Considerations for Reverse-Time migration

and

Modelling, migration, and inversion with Multilinear Algebra

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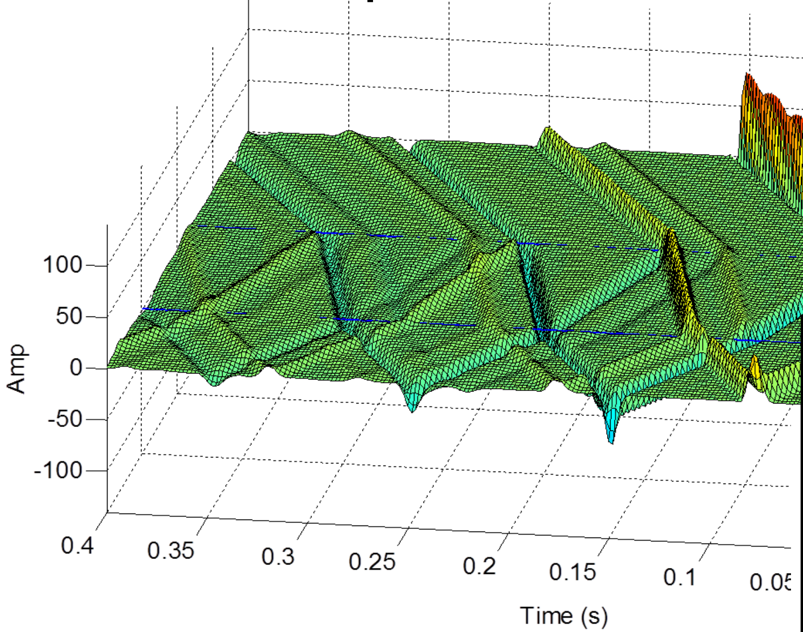
CREWES 2013

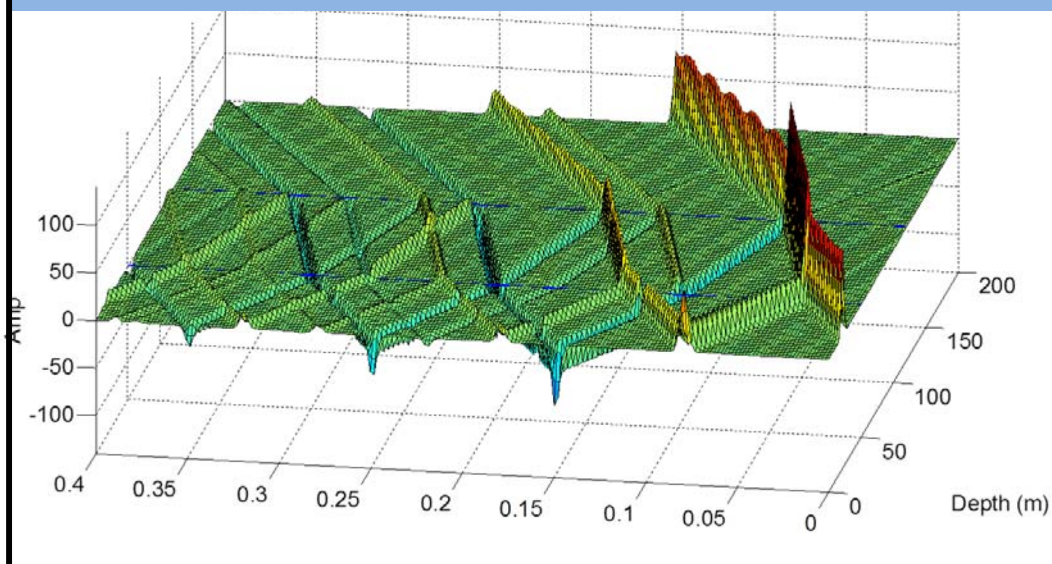
Motivation

Complete forward model



- Downgoing
- Primaries
- Surface multiples
- Interbed multiples
- Reflecting boundary
- Absorbing

Reconstructed wavefield: downward cont.



Outline

- Diffractions and Multilinear Algebra
- Modelling, migration and inversion
- Reverse-time migration problems
- Do we want the full wavefield
- Visualizing 1D wave propagation
- Finite difference
- Reverse time and downward continuation
- Imaging condition1D Modelling

Diffractions

a.

$$T^2 = T_0^2 + \frac{4x^2}{v^2}$$

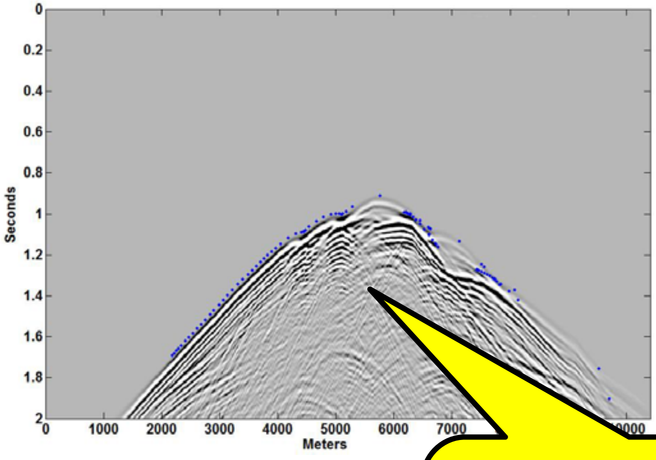
b.

$$a)\ \begin{bmatrix} 3 & 2 & 2 & 2 & 3 \end{bmatrix} \text{ time or sample number}$$

$$b)\ \begin{bmatrix} 0.2 & 0.5 & 1.0 & 0.5 & 0.2 \end{bmatrix} \text{ spatial amplitude of the diffraction}$$

c.

$$\begin{bmatrix} . & . & . & . & . \\ . & 0.5 & 1.0 & 0.5 & . \\ 0.2 & . & . & . & 0.2 \\ . & & . & . & . \end{bmatrix}$$



Wasteful

OK

Modelling and migration

Reflectivity

$$\mathbf{r} = \begin{matrix} 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

Modelled

$$\mathbf{s} = \begin{matrix} 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & 2 & 4 & 4 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 4 & 0 & 0 & 2 \end{matrix}$$

$$\mathbf{s_v} = \mathbf{D_{2D} r_v}$$

Migrated

$$\mathbf{m} = \begin{matrix} 4 & 10 & 4 & 6 & 12 \\ 6 & 8 & 6 & 10 & 16 \\ 4 & 8 & 6 & 6 & 10 \end{matrix}$$

$$\mathbf{m} = \mathbf{D_{2D}}^{\mathbf{T}} \mathbf{s_v}$$

Least squares inversion

Reflectivity

$$\mathbf{r} = \begin{matrix} 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

Least squares inverted data

-2.2204e-016	**2.0000e+000**	2.6645e-015	-6.6613e-016	-6.6613e-016
-8.4377e-015	9.4369e-015	-4.4409e-015	-1.7764e-015	**4.0000e+000**
1.0936e-014	-8.4377e-015	-1.5543e-015	8.8818e-016	2.8866e-015

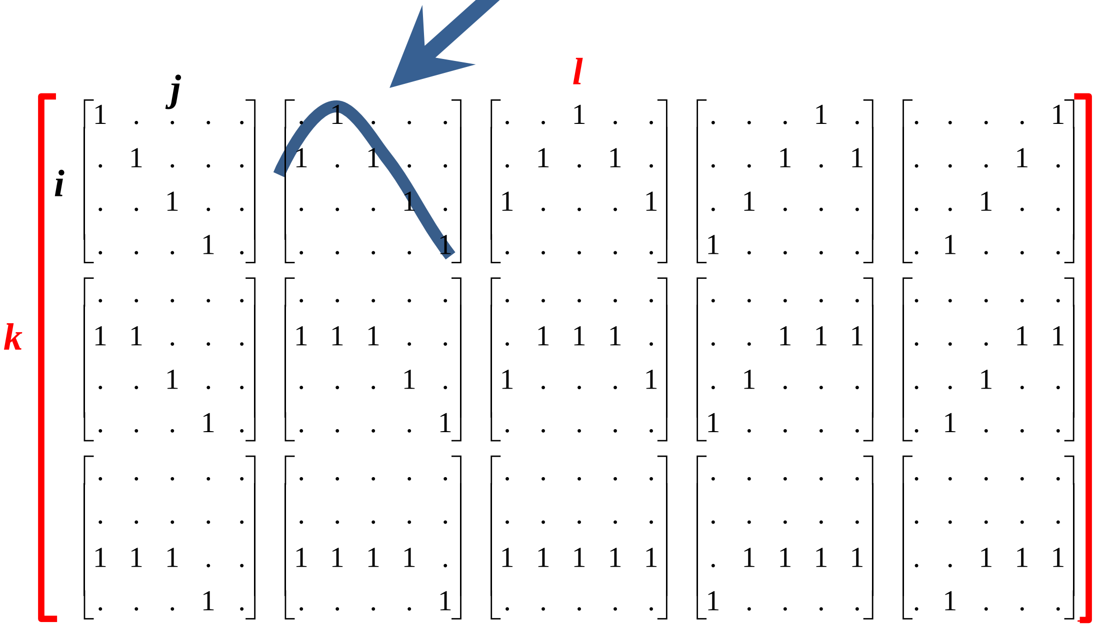
$$\mathbf{r_v} = \left(\mathbf{D_{2D}}^{\mathbf{T}}\mathbf{D_{2D}}\right)^{-1}\mathbf{D_{2D}}^{\mathbf{T}}\mathbf{s_v}$$

Only works for 2D matrix and two vectors

Diffraction matrix

Reflectivity

$$\mathbf{r} = \begin{matrix} 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$



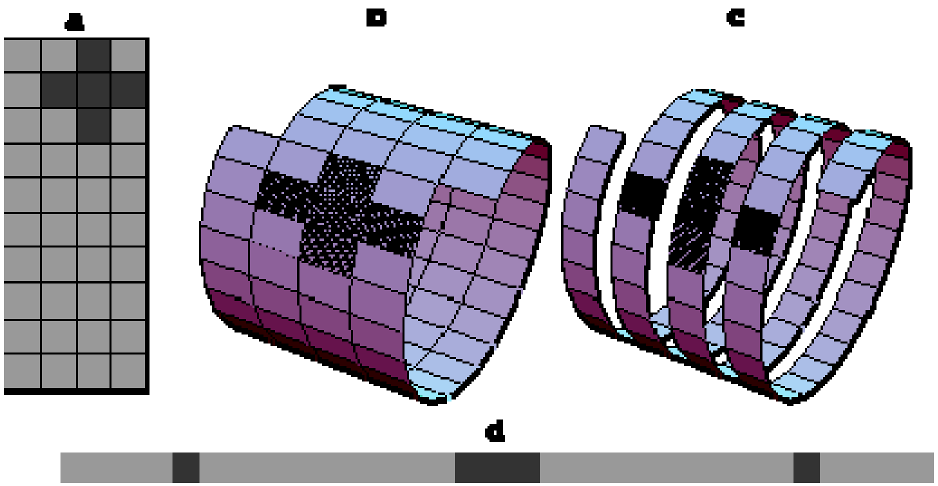
4D diffraction matrix **D**

Reflectivity matrix to vector

Reflectivity

$$r = \begin{matrix} 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$$r_v = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 4 \\ 0 \end{bmatrix}$$



Convert a 2D matrix to a vector
Unwrapping

Reflectivity matrix to vector

Reflectivity

$$
r = \begin{matrix} 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}
$$

$$
r_v = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 4 \\ 0 \end{bmatrix}
$$

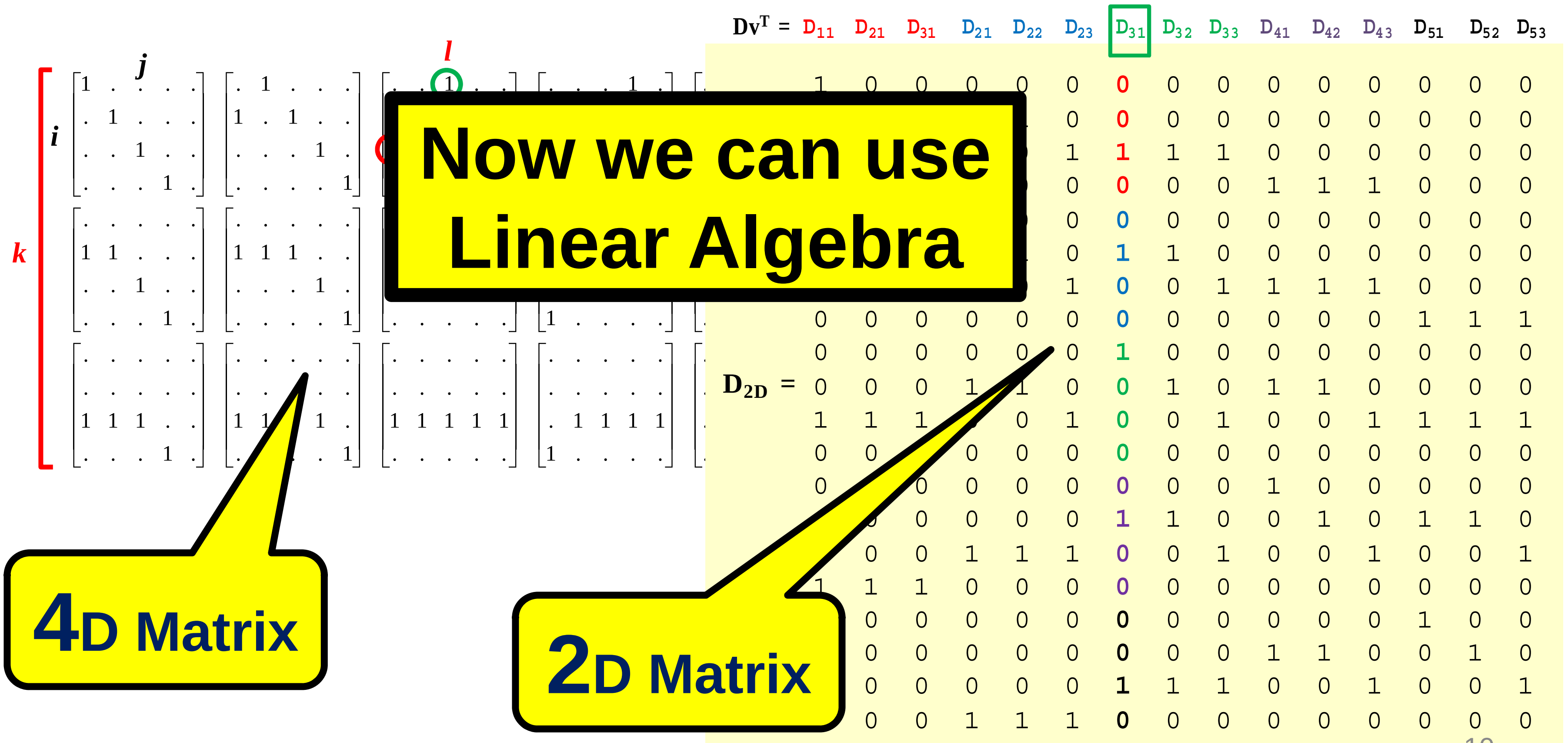
Array of diffraction matrices

$$
Dv^T = D_{11} \quad D_{21} \quad D_{31} \quad D_{21} \quad D_{22} \quad D_{23} \quad D_{31} \quad D_{32} \quad D_{33} \quad D_{41} \quad D_{42} \quad D_{43} \quad D_{51} \quad D_{52} \quad D_{53}
$$

Diffraction matrix

Reflectivity

$$r = \begin{matrix} 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

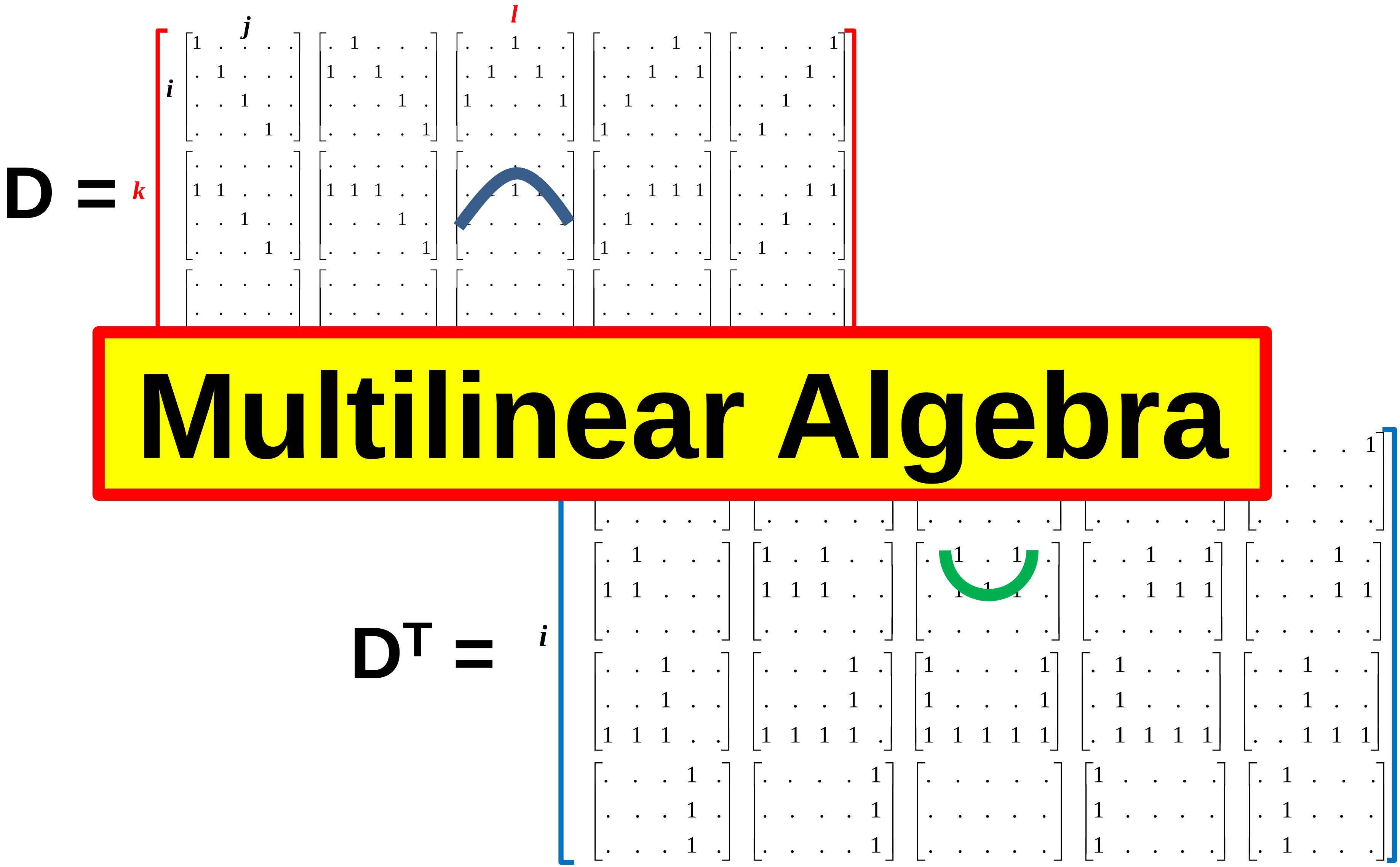


Comments on linear algebra

- Cannot handle arrays > 2D
- Diffractions 2D matrix
- Reflectivity 1D vector
- Seismic 1D vector
- Transpose, Least-squares 2D and 1D

- Unwrap higher dimensions to 1D and 2D
- Data stored in a computer is 1D (???)

2D transpose of 4D diff. matrix



Multilinear Algebra

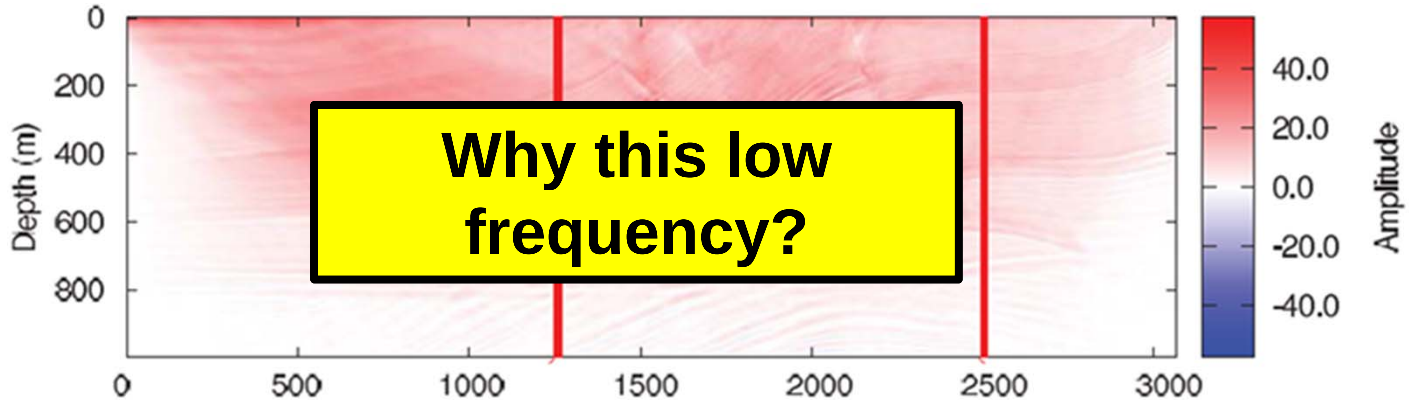
Intermission

Problems with Reverse-time migration

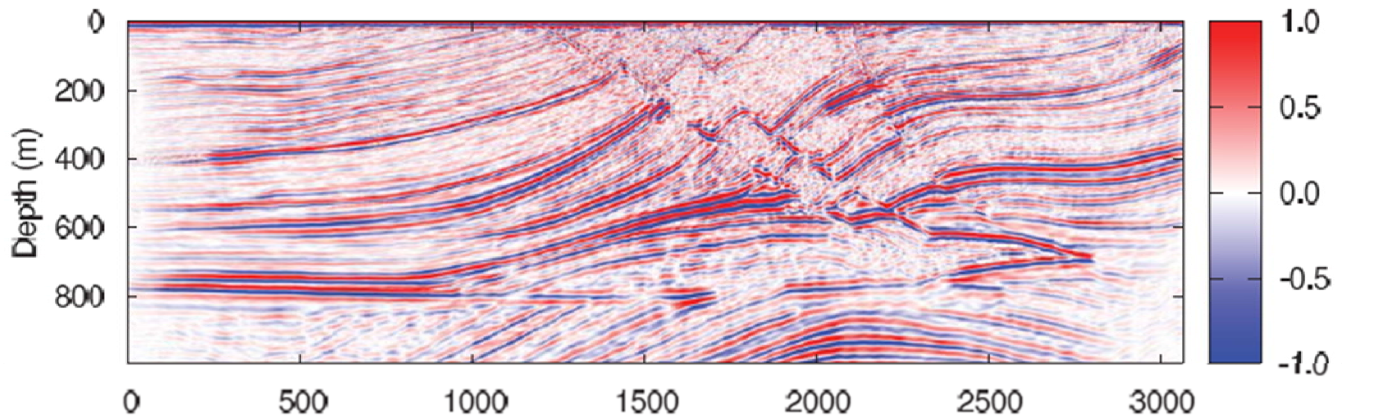
- Cross correlation
  - Only zero lag
- DC bias on the cross-correlation
- Does not reconstruct the complete wavefield
- Believed to be required for FWI

Problems with Reverse-time migration

Inversion RTM Claerbout's Imaging condition



Highpass filtered



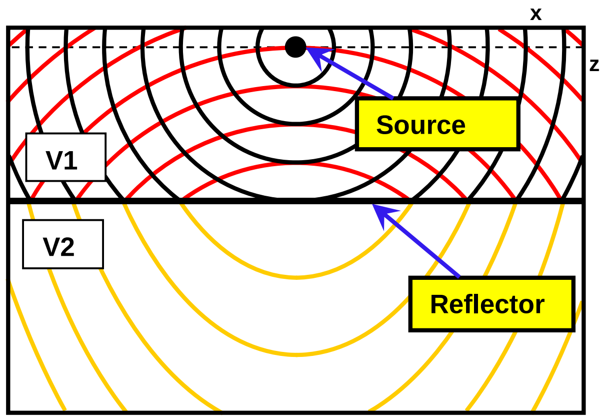
Zaiming Jiang

Other filter option:

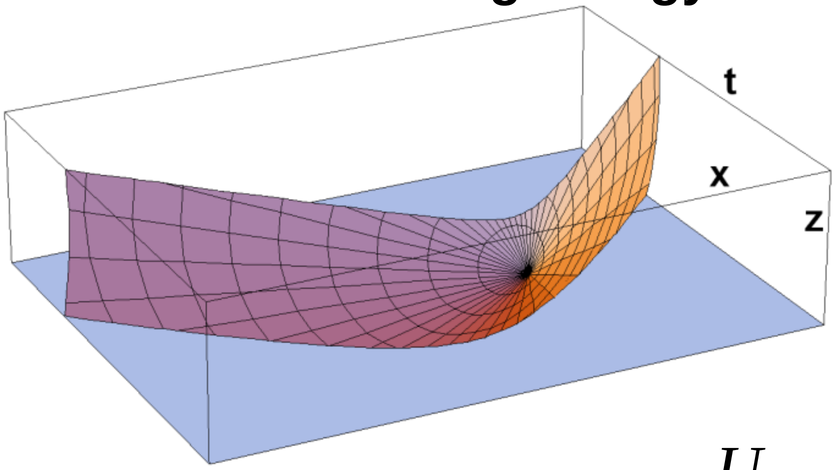
Laplacian [1 -2 1] (second derivative)

Derivative [1 -1]

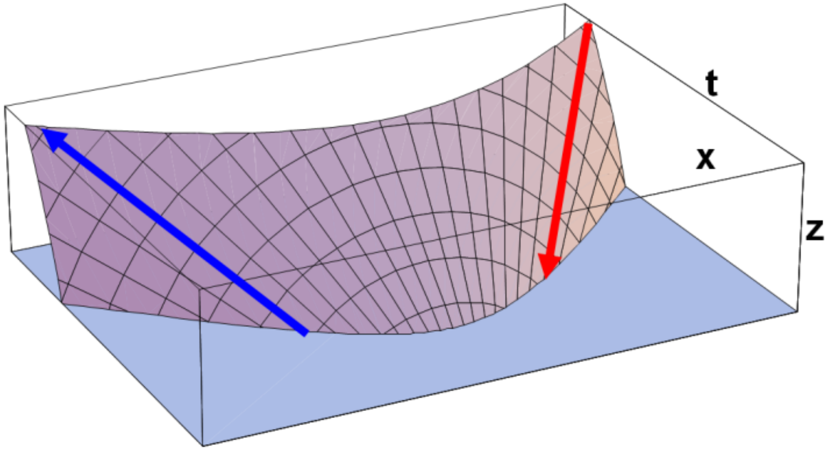
Claerbout's imaging condition



Forward radiating energy D

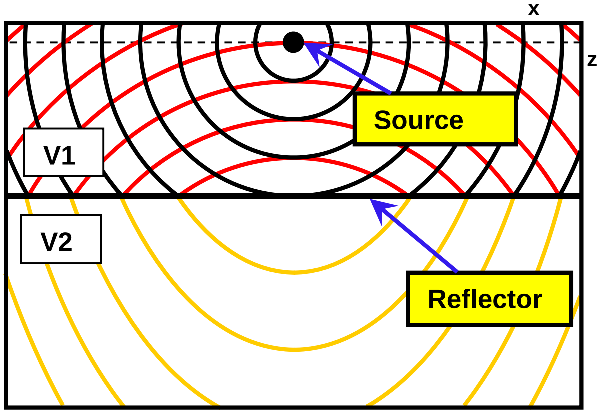


Reflected energy U



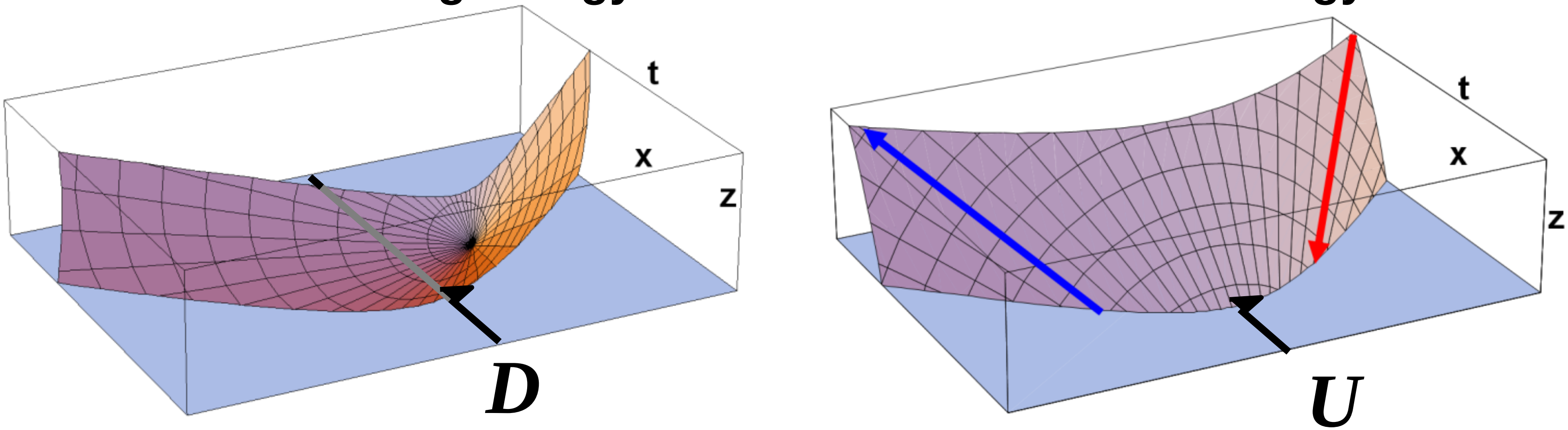
$$R = \frac{U}{D} \approx kU \otimes D^{*}$$

Claerbout's imaging condition



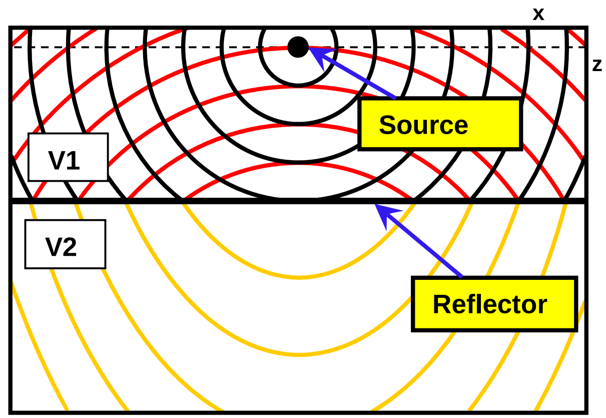
Forward radiating energy D

Reflected energy U

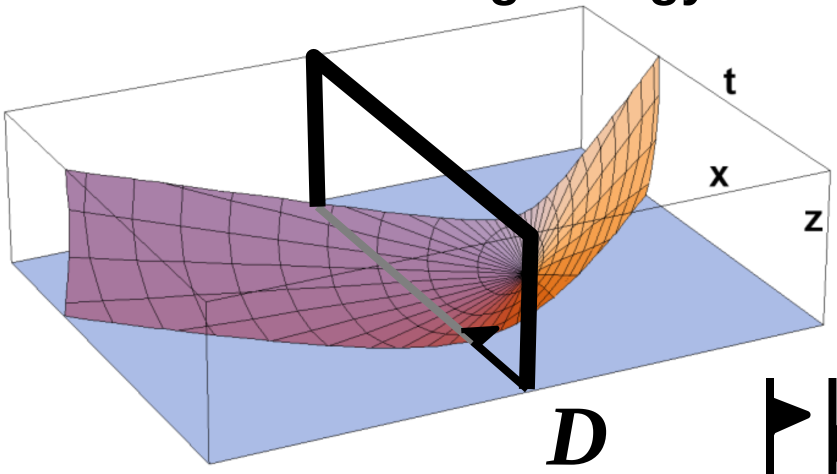


$$R = \frac{U}{D} \approx kU \otimes D^{*}$$

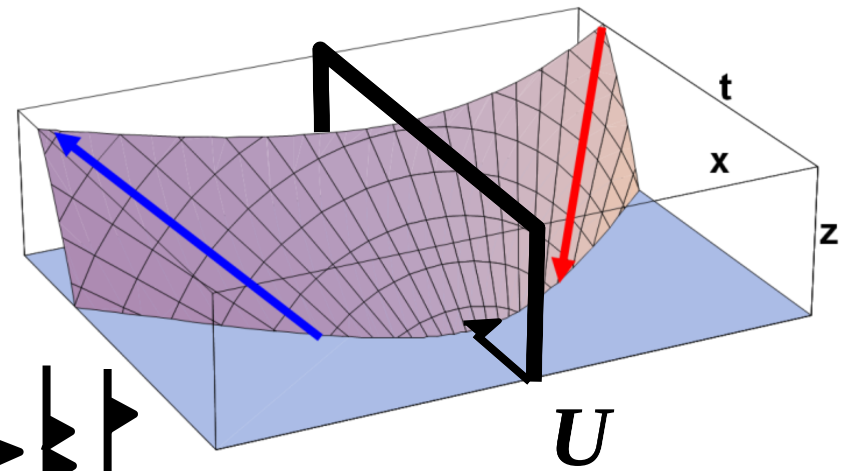
Claerbout's imaging condition

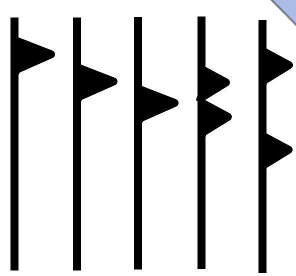


Forward radiating energy D



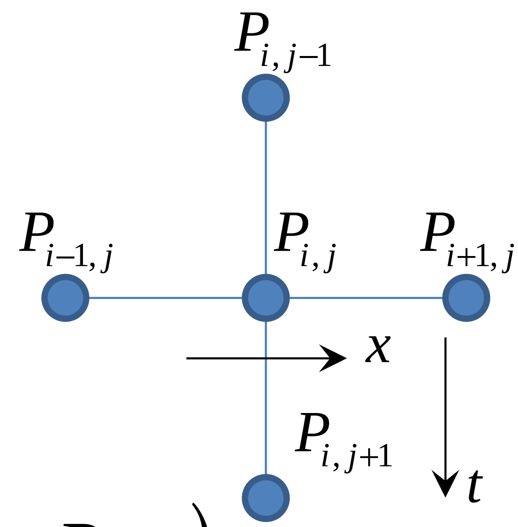
Reflected energy U





Waves on 1D model

$$\frac{\partial^2 P}{\partial z^2} = \frac{1}{v}\frac{\partial^2 P}{\partial t^2}$$



Finite difference solution

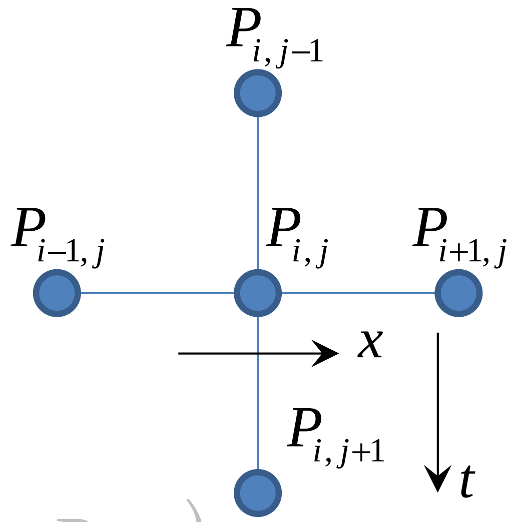
$$P_{i-1,j} - 2P_{i,j} + P_{i+1,j} = \frac{\delta x^2}{v^2 \delta t^2}\left(P_{i,j-1} - 2P_{i,j} + P_{i,j+1}\right)$$

Locally constant
Very fine sampling

Forward time
Reverse time
Downward cont.
Upward cont.

Waves on 1D model

$$\frac{\partial^2 P}{\partial z^2} = \frac{1}{v}\frac{\partial^2 P}{\partial t^2}$$



Finite difference solution

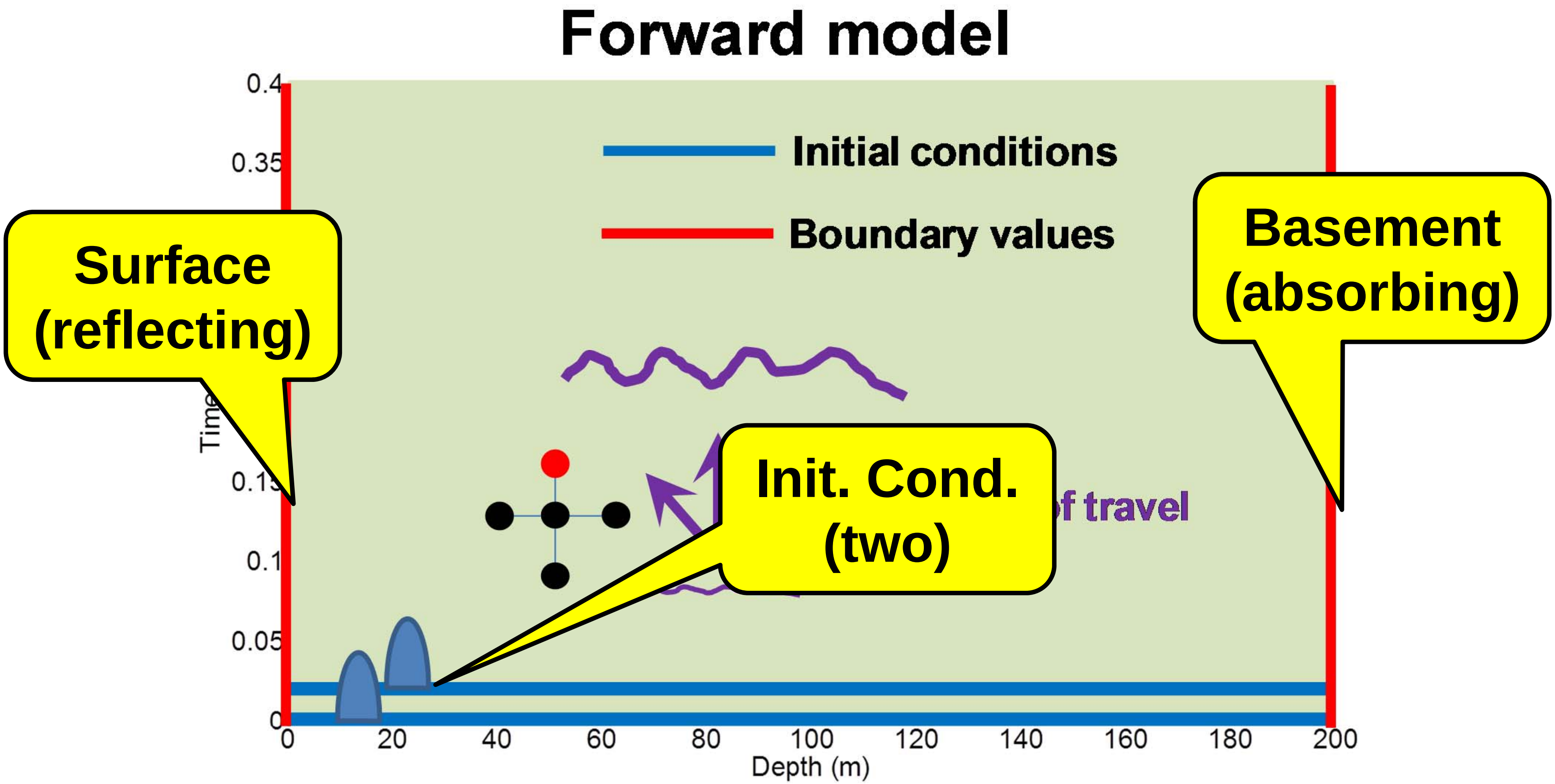
$$P_{i-1,j} - 2P_{i,j} + P_{i+1,j} = \frac{\delta x^2}{v^2 \delta t^2}\left(P_{i,j-1} - 2P_{i,j} + P_{i,j+1}\right)$$

Phase-shift solution

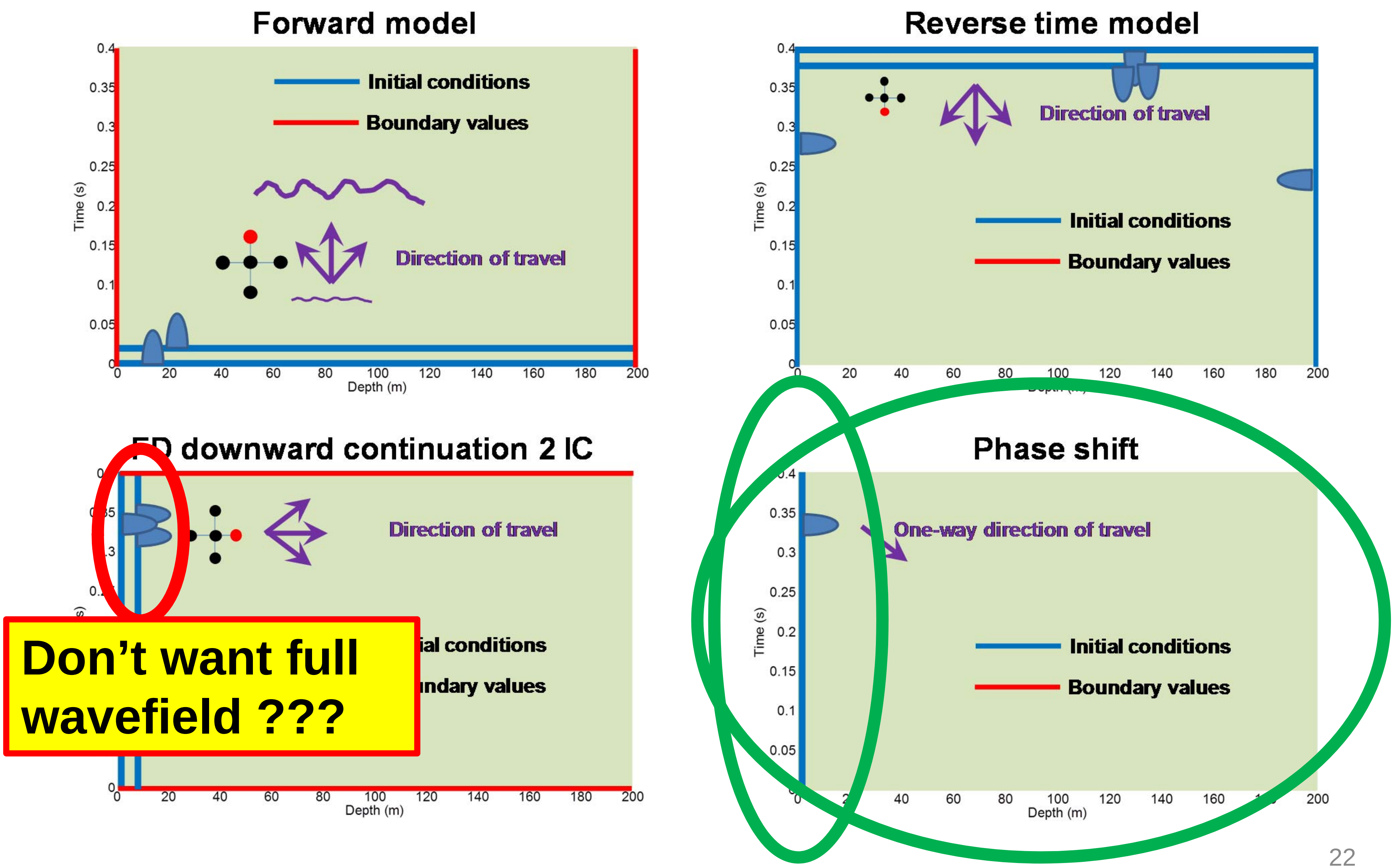
$$P\left(z + \delta z, \omega\right) = P\left(z, \omega\right) e^{\pm \frac{i\delta z \omega}{v}}$$

Downward cont.
Upward cont.

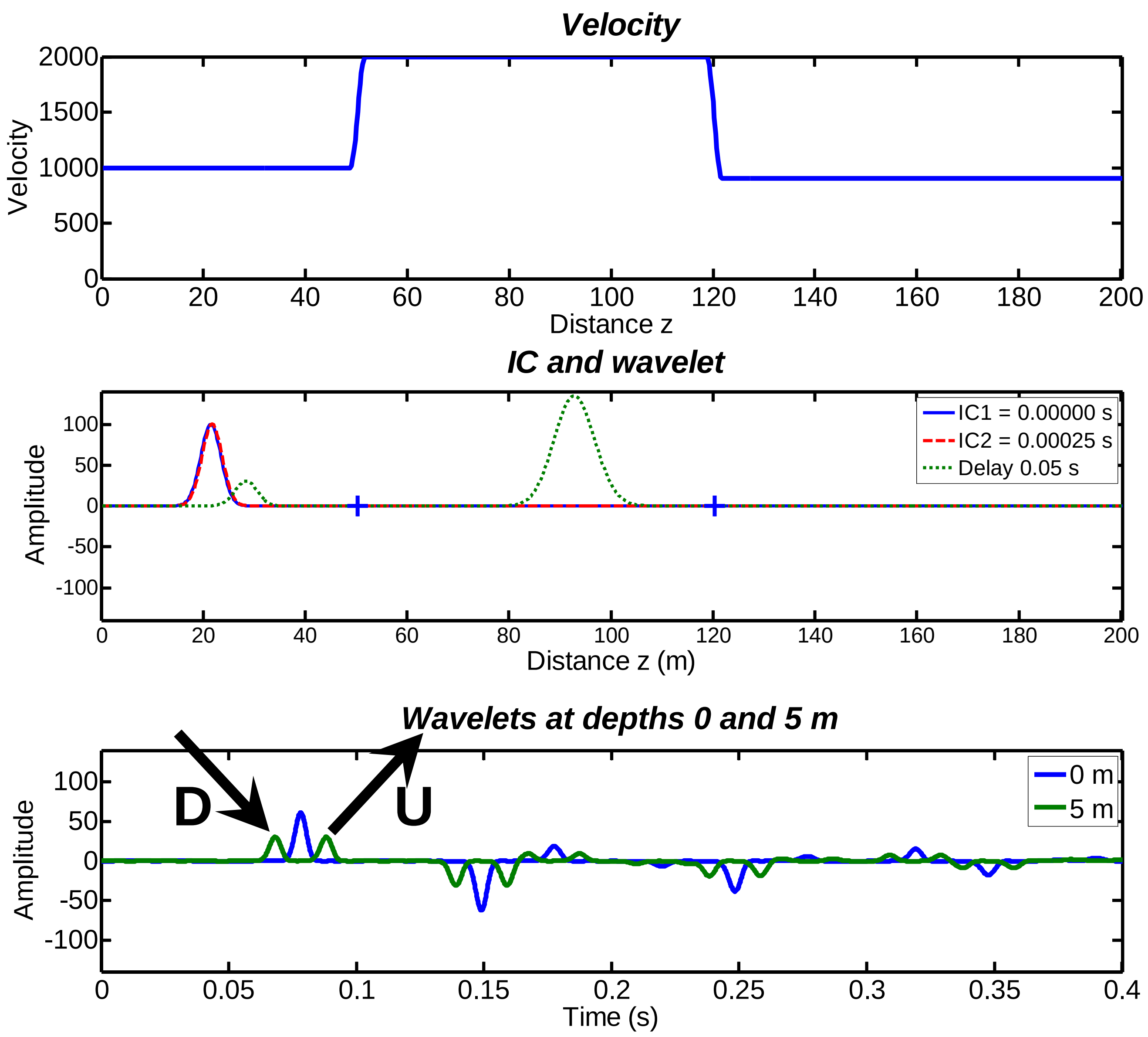
Waves: Forward modelling



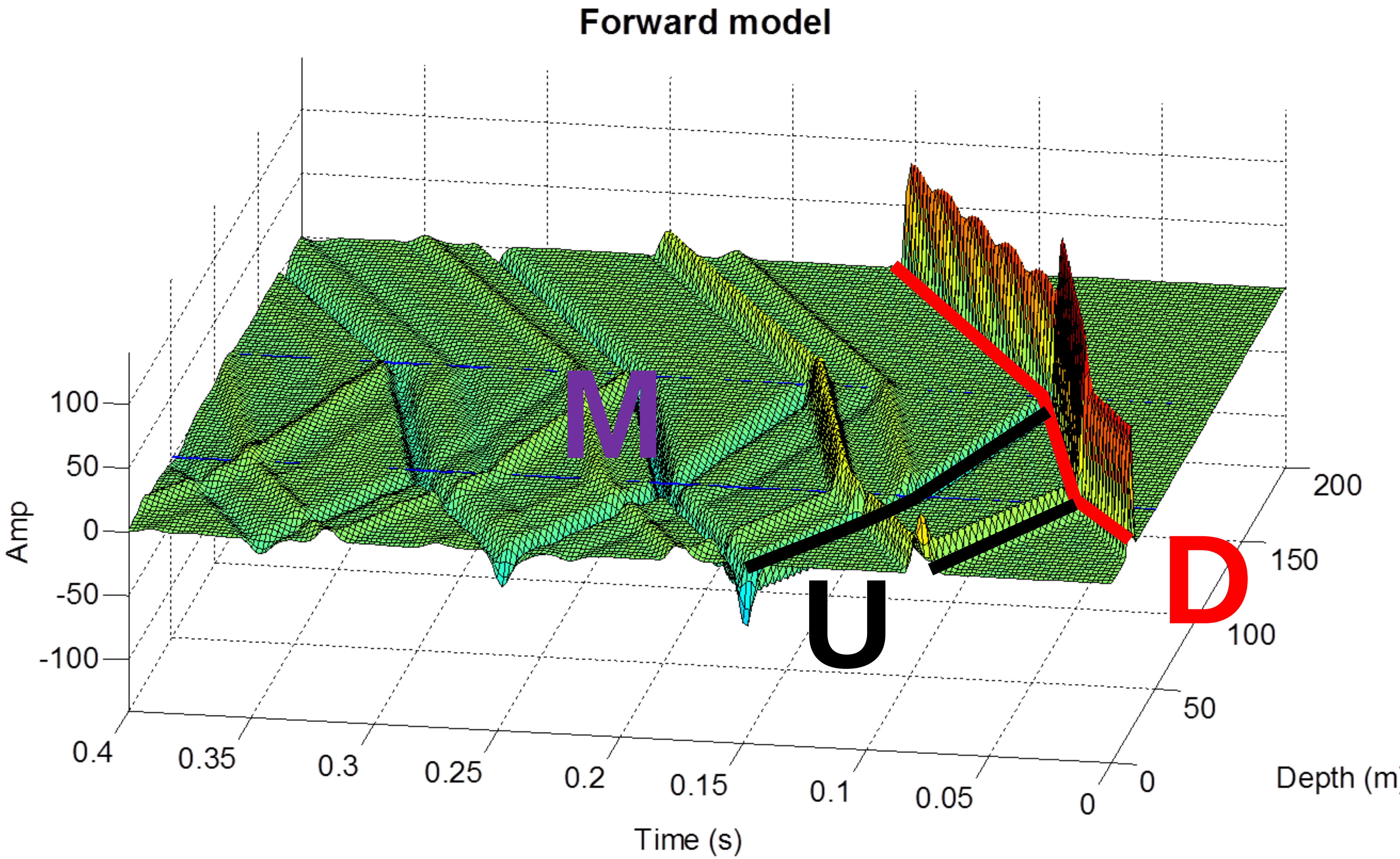
Wavefield reconstruction B.C.



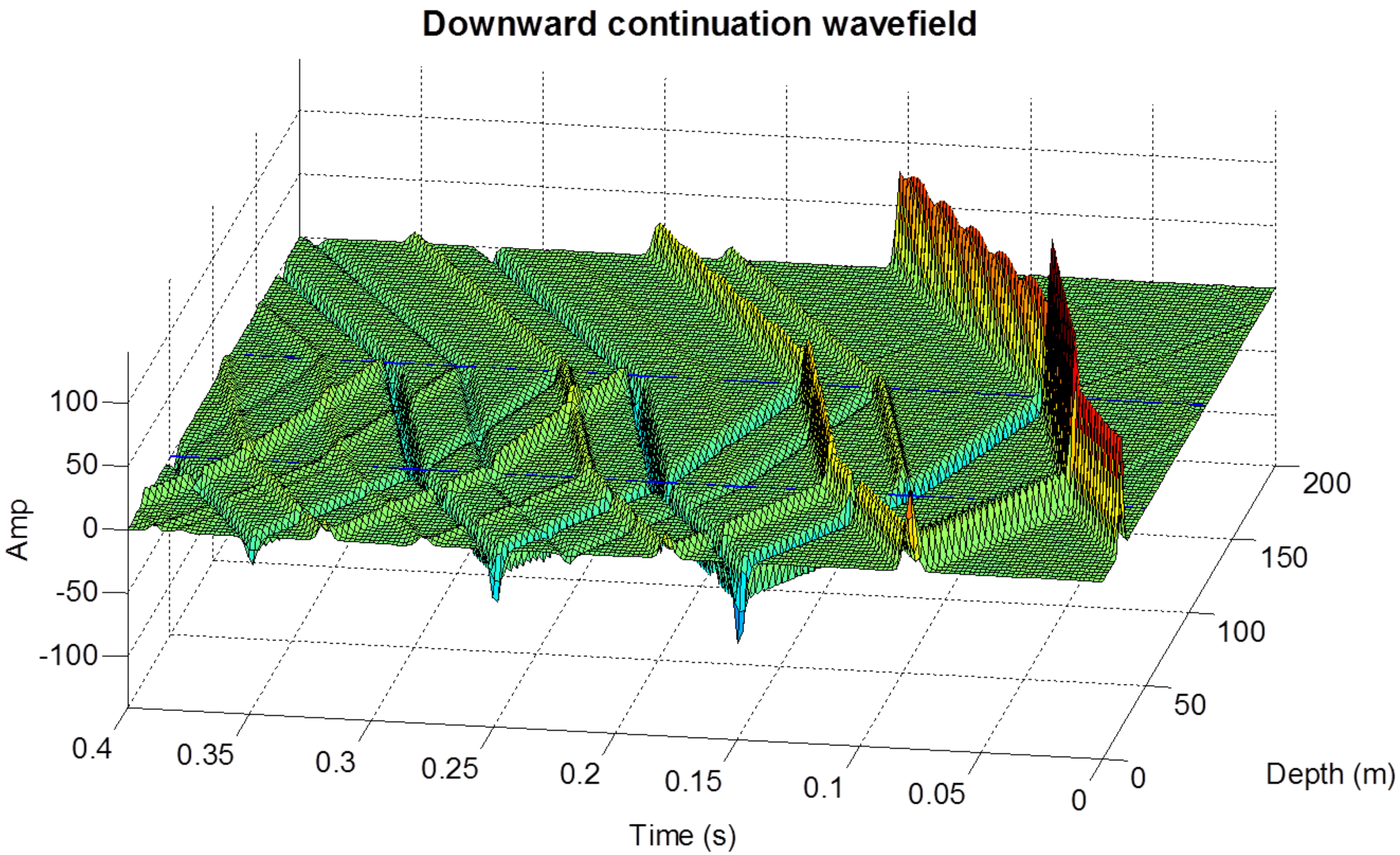
Waves on 1D model



Waves on 1D model

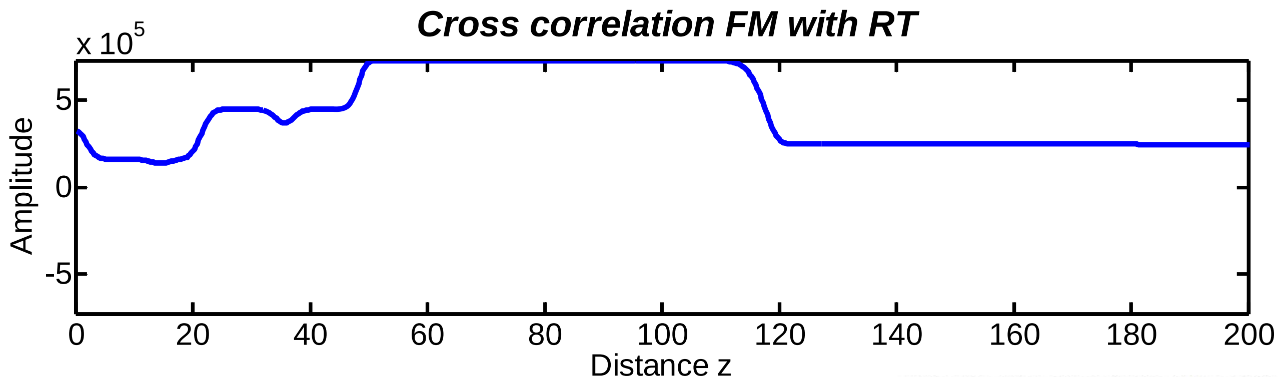


Waves: Downward continuation

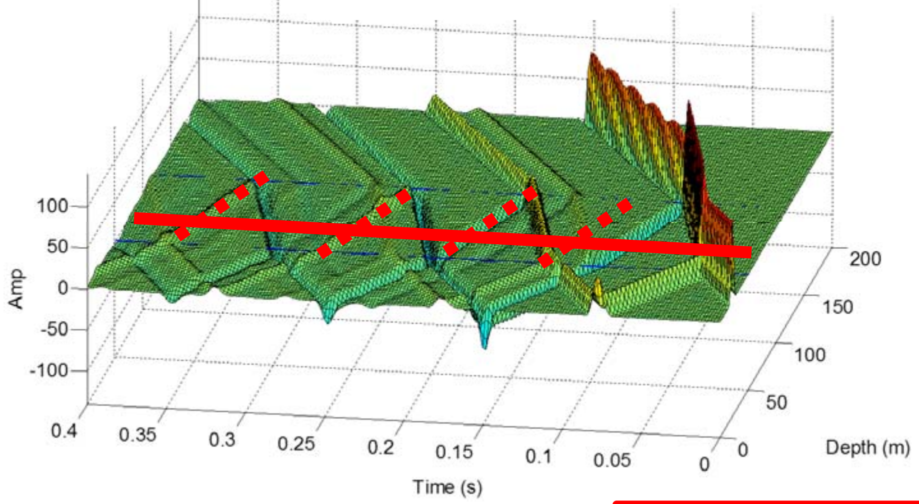


<u>Complete recovery of the wavefield</u>

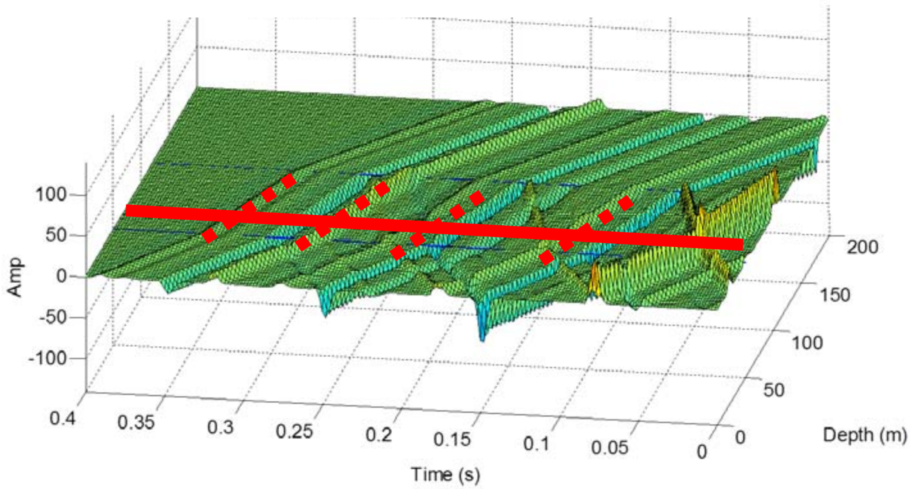
Cross-correlation



Forward model

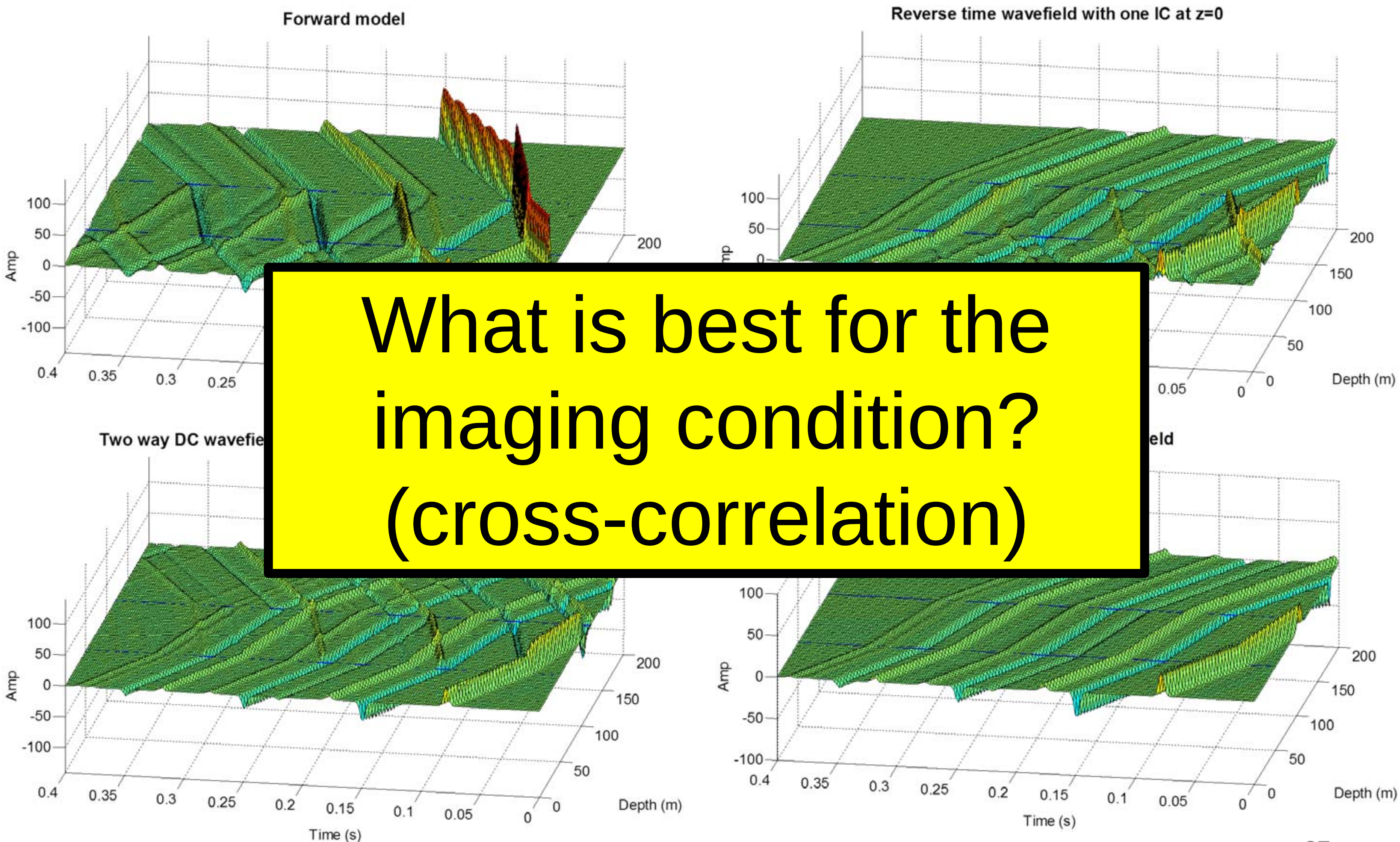


Reverse time reconstruction



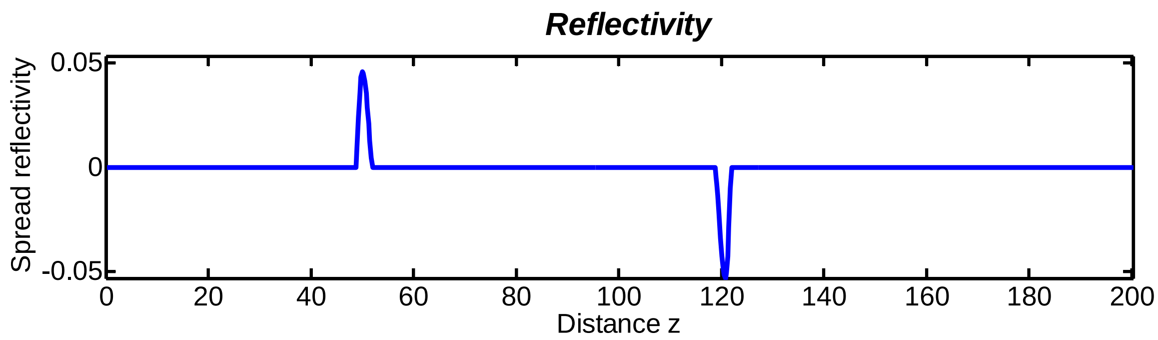
Multiples coherent

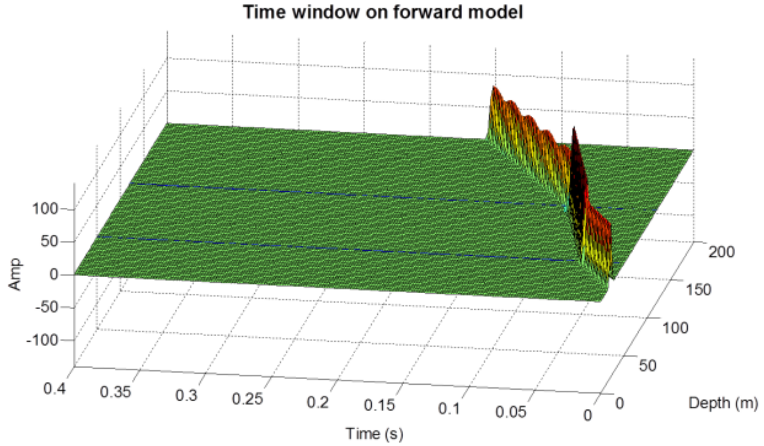
Waves: RT, DC, Phase-shift

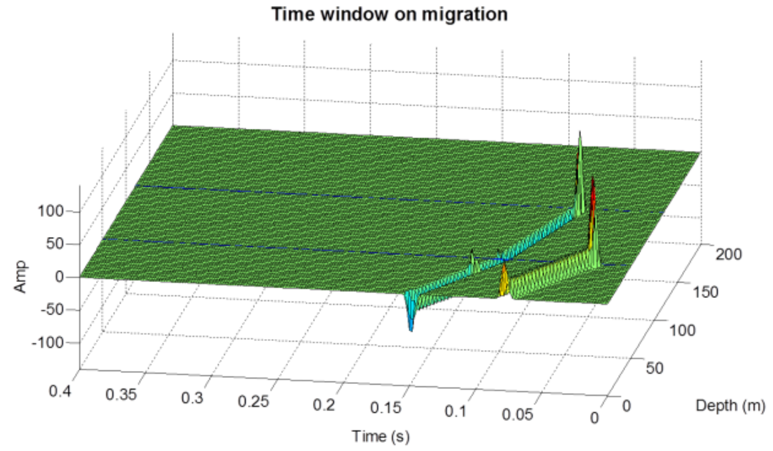


What is best for the imaging condition? (cross-correlation)

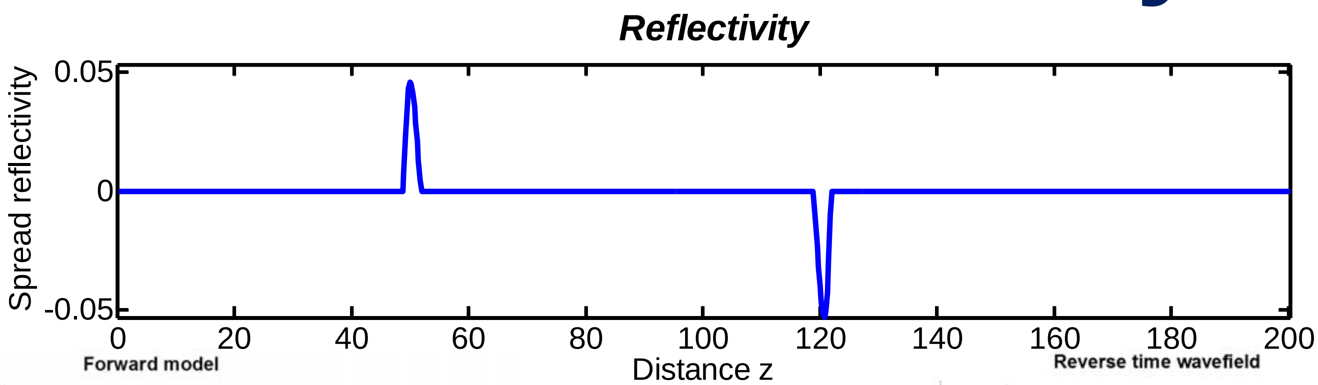
Waves: Reflectivity

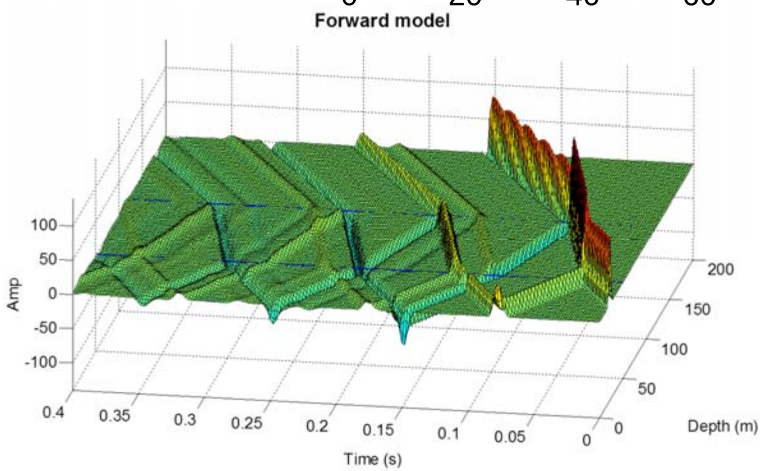


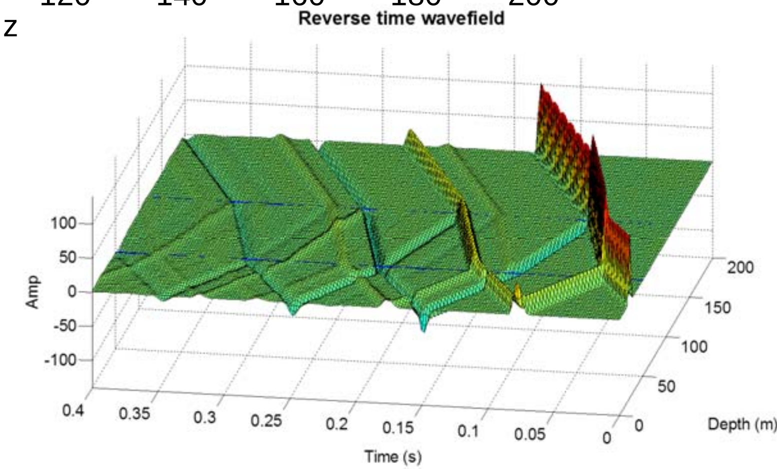


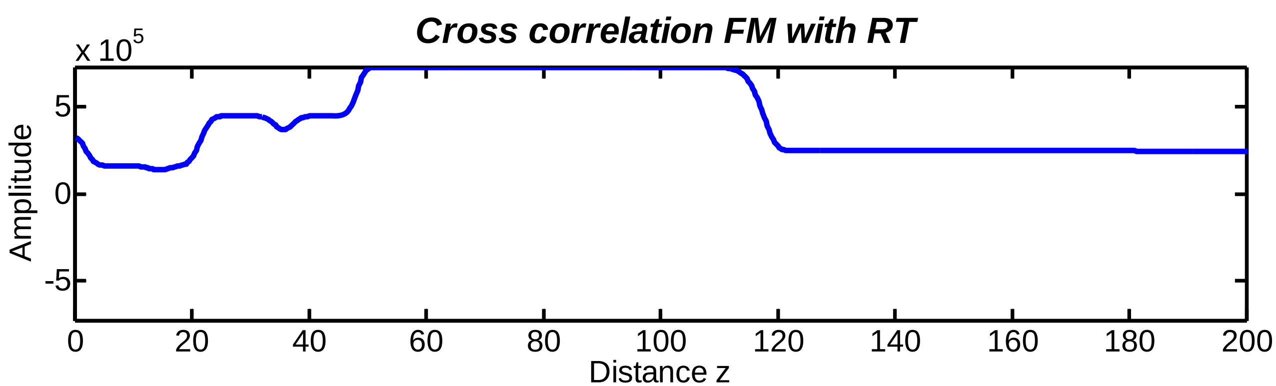


Waves: Reflectivity

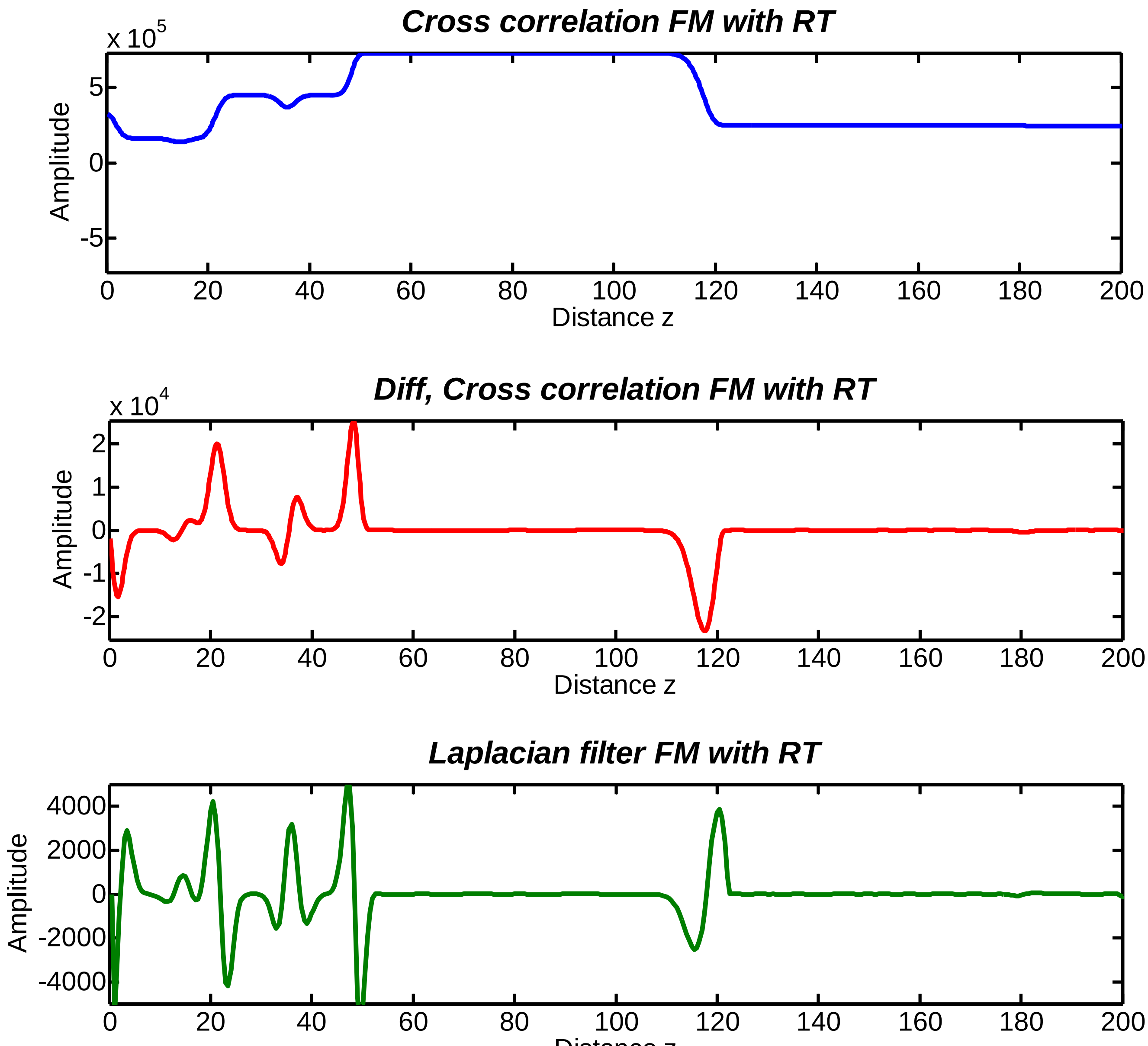




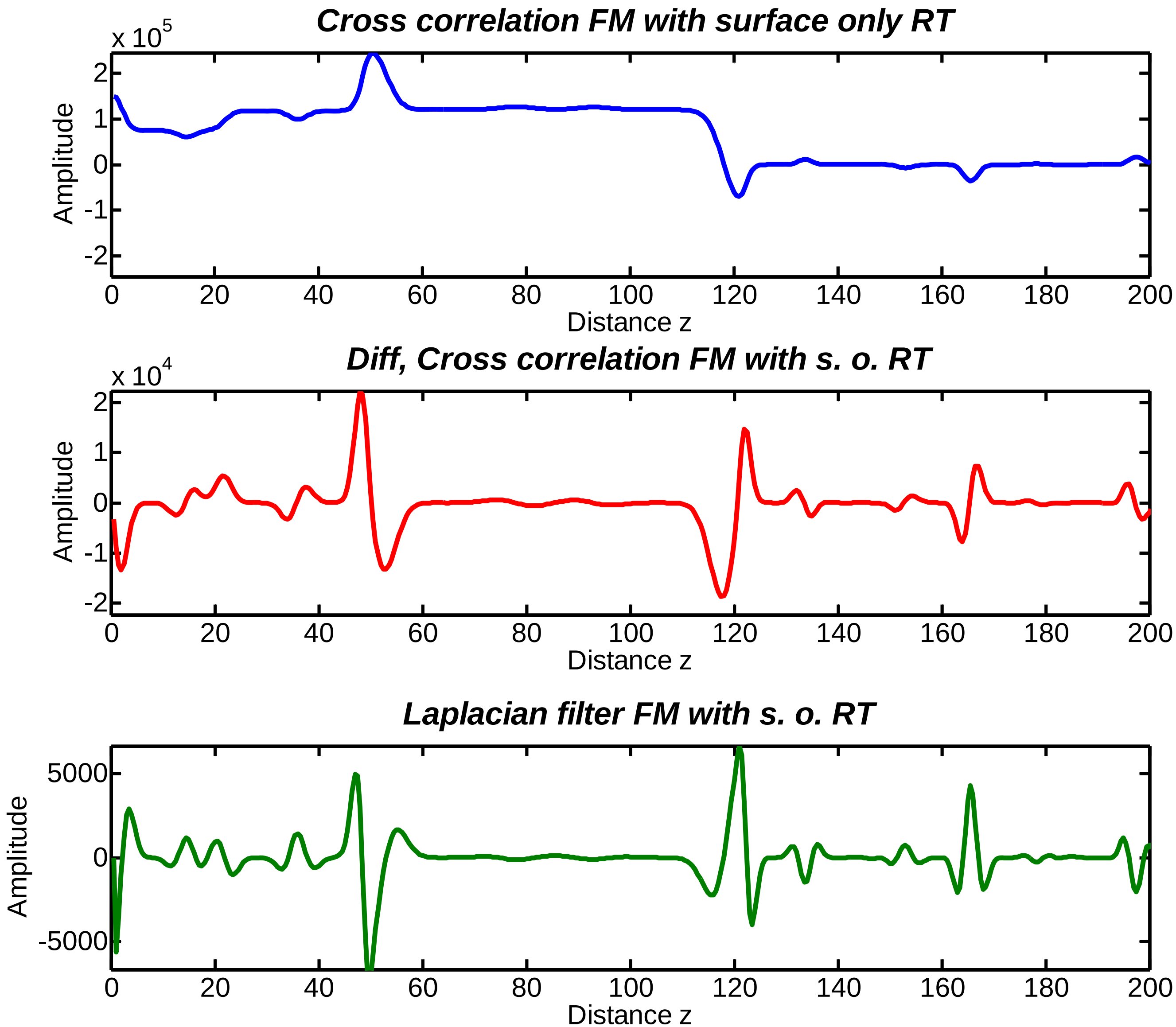




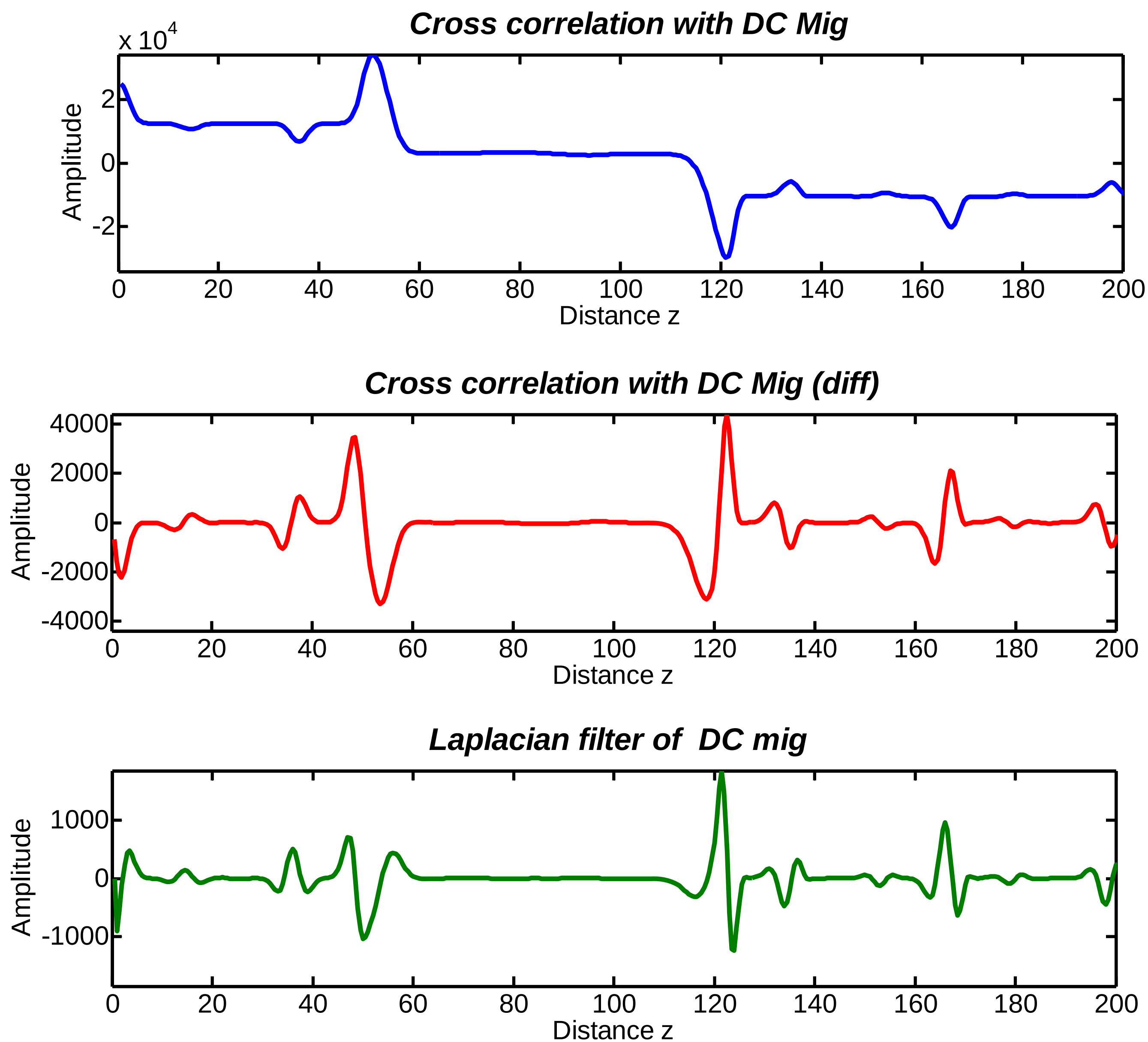
Reflectivity: FM with RT



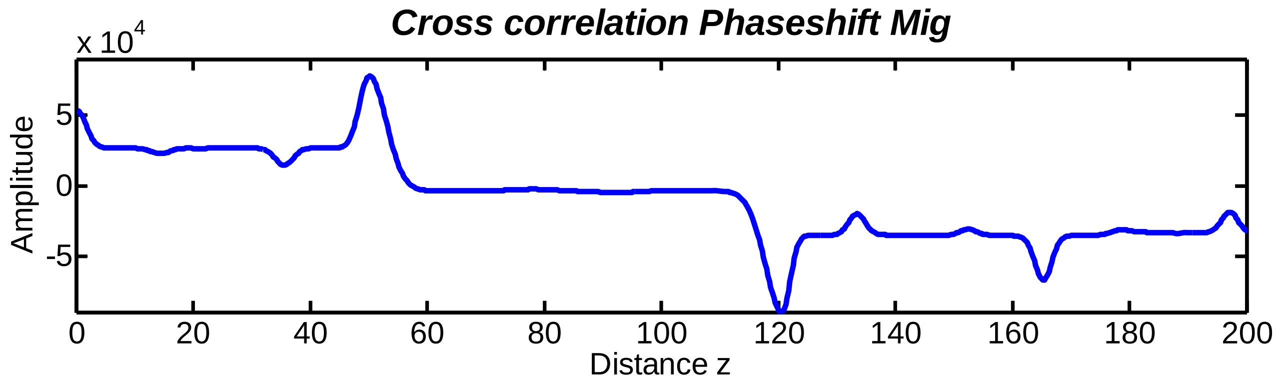
Reflectivity: FM with surf. RT

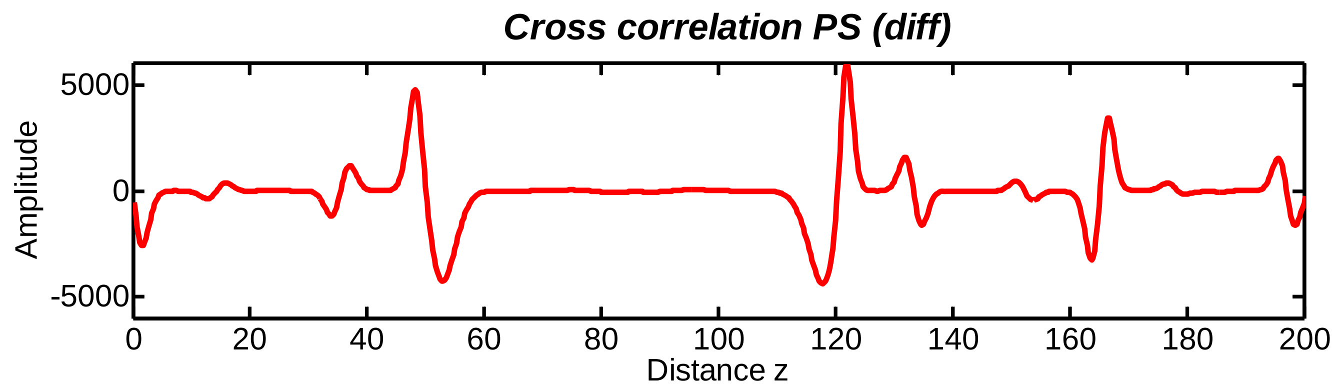


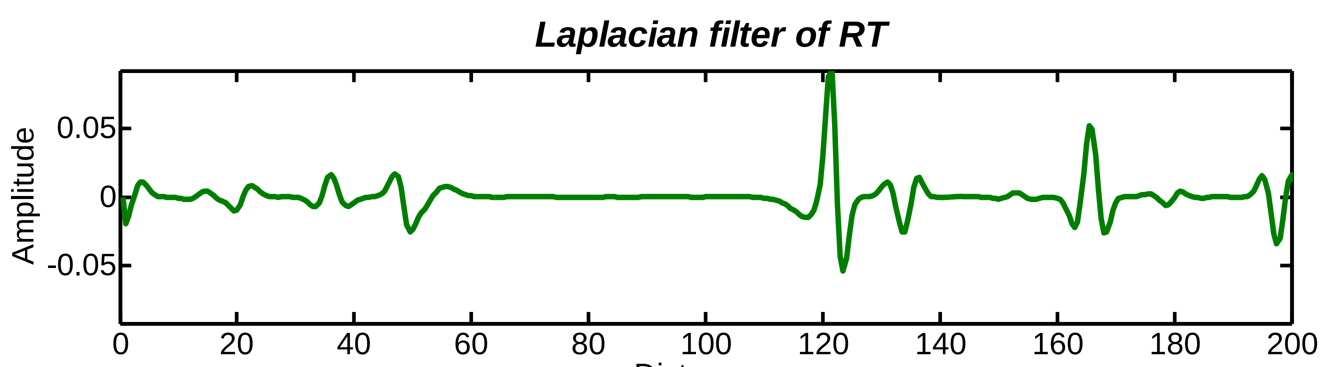
Reflectivity: FM with surf. DC



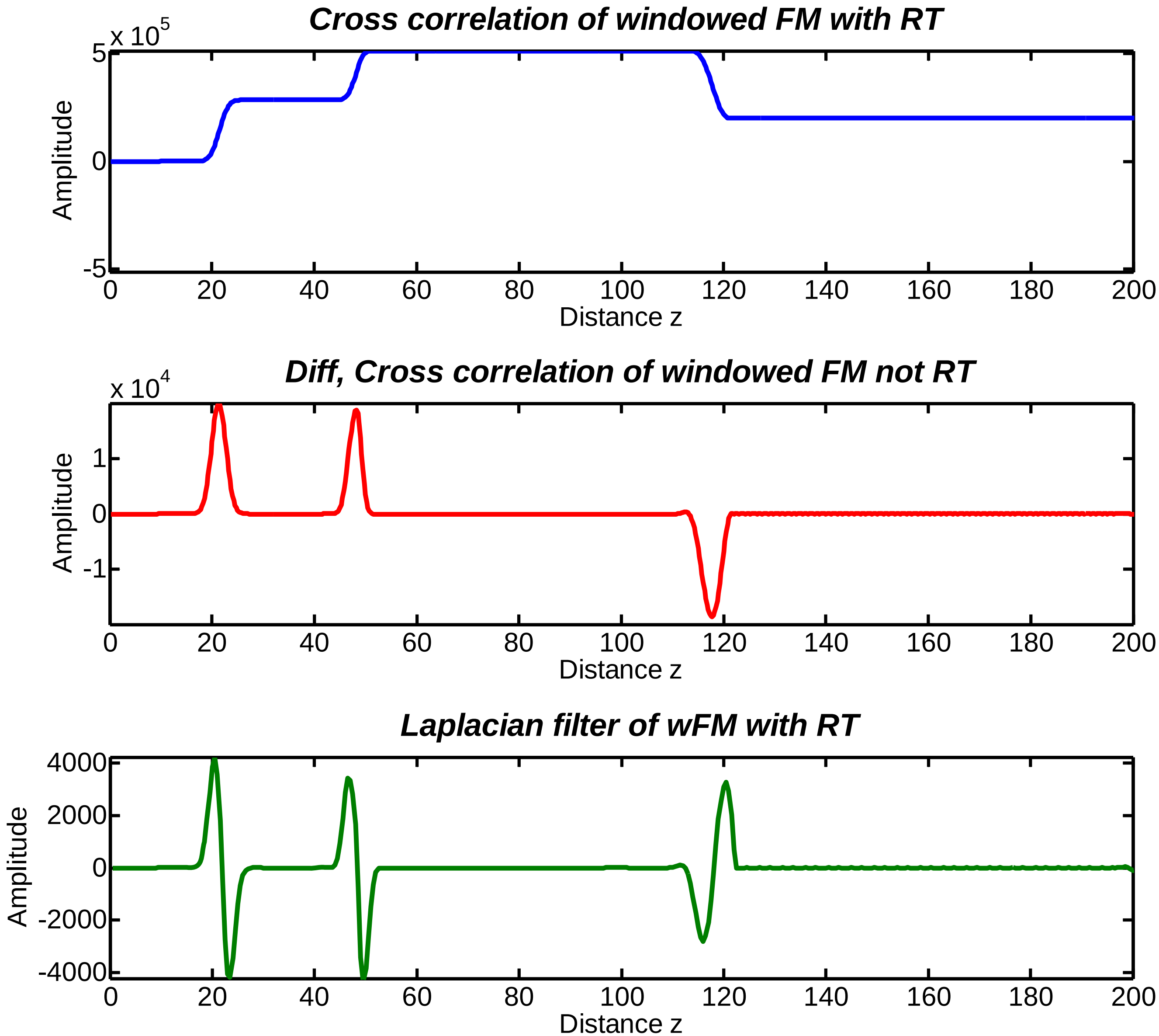
Reflectivity: FM with Phase-shift



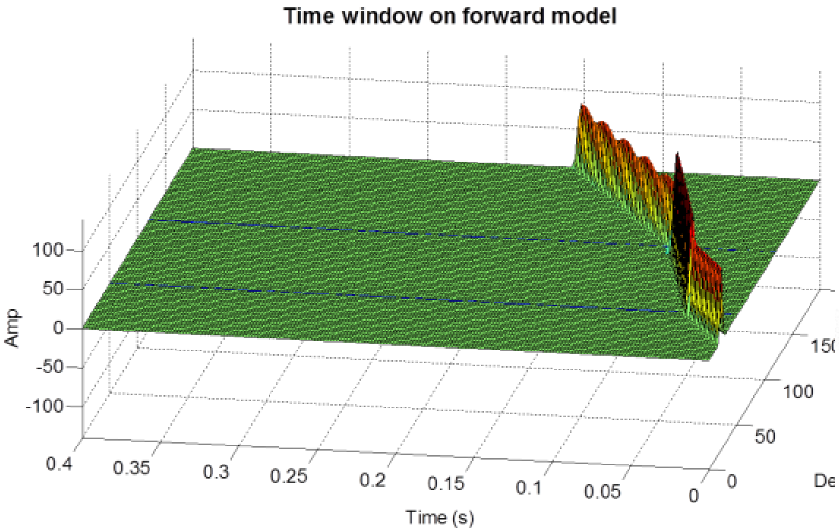


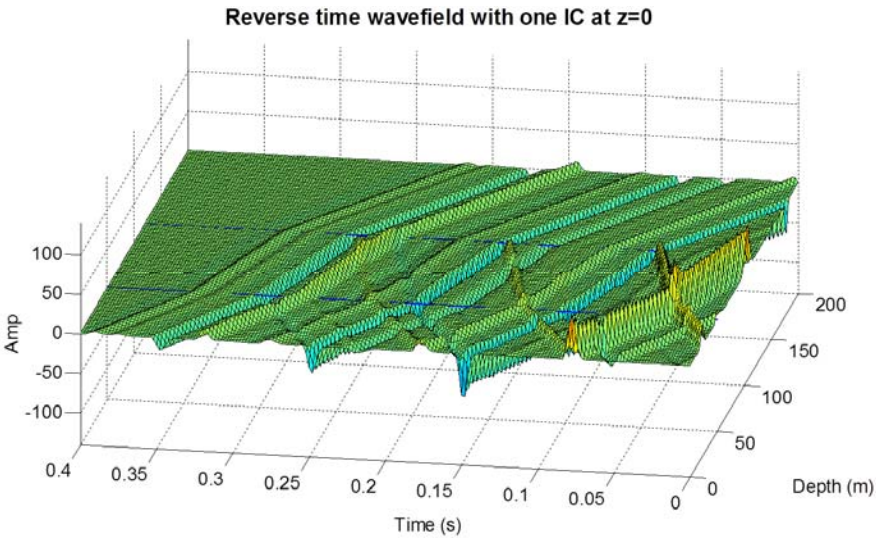


Reflectivity: windowed FM with RT



Reflectivity: windowed FM with RT





Maybe both with one-way propagation

Observations for Imm. Cond.

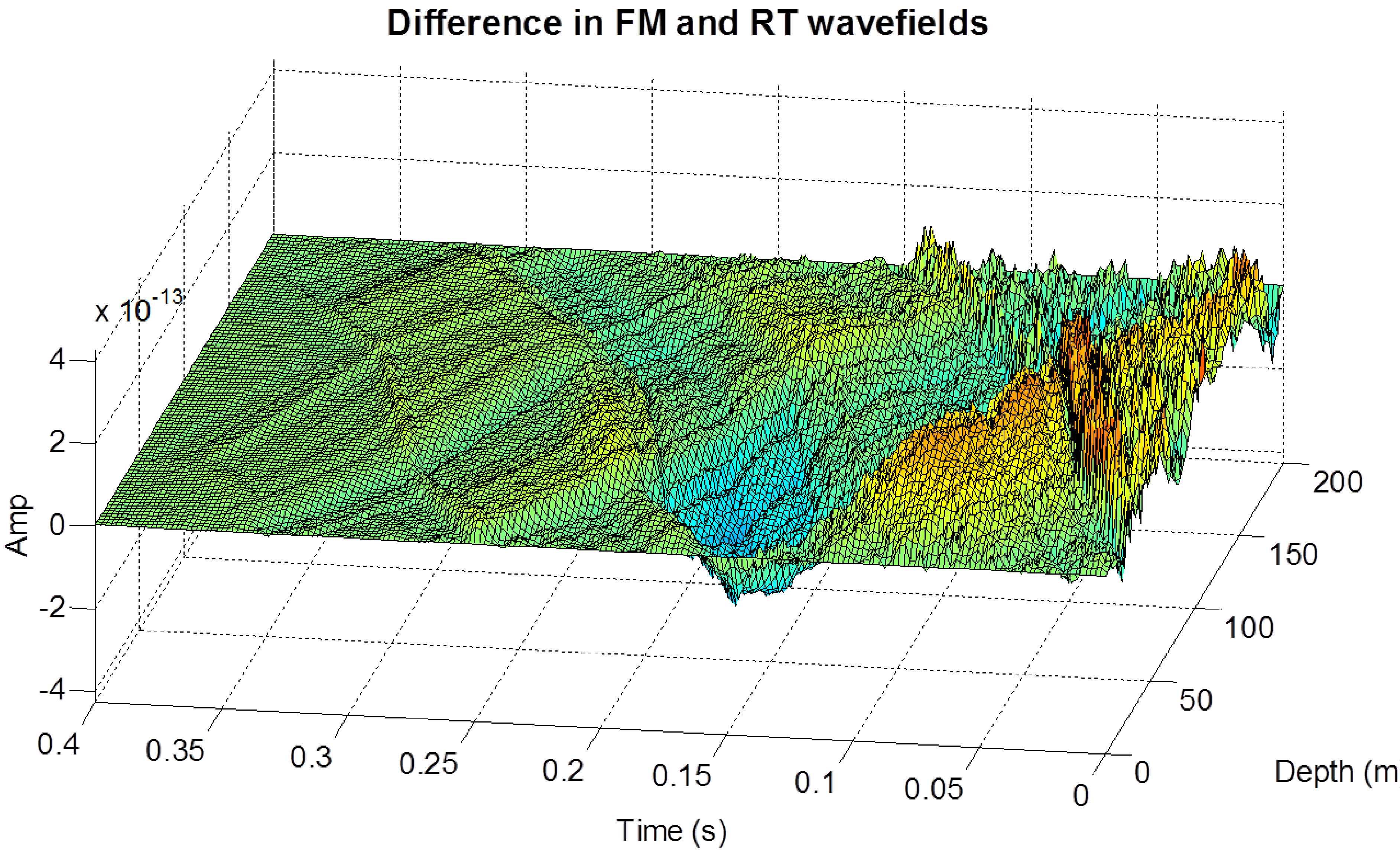
- Corresponding movement of multiples causes constant value.
- Surface IC establishes one-way motion
- Reverse time not necessary.
  - Only one lag in CC.
- One-way is preferable.
- Downward continuation good.
  - Allows true cross-correlation.
- Laplacian filter very poor. [1 -2   1]
- Windowing the forward model may be of value.

Conclusions

- ID modelling aids in choosing an algorithm.
- Only part of the wave field is recovered.
- Multiple energy aligns to cause DC.
- Any migration is OK.
- Windowing the forward model is of value.
- Alternate algorithms to RT should be considered.

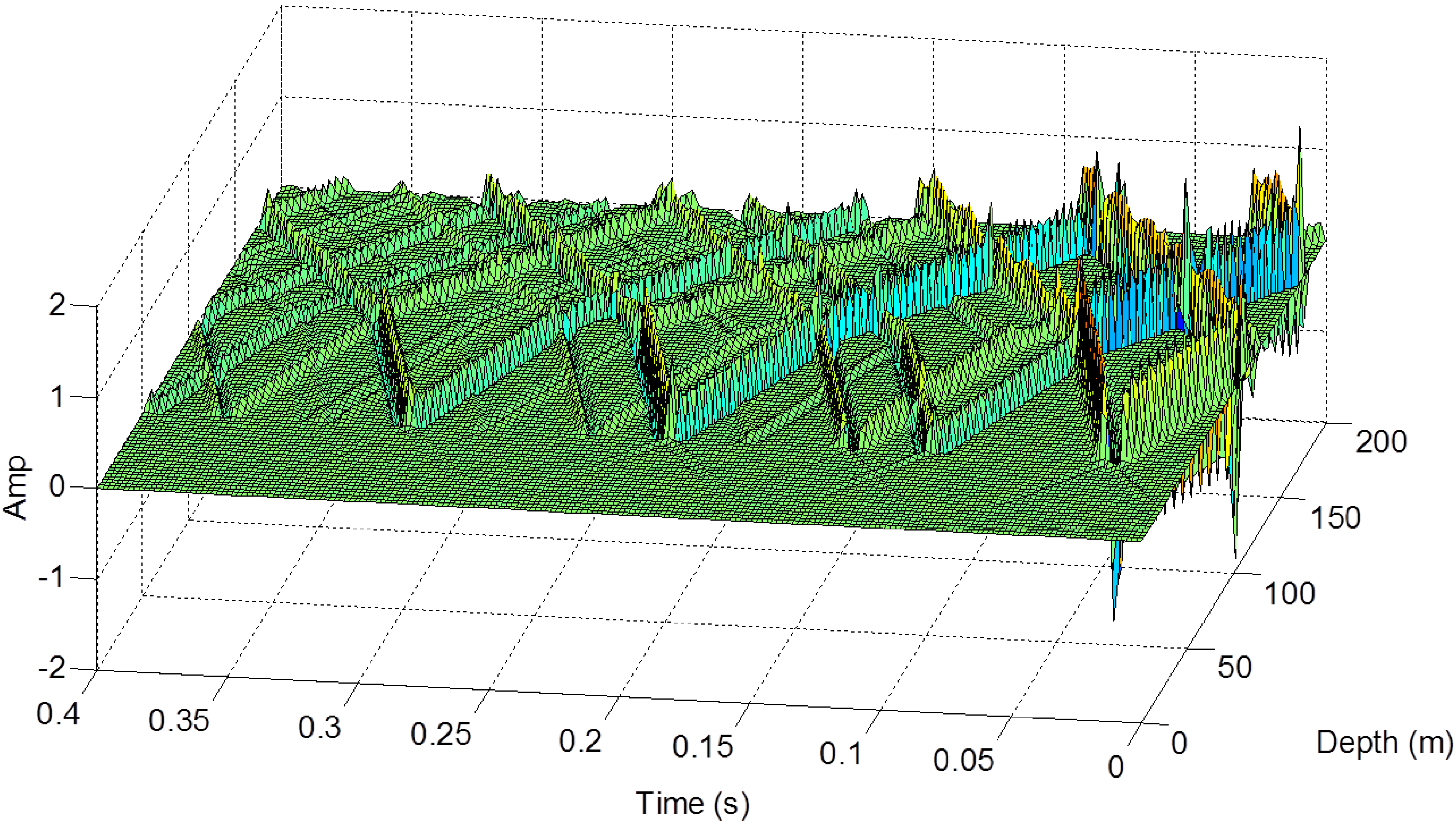
Acknowledgements


END



Difference in FM and RT wavefields
x 10^-13
4
2
0
-2
-4
Amp
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Time (s)
200
150
100
50
0
Depth (m)

Percent difference in FM and DC wavefields



Bipolar wavelet