3D dipole borehole-source wavefield simulation

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Outline

Introduction

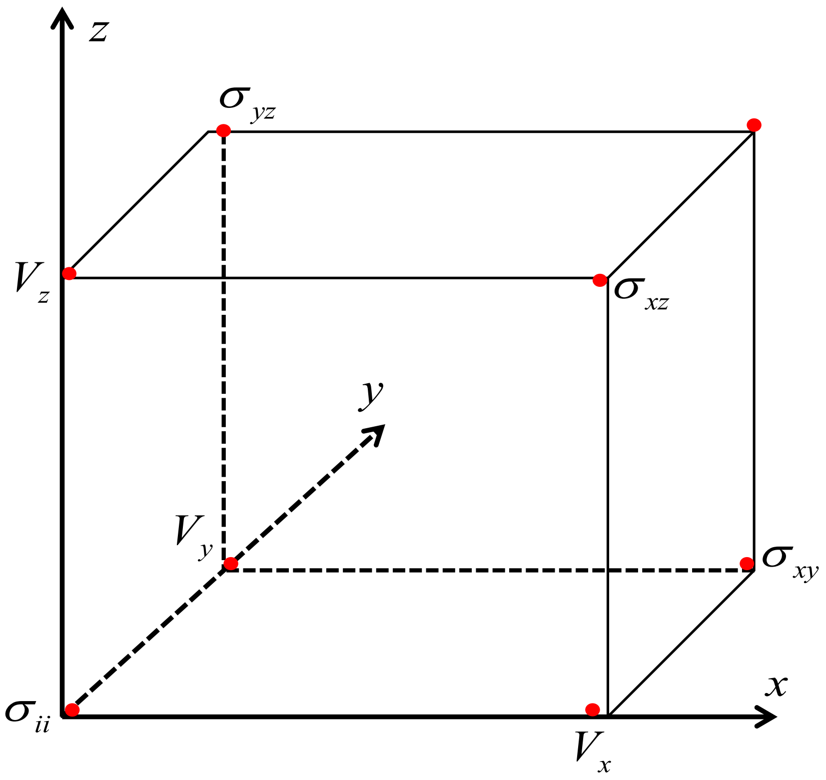
- Great potential has been reviewed for acoustic reflection imaging logging to detect unconventional subtle reservoirs like fractures and vugs.

- Dipole source is capable of unveiling azimuth and dip information of structures outside borehole.

- 3D staggered-grid finite difference method with hybrid perfectly matched layer absorbing scheme.

- Based on the convolutional model of received waveforms, the relationships between the reflection amplitude and the offset as well as the relationships between the reflection reception response and the azimuth angle are analyzed.

Staggered-grid finite difference



$$\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \sigma_{xy}}{\partial y}+\frac{\partial \sigma_{xz}}{\partial z}=\rho\frac{\partial V_x}{\partial t}$$

$$\frac{\partial \sigma_{yx}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}+\frac{\partial \sigma_{yz}}{\partial z}=\rho\frac{\partial V_y}{\partial t}$$

$$\frac{\partial \sigma_{zx}}{\partial x}+\frac{\partial \sigma_{zy}}{\partial y}+\frac{\partial \sigma_{zz}}{\partial z}=\rho\frac{\partial V_z}{\partial t}$$

$$\frac{\partial \sigma_{xx}}{\partial t}=c_{11}\frac{\partial V_x}{\partial x}+(c_{11}-2c_{66})\frac{\partial V_y}{\partial y}+c_{13}\frac{\partial V_z}{\partial z}$$

$$\frac{\partial \sigma_{yy}}{\partial t}=(c_{11}-2c_{66})\frac{\partial V_x}{\partial x}+c_{11}\frac{\partial V_y}{\partial y}+c_{13}\frac{\partial V_z}{\partial z}$$

$$\frac{\partial \sigma_{zz}}{\partial t}=c_{13}\frac{\partial V_x}{\partial x}+c_{13}\frac{\partial V_y}{\partial y}+c_{33}\frac{\partial V_z}{\partial z}$$

$$\frac{\partial \sigma_{yz}}{\partial t}=c_{44}\left(\frac{\partial V_y}{\partial z}+\frac{\partial V_z}{\partial y}\right)$$

$$\frac{\partial \sigma_{xz}}{\partial t}=c_{44}\left(\frac{\partial V_x}{\partial z}+\frac{\partial V_z}{\partial x}\right)$$

$$\frac{\partial \sigma_{xy}}{\partial t}=c_{66}\left(\frac{\partial V_x}{\partial y}+\frac{\partial V_y}{\partial x}\right)$$

$$\delta_x\sigma_{xx}^n\left(l_x+\frac{1}{2},l_y,l_z\right)=\frac{1}{\Delta x}\sum_{m=0}^{N-1}a_m\left[\sigma_{xx}^n(l_x+m+1,l_y,l_z)-\sigma_{xx}^n(l_x+m,l_y,l_z)\right]$$

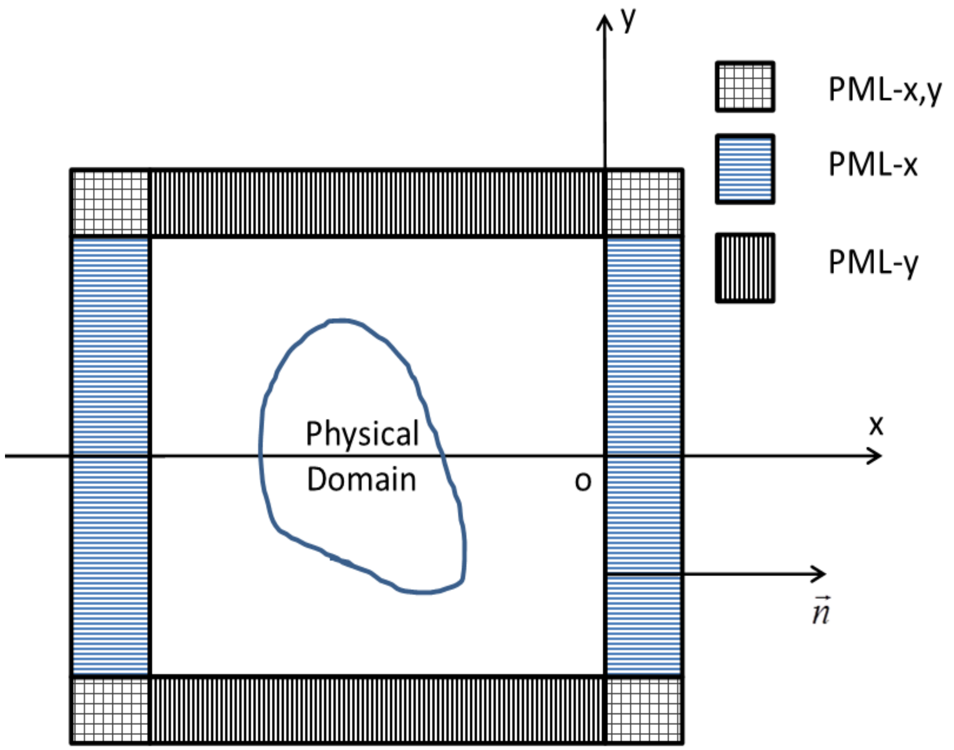
The perfectly matched layer (PML) (Bérenger, 1994)

Take x direction as an example

$$s_x = \frac{i\omega + d_x}{i\omega} = 1 + \frac{d_x}{i\omega}.$$

$$\frac{\partial}{\partial \tilde{x}} = \frac{1}{s_x}\frac{\partial}{\partial x}$$

$\vec{S} = Ae^{-}$ $\cdot$ $\frac{\gamma_x}{\omega}k_x]$ $\tilde{}$ $i\omega$



Drawbacks:

1. Requires the use of split fields
2. Its efficiency becomes poor at grazing incidence after discretization.

The convolutional-PML(C-PML) (Kuzuoglu and Mittra, 1996)

$$s_x = \kappa_x + \frac{d_x}{\alpha_x + i\omega}$$

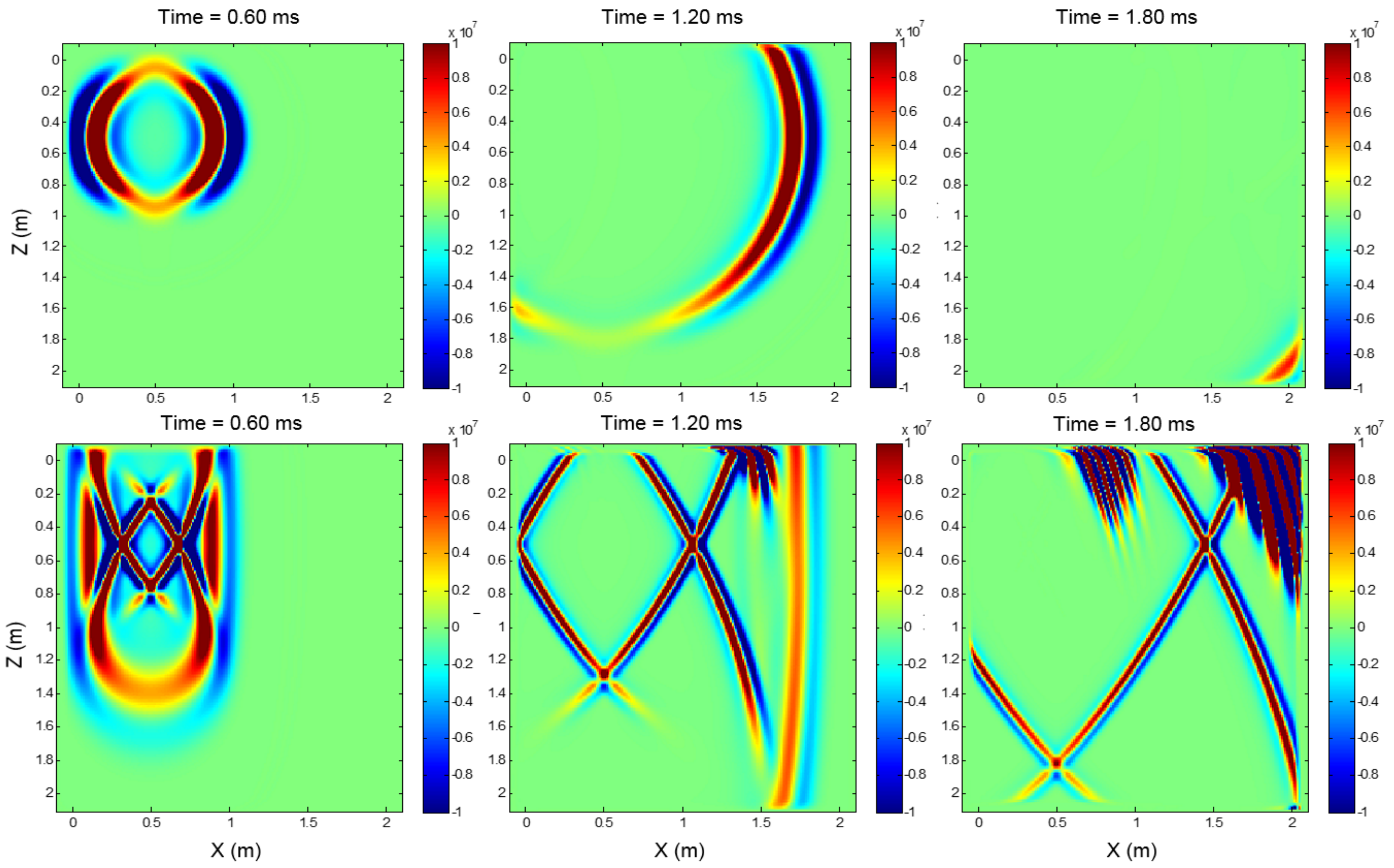
Drawbacks:

Suffers instability either because of its frequency-dependent term or the convolution operations

C-PML

$$C = \begin{bmatrix} 4 & & \\ & 4 & \\ & & 2 \end{bmatrix} GPa$$

$$C = \begin{bmatrix} 4 & 7.5 & \\ 7.5 & 20 & \\ & & 2 \end{bmatrix} GPa$$

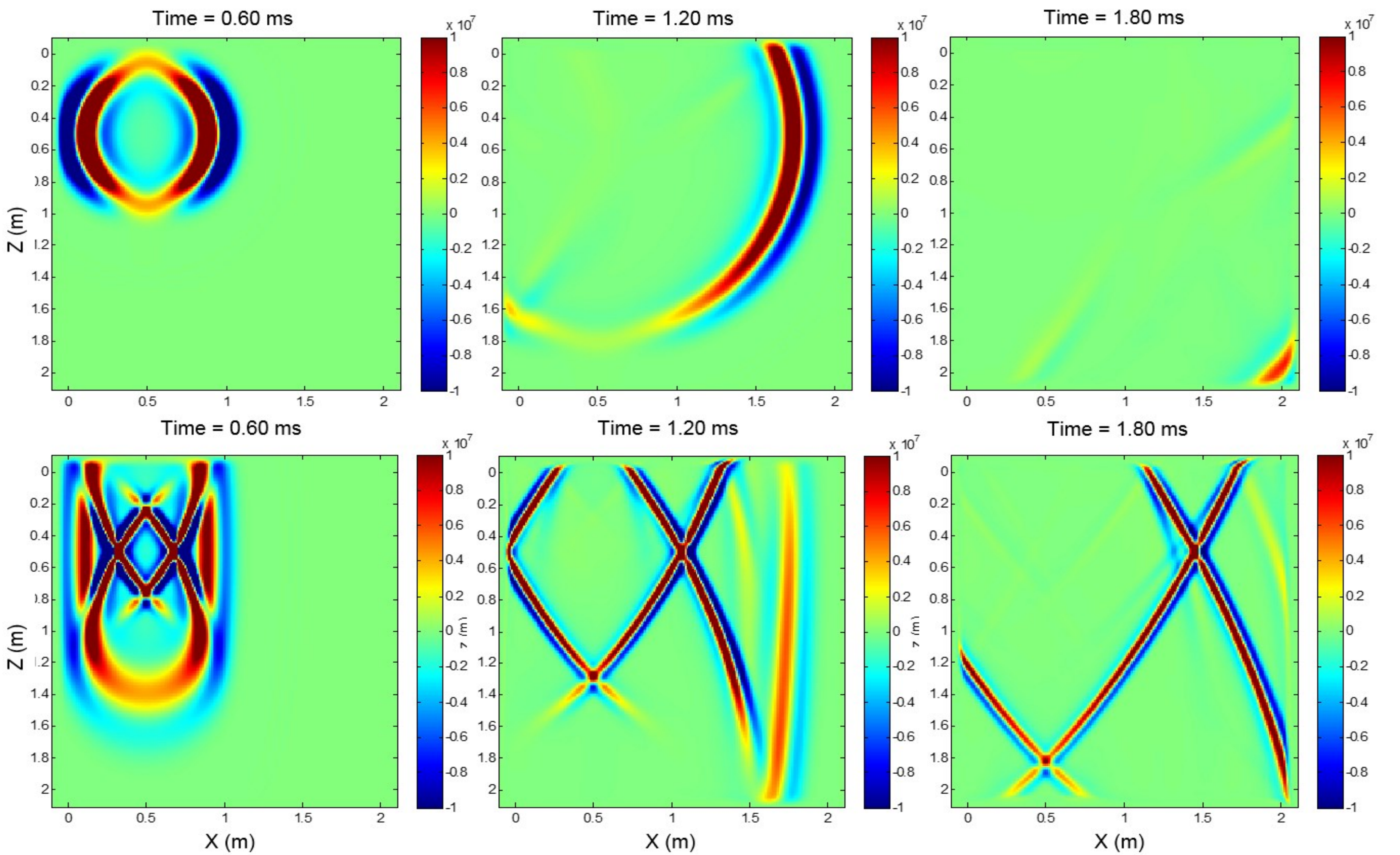


M-PML

$$C=\begin{bmatrix}4 & & \\ & 4 & \\ & & 2\end{bmatrix}GPa$$

$$s_x=1+\frac{d_x+m\cdot d_y+m\cdot d_z}{i\omega}$$

$$C=\begin{bmatrix}4 & 7.5 & \\ 7.5 & 20 & \\ & & 2\end{bmatrix}GPa$$

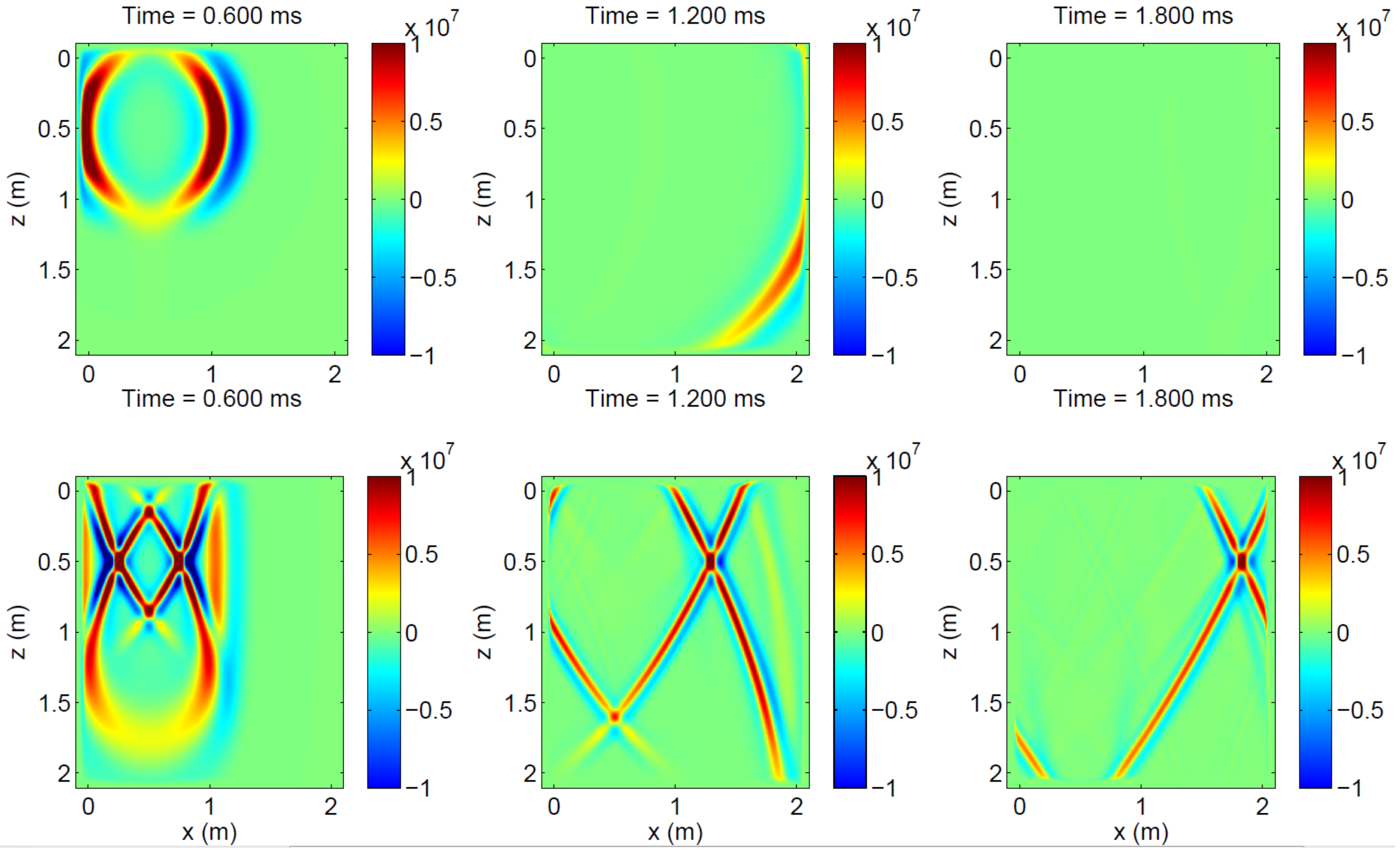


H-PML

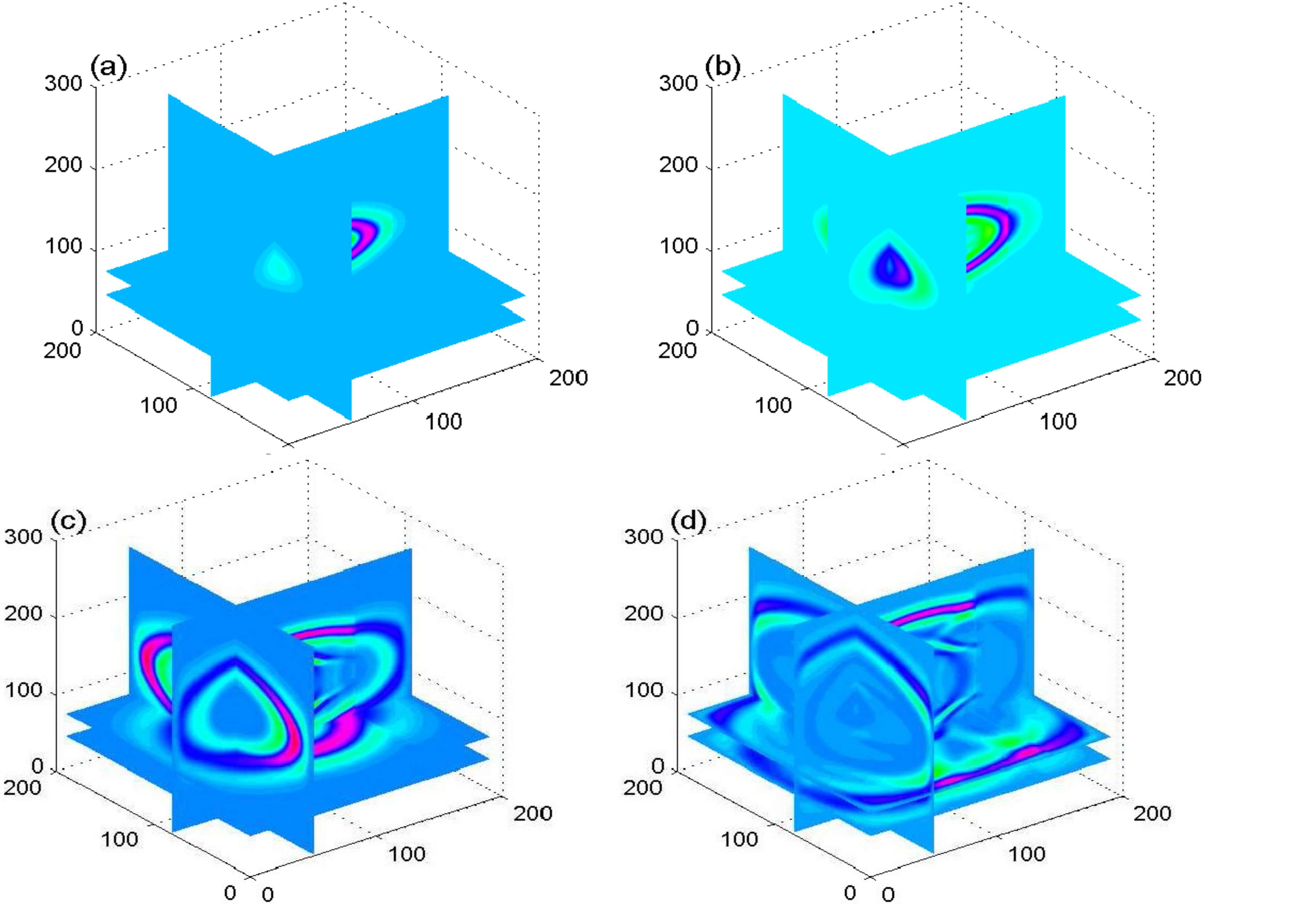
$$C = \begin{bmatrix} 4 & & \\ & 4 & \\ & & 2 \end{bmatrix} GPa$$

$$s_x = \kappa_x + \frac{d_x + m \cdot d_y + m \cdot d_z}{\alpha_x + i\omega}$$

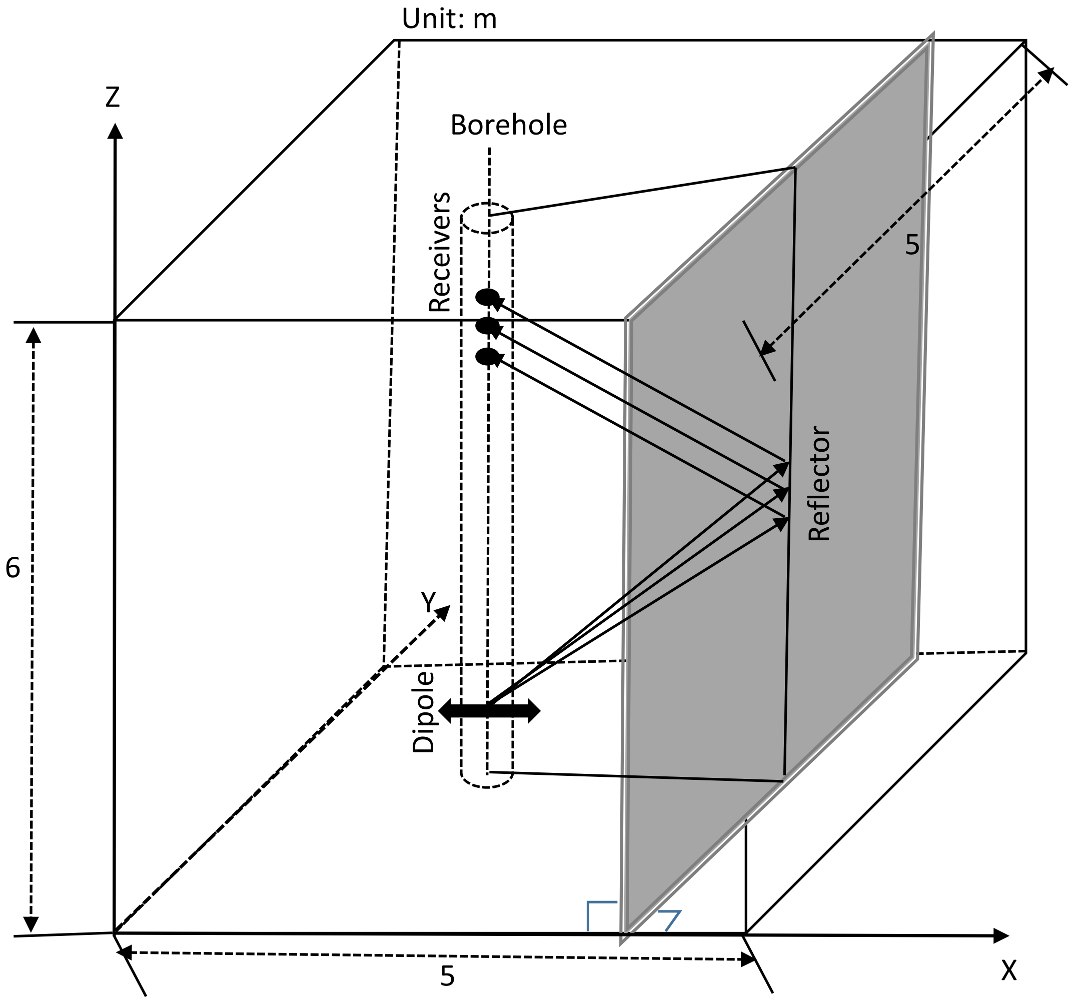
$$C = \begin{bmatrix} 4 & 7.5 & \\ 7.5 & 20 & \\ & & 2 \end{bmatrix} GPa$$



Snapshots in isotropic media

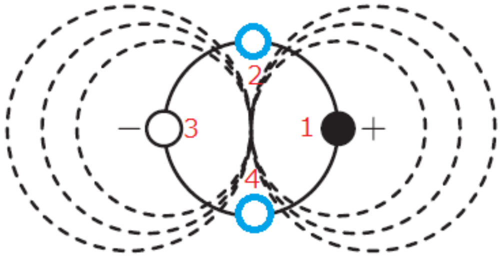


Borehole wave field reception



	$V_{f(m/s)}$	$V_{P(m/s)}$	$V_{S(m/s)}$	$\rho_{(k/cm^3)}$
Borehole	1500	-	-	1.0
Near Borehole formation	-	3000	1200	2.5
Second layer	-	4000	2300	2.5

dx=0.01m, dt=1us, f_0=3k



$$RD_{SH} = i\rho\beta\omega D(\omega, k_0)\sin\theta\cos\phi,$$

(Meredith, 1990)

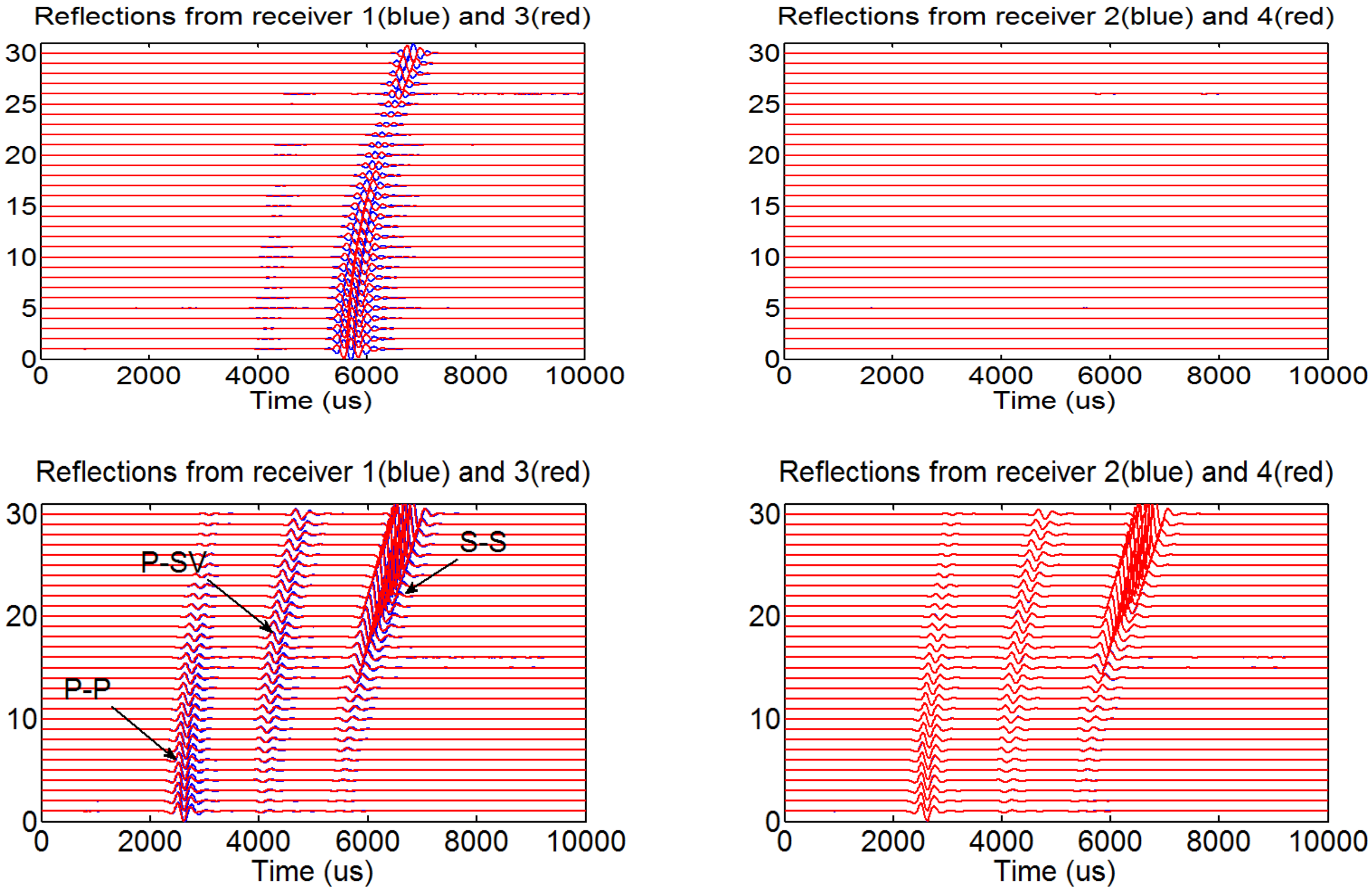
$$RD_{SV} = \rho\beta\omega F(\omega, k_0)\sin\theta\sin\phi,$$

$$RC(\omega,\theta) = RD(\omega,\theta); \quad \text{(Peng et al., 1993)}$$

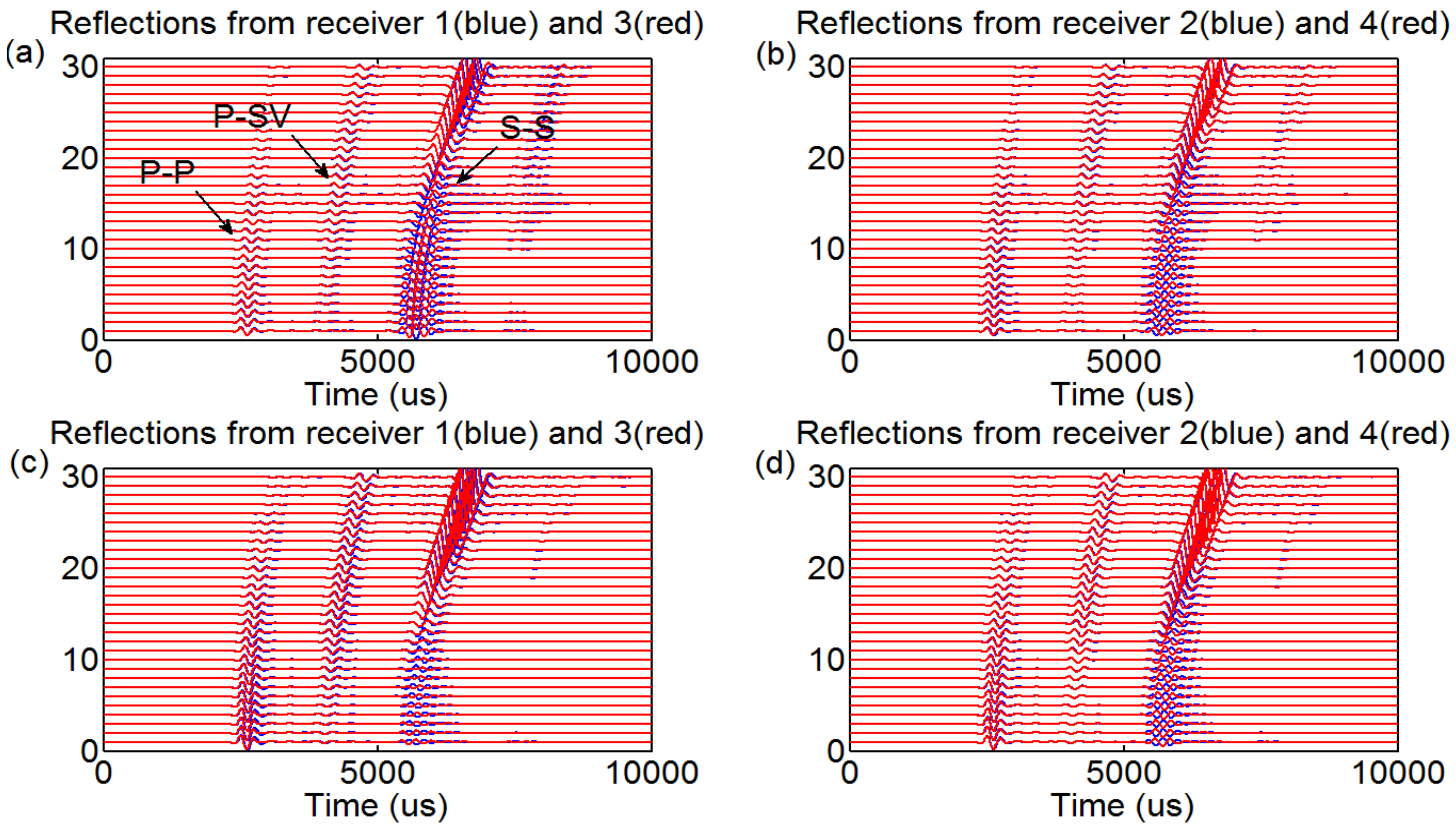
$$R_{(SH)} = \frac{\rho_1\beta_1\cos\varphi_1 - \rho_2\beta_2\cos\varphi_2}{\rho_1\beta_1\cos\varphi_1 + \rho_2\beta_2\cos\varphi_2}$$

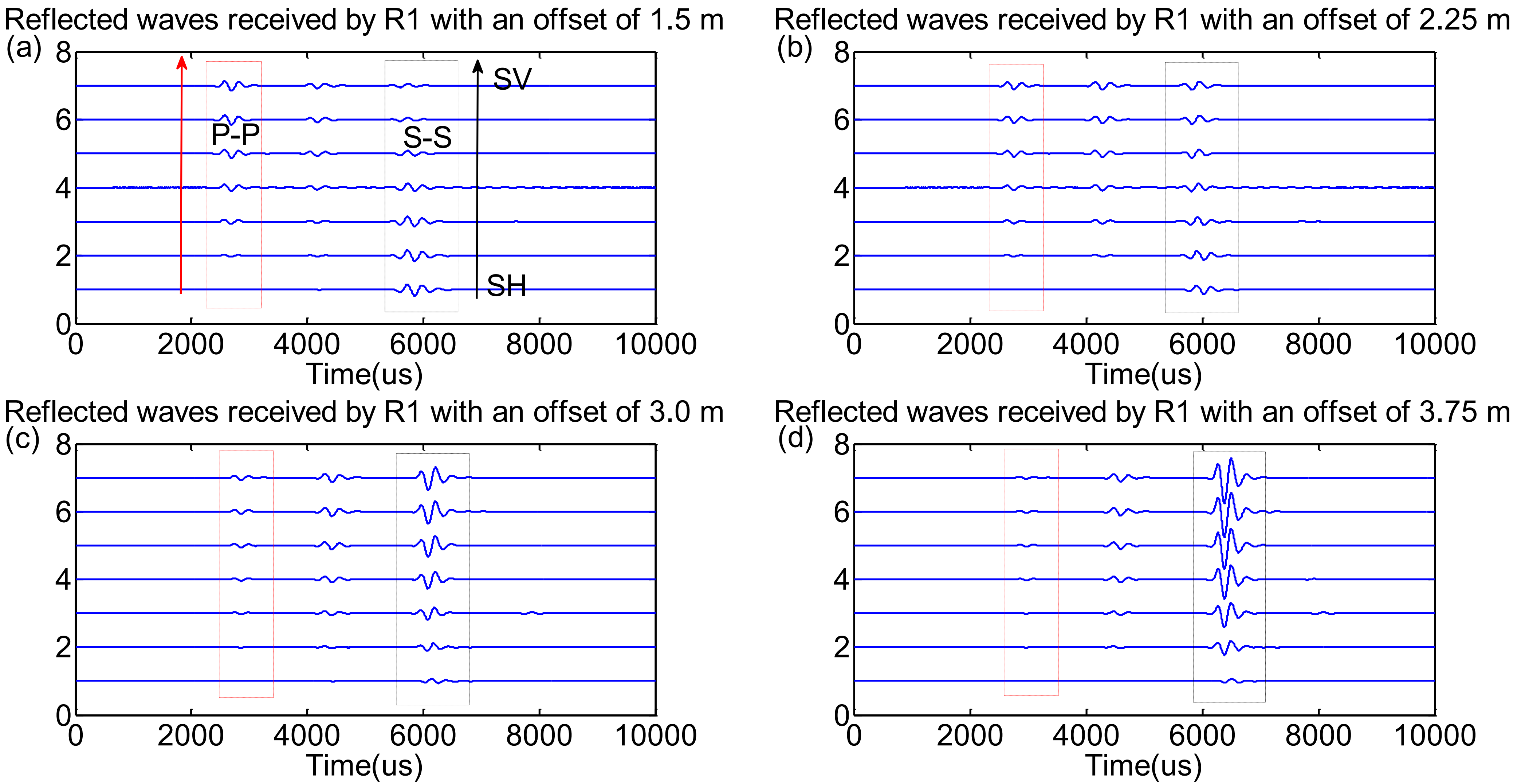
$$RWV(\omega) = S(\omega) * RD(\omega) * RF(\omega) * RC(\omega)\frac{e^{i\omega D/\beta}}{D}e^{-\frac{\omega D}{2Q_\beta\beta}}$$

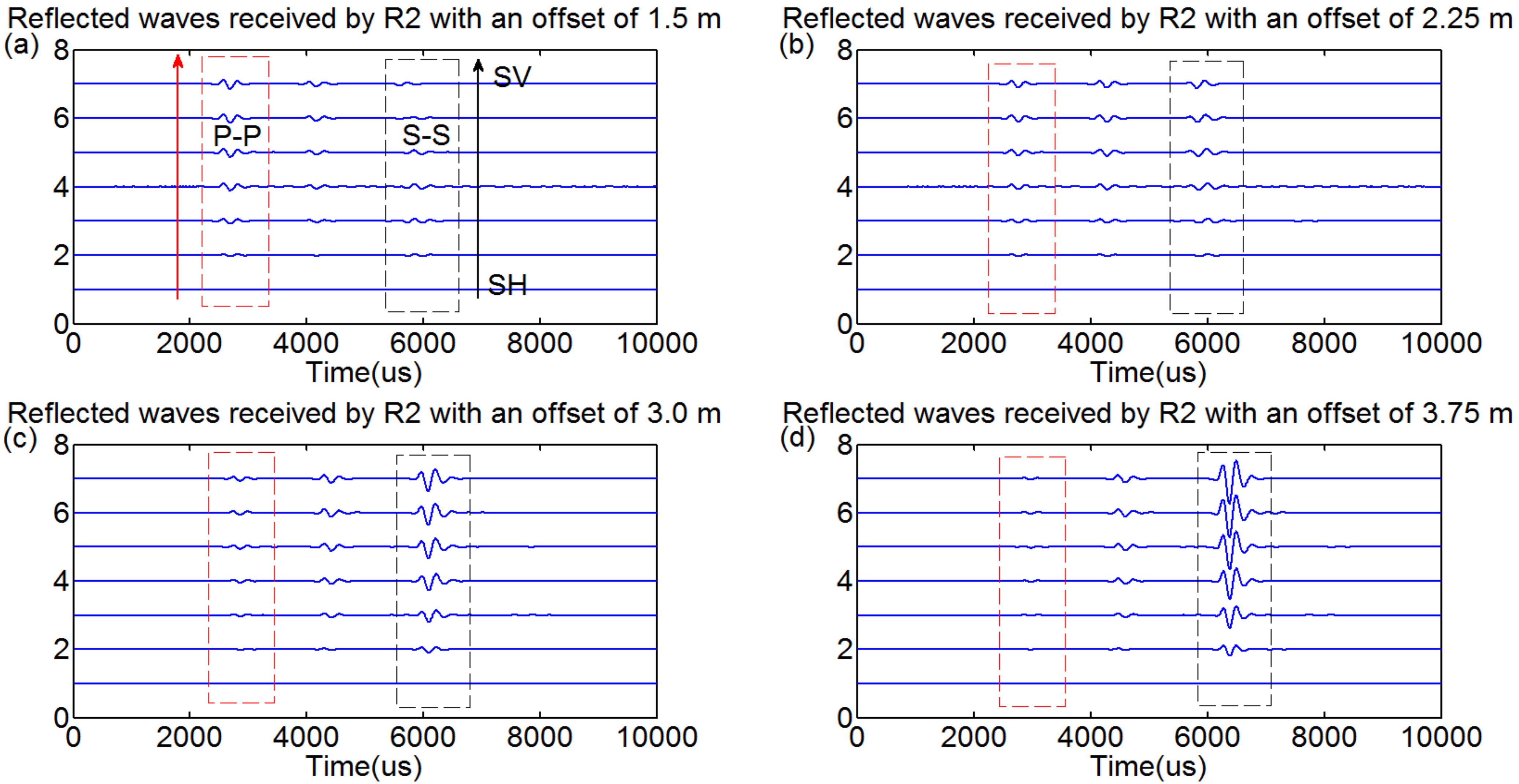
Received reflections when azimuth is 0 and 90 degrees

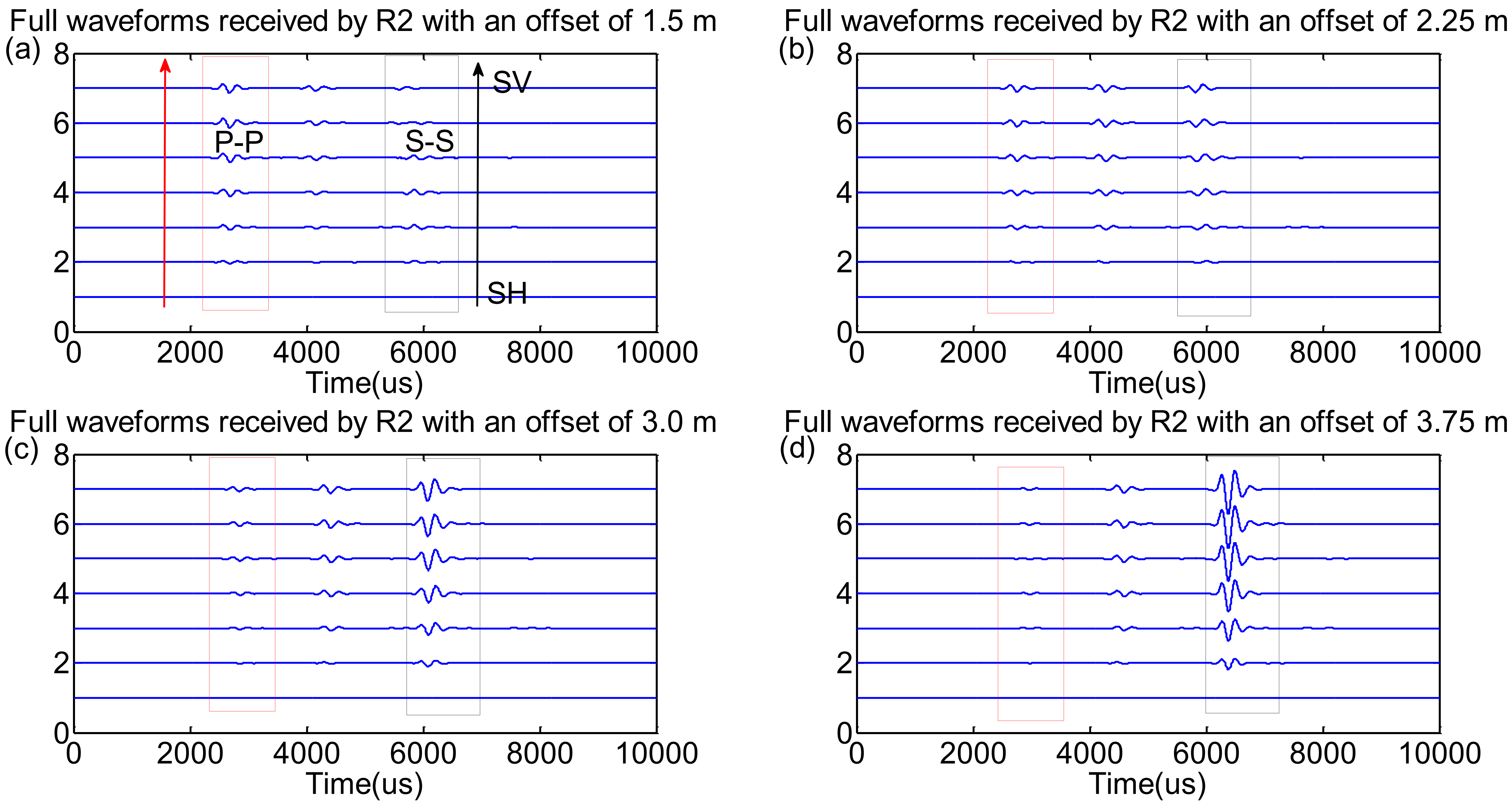


Received reflections when azimuth is 30 and 60 degrees




Reflected waves received by R1 with an offset of 1.5 m
(a)
P-P
S-S
SV
SH
Time(us)
Reflected waves received by R1 with an offset of 2.25 m
(b)
Time(us)
Reflected waves received by R1 with an offset of 3.0 m
(c)
Time(us)
Reflected waves received by R1 with an offset of 3.75 m
(d)
Time(us)


Reflected waves received by R2 with an offset of 1.5 m
(a)
P-P
S-S
SV
SH
Time(us)
Reflected waves received by R2 with an offset of 2.25 m
(b)
Time(us)
Reflected waves received by R2 with an offset of 3.0 m
(c)
Time(us)
Reflected waves received by R2 with an offset of 3.75 m
(d)
Time(us)

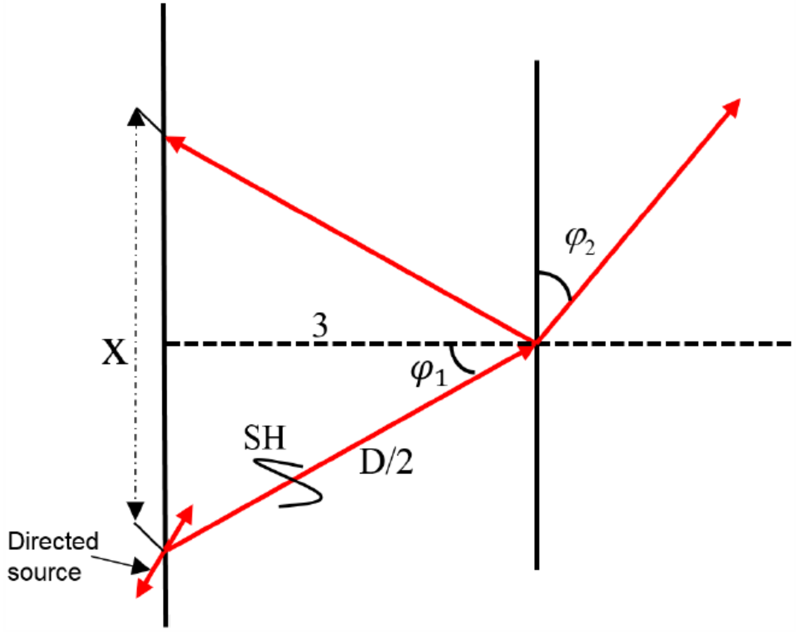

Full waveforms received by R2 with an offset of 1.5 m
(a)
P-P
S-S
SV
SH
Time(us)
Full waveforms received by R2 with an offset of 2.25 m
(b)
Time(us)
Full waveforms received by R2 with an offset of 3.0 m
(c)
Time(us)
Full waveforms received by R2 with an offset of 3.75 m
(d)
Time(us)
0
2
4
6
8
0
2000
4000
6000
8000
10000

$$RWV_{SH}(\omega) = S(\omega) * RD_{SH} * R_{(SH)} * RD_{SH}\frac{e^{i\omega D/\beta}}{D}$$

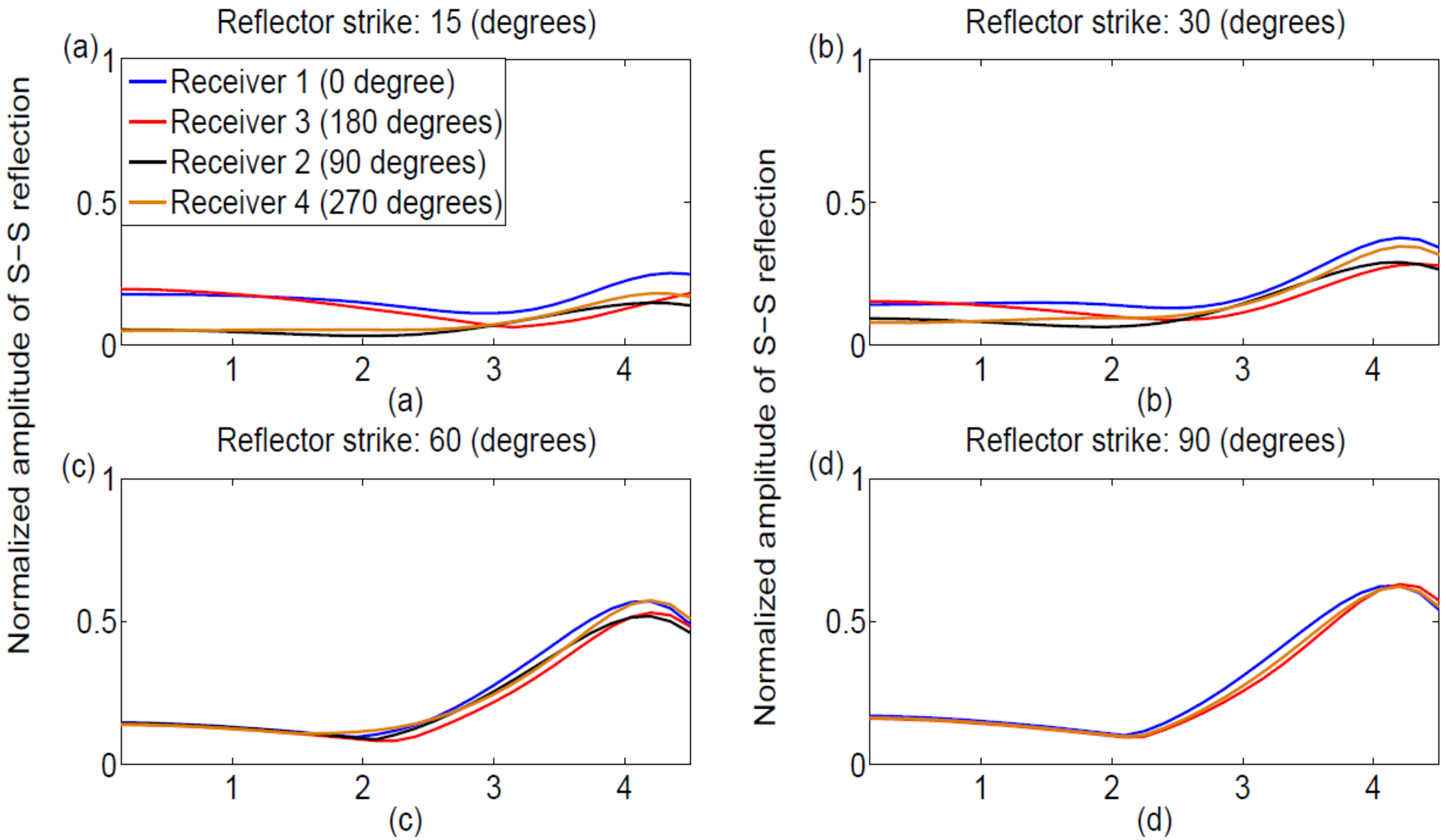
$$H(\omega) = S(\omega) * RD_{SH} * R_{(SH)} * RD_{SH}$$

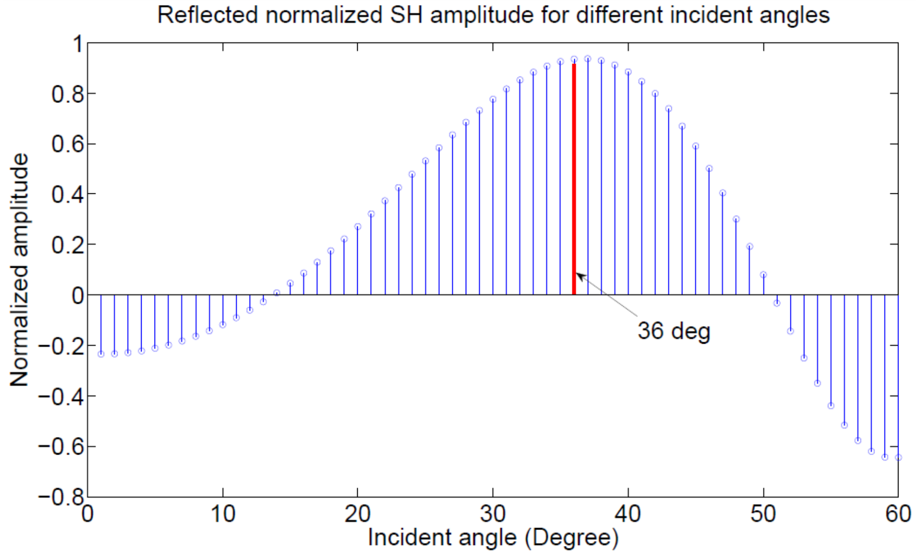
$$H(\omega)\frac{e^{i\omega D/\beta}}{D} = \frac{1}{D}\int_{-\infty}^{+\infty} h(t)e^{i\omega(t-D/\beta)}dt = RWV(\omega)$$

$$h(t) = s(t)rd^2_{(SH)}r_{(SH)}$$



$$RWV(\omega) = r_{(SH)}\frac{-\frac{1}{4\pi}\rho^2\beta^2\cos^2\phi}{D}\int_{-\infty}^{+\infty} s(t)dt\int_{-\infty}^{+\infty}\omega^2 D^2(\omega, k_0)e^{iw(3t-D/\beta)}d\omega$$



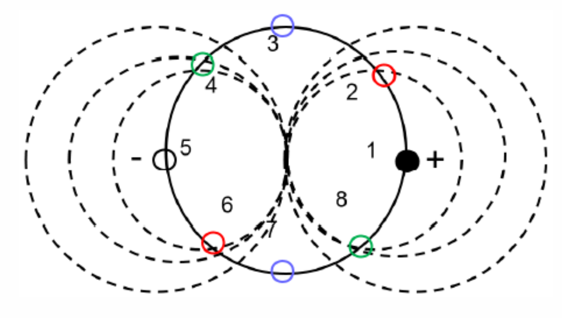


$$RWV(\omega) = r_{(SH)} \frac{-\frac{1}{4\pi}\rho^2\beta^2\cos^2\phi}{D} \int_{-\infty}^{+\infty} s(t)dt \int_{-\infty}^{+\infty} \omega^2 D^2(\omega, k_0) e^{iw(3t-D/\beta)} d\omega$$

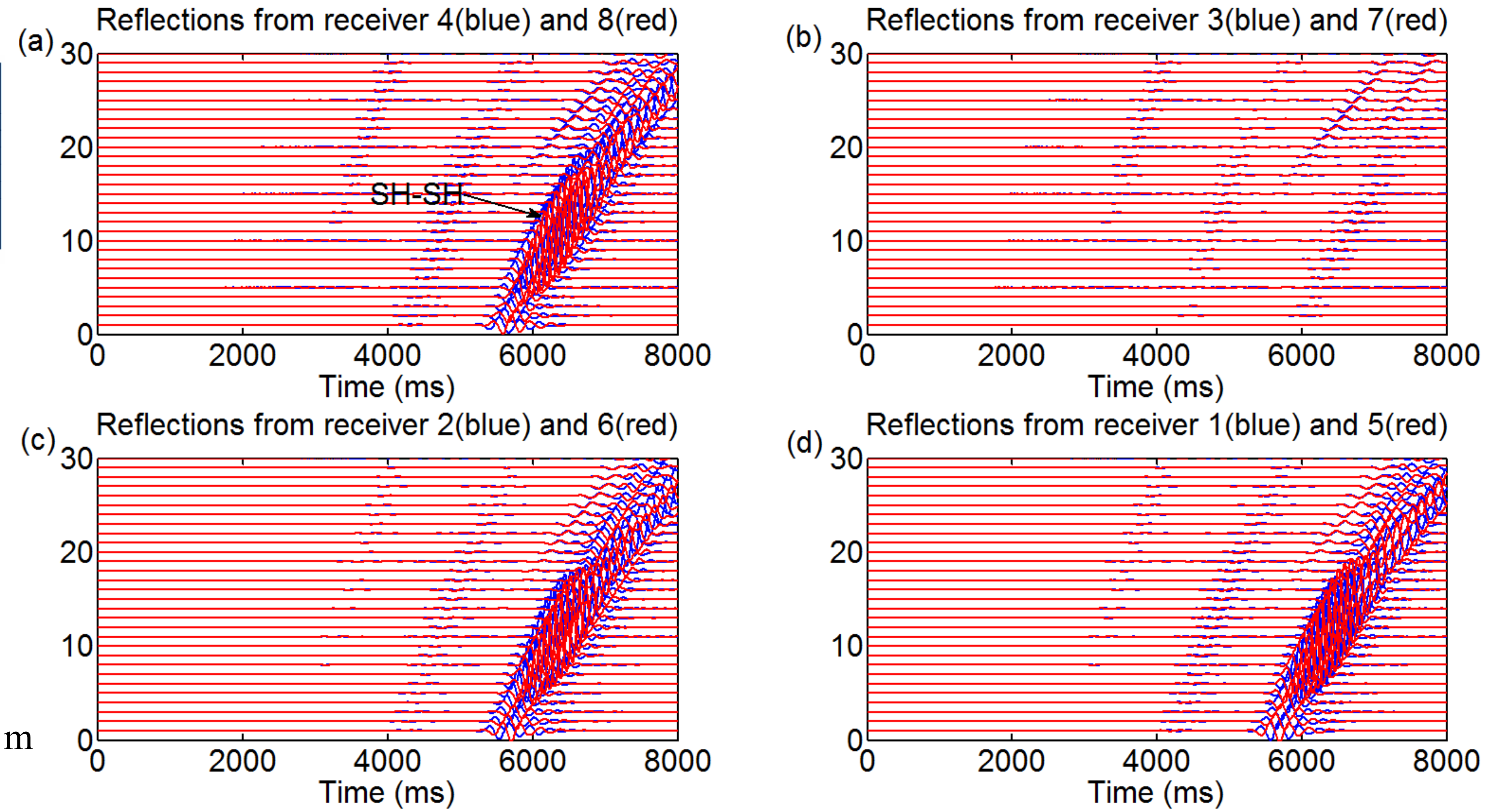
$$\cos\varphi_2 = \sqrt{1 - \frac{\beta_2^2}{\beta_1^2}\left(1 - \frac{36}{D^2}\right)}$$

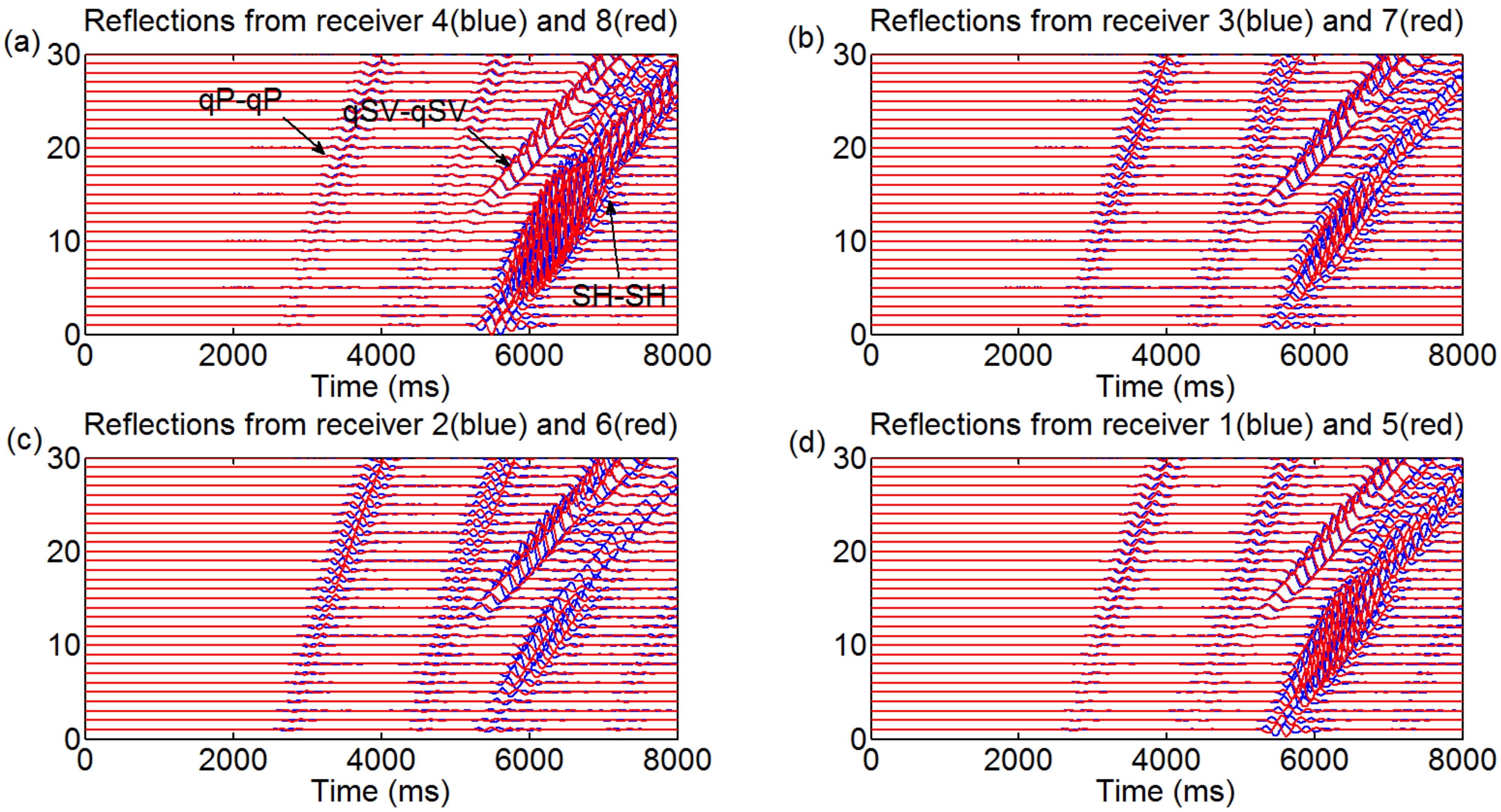
Simulation for a dipole in VTI media

$$c_{VTI} = \begin{bmatrix} 23.87 & 15.33 & 9.79 & 0 & 0 & 0 \\ 15.33 & 23.87 & 9.79 & 0 & 0 & 0 \\ 9.79 & 9.79 & 15.33 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.77 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.77 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.27 \end{bmatrix}$$

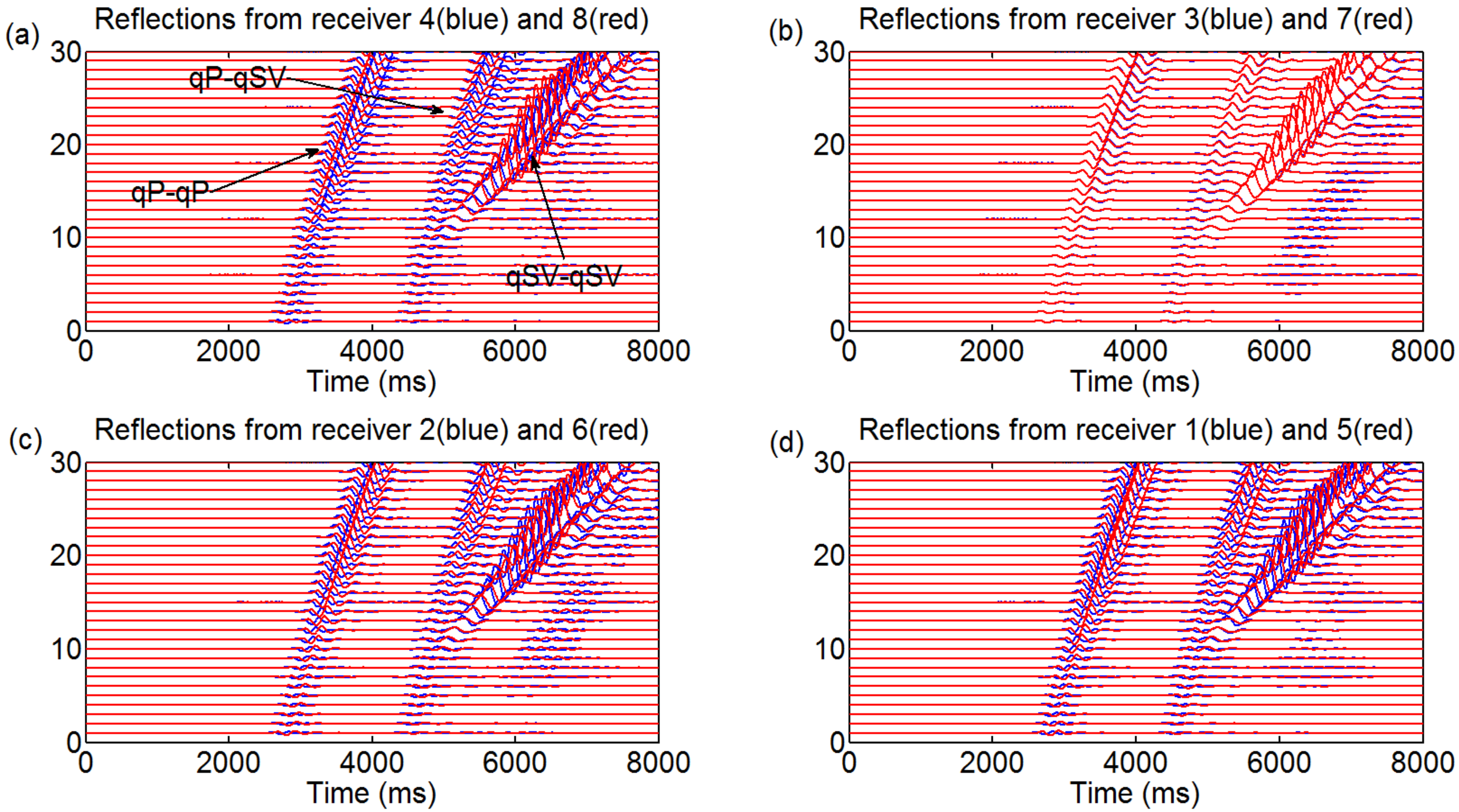


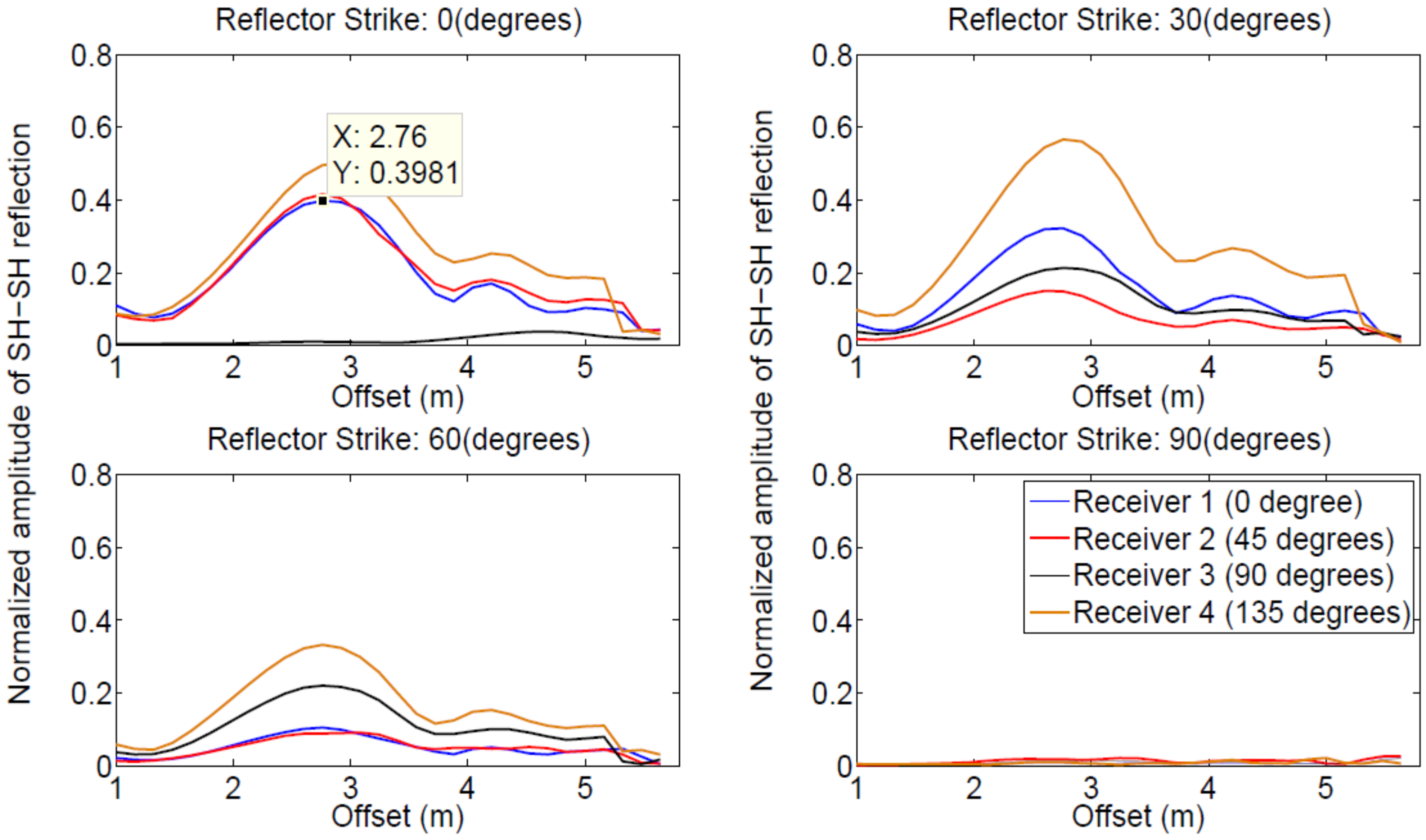
dx=0.01m, dt=1us, f_0=3k
Receiver spacing: 0.16 m
Offset of receivers: 1 to 5.64 m





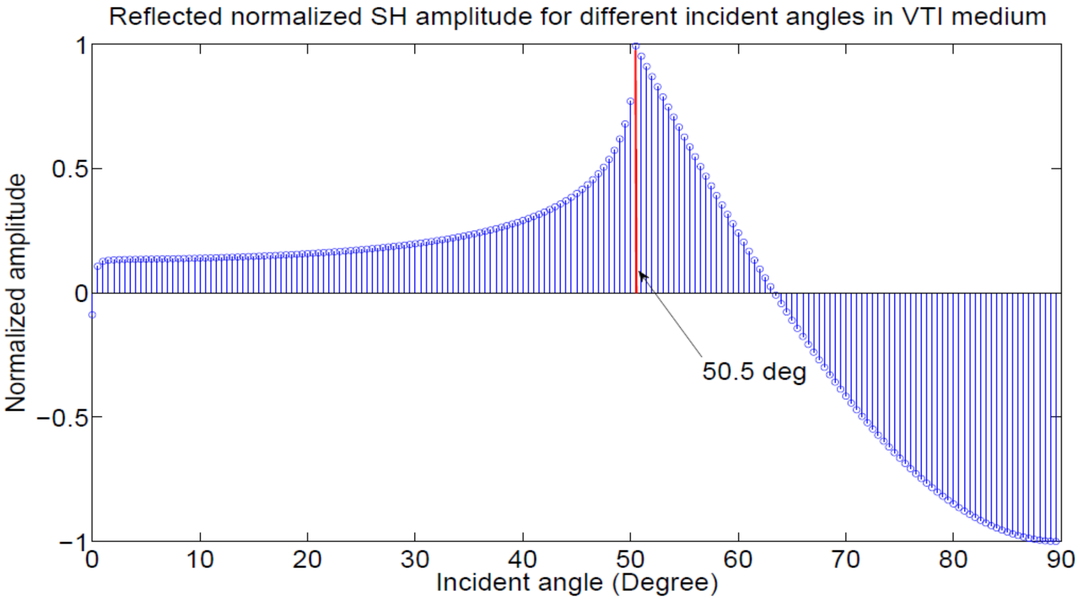
(a)
Reflections from receiver 4(blue) and 8(red)
qP-qP
qSV-qSV
SH-SH
Time (ms)
(b)
Reflections from receiver 3(blue) and 7(red)
Time (ms)
(c)
Reflections from receiver 2(blue) and 6(red)
Time (ms)
(d)
Reflections from receiver 1(blue) and 5(red)
Time (ms)


(a)
Reflections from receiver 4(blue) and 8(red)
qP-qSV
qP-qP
qSV-qSV
(b)
Reflections from receiver 3(blue) and 7(red)
(c)
Reflections from receiver 2(blue) and 6(red)
(d)
Reflections from receiver 1(blue) and 5(red)
0
10
20
30
0
2000
4000
6000
8000
Time (ms)



$$R_{SH} = \frac{\dfrac{\sqrt{\rho_1} c_{44}^{I} \cos\varphi_1}{\sqrt{c_{66}^{I} \sin\varphi_1{}^2 + c_{44}^{I} \cos\varphi_1{}^2}} - \dfrac{\sqrt{\rho_2} c_{44}^{II} \cos\varphi_2}{\sqrt{c_{66}^{II} \sin^2\varphi_2 + c_{44}^{II} \cos^2\varphi_2}}}{\dfrac{\sqrt{\rho_1} c_{44}^{I} \cos\varphi_1}{\sqrt{c_{66}^{I} \sin^2\varphi_1 + c_{44}^{I} \cos^2\varphi_1}} + \dfrac{\sqrt{\rho_2} c_{44}^{II} \cos\varphi_2}{\sqrt{c_{66}^{II} \sin^2\varphi_2 + c_{44}^{II} \cos^2\varphi_2}}}$$

$$\varphi_2 = \arcsin\sqrt{\frac{\rho_1 c_{44}^{II} \sin^2\varphi_1}{[\rho_2(c_{66}^{I} - c_{44}^{I}) - \rho_1(c_{66}^{II} - c_{44}^{II})]\sin^2\varphi_1 + \rho_2 c_{44}^{I}}}$$



Conclusions

- A hybrid PML based on the C-PML and M-PML is proposed and used in 3D staggered-grid FD method.

- The wavefield simulation for a directional dipole source in isotropic media is analyzed. And a transition is detected between the SH-SH reflection and SV-SV reflection with the increase of the offset. Based on the cross-plot of maximum amplitude versus receiver offsets, both the distance between the borehole and the reflector and the critical angle can be calculated.

- For a further discussion, the wavefield simulation for a directional dipole source in VTI media is discussed. The SH-SH reflection coefficient in the VTI medium is introduced and used to calculate the relationship between the incident angle and reflected amplitude.

Discussion and future work

SH-SH reflection imaging

$$c_{66}k_r^2 + c_{44}k_z^2 - \rho\omega^2 = 0 \qquad v_{sh}^2(\theta) = v_{so}^2(1 + 2\gamma \sin^2\theta)$$

$$\frac{\partial^2 U_{SH}(\mathbf{k},t)}{\partial t^2} = -v_{so}^2(\mathbf{k}^2 + 2\gamma k_r^2)U_{SH}(\mathbf{k},t)$$

$$U_{SH}(\mathbf{k}, t+\Delta t) = 2U_{SH}(\mathbf{k},t) - v_{so}^2\Delta^2 t(\mathbf{k}^2 + 2\gamma k_r^2)U_{SH}(\mathbf{k},t) - U_{SH}(\mathbf{k}, t-\Delta t)$$

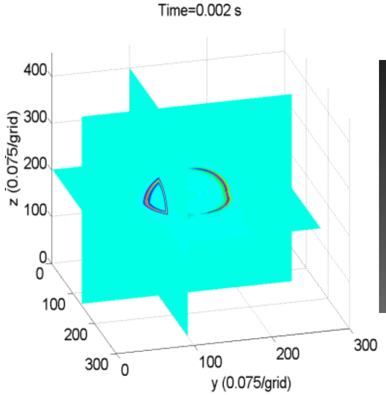
Coordinate Stretching scheme

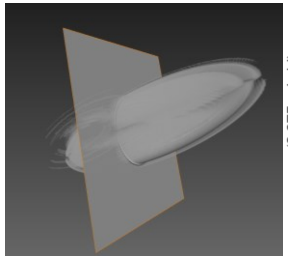
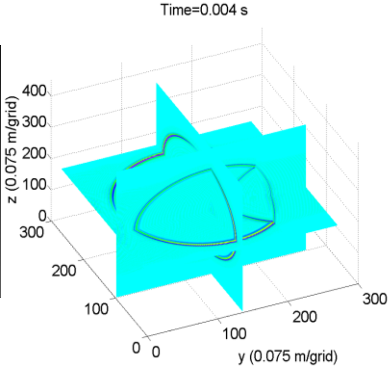
$$-\rho\omega^2\varepsilon_x\varepsilon_y\varepsilon_z\mathbf{u} = \nabla\cdot\tilde{\mathbf{T}}$$

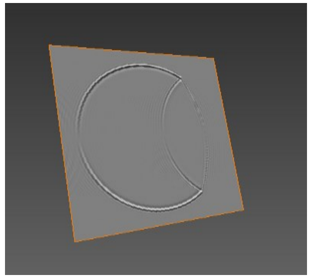
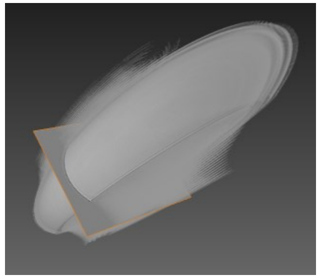
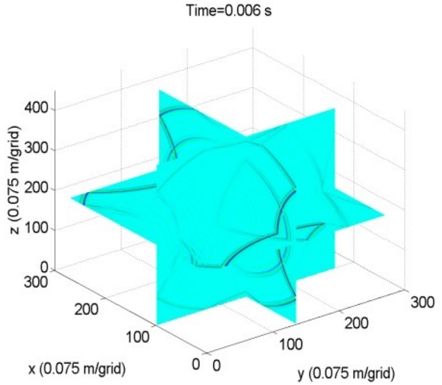
$$\tilde{c}_{ijkl} = c_{ijkl}\frac{\varepsilon_x\varepsilon_y\varepsilon_z}{\varepsilon_i\varepsilon_k}, (i,j,k,l = x,y,z)$$

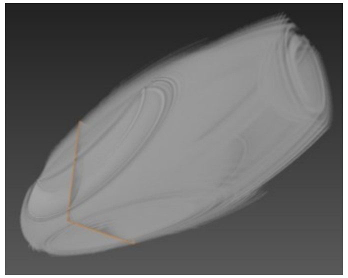
$$\partial/\partial\tilde{x}_i = (1/\varepsilon_i)\partial/\partial x_i$$

$$\varepsilon_x = 1 - i\gamma \qquad \gamma = \gamma_{max}\frac{(n-b)^2}{(M-b)^2}$$





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Questions & Comments