L1 nonstationary adaptive subtraction

Application to internal multiple attenuation

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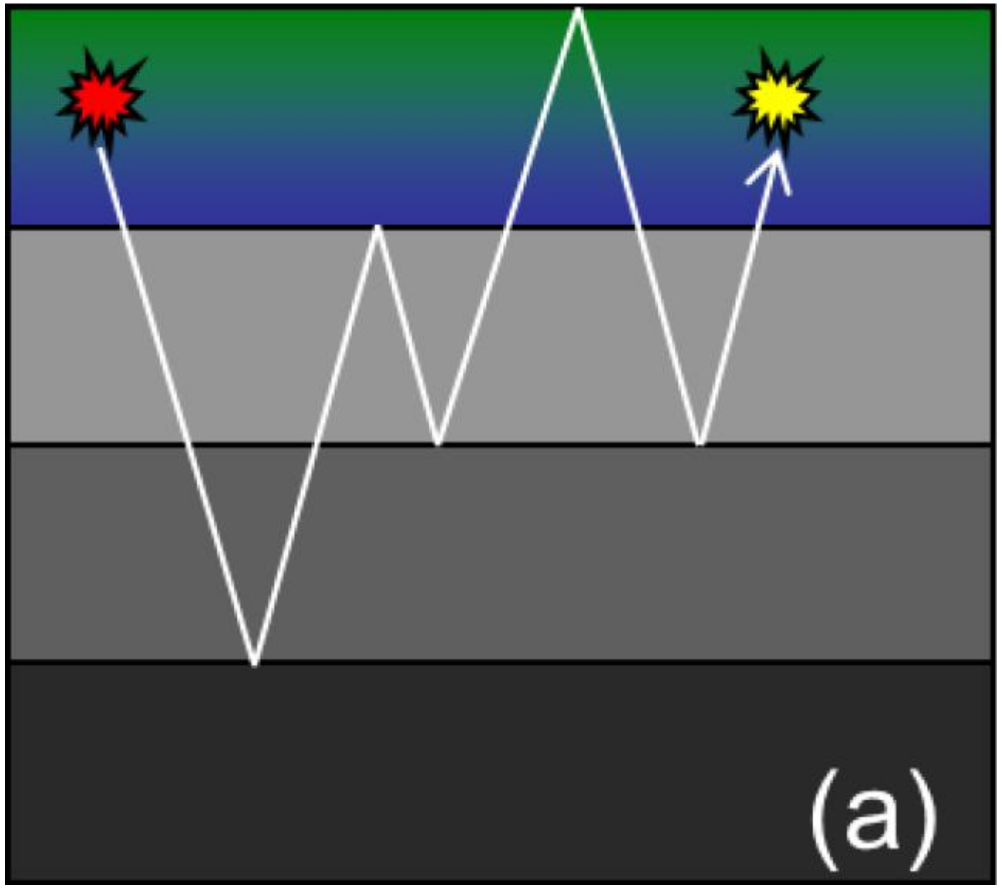
Multiples

- Multiples in seismic data are often undesirable, harming both processing and interpretation.
- There exist computationally cheap multiple removal methods, such as predictive deconvolution and filtering based multiple removal.
- These can fail when confronted with complex geology.
- For complicated internal multiples, inverse scattering multiple prediction can be a preferable method for multiple removal.

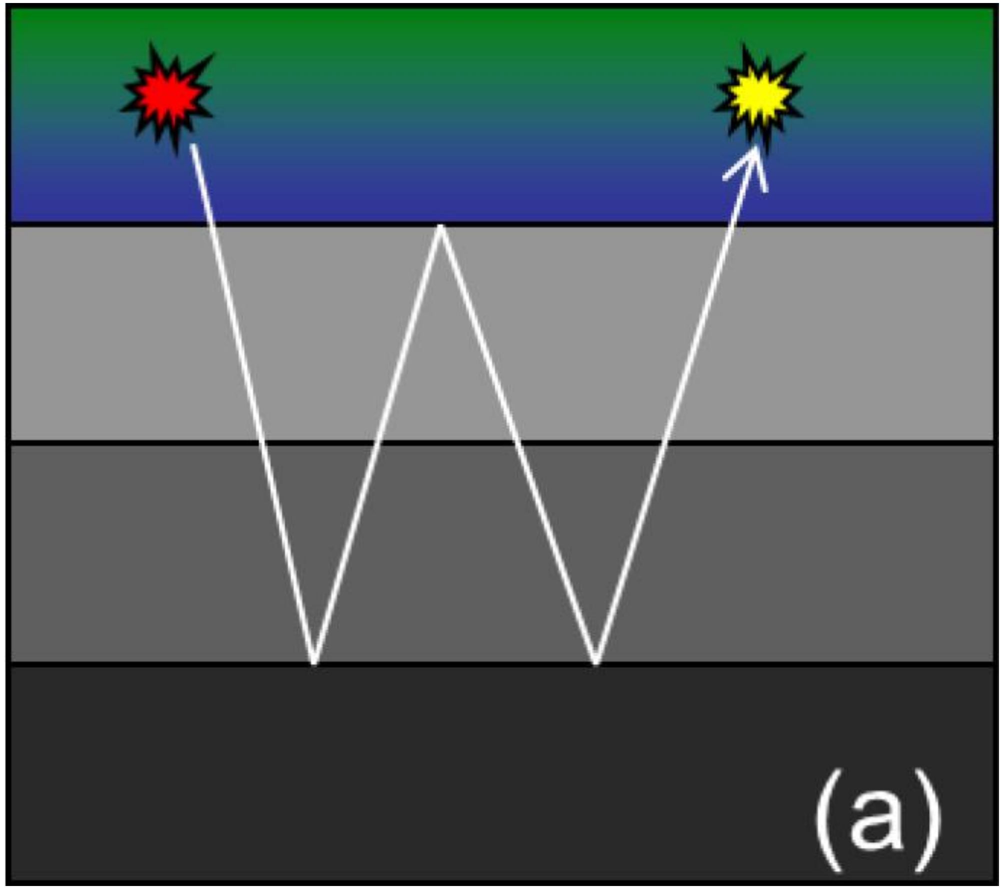
Types of Multiples

- There are two major types of multiples.
- Free surface multiples experience at least one downward reflection at the free surface.
- Internal multiples are multiples that do not have any reflection at the free surface.
- Internal multiples are typically of greater importance in land data, as the near surface scatters away most of the energy associated with free surface multiples.

Free Surface Multiple



Internal Multiple



Figures from Weglein and Dragoset, 2005

Inverse Scattering Multiple Prediction

- Inverse scattering multiple prediction generates an internal multiple prediction based on the seismic data alone, and is capable of dealing with very complex geologies.
- CREWES is working on several multidimensional implementations of inverse scattering in a variety of novel domains, such as work on plane wave domain prediction (Sun, 2015) and on space-time domain prediction (Innanen, 2015).

Inverse Scattering Multiple Prediction

- Inverse scattering multiple prediction works by identifying the subset of the Born series which contributes to the generation of internal multiples, and calculating the first term of this set.
- To ensure that only physically valid multiples are predicted, the scattering events are limited to a lower – higher – lower relation.
- In 1.5D for example, the prediction is given by

$$m(k_g,\omega) = \int_{-\infty}^{\infty} dz e^{ik_z z} b_1(k_g, z) \int_{-\infty}^{z-\epsilon} dz' e^{-ik_z z'} b_1(k_g, z') \int_{z'+\epsilon}^{\infty} dz'' e^{ik_z z''} b_1(k_g, z'') \quad .$$

Inverse Scattering Multiple Prediction

- Inverse scattering multiple prediction is in practice generated using a truncation of an infinite series, and so is not exact.
- Additionally, theoretically necessary preprocessing steps such as deconvolution and deghosting are often neglected, introducing additional errors in both amplitude and phase (Pan, 2015).

Adaptive Subtraction

- In order to remove these predicted multiples, we need to match them to the observed multiples and subtract them.
- This is called adaptive subtraction.
- This matching is typically done by applying a filter to the predicted multiple.
- As we do not know the correct multiple, we need to determine some way of choosing the filter we apply.

Least squares adaptive subtraction

- One of the simplest methods of adaptive subtraction is to choose the filter which gives the multiple prediction that minimizes the total energy in the data. (Verschuur et. al. 1992)
- This hinges on the assumption that multiples and primaries do not overlap in the data, and so eliminating the multiples yields the minimum energy.

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Least squares adaptive subtraction

- Minimizing the energy is equivalent to minimizing the L_2 norm, given by

$$\sum_{i=0}^{t_n} r_i^2 = \sum_{i=0}^{t_n} (\mathbf{d} - \mathbf{Mf})_i^2$$

where r is the data after subtraction, given by

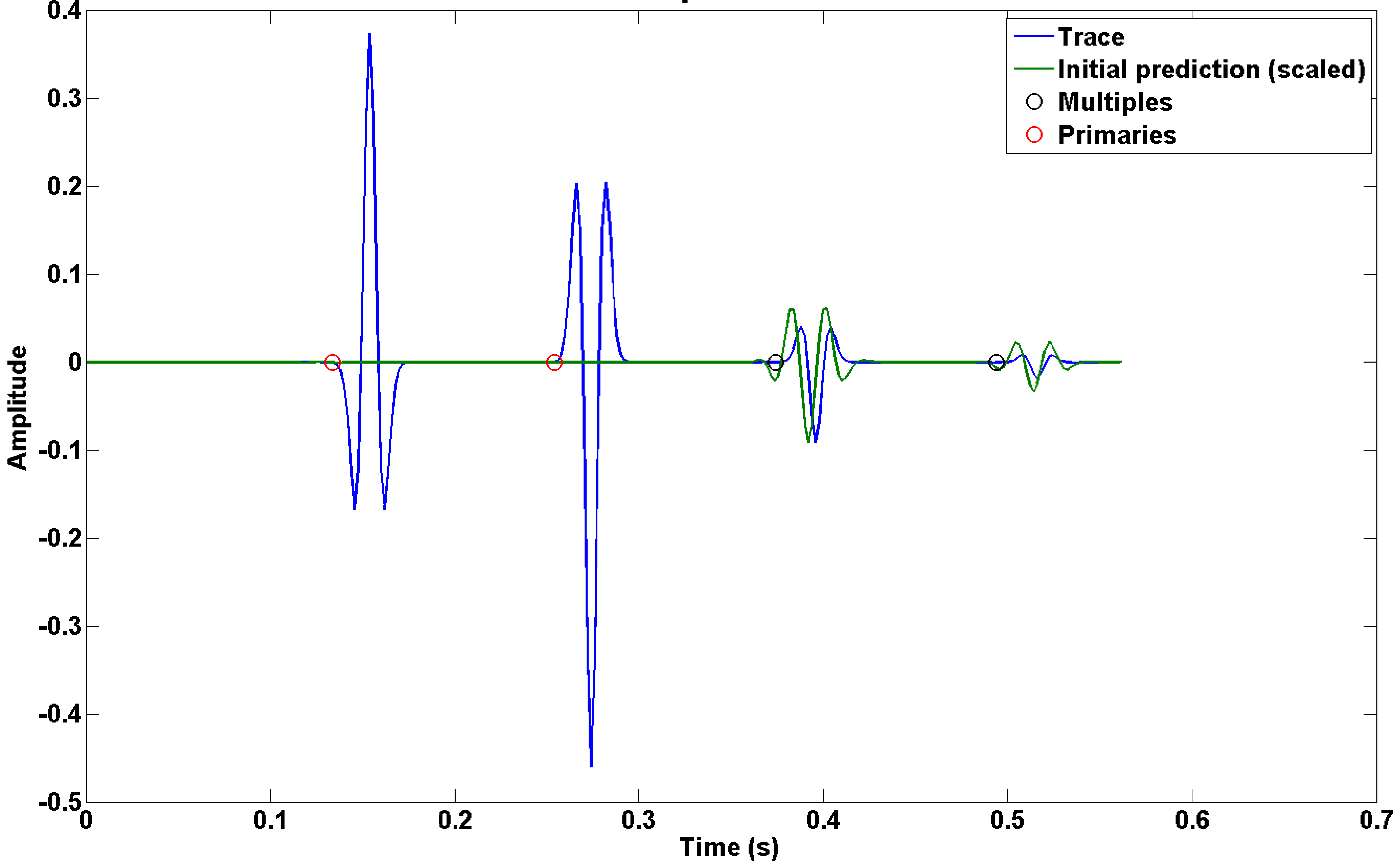
$$\mathbf{r} = \mathbf{d}\text{-}\mathbf{Mf}$$

where M is the matrix representing convolution with the predicted multiple trace, d is the data trace, and f is the filter.

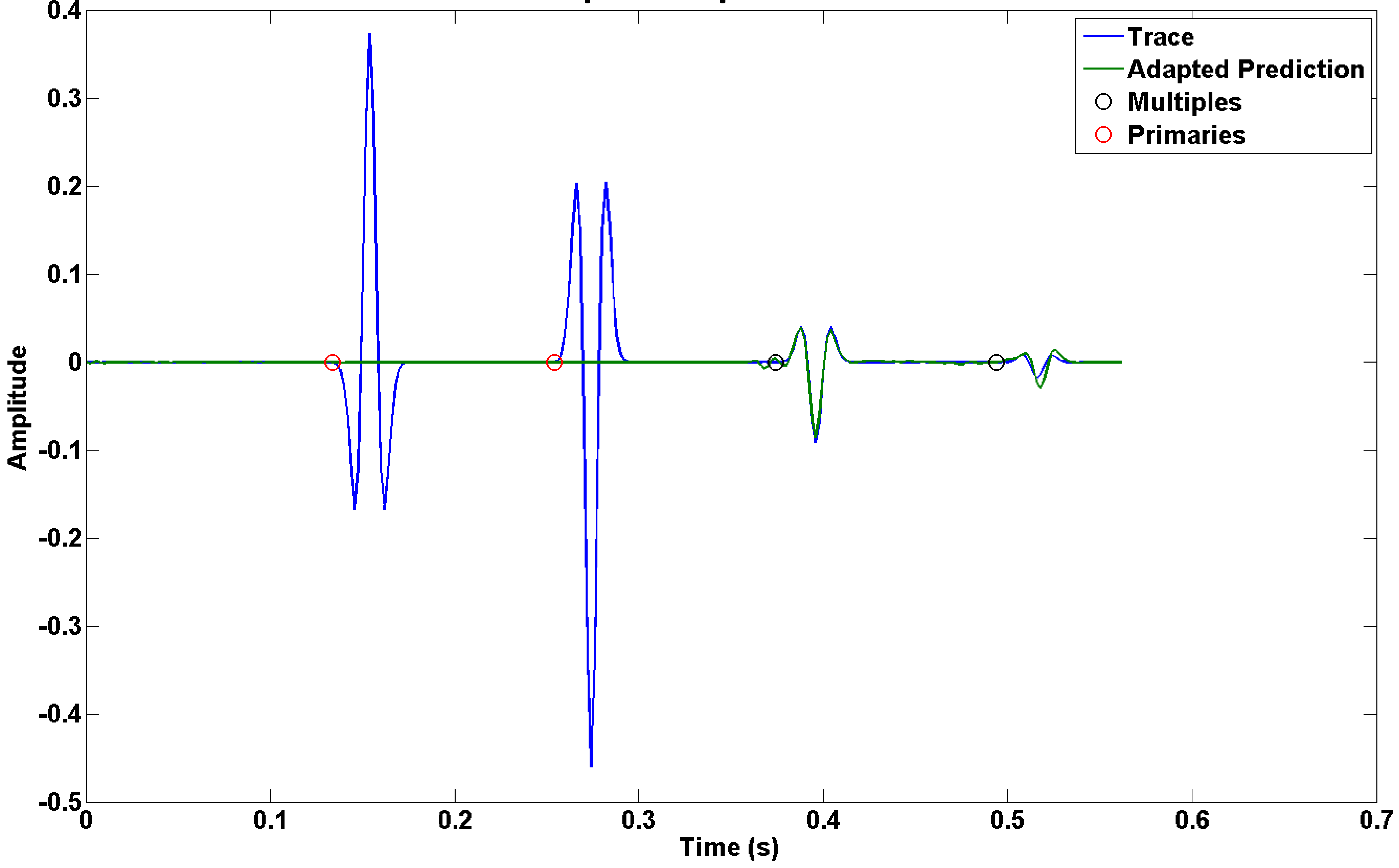
- To find the filter, we wish to solve $\mathbf{Mf} = \mathbf{d}$ We do this by least squares:

$$\mathbf{f} = (\mathbf{M}^T\mathbf{M})^{-1}\mathbf{M}^T\mathbf{d} \quad .$$

Initial prediction



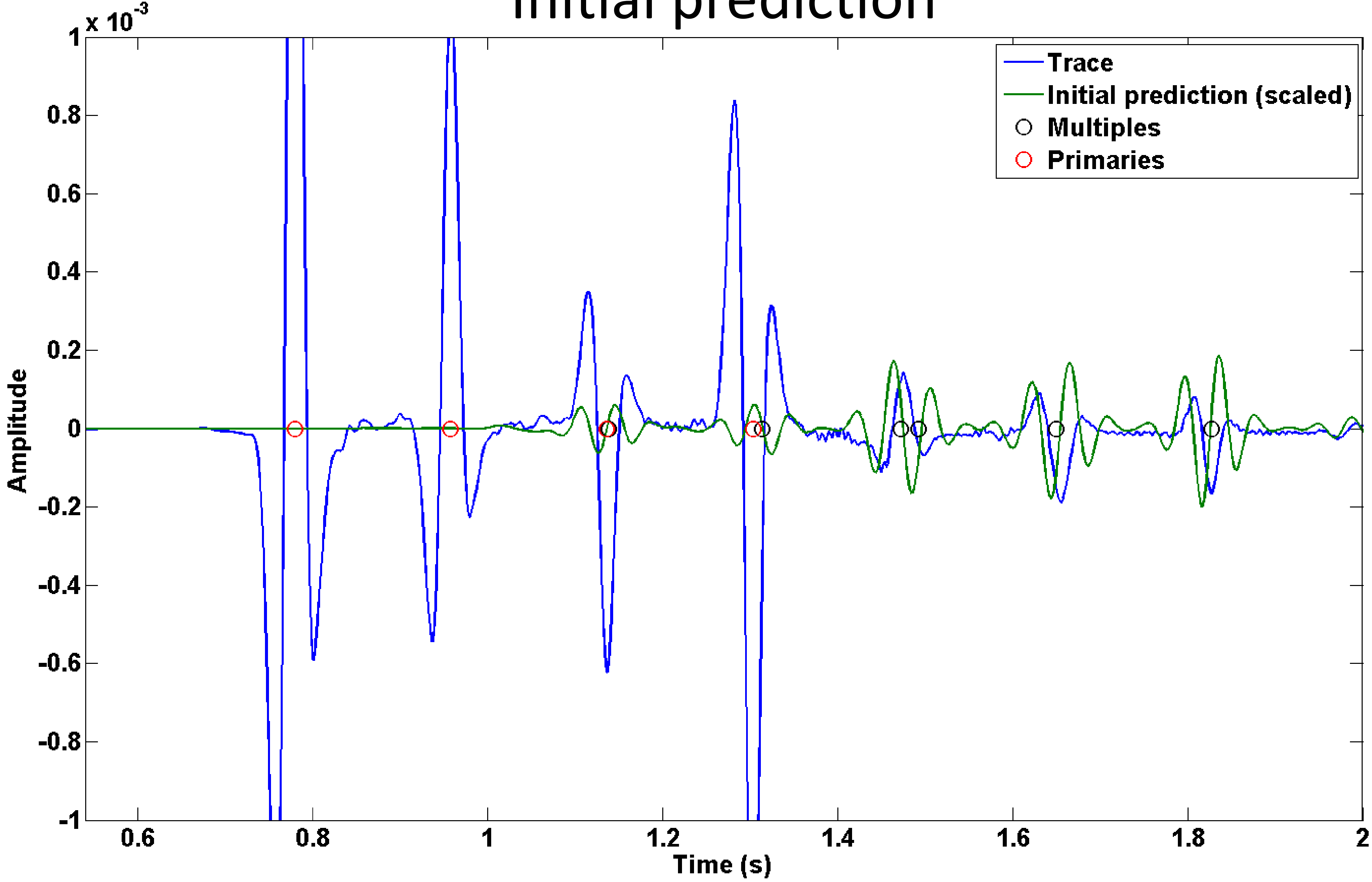
Adapted prediction



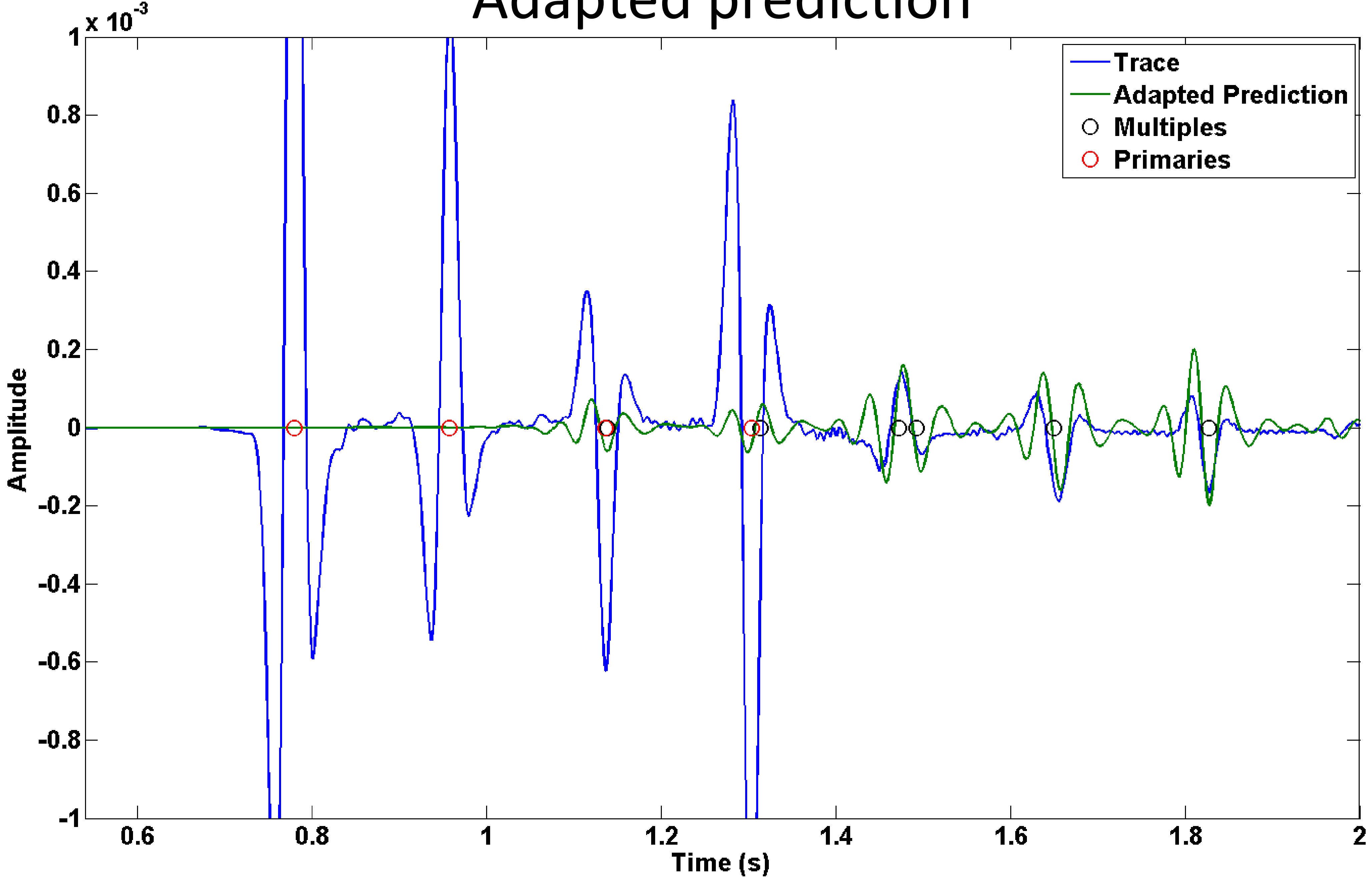
Least squares adaptive subtraction

- Unfortunately, multiples and primaries often overlap to some extent.
- Given that primaries typically have greater energy than multiples, least squares subtraction will prioritize the removal of primaries in the case of primary-multiple overlap.
- This can lead to poor multiple removal, and worse, removal of signal from the primaries .

Initial prediction



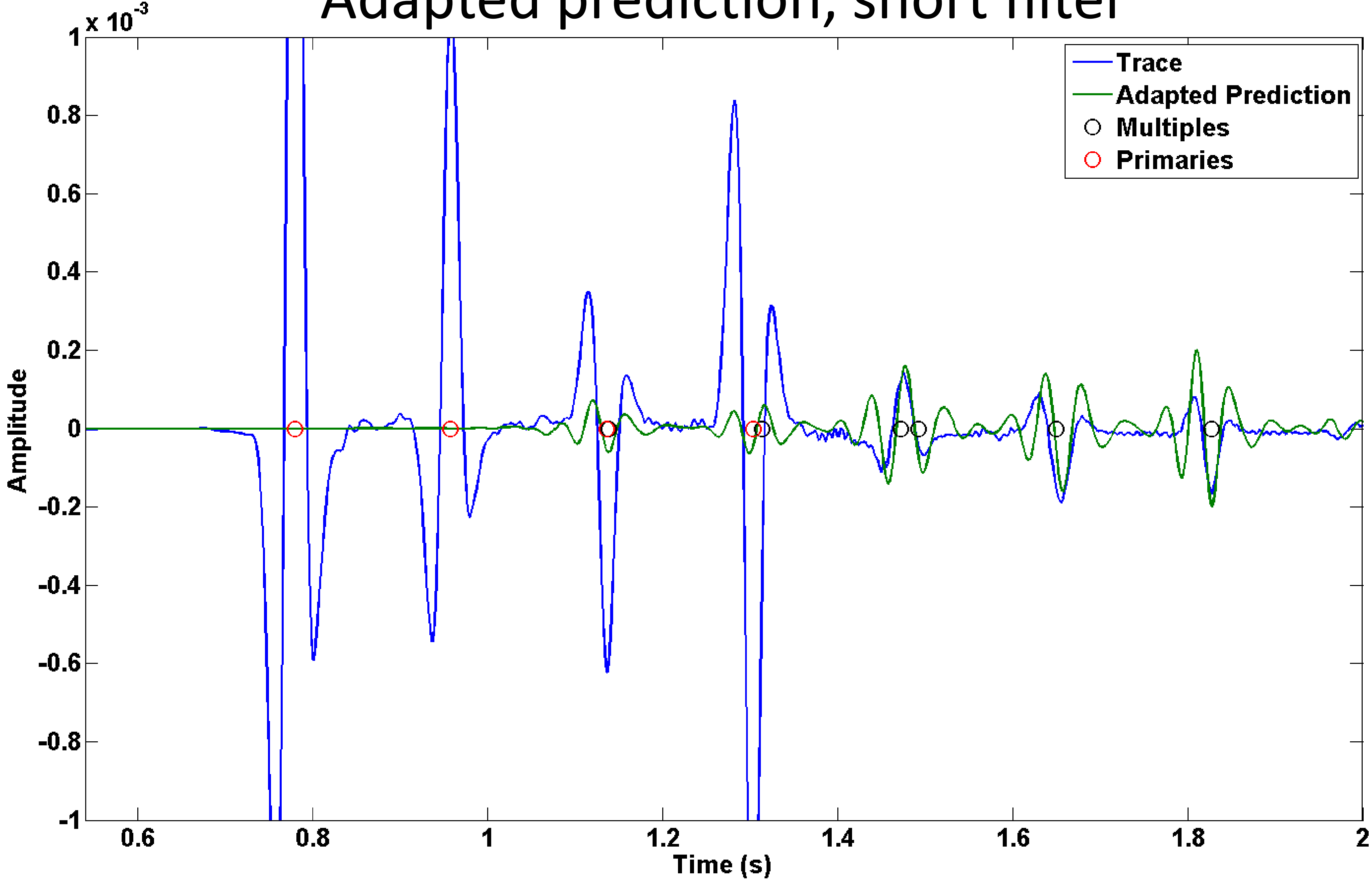
Adapted prediction



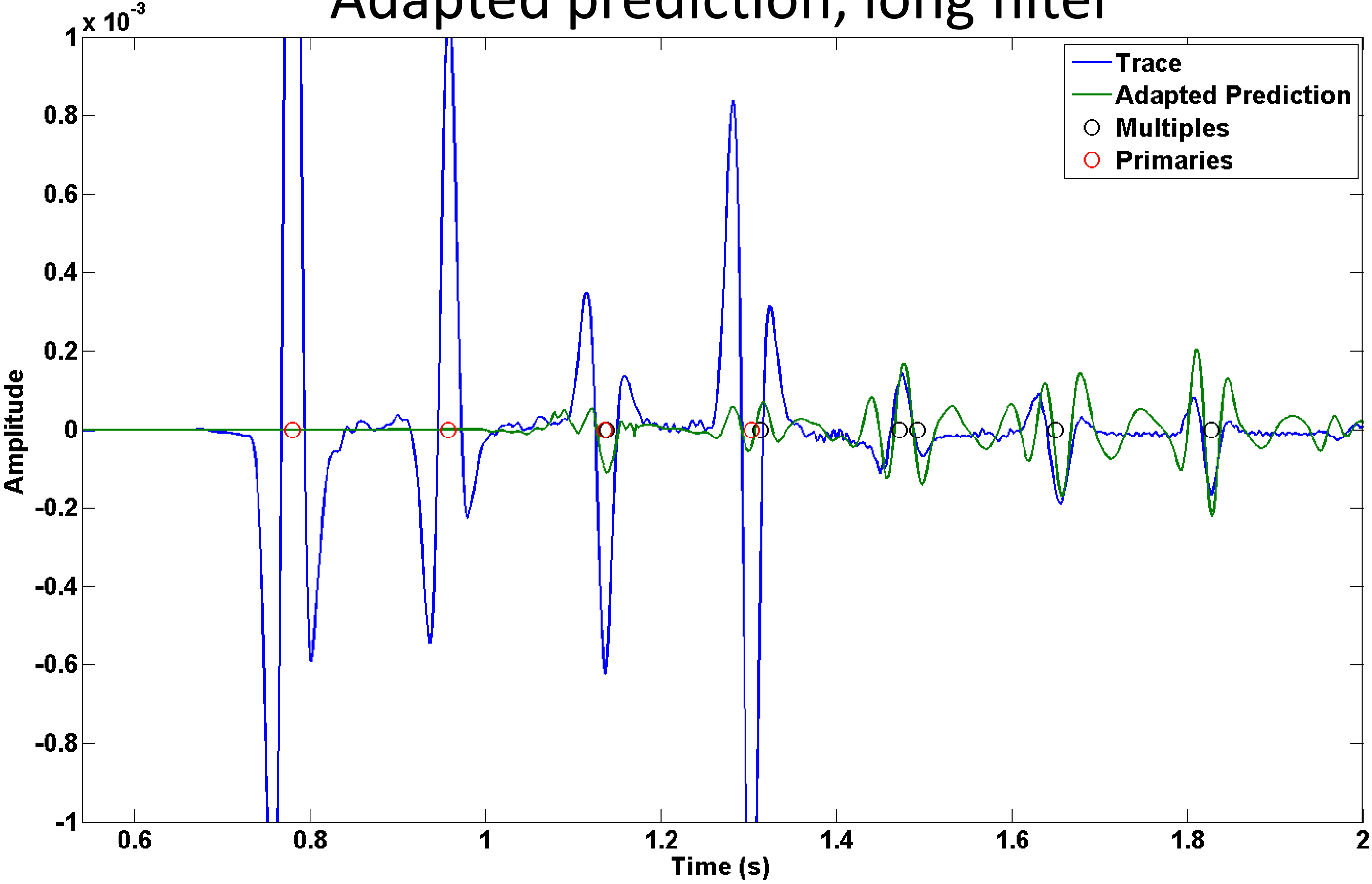
Least squares adaptive subtraction

- An important parameter in adaptive subtraction is the filter length.
- With a sufficiently long filter, the predicted multiples can be made to match any signal.
- A longer filter can be advantageous, as it increases the level of matching, and allows for more complete removal of multiples
- Longer filters also increase the chance that primary data will be removed

Adapted prediction, short filter



Adapted prediction, long filter



L_1 Norm

- The major problem in the least squares adaptive subtraction method is the tendency for predicted multiples to be matched to primaries.
- This largely arises due to the large amplitudes of the primaries.
- If the same amount of signal is subtracted from a large signal and a smaller signal, the subtraction will lower the L_2 norm more in the case where the large signal is reduced.
- This means that the energy minimizing filter often does its best to match primaries over a small time period, at cost to the matching to multiples over a longer period.

L_1 Norm

- An alternative to the L_2 norm for adaptive subtraction is the L_1 norm (Guitton and Verschuur, 2004).
- When a constant amount of signal is reduced from a large signal or a small signal, the L_1 norm reduces by the same amount.
- Consequently, high amplitude primaries are of dramatically less relevance in L_1 norm minimization.
- The L_1 norm is given by

$$\sum_{i=0}^{t_n} |r_i| = \sum_{i=0}^{t_n} |\mathbf{d} - \mathbf{M}\mathbf{f}|_i$$

L_1 Norm

- The L_1 norm minimizing filter is found by solving the following least-squares equation $$\mathbf{f} = (\mathbf{M}^T\mathbf{W}\mathbf{M})^{-1}\mathbf{M}^T\mathbf{W}\mathbf{d} \quad ,$$

  where W is a diagonal matrix whose elements W_{ii} are related to the residual at time *i* by $w_{ii} = r_i^{-1}$, where

$$\mathbf{r} = \mathbf{d}\text{-}\mathbf{M}\mathbf{f}$$

- Unfortunately, this expression is singular where r_i is zero.

L_1 / L_2 Norm

- Bube and Langan (1997) propose an L_1/L_2 hybrid norm.
- As the L_2 norm is well behaved as the residual approaches zero, the hybrid norm is non-singular everywhere.
- The expression for the filter which minimizes the hybrid norm is again

$$\mathbf{f} = (\mathbf{M}^T\mathbf{W}\mathbf{M})^{-1}\mathbf{M}^T\mathbf{W}\mathbf{d} \quad ,$$

but with

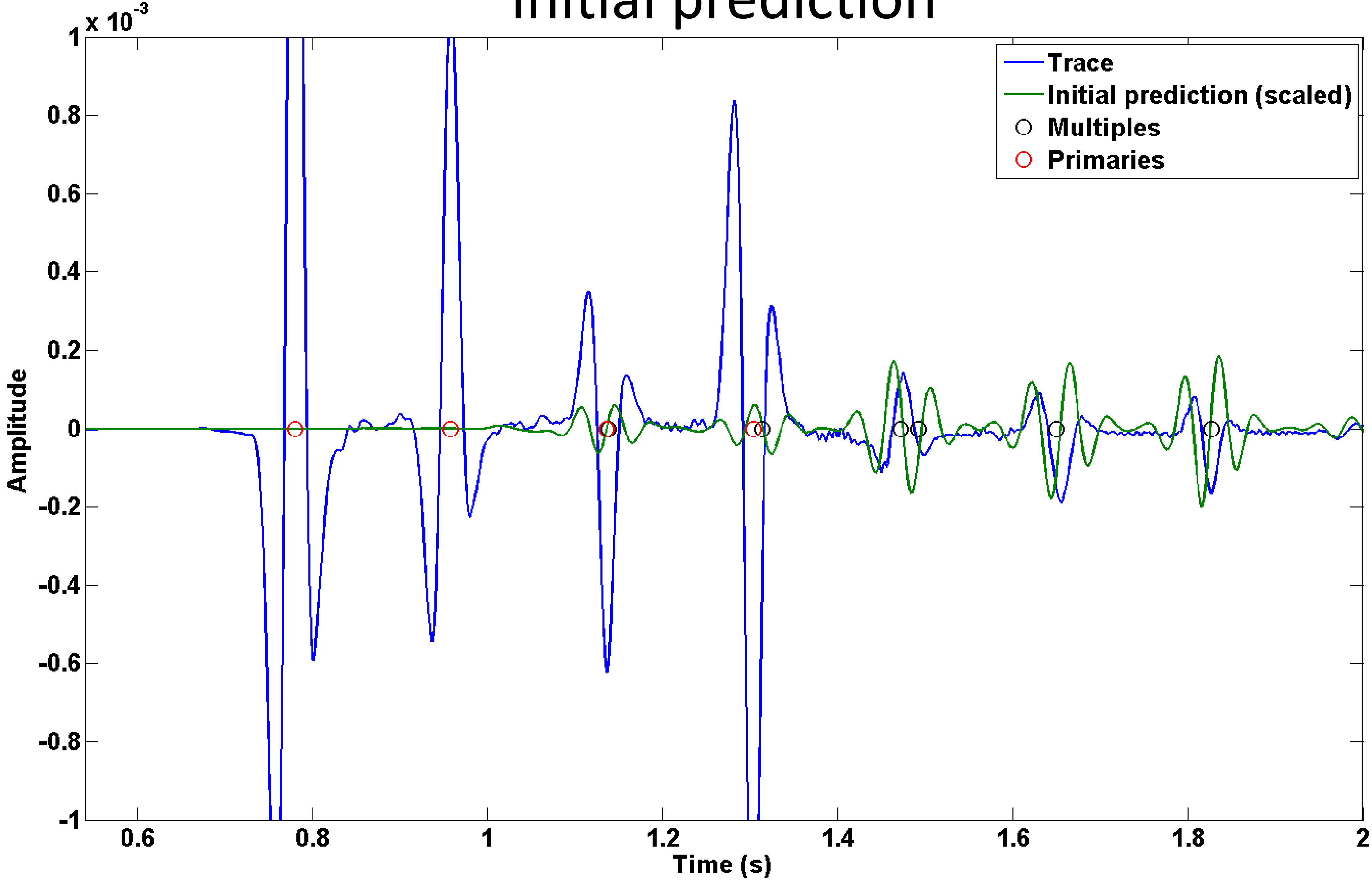
$$w_{ii} = \left(\frac{1}{1+(\frac{r}{\sigma})^2}\right)^{\frac{1}{2}}$$

where r are the residuals and sigma is a chosen factor.

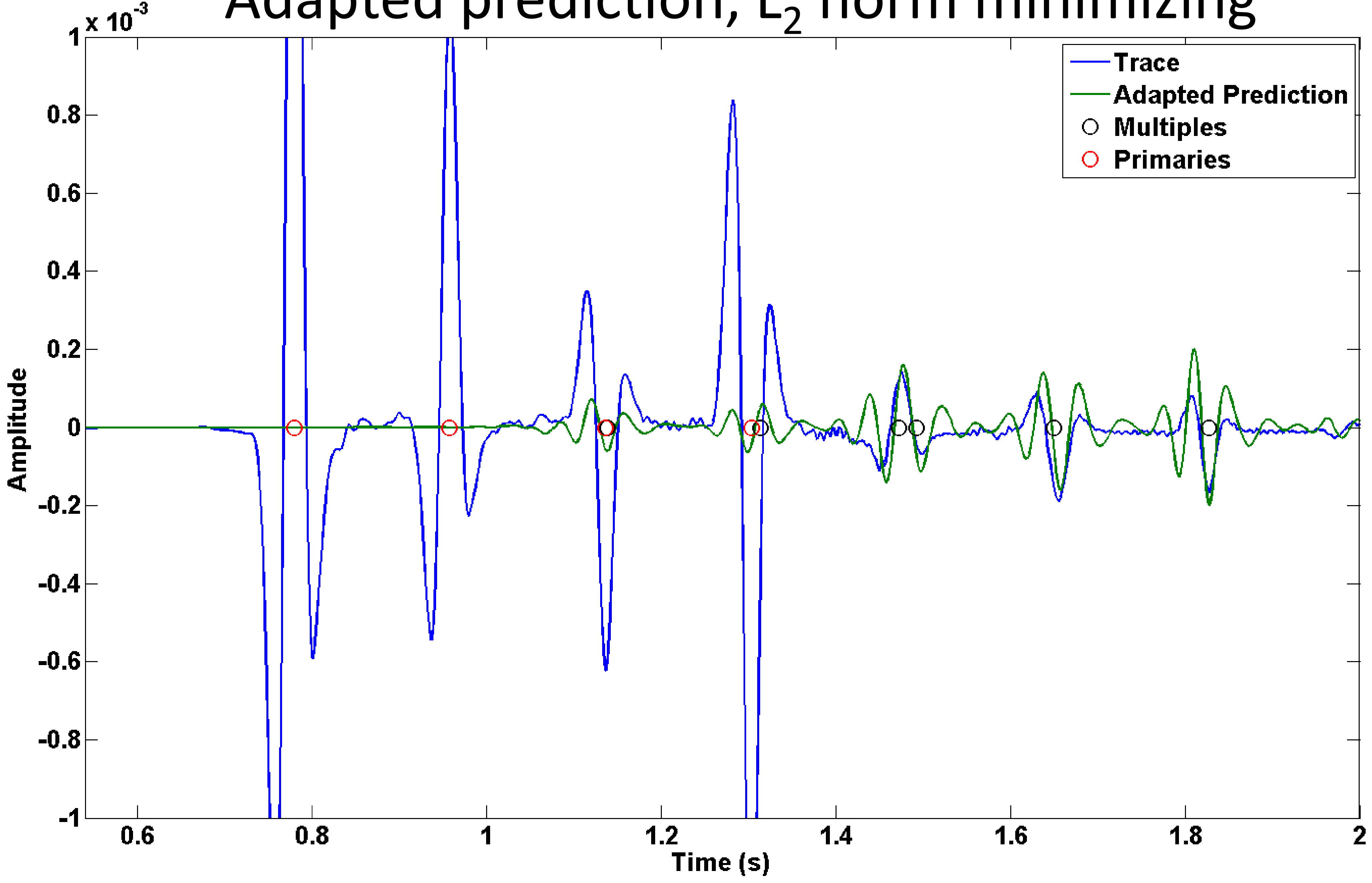
L_1 / L_2 Norm

- In effect this equation minimizes J, where $$J = \sqrt{1 + \left(\frac{r}{\sigma}\right)^2} - 1$$
- Small sigma will closely emulate the L_1 norm, while large sigma will approximate L_2.
- The factor sigma must be decided on by the user.
- The expression for the L_1/L_2 norm is nonlinear.
- It can be solved iteratively.

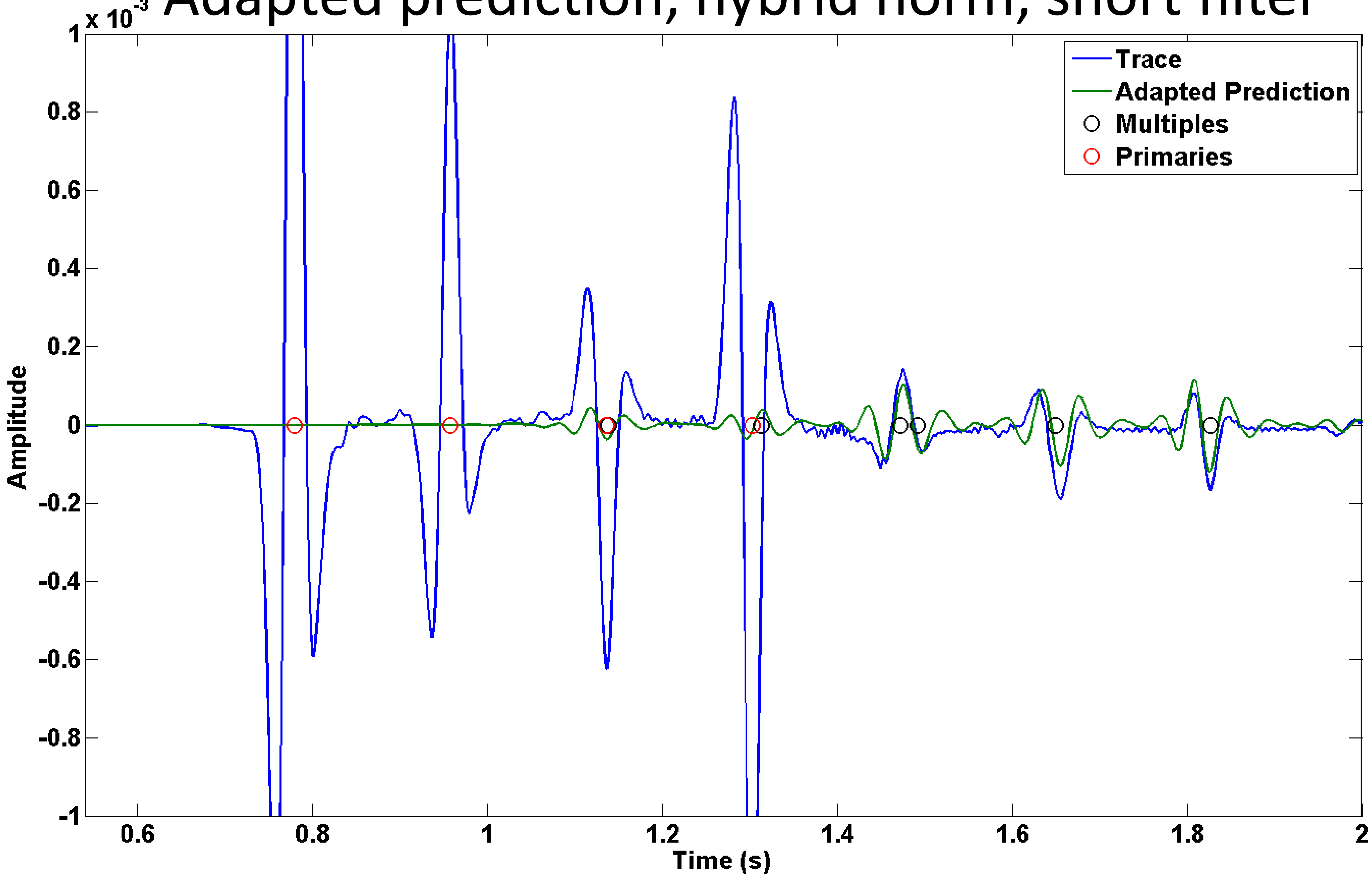
Initial prediction



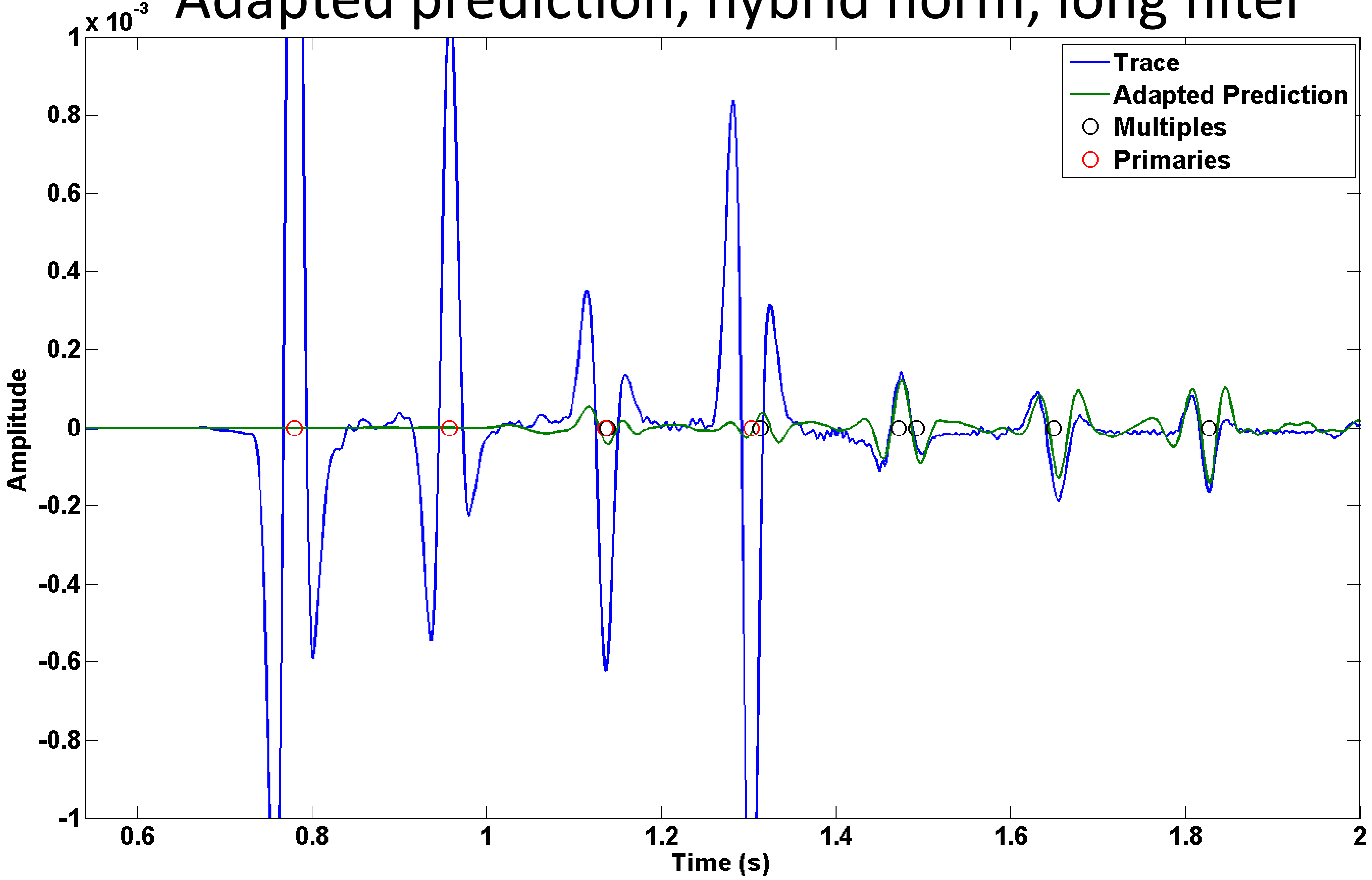
Adapted prediction, L_2 norm minimizing



Adapted prediction, hybrid norm, short filter



Adapted prediction, hybrid norm, long filter

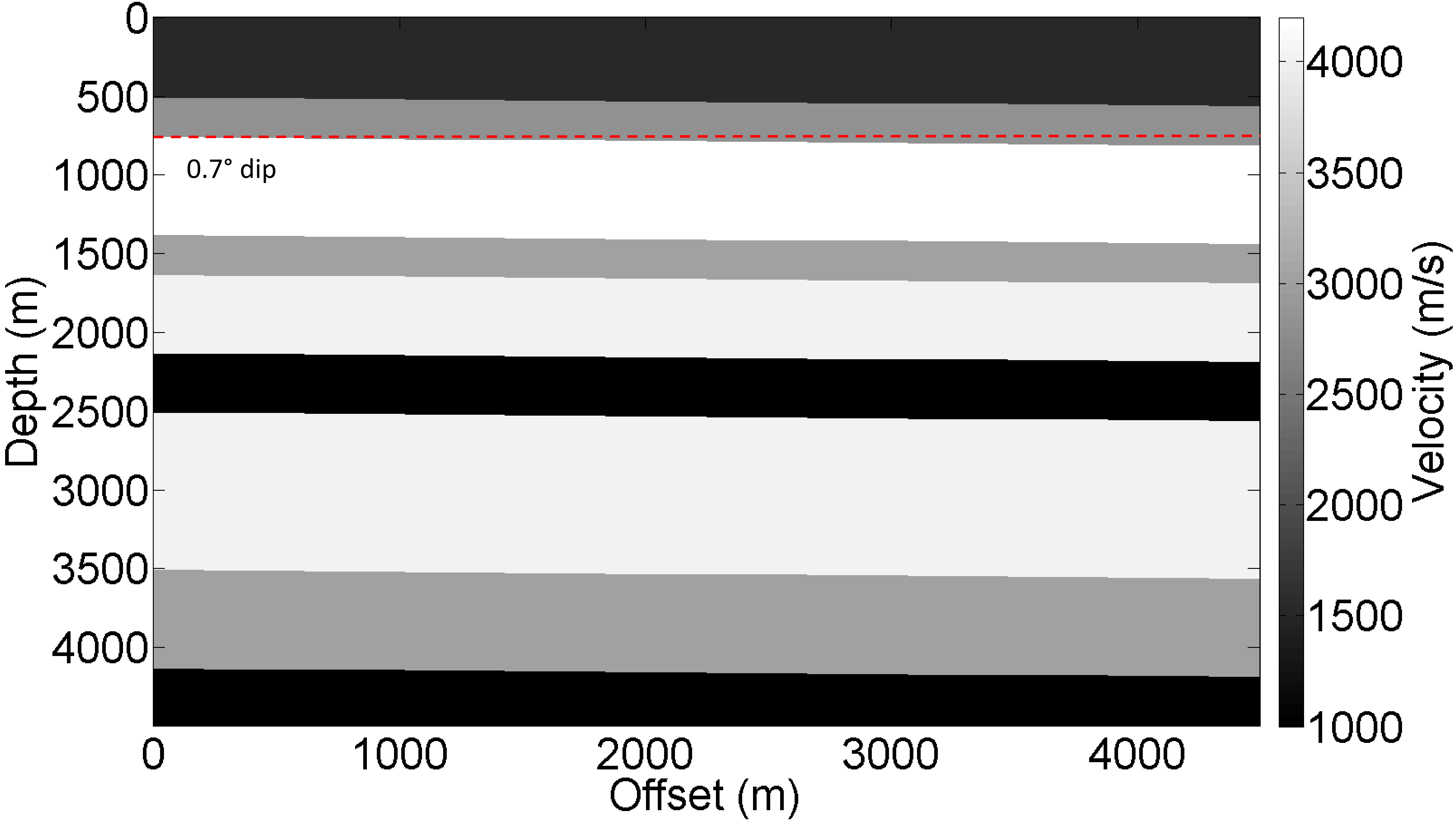


Nonstationarity

- Seismic data are often nonstationary.
- Additionally, approximations may be made in multiple prediction whose validity varies in space or time.
- These factors can lead to a multiple prediction which is not related to the true multiples by a single, stationary filter.
- In this case, a time and/or space variant filter is necessary.

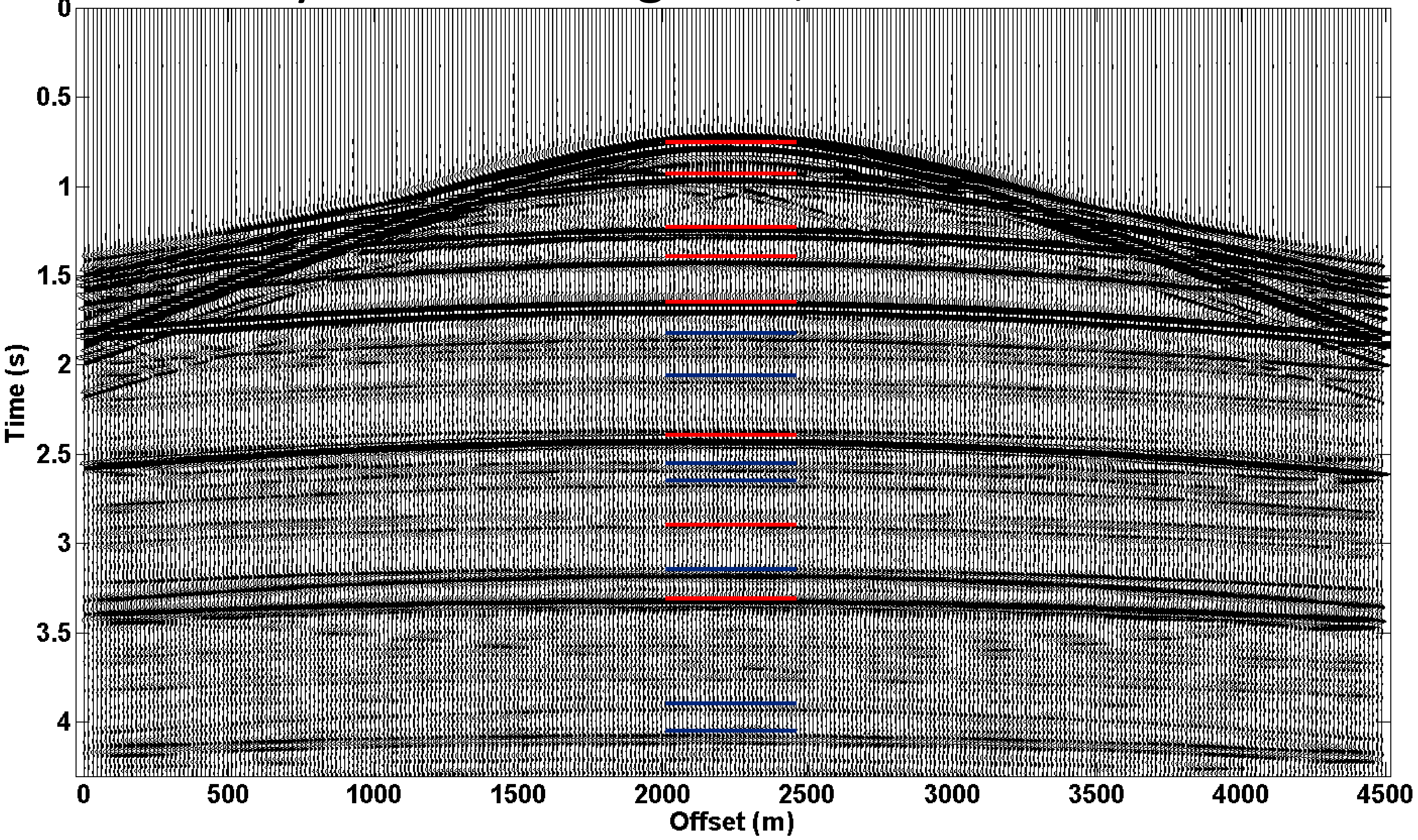
Nonstationarity

- A nonstationary filter can be created by windowing the data about each point in succession, and calculating the filter which works best in each window.
- This creates a different filter for each point.
- The window size controls how quickly the filter is allowed to vary.
- A Gaussian-shaped window was found to give good results.

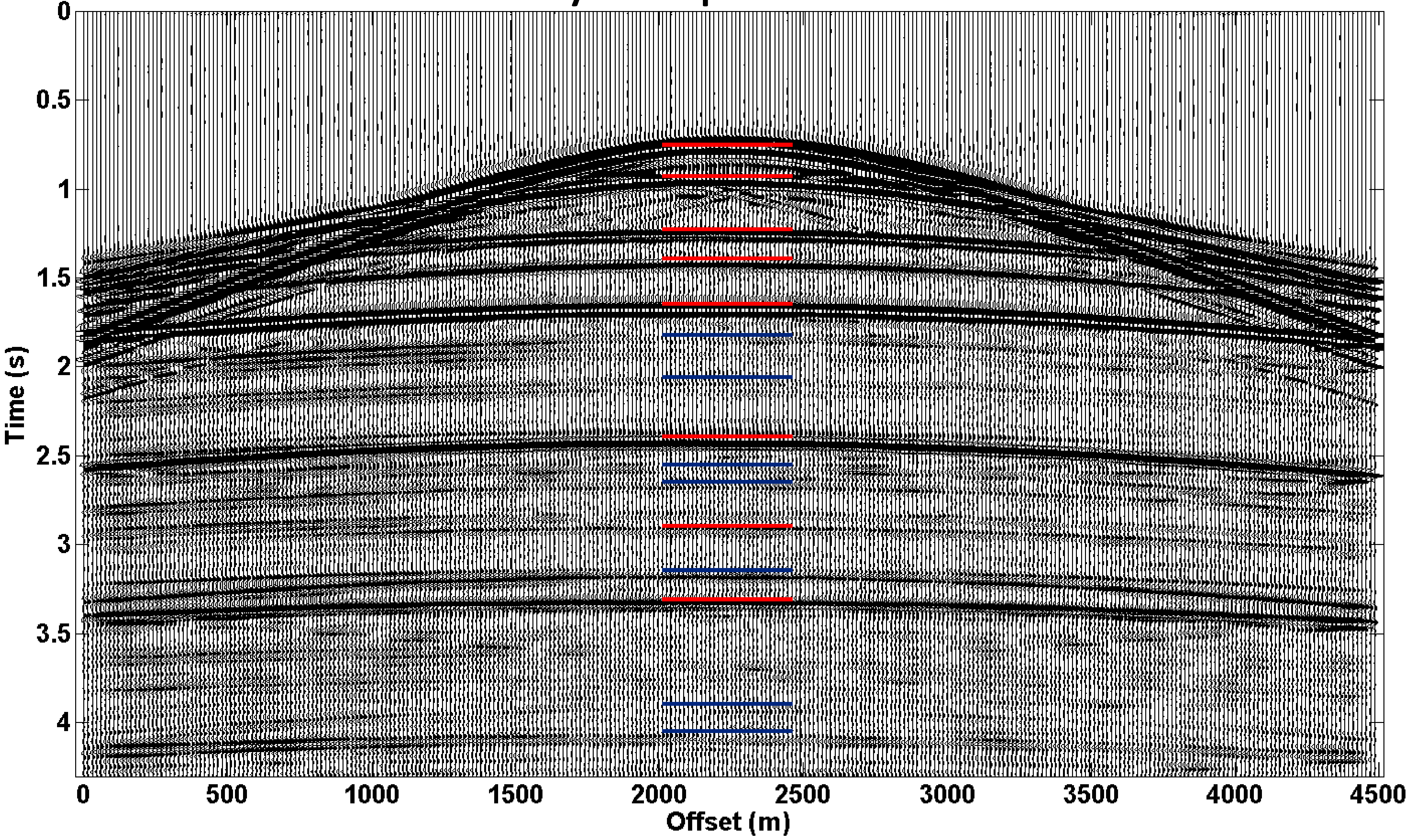


0.7° dip
Depth (m)
0
500
1000
1500
2000
2500
3000
3500
4000
Offset (m)
0
1000
2000
3000
4000
Velocity (m/s)
4000
3500
3000
2500
2000
1500
1000

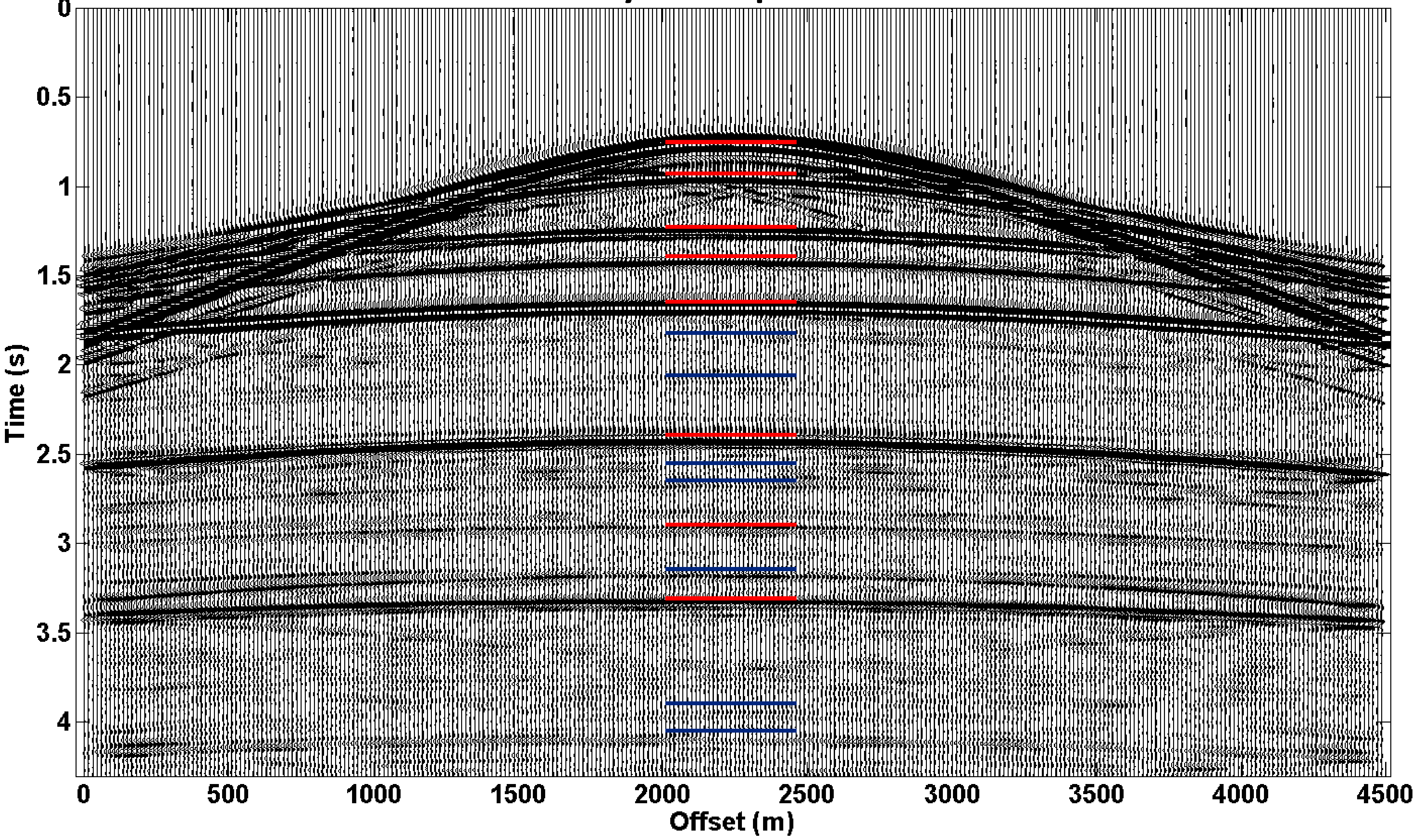
Synthetic shot gather, measured data

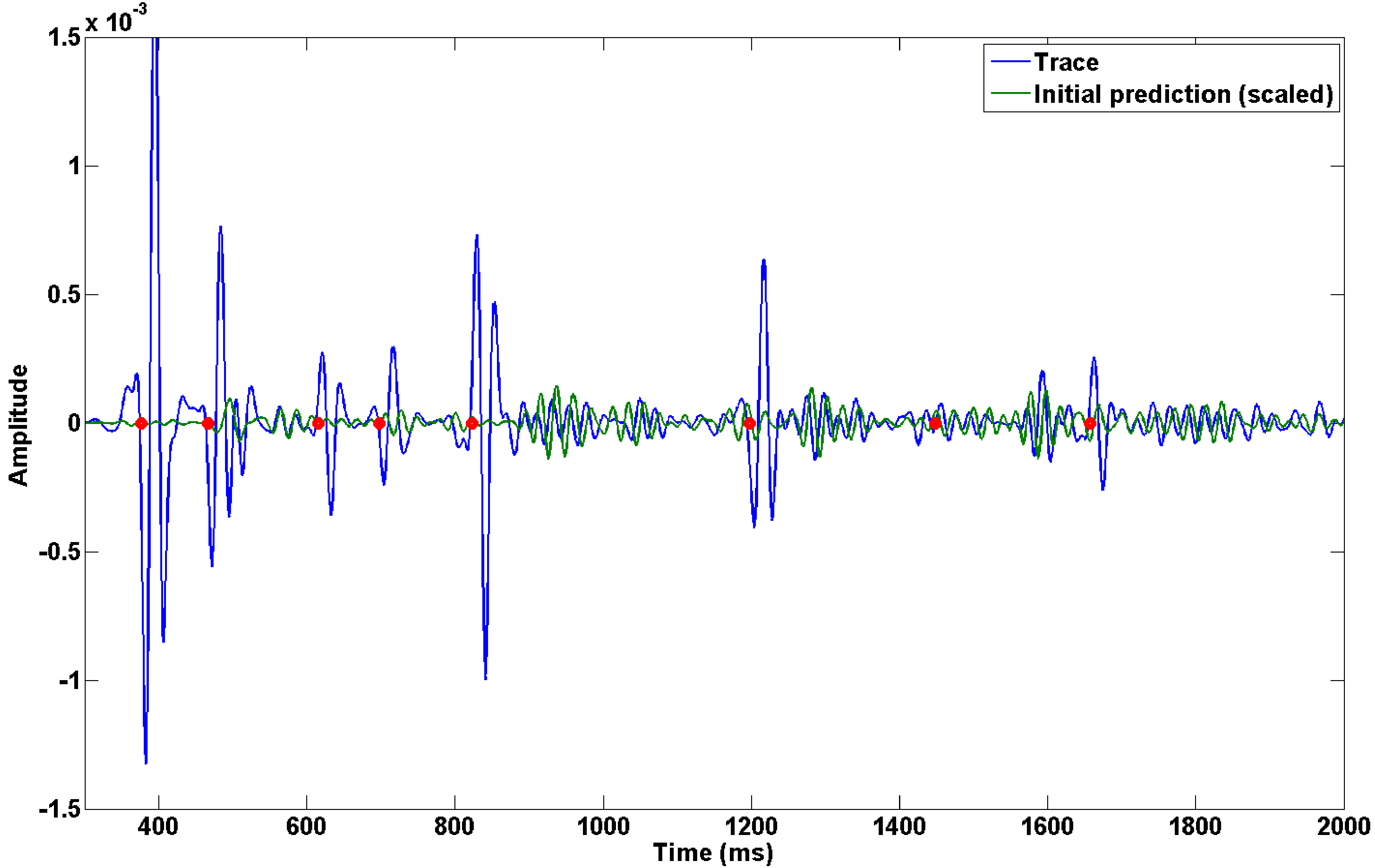


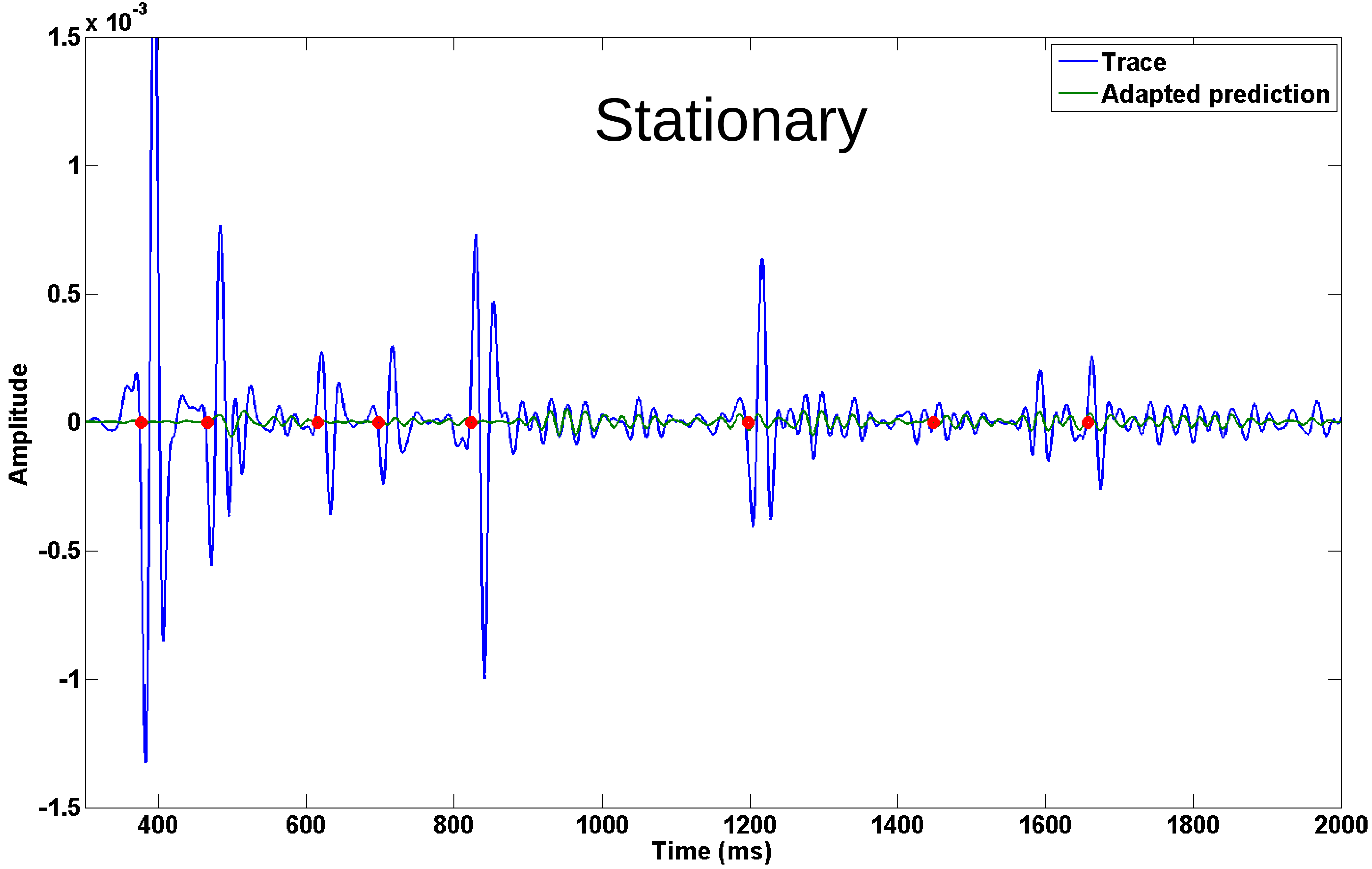
Stationary adaptive subtraction

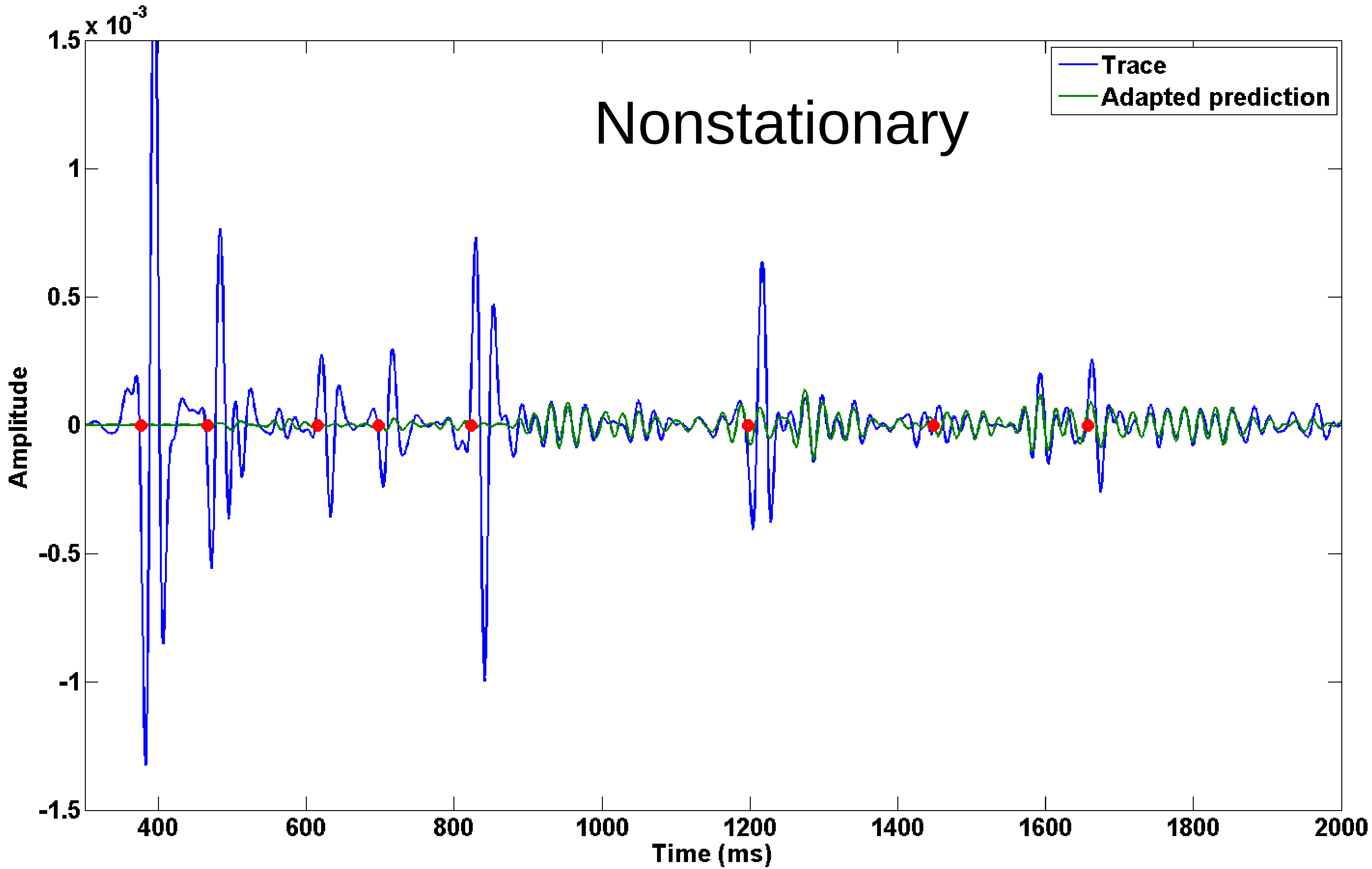


Nonstationary adaptive subtraction



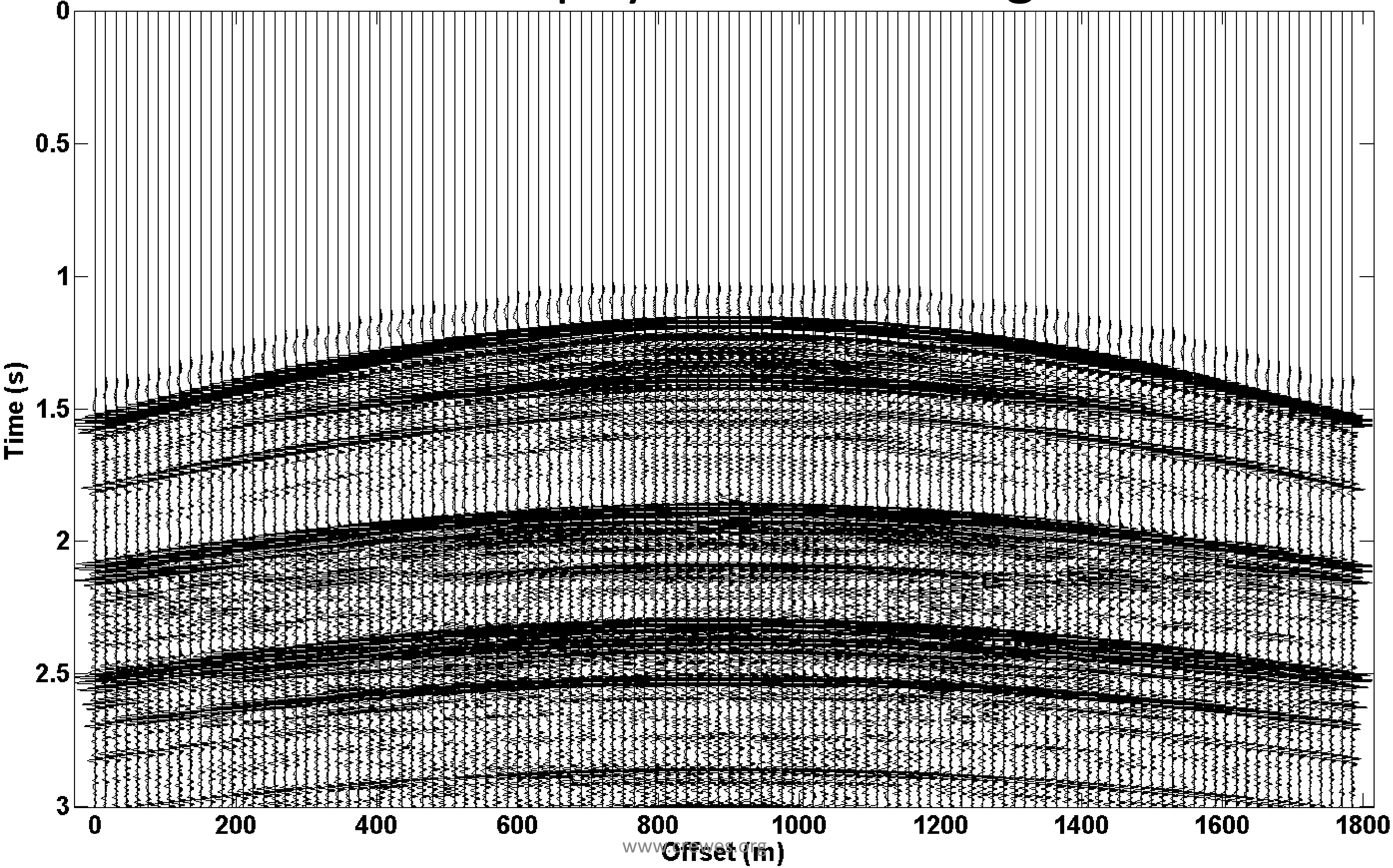

x 10^-3
1.5
1
0.5
0
-0.5
-1
-1.5
Amplitude
400
600
800
1000
1200
1400
1600
1800
2000
Time (ms)
Trace
Initial prediction (scaled)


Stationary
Trace
Adapted prediction
x 10^-3
1.5
1
0.5
0
-0.5
-1
-1.5
Amplitude
400
600
800
1000
1200
1400
1600
1800
2000
Time (ms)

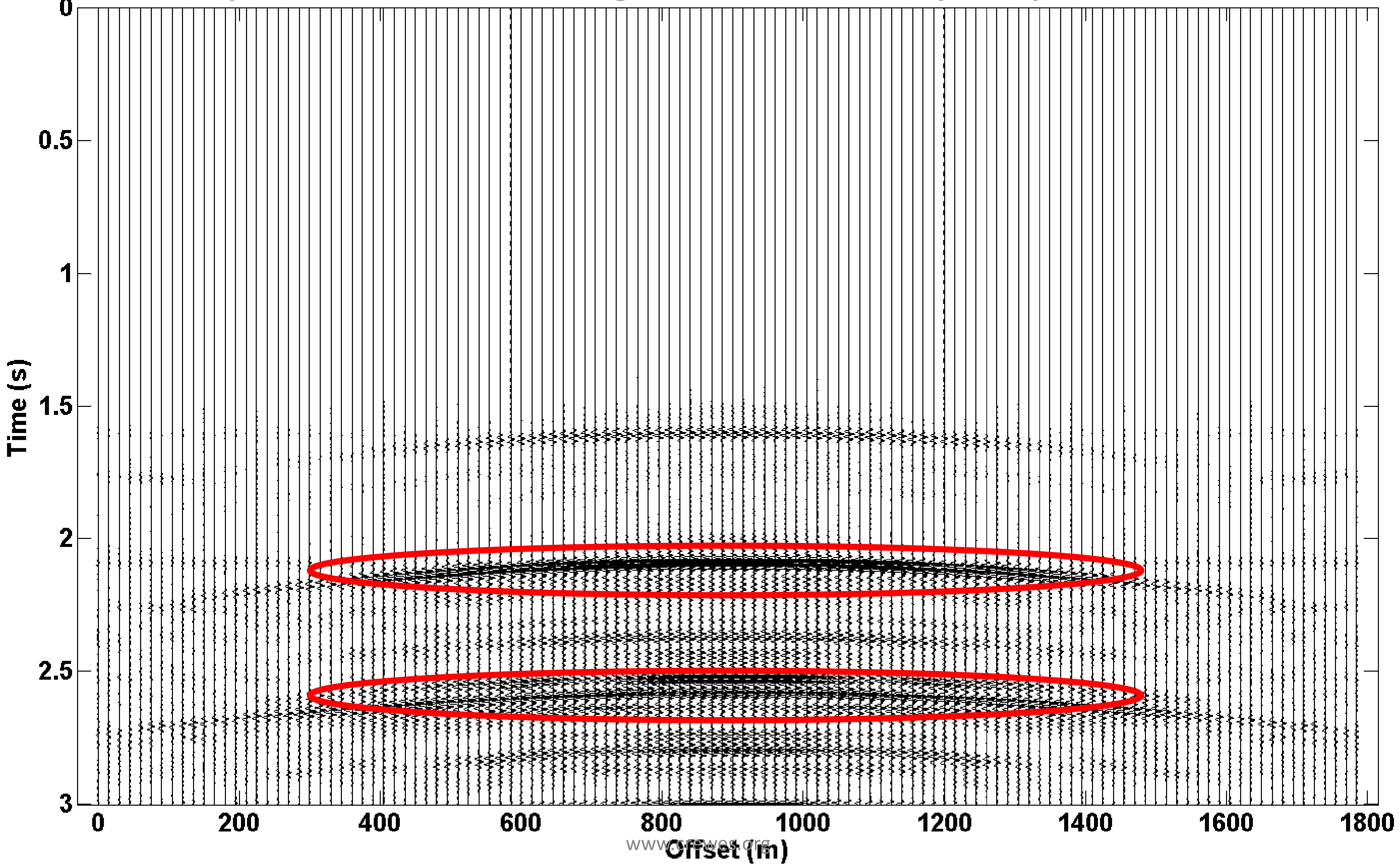


Nonstationary
Trace
Adapted prediction
x 10^-3
1.5
1
0.5
0
-0.5
-1
-1.5
Amplitude
400
600
800
1000
1200
1400
1600
1800
2000
Time (ms)

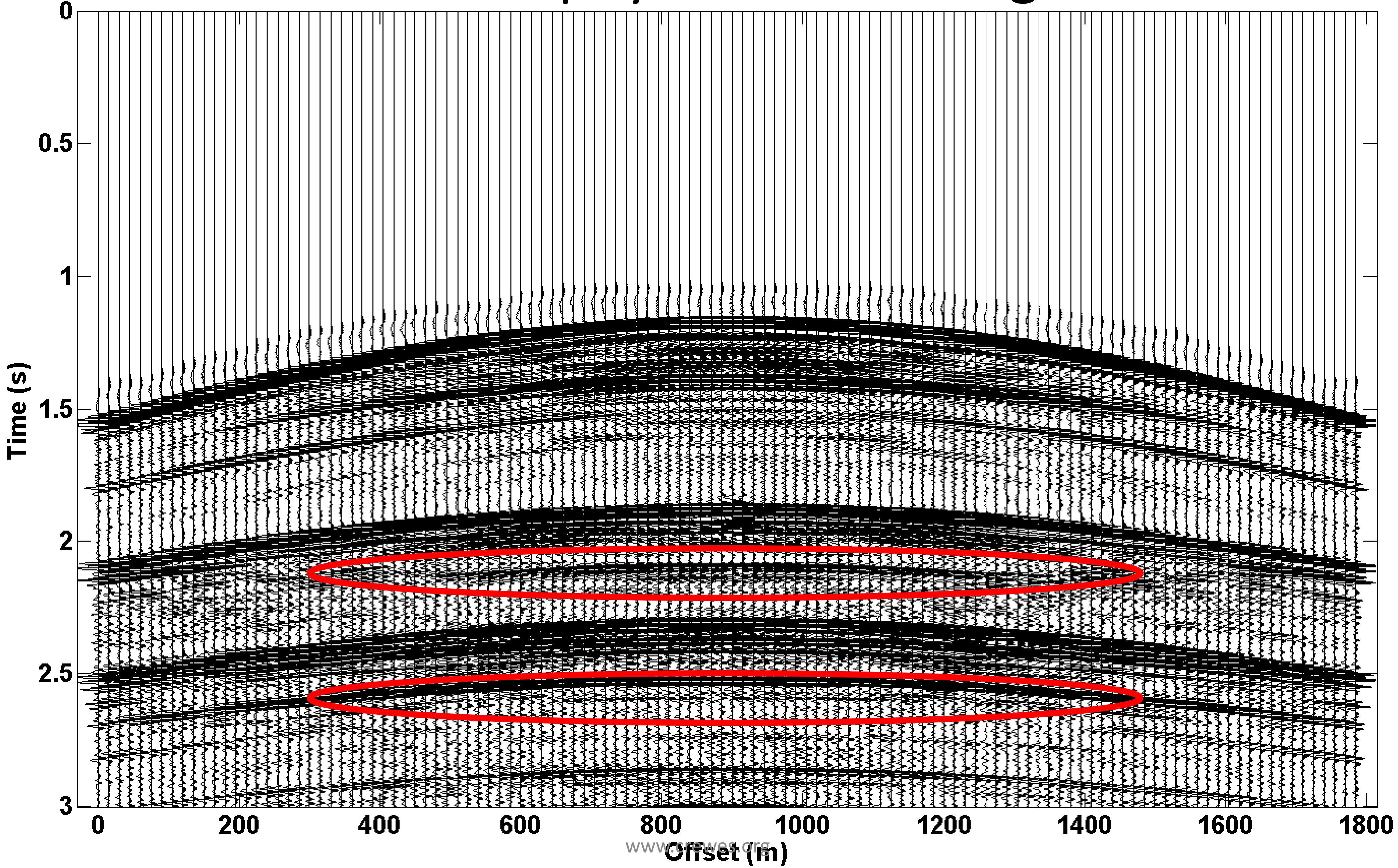
Measured physical modelling data



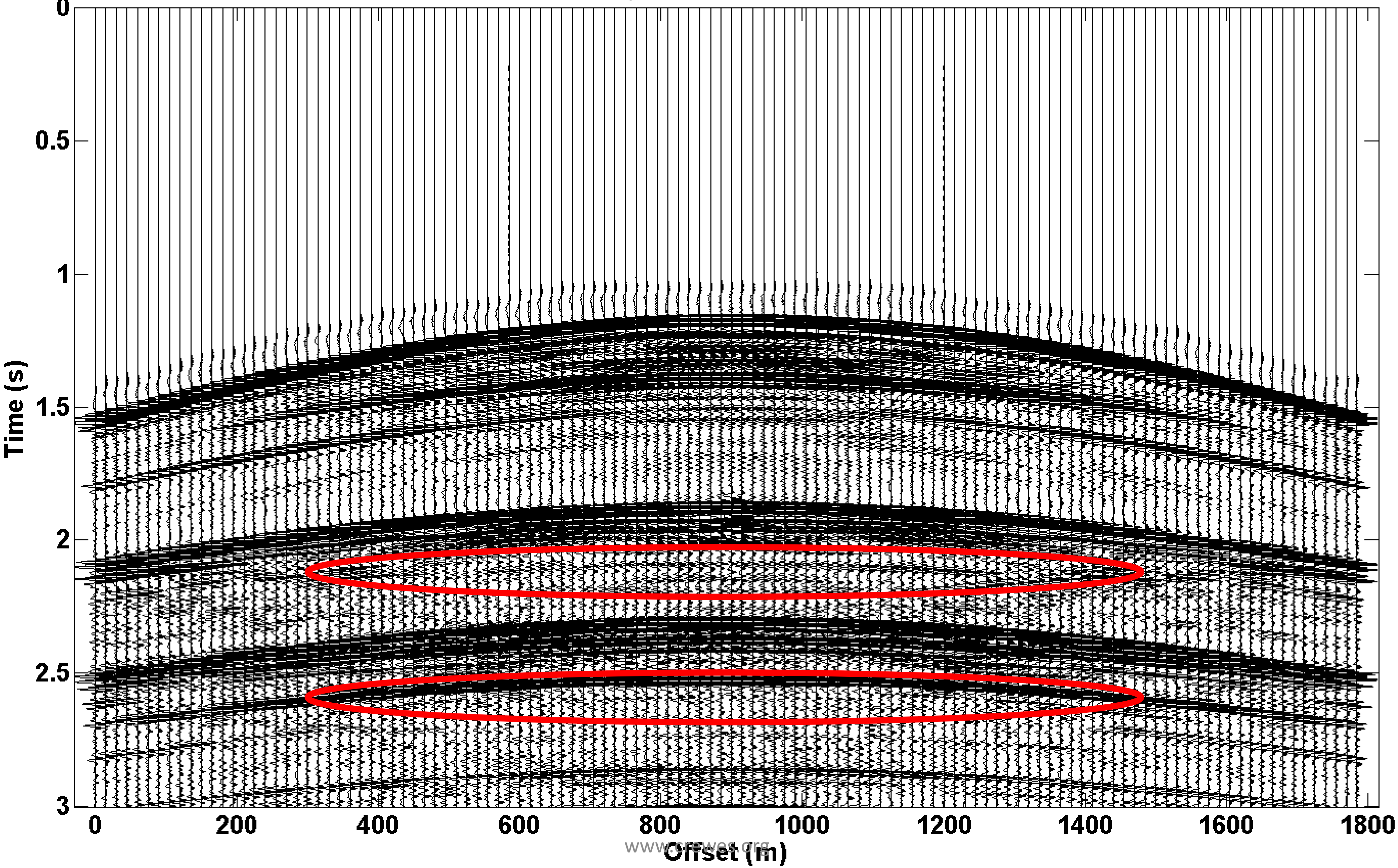
Physical modelling data, multiple prediction



Measured physical modelling data



After adaptive subtraction



Conclusions

- Inverse scattering multiple predictions are inexact in practice, and need to be modified before they can be subtracted from the data.
- This modification can be done by convolving the data with a filter.
- An L_1 minimizing, nonstationary filter provides a means of achieving a reliable and robust adaptive subtraction.

Future Work

- Integrate adaptive subtraction with prediction implementations in tau-pg-ps, kg-t and xg-t domains.
- Apply multiple prediction and adaptive subtraction to field data.

References

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- Guitton, J., and Verschuur, D., 2004, Adaptive subtraction of multiples using the L1-norm, Geophysical Prospecting, **52**, 27-38
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- Pan, P., 2015, 1.5D Internal Multiple Prediction: an Application on Synthetic Data, Physical Modelling Data and Land Data Synthetics.
- Sun, J., 2014,1.5D Internal multiple prediction in the plane wave domain, CREWES Annual Report, **26**.

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