Setting up multicomponent FWI:
parameters, modes & nonlinearity

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Open questions

Can we parameterize FWI / IMMI with the same versatility* with which we do AVO / AVA?

Does FWI accommodate multicomponent data (e.g., PP + PS modes) jointly? Independently? Either?

If high-angle reflectivity is key, linearizations• are likely problematic – where in FWI do they lurk?

How do we adapt sensitivity analysis to account for large contrasts / large angles?

*See Anagaw, 2014 (Phd Thesis, U of A)

•With average angles (Downton & Ursenbach 2006) being unavailable to FWI

Outline

Elastic scattering framework

Sensitivities in (γ,μ,ρ) and beyond

Anatomy of a FWI update

Multicomponent elastic updates

Nonlinear sensitivities & reflectivity

Elastic scattering

Move towards general sensitivity formulas. AVO prototype:

$$R_{\mathrm{PP}}(\theta) \approx \frac{1}{2}\left(1+\tan^2\theta\right)\frac{\Delta V_P}{V_P} - 4\frac{V_S^2}{V_P^2}\sin^2\theta\frac{\Delta V_S}{V_S} + \frac{1}{2}\left(1+4\frac{V_S^2}{V_P^2}\sin^2\theta\right)\frac{\Delta\rho}{\rho}$$

Elastic scattering

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Elastic scattering

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$$\frac{\partial u}{\partial m_{\mathrm{P}}} \approx \frac{1}{2}\left(1 + \tan^2\theta\right)$$

Elastic scattering

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u $\qquad$ m_{P}

$$\frac{\partial u}{\partial m_{\mathrm{P}}} \approx \frac{1}{2}\left(1+\tan^2\theta\right)$$

"Frechet kernel" / "sensitivity" plays the same role as that played by AVO coefficients– how to update (e.g., in V_P), how to understand characteristics of inversion

Elastic scattering

Weglein and Stolt (γ,μ,ρ)

displacement $$\delta \mathbf{u} = \mathcal{G}_0 \mathcal{V} \mathbf{u}_0$$

$$\downarrow$$

PP, PS, SP, SS $$\delta \mathbf{G} = \mathbf{G}_0 \mathbf{V} \mathbf{G}$$

Elastic scattering

Weglein and Stolt (γ,μ,ρ)

displacement $$\delta \mathbf{u} = \mathcal{G}_0 \mathcal{V} \mathbf{u}_0$$

$$\mathcal{V} = \mathcal{L} - \mathcal{L}_0$$

PP, PS, SP, SS $$\delta \mathbf{G} = \mathbf{G}_0 \mathbf{V} \mathbf{G}$$

$$\mathcal{L} = \left[\rho\omega^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \partial_x \gamma \partial_x + \partial_z \mu \partial_z & \partial_x (\gamma - 2\mu) \partial_z + \partial_z \mu \partial_x \\ \partial_z (\gamma - 2\mu) \partial_x + \partial_x \mu \partial_z & \partial_z \gamma \partial_z + \partial_x \mu \partial_x \end{pmatrix} \right]$$

Elastic scattering

Weglein and Stolt (γ,μ,ρ)

displacement $$\delta \mathbf{u} = \mathcal{G}_0 \mathcal{V} \mathbf{u}_0$$

$$\downarrow \qquad \Pi = \begin{bmatrix} \partial_x & \partial_z \\ -\partial_z & \partial_x \end{bmatrix}$$

PP, PS, SP, SS $$\delta \mathbf{G} = \mathbf{G}_0 \mathbf{V} \mathbf{G}$$

Elastic scattering

Weglein and Stolt (γ,μ,ρ)

displacement $$\delta \mathbf{u} = \mathcal{G}_0 \mathcal{V} \mathbf{u}_0$$

$$\downarrow$$

ideal for flexible parameterization

PP, PS, SP, SS $$\delta \mathbf{G} = \mathbf{G}_0 \mathbf{V} \mathbf{G}$$

ideal for multi-component seismic

Elastic scattering

Weglein and Stolt (γ,μ,ρ)

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$$\downarrow$$

ideal for flexible parameterization

PP, PS, SP, SS $$\delta \mathbf{G} = \mathbf{G}_0 \mathbf{V} \mathbf{G}$$

ideal for multi-component seismic

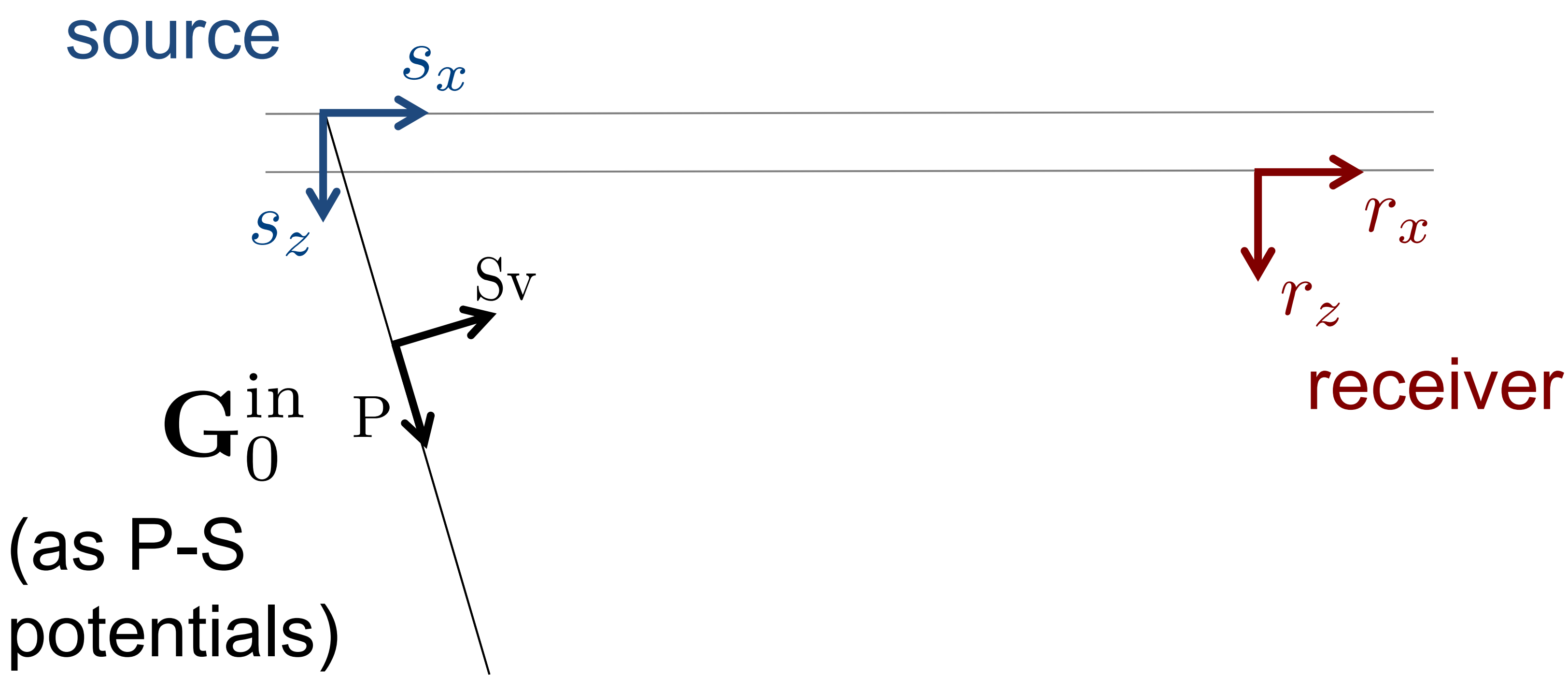
...does this mean we have a problem?

Sensitivities from linearized scattering

The goal

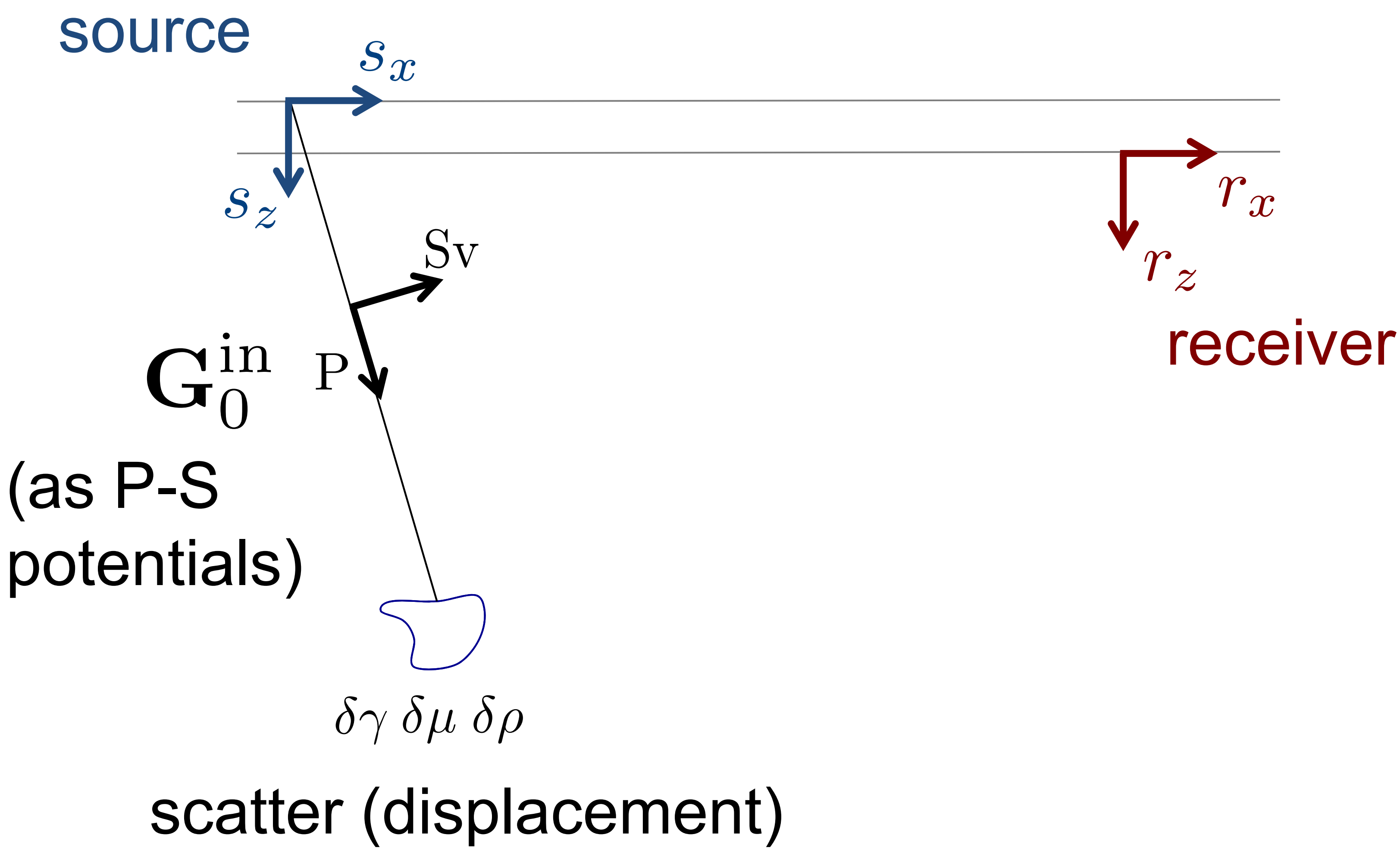
Sensitivities from linearized scattering

The goal



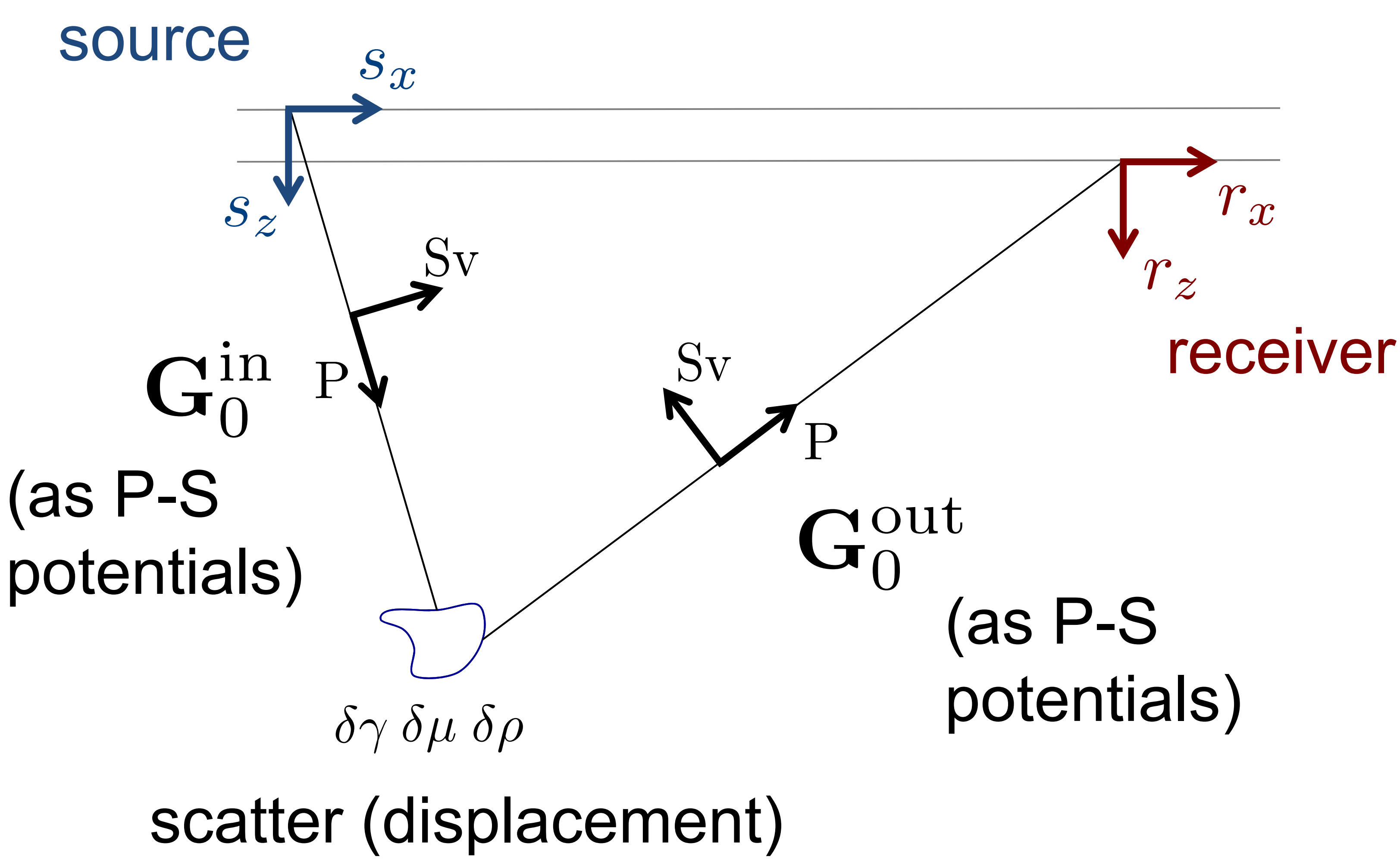
Sensitivities from linearized scattering

The goal



Sensitivities from linearized scattering

The goal



Sensitivities from linearized scattering

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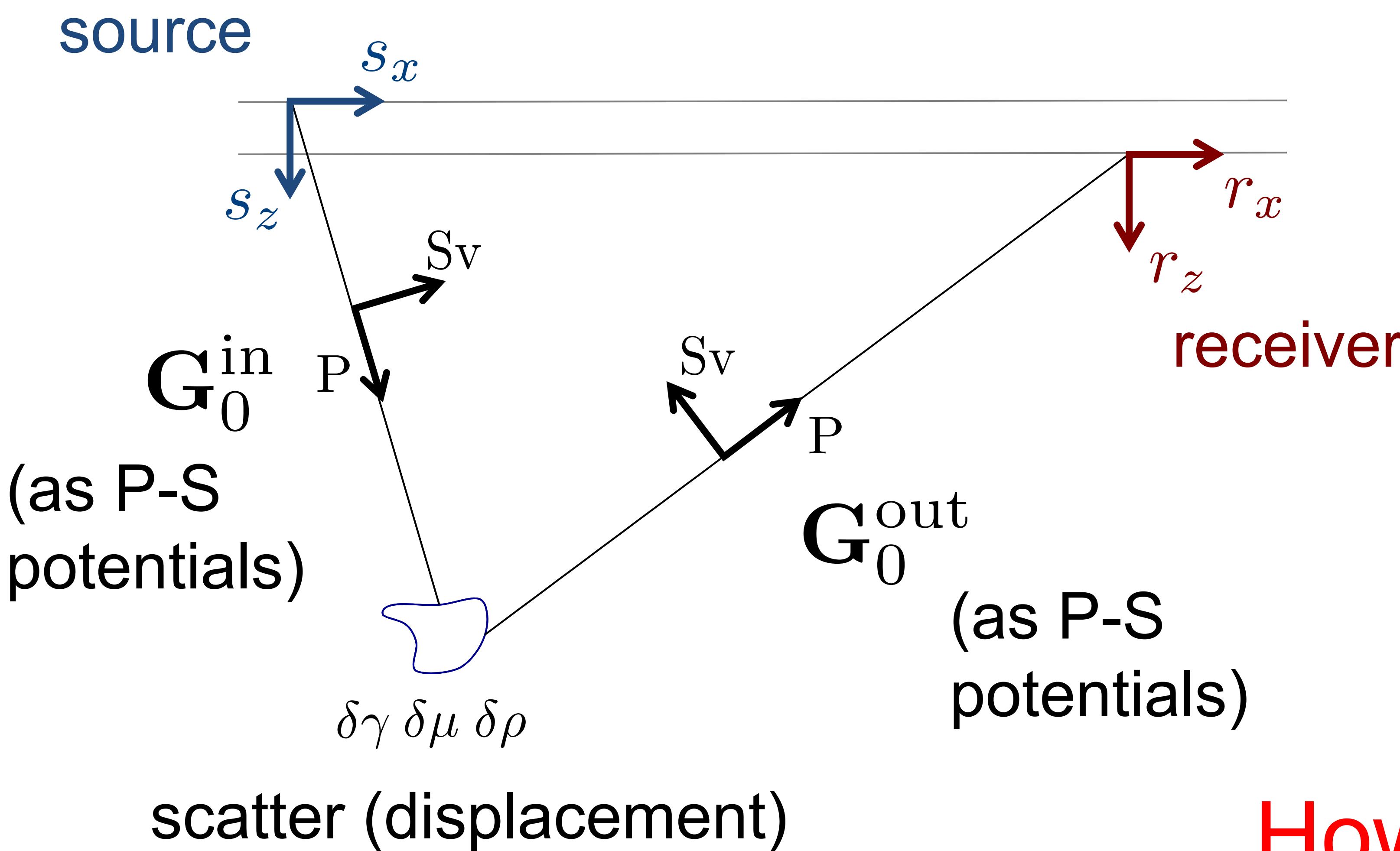

How?

Sensitivities from linearized scattering

Sensitivities, base parameters

$$\frac{\partial \mathbf{G}(k_g, k_s)}{\partial s_\gamma(x,z)} = \begin{bmatrix} \partial_x^2 G_{\mathrm{P}_0} & \partial_z^2 G_{\mathrm{P}_0} \\ -\partial_x \partial_z G_{\mathrm{S}_0} & \partial_z \partial_x G_{\mathrm{S}_0} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \partial_x^2 G'_{\mathrm{P}_0} & -\partial_x \partial_z G'_{\mathrm{S}_0} \\ \partial_z^2 G'_{\mathrm{P}_0} & \partial_z \partial_x G'_{\mathrm{S}_0} \end{bmatrix}$$

2x2 matrix of sensitivities of G_{PP}, G_{PS}, G_{SP}, G_{SS} with respect to γ

Sensitivities from linearized scattering

Sensitivities, base parameters

$$\frac{\partial \mathbf{G}(k_g, k_s)}{\partial s_\gamma(x,z)} = \begin{bmatrix} \partial_x^2 G_{\mathrm{P}_0} & \partial_z^2 G_{\mathrm{P}_0} \\ -\partial_x \partial_z G_{\mathrm{S}_0} & \partial_z \partial_x G_{\mathrm{S}_0} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \partial_x^2 G'_{\mathrm{P}_0} & -\partial_x \partial_z G'_{\mathrm{S}_0} \\ \partial_z^2 G'_{\mathrm{P}_0} & \partial_z \partial_x G'_{\mathrm{S}_0} \end{bmatrix}$$

Via IBP, all derivatives act on the incoming and outgoing waves

Sensitivities from linearized scattering

Sensitivities, base parameters

$$\frac{\partial \mathbf{G}(k_g, k_s)}{\partial s_\gamma(x,z)} = \begin{bmatrix} \partial_x^2 G_{\mathrm{P}_0} & \partial_z^2 G_{\mathrm{P}_0} \\ -\partial_x \partial_z G_{\mathrm{S}_0} & \partial_z \partial_x G_{\mathrm{S}_0} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \partial_x^2 G'_{\mathrm{P}_0} & -\partial_x \partial_z G'_{\mathrm{S}_0} \\ \partial_z^2 G'_{\mathrm{P}_0} & \partial_z \partial_x G'_{\mathrm{S}_0} \end{bmatrix}$$

Simple intermediary mixing matrix

Sensitivities from linearized scattering

Jumping to other parameters

(e.g., Goodway's LMR)

$$\delta s_\lambda \rightarrow \left(\frac{s_{\lambda_0}}{s_{\lambda\rho_0}}\right)\delta s_{\lambda\rho} - \left(\frac{s_{\lambda_0}}{s_{\rho_0}}\right)\delta s_\rho$$

$$\delta s_\mu \rightarrow \left(\frac{s_{\mu_0}}{s_{\mu\rho_0}}\right)\delta s_{\mu\rho} - \left(\frac{s_{\mu_0}}{s_{\rho_0}}\right)\delta s_\rho$$

$$\delta s_\rho \rightarrow \delta s_\rho$$

Sensitivities from linearized scattering

Jumping to other parameters

(e.g., Goodway's LMR)

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$$\delta s_{\rho} \rightarrow \delta s_{\rho}$$

starting from (λ,μ,ρ)

Sensitivities from linearized scattering

Jumping to other parameters

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$$\delta s_{\lambda} \rightarrow \left(\frac{s_{\lambda_0}}{s_{\lambda\rho_0}}\right) \delta s_{\lambda\rho} - \left(\frac{s_{\lambda_0}}{s_{\rho_0}}\right) \delta s_{\rho}$$

$$\delta s_{\mu} \rightarrow \left(\frac{s_{\mu_0}}{s_{\mu\rho_0}}\right) \delta s_{\mu\rho} - \left(\frac{s_{\mu_0}}{s_{\rho_0}}\right) \delta s_{\rho}$$

$$\delta s_{\rho} \rightarrow \delta s_{\rho}$$

starting from (λ,μ,ρ)

now involve ratios of reference parameters with ($\lambda\rho$,$\mu\rho$,ρ)

Anatomy of a FWI update

Univariate prototype

forward modeling $F(x)$

$$d = F(x^*)$$ datum, true model

objective function $$\phi(x) = \frac{1}{2}\left[F(x) - d\right]^2$$

$$\Delta x_N = -\frac{\phi'(x)}{\phi''(x)}$$ Newton step from x towards x^*

Anatomy of a FWI update

Univariate prototype

explicitly, $$\Delta x_N = -\frac{F'(x)[F(x)-d]}{[F'(x)]^2 + F''(x)[F(x)-d]}$$

$$\Delta x_N = -\frac{J(x)r(x)}{J(x)J(x) + H_{NL}(x)}$$

$J(x)r(x)$ } gradient

$J(x)J(x) + H_{NL}(x)$ } Hessian

Anatomy of a FWI update

Univariate prototype

explicitly, $$\Delta x_N = -\frac{F'(x)[F(x)-d]}{[F'(x)]^2 + F''(x)[F(x)-d]}$$

residuals

Jacobian / sensitivity matrix

$$\Delta x_N = -\frac{J(x)r(x)}{J(x)J(x) + H_{NL}(x)}$$

gradient

Hessian

Anatomy of a FWI update

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explicitly, $$\Delta x_N = -\frac{F'(x)[F(x) - d]}{[F'(x)]^2 + F''(x)[F(x) - d]}$$

residuals

Jacobian / sensitivity matrix

$$\Delta x_N = -\frac{J(x)r(x)}{J(x)J(x) + H_{NL}(x)}$$

gradient

Hessian

Gauss-Newton part of H

nonlinear, *r(x)*-dependent part of H

Anatomy of a FWI update

FWI nonlinearity

Seek nonlinearity in the

1. Residuals (forward modeling)
2. Sensitivities ($J(x) = a + bx + cx^2 + \ldots \approx J_0(x)$)
3. Residual dependent part of H

$$\Delta x_N = -\frac{J(x)r(x)}{J(x)J(x) + H_{NL}(x)}$$

Anatomy of a FWI update

Derive forms for

Multicomponent elastic FWI (flexible parameterization, linearized sensitivities)

$$\Delta x_{GN} = -\frac{J_0(x)r(x)}{J_0(x)J_0(x)}$$

Scalar FWI (nonlinear sensitivities)

$$\Delta x_{2nd} = -\frac{[J_0(x) + J_1(x)]r(x)}{J_0(x)J_0(x)}$$

Multicomponent elastic updates

Elastic objective function

$$\phi = \frac{1}{2} \int d\omega \sum_{k_g, k_s} \mathrm{tr}\left(\delta \mathbf{P}^H \delta \mathbf{P}\right)$$

$$\delta \mathbf{P}(k_g, k_s) = \begin{bmatrix} \delta P_{\mathrm{PP}}(k_g, k_s) & \delta P_{\mathrm{SP}}(k_g, k_s) \\ \delta P_{\mathrm{PS}}(k_g, k_s) & \delta P_{\mathrm{SS}}(k_g, k_s) \end{bmatrix}$$

discrete k_g, k_s, continuous ω

Multicomponent elastic updates

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Frobenius product $\operatorname{tr}(\mathrm{A}^{\mathrm{T}}\mathrm{B})$

Multicomponent elastic updates

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discrete k_g, k_s, continuous ω

Frobenius product $\mathrm{tr}\left(A^T B\right)$

joint or independent PP, PS, etc. inversion

Multicomponent elastic updates

Multicomponent updates

$$\begin{bmatrix} \delta s_X(x,z) \\ \delta s_Y(x,z) \\ \delta s_Z(x,z) \end{bmatrix} = \int dx' \int dz' \mathcal{H}^{-1}(x,z,x',z')\mathbf{g}(x',z')$$

vector of gradient functions

$$\mathbf{g}(x,z) = \begin{bmatrix} g_X \\ g_Y \\ g_Z \end{bmatrix}$$

$$\mathcal{H}(x,z,x',z') = \begin{bmatrix} H_{XX} & H_{XY} & H_{XZ} \\ H_{YX} & H_{YY} & H_{YZ} \\ H_{ZX} & H_{ZY} & H_{ZZ} \end{bmatrix}$$

matrix of Hessian functions

Multicomponent elastic updates

Multicomponent updates

$$H_{XY}(x,z,x',z') = \int d\omega \sum_{k_g,k_s} \mathrm{tr}\left\{ \left[\frac{\partial \mathbf{G}(k_g,k_s)}{\partial s_X(x',z')}\right]^H \left[\frac{\partial \mathbf{G}(k_g,k_s)}{\partial s_Y(x,z)}\right] \right\}$$

$$g_X(x,z) = -\int d\omega \sum_{k_g,k_s} \mathrm{tr}\left\{ \left[\frac{\partial \mathbf{G}(k_g,k_s)}{\partial s_X(x,z)}\right]^T \delta\mathbf{P}^* \right\}$$

Involving our (1) flexible sensitivities

and (2) joint or independent use of PP, PS, SP, SS

Nonlinear sensitivities and reflectivity

FWI nonlinearities affecting reflectivity

$$\Delta x_N = -\frac{J(x)r(x)}{J(x)J(x) + H_{NL}(x)}$$

Reduce from full Newton update...

Nonlinear sensitivities and reflectivity

FWI nonlinearities affecting reflectivity

$$\Delta x_{2nd} = -\frac{[J_0(x) + J_1(x)]r(x)}{J_0(x)J_0(x)}$$

...to Gauss-Newton with 2nd order sensitivities

Nonlinear sensitivities and reflectivity

FWI nonlinearities affecting reflectivity

$$\left(\frac{\partial G(\mathbf{r}_g,\mathbf{r}_s)}{\partial s(\mathbf{r})}\right)_N = \lim_{\delta s \to 0} \frac{\delta G(\mathbf{r}_g,\mathbf{r}_s)}{\delta s(\mathbf{r})}, \quad \delta s(\mathbf{r}) \approx \sum_{n=1}^{N} \delta s_n(\mathbf{r})$$

Nonlinear sensitivities and reflectivity

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Reduction to 1st order sensitivity

$$\left(\frac{\partial G(\mathbf{r}_g,\mathbf{r}_s)}{\partial s(\mathbf{r})}\right)_1 = \lim_{\delta s \to 0} \frac{\delta G(\mathbf{r}_g,\mathbf{r}_s)}{\delta s(\mathbf{r})}$$

Nonlinear sensitivities and reflectivity

FWI nonlinearities affecting reflectivity

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$$-\delta G(\mathbf{r}_g,\mathbf{r}_s)[\omega^2 G(\mathbf{r}_g,\mathbf{r})G(\mathbf{r},\mathbf{r}_s)]^{-1}$$

Nonlinear sensitivities and reflectivity

FWI nonlinearities affecting reflectivity

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Nonlinear sensitivities and reflectivity

FWI nonlinearities affecting reflectivity

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Reduction to 1st order sensitivity

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$$= \lim_{\delta s \to 0} \left(\frac{\delta G(\mathbf{r}_g, \mathbf{r}_s)}{\delta G(\mathbf{r}_g, \mathbf{r}_s)\left[-\omega^2 G(\mathbf{r}_g, \mathbf{r}) G(\mathbf{r}, \mathbf{r}_s)\right]^{-1}}\right)$$

$$= -\omega^2 G(\mathbf{r}_g, \mathbf{r}) G(\mathbf{r}, \mathbf{r}_s)$$

(consistent with, e.g., Tarantola 1984)

Nonlinear sensitivities and reflectivity

FWI nonlinearities affecting reflectivity

$$\left(\frac{\partial G(\mathbf{r}_g,\mathbf{r}_s)}{\partial s(\mathbf{r})}\right)_N = \lim_{\delta s\to 0}\frac{\delta G(\mathbf{r}_g,\mathbf{r}_s)}{\delta s(\mathbf{r})},\quad \delta s(\mathbf{r}) \approx \sum_{n=1}^{N}\delta s_n(\mathbf{r})$$

2nd order “collocated scattering” sensitivity

$$\left(\frac{\partial G(\mathbf{r}_g,\mathbf{r}_s)}{\partial s(\mathbf{r})}\right)_2 = -\omega^2 G(\mathbf{r}_g,\mathbf{r})G(\mathbf{r},\mathbf{r}_s)\left(1+\frac{\delta P^*(\mathbf{r}_g,\mathbf{r}_s)G(\mathbf{r},\mathbf{r})}{G^*(\mathbf{r}_g,\mathbf{r})G^*(\mathbf{r},\mathbf{r}_s)}\right)$$

Nonlinear sensitivities and reflectivity

FWI nonlinearities affecting reflectivity

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2nd order "collocated scattering" sensitivity

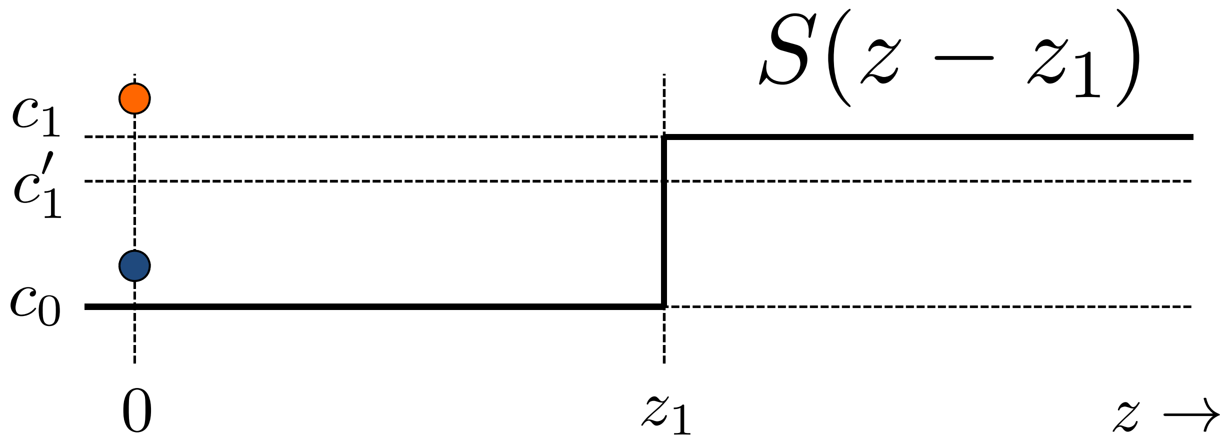
$$\left(\frac{\partial G(\mathbf{r}_g,\mathbf{r}_s)}{\partial s(\mathbf{r})}\right)_2 = -\omega^2 G(\mathbf{r}_g,\mathbf{r})G(\mathbf{r},\mathbf{r}_s)\left(1+\frac{\delta P^*(\mathbf{r}_g,\mathbf{r}_s)G(\mathbf{r},\mathbf{r})}{G^*(\mathbf{r}_g,\mathbf{r})G^*(\mathbf{r},\mathbf{r}_s)}\right)$$

engages the data nonlinearly through appearance of δP

consistent with 1st order sensitivities in the limit of small residuals

Nonlinear sensitivities and reflectivity

Analytic / numerical example



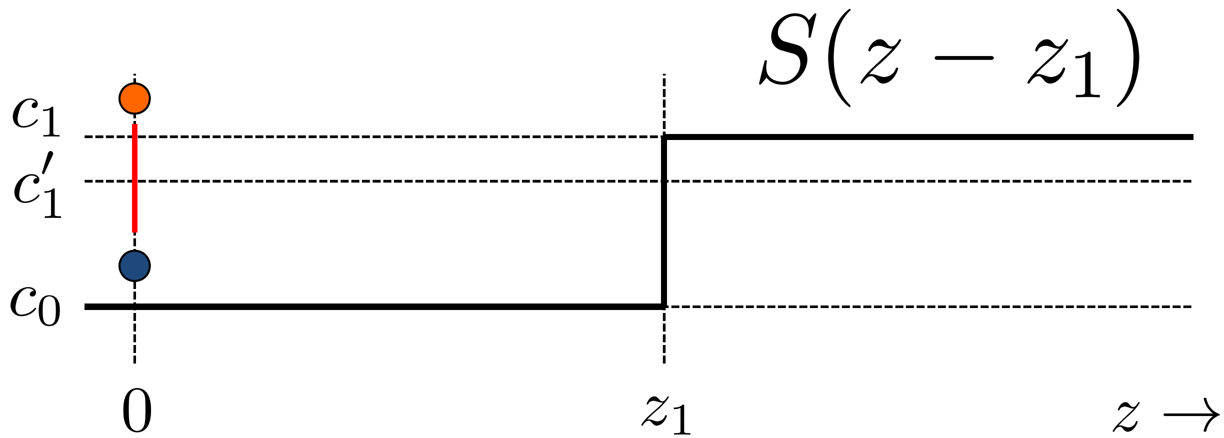
Scalar (V_P) model with one reflecting interface

Source, receiver collocated at $z = 0$

$$P(z_g = 0, z_s = 0) = \frac{1}{i2k} + R\frac{e^{i2kz_1}}{i2k}$$

Nonlinear sensitivities and reflectivity

Analytic / numerical example



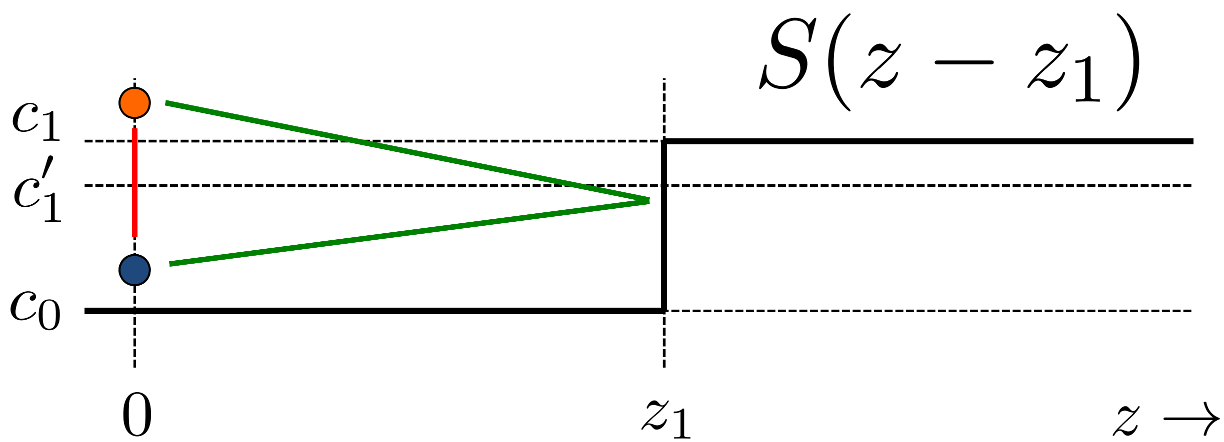
Scalar (V_P) model with one reflecting interface

Source, receiver collocated at $z = 0$

$$P(z_g = 0, z_s = 0) = \underbrace{\frac{1}{i2k}}_{\text{direct}} + R\frac{e^{i2kz_1}}{i2k}$$

Nonlinear sensitivities and reflectivity

Analytic / numerical example



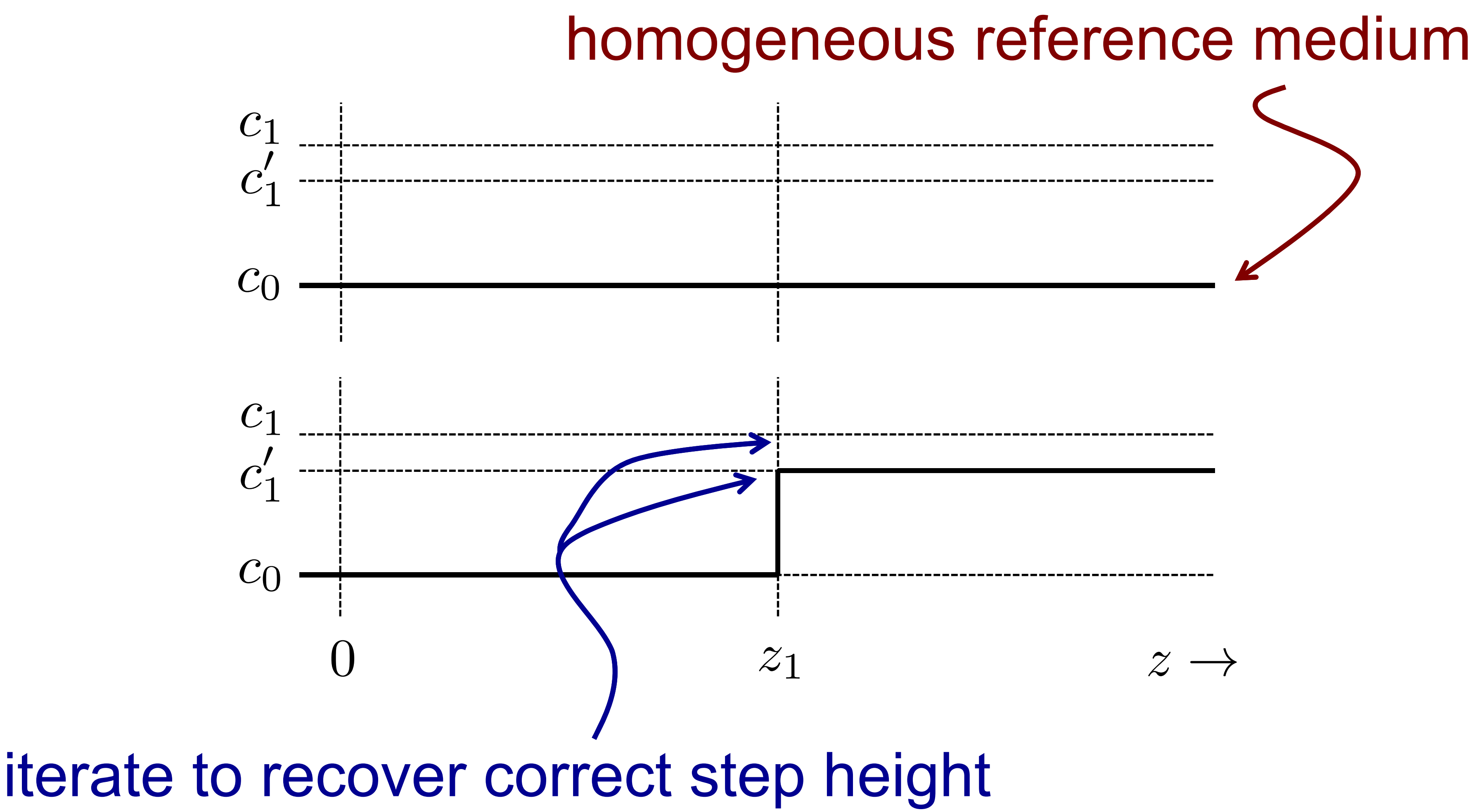
Scalar (V_P) model with one reflecting interface

Source, receiver collocated at $z = 0$

$$P(z_g = 0, z_s = 0) = \underbrace{\frac{1}{i2k}}_{\text{direct}} + \underbrace{R\frac{e^{i2kz_1}}{i2k}}_{\text{reflected}}$$

Nonlinear sensitivities and reflectivity

Analytic / numerical example



Nonlinear sensitivities and reflectivity

gradient based on 1st order sensitivities

$$g_1(z) = \frac{\pi c_0^3 R}{4} S(z - z_1)$$

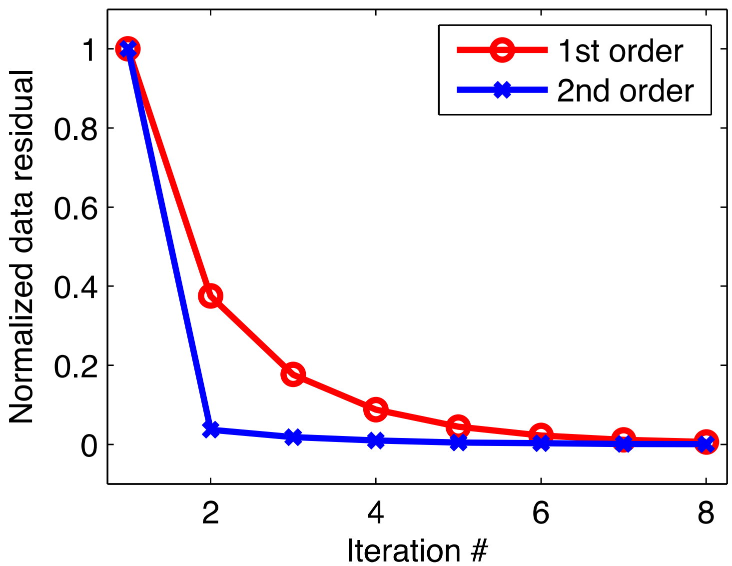
$$g_2(z) = \frac{\pi c_0^3}{4} [R - 2R^2] S(z - z_1)$$

gradient based on 2nd order sensitivities

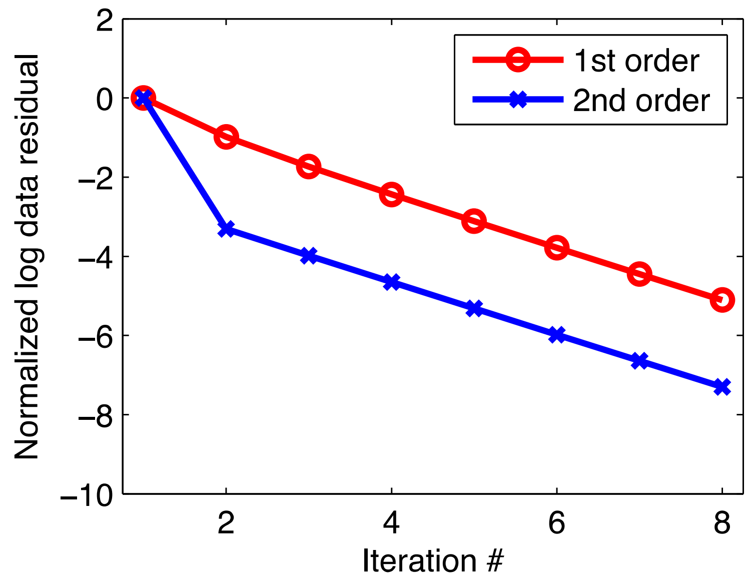
$$H_{\mathrm{GN}}^{-1}(z, z') = \frac{16}{c_0^5 \pi} \delta(z - z')$$

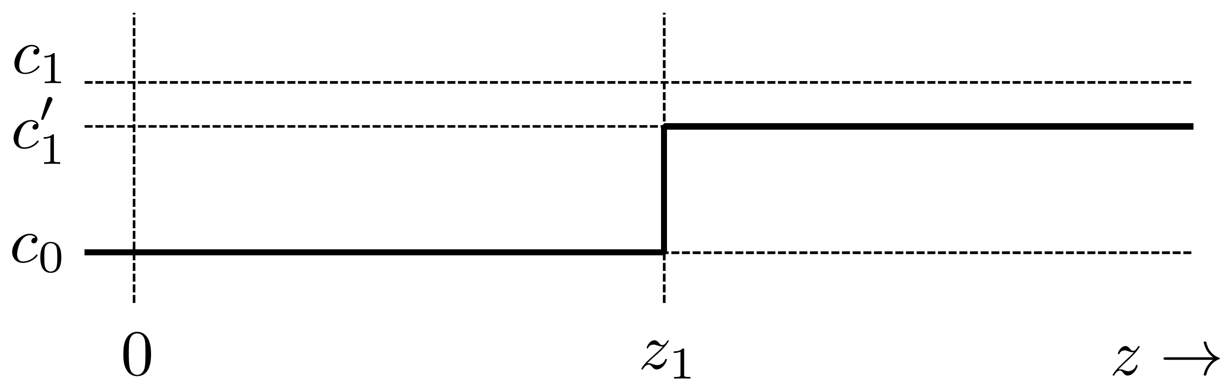
Nonlinear sensitivities and reflectivity

Data residuals



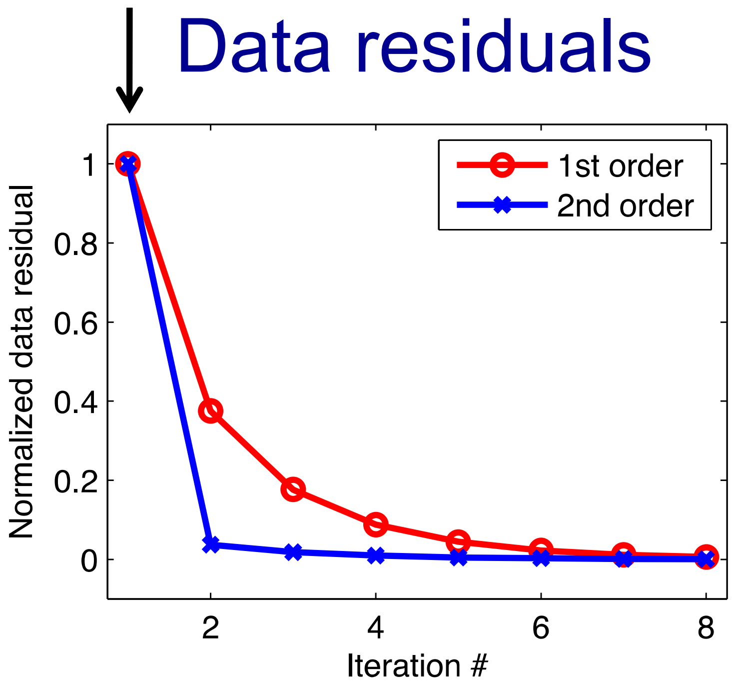
Log data residuals



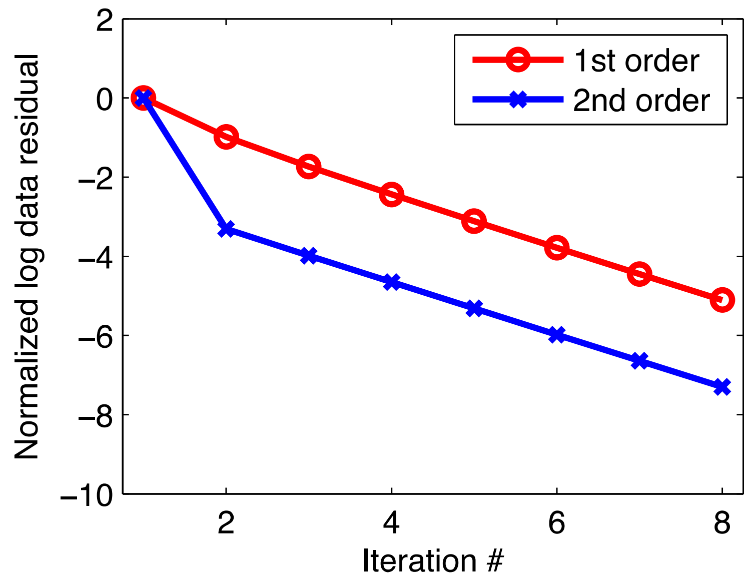


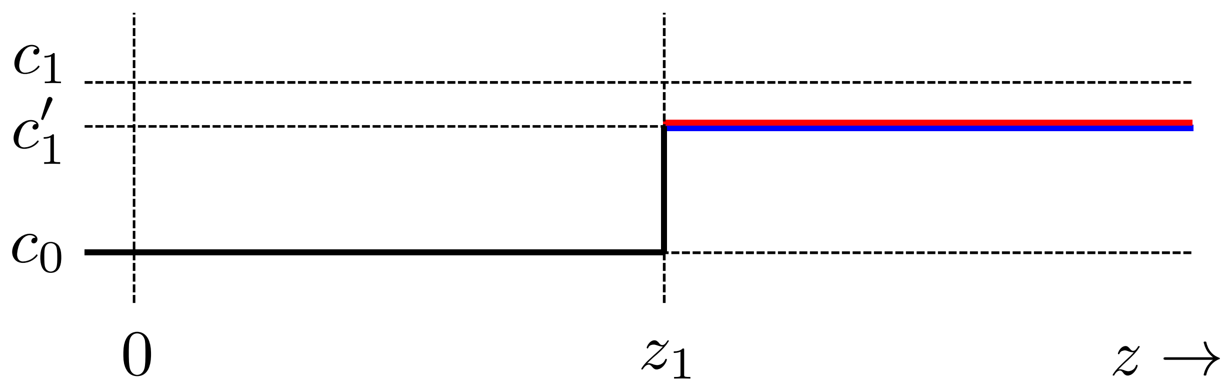
Nonlinear sensitivities and reflectivity

Data residuals



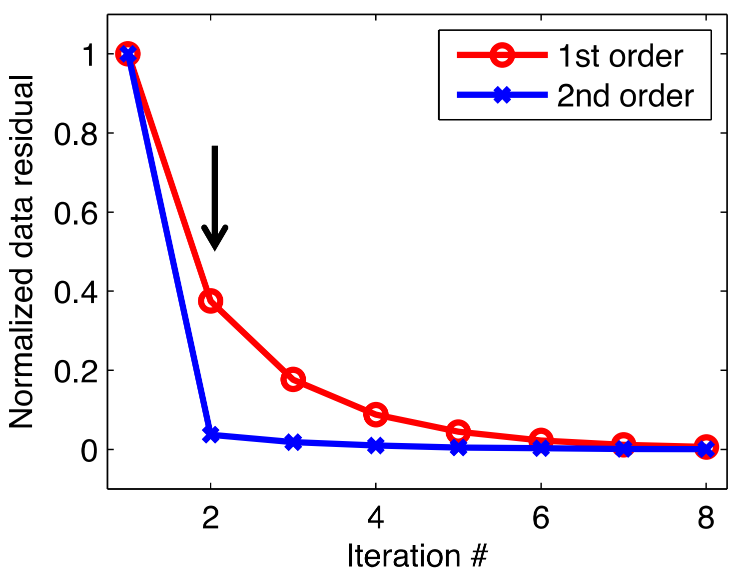
Log data residuals



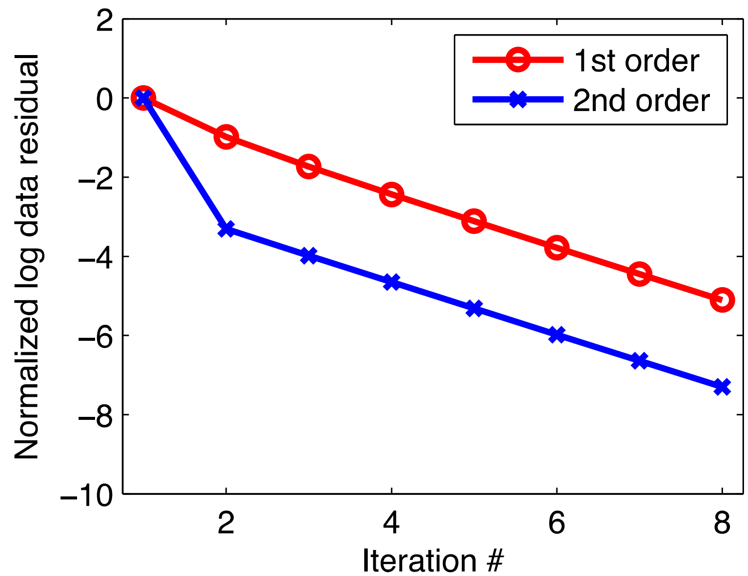


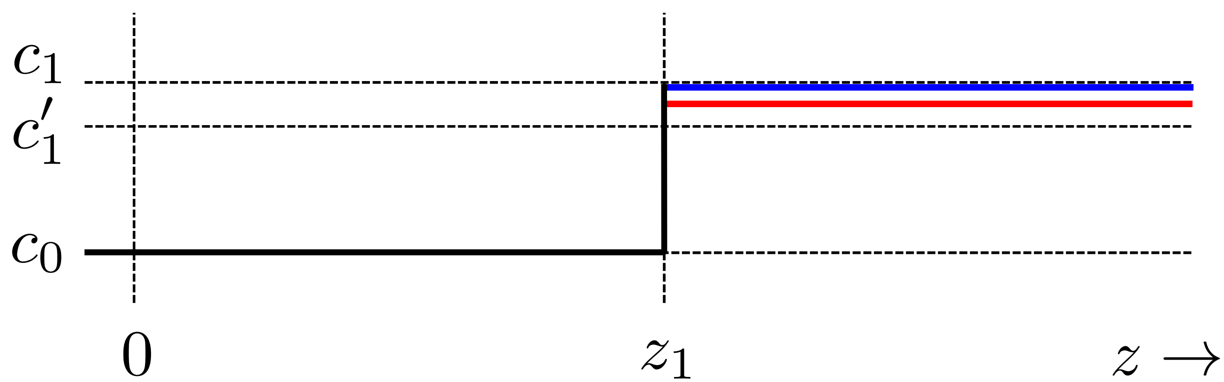
Nonlinear sensitivities and reflectivity

Data residuals



Log data residuals





Conclusions

Flexible elastic / petrophysical parameterization, joint and/or independent use of PP, PS, SS modes, are included in FWI / IMMI formulation.

Goal: what we know about AVO inversion and quantitative interpretation becomes internal to FWI

Key geological information resides in high angle AVO, where, from FWI perspective, nonlinearity reigns

Nonlinear sensitivities impact nonlinear reflectivity; route to pulling high angle information into updates? Nonlinear Hessian – transmission nonlinearity?

Acknowledgments

CREWES sponsors & researchers

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