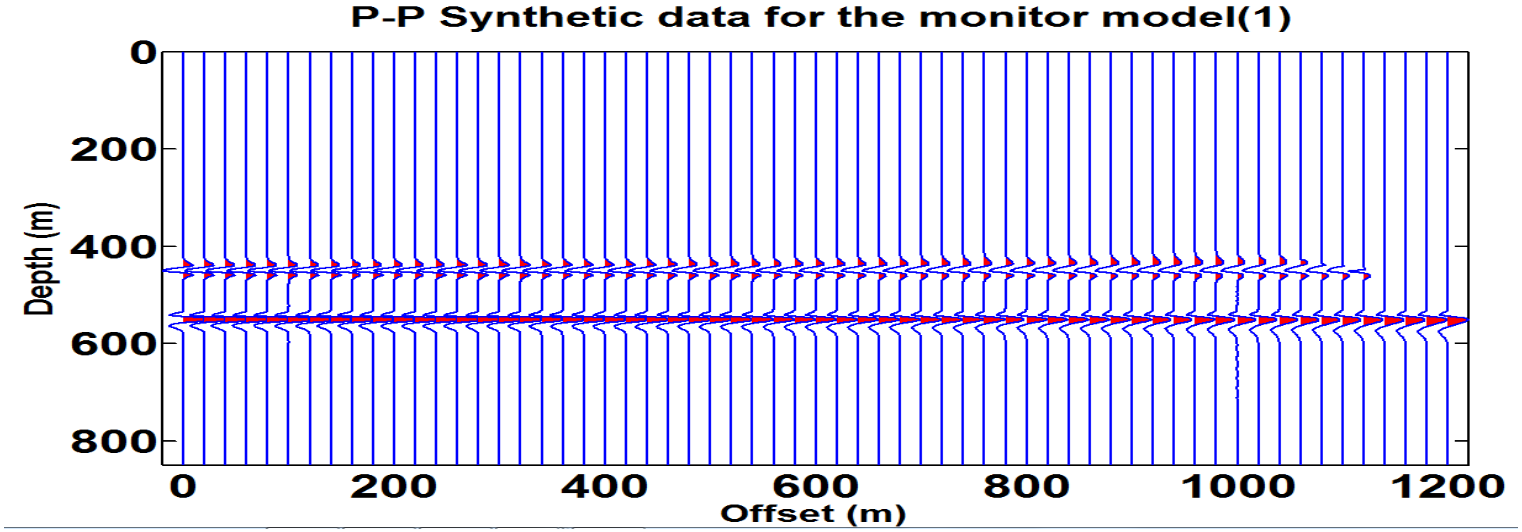


Fig.6. Synthetic P-P data for the base and monitor models

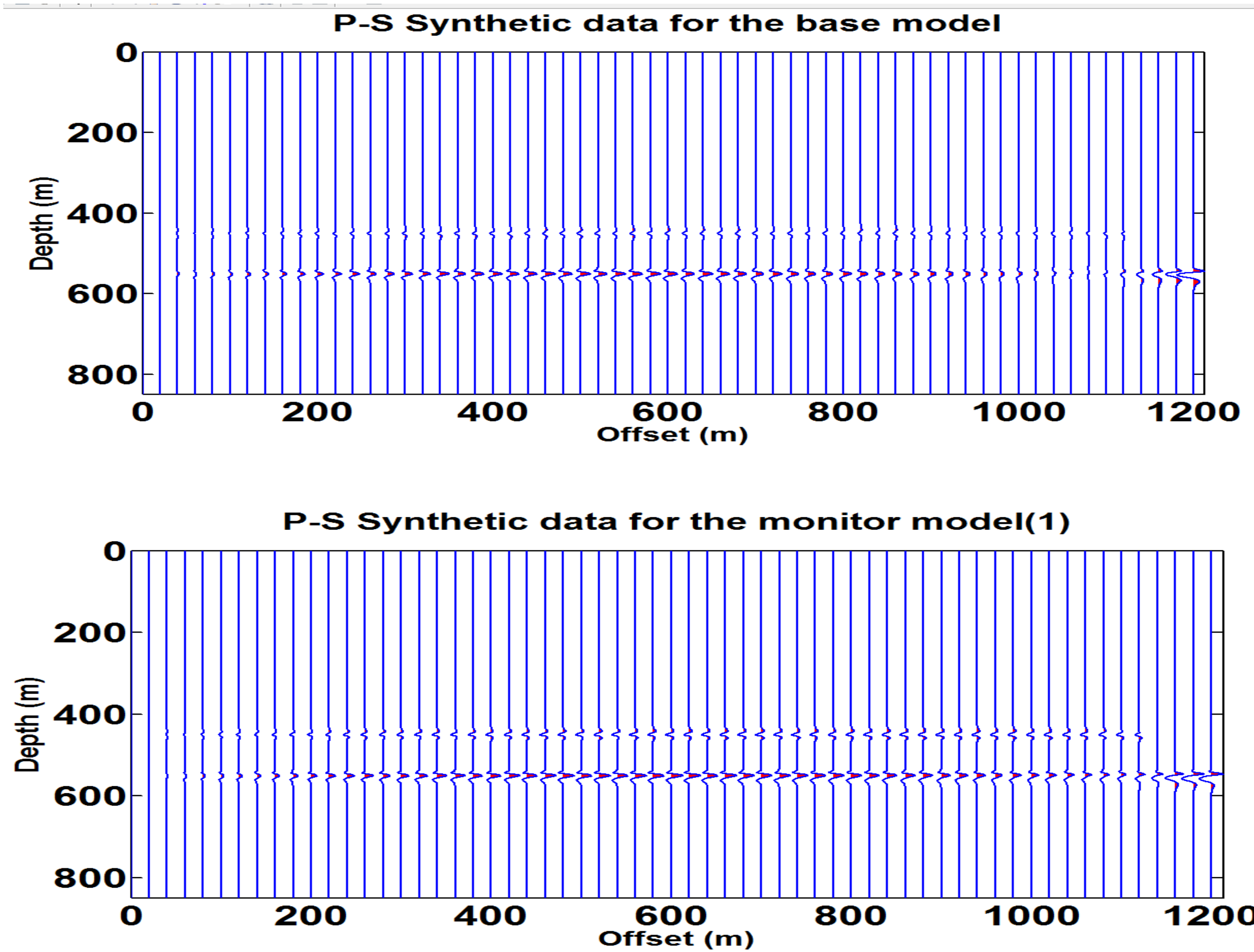
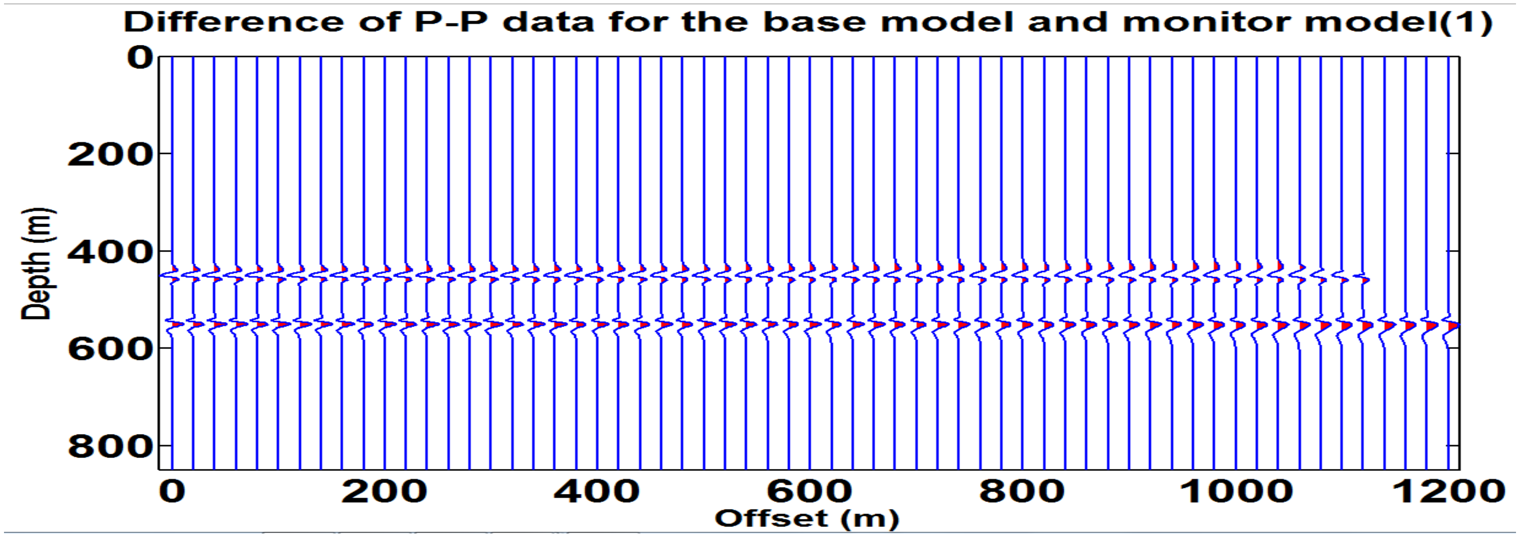


Fig.7. Synthetic P-S data for the base and monitor models



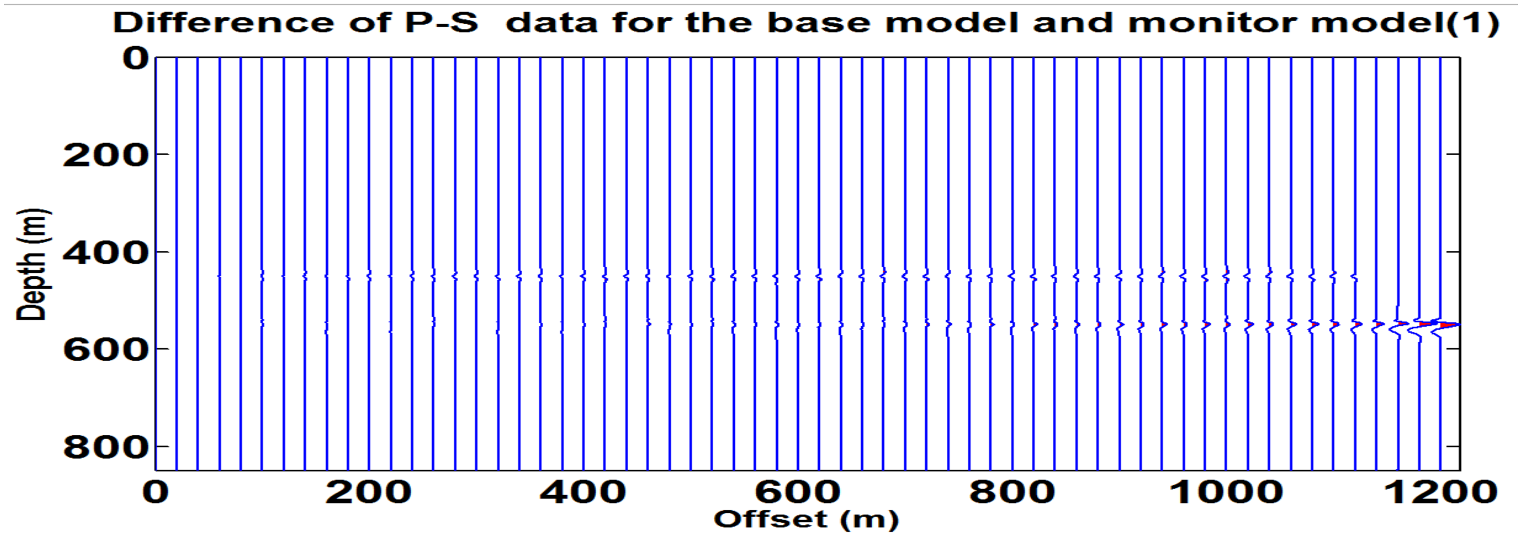


Fig.8. Difference of synthetic data for the base and monitoring models (top P-P) Bottom (P-S)

AVO Inversion

Modified from Aki – Richards (1980)

$$R_{PP}(\theta) \approx \frac{\left(1+\tan^2\theta\right)}{2}\frac{\Delta I}{I} - 4\frac{\beta^2}{\alpha^2}\sin^2\theta\frac{\Delta J}{J} - \left[\frac{1}{2}\tan^2\theta - 2\frac{\beta^2}{\alpha^2}\sin^2\theta\right]\frac{\Delta\rho}{\rho}$$

$$R_{PS}(\theta,\varphi) \approx \frac{-\alpha\tan\varphi}{2\beta}\left[\left(1+2\sin^2\varphi - \frac{2\beta}{\alpha}\cos\theta\cos\varphi\right)\frac{\Delta\rho}{\rho} - \left(4\sin^2\varphi - \frac{4\beta}{\alpha}\cos\theta\cos\varphi\right)\frac{\Delta J}{J}\right].$$

$$\boldsymbol{d} = \boldsymbol{G}\boldsymbol{m}$$

$\boldsymbol{d}$ data, $\boldsymbol{G}$ forward operator, $\boldsymbol{m}$ model parameters

AVO Inversion … continued

- **Constrained least-squares inversion**

$$(G^T G + \lambda\, W_m^T W_m) m = G^T d$$

$$\left[\left(G_{pp}^T G_{pp} + \lambda_1 W_m^T W_m\right) + \left(G_{ps}^T G_{ps} + \lambda_2 W_m^T W_m\right)\right] m = \left[\left(G_{pp}^T d_{pp}\right) + \left(G_{ps}^T d_{ps}\right)\right]$$

1- **Smoothness regularization** – Tikhonov (0, 1, 2 order)

2- **Compactness regularization** - (Last and Kubik, (1983)
Minimizes the area where strong variation in model parameter or discontinuity occur (spatially variable damping matrix - high in small m).

Time-lapse AVO inversion

$$G_0 m_0 = d_0 \qquad (1)$$

$$G_i m_i = d_i \qquad (2)$$

- Least squares inversion - minimization of cost functions

$$J(m_i) = \left\| G_i m_i - d_i \right\|_2 + \lambda_i^2 \left\| W m_i \right\|_2$$

$$m_i = (G_i^T G_i + \lambda W_m^T W_m)^{-1} G_i^T d_i$$

$$\Delta m = m_i - m_0 \quad or \qquad \Omega_m = \frac{\Delta m}{m_0} \cdot 100$$

Practical time-lapse AVO inversion - i

$$G_1 m_1 - G_0 m_0 = d_1 - d_0 \qquad \text{(A)}$$

- **1) Total inversion of differences:** inverting for m_1 & Δm using ($\Delta G = G_1 - G_0$) , substitute for G_1 and re-arrange

$$\Delta G m_1 + G_0 \, \Delta m = \Delta d \qquad \text{(B)}$$

$$J(m_1, \Delta m) = \left\| \begin{bmatrix} \Delta G & 0 \\ 0 & G_0 \end{bmatrix} \begin{bmatrix} m_1 \\ \Delta m \end{bmatrix} - \begin{bmatrix} d_1 \\ d_0 \end{bmatrix} \right\|_2 + \left\| \begin{bmatrix} \lambda W_1 & 0 \\ 0 & \lambda W_0 \end{bmatrix} \begin{bmatrix} m_1 \\ \Delta m \end{bmatrix} \right\|_2$$

$m_1 = \frac{\Delta I}{I}$, $\frac{\Delta J}{J}$ and $\frac{\Delta \rho}{\rho}$

$\Delta m = \Delta(\frac{\Delta I}{I})$, $\Delta(\frac{\Delta J}{J})$ and $\Delta(\frac{\Delta \rho}{\rho})$

Practical time-lapse AVO inversion - i

$$G_1 m_1 - G_0 m_0 = d_1 - d_0 \qquad \text{(A)}$$

- 2) **Total inversion of differences:** inverting for m_0 & Δm using ($\Delta m = m_1 - m_0$) , substitute for m_1 and re-arrange

$$\Delta G m_0 + G_1 \Delta m = \Delta d \qquad \text{(c)}$$

$$J(m_0, \Delta m) = \left\| \begin{bmatrix} \Delta G & 0 \\ 0 & G_1 \end{bmatrix} \begin{bmatrix} m_0 \\ \Delta m \end{bmatrix} - \begin{bmatrix} d_0 \\ d_1 \end{bmatrix} \right\|_2 + \left\| \begin{bmatrix} \lambda W_0 & 0 \\ 0 & \lambda W_1 \end{bmatrix} \begin{bmatrix} m_0 \\ \Delta m \end{bmatrix} \right\|_2$$

$$m_0 = \frac{\Delta I}{I}, \frac{\Delta J}{J} \text{ and } \frac{\Delta \rho}{\rho}$$

$$\Delta m = \Delta\left(\frac{\Delta I}{I}\right), \Delta\left(\frac{\Delta J}{J}\right) \text{ and } \Delta\left(\frac{\Delta \rho}{\rho}\right)$$

➢ Total inversion of differences – (IP, IS, ρ)

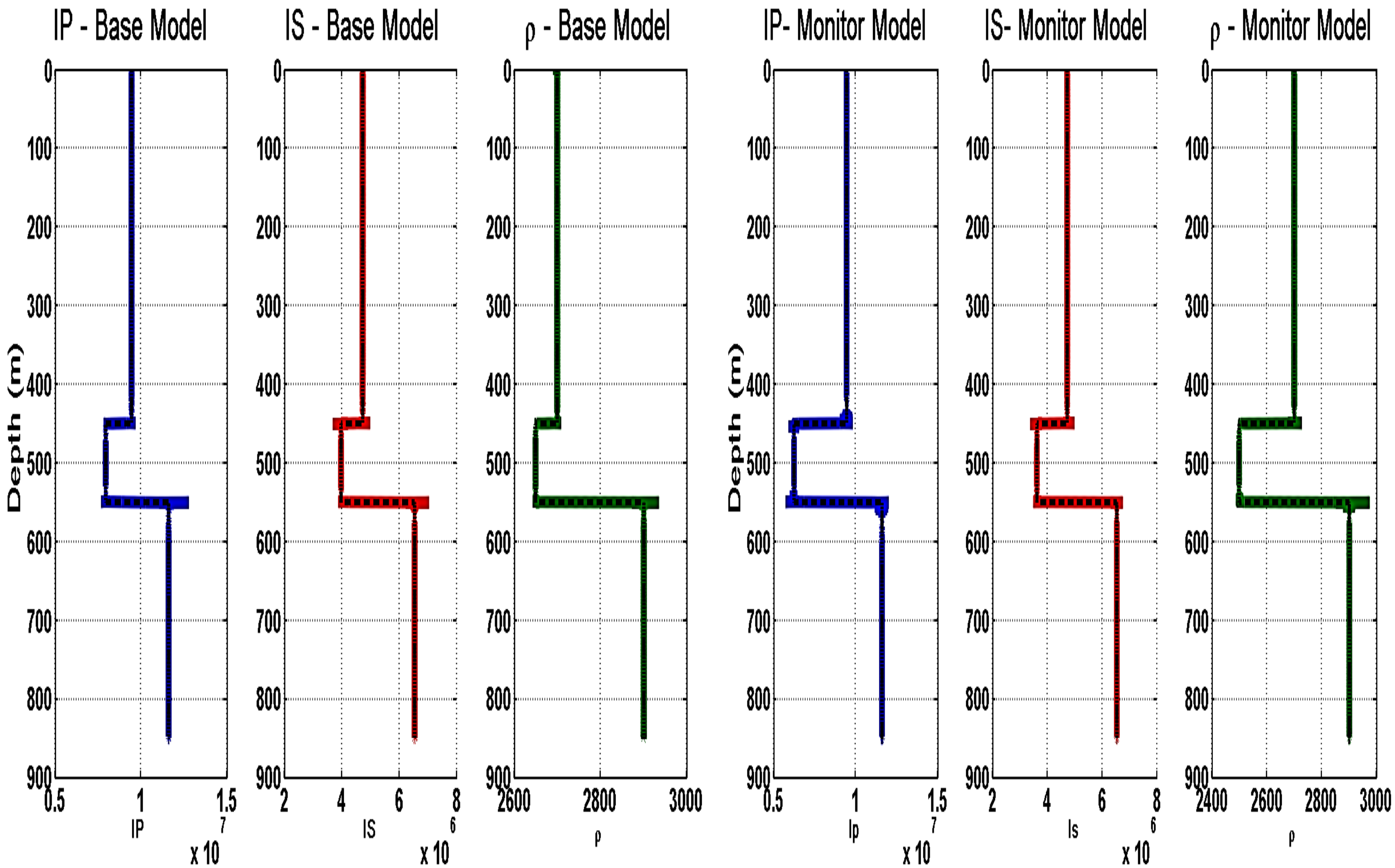


Fig.9. Actual and inverted elastic impedances for the Base (left) and Monitor (right).

➢ **Total inversion of differences –** (Δ**IP**, Δ**IS** and Δ**ρ**)

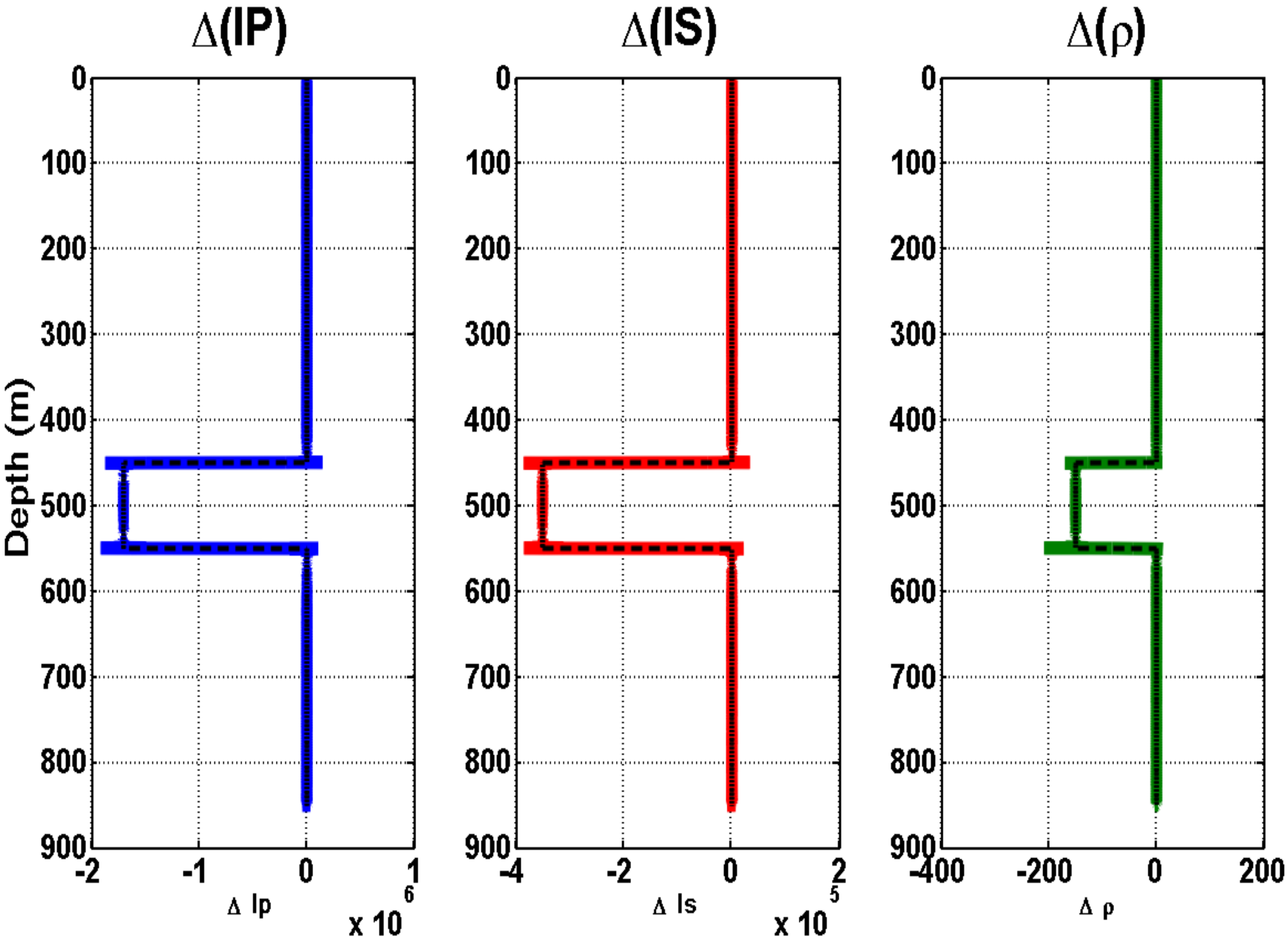


Fig.10. Actual and inverted elastic impedances differences.

Practical time-lapse AVO inversion - ii

$$\Delta\mathbf{G}\boldsymbol{m}_1 + \mathbf{G}_0\,\Delta\boldsymbol{m} = \Delta\mathbf{d} \qquad \textbf{(B)}$$

$$\Delta\mathbf{G}\boldsymbol{m}_\mathbf{0} + \mathbf{G}_\mathbf{1}\Delta\boldsymbol{m} = \Delta\mathbf{d} \qquad \textbf{(C)}$$

When $\Delta\mathbf{G} \approx \mathbf{0}$

- **Inversion of seismic difference ($\Delta\mathbf{d}$) only:**

$$\mathbf{G}_0\,\Delta\boldsymbol{m} = \Delta\mathbf{d} \qquad \textbf{(D)}$$

$$\mathbf{G}_\mathbf{1}\Delta\boldsymbol{m} = \Delta\mathbf{d} \qquad \textbf{(E)}$$

$$\left(\boldsymbol{G}_{\boldsymbol{i}}^{\boldsymbol{T}}\boldsymbol{G}_{\boldsymbol{i}} + \lambda^2\boldsymbol{W}_{i}^{\boldsymbol{T}}\boldsymbol{W}_{i}\right)\Delta\mathbf{m} = \Delta\mathbf{d}$$

$\Delta\mathbf{m} = \boldsymbol{\Delta}(\frac{\Delta \boldsymbol{I}}{\boldsymbol{I}})$, $\boldsymbol{\Delta}(\frac{\Delta \boldsymbol{J}}{\boldsymbol{J}})$ and $\boldsymbol{\Delta}(\frac{\Delta \boldsymbol{\rho}}{\boldsymbol{\rho}})$

➢ Inversion of seismic difference (Δd) only:

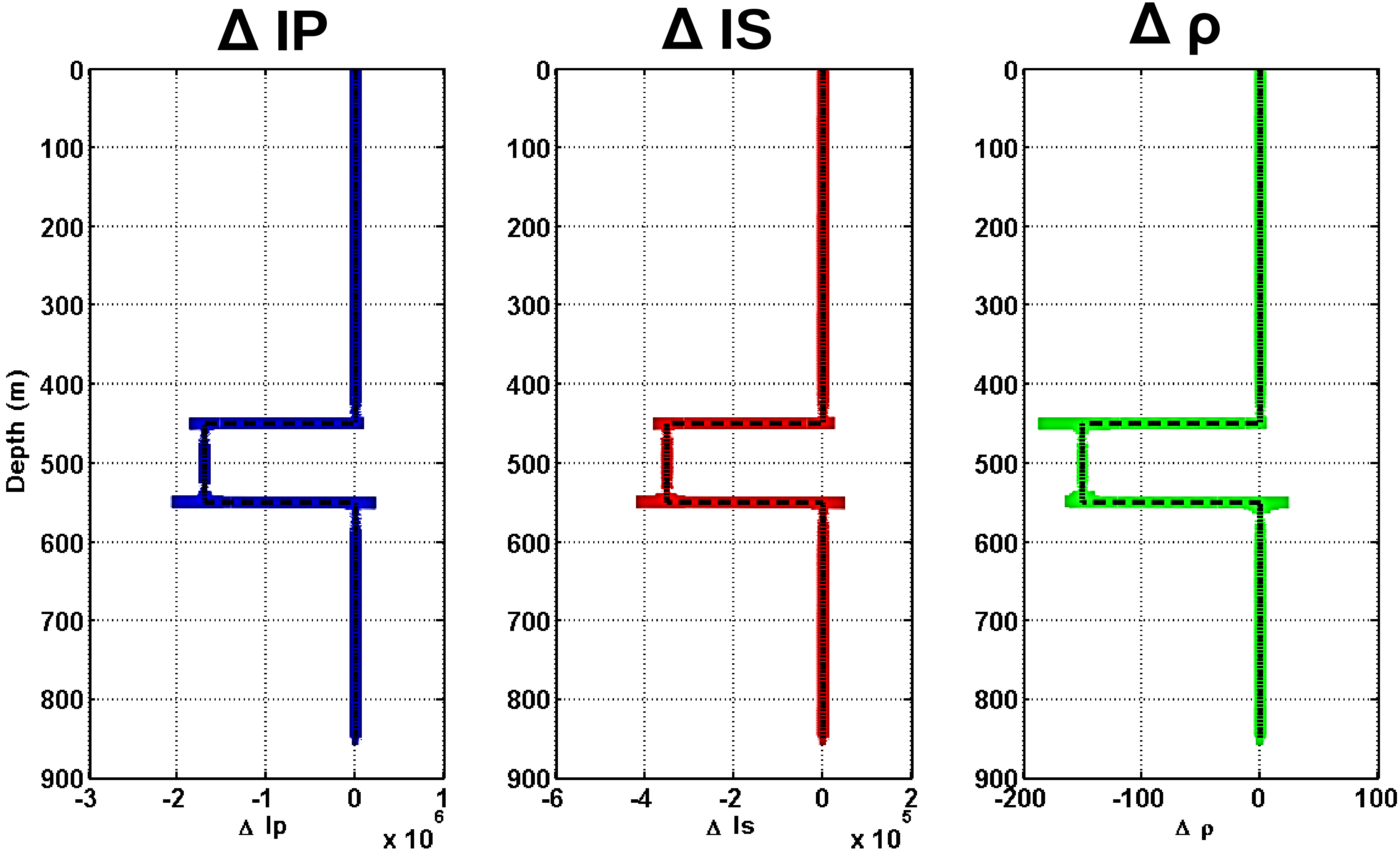


Fig.11. Actual and inverted elastic impedances differences (ΔIP, ΔIS & Δρ).

Practical time-lapse AVO inversion - iii

➢ **Sequential reflectivity-constrained Inversion**

$$\left(\left[\; G^{T}G + <_{1} W^{T}W + <_{2} V^{T}V \right)\right] m_{i} = \left[G^{T}d + <_{2} V^{T}V(\; m^{M}_{i-1} - m^{B}_{0})^{T} \right]$$

$$V^{i}_{Monit} = diag\left(\left[\; m^{M}_{i-1} - m^{B}_{0} \right)\right]$$

$$\frac{\left\| \mathbf{m}^{\mathbf{i+1}} - \mathbf{m}^{\mathbf{i}} \right\|_{2}}{1 + \left\| \mathbf{m}^{\mathbf{i+1}} \right\|_{2}} \langle \tau$$

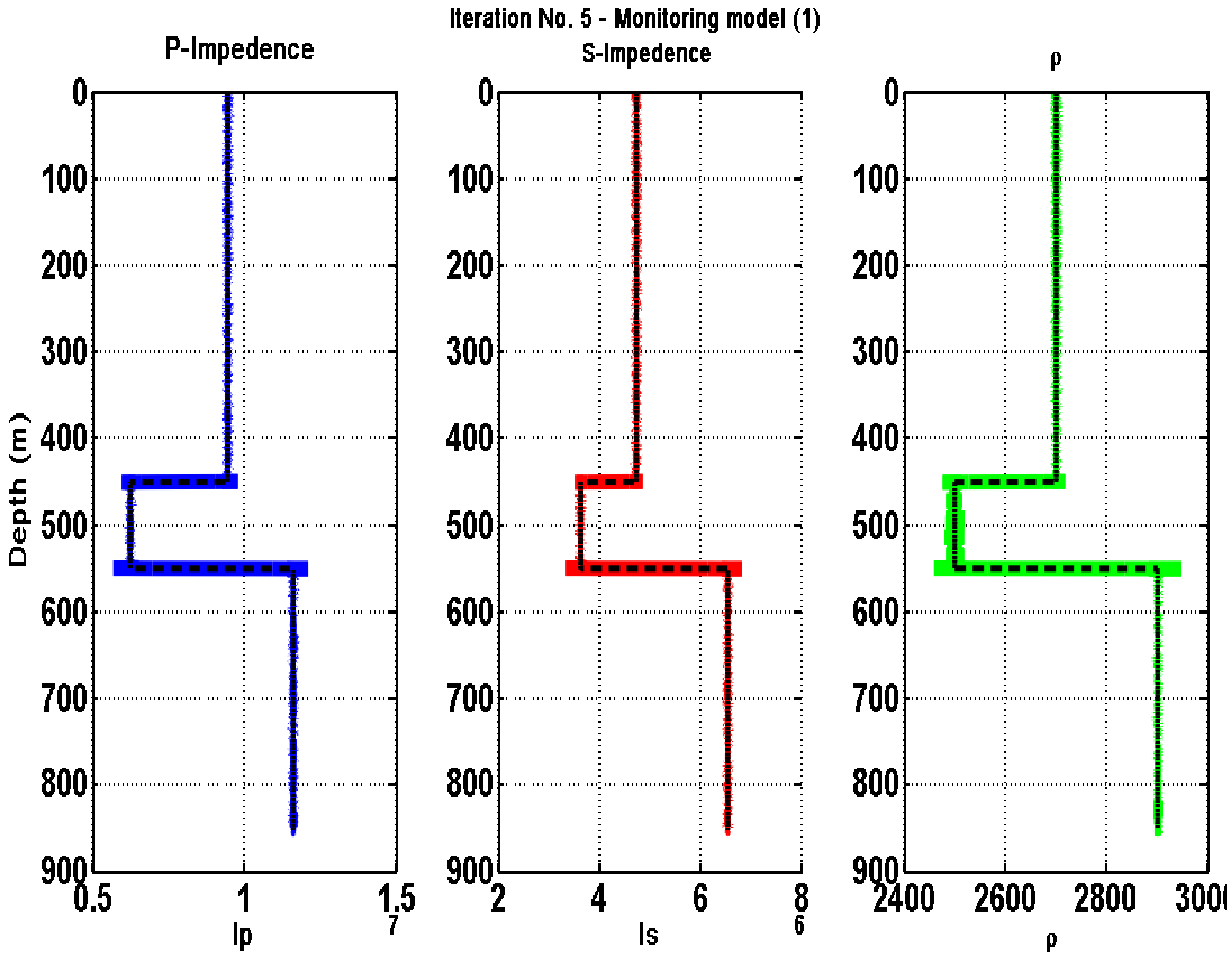


Fig.12. Inverted impedances for monitor noisy data using sequential reflectivity-constrained Inversion scheme.

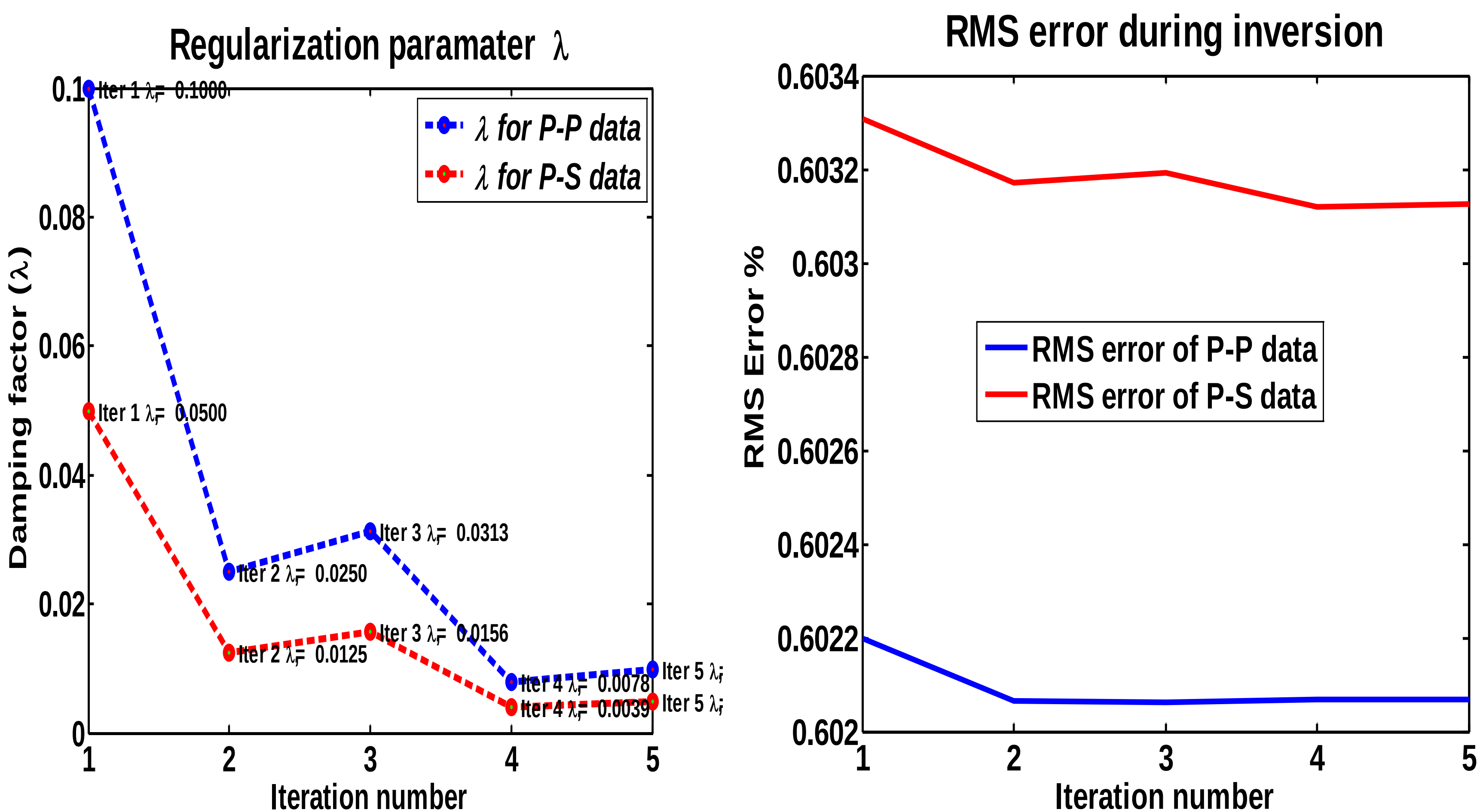


Fig.13. Regularization parameter (left), and RMS error (right) during inversion using sequential reflectivity-constrained Inversion.

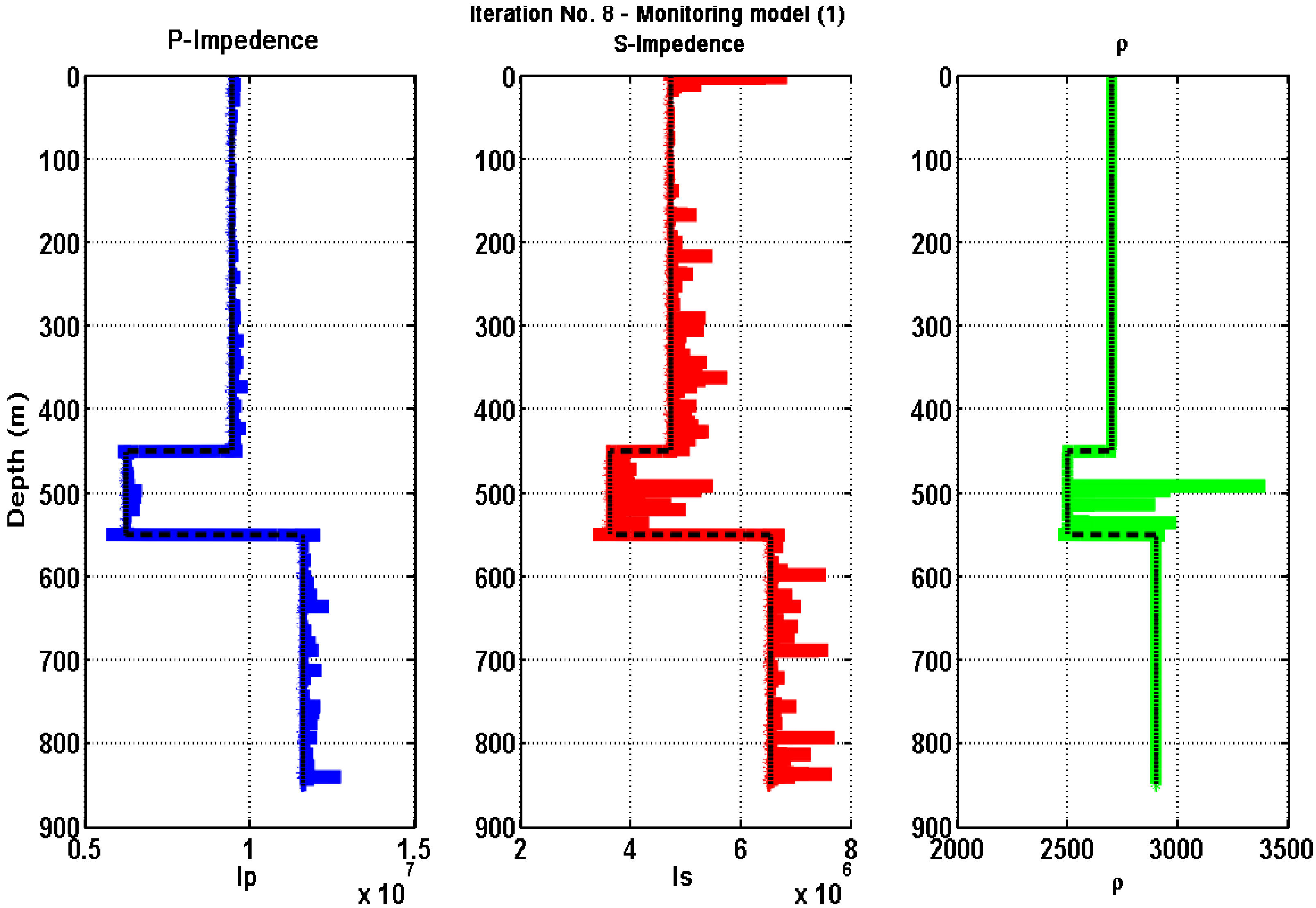


Fig.14. Elastic parameters (**IP**, **IS** and **ρ**) using **sequential reflectivity-constrained inversion** of the noisy (10 times amount of noise in figure 12) **monitor** model.

Conclusions

- **Well log analysis assist in lithology discriminations.**
- **Introduced new time-lapse AVO inversion schemes.**
- **Establishing IRLS AVO inversion to refine reflectivity model parameters - 60% of computation time in the1st iteration.**
- **Effects of incorporating constraints in the inverse formula.**

Future work….

- **Apply proposed inverse schemes using time-lapse seismic (1991 & 2000) surveys of Pikes Peak seismic.**
- **IRLS - AVO**
- **Estimate Saturation & pressure changes in time-lapse AVO inversion (LandrØ's method).**
- **Quadratic programming for total inversion of differences.**

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