2D Internal multiple prediction in the $\tau - p_s - p_g$ domain

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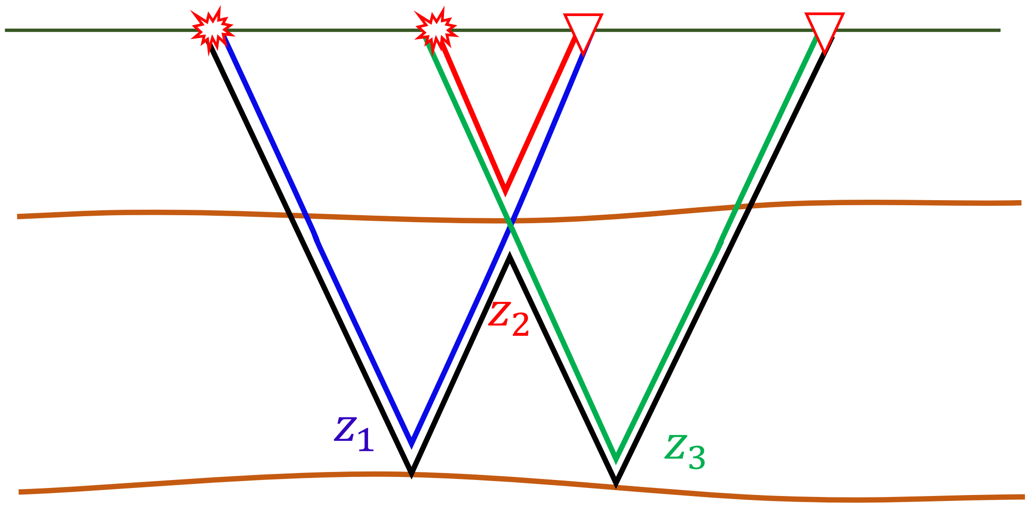
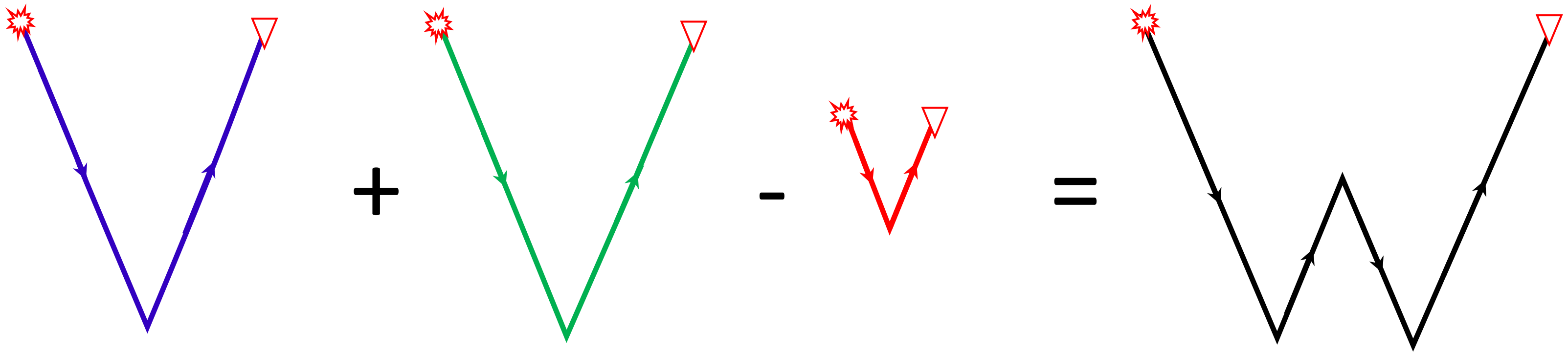
Outline

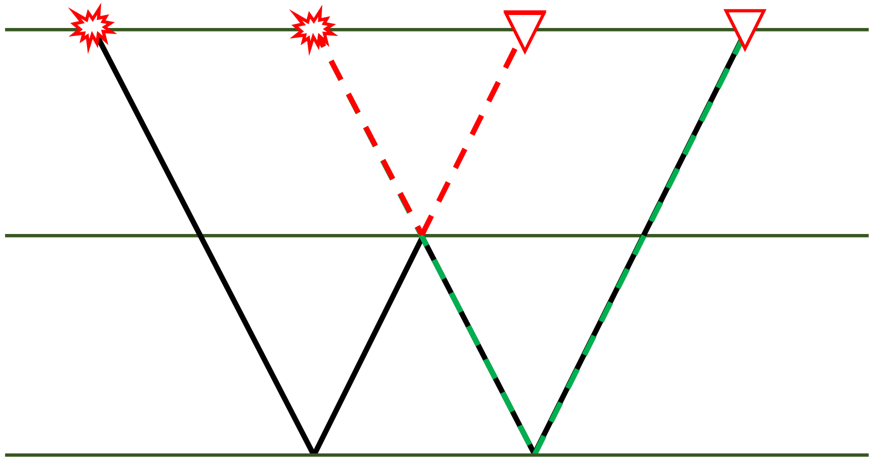
- Review of Inverse scattering series (ISS) in multiple prediction
- Internal multiples (IMs) prediction: 1.5D to 2D
- Double $\tau - p_s - p_g$ transform
- Synthetic example of 2D IMs prediction
- Conclusion and future work
- Acknowledgements

Inverse scattering series





IMs prediction: 1.5D to 2D



$$b_{3_{IM}}(p,\omega) = \int_{-\infty}^{+\infty} d\tau e^{i\omega\tau} b_1(p,\tau) \int_{-\infty}^{\tau-\epsilon} d\tau' e^{-i\omega\tau'} b_1(p,\tau') \int_{\tau'+\epsilon}^{+\infty} d\tau'' e^{i\omega\tau''} b_1(p,\tau'')$$

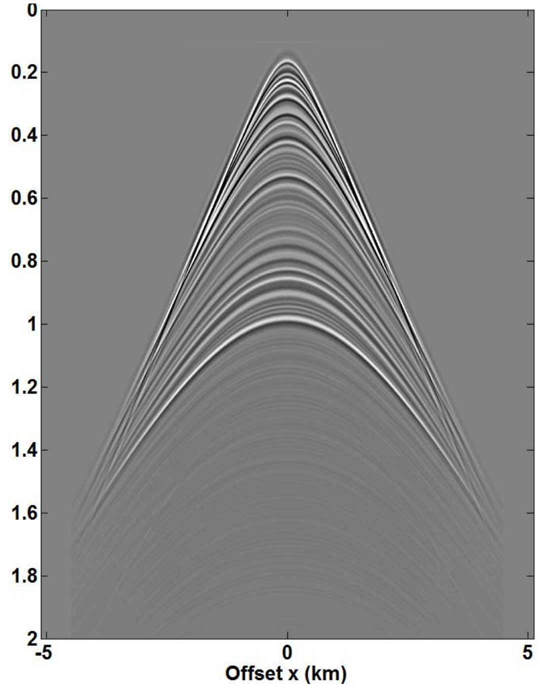
$where$ p is the horizontal slowness, or ray parameter,

τ is the intercept times for primaries.

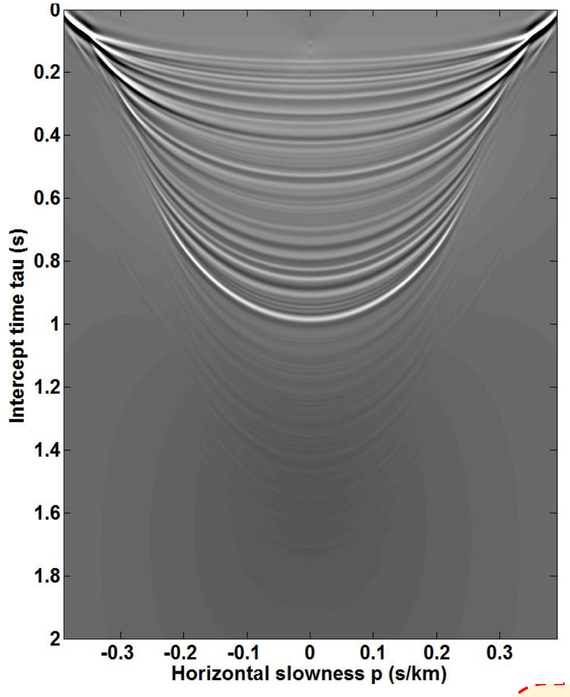
$$b_1(p,\tau) = -i2q_s \mathrm{D}(p,\tau)$$

IMs prediction: 1.5D to 2D

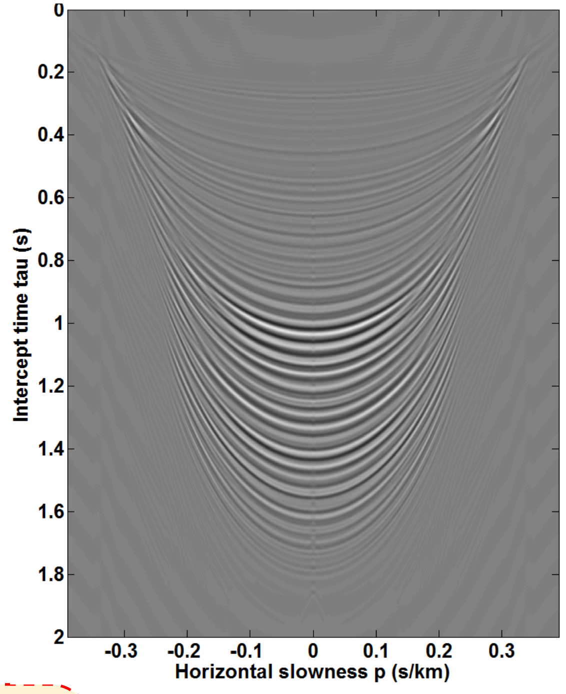
Synthetic in offset domain



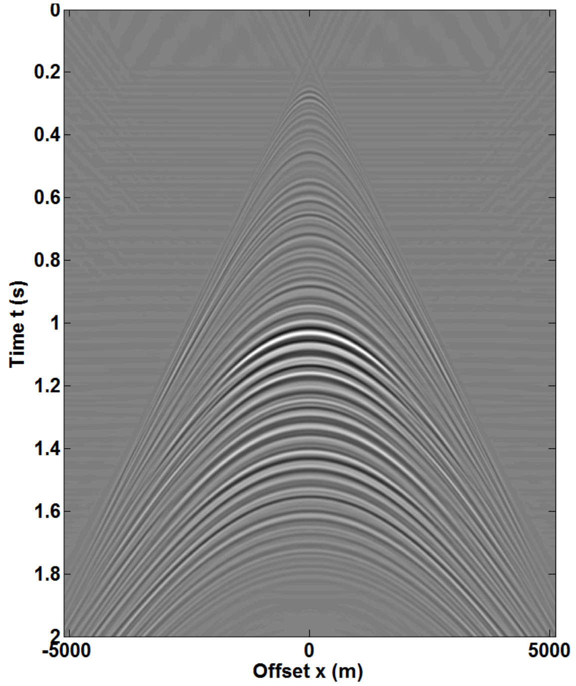
Synthetic in $\tau - p$ domain



Prediction in $\tau - p$ domain



Prediction in offset domain



$\mathrm{d}(x,t)$ → $D(p,\tau)$ → $b_1(p,\tau)$ → $b_{3_{IM}}(p,\omega)$ → $b_{3_{IM}}(p,\tau)$ → $b_{3_{IM}}(x,t)$

IMs prediction: 1.5D to 2D

Weglein (1993, 1997):

$$b_{3_{IM}}(k_g, k_s, \omega) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{+\infty} dk_1 e^{-iq_1(\varepsilon_g - \varepsilon_s)} dk_2 e^{-iq_2(\varepsilon_g - \varepsilon_s)} \int_{-\infty}^{+\infty} dz e^{i(q_g + q_1)z} b_1(k_g, k_1, z) \times \int_{-\infty}^{z-\epsilon} dz' e^{-i(q_1+q_2)z'} b_1(k_1, k_2, z') \int_{z'+\epsilon}^{+\infty} dz'' e^{i(q_2+q_s)z''} b_1(k_2, k_s, z'')$$

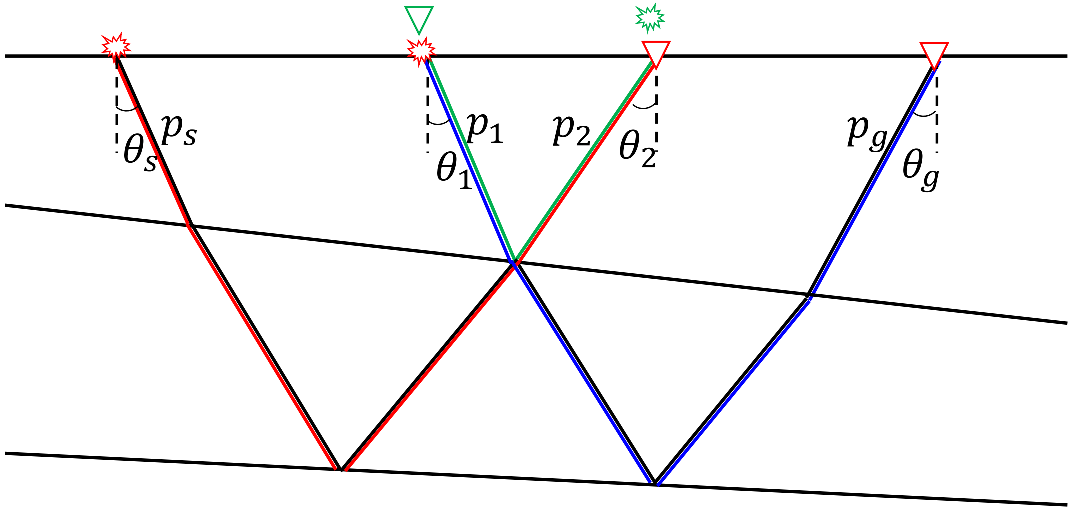
Where,

$$q_x = \frac{\omega}{c_0}\sqrt{1 - \frac{{k_x}^2 {c_0}^2}{\omega^2}};$$

$$k_z = q_g + q_s;$$

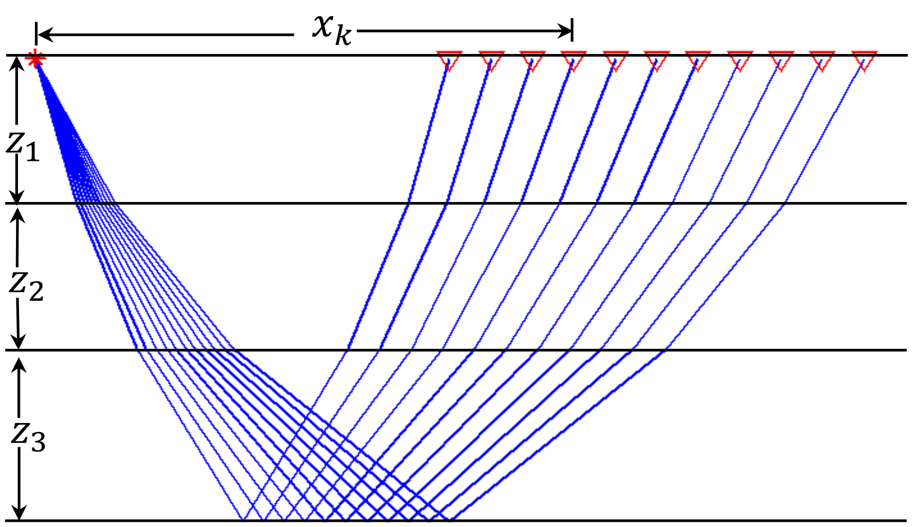
Coates (1996):

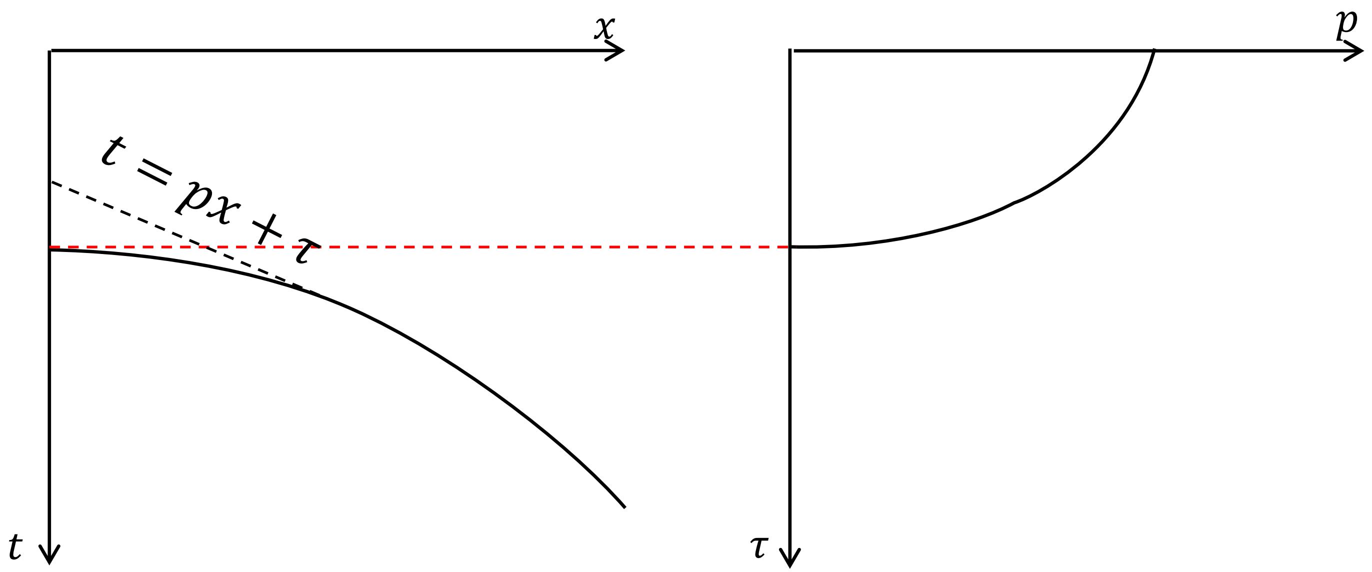
$$b_{3_{IM}}(p_g, p_s, \omega) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{+\infty} dp_1 e^{-i\omega(\tau_{1g} - \tau_{1s})} dp_2 e^{-i\omega(\tau_{2g} - \tau_{2s})} \int_{-\infty}^{+\infty} d\tau e^{i\omega\tau} b_1(p_g, p_1, \tau) \times \int_{-\infty}^{\tau-\epsilon} d\tau' e^{-i\omega\tau'} b_1(p_1, p_2, \tau') \int_{\tau'+\epsilon}^{+\infty} d\tau'' e^{i\omega\tau''} b_1(p_2, p_s, \tau'')$$



$$\boldsymbol{b_1(p_g, p_s, \tau) = -i2q_s D(p_g, p_s, \tau)}$$

Double $\tau - p_s - p_g$ transform





$$t = px + 2\sum_i z_{si} q_i$$

$$\tau = 2\sum_i z_{si} q_i$$

$$\begin{cases} \psi(p,\tau) = \int \Psi(x, \tau + px)dx & \text{Time domain} \\ \phi(p,\omega) = \int \varphi(x,\omega)e^{i\omega px}dx & \text{Frequency domain} \end{cases}$$

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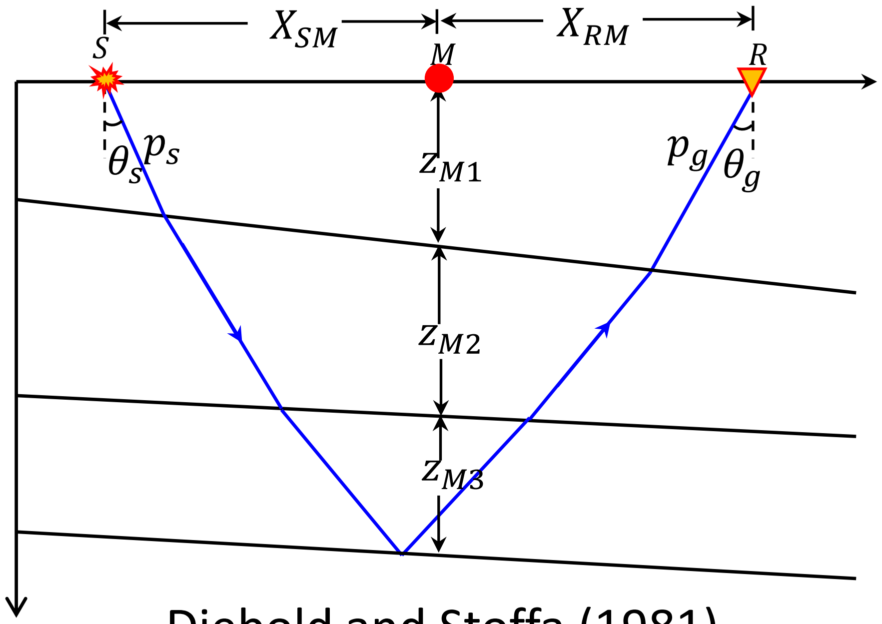
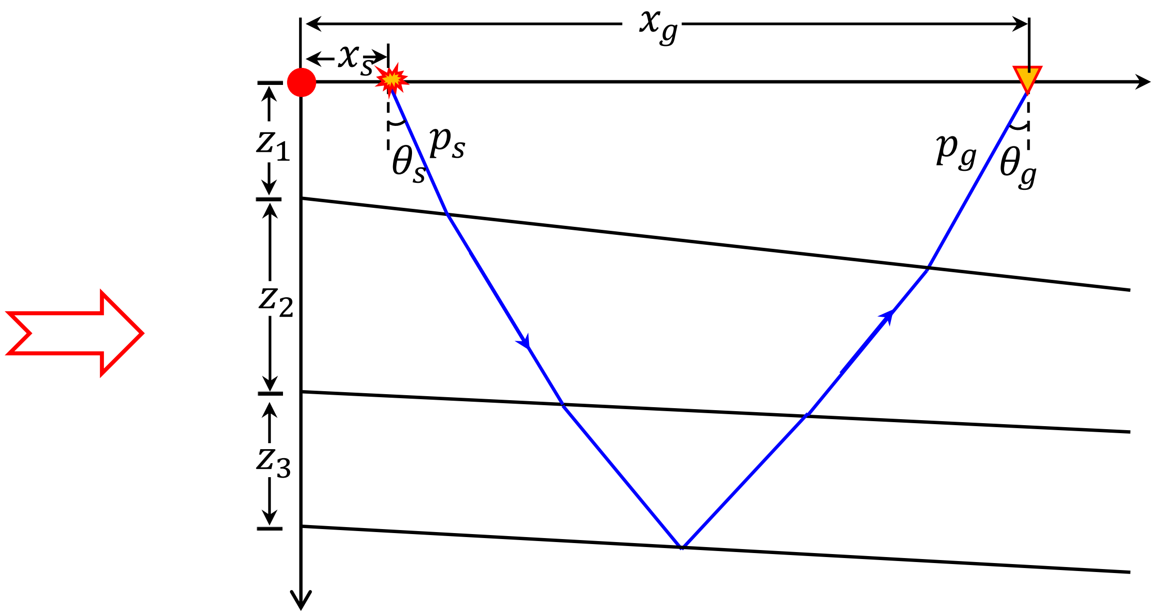
Double $\tau - p_s - p_g$ transform



Diebold and Stoffa (1981)



$$X_{SM} + X_{RM} = X$$

$$T = X_{SM}p_s + X_{RM}p_g + \sum_i z_{Mi}(q_{si} + q_{gi})$$

$$p_s = \frac{sin\theta_s}{v_0} > 0; \qquad p_g = \frac{sin\theta_g}{v_0} > 0.$$

$$x_g - x_s = X$$

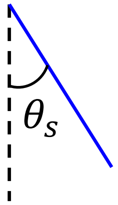
$$T = (-x_s)p_s + x_g p_g + \sum_i z_i(q_{si} + q_{gi})$$

$$p_s = \frac{sin\theta_s}{v_0} > 0; \qquad p_g = \frac{sin\theta_g}{v_0} > 0.$$

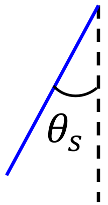
Double $\tau - p_s - p_g$ transform

$$T = -x_s p_s + x_g p_g + \sum_i z_i (q_{si} + q_{gi})$$

For source:

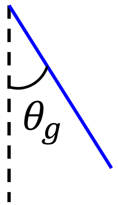


$\Rightarrow$ $\theta_s > 0$ $\Rightarrow$ $p_s = \frac{sin\theta_s}{v_0} > 0$

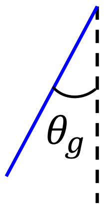


$\Rightarrow$ $\theta_s < 0$ $\Rightarrow$ $p_s = \frac{sin\theta_s}{v_0} < 0$

For receiver:

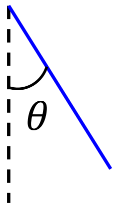


$\Rightarrow$ $\theta_g < 0$ $\Rightarrow$ $p_g = \frac{sin\theta_g}{v_0} < 0$

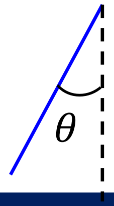


$\Rightarrow$ $\theta_g > 0$ $\Rightarrow$ $p_s = \frac{sin\theta_g}{v_0} > 0$

For source and receiver:



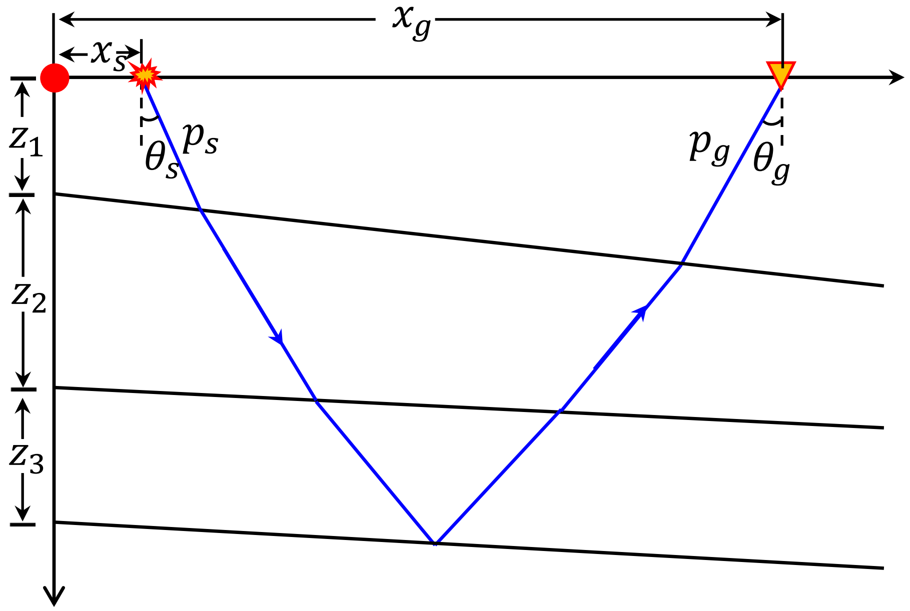
$\Rightarrow$ $\theta < 0$ $\Rightarrow$ $p = \frac{sin\theta}{v_0} < 0$



$\Rightarrow$ $\theta > 0$ $\Rightarrow$ $p = \frac{sin\theta}{v_0} > 0$

$$T = x_s p_s + x_g p_g + \sum_i z_i (q_{si} + q_{gi})$$

Double $\tau - p_s - p_g$ transform



$$\theta_s < 0 \quad \Rightarrow \quad p_s = \frac{sin\theta_s}{v_0} < 0$$

$$\theta_g > 0 \quad \Rightarrow \quad p_g = \frac{sin\theta_g}{v_0} > 0$$

$$T = x_s p_s + x_g p_g + \sum_i z_i (q_{si} + q_{gi})$$

Time domain:

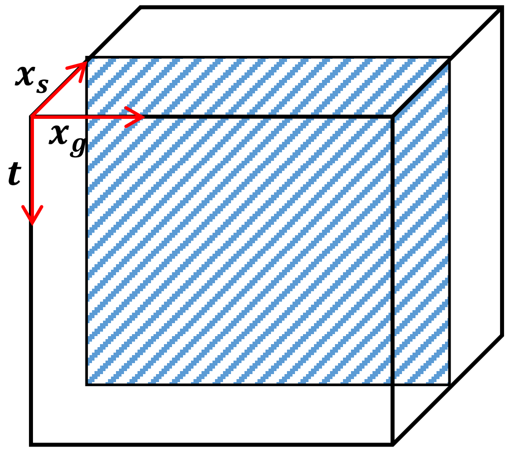
$$D(p_s, p_g, \tau) = \iint d(x_s, x_g, \tau + p_s x_s + p_g x_g) dx_s dx_g$$

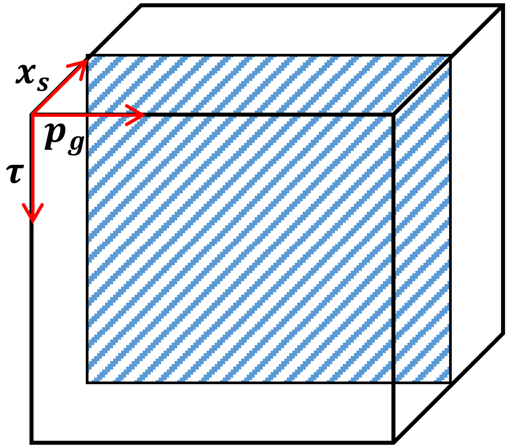
Frequency domain:

$$\tilde{D}(p_s, p_g, \omega) = \iint_{-\infty}^{+\infty} d(x_s, x_g, \omega) e^{+i\omega(p_s x_s + p_g x_g)} dx_s dx_g$$

Double $\tau - p_s - p_g$ transform

$d(x_s, x_g, t)$

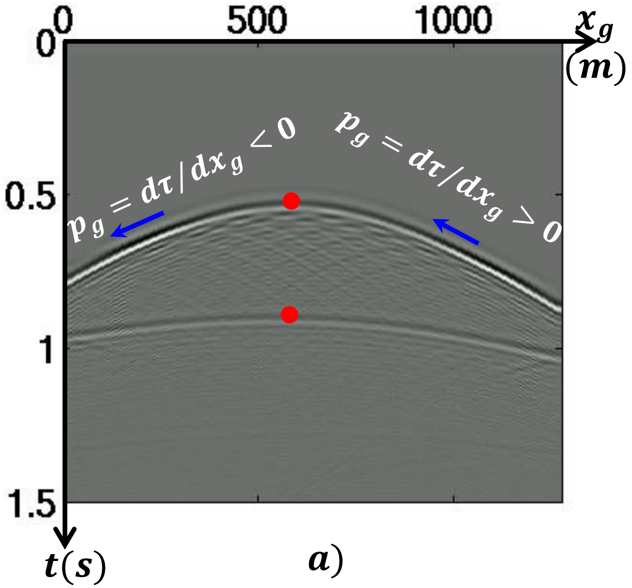


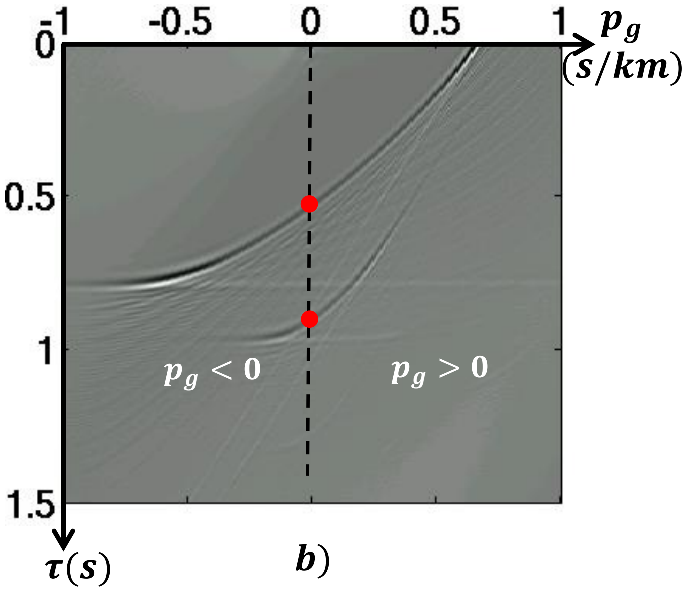


$\hat{d}(x_s, p_g, \tau)$

$d(x_g, t)$

$x_s = 636m$



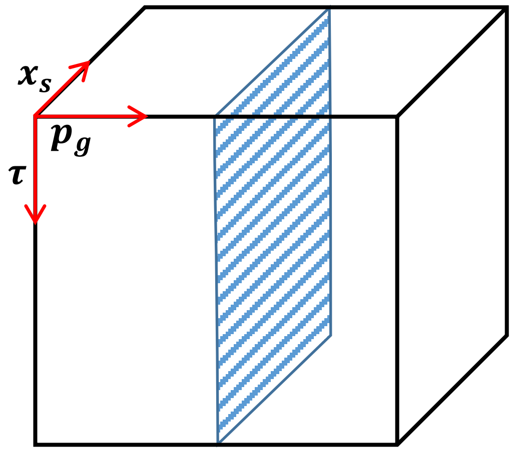


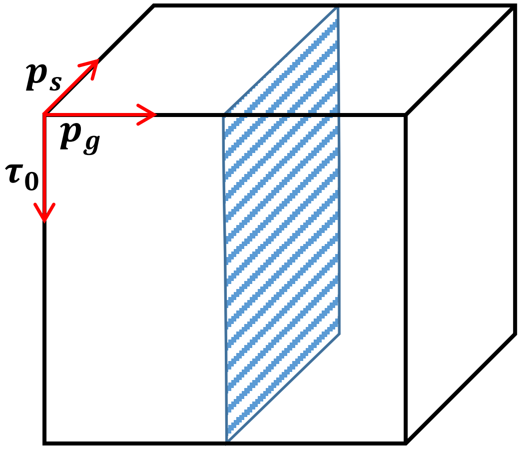
$\hat{d}(p_g, \tau)$

$x_s = 636m$

Double $\tau - p_s - p_g$ transform

$\hat{d}(x_s, p_g, \tau)$

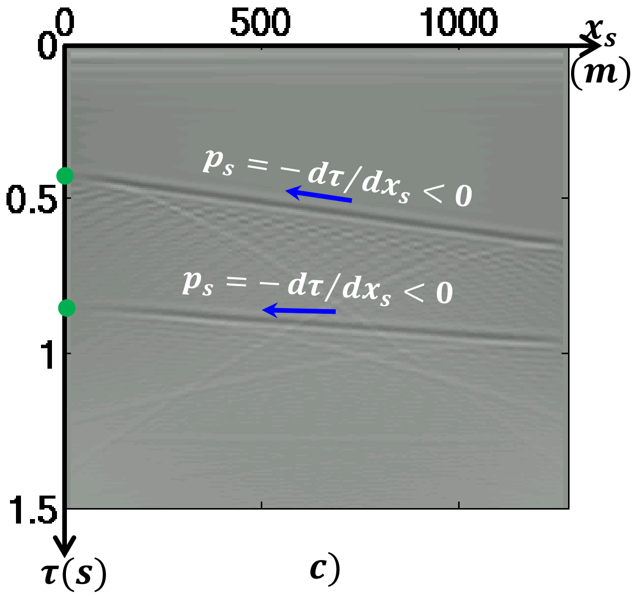


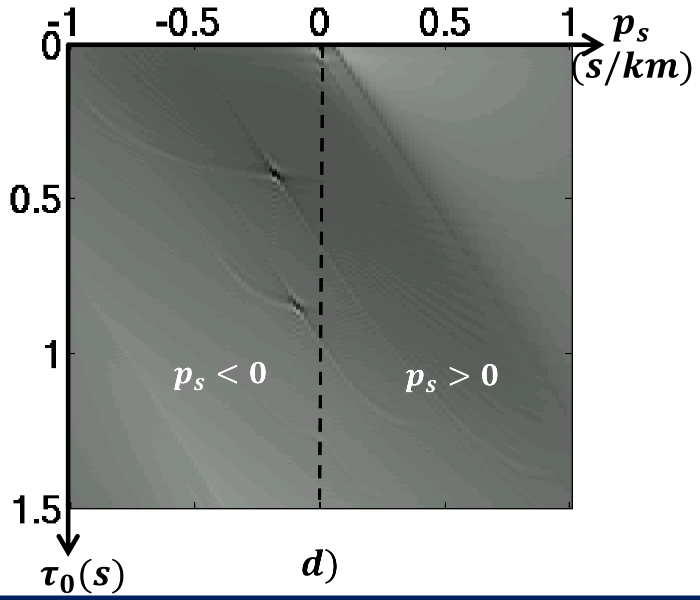


$D(p_s, p_g, \tau_0)$

$\hat{d}(x_s, \tau)$

$p_g = 0$

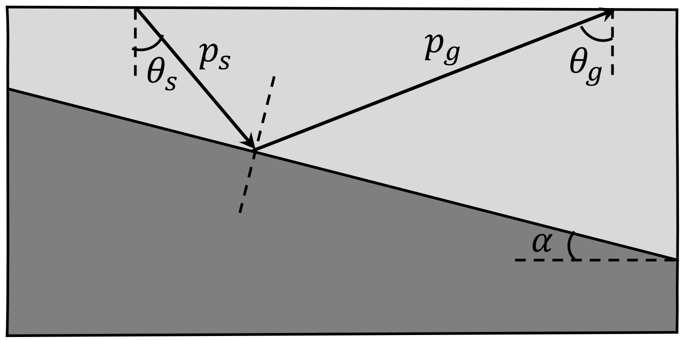




$D(p_s, \tau_0)$

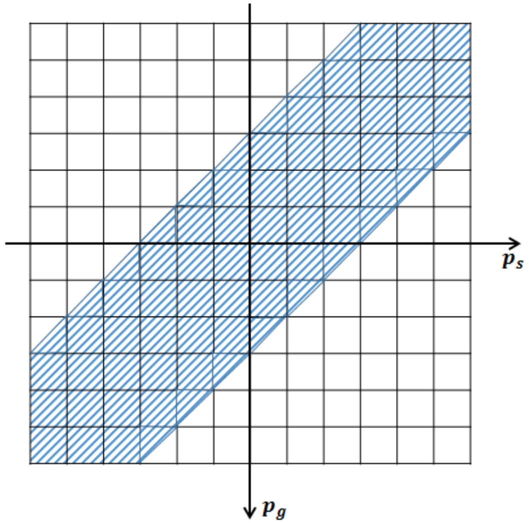
$p_g = 0$

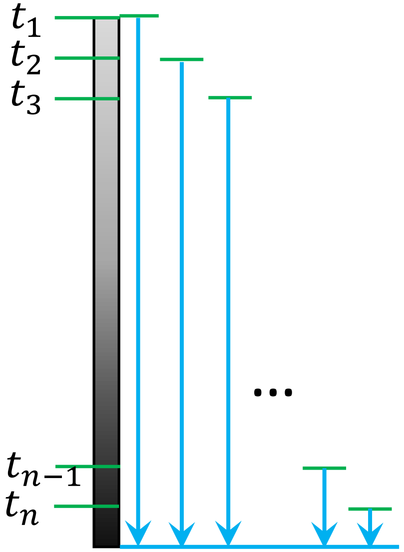
Matrix multiplication

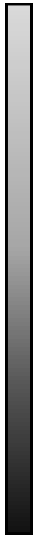


$$\left||p_s| - |p_g|\right| \le \frac{2\sin\alpha}{v_{min}}$$

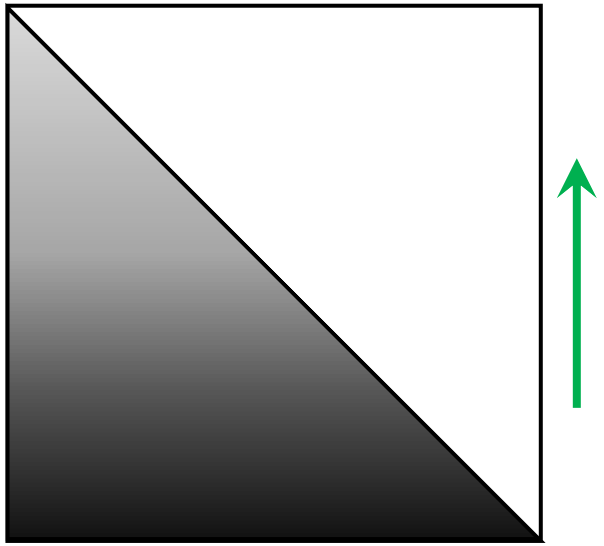
(Liu et al., 2000)





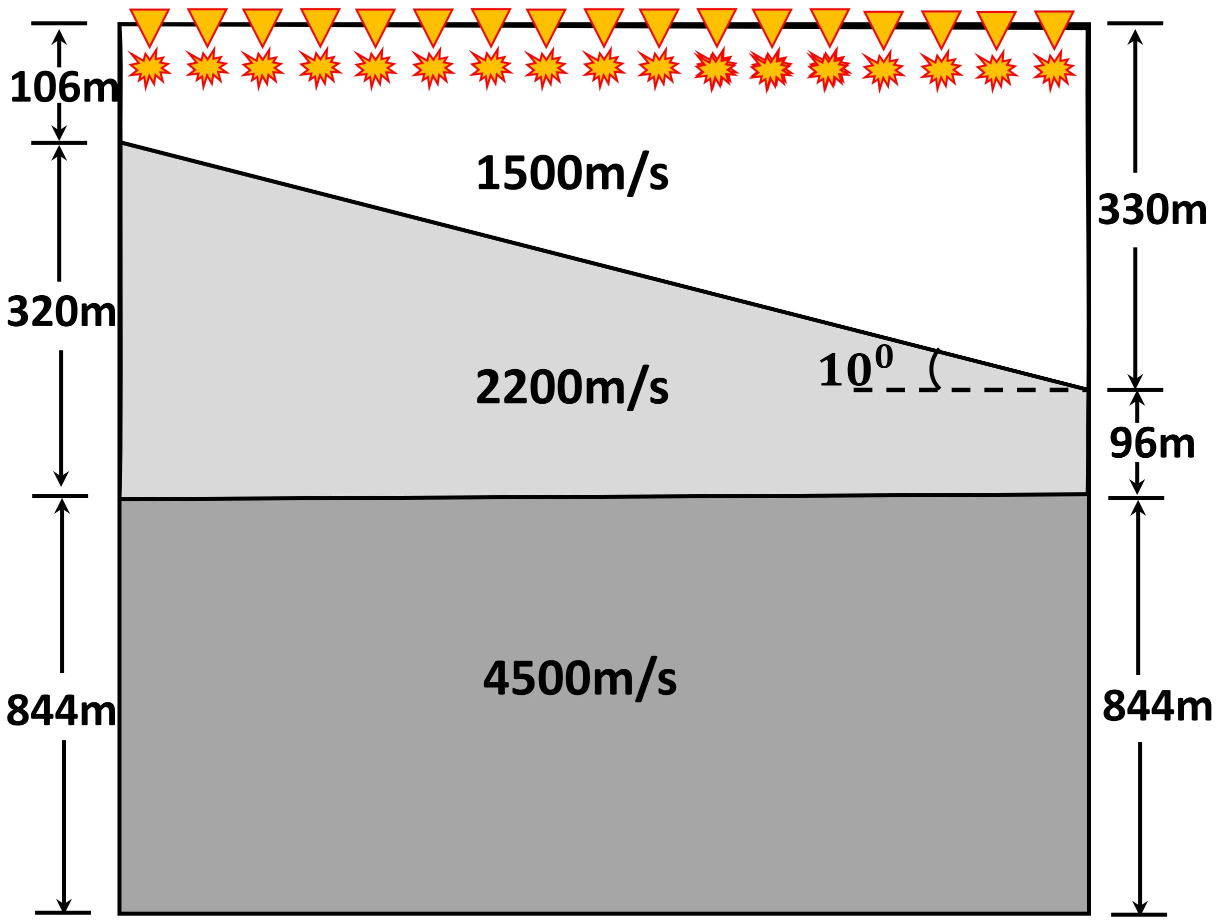


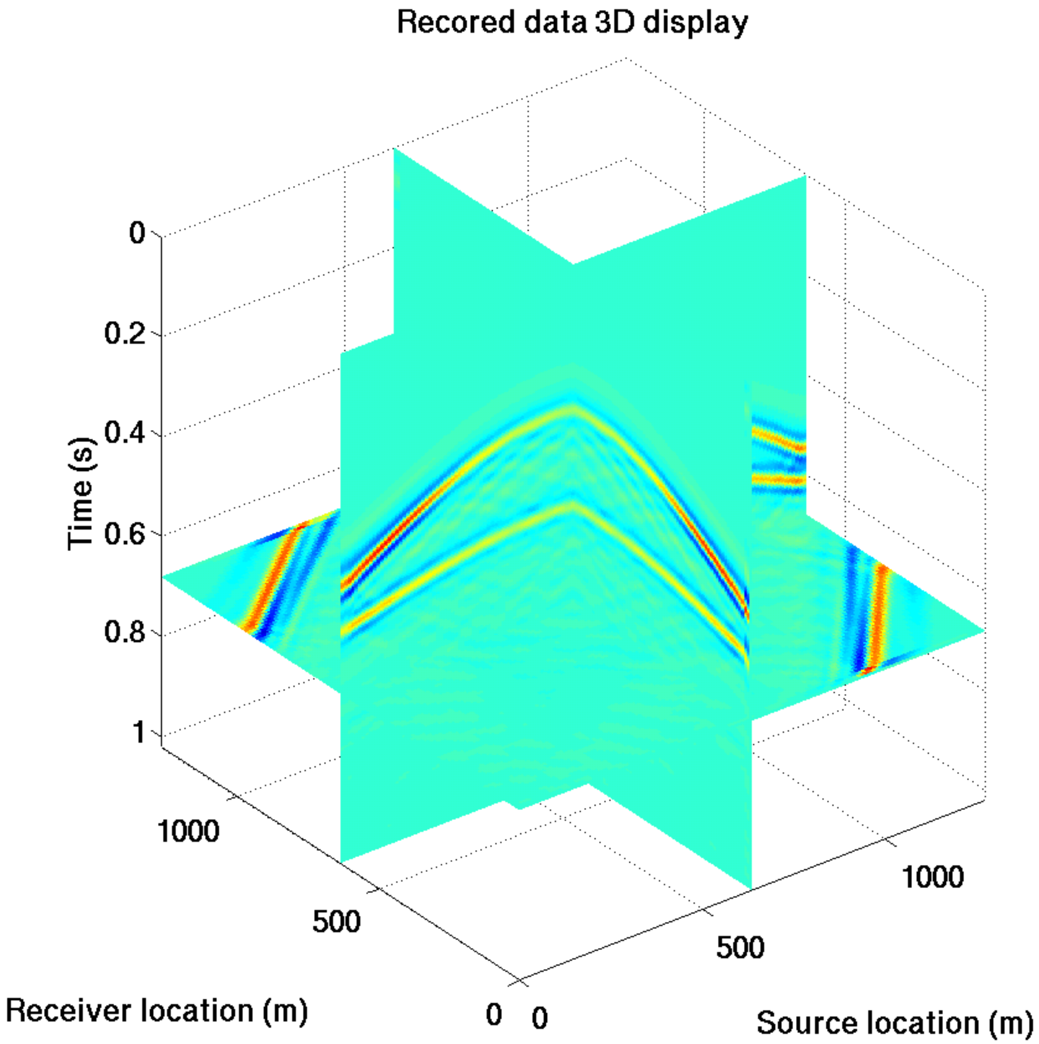
$$\times \begin{bmatrix} 1 & & & & & \\ 1 & 1 & & & & \\ 1 & 1 & 1 & & & \\ \vdots & & & \ddots & & \\ 1 & 1 & \cdots & 1 & 1 & \\ 1 & 1 & 1 & \cdots & 1 & 1 \end{bmatrix} =$$



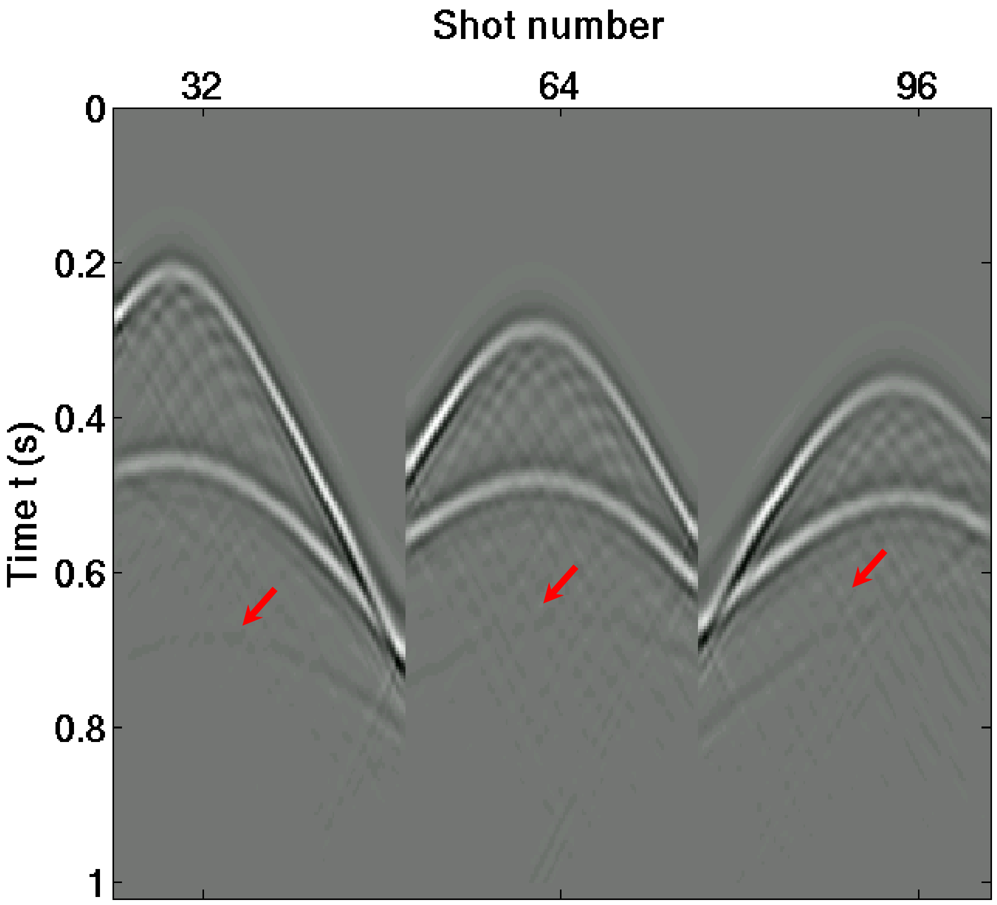
Summation

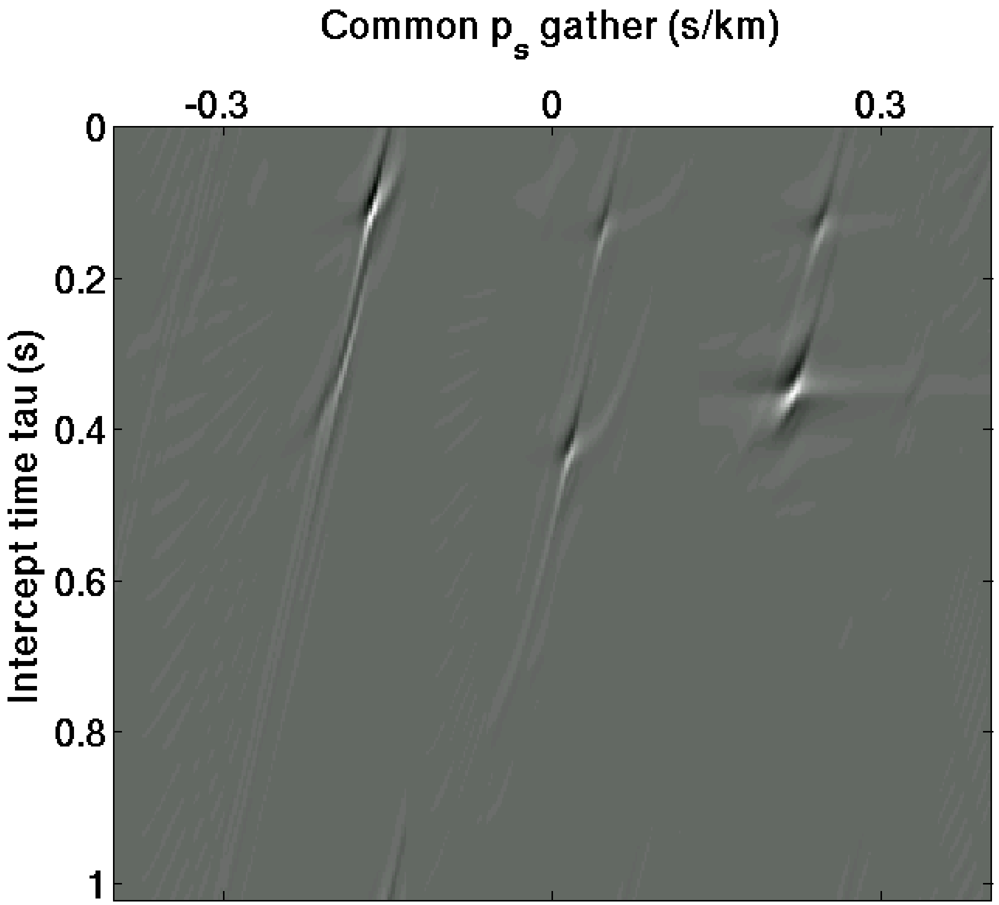
Synthetic example





Synthetic example

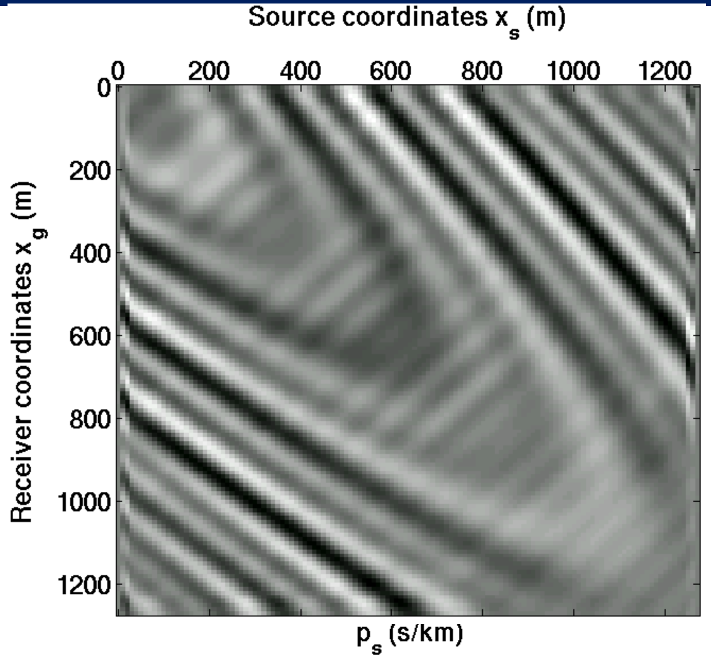


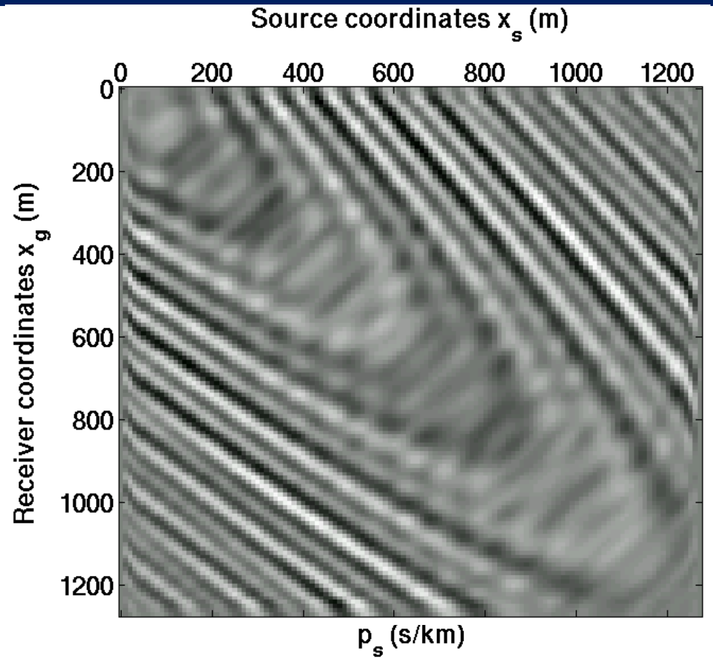


Synthetic example

$15Hz$

$d(x_s, x_g)$



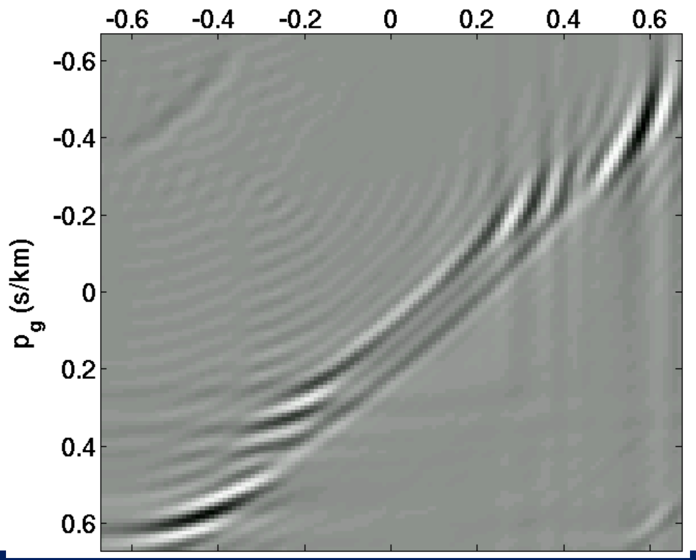


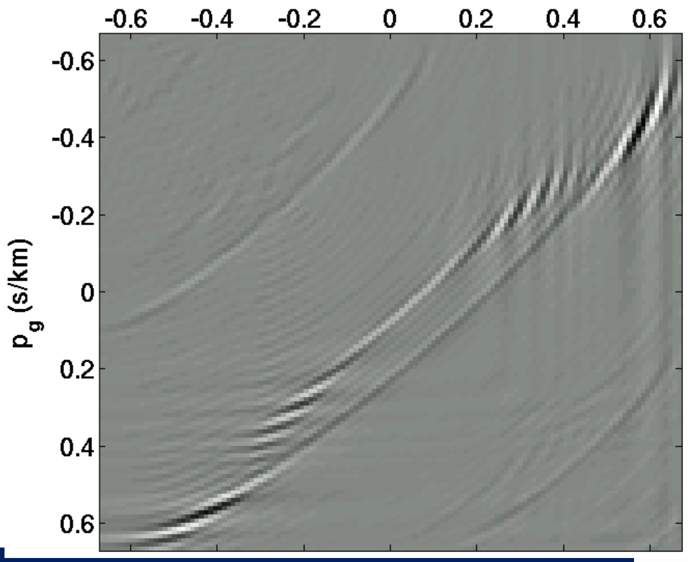
$25Hz$

$d(x_s, x_g)$

$15Hz$

$D(p_s, p_g)$





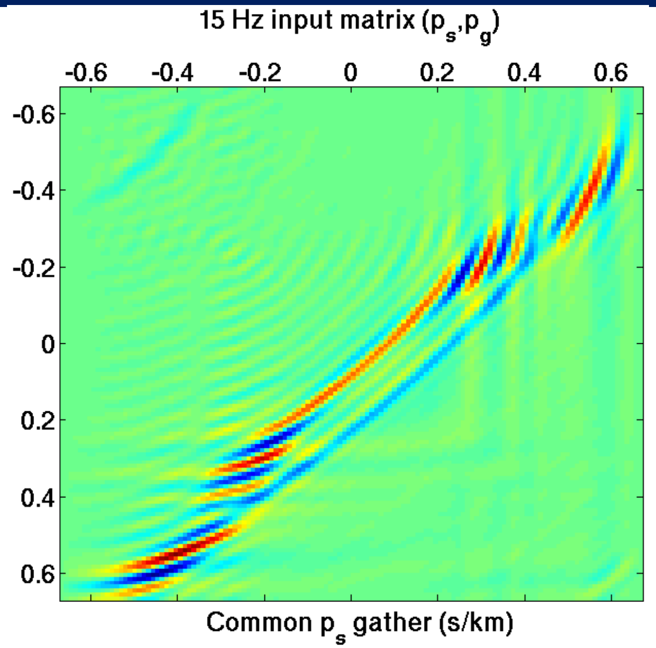
$25Hz$

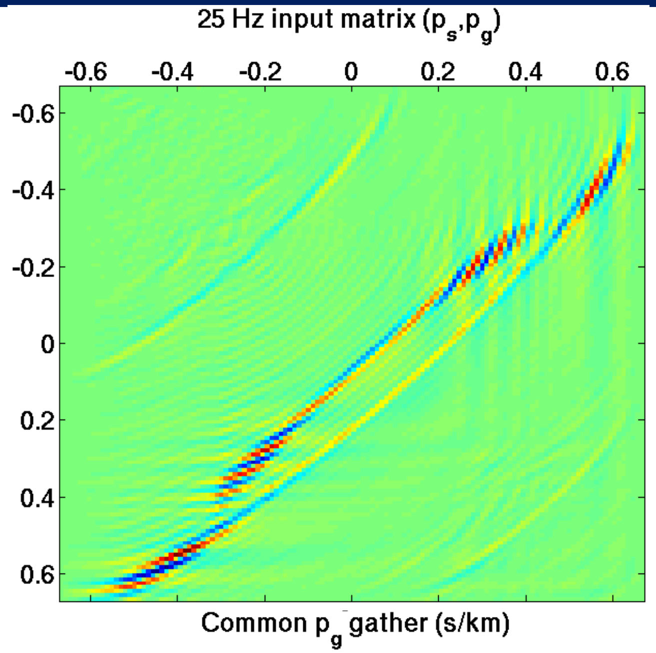
$D(p_s, p_g)$

Synthetic example

$15Hz$

$b1(p_s, p_g)$



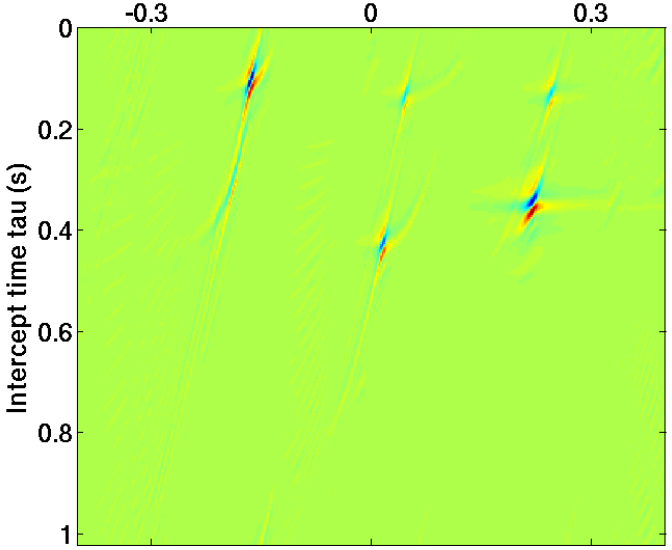


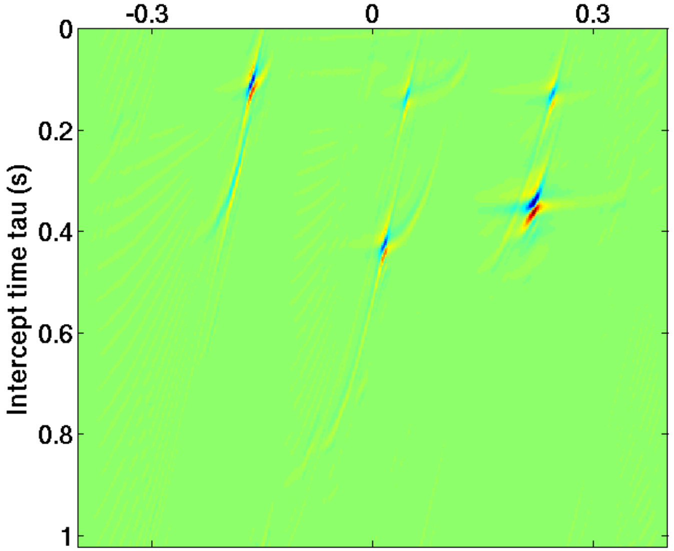
$25Hz$

$b1(p_s, p_g)$

$b1(p_s, \tau)$

$p_s = -0.3, 0, 0.3$

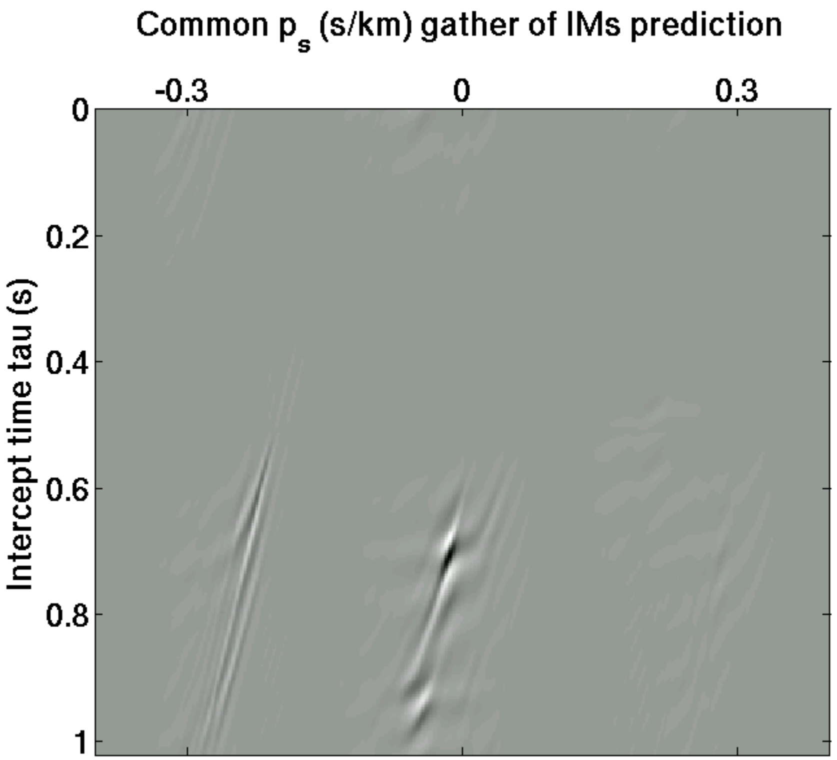


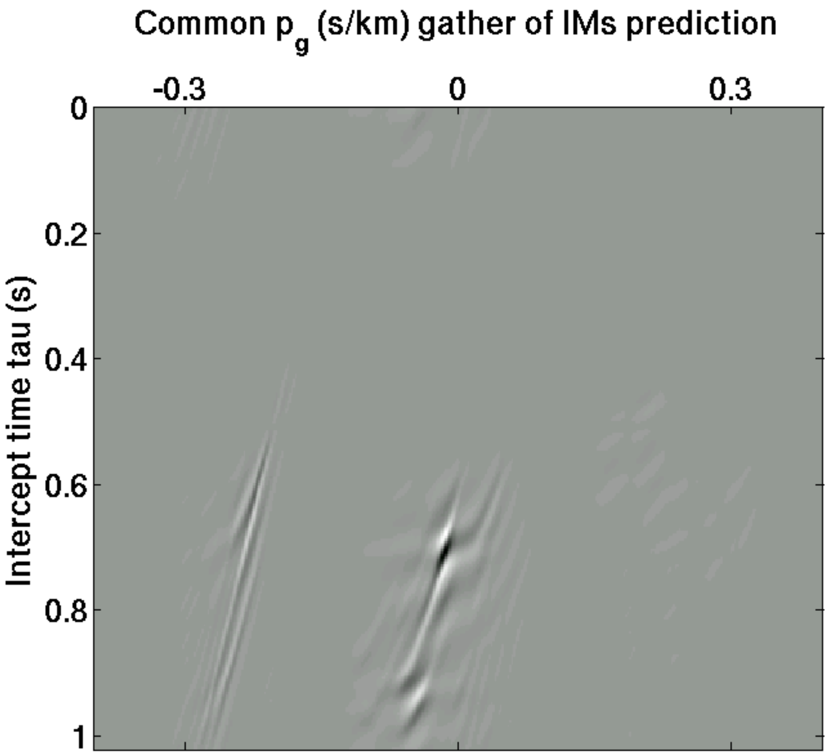


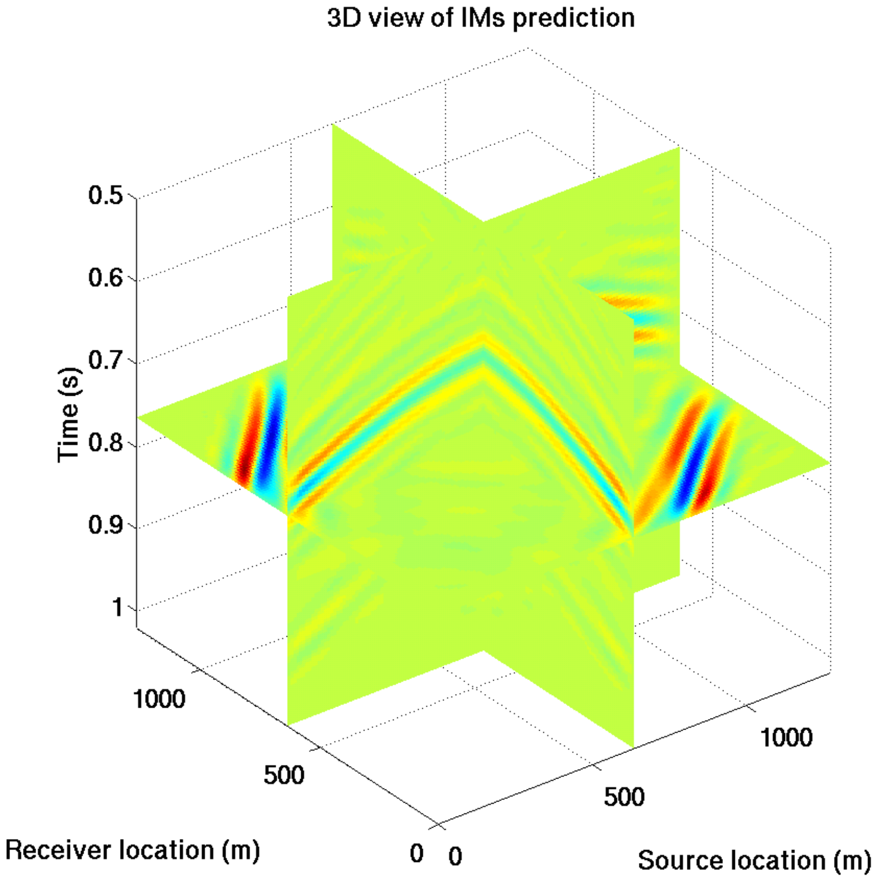
$b1(p_g, \tau)$

$p_g = -0.3, 0, 0.3$

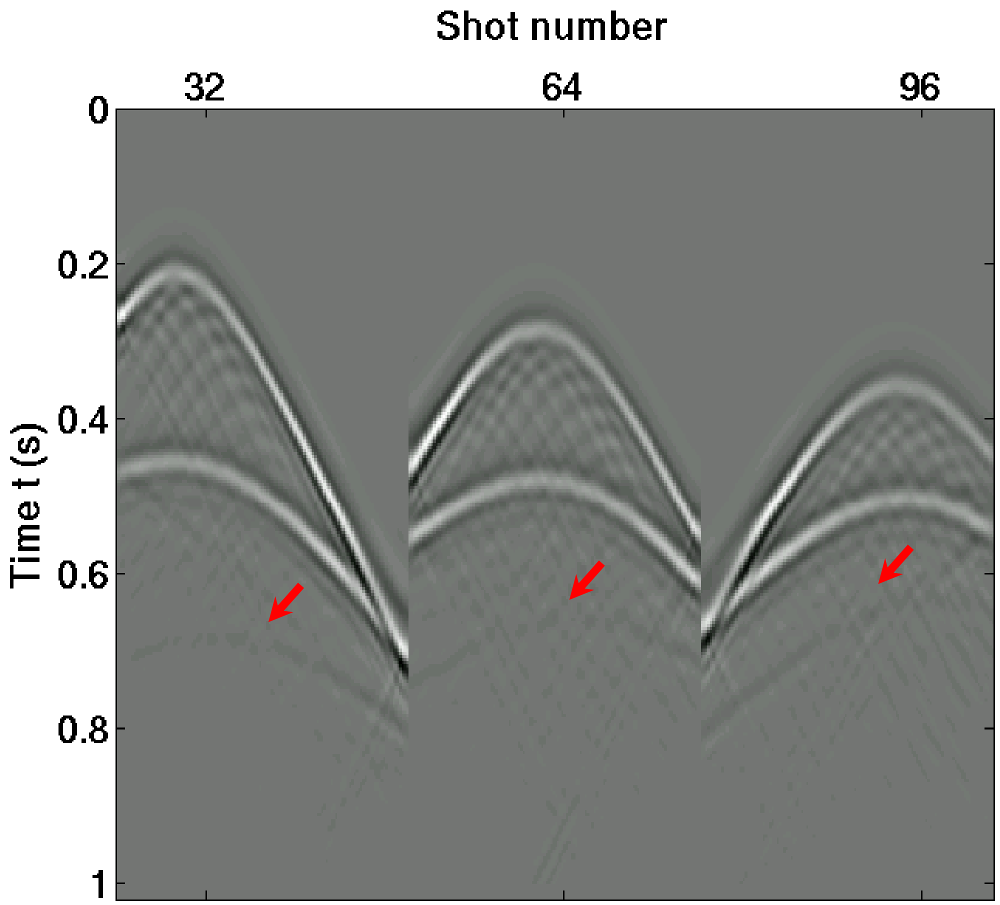
Synthetic example

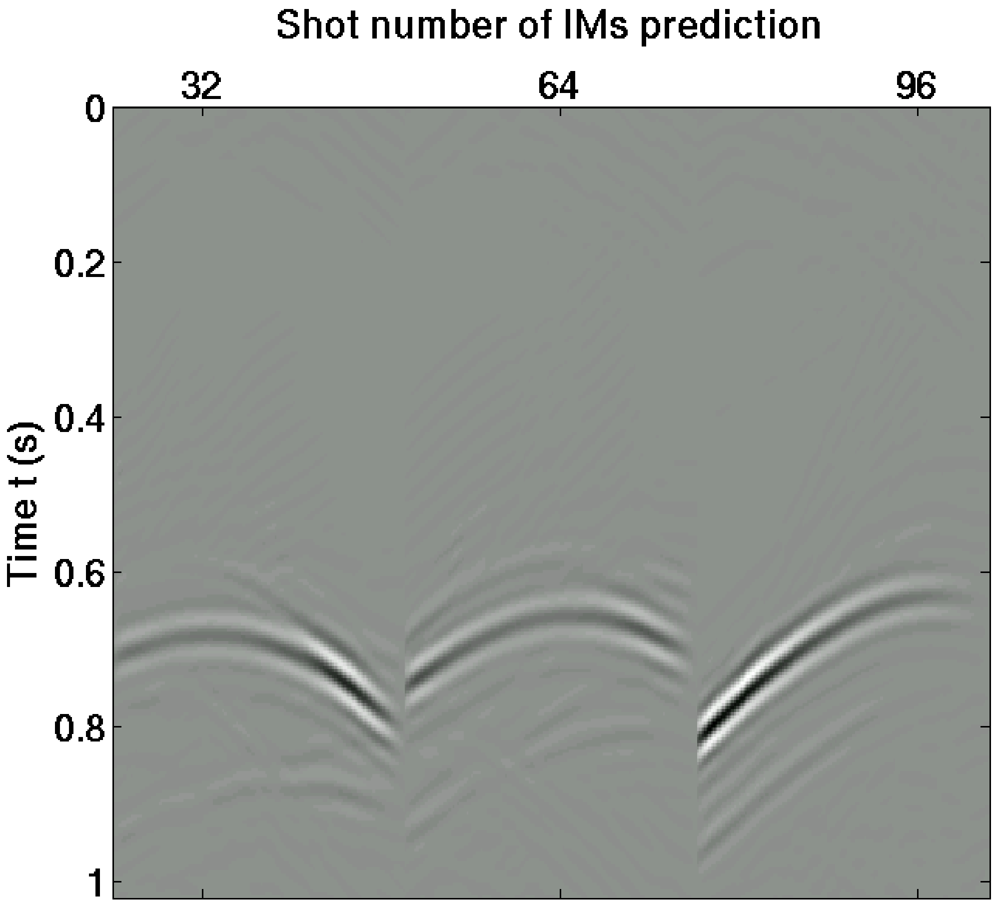




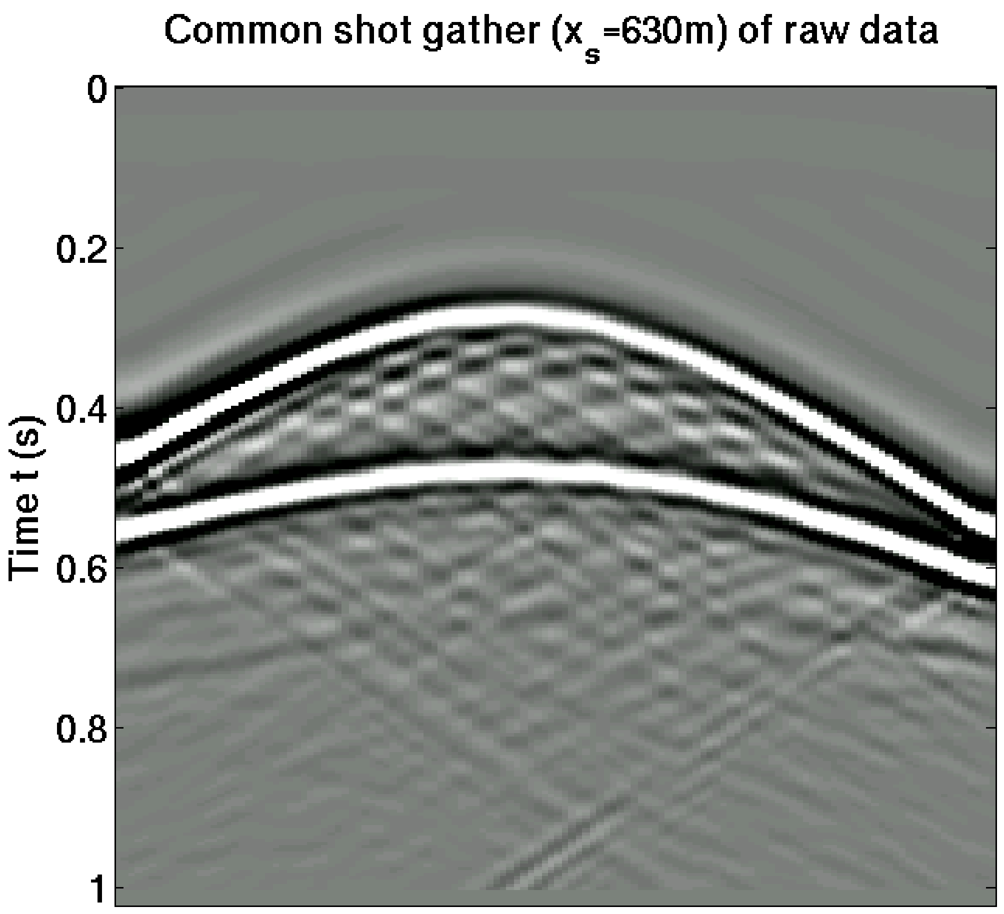


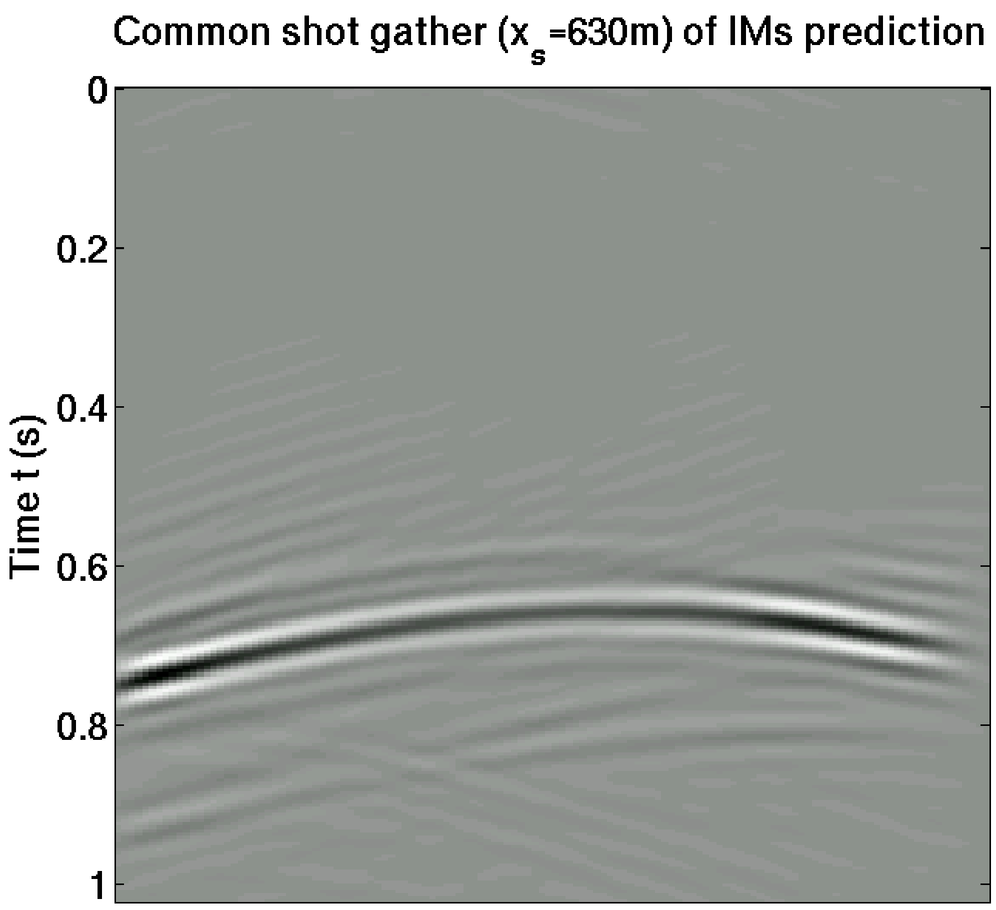
Synthetic example





Synthetic example





Conclusion and future work

- Internal multiples can be reconstructed using inverse scattering series (ISS) algorithm in an automatic and stepwise way.
- Double $\tau - p_s - p_g$ transform with respect to source, receiver coordinates was discussed and applied to prepare the input data for ISS algorithm.
- 2D internal multiple prediction using ISS algorithm in double plane wave domain was performed.
- Preliminary results exemplify that ISS algorithm in double plane wave domain can provide more relevant and practical benefits.

Future work:

- Computation burden and practical tests.

Acknowledgements

- All CREWES sponsors
- All CREWES staff and students

Thank You!