On the Role of Deconvolution Imaging Condition in Full Waveform Inversion

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CREWES, University of Calgary

Motivation

- ❑ **CREWES is moving towards pratical application of FWI in recent years (Margrave et al. 2013).**
- ❑ **Full Waveform Inversion (FWI) and Standard Inversion Methodology (SM) (Margrave et al. 2012).**
- ❑ **Inverse Hessian and Illumination Compensation.**
- ❑ **Gradient with Illumination Compensation, Deconvolution Imaging Condition and Reflectivity.**
- ❑ **Impedance Perturbation Estimation**
- ❑ **Well Control**
- ❑ **Iterative Modeling and Migration and Inversion Method (Margrave et al. 2012).**

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Deconvolution Imaging Condition

$$I(\mathbf{r}) = \int_{\omega}^{\omega_N} d\omega \frac{U(\mathbf{r},\omega)}{D(\mathbf{r},\omega)}$$

The reflectivity is given as the ratio of the upgoing wavefields and downgoing wavefields.

$$I(\mathbf{r}) = \int_{\omega}^{\omega_N} d\omega \frac{D^*(\mathbf{r},\omega)U(\mathbf{r},\omega)}{D^*(\mathbf{r},\omega)D(\mathbf{r},\omega) + \lambda A_{max}}$$

The complex conjugate of the downgoing wavefields D^* is always multiplied by in the denominator and numerator to make the imaging condition stable.

Cross-correlation Imaging Condition

$$
\begin{aligned}
I(\mathbf{r}) &\simeq \frac{1}{A_D^2} \int_{\omega}^{\omega_N} d\omega D^*(\mathbf{r},\omega) U(\mathbf{r},\omega) \\
&\simeq \int_{\omega}^{\omega_N} d\omega D^*(\mathbf{r},\omega) U(\mathbf{r},\omega)
\end{aligned}
$$

The auto-correlation of the downgoing wavefields can be taken out from the integration.

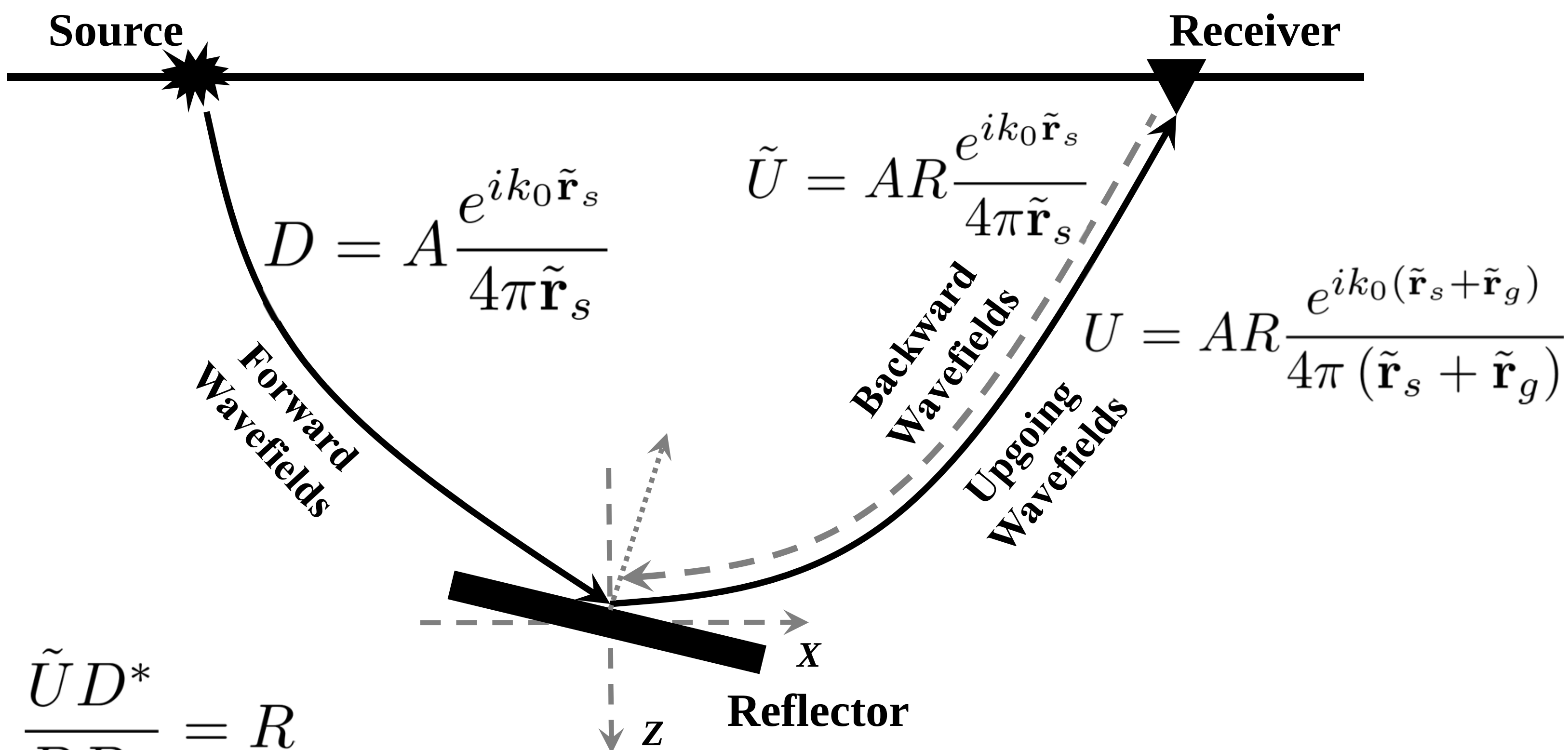
$$
I_{cross}(\mathbf{r}) = \sum_{\mathbf{r}_s} \int d\omega \Re\{\omega^2 \mathcal{F}_s(\omega) G(\mathbf{r},\mathbf{r}_s,\omega) G(\mathbf{r}_g,\mathbf{r},\omega) \psi^*(\omega)\}
$$

Crosscorrelation imaging condition

$$
I_{dec}(\mathbf{r}) = \frac{\sum_{\mathbf{r}_s} \int d\omega \Re\{\omega^2 \mathcal{F}_s(\omega) G(\mathbf{r},\mathbf{r}_s,\omega) G(\mathbf{r}_g,\mathbf{r},\omega) \psi^*(\omega)\}}{\sum_{\mathbf{r}_s} \int d\omega \Re\{\omega^4 |\mathcal{F}_s(\omega)|^2 |G(\mathbf{r},\mathbf{r}_s,\omega)|^2 \psi^*(\omega)\} + \lambda A_{max}}
$$

Deconvolution imaging condition

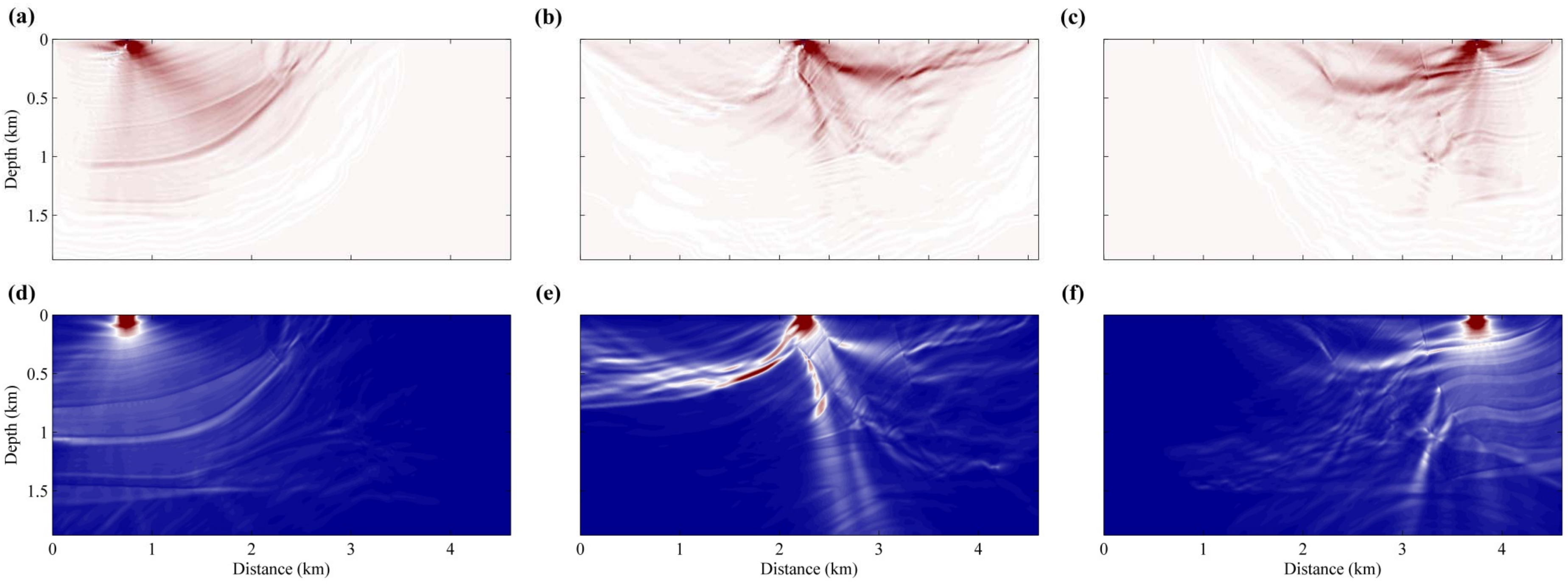
Analytic Solution of the Imaging Conditions



$$I_{dec} = \frac{\tilde{U}D^*}{DD^*} = R$$

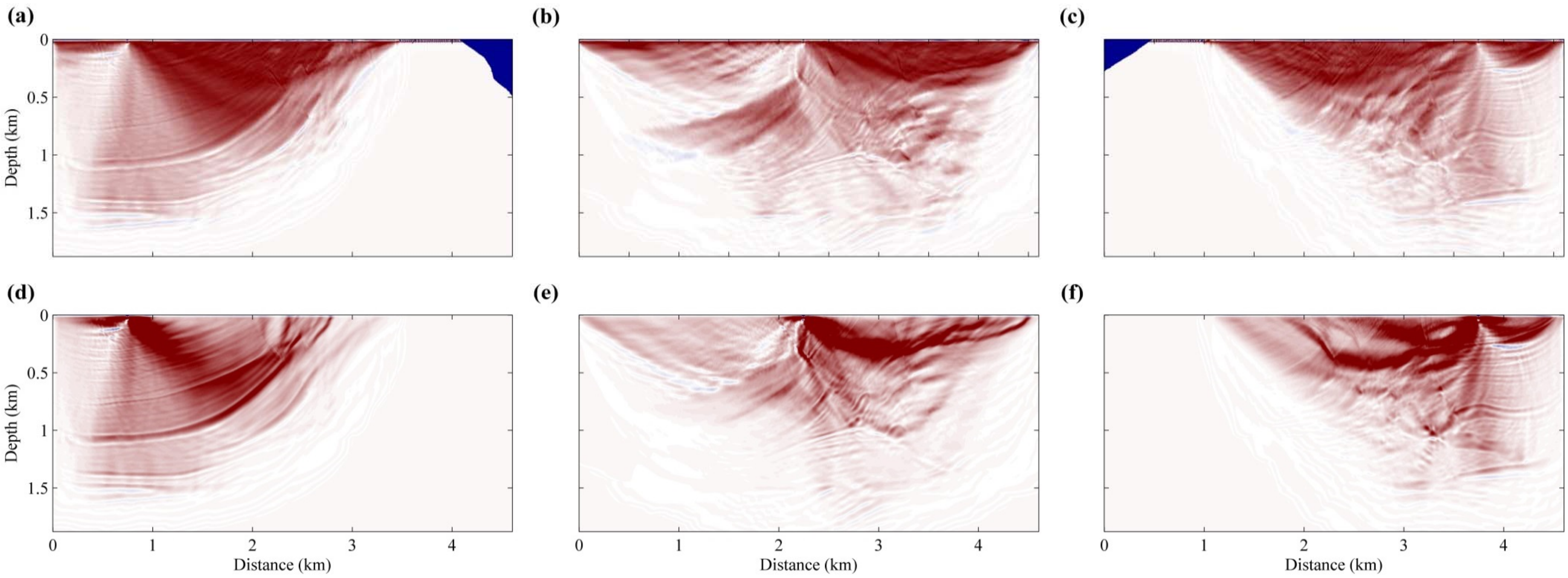
$$I_{cross} = D\tilde{U}^* = \frac{1}{\tilde{\mathbf{r}}_s^2}\frac{A^2}{(4\pi)^2}R$$

Numerical Example



$0.75km$ $2.25km$ $3.75km$

Numerical Example



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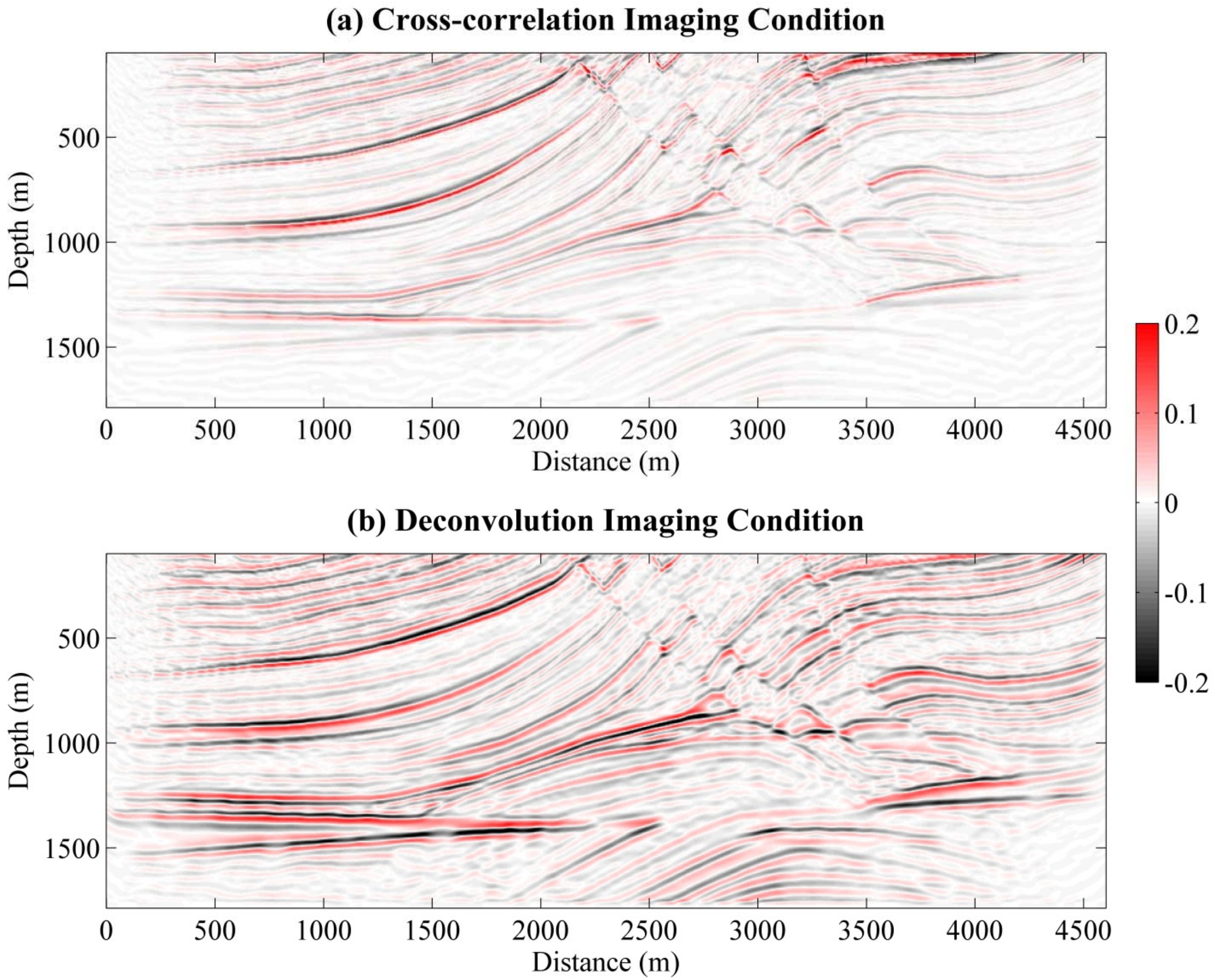

A Review of General Principle for FWI

$$\phi = \frac{1}{2}\|\mathbf{d} - \mathbf{B}\mathbf{u}\|_2$$

Misfit function in matrix form

$$g = \frac{\partial \phi}{\partial m} = \mathbf{J}^T \Delta d^*$$

Gradient

$$H_a = \mathbf{J}^T \mathbf{J}$$

Approximate Hessian

A Review of General Principle for FWI

$$\delta m = -(\mathbf{J}^T\mathbf{J} + \lambda I)^{-1}(\mathbf{J}^T \Delta d^*) \simeq \delta m = -\frac{\mathbf{J}^T \Delta d^*}{\mathbf{J}^T\mathbf{J} + \lambda I}$$

Model Perturbation

$$\mathbf{B}(\mathbf{r}, \omega)\mathbf{u}(\mathbf{r}, \omega) = f(\mathbf{r}, \omega)$$

Wave Equation

A Review of General Principle for FWI

$$\delta m = -(\mathbf{J}^T\mathbf{J} + \lambda I)^{-1}(\mathbf{J}^T \Delta d^*) \simeq \delta m = -\frac{\mathbf{J}^T \Delta d^*}{\mathbf{J}^T\mathbf{J} + \lambda I}$$

Model Perturbation

$$\left.\mathbf{B}\frac{\partial \mathbf{u}}{\partial m} = -\frac{\partial \mathbf{B}}{\partial m}\mathbf{u}\right\} \text{SECONDARY SOURCE}$$

Scattered Source

A Review of General Principle for FWI

$$\delta m = -(\mathbf{J}^T\mathbf{J} + \lambda I)^{-1}(\mathbf{J}^T \Delta d^*) \simeq \delta m = -\frac{\mathbf{J}^T \Delta d^*}{\mathbf{J}^T\mathbf{J} + \lambda I}$$

Model Perturbation

$$\mathbf{J} = \frac{\partial \mathbf{u}}{\partial m} = -\mathbf{B}^{-1}\frac{\partial \mathbf{B}}{\partial m}\mathbf{u}$$

Jacobian Matrix

A Review of General Principle for FWI

$$g = -\sum_{\omega,\mathbf{r}_s} \omega^2 \Re\left\{\mathbf{u}^T \otimes \left(\mathbf{B}^{-1}\right)^T \Delta d^*\right\}$$

Gradient

$$H_a = \left(\mathbf{B}^{-1}\frac{\partial \mathbf{B}}{\partial m}\mathbf{u}\right)^* \left(\mathbf{B}^{-1}\frac{\partial \mathbf{B}}{\partial m}\mathbf{u}\right)$$

Approximate Hessian

A Review of General Principle for FWI

$$g = -\sum_{\omega,\mathbf{r}_s} \omega^2 \Re\left\{\mathbf{u}^T \otimes \left(\mathbf{B}^{-1}\right)^T \Delta d^*\right\}$$

Gradient

$$H_a = \omega^4 \Re\left(\mathbf{u}^* \mathbf{u}\left(\mathbf{B}^{-1}\right)^* \left(\mathbf{B}^{-1}\right)\right)$$

Approximate Hessian

A Review of General Principle for FWI

$$g = -\sum_{\omega,\mathbf{r}_s} \omega^2 \Re \left\{ \mathbf{u}^T \otimes \left(\mathbf{B}^{-1}\right)^T \Delta d^* \right\}$$

Gradient

$$H_a = \omega^4 \Re \left(\mathbf{u}^* \mathbf{u} \left(\mathbf{B}^{-1}\right)^* \left(\mathbf{B}^{-1}\right) \right)$$

Approximate Hessian

$$H_{pseudo} = f^*_{virtual} f_{virtual} = \left(\frac{\partial \mathbf{B}}{\partial m} \mathbf{u} \right)^* \left(\frac{\partial \mathbf{B}}{\partial m} \mathbf{u} \right) = \omega^4 \Re \left(\mathbf{u}^* \mathbf{u} \right)$$

Pseudo Hessian

A Review of General Principle for FWI

$$\delta m = -\Re\left\{ \underbrace{\cancel{\frac{1}{\left(\mathbf{B}^{-1}\right)^{*}\left(\mathbf{B}^{-1}\right)}}}_{\text{BACK PROPAGATION OPERATOR}} \frac{\overbrace{\sum_{\omega,\mathbf{r}_s} \omega^{2}\Re\left\{\mathbf{u}^{T}\otimes\left(\mathbf{B}^{-1}\right)^{T}\Delta d^{*}\right\}}^{\text{CROSS-CORRELATION BASED GRADIENT}}}{\underbrace{\omega^{4}\mathbf{u}^{*}\mathbf{u}}_{\text{PSEUDO HESSIAN}}} \right\}$$

Model Perturbation By Gary et.al (2011)

A Review of General Principle for FWI

$$\delta m \simeq -\Re\left\{\overbrace{\frac{\sum_{\omega,\mathbf{r}_s}\omega^2\Re\left\{\mathbf{u}^T\otimes\left(\mathbf{B}^{-1}\right)^T\Delta d^*\right\}}{\underbrace{\mathbf{Diag}\left(\omega^4\mathbf{u}^*\mathbf{u}\right)}_{\text{SOURCE ILLUMINATION}}}}^{\text{DECONVOLUTION IMAGING CONDITION}}\right\}$$

Model Perturbation

A Review of General Principle for FWI

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$$g = -\sum_{\omega,\mathbf{r}_s} \omega^2 \Re\left\{ A\frac{e^{ik_0\tilde{\mathbf{r}}_s}}{4\pi\tilde{\mathbf{r}}_s} \otimes AR\frac{e^{-ik_0\tilde{\mathbf{r}}_s}}{4\pi\tilde{\mathbf{r}}_s} \right\} = -\sum_{\omega,\mathbf{r}_s} \omega^2 \Re\left\{ \frac{1}{\tilde{\mathbf{r}}_s^2}\frac{A^2}{16\pi^2}R \right\}$$

ENERGY DECAY

Analytic Expression of the Gradient

$$H_{pseudo} = \Re\left\{\omega^4 \mathbf{u}^*(\mathbf{r}'_s,\omega)\mathbf{u}(\mathbf{r}''_s,\omega)\right\} = \Re\left\{ \omega^4 \left(A\frac{e^{-ik_0\tilde{\mathbf{r}}'_s}}{4\pi\tilde{\mathbf{r}}'_s} \right) \left(A\frac{e^{ik_0\tilde{\mathbf{r}}''_s}}{4\pi\tilde{\mathbf{r}}''_s} \right) \right\}$$

Analytic Expression of Pseudo Hessian

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A Review of General Principle for FWI

$$g = -\sum_{\omega,\mathbf{r}_s} \omega^2 \Re\left\{ A\frac{e^{ik_0\tilde{\mathbf{r}}_s}}{4\pi\tilde{\mathbf{r}}_s} \otimes AR\frac{e^{-ik_0\tilde{\mathbf{r}}_s}}{4\pi\tilde{\mathbf{r}}_s} \right\} = -\sum_{\omega,\mathbf{r}_s} \omega^2 \Re\left\{ \frac{1}{\tilde{\mathbf{r}}_s^2} \frac{A^2}{16\pi^2} R \right\}$$

Analytic Expression of the Gradient

ENERGY DECAY

$$\mathbf{Diag}\left(H_{pseudo}\right) = \Re\left\{\omega^4 \mathbf{u}^*(\mathbf{r}_s,\omega)\mathbf{u}(\mathbf{r}_s,\omega)\right\} = \Re\left\{\omega^4 \left(A\frac{e^{-ik_0\tilde{\mathbf{r}}_s}}{4\pi\tilde{\mathbf{r}}_s}\right)\left(A\frac{e^{ik_0\tilde{\mathbf{r}}_s}}{4\pi\tilde{\mathbf{r}}_s}\right)\right\}$$

Analytic Expression of Diagonal Pseudo Hessian

A Review of General Principle for FWI

$$\delta m \simeq \Re\left\{\frac{-\sum_{\omega,\mathbf{r}_s}\omega^2\frac{1}{\tilde{\mathbf{r}}_s^2}\frac{A^2}{16\pi^2}R}{\omega^4\frac{A^2}{16\pi^2\tilde{\mathbf{r}}_s^2}}\right\} = -\Re\left\{\sum_{\omega}\omega^2 R\right\}$$

Analytic Expression of Model Perturbation **REFLECTIVITY**

Impedance Perturbation Estimation

$$R_n = \frac{I_{n+1} - I_n}{I_{n+1} + I_n} = \frac{\Delta I}{I_{n+1} + I_n}$$

Recall the relationship between the reflectivity and impedance at normal incidence

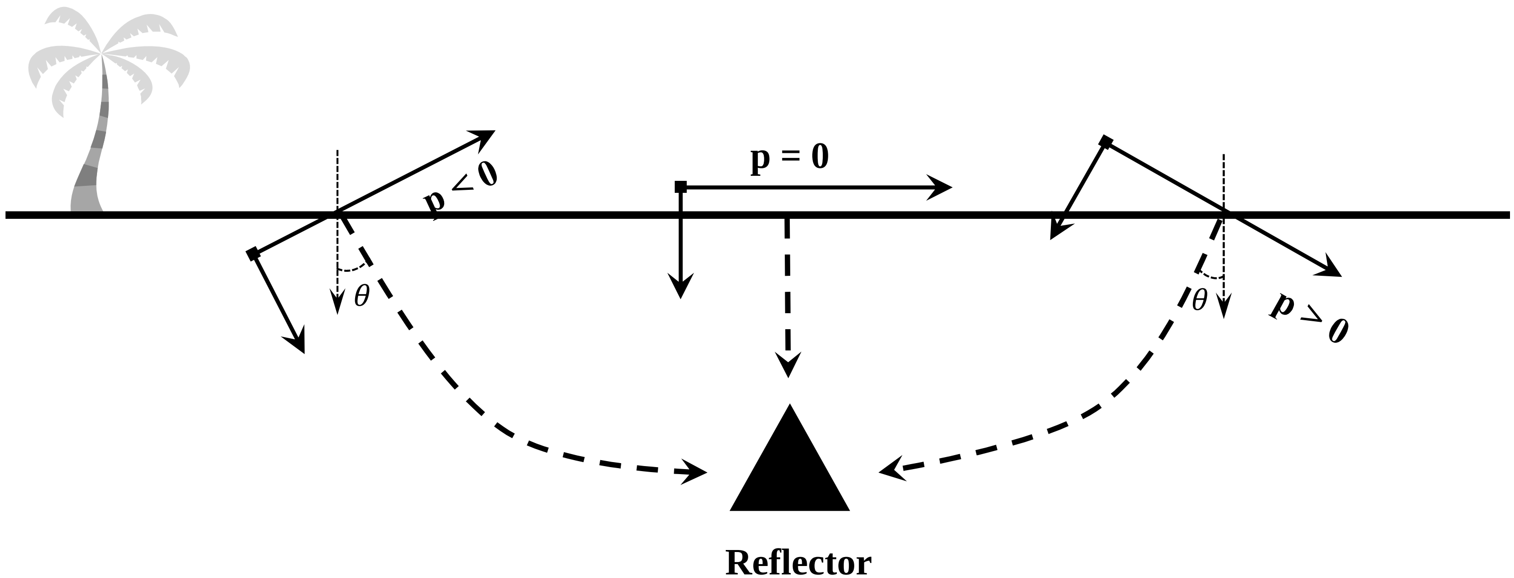
$$\Delta I \simeq 2 I_n R_n$$

Impedance Perturbation or Impedance Imaging Condition

$$I_{k+1} = I_k + \Delta I_k = I_k + 2 I_k R_k$$

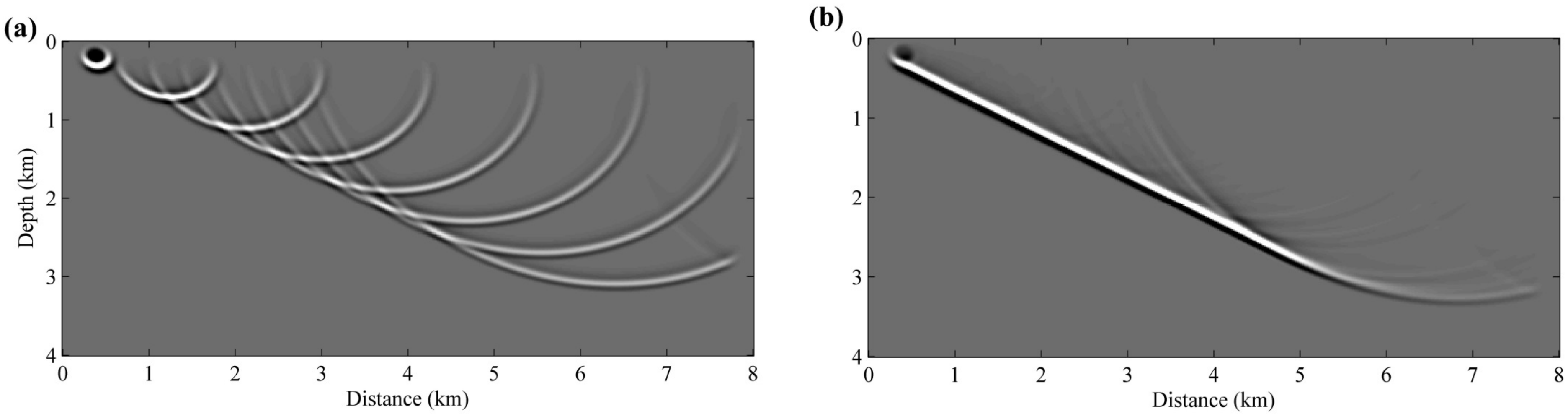
Impedance can be updated iteratively

Phase Encoded Gradient and Phase Encoded Pseudo-Hessian



The slant gradient with different ray parameters are responsible to update the subsurface layers with different steep angles

Phase Encoded Gradient and Phase Encoded Pseudo-Hessian



Phase Encoded Source Wavefields and Plane-wave Source Wavefields
Reproduced from Shan et al. (2006)

Phase Encoded Gradient and Phase Encoded Pseudo-Hessian

$$\tilde{g}(p_i^g, \omega) = -\int d\omega \Re \left\{ \omega^2 f_s(\omega) G(\mathbf{r}, \mathbf{r}_s, \omega) G(\mathbf{r}_g, \mathbf{r}, \omega) e^{i\omega p_i^g (x_s - x_g)} \delta P^* \right\}$$

Phase Encoded Gradient

$$\tilde{H}_{pseudo}\left(\mathbf{p}^H, \omega\right) = \sum_{\mathbf{r}_s} \sum_{i=1}^{N_p^H} \int d\omega \Re \left\{ \omega^4 G(\mathbf{r}'', \mathbf{r}'_s, \omega) G^*(\mathbf{r}', \mathbf{r}_s, \omega) e^{i\omega (p_i^H + \varepsilon \triangle p)(x'_s - x_s)} \right\}$$

Phase Encoded Pseudo-Hessian

Phase Encoded Gradient and Phase Encoded Pseudo-Hessian

$$\tilde{g}(p_i^g, \omega) = -\int d\omega \Re \left\{ \omega^2 f_s(\omega) G(\mathbf{r}, \mathbf{r}_s, \omega) G(\mathbf{r}_g, \mathbf{r}, \omega) e^{i\omega p_i^g (x_s - x_g)} \delta P^* \right\}$$

Phase Encoded Gradient

$$\mathbf{Diag}\left(\tilde{H}_{pseudo} \left(\mathbf{p}^H, \omega \right) \right) = \sum_{\mathbf{r}_s} \sum_{i=1}^{N_p^H} \int d\omega \Re \left\{ \omega^4 G(\mathbf{r}, \mathbf{r}_s', \omega) G^*(\mathbf{r}, \mathbf{r}_s, \omega) e^{i\omega (p_i^H + \varepsilon \triangle p)(x_s' - x_s)} \right\}$$

Phase Encoded Source Illumination or Phase Encoded Diagonal Pseudo-Hessian

Phase Encoded Gradient and Phase Encoded Pseudo-Hessian

$$\tilde{R} = \Re\left\{ \frac{\tilde{g}\left(p_i^g, \omega\right)}{\mathbf{Diag}\left(\tilde{H}_{pseudo}\left(\mathbf{p}^H, \omega\right)\right)/N_p^H + \lambda A_{max}} \right\}$$

Reflectivity Approximation Estimation or Phase Encoded Deconvolution Imaging Condition

$$\Delta I_k = 2 I_k \tilde{R}_k$$

Impedance Perturbation Estimation

Phase Encoded Gradient and Phase Encoded Pseudo-Hessian

$$\tilde{R} = \Re\left\{ \frac{\tilde{g}\left(p_i^g, \omega\right)}{\mathbf{Diag}\left(\tilde{H}_{pseudo}\left(\mathbf{p}^H, \omega\right)\right)/N_p^H + \lambda A_{max}} \right\}$$

Reflectivity Approximation Estimation or Phase Encoded Deconvolution Imaging Condition

$$I_{k+1} = I_k + \Delta I_k = I_k + \mu_k \left(2 I_k \tilde{R}_k\right)$$

Iterative Impedance Update

Iterative Modeling Migration and Inversion (IMMI)

As proposed by Gary et.al (2012), the estimation of the reflectivity using deconvolution imaging condition enables us to combine FWI with SM, which forms the *Iterative Modeling Migration and Inversion* (IMMI) method.

Table 1. Pseudo Code of IMMI Method

BEGIN $\leftarrow I_0$, initial model;

WHILE $\epsilon \leq \epsilon_{min}$ or $k \leq k_{max}$

Modeling

1. Identify the ray parameter $p^g_{i,k}$ for constructing the phase encoded gradient
2. Identify the frequency band $f^k = f_0 \rightarrow f_{max}$, $f_{interval}$, every n iterations
3. Generate the data residual δP and apply low-pass filtering $\delta \tilde{P} = \textbf{low\_pass}\left(\delta P, f^k\right)$

Migration

4. Create the phase encoded gradient $\tilde{g}_k\left(p^g_{i,k}, \omega\right)$
5. **FOR** $i = 1$ to N^H_p, every 1 or m iterations

   Construct the diagonal part of the phase encoded pseudo-Hessian: $\textbf{Diag}\left(\tilde{H}^k_{pseudo}\right)$

   END FOR

Inversion

6. Reflectivity Estimation with Well Control: **Phase Shift** and **Phase Rotation**
7. Impedance Perturbation Estimation:

$$\Delta I = 2 I_k \Re\left\{\frac{\tilde{g}_k\left(p^g_{i,k}, \omega\right)}{\textbf{Diag}\left(\tilde{H}^k_{pseudo}(\mathbf{p}^H, \omega)\right)/N^H_p + \lambda A_{max}}\right\}$$

8. Calculate the step length μ_k using the line search method
9. Update the impedance:

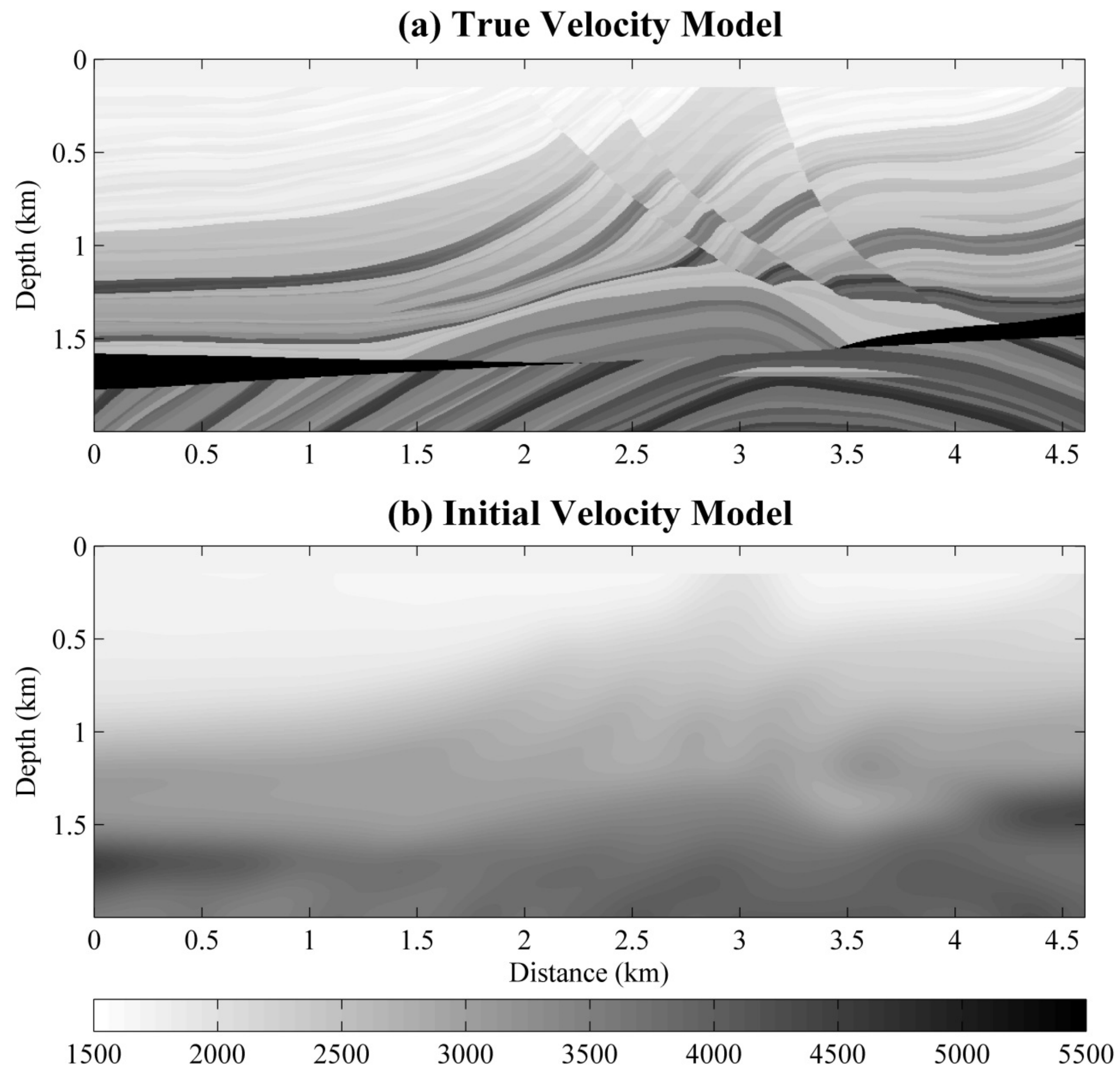
$$I_{k+1} = I_k + 2\mu_k I_k \Re\left\{\left(\textbf{Diag}\left(\tilde{H}^k_{pseudo}\left(\mathbf{p}^H, \omega\right)\right)/N^H_p + \lambda \tilde{A}_{max}\right)^{-1} \tilde{g}_k\left(p^g_{i,k}, \omega\right)\right\}$$

10. Calculate the relative least-squares error:

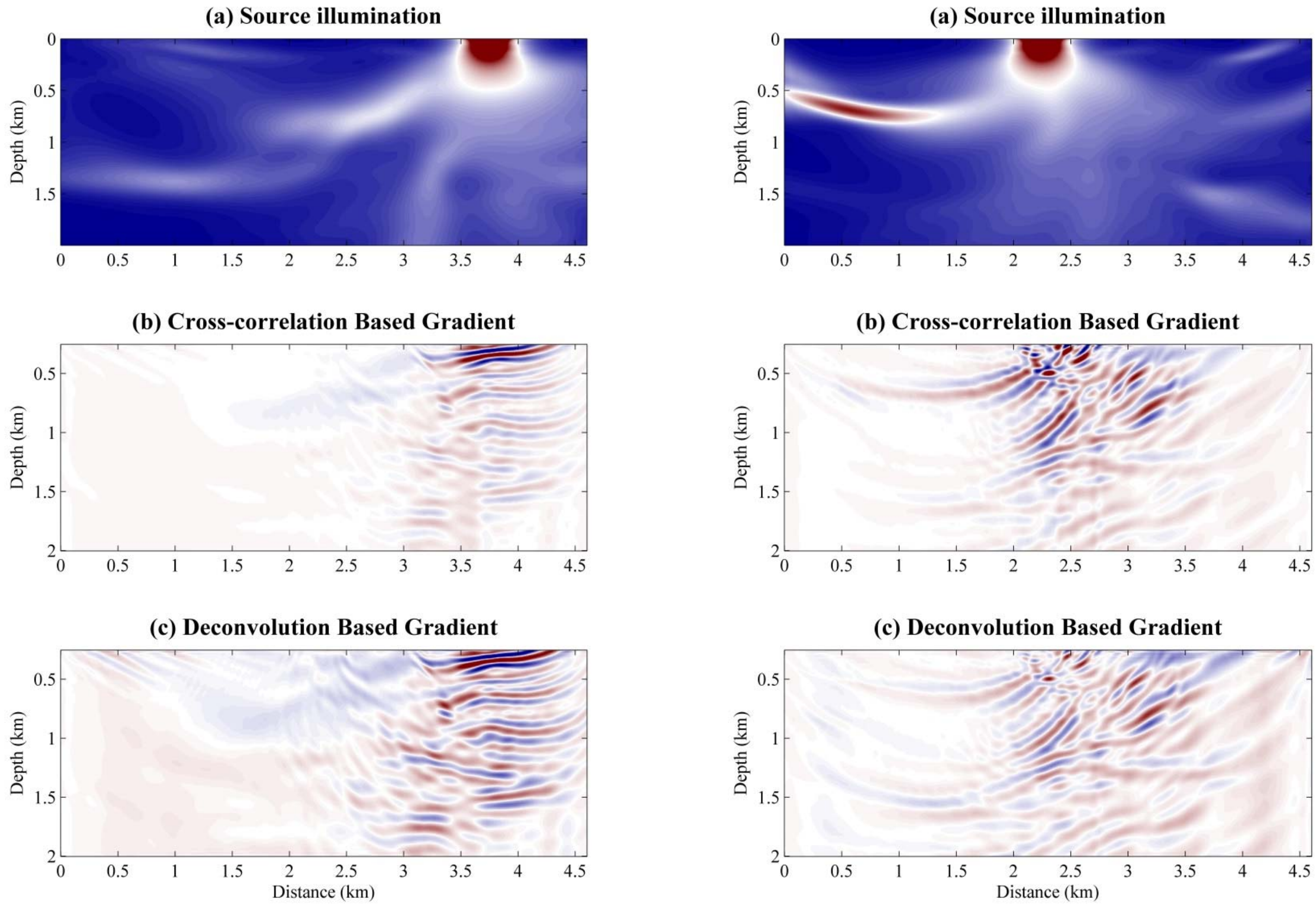
$$\epsilon = \frac{\|I_{k+1} - I_{true}\|_2}{\|I_{true}\|_2}$$

END WHILE

Numerical Experiment

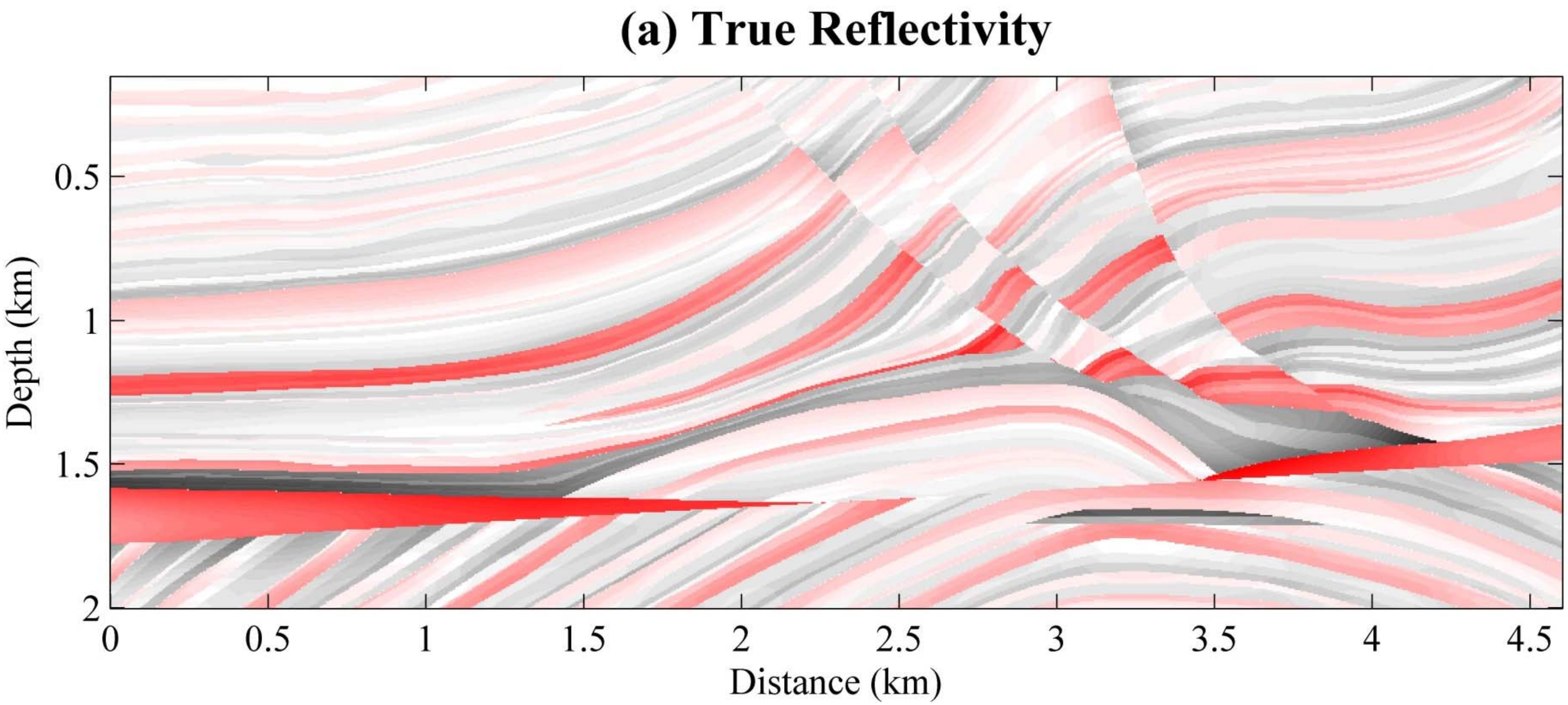


Numerical Experiment



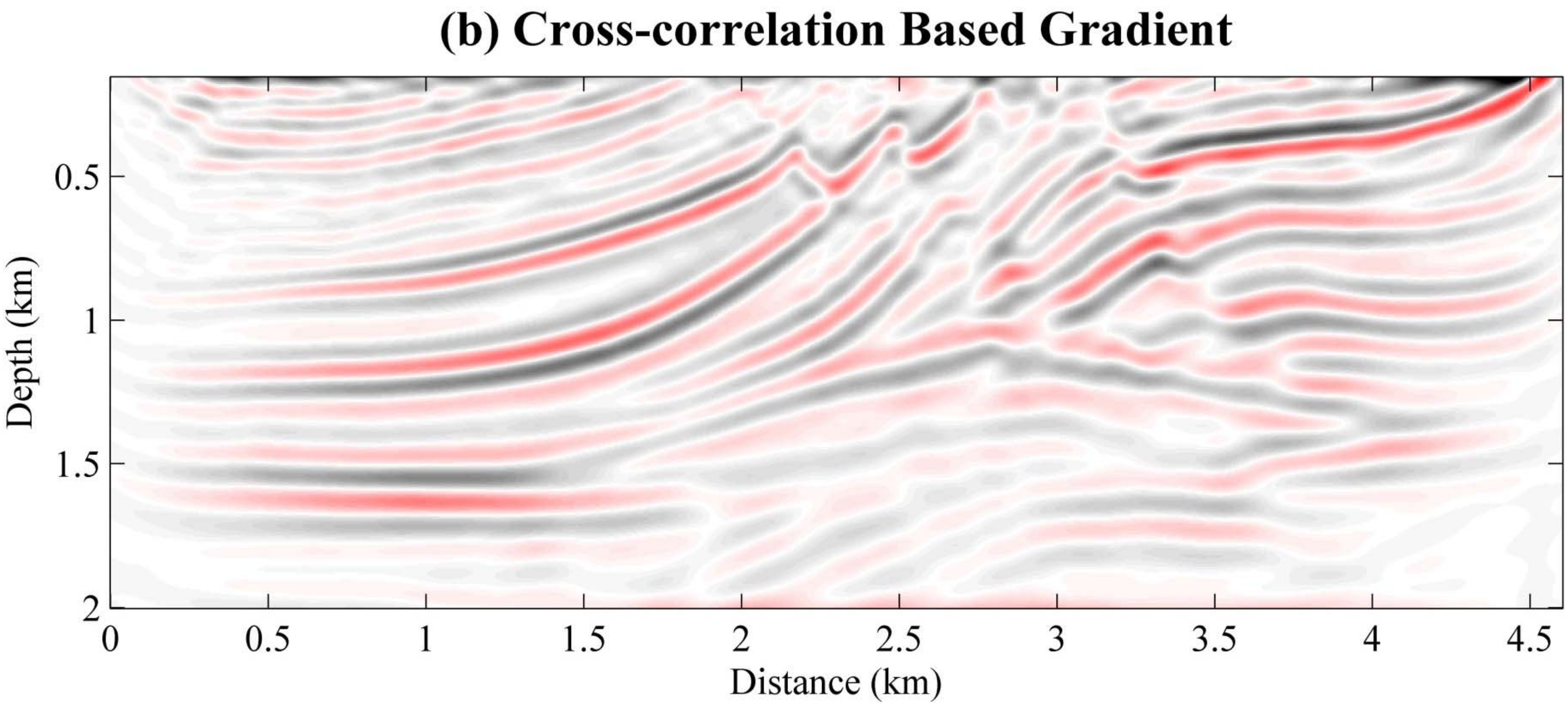
Gradient with Single Shot

Gradient Comparison



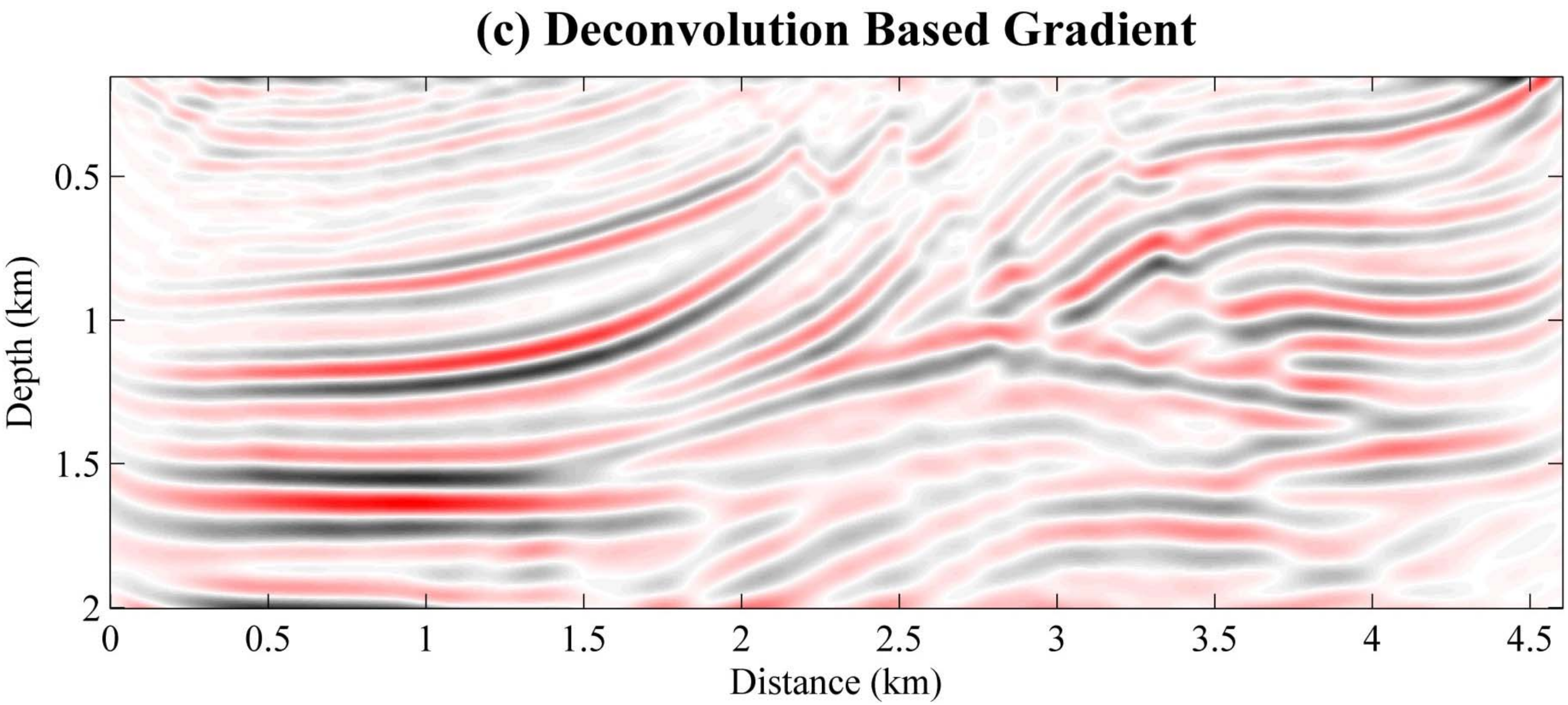
Normalized Amplitude

Gradient Comparison



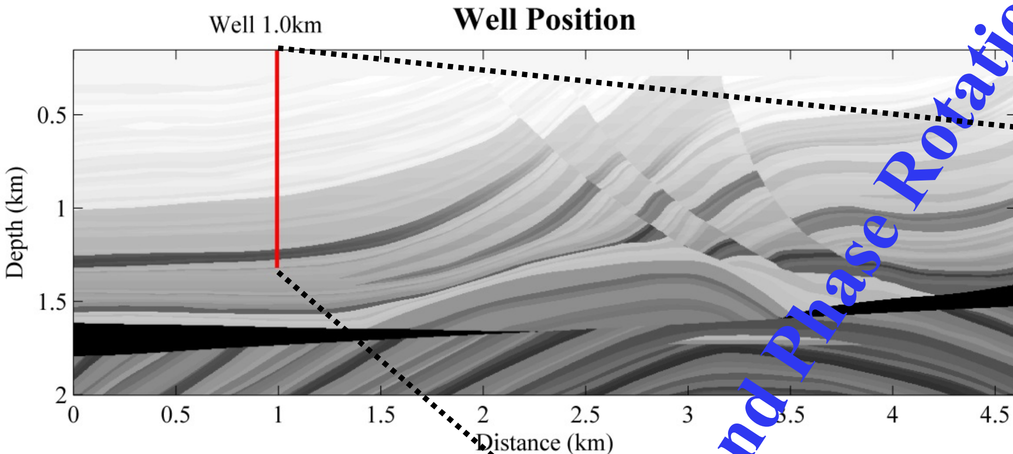
Normalized Amplitude

Gradient Comparison

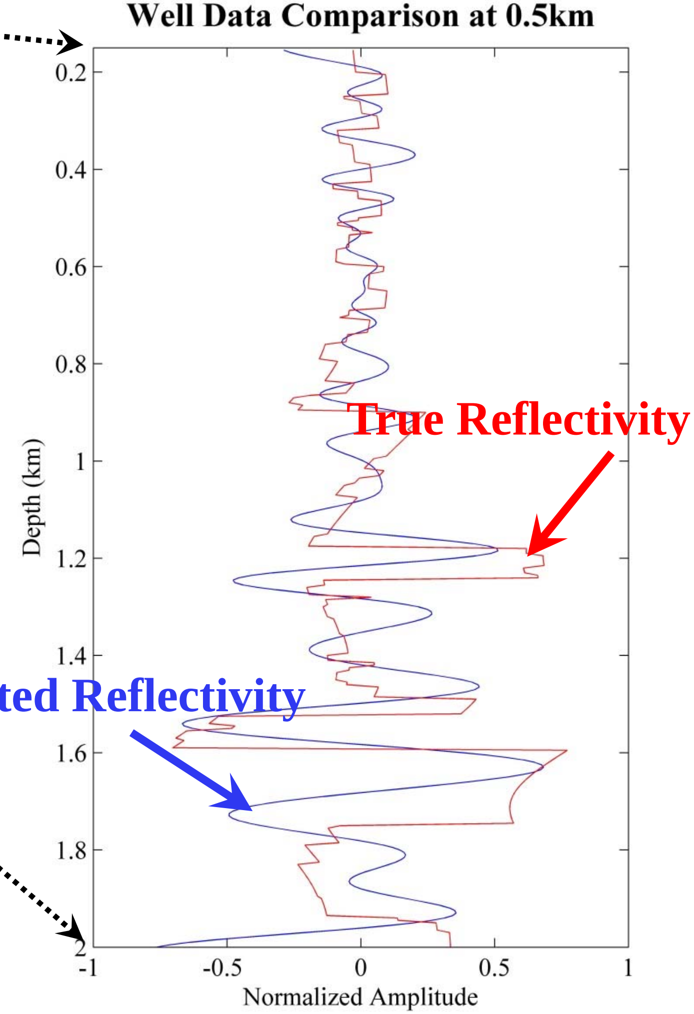


Normalized Amplitude

Well Control



Phase Shift and Phase Rotation



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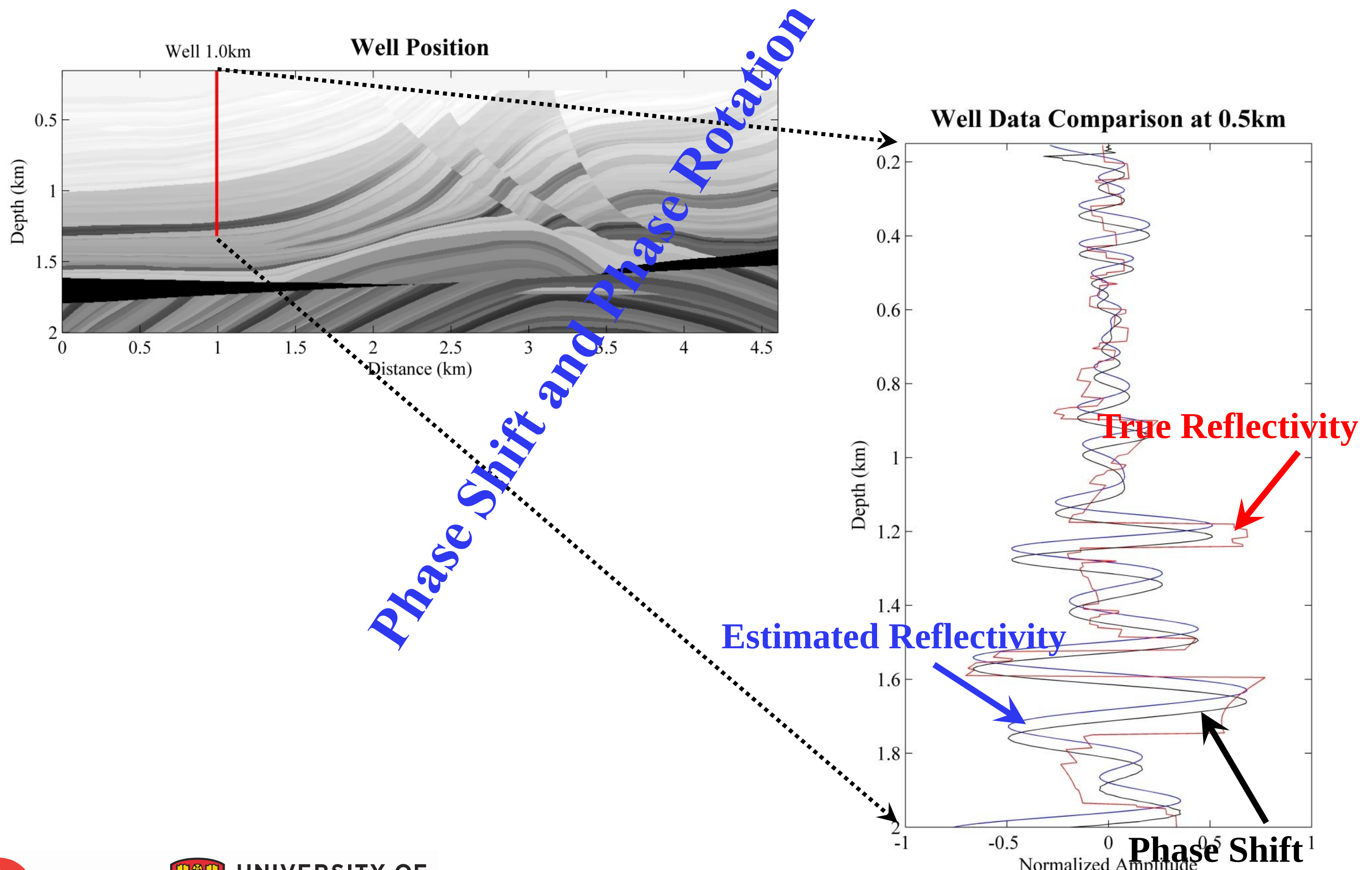
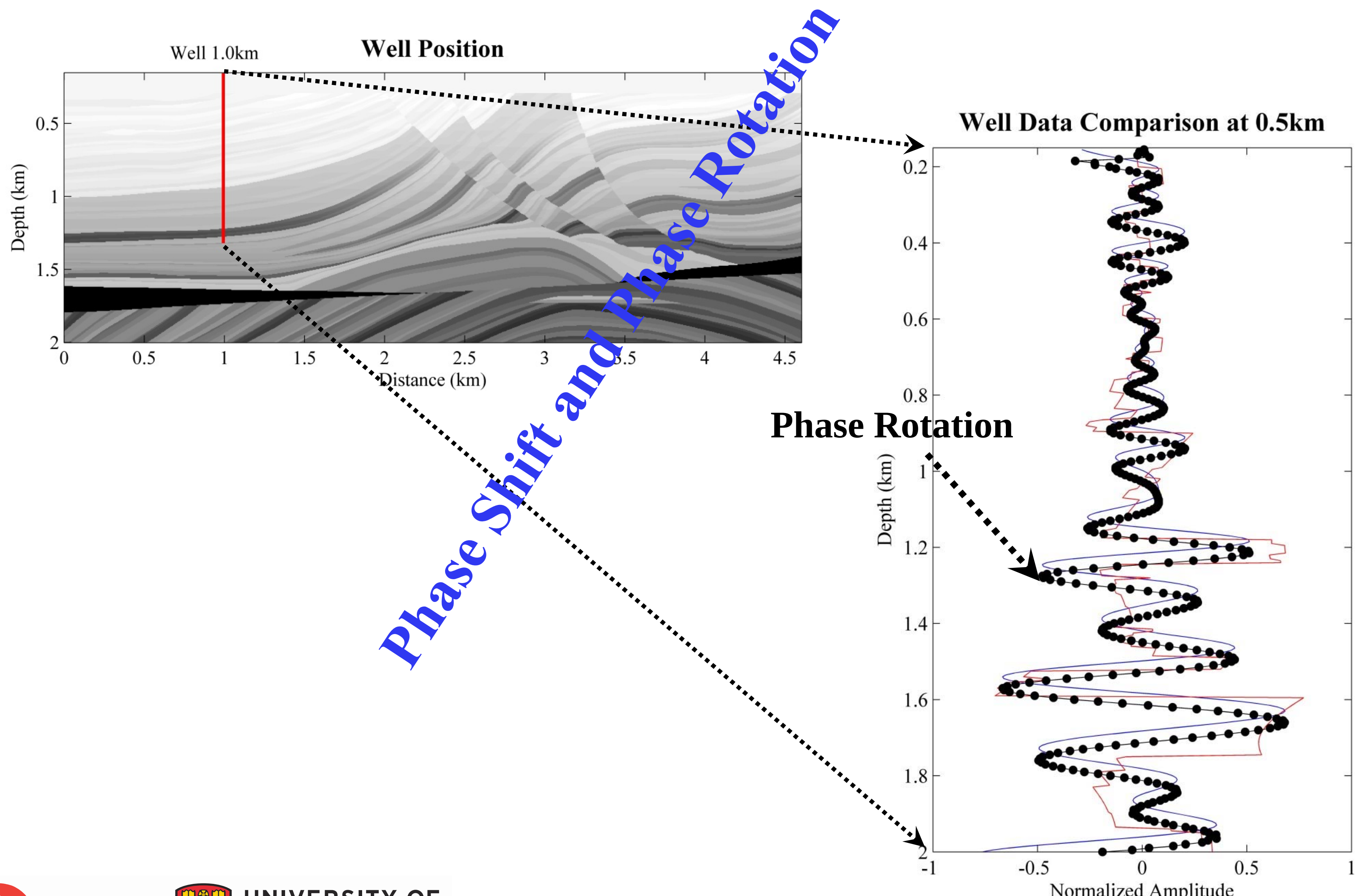
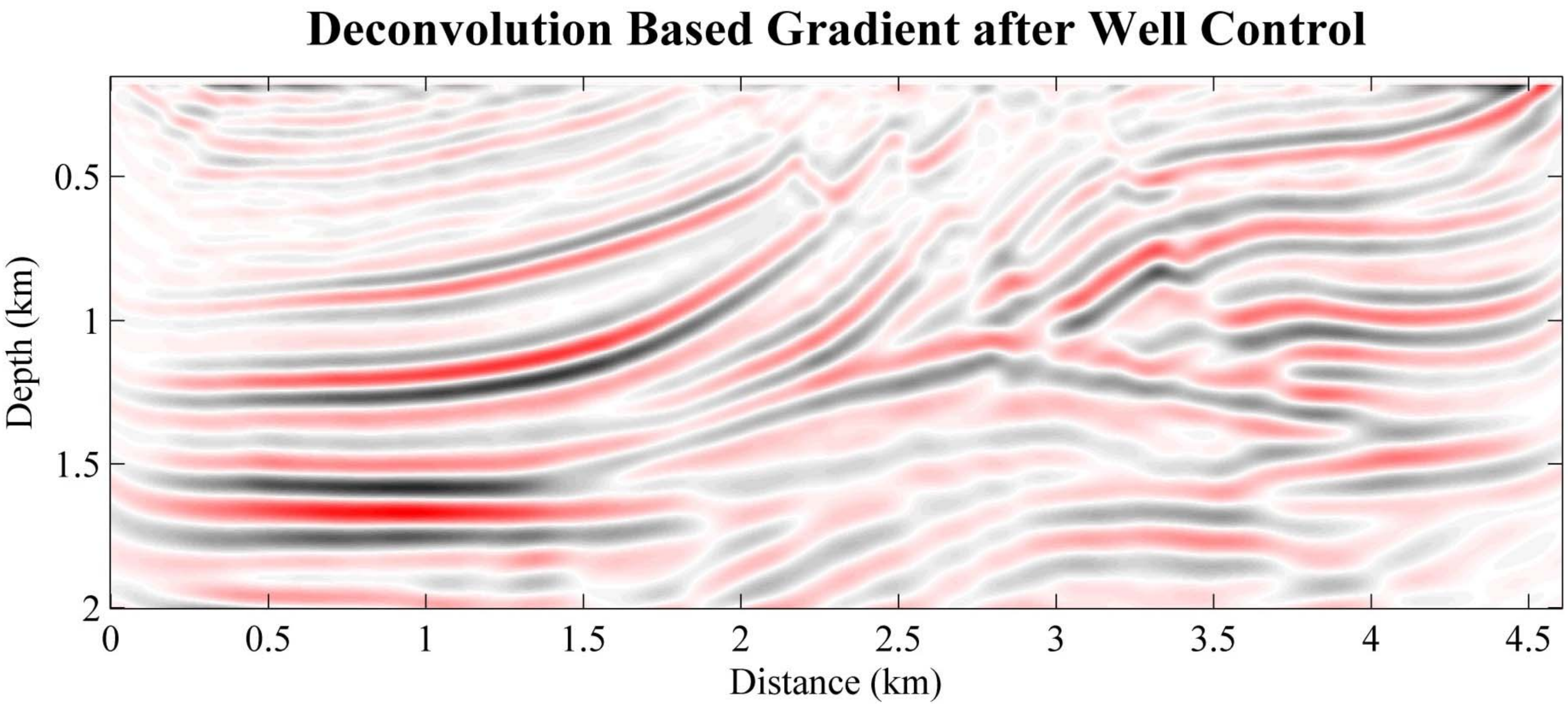
Well Control



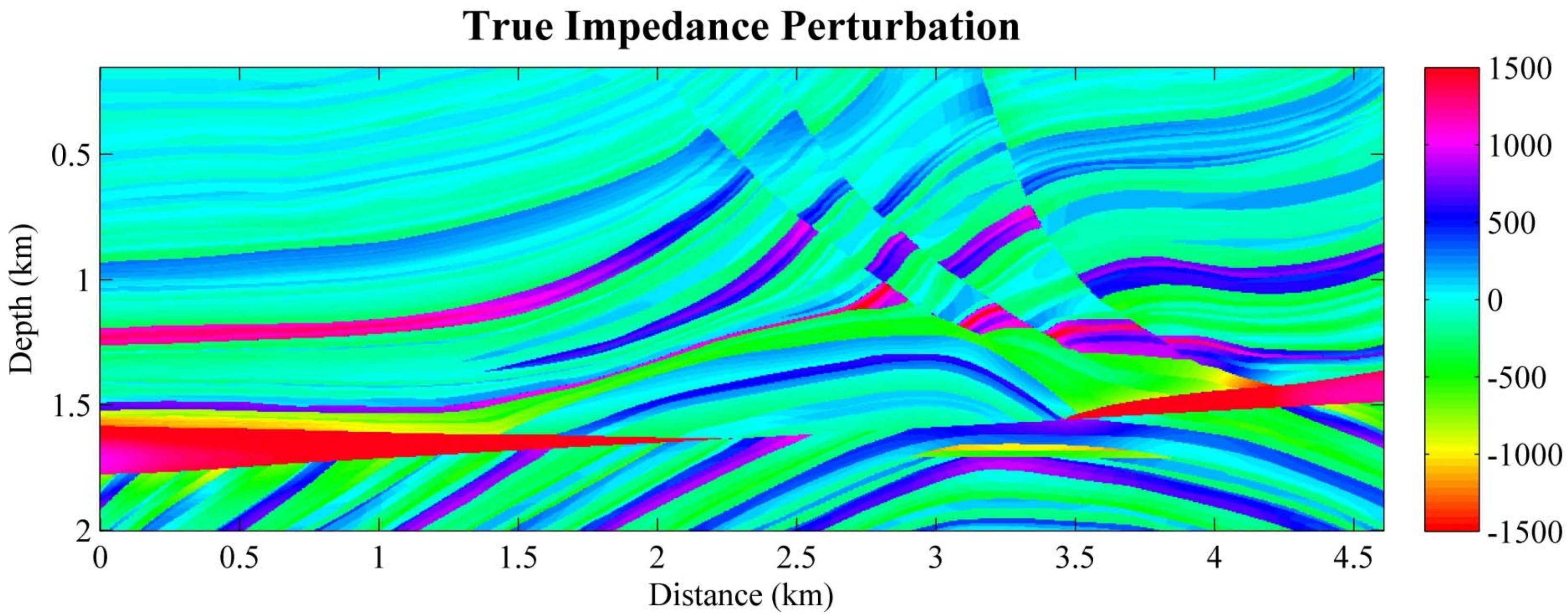
Well Control



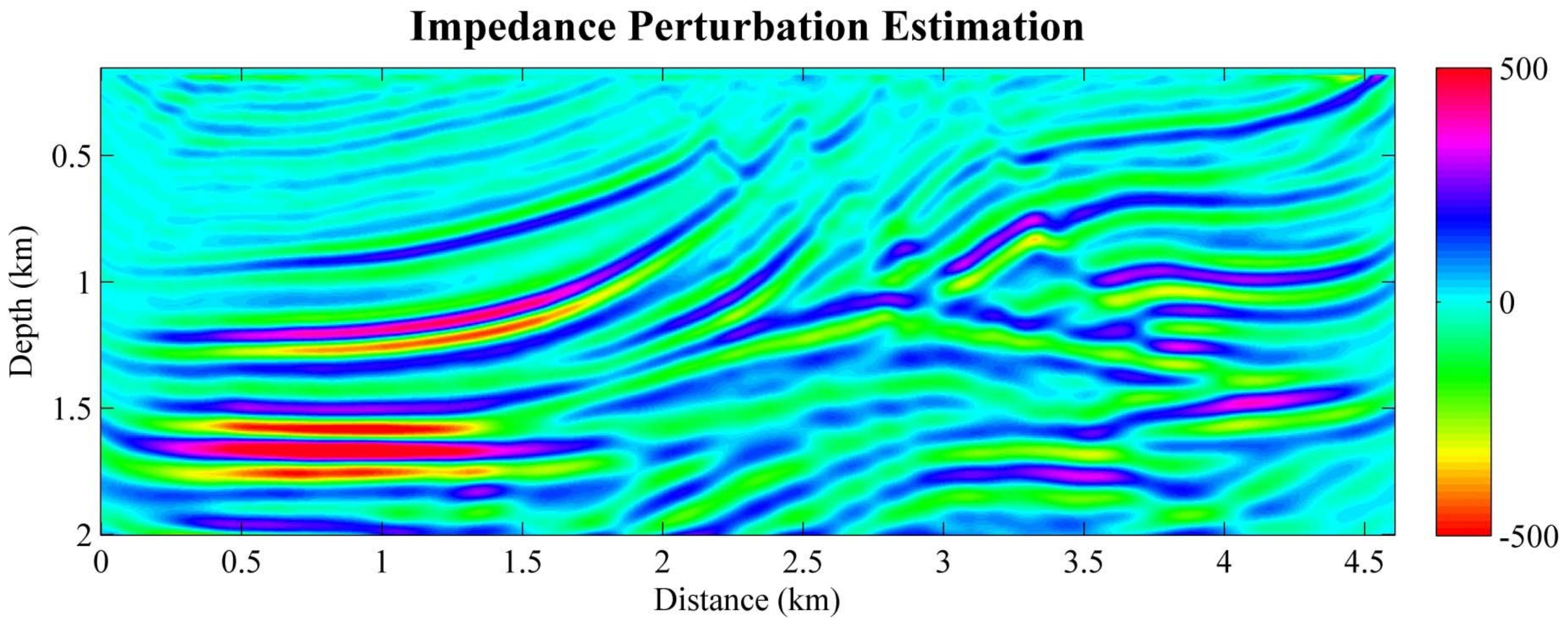
Estimated Reflectivity



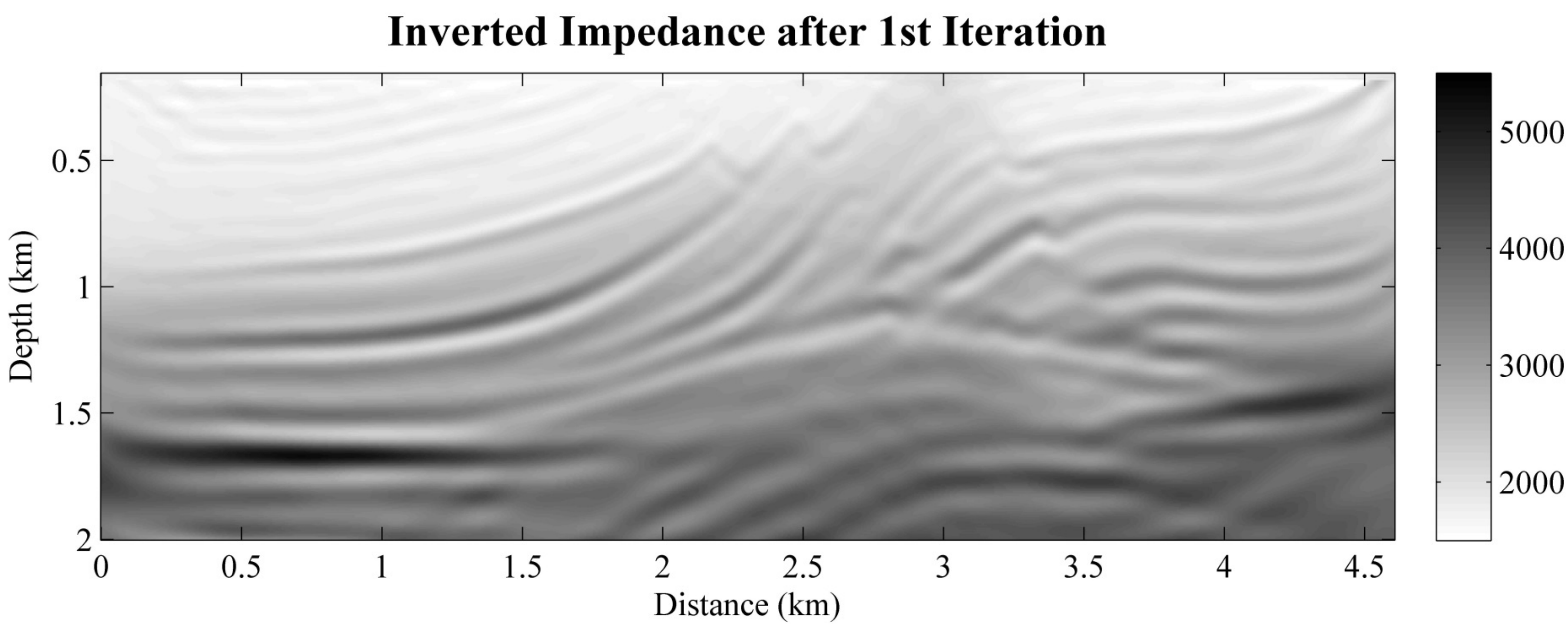
Impedance Perturbation Estimation



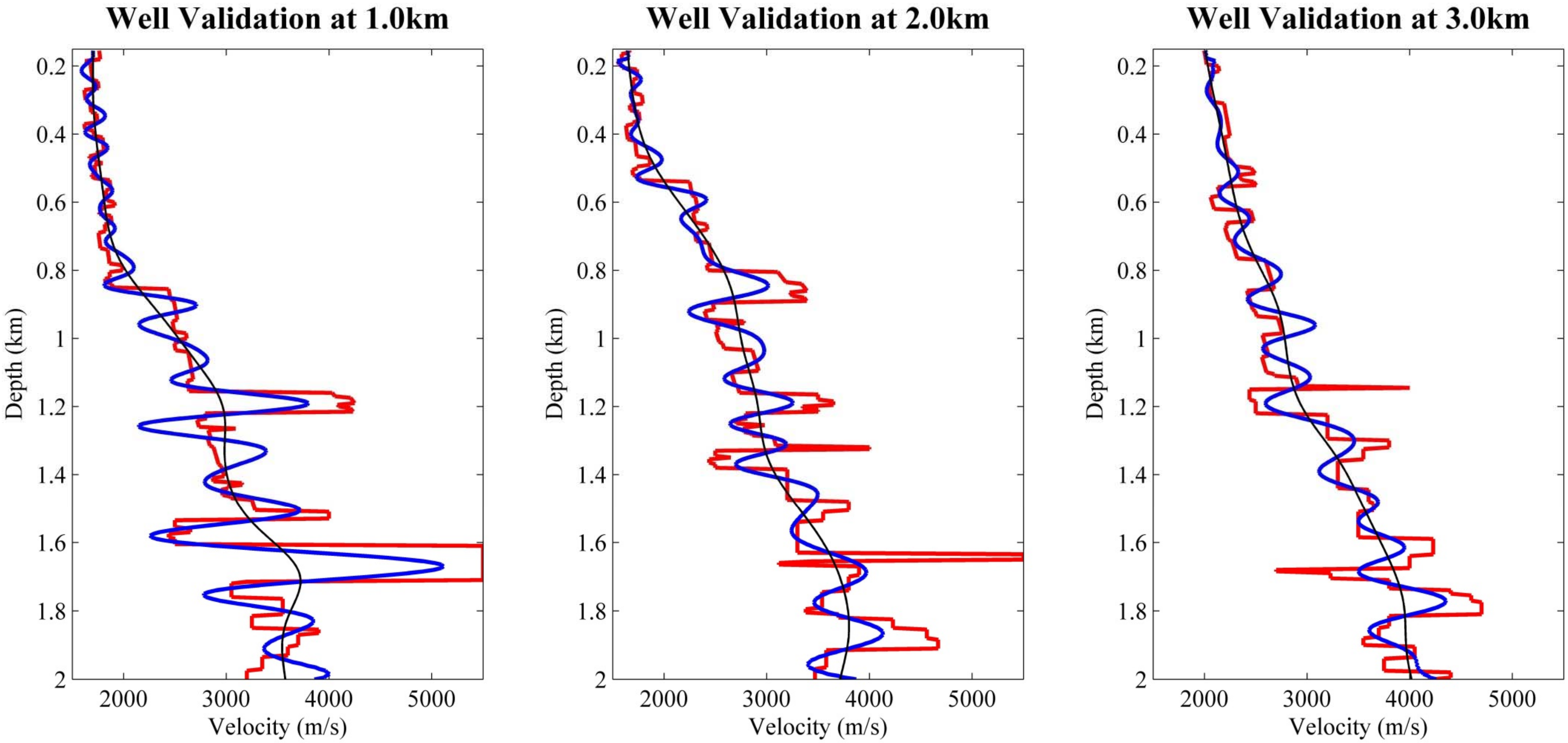
Impedance Perturbation Estimation



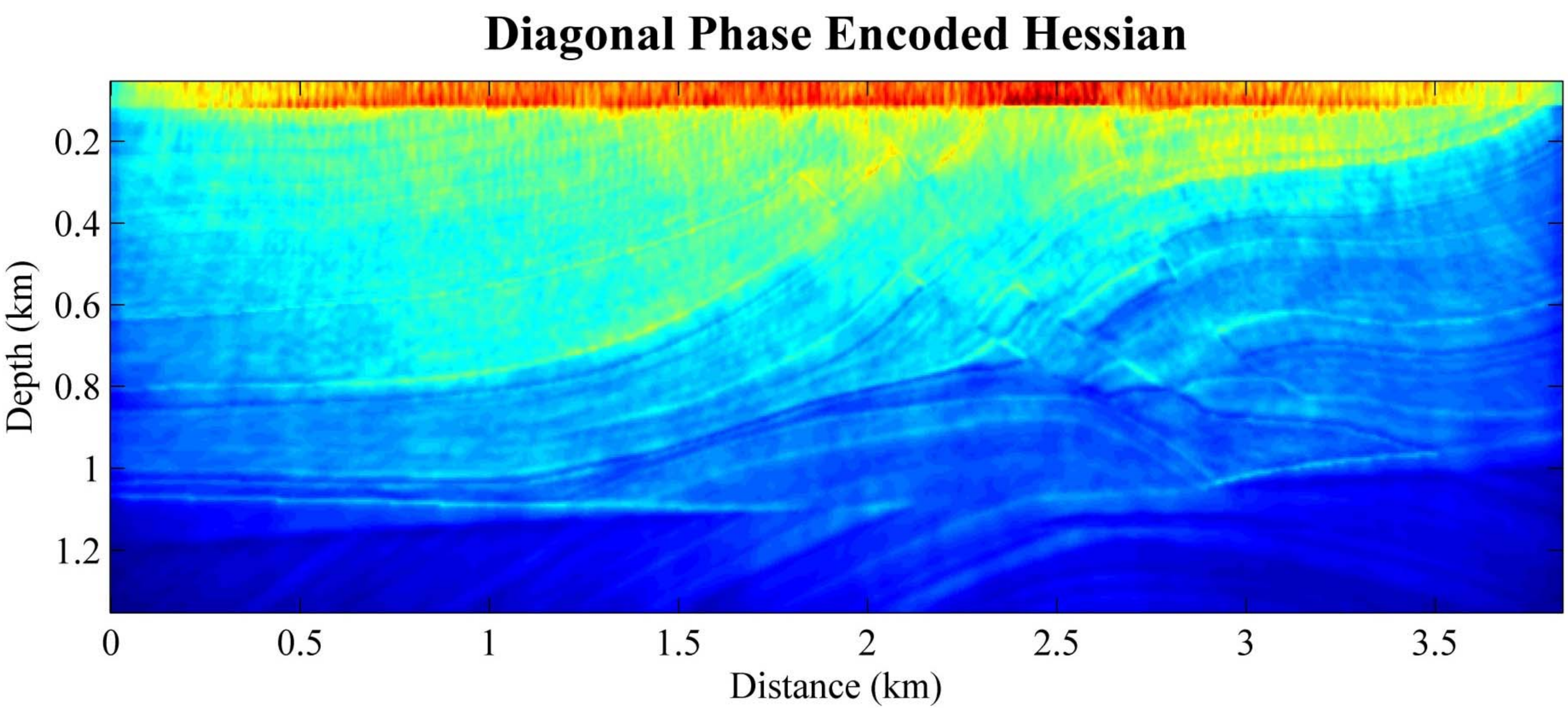
Impedance Perturbation Estimation



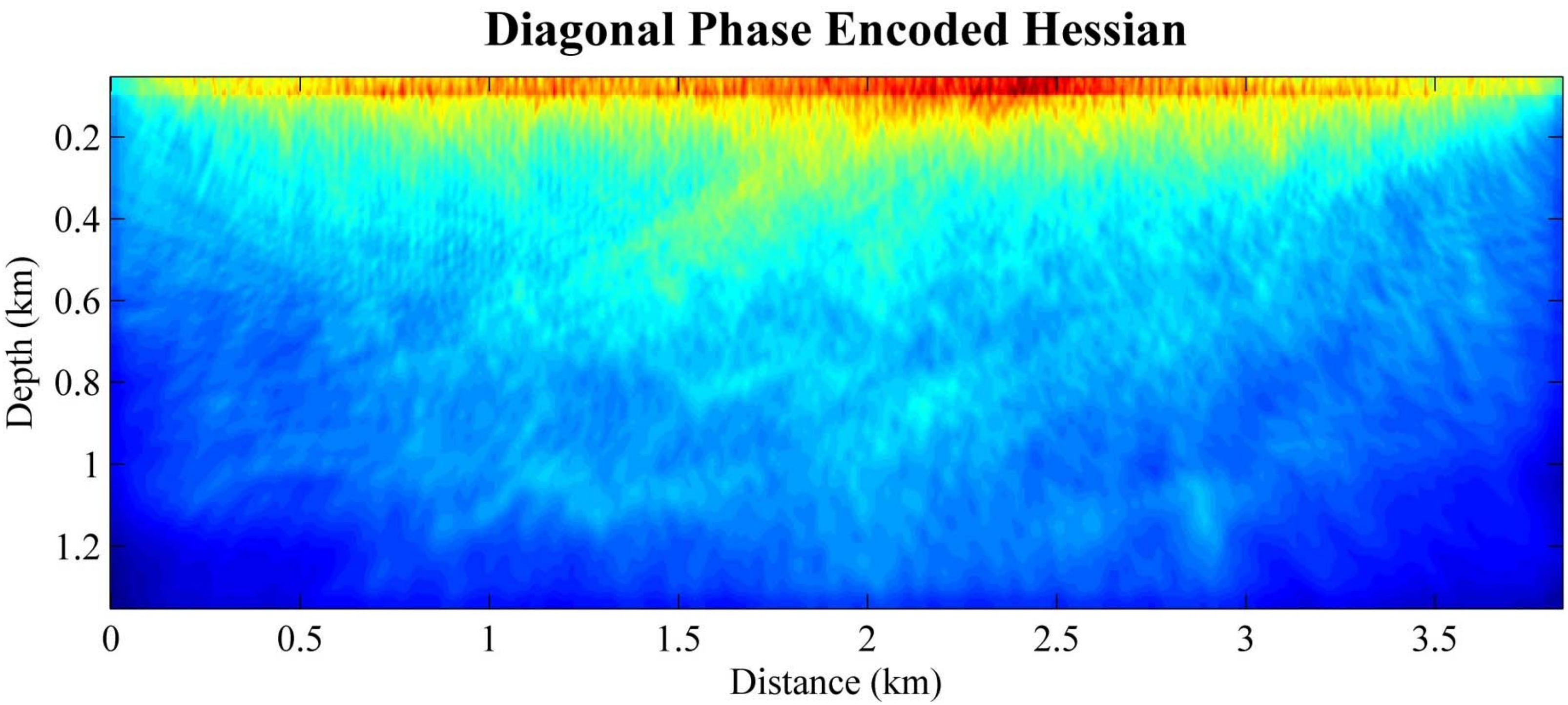
Well Data Comparison



Phase Encoded Pseudo-Hessian

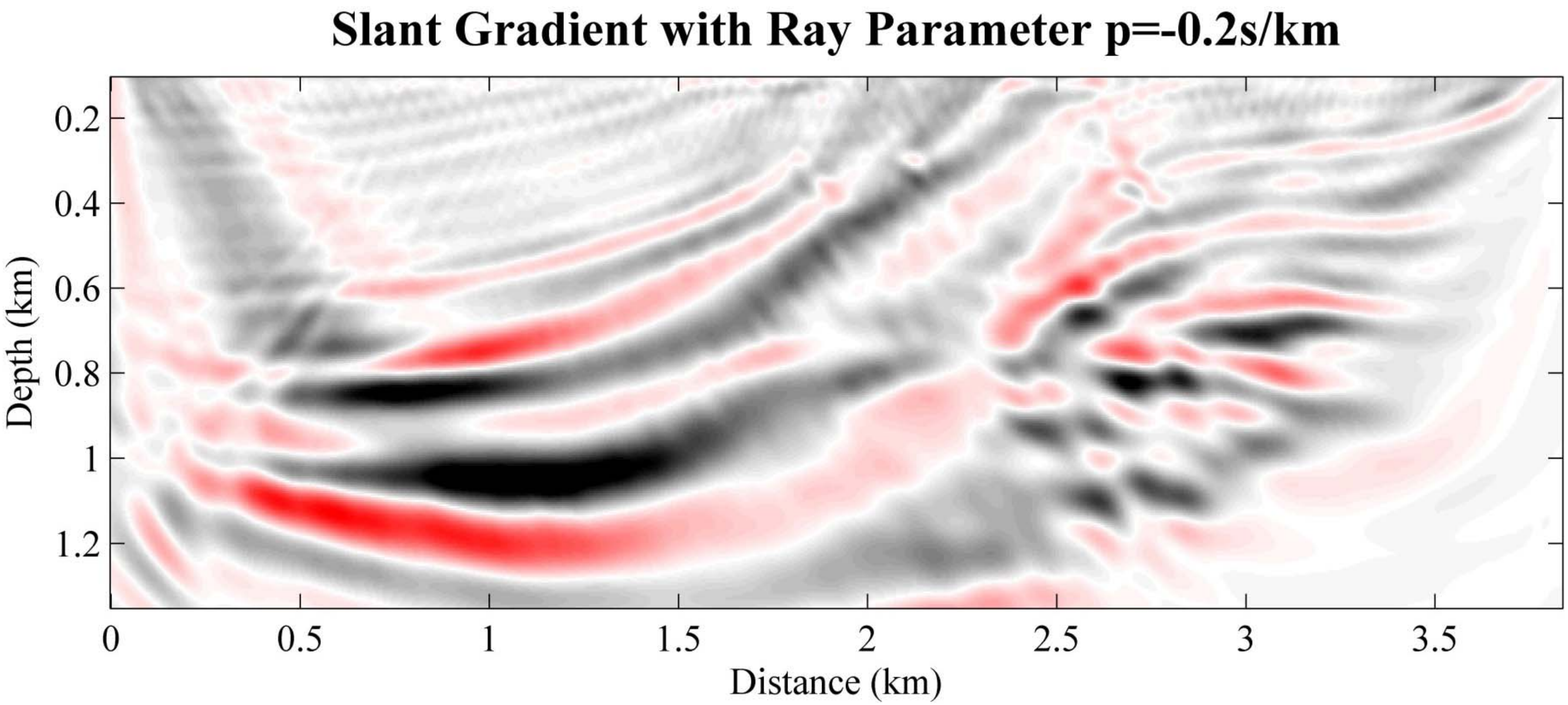


Phase Encoded Pseudo-Hessian



Slant Gradient

CREWES



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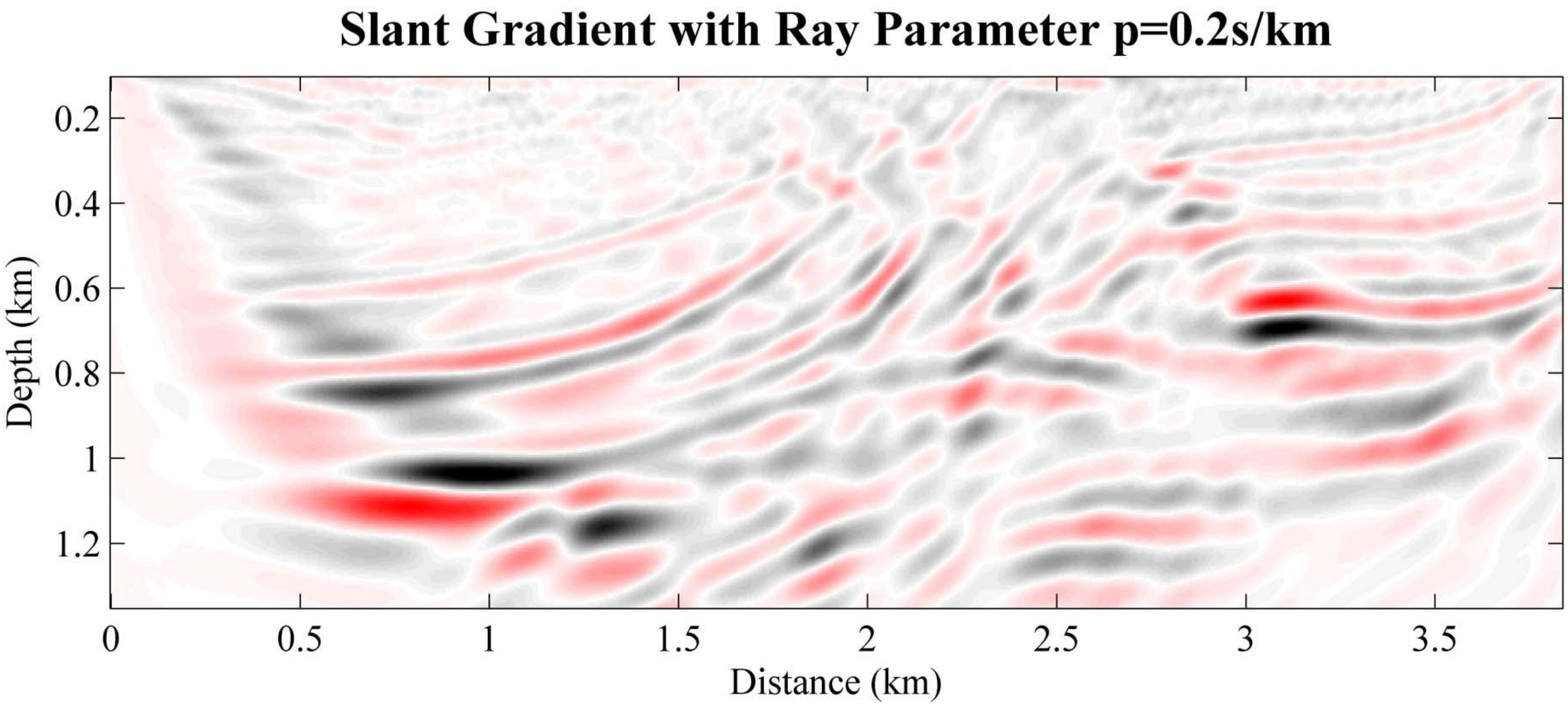
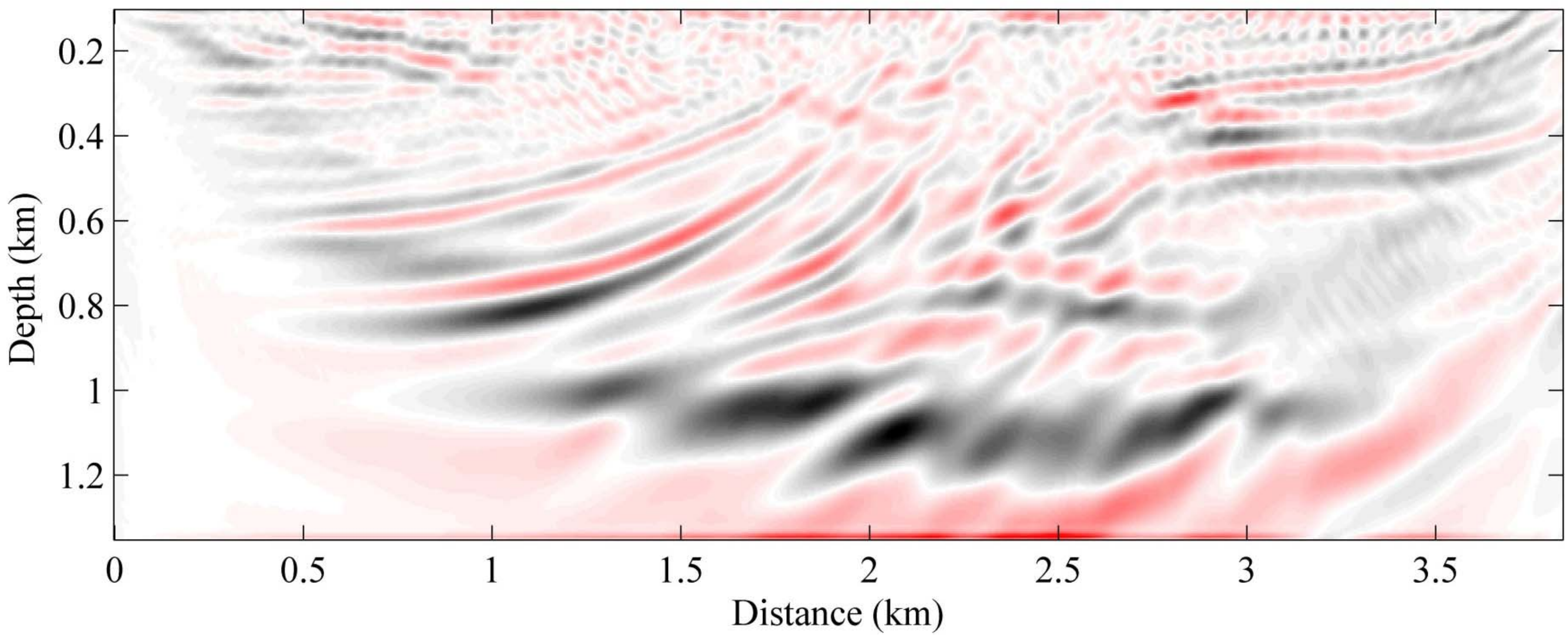
Slant Gradient

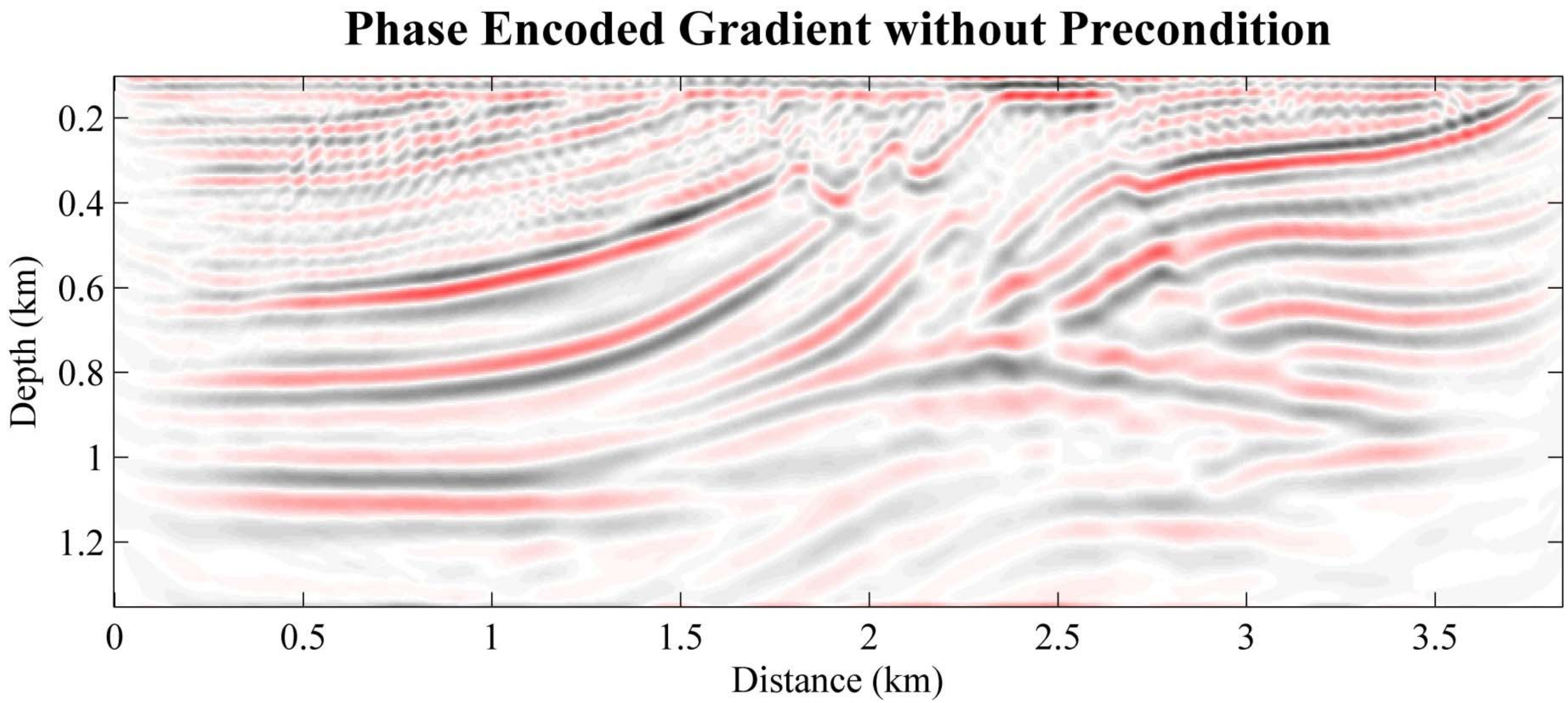


Slant Gradient

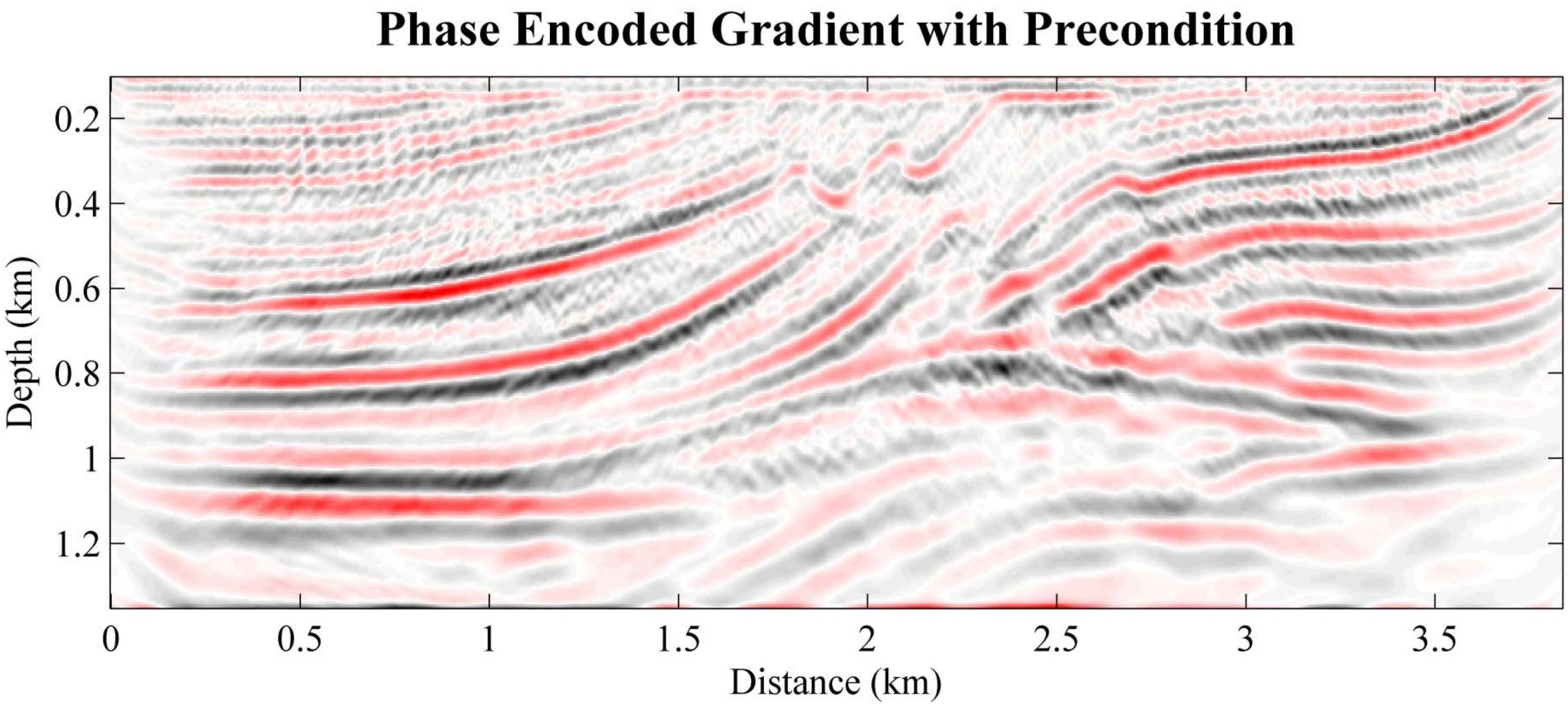
Slant Gradient with Ray Parameter p=-0.3s/km



Phase Encoded Gradient



Phase Encoded Gradient



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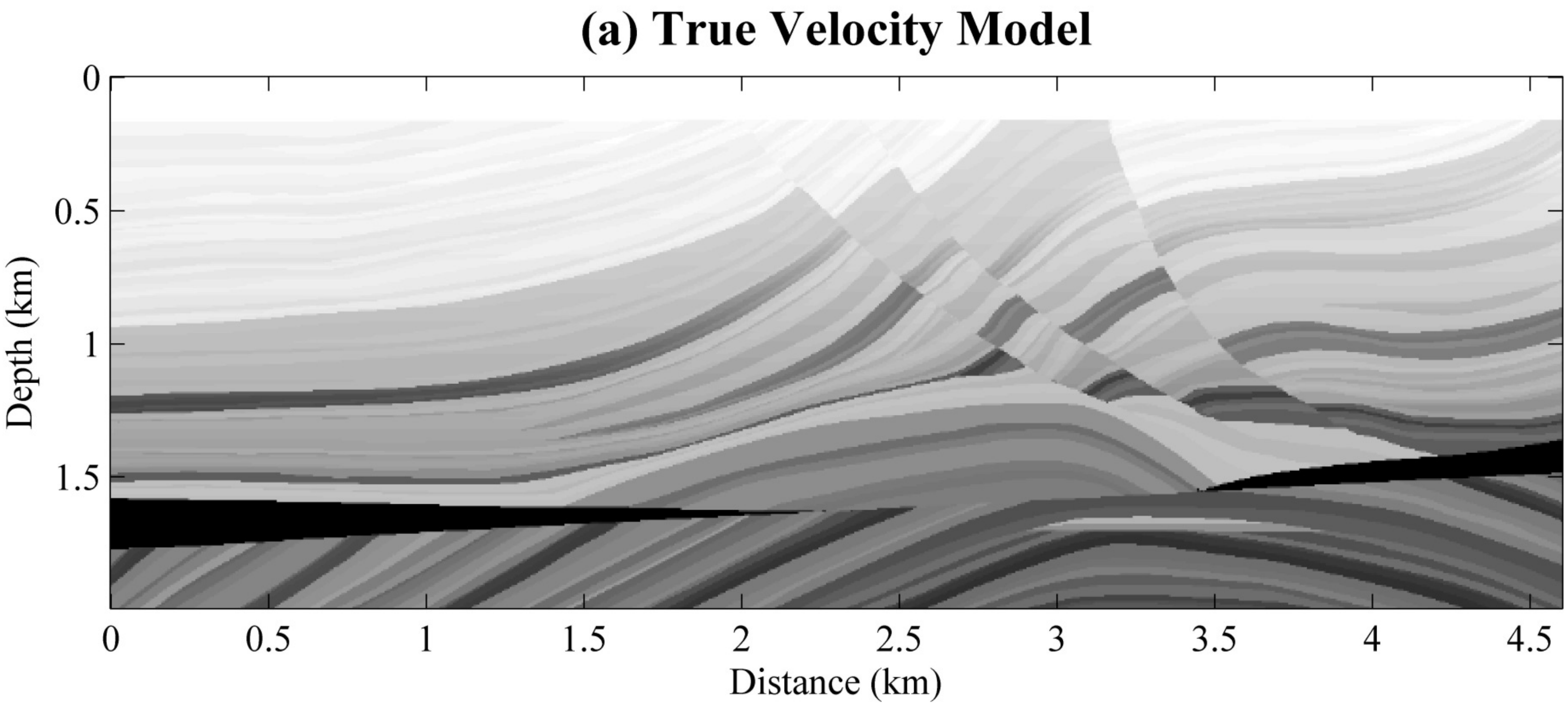
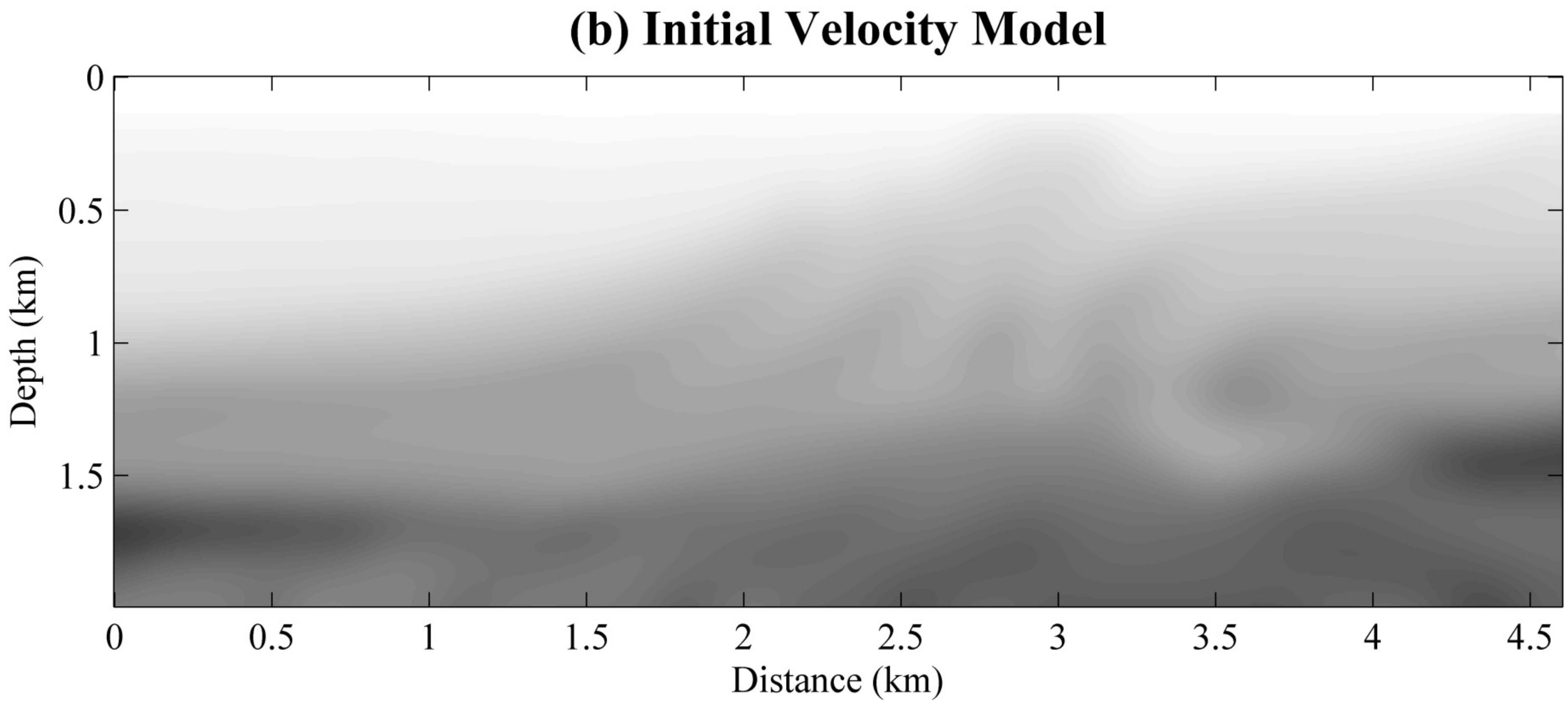
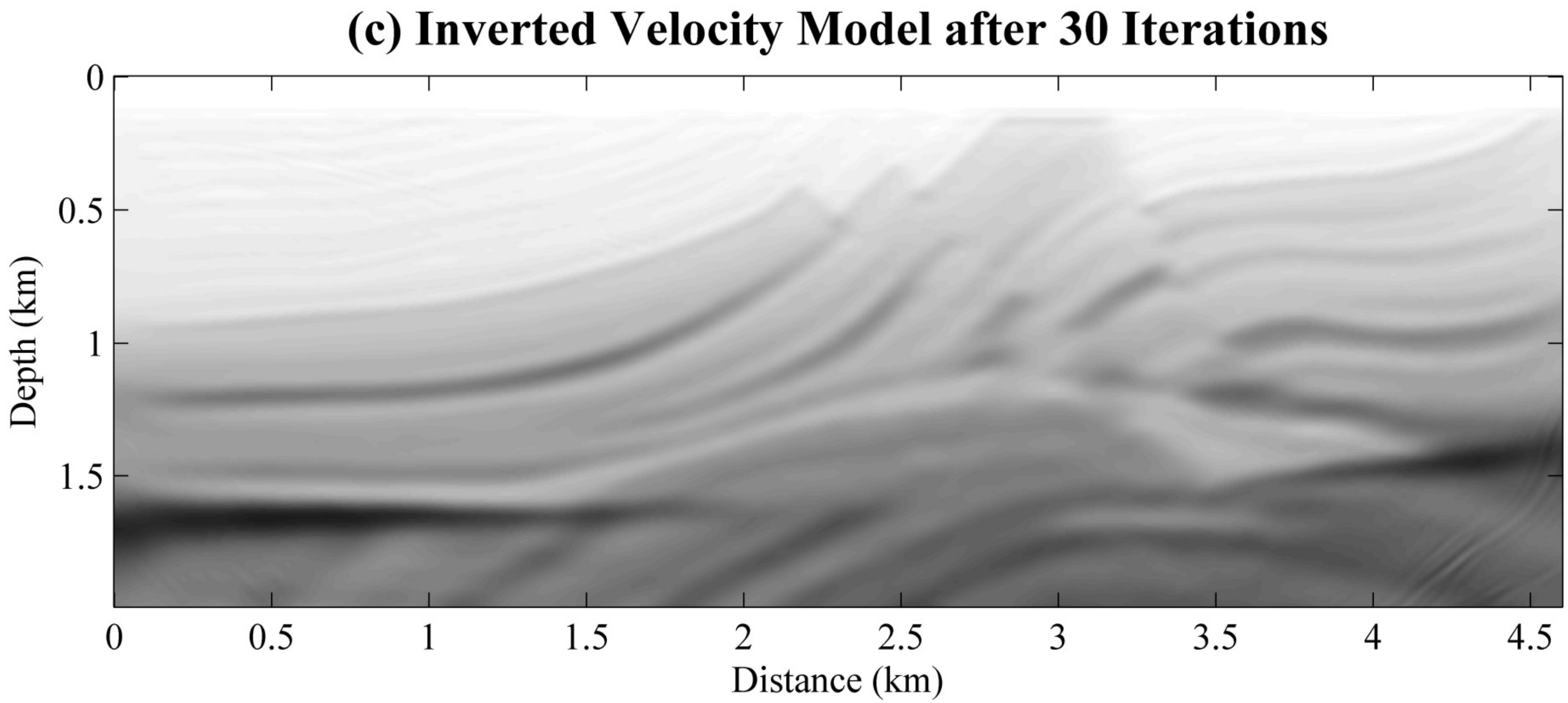
Inversion Result



Inversion Result

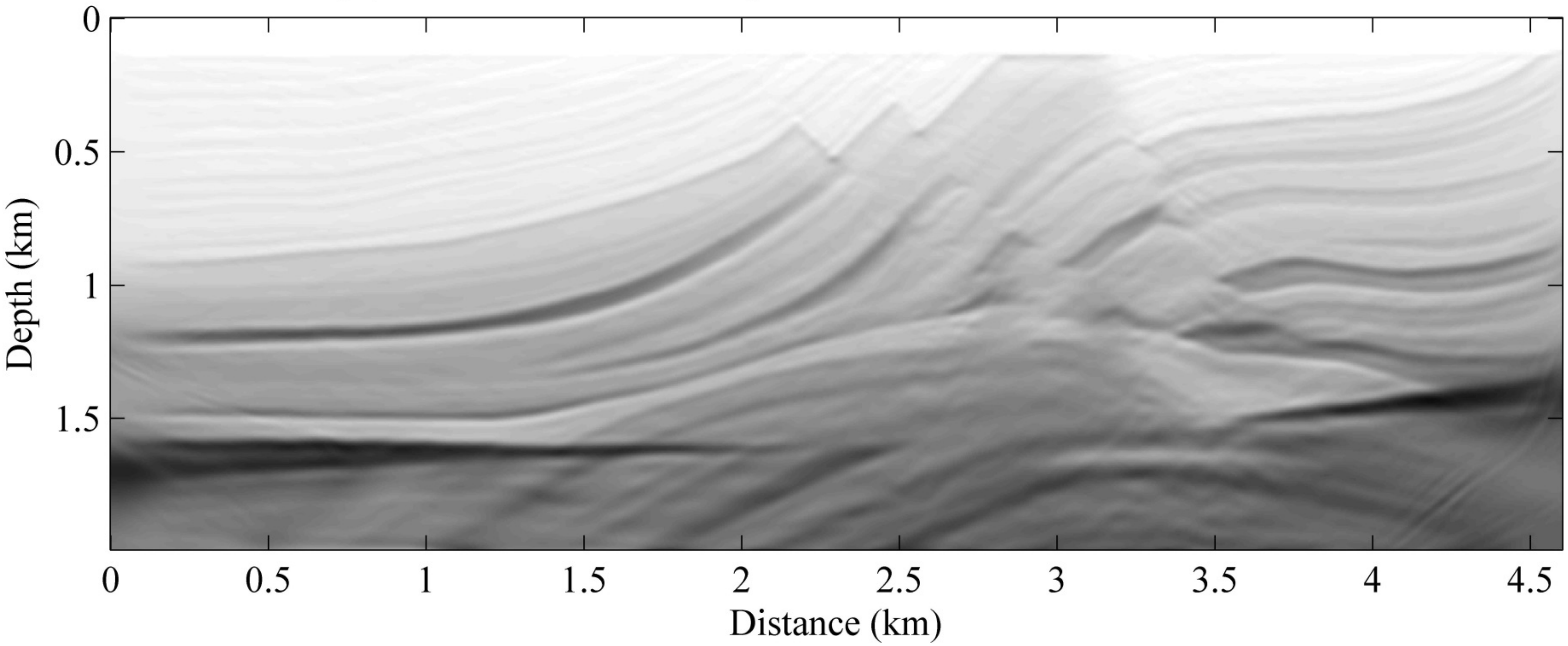


Inversion Result

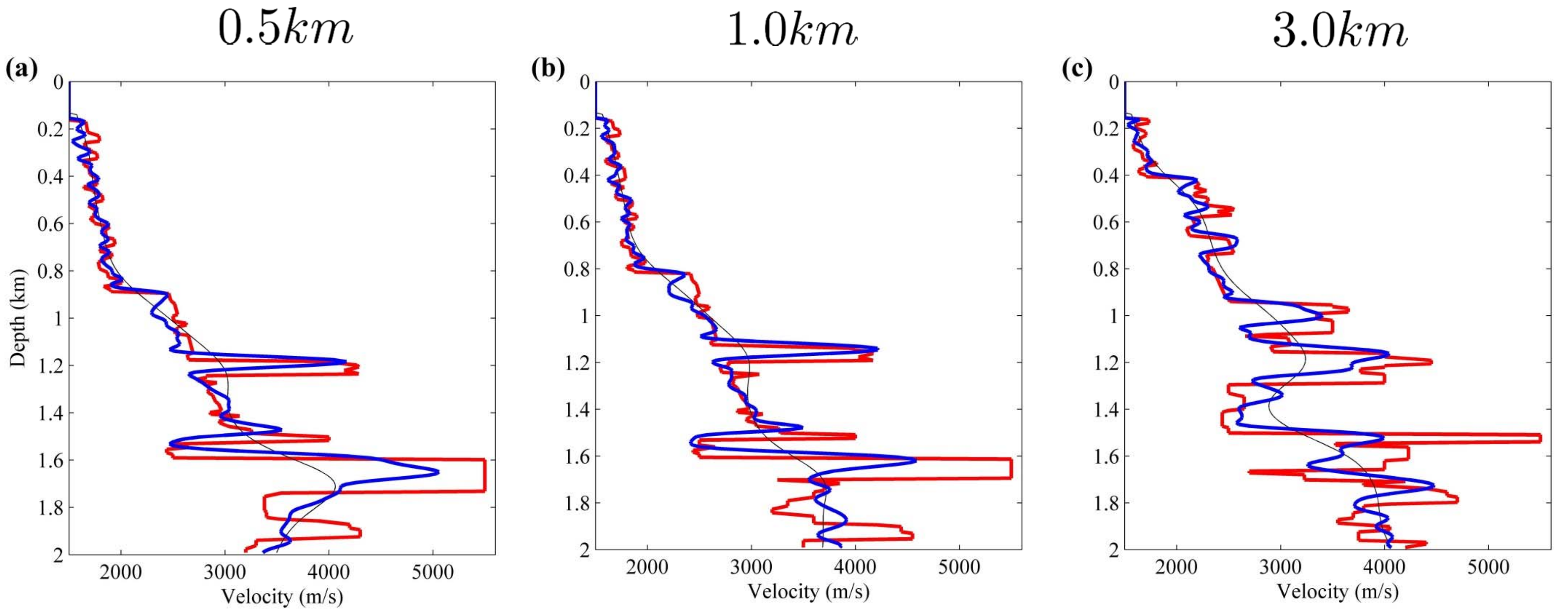


Inversion Result

(d) Inverted Velocity Model after 100 Iterations



Inversion Result



The **Bold-Red** **line,** Black **line and** **Bold-Blue** **line indicate the** **True Velocity Model**, Initial Velocity Model **and** **Inverted Velocity Model** **respectively.**

Conclusions

- The gradient with illumination compensation can compensate the geometrical spreading effects, recover the amplitudes of the deep reflectors and can estimate the reflectivity directly.
- The phase encoding method can reduce the computational cost effectively.
- The IMMI method, which combines FWI and traditional impedance inversion method, is efficient and stable to reconstruct the velocity model.

Further Research Plan

- Apply the strategies proposed in this research on the Hussar practical datasets.
- Incorporate AVO information in practical FWI (Innanen 2013).

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Thank You !