Convergence of a FWI scheme based on PSPI migration

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Dr. Robert Ferguson

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Outline

- Introduction
- Line search vs least squares step length
- Gradient
- Conjugate gradient
- Impedance inversion
- Conclusions
- Future work

Introduction

Introduction

Least Squares

Introduction

Least Squares

Minimize $C(m) \equiv \|\Delta d_m\|^2$

Introduction

Least Squares

Minimize $C(m) \equiv \|\Delta d_m\|^2$

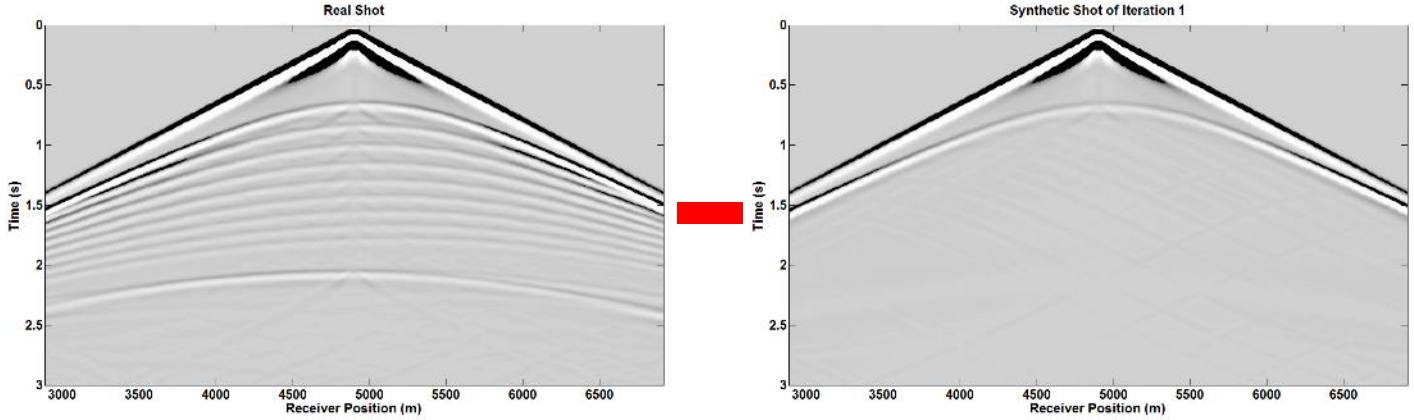
$\boldsymbol{d_{obs} - d_{syn}}$

Introduction

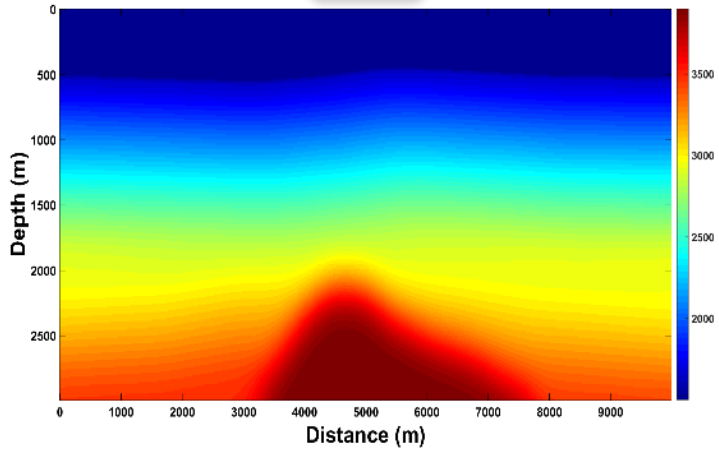
Least Squares

Minimize $C(m) \equiv |\Delta d_m|^2$

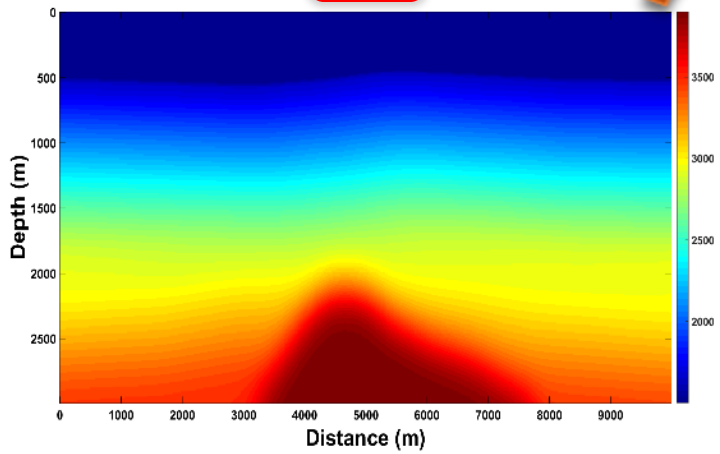
$$d_{obs} - d_{syn}$$

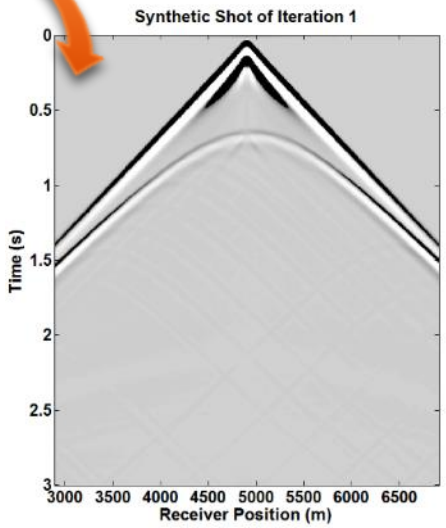


m_0

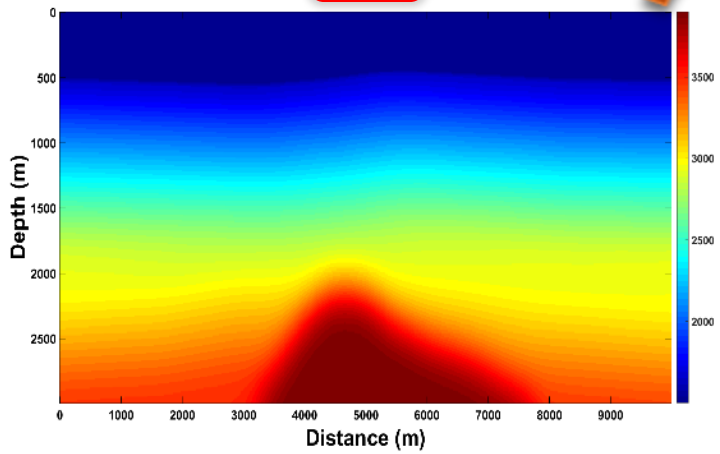

Depth (m)
0
500
1000
1500
2000
2500
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
Distance (m)
3500
3000
2500
2000

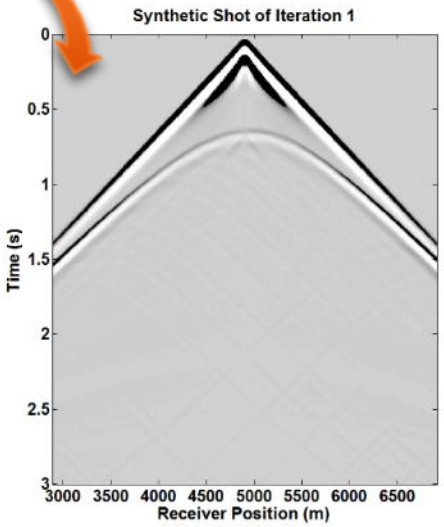
m_0

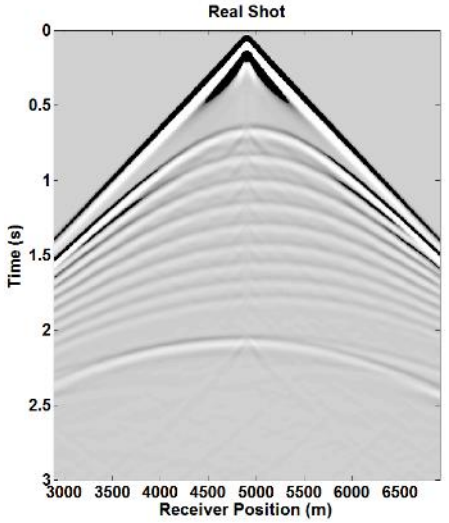

Depth (m)
Distance (m)
0
500
1000
1500
2000
2500
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
3500
3000
2500
2000

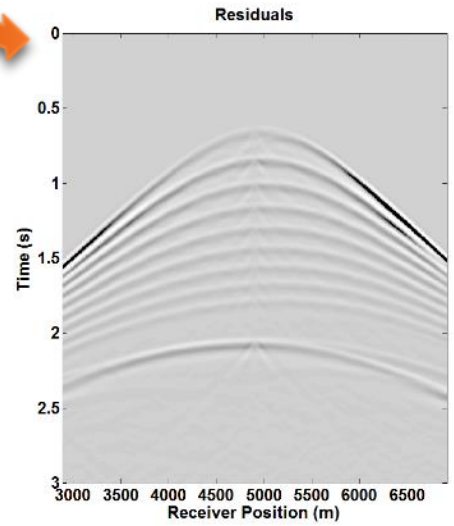

Synthetic Shot of Iteration 1
Time (s)
Receiver Position (m)
0
0.5
1
1.5
2
2.5
3
3000
3500
4000
4500
5000
5500
6000
6500

m_0









Δd_m

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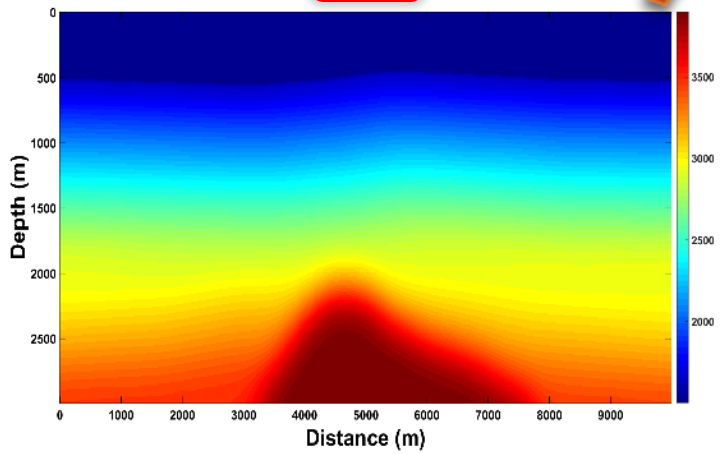
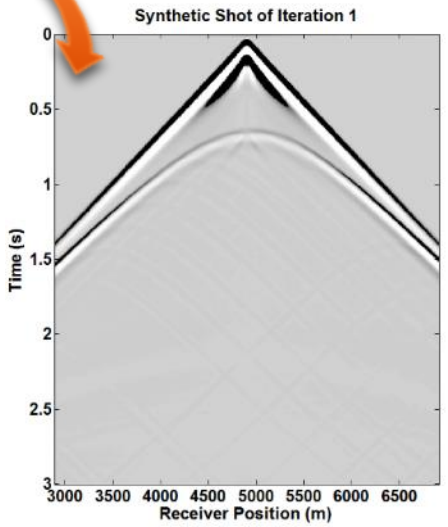
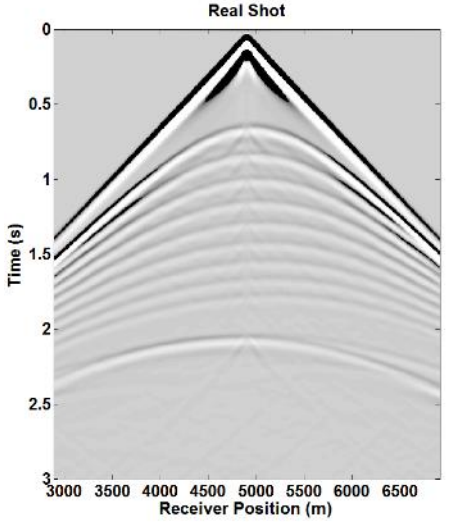
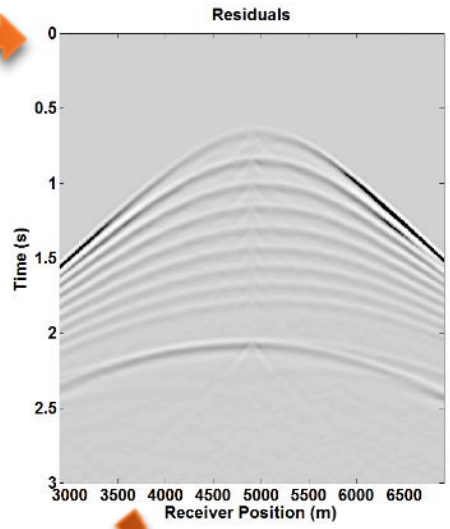
m_0









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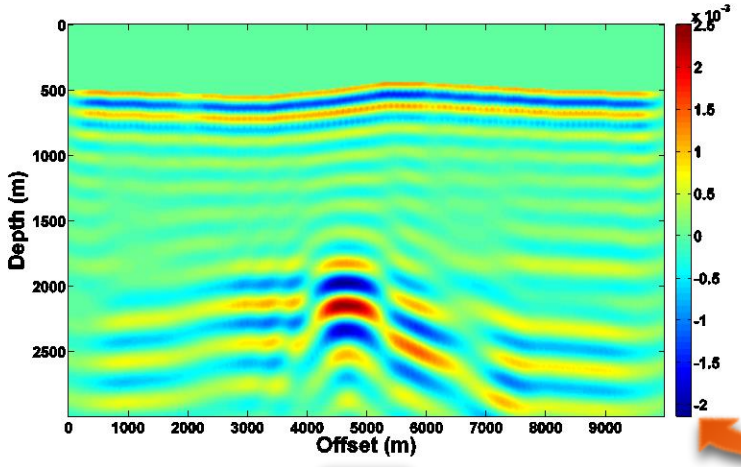
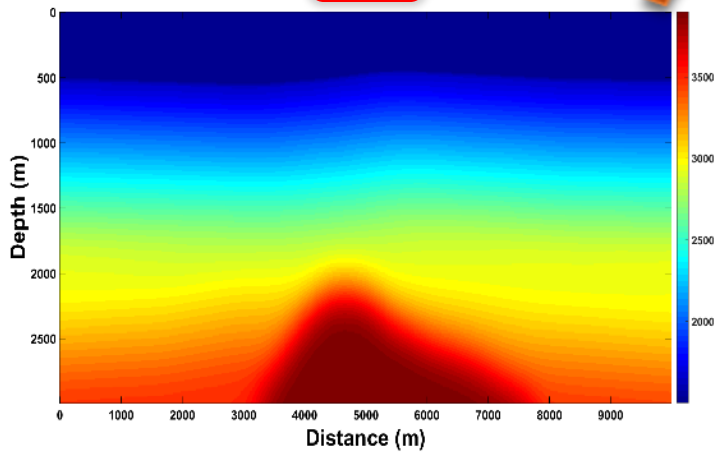
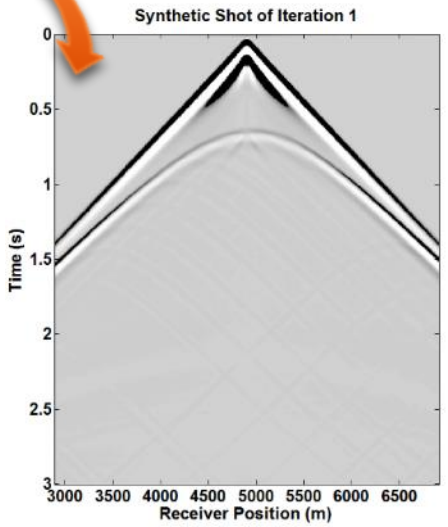
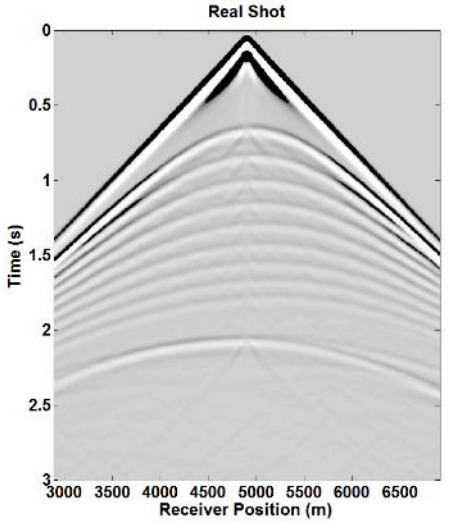


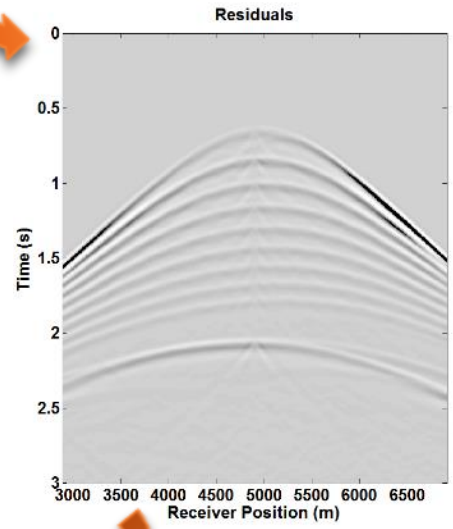
g_0

m_0

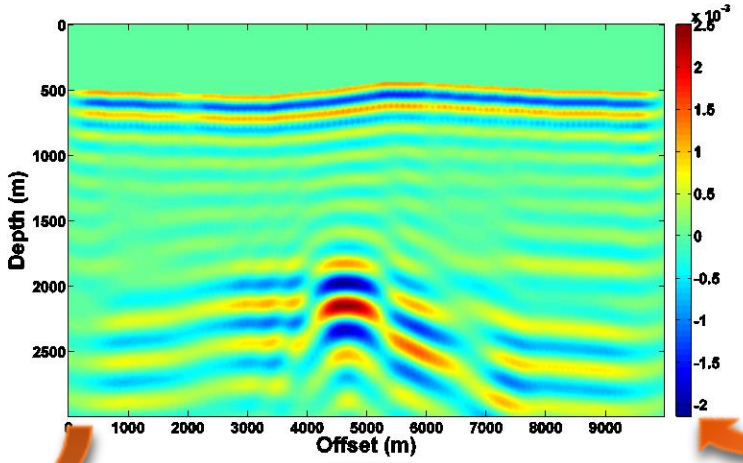








Δd_m



g_0

α_0

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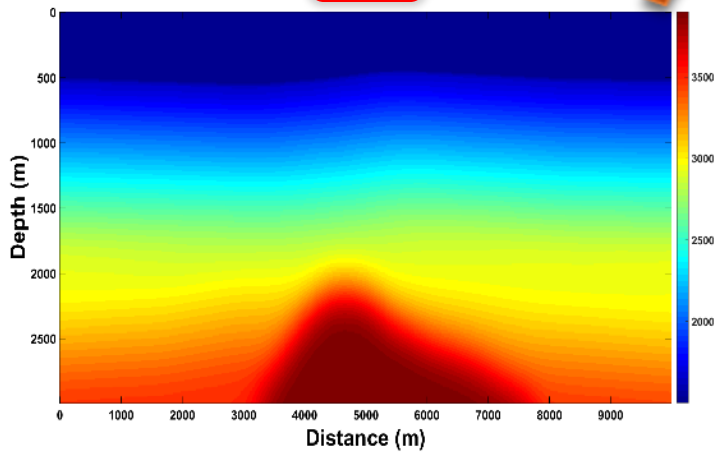
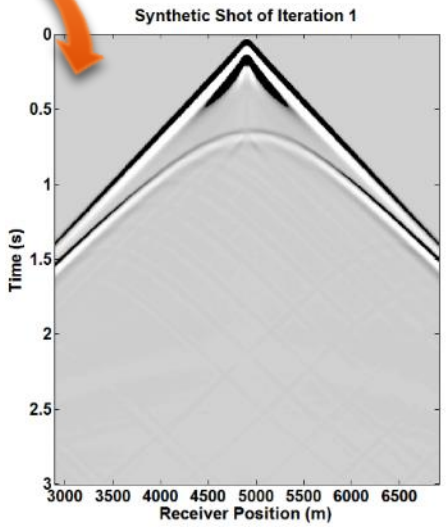
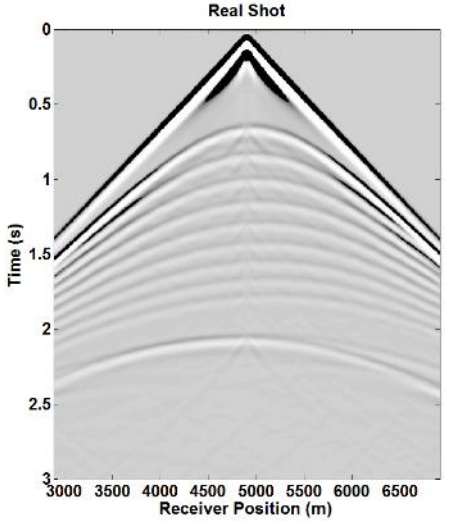
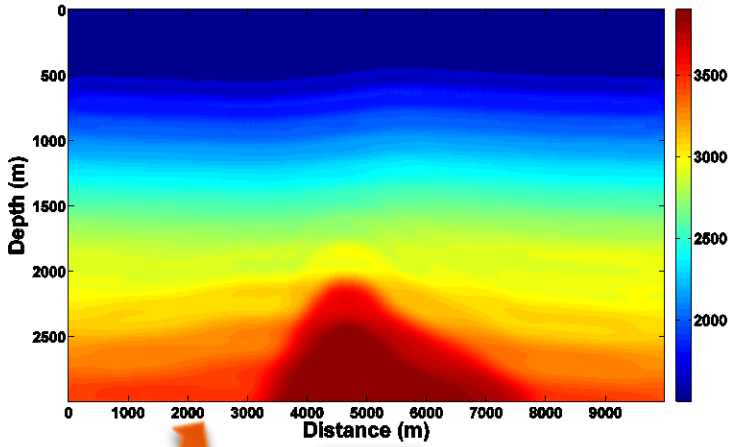
m_0

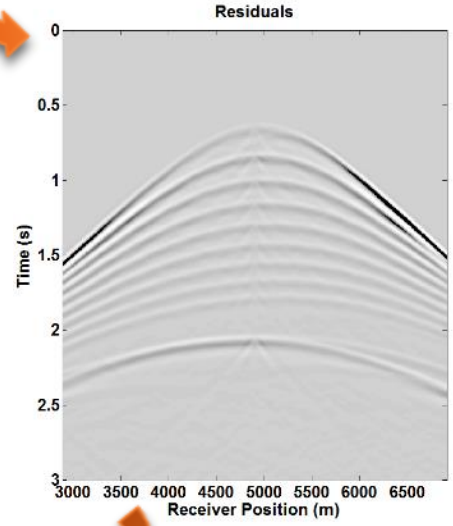




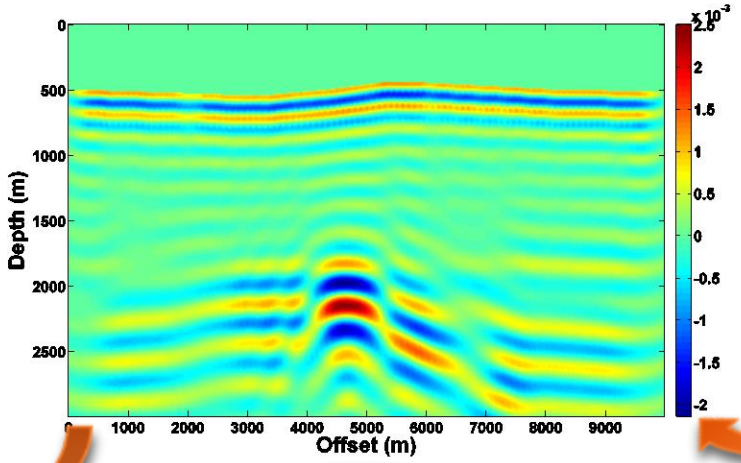




m_1



Δd_m



g_0

α_0

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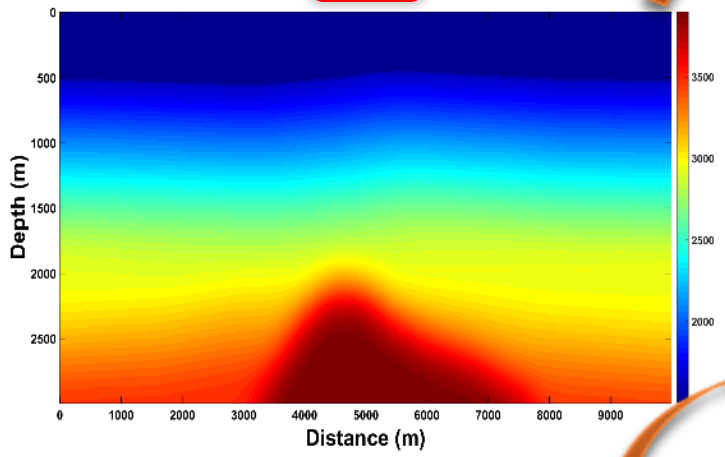
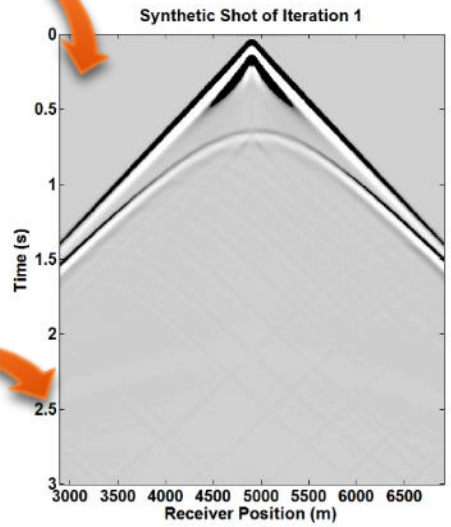
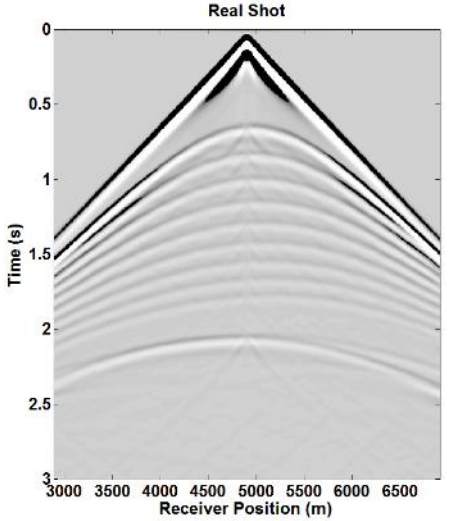
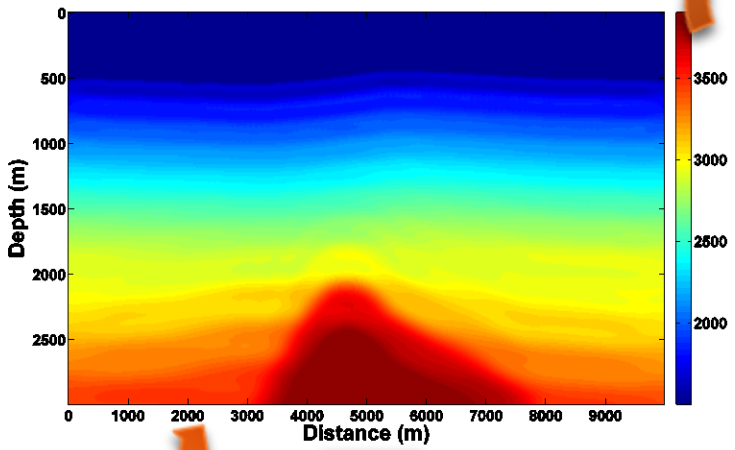
m_0









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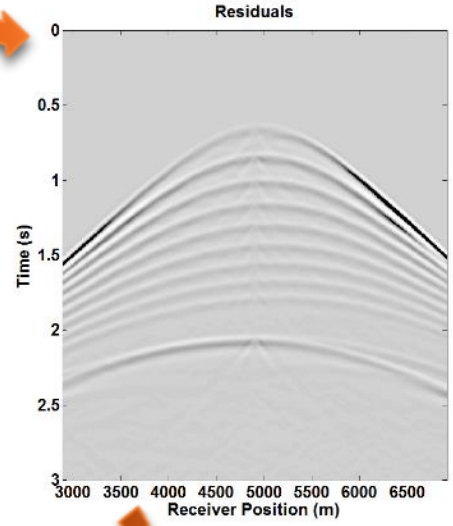
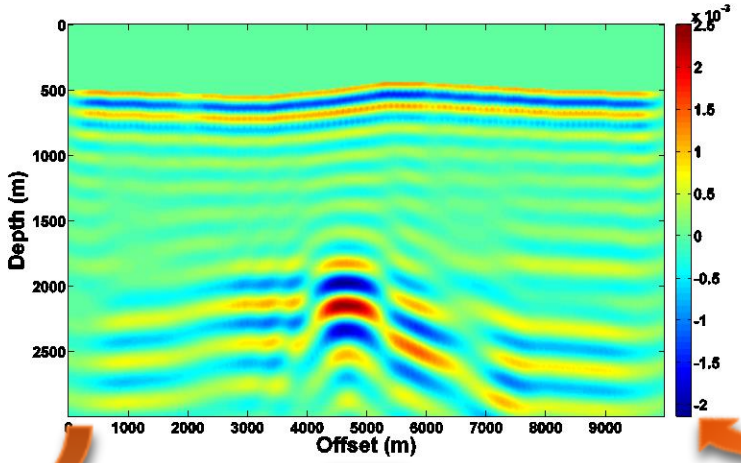


Δd_m



g_0

α_0

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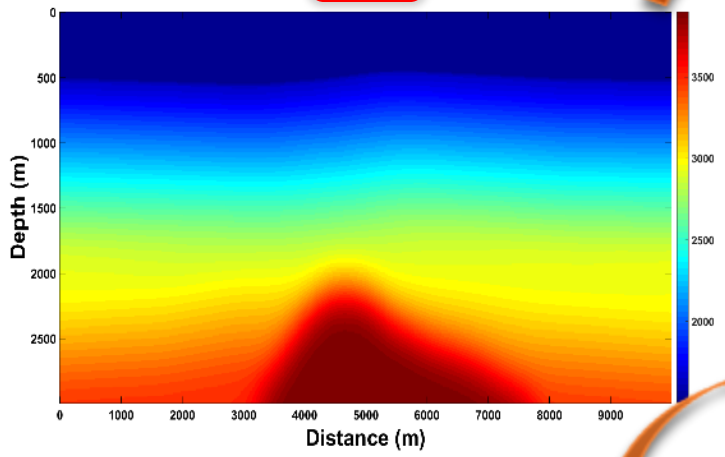
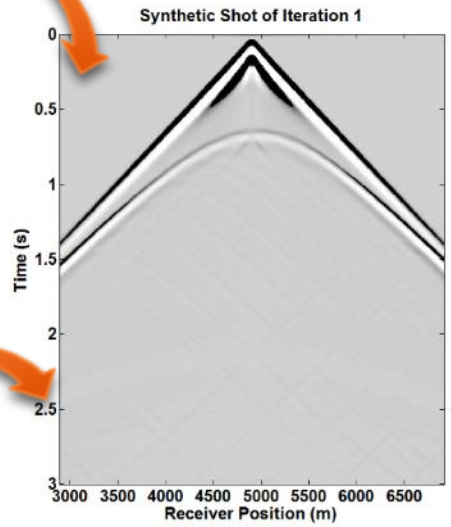
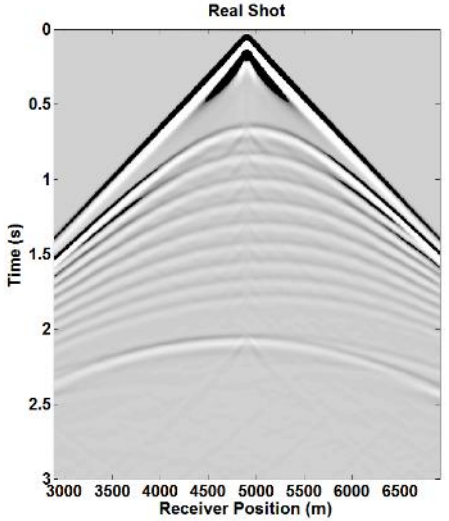
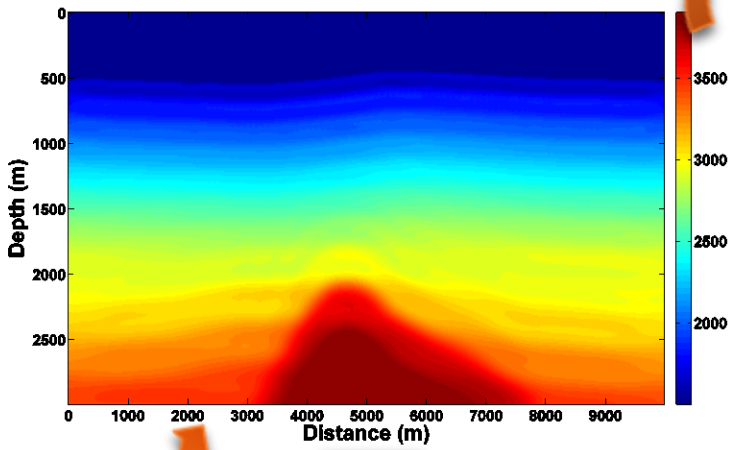
m_0



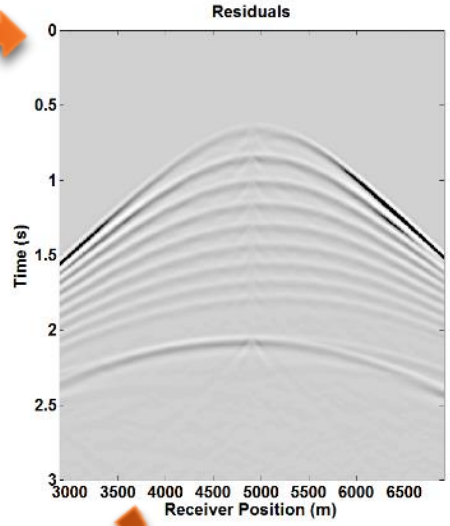




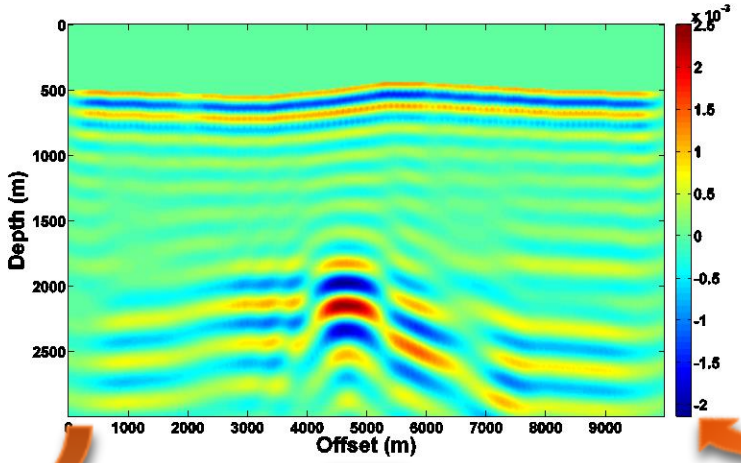


m_1

$$m_{(n+1)} = m_{(n)} + \alpha_{(n)} g_{(n)}$$



Δd_m



g_0

α_0

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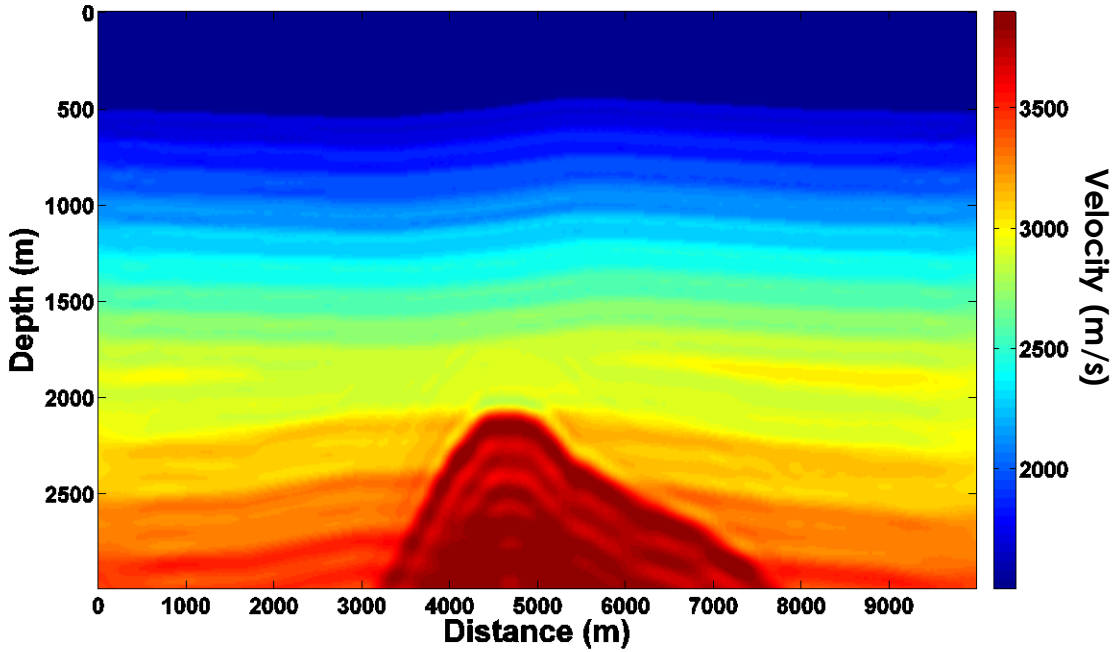
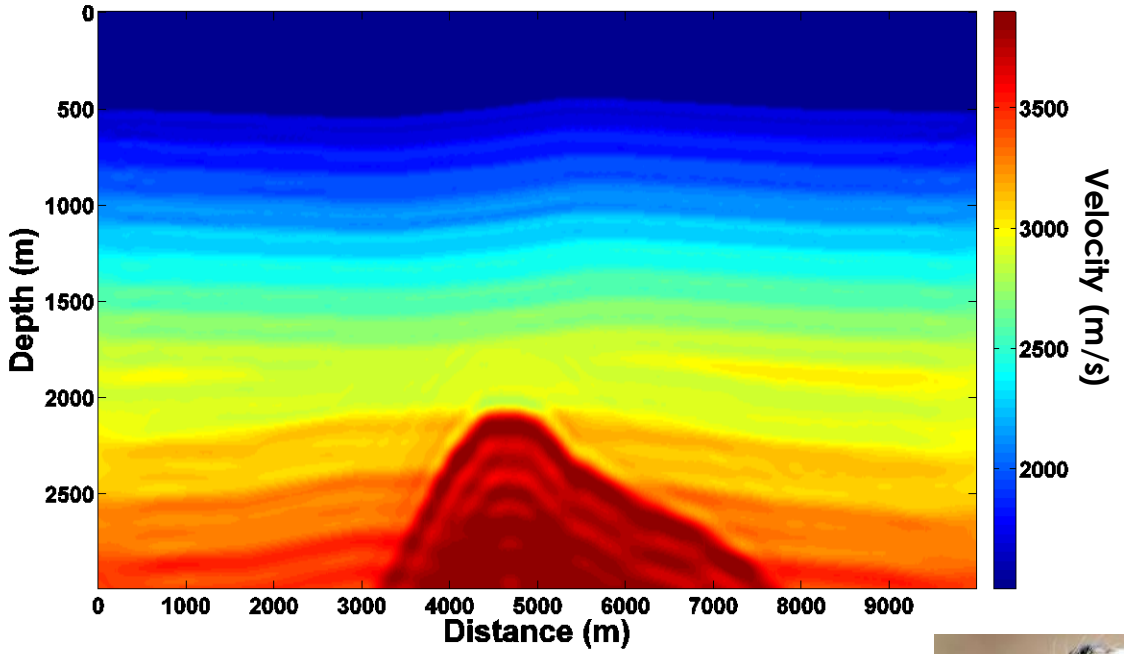
CREWES

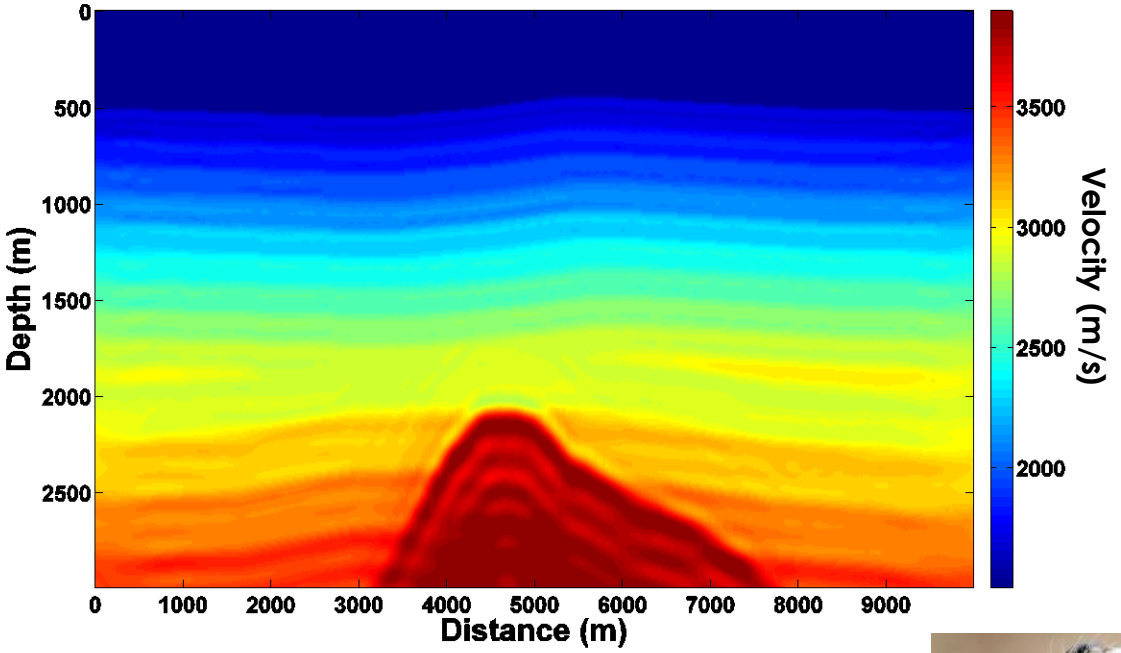
Introduction

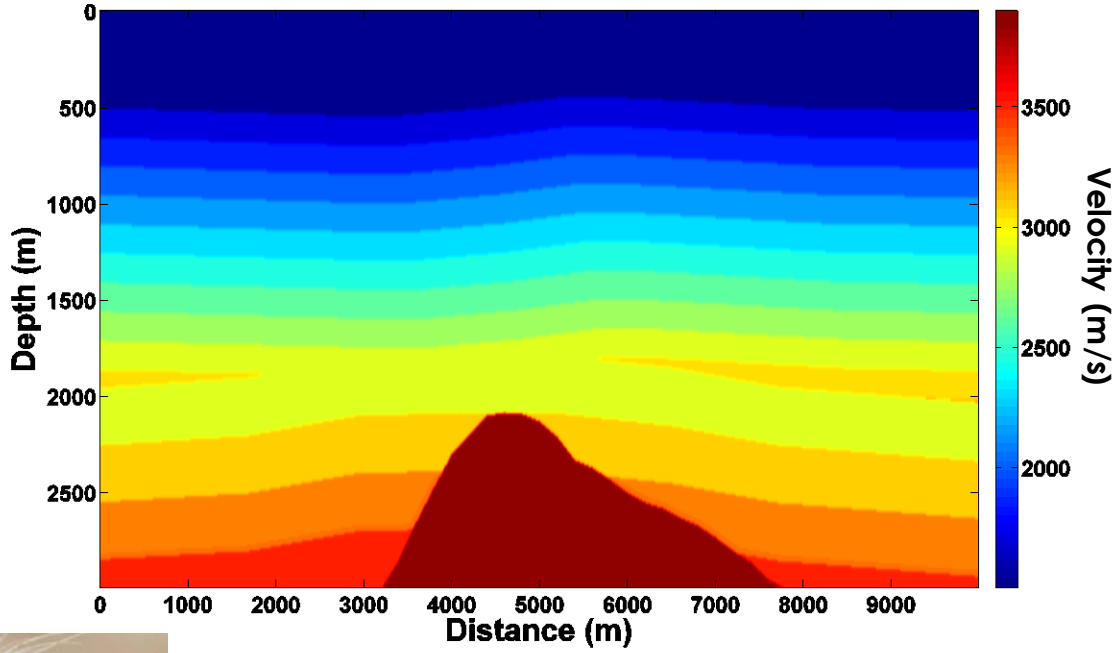


Introduction



Introduction

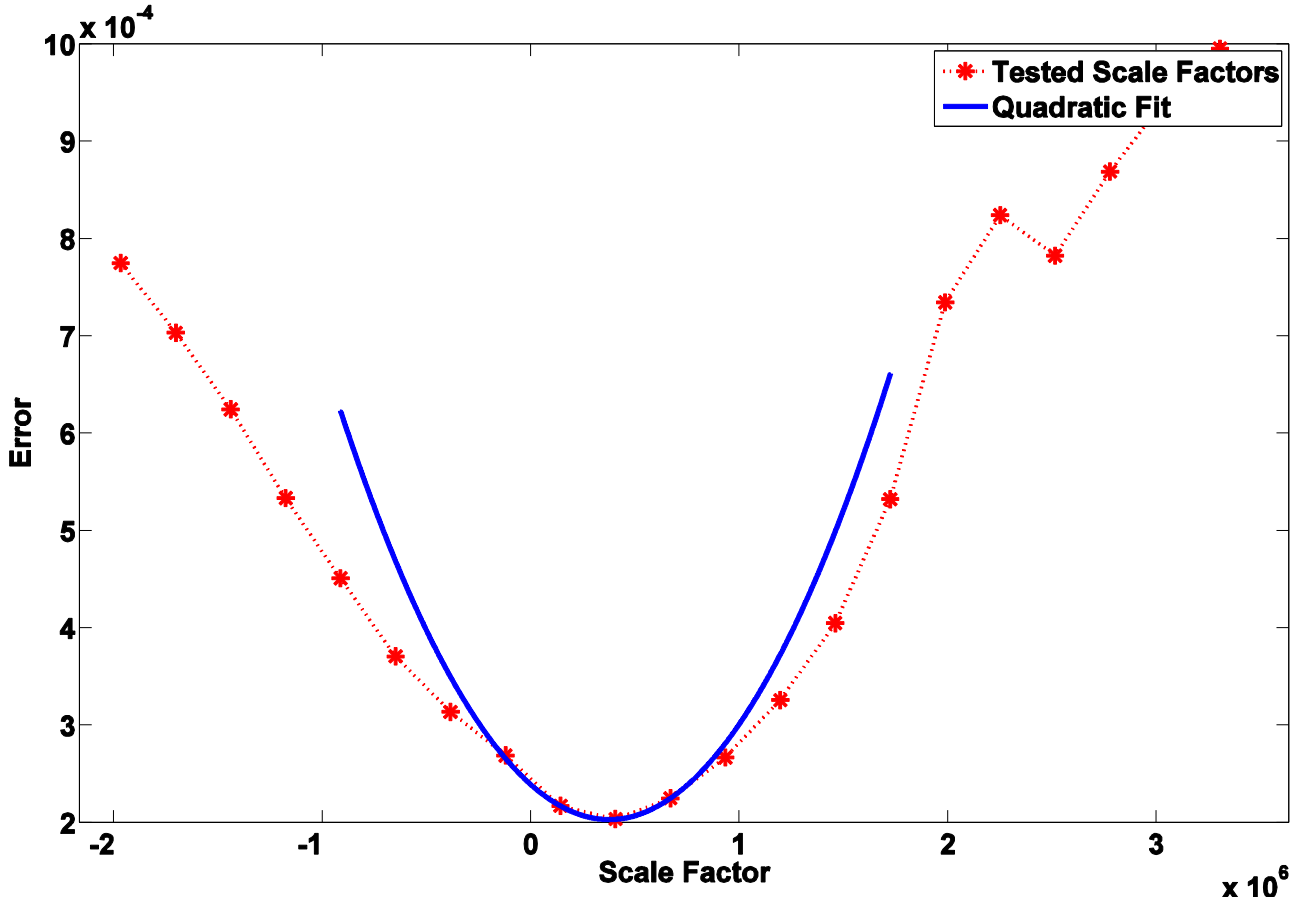




Line search vs least squares

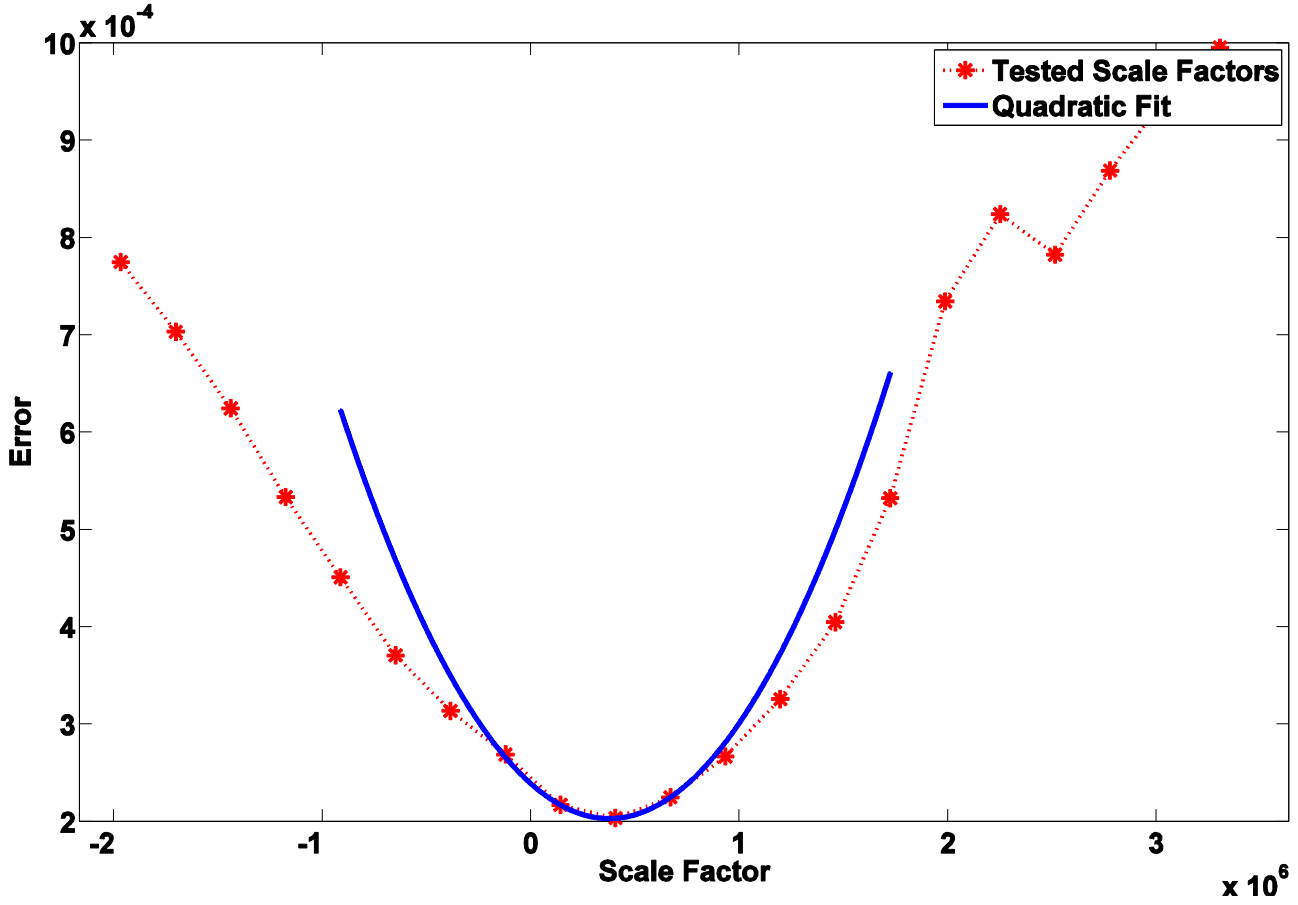
Step length

Line search vs least squares



- Line search
- Test scale factors
- Update model
- Forward modeling
- Norm-2 error
- Quadratic fit
- Lowest error

Line search vs least squares



- Line search
- Test scale factors
- Update model
- Forward modeling
- Norm-2 error
- Quadratic fit
- Lowest error
- **Expensive**

Line search vs least squares

- Least Squares:

$$C(\boldsymbol{m}_{n+1}) = \Delta \boldsymbol{d}_m{}^T \Delta \boldsymbol{d}_m$$

Line search vs least squares

- Least Squares:

$$\boldsymbol{C}(\boldsymbol{m}_{n+1}) = \Delta \boldsymbol{d}_m{}^T \Delta \boldsymbol{d}_m$$

$$\boldsymbol{C}(\boldsymbol{m}_{n+1}) = \boldsymbol{C}(\boldsymbol{m}_n + \boldsymbol{\alpha}_n \boldsymbol{g}_n) = \left[\boldsymbol{d}_{obs} - \boldsymbol{d}_{syn}(\boldsymbol{m}_n + \boldsymbol{\alpha}_n \boldsymbol{g}_n)\right]^T \left[\boldsymbol{d}_{obs} - \boldsymbol{d}_{syn}(\boldsymbol{m}_n + \boldsymbol{\alpha}_n \boldsymbol{g}_n)\right]$$

Line search vs least squares

- Least Squares:

$$C(m_{n+1}) = \Delta d_m{}^T \Delta d_m$$

$$C(m_{n+1}) = C(m_n + \alpha_n g_n) = \left[d_{obs} - d_{syn}(m_n + \alpha_n g_n)\right]^T \left[d_{obs} - d_{syn}(m_n + \alpha_n g_n)\right]$$

- Step length:

$$\alpha_n = \frac{[F_n g_n]^T \left[d_{obs} - d_{syn}(m_n)\right]}{[F_n g_n]^T [F_n g_n]}$$

Line search vs least squares

- Least Squares:

$$C(m_{n+1}) = \Delta d_m{}^T \Delta d_m$$

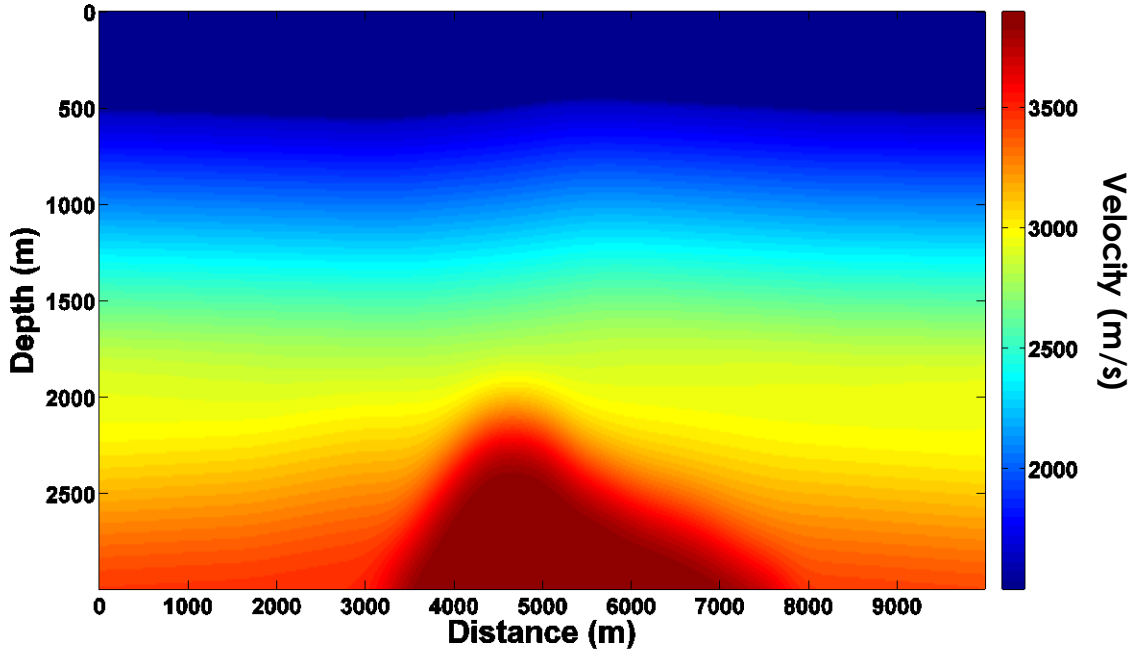
$$C(m_{n+1}) = C(m_n + \alpha_n g_n) = \left[d_{obs} - d_{syn}(m_n + \alpha_n g_n)\right]^T \left[d_{obs} - d_{syn}(m_n + \alpha_n g_n)\right]$$

- Step length:

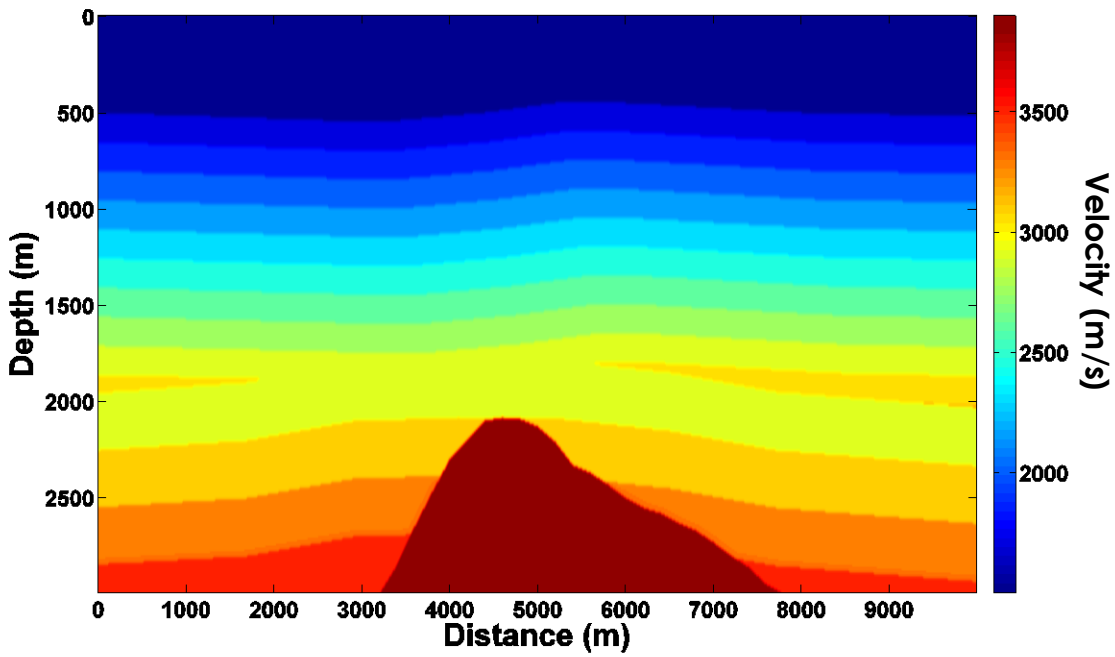
$$\alpha_n = \frac{[F_n g_n]^T \left[d_{obs} - d_{syn}(m_n)\right]}{[F_n g_n]^T [F_n g_n]} \longrightarrow F_n g_n = \lim_{\varepsilon \to 0} \frac{d(m_n + \varepsilon g_n) - d(m_n)}{\varepsilon}$$

Line search vs least squares

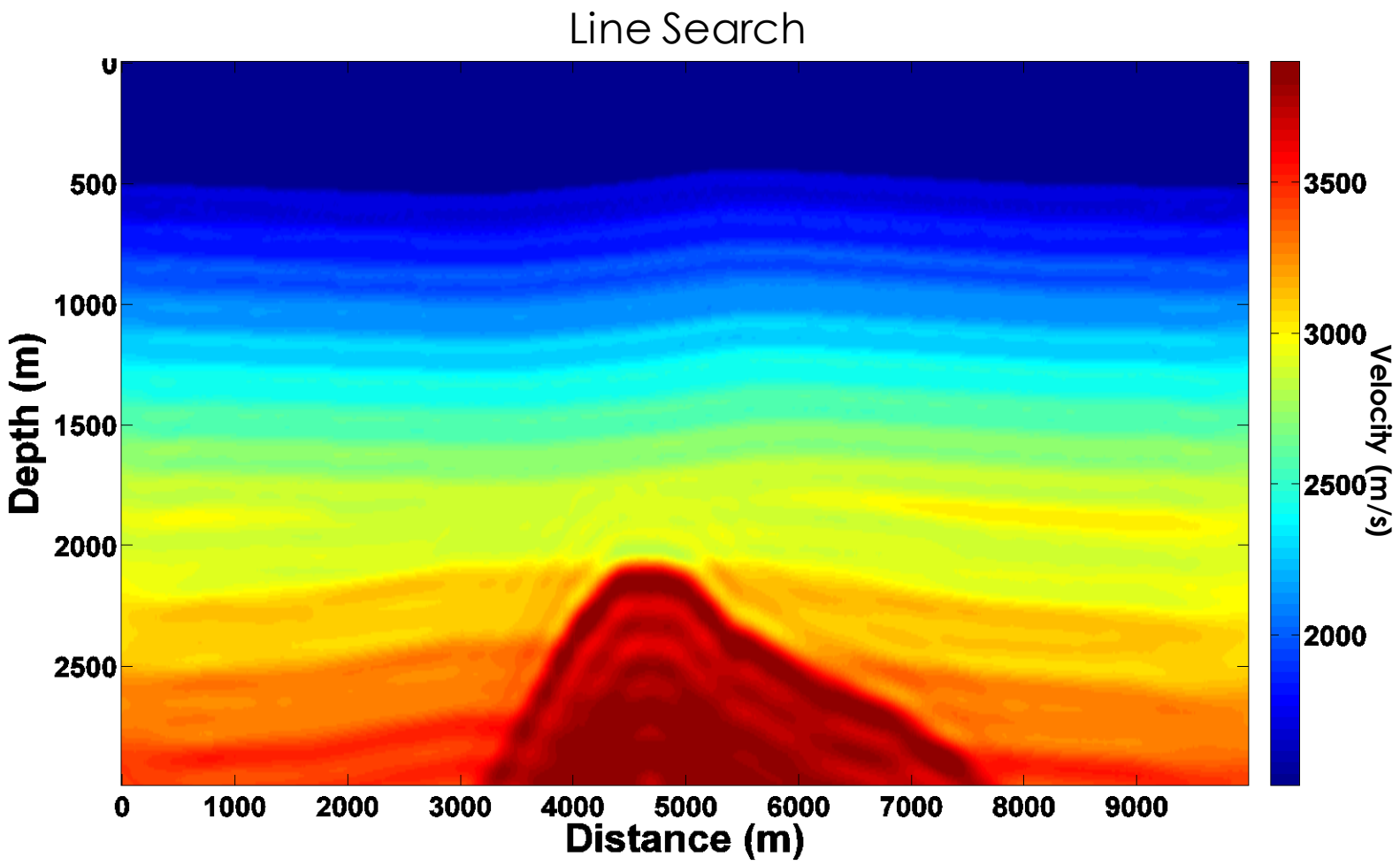
Initial Model



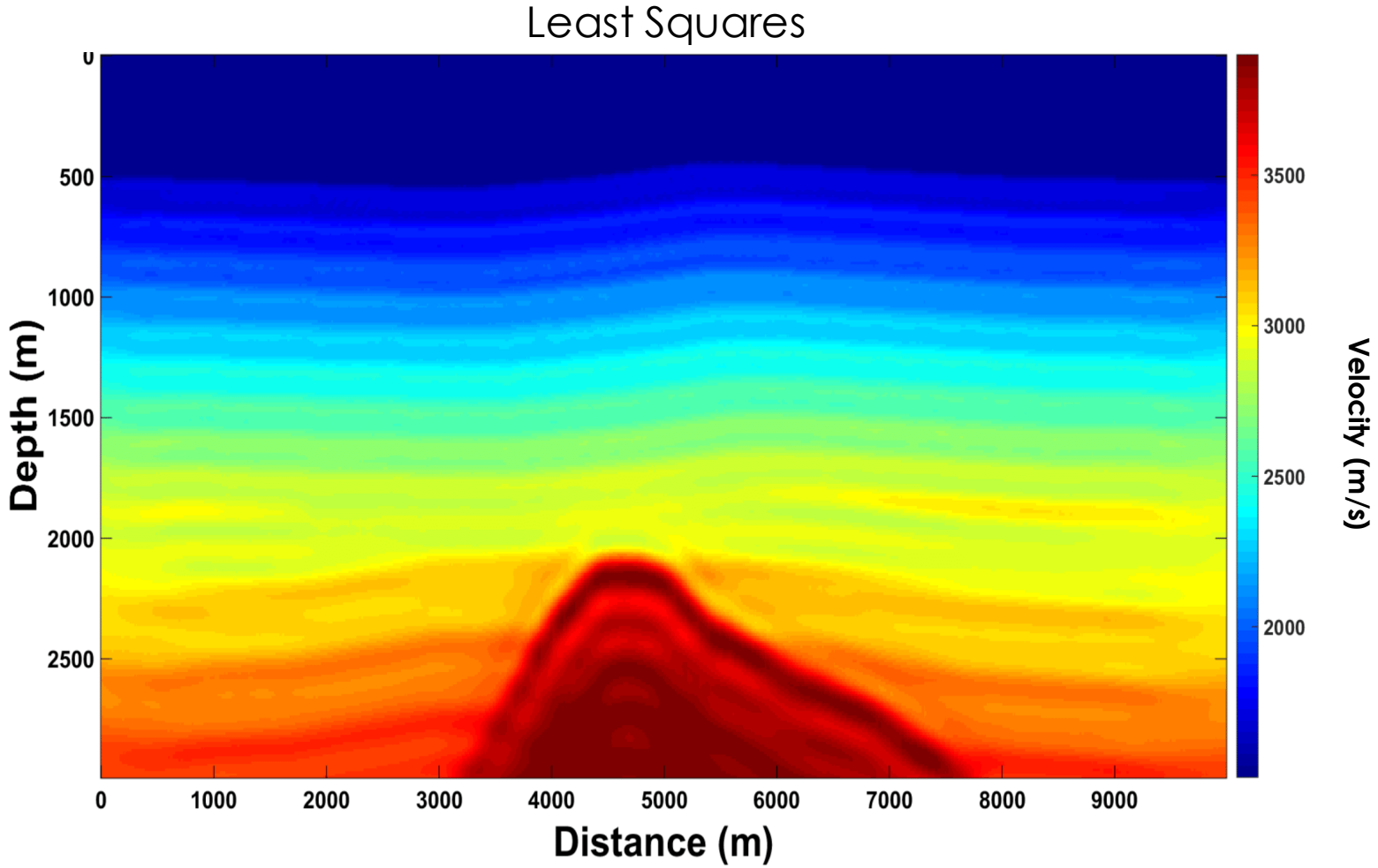
CorrectModel



Line search vs least squares



Line search vs least squares



Gradient

Gradient

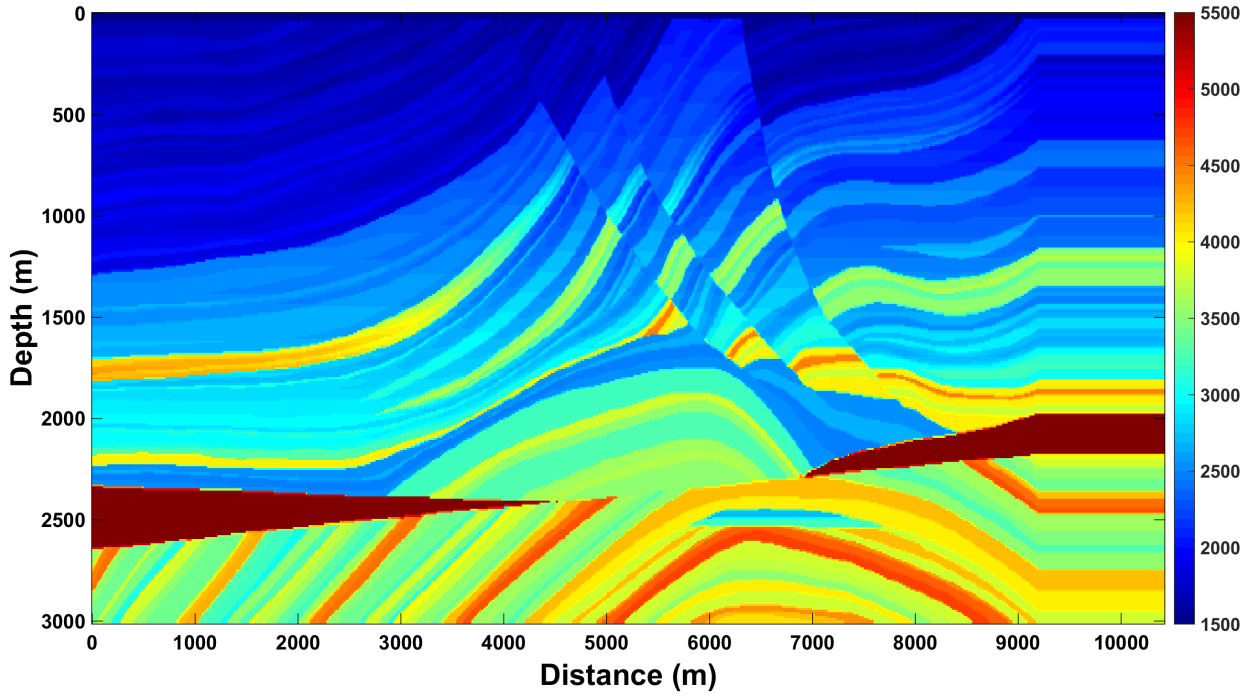
- Monochromatic PSPI migration of the residuals
- One step length per frequency
- Average scaled monochromatic gradients

Gradient

- Monochromatic PSPI migration of the residuals
- One step length per frequency
- Average scaled monochromatic gradients

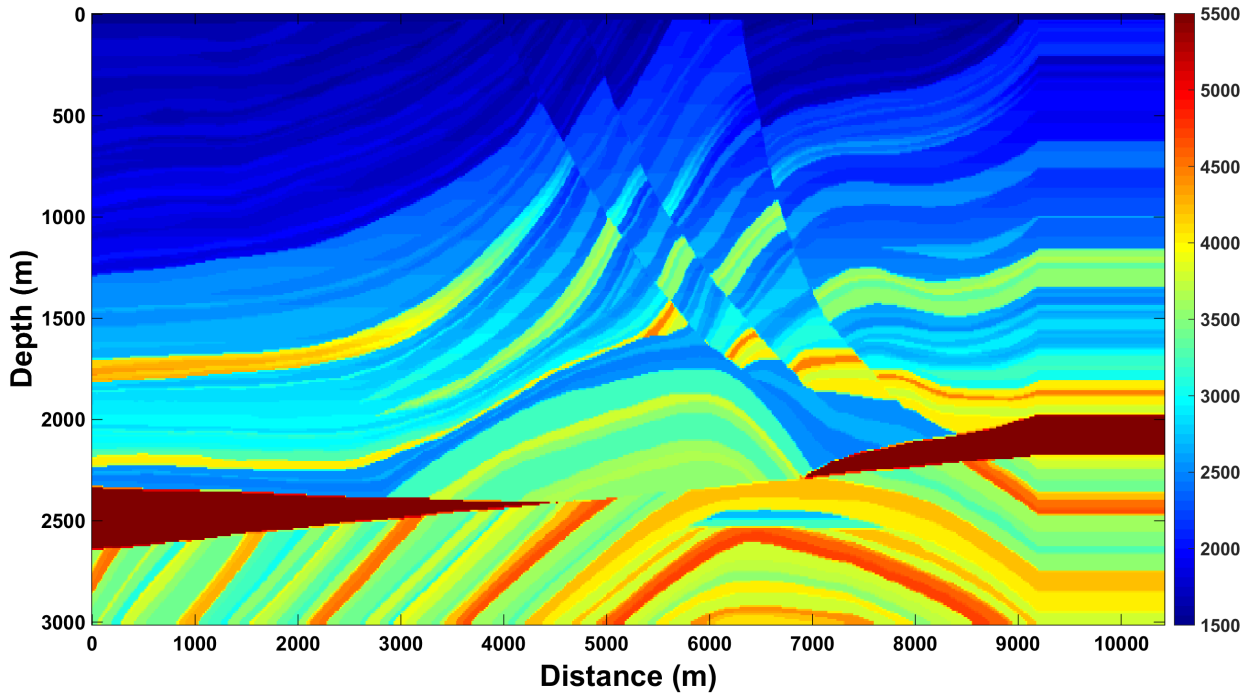
$$\boldsymbol{m}_{n+1} = \boldsymbol{m}_n + \frac{\mathbf{1}}{N}\sum_{i=1}^{N} \alpha_n(\omega_i)\boldsymbol{g}_n(\omega_i)$$

Gradient



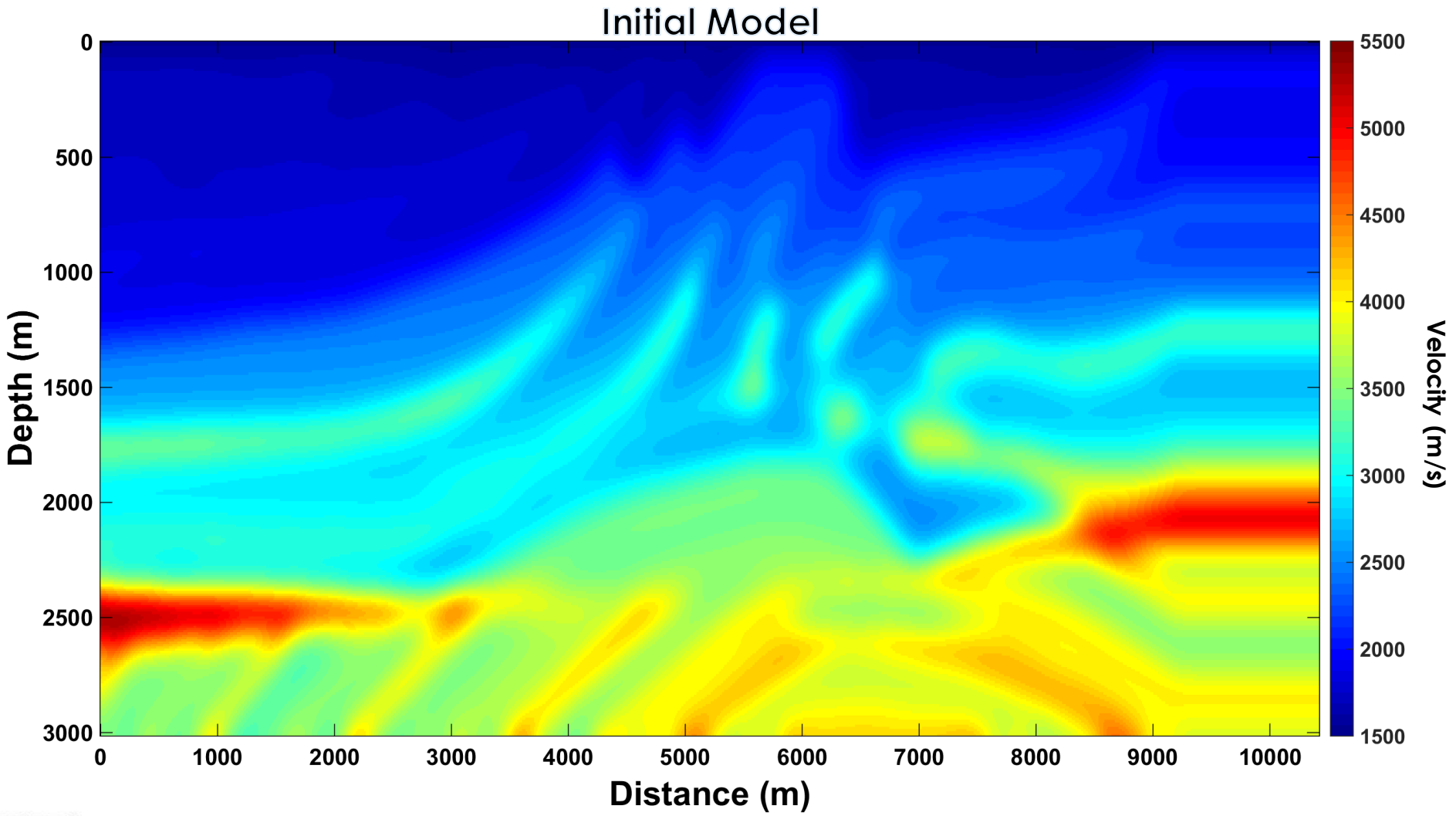
Shots parameters	
Model	Marmousi
Number of shots	105
Shot spacing	100m
Shot depth	0m
# of receivers	603
Receiver spacing	10m
Receiver depth	0m
Frequency	5Hz

Gradient

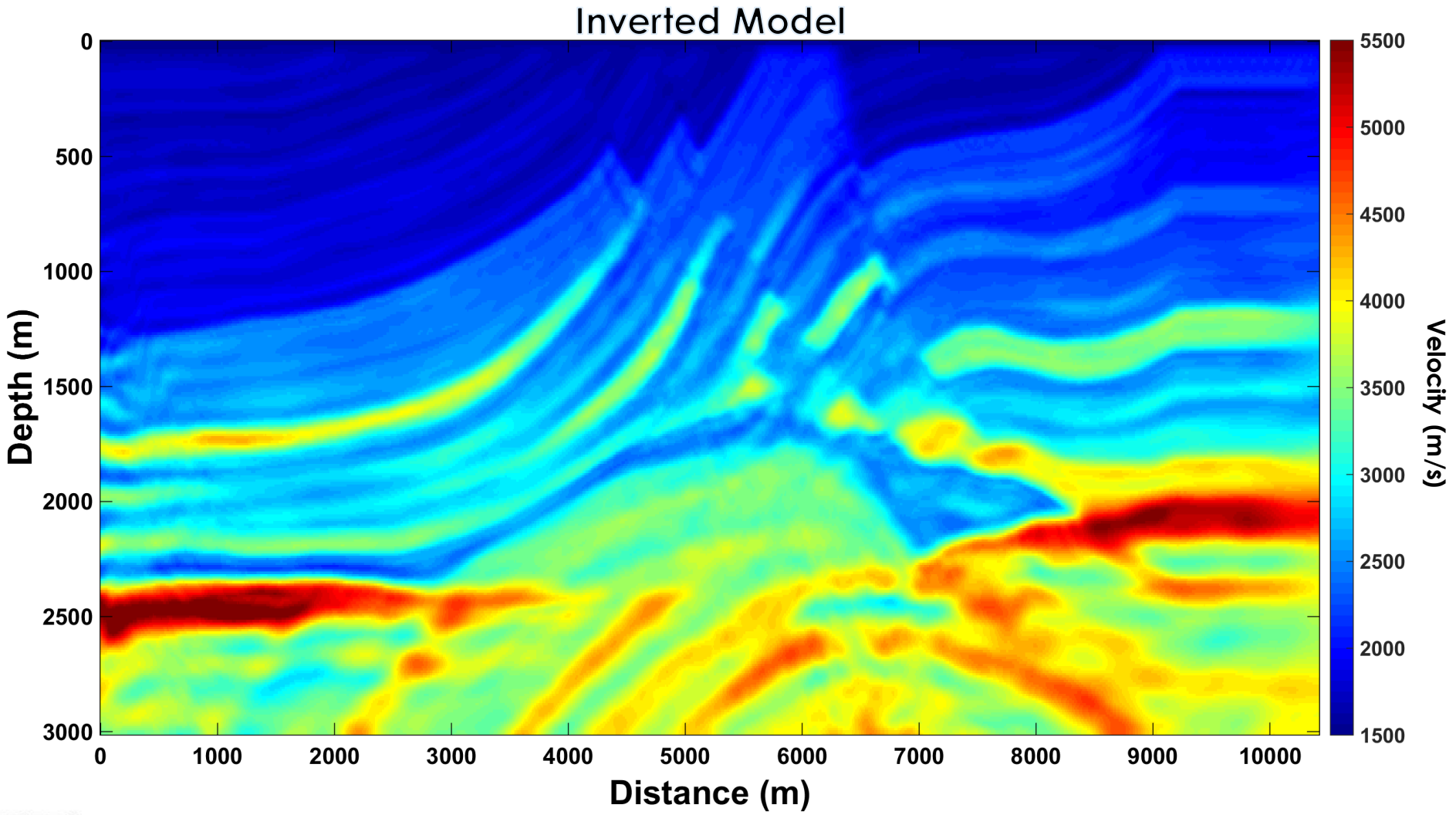


- Starting: low frequency
- Increase frequency band
- Convergence
- By 2Hz
- Maximum: 60Hz
- More migrations as frequency band gets larger

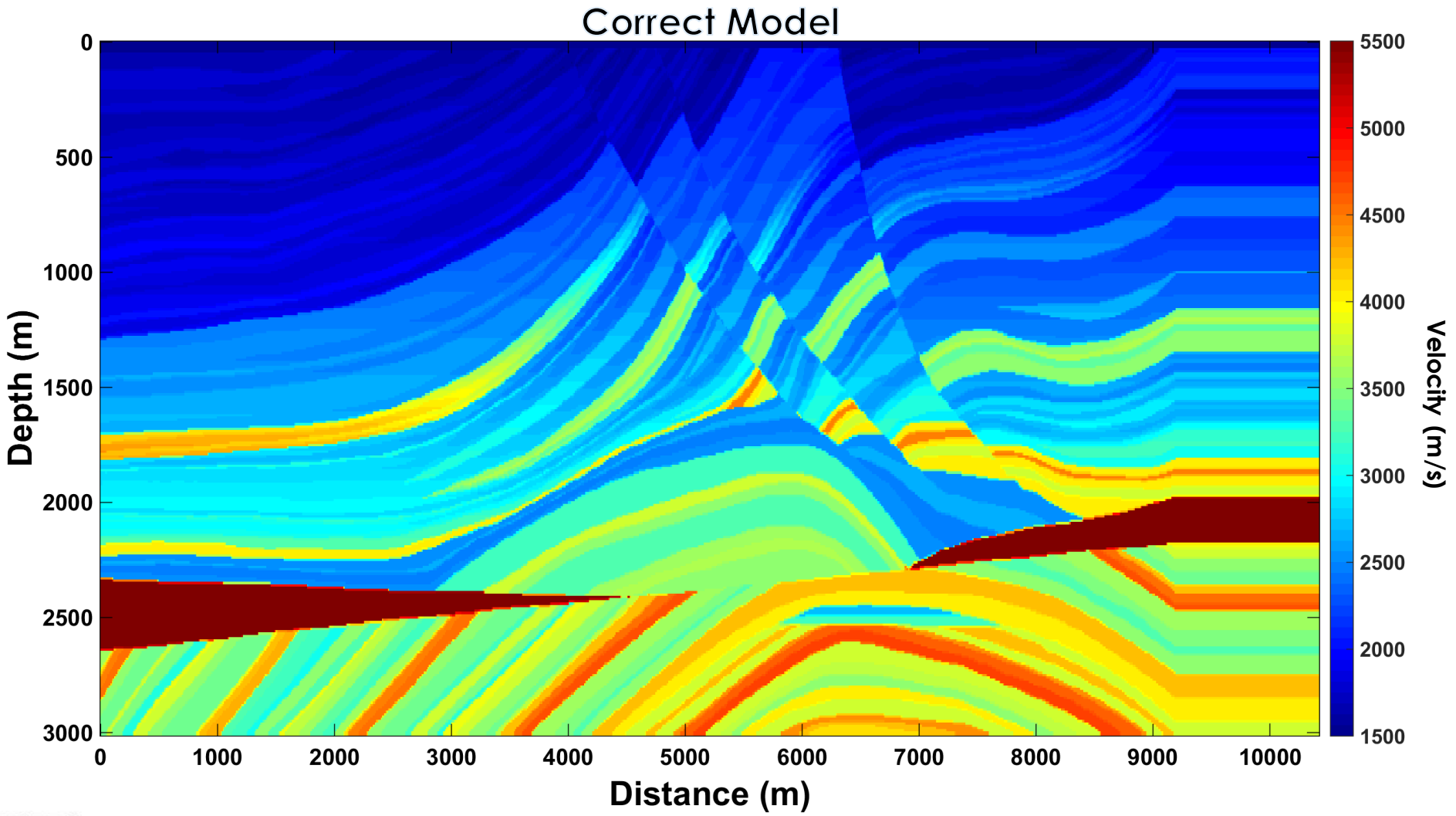
Gradient


Initial Model
Depth (m)
0
500
1000
1500
2000
2500
3000
Distance (m)
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
Velocity (m/s)
5500
5000
4500
4000
3500
3000
2500
2000
1500

Gradient


Inverted Model
Depth (m)
0
500
1000
1500
2000
2500
3000
Distance (m)
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
Velocity (m/s)
5500
5000
4500
4000
3500
3000
2500
2000
1500

Gradient


Correct Model
Depth (m)
0
500
1000
1500
2000
2500
3000
Distance (m)
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
Velocity (m/s)
5500
5000
4500
4000
3500
3000
2500
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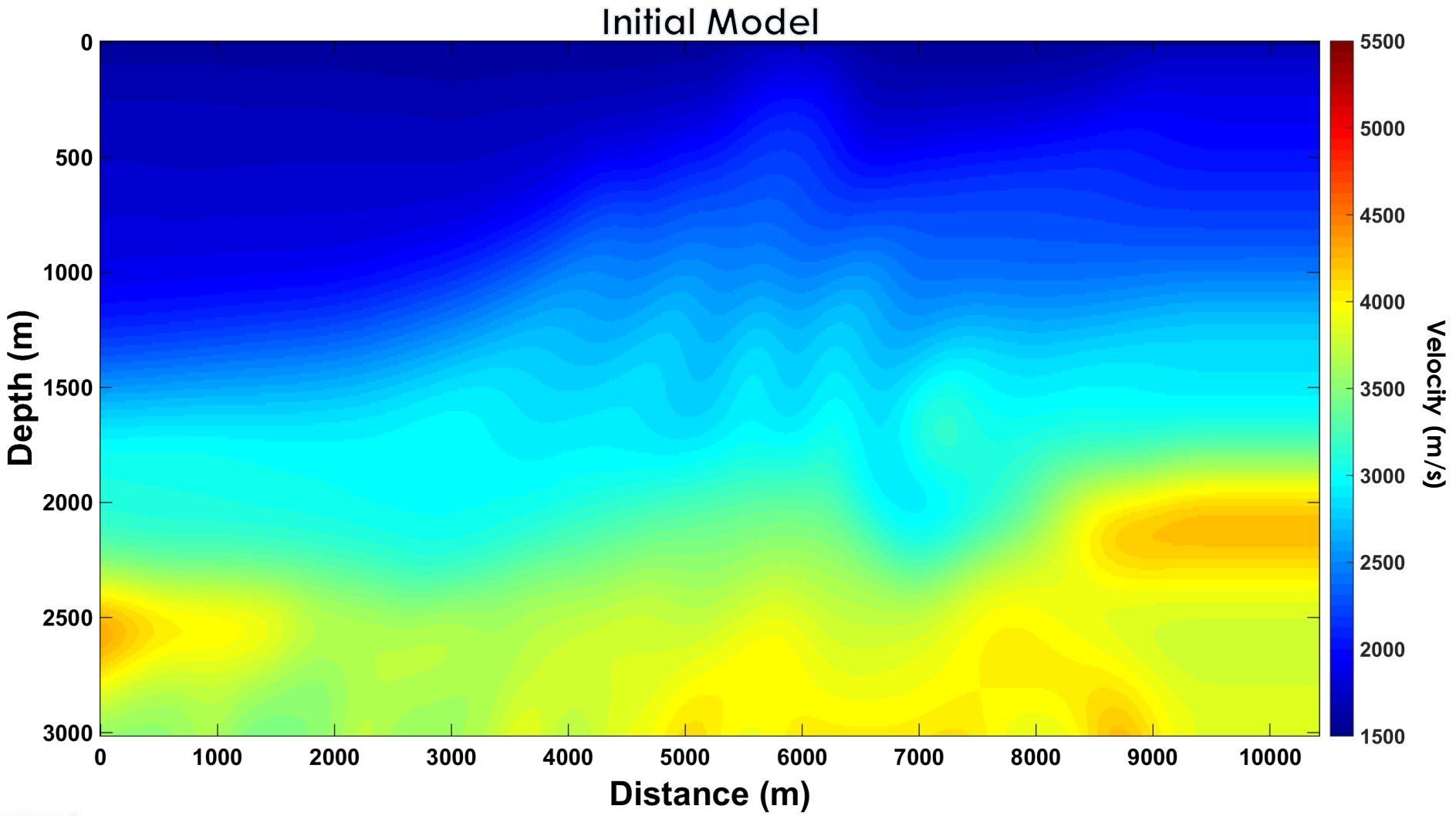
Gradient


Initial Model
Depth (m)
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1000
1500
2000
2500
3000
Distance (m)
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
Velocity (m/s)
5500
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4500
4000
3500
3000
2500
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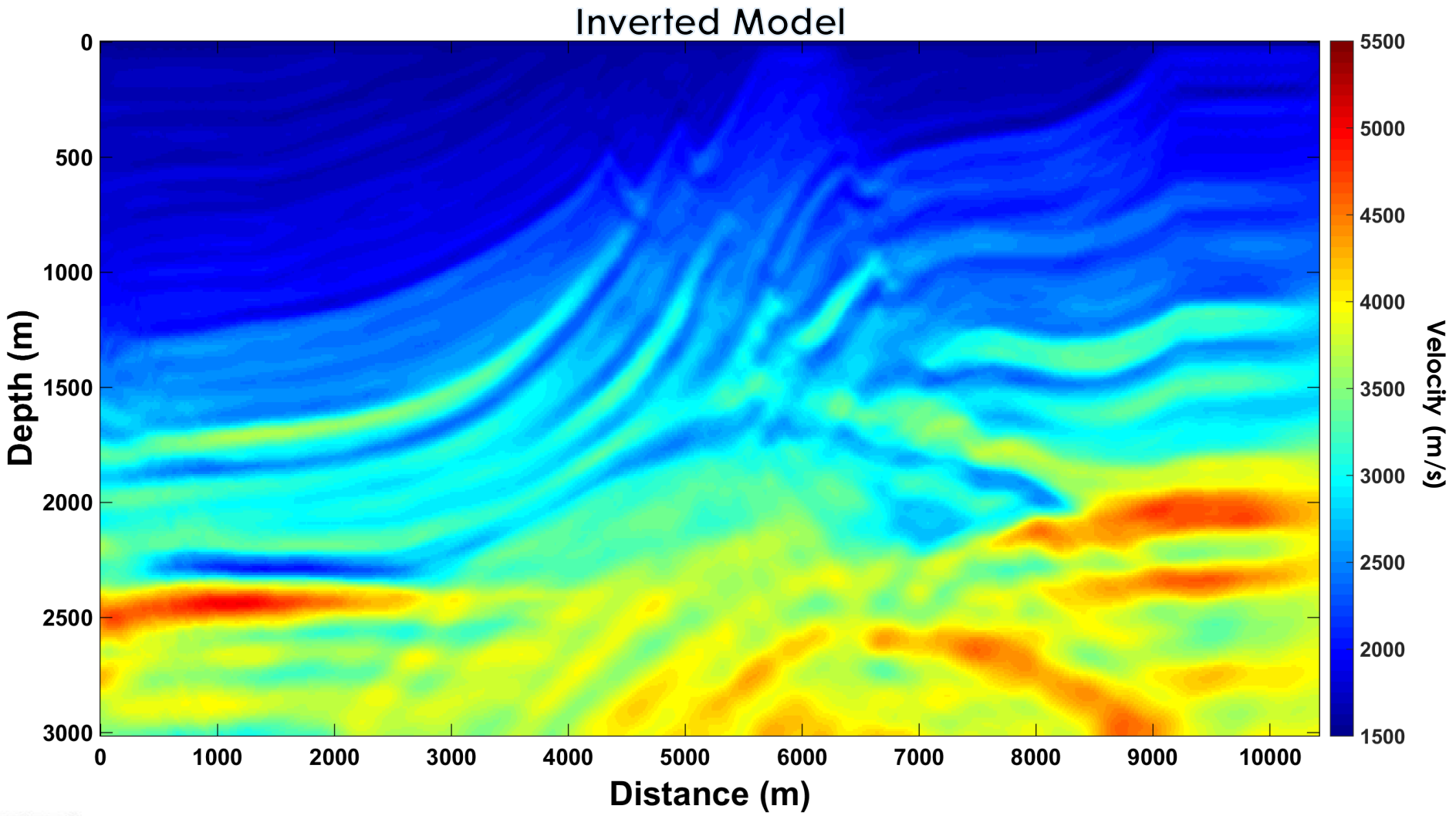
1500

Gradient


Inverted Model
Depth (m)
0
500
1000
1500
2000
2500
3000
Distance (m)
0
1000
2000
3000
4000
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6000
7000
8000
9000
10000
Velocity (m/s)
5500
5000
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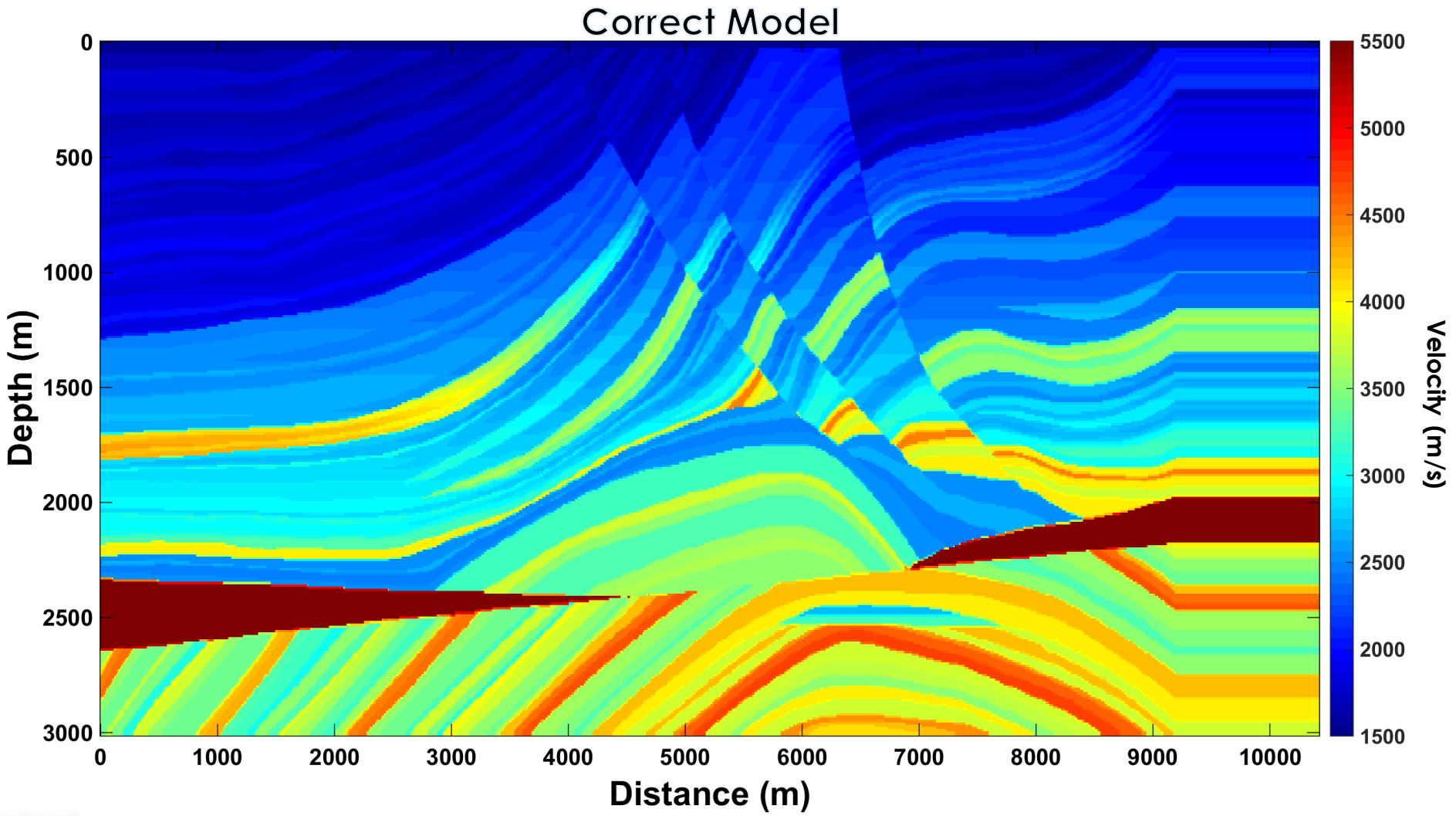
3500
3000
2500
2000
1500

Gradient


Correct Model
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10000
Velocity (m/s)
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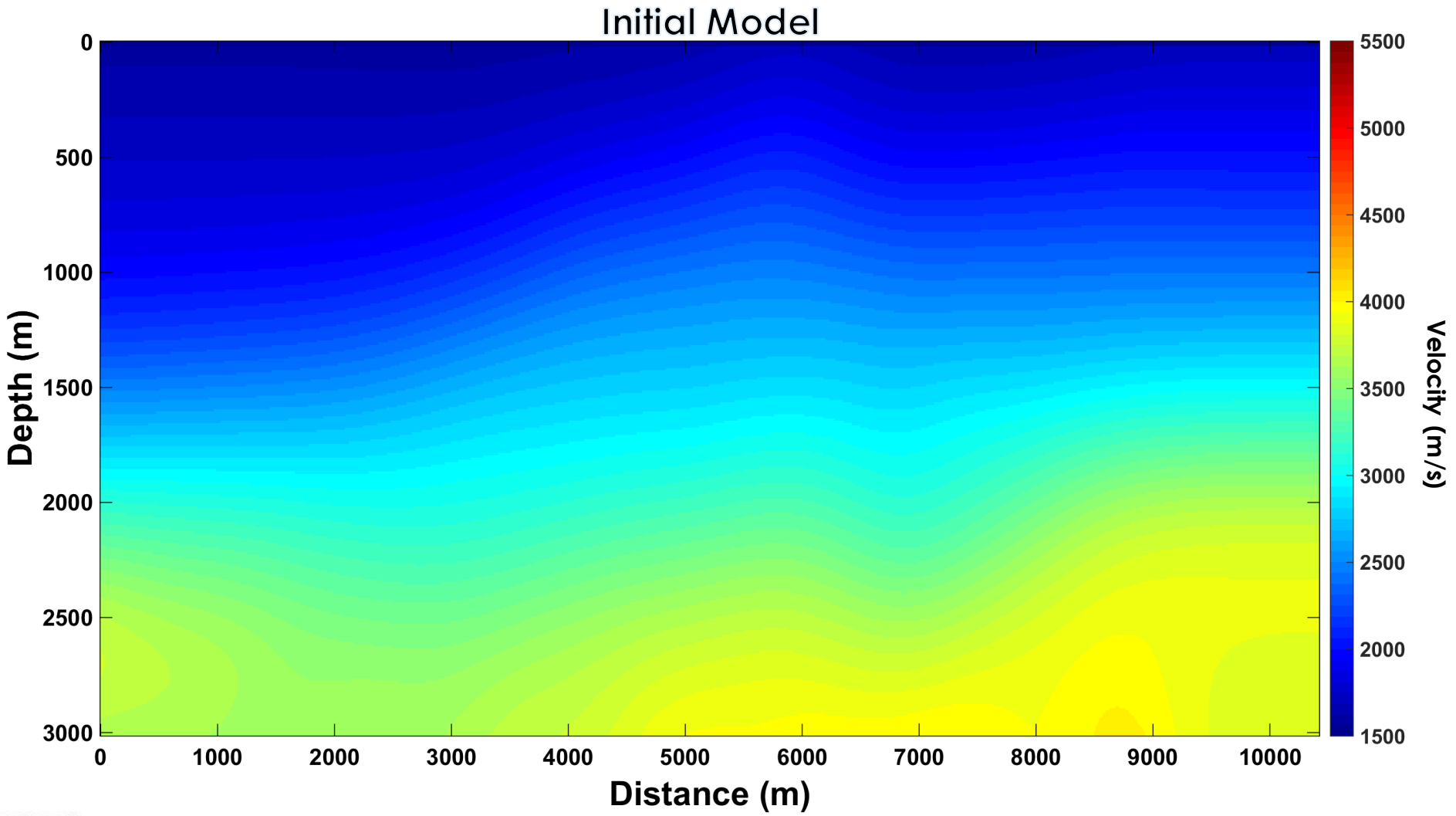
Gradient


Initial Model
Depth (m)
0
500
1000
1500
2000
2500
3000
Distance (m)
0
1000
2000
3000
4000
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6000
7000
8000
9000
10000
Velocity (m/s)
5500
5000
4500
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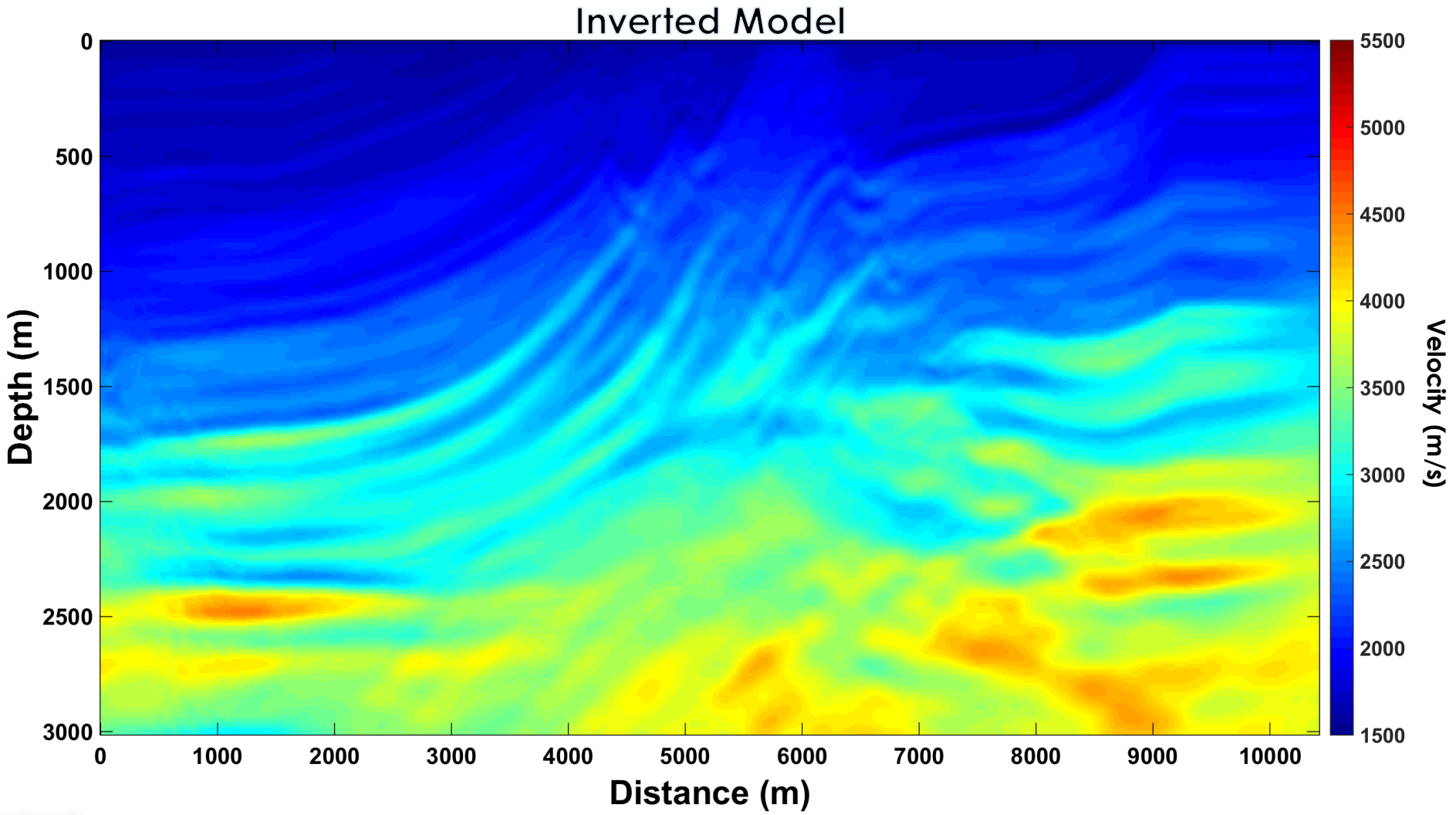
3500
3000
2500
2000
1500

Gradient


Inverted Model
Depth (m)
0
500
1000
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2000
2500
3000
Distance (m)
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1000
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4000
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9000
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Velocity (m/s)
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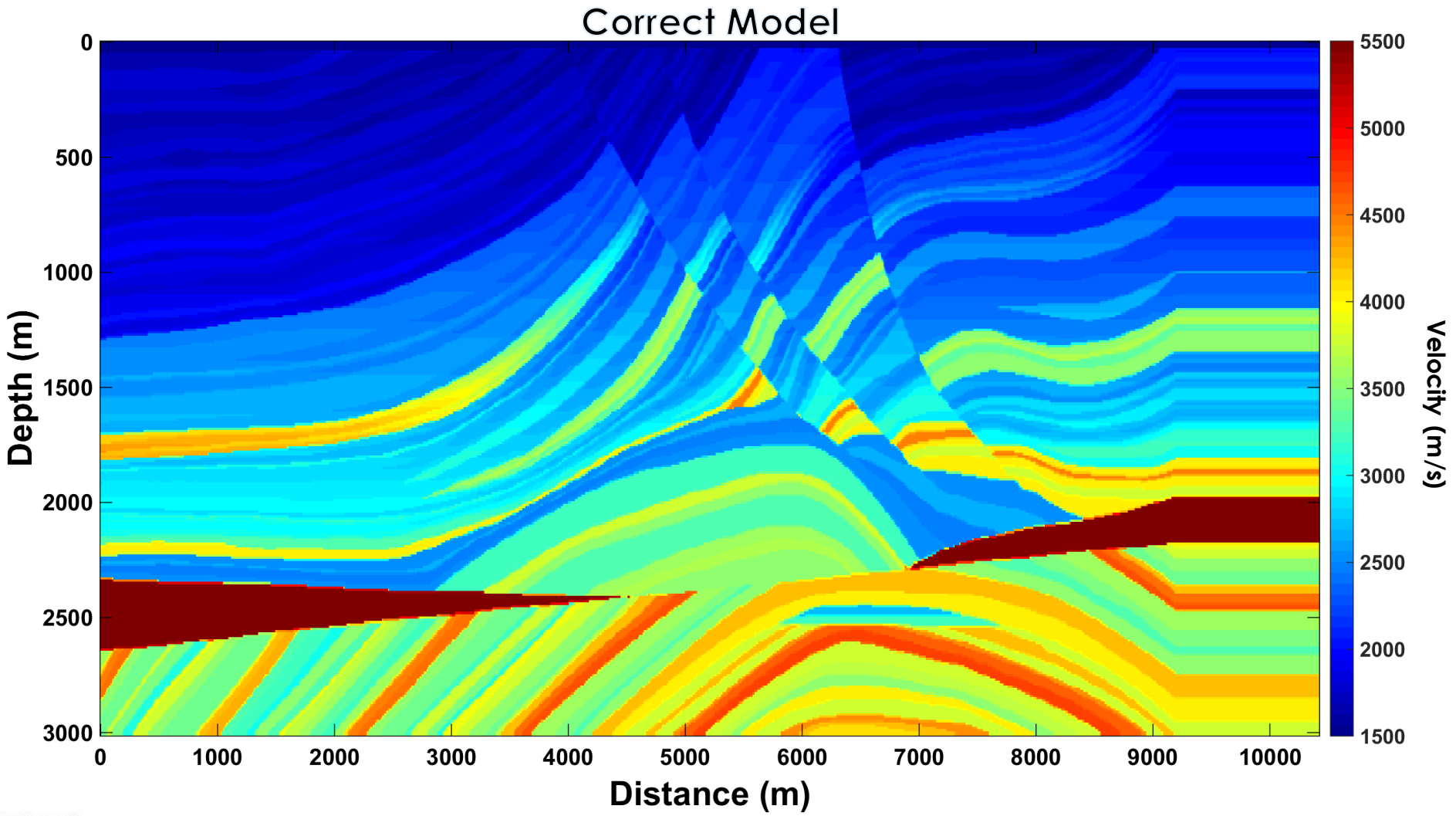
Gradient


Correct Model
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Velocity (m/s)
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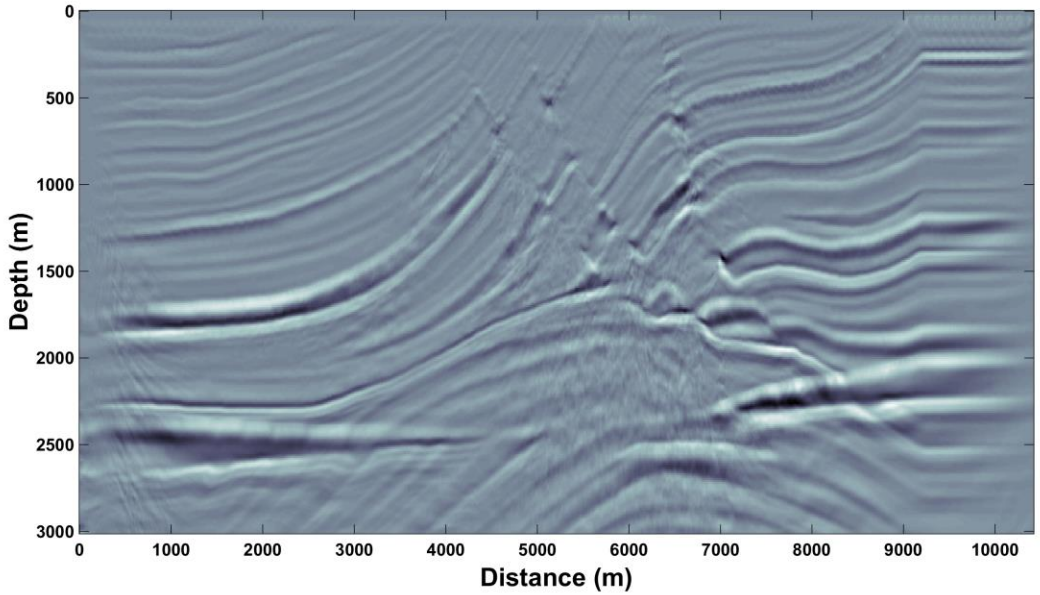
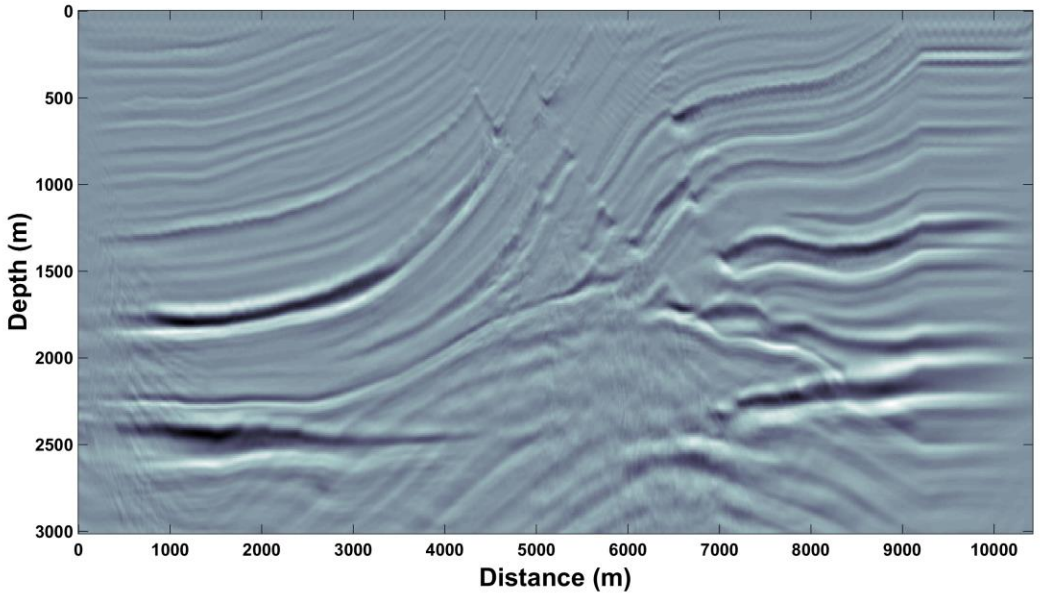
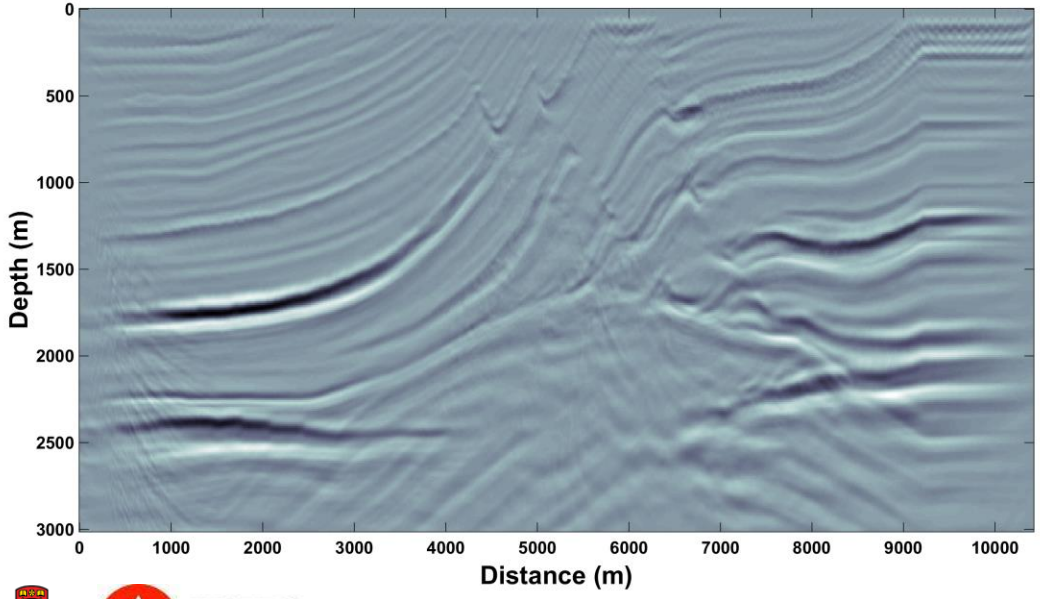
Marmousi model



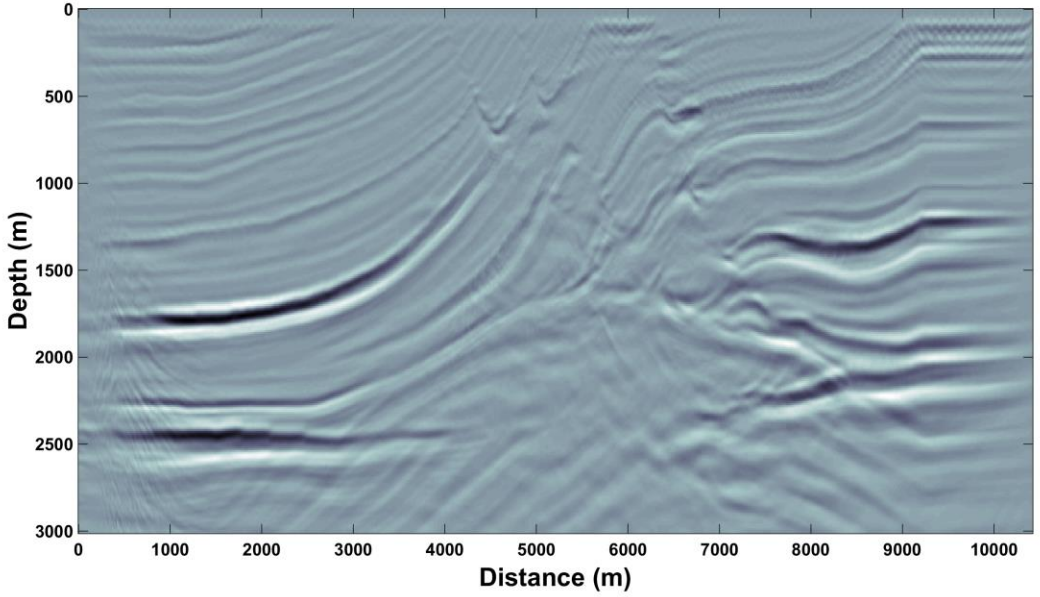
First model



Second model



Third model



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Conjugate gradient

Conjugate gradient

$$m_{n+1} = m_n + \alpha_n h_n$$

Conjugate gradient

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$$h_0 = g_0$$

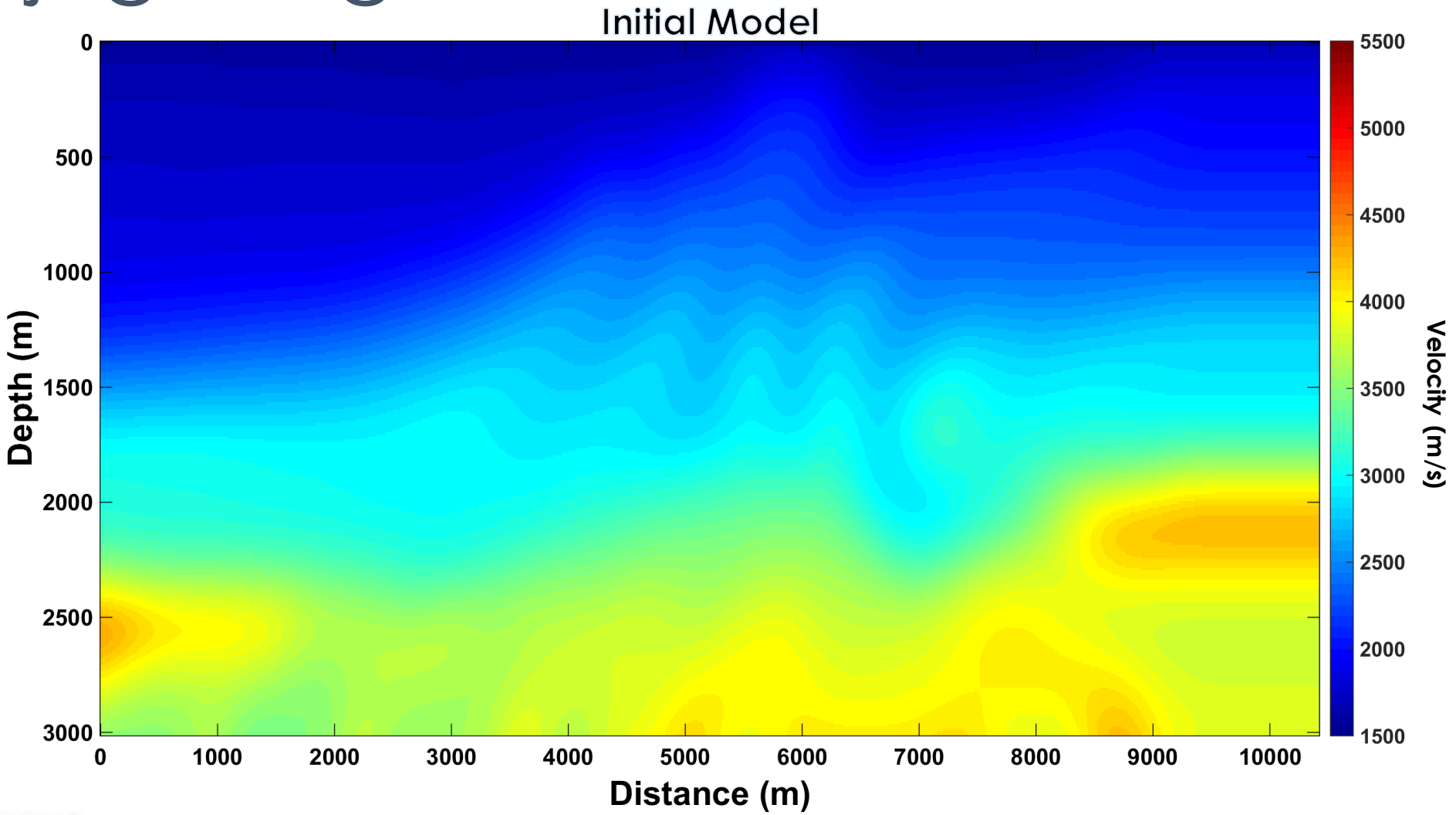
Conjugate gradient

$$m_{n+1} = m_n + \alpha_n h_n$$

$$h_0 = g_0$$

$$h_n = g_n + \beta_n h_{n-1} \longrightarrow \beta_n = \frac{g_n^T (g_n - g_{n-1})}{g_{n-1}^T g_{n-1}}$$

Conjugate gradient



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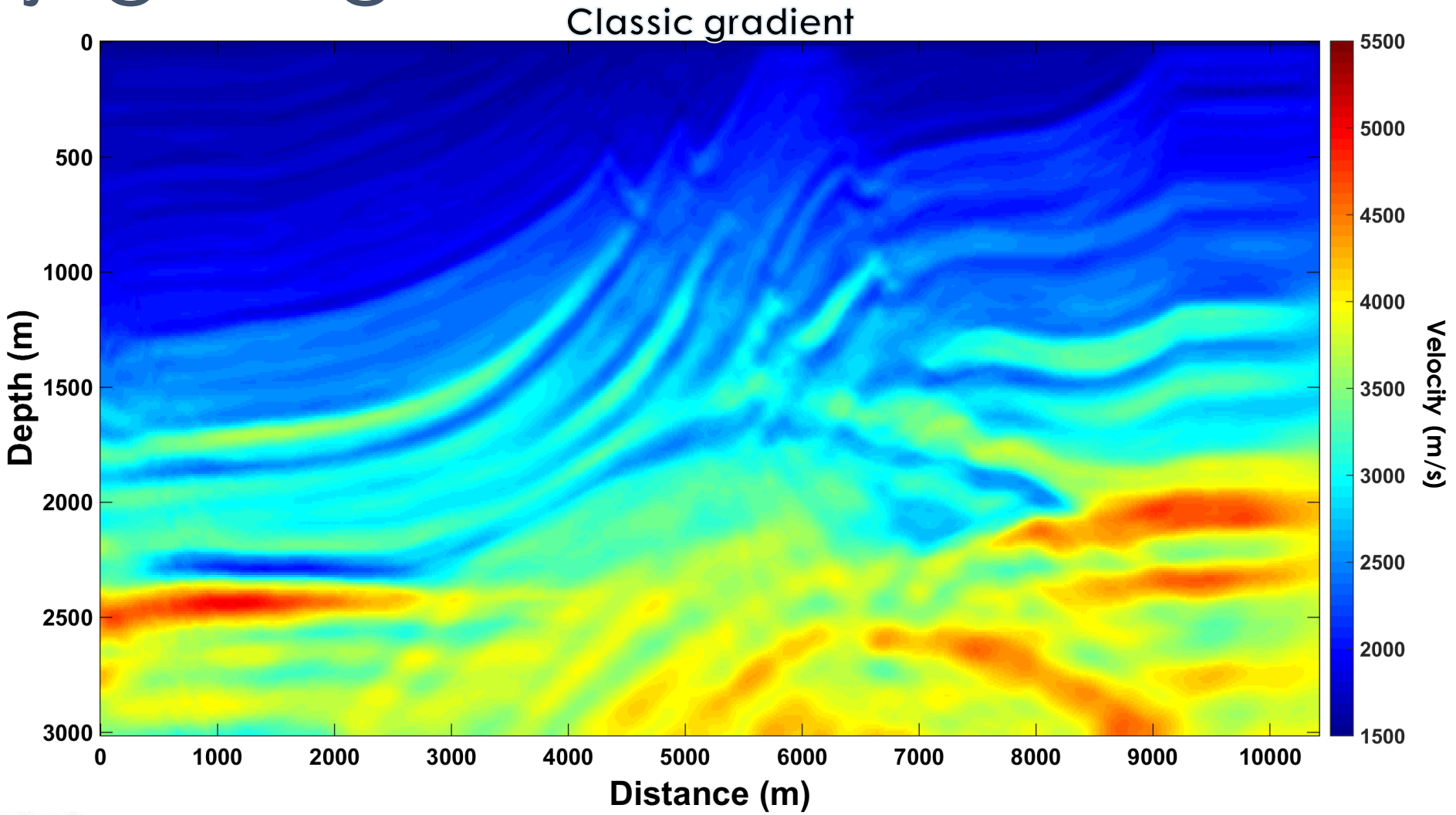
Conjugate gradient



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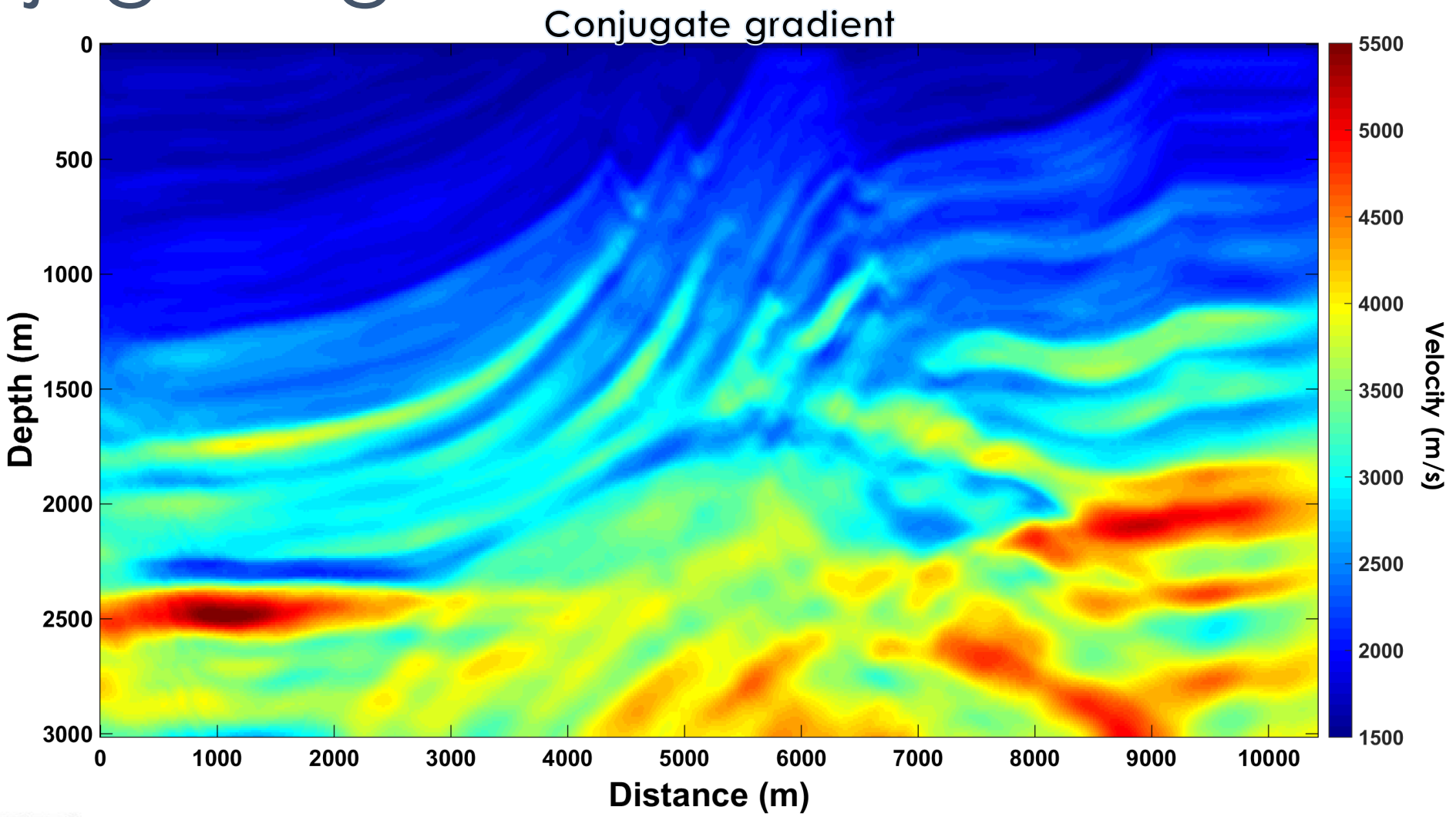
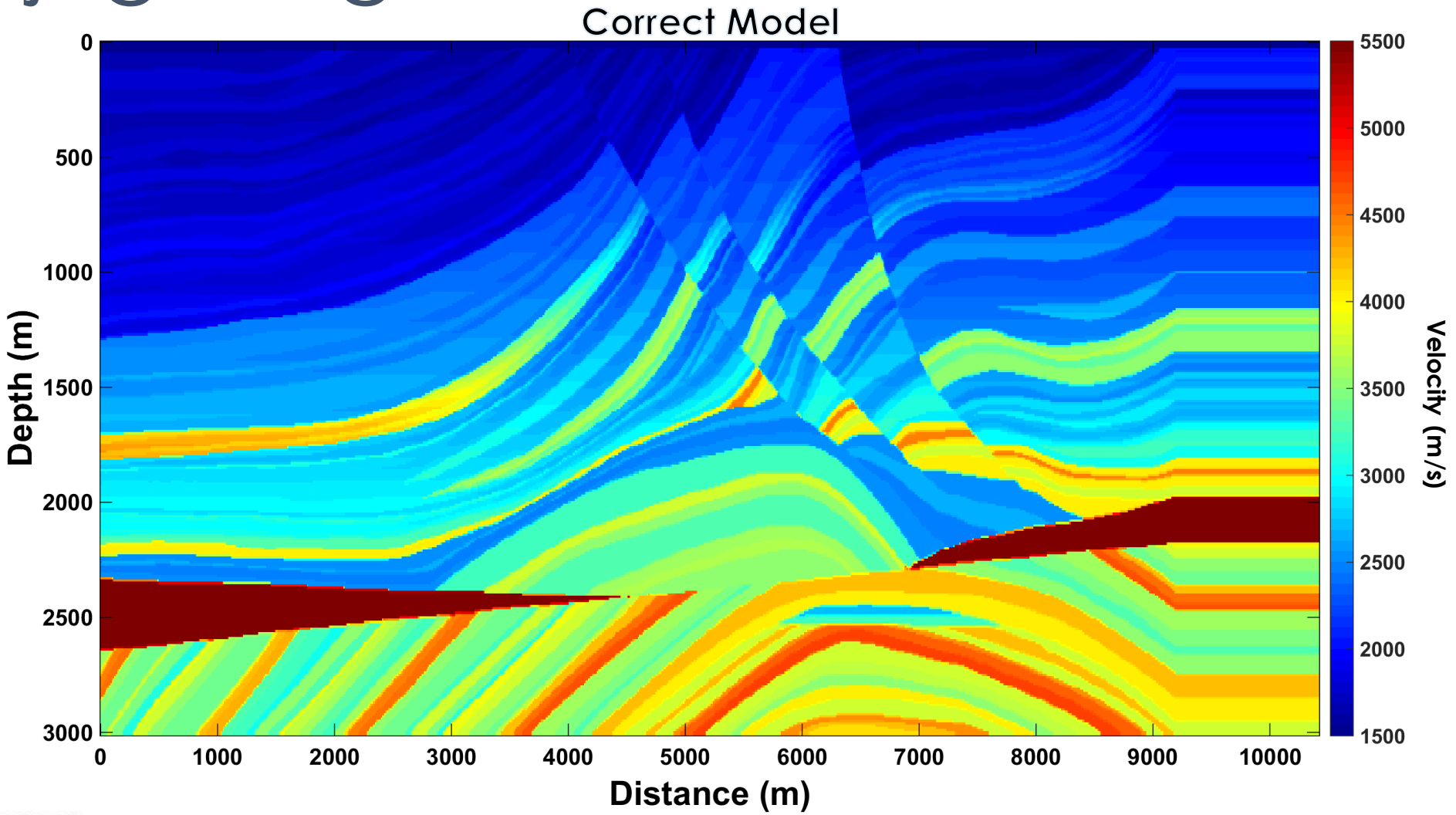
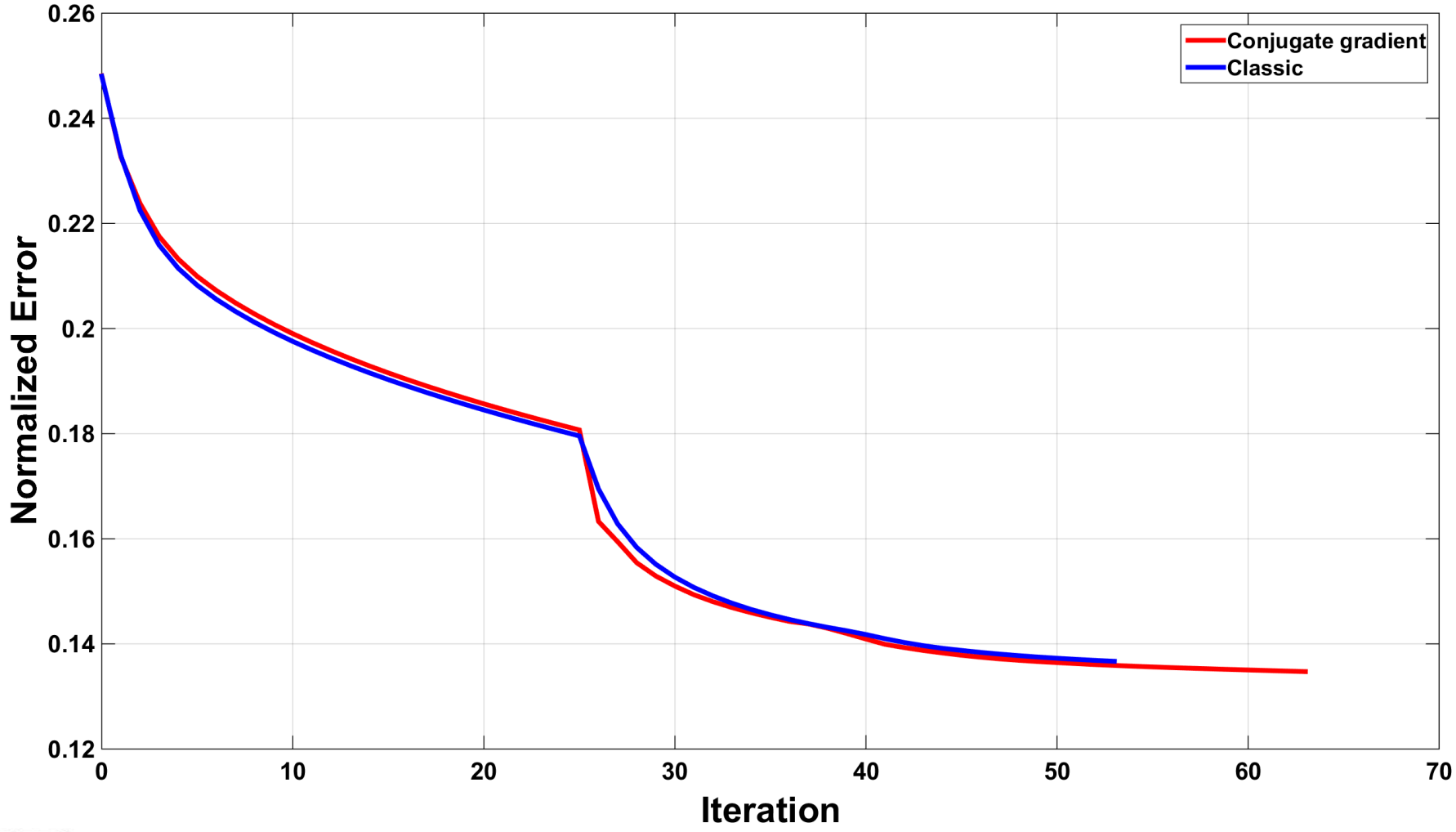
Conjugate gradient



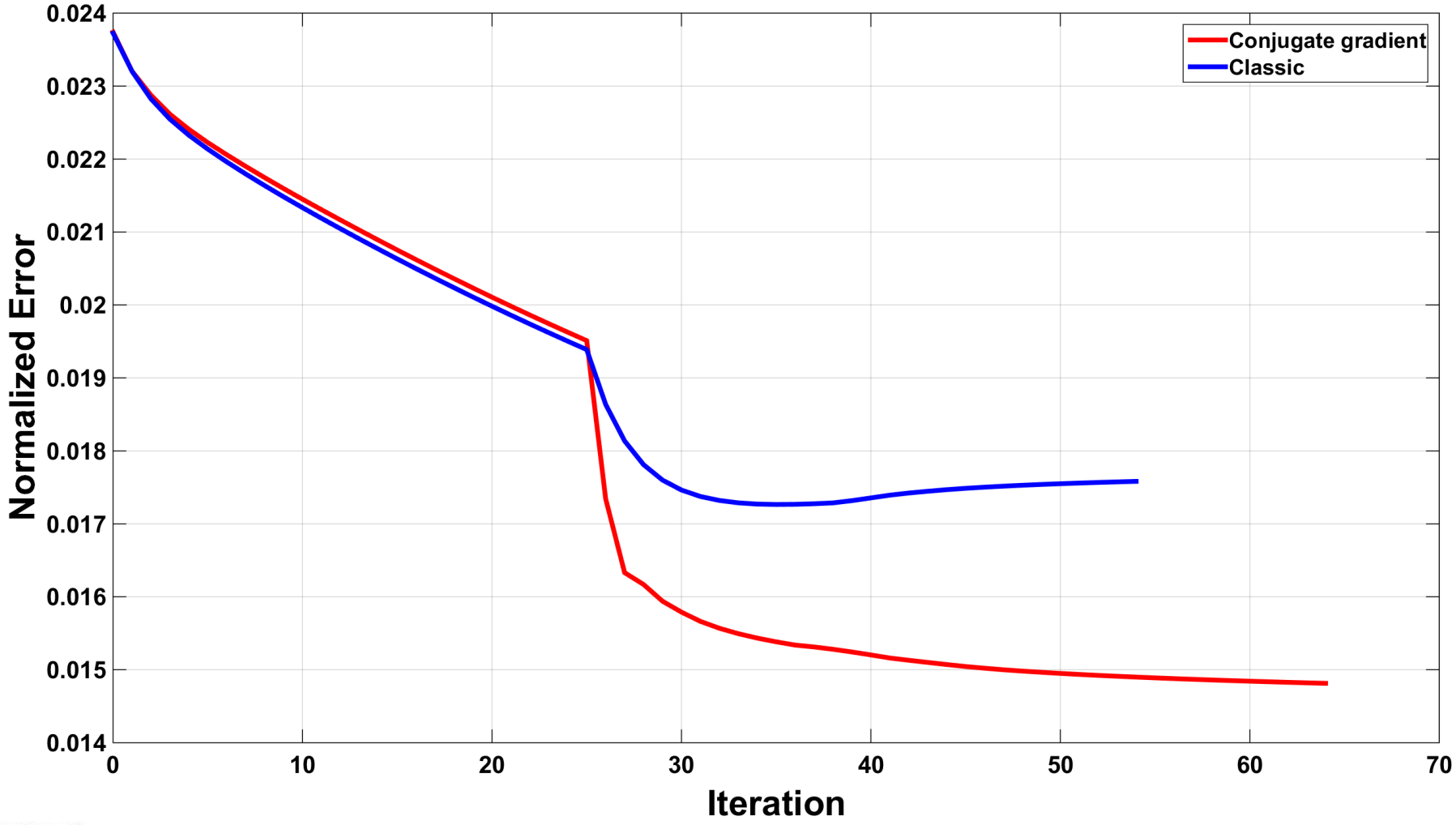
Conjugate gradient



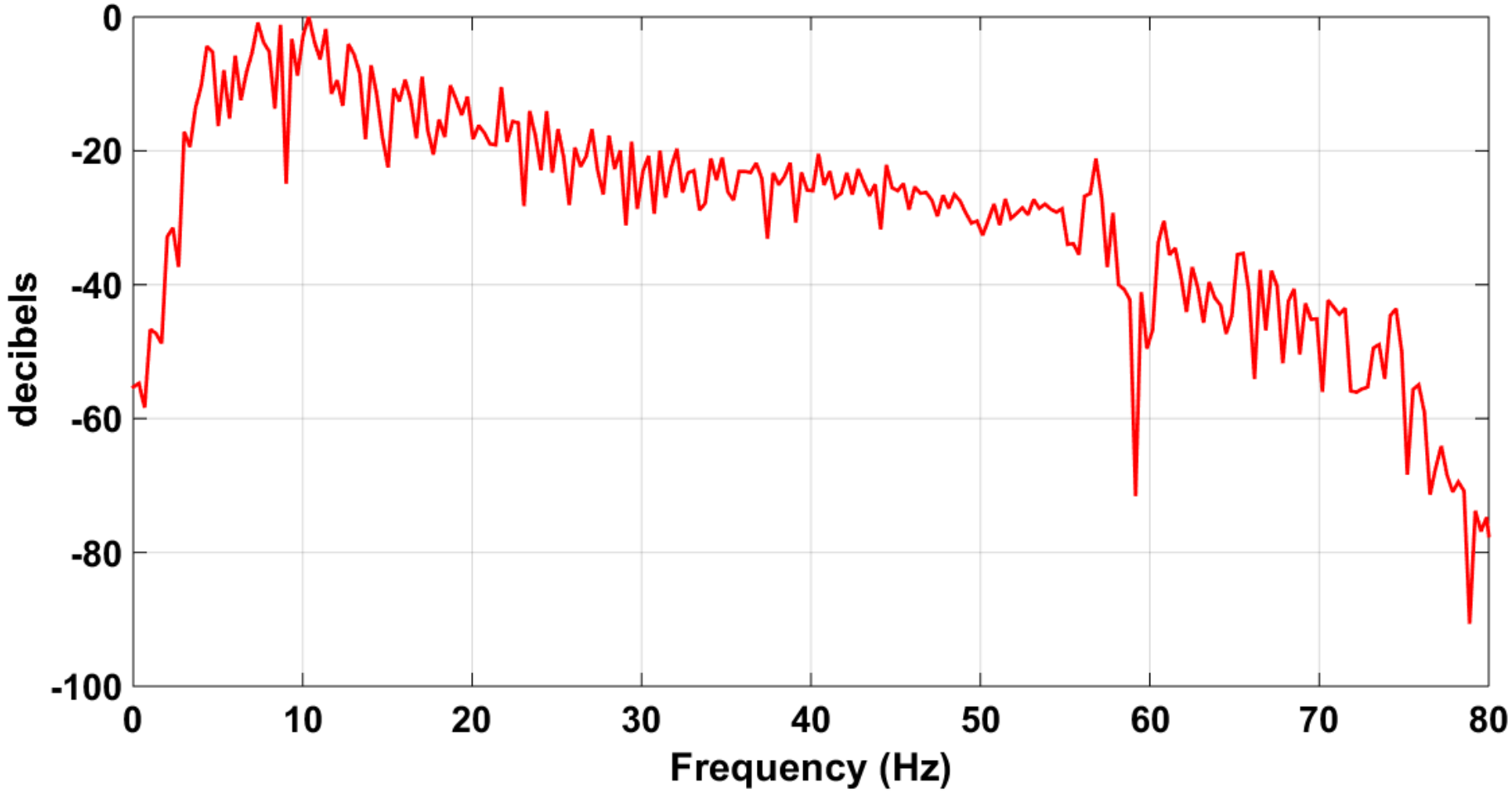
Conjugate gradient – shots errors



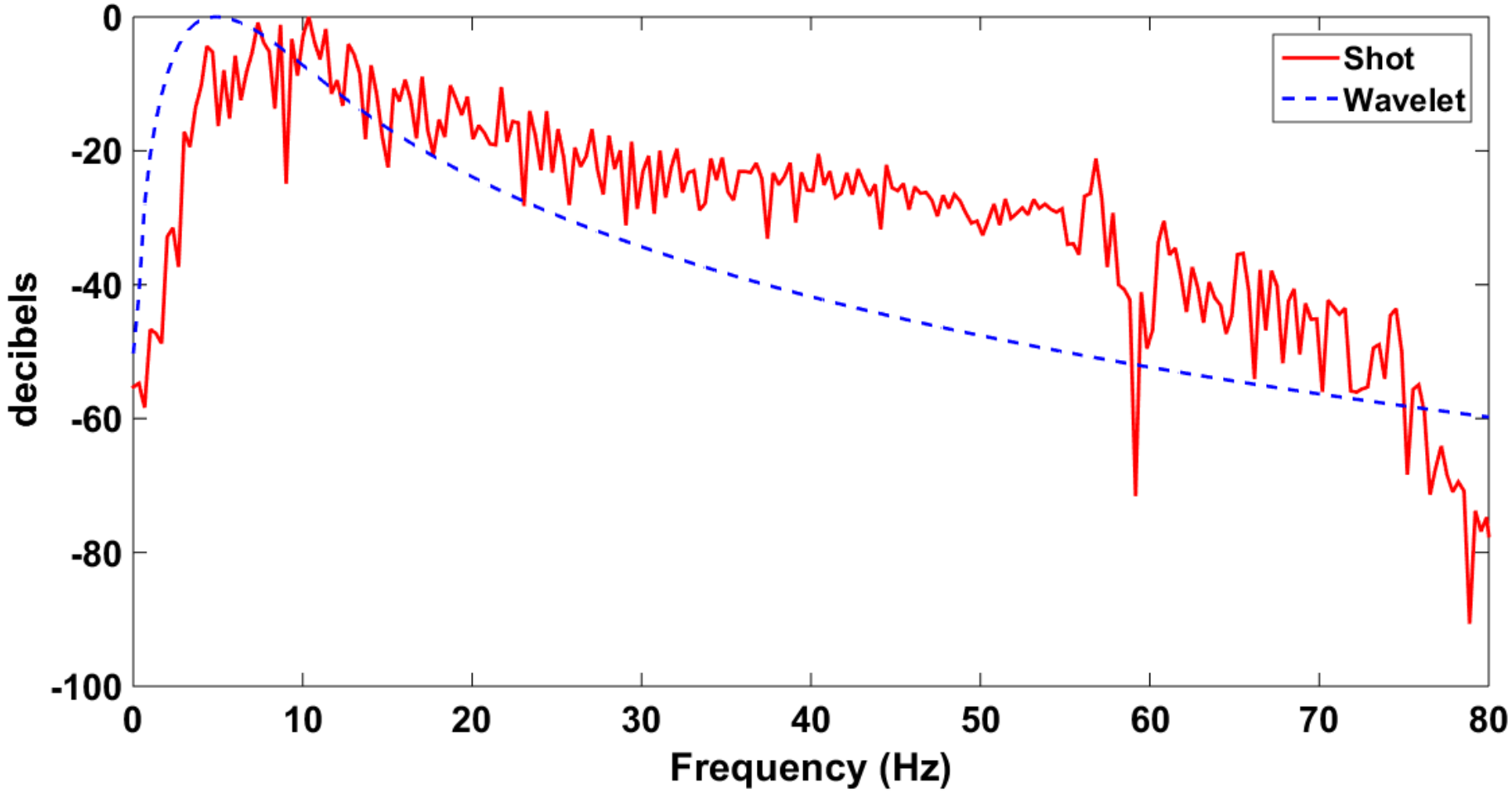
Conjugate gradient – models errors



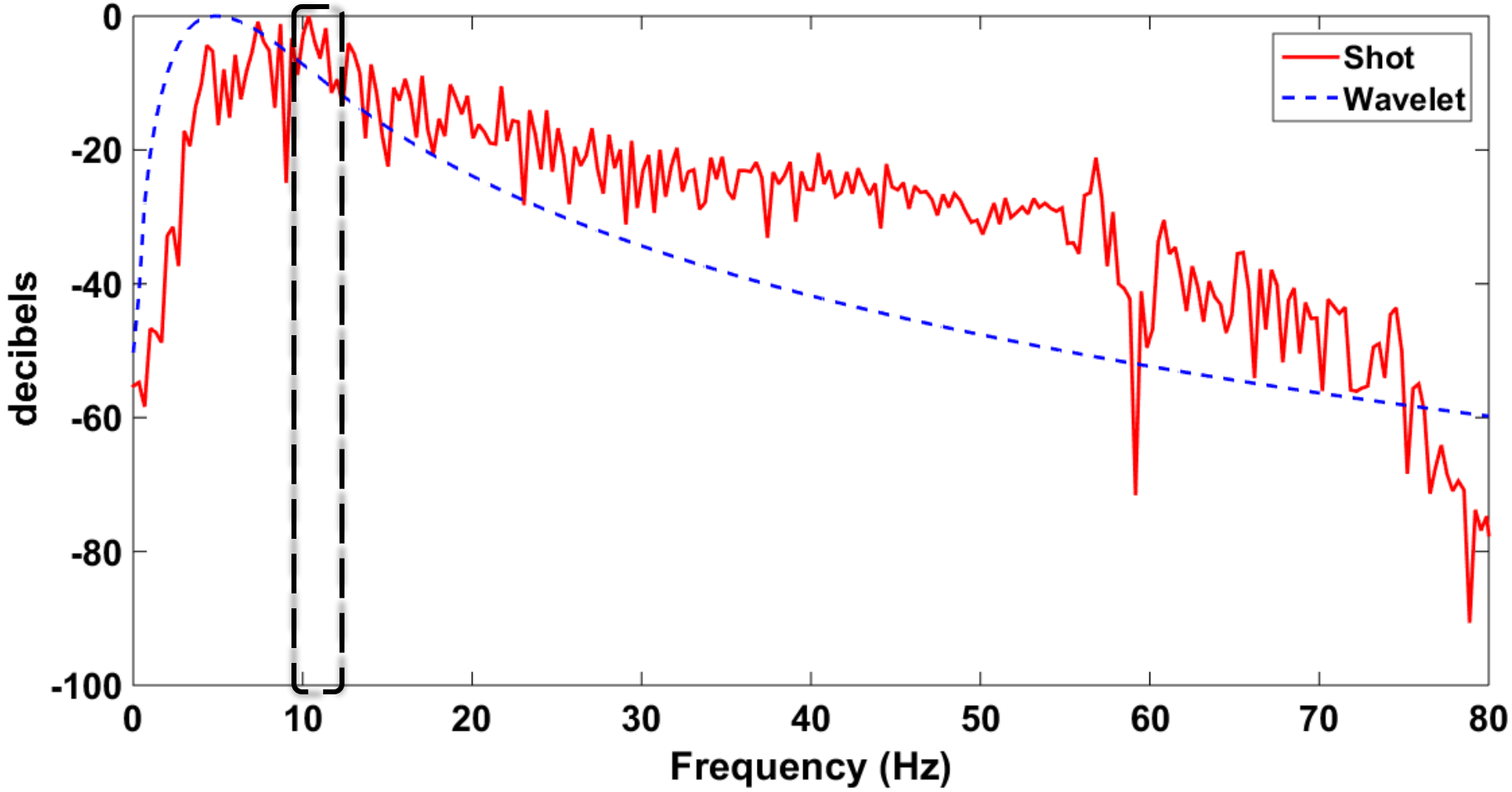
Conjugate gradient – amplitude



Conjugate gradient – amplitude



Conjugate gradient – amplitude



Impedance inversion

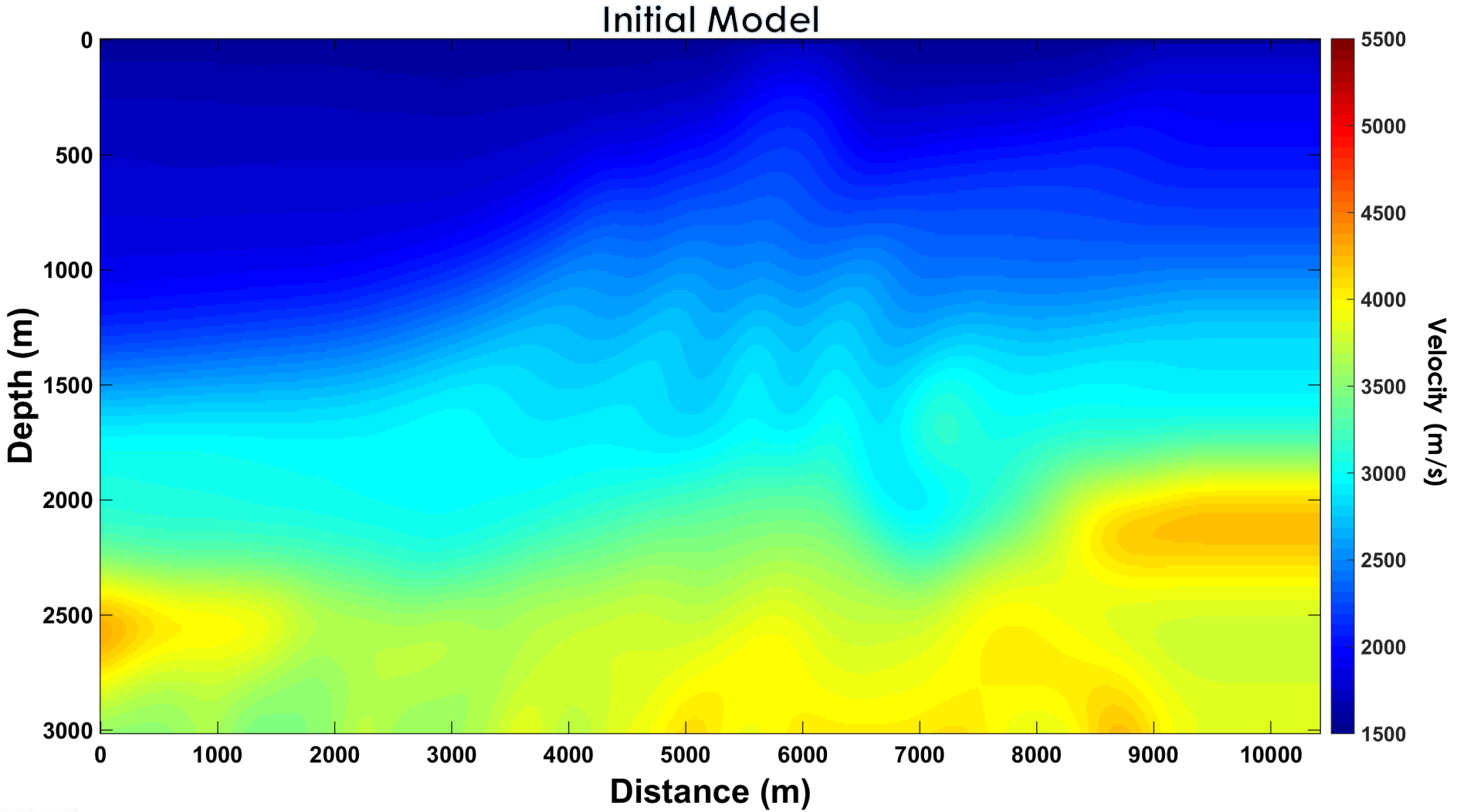
Impedance inversion

- Reflection coefficients → impedance
- Small contrasts of impedance
- Acoustic (constant density)

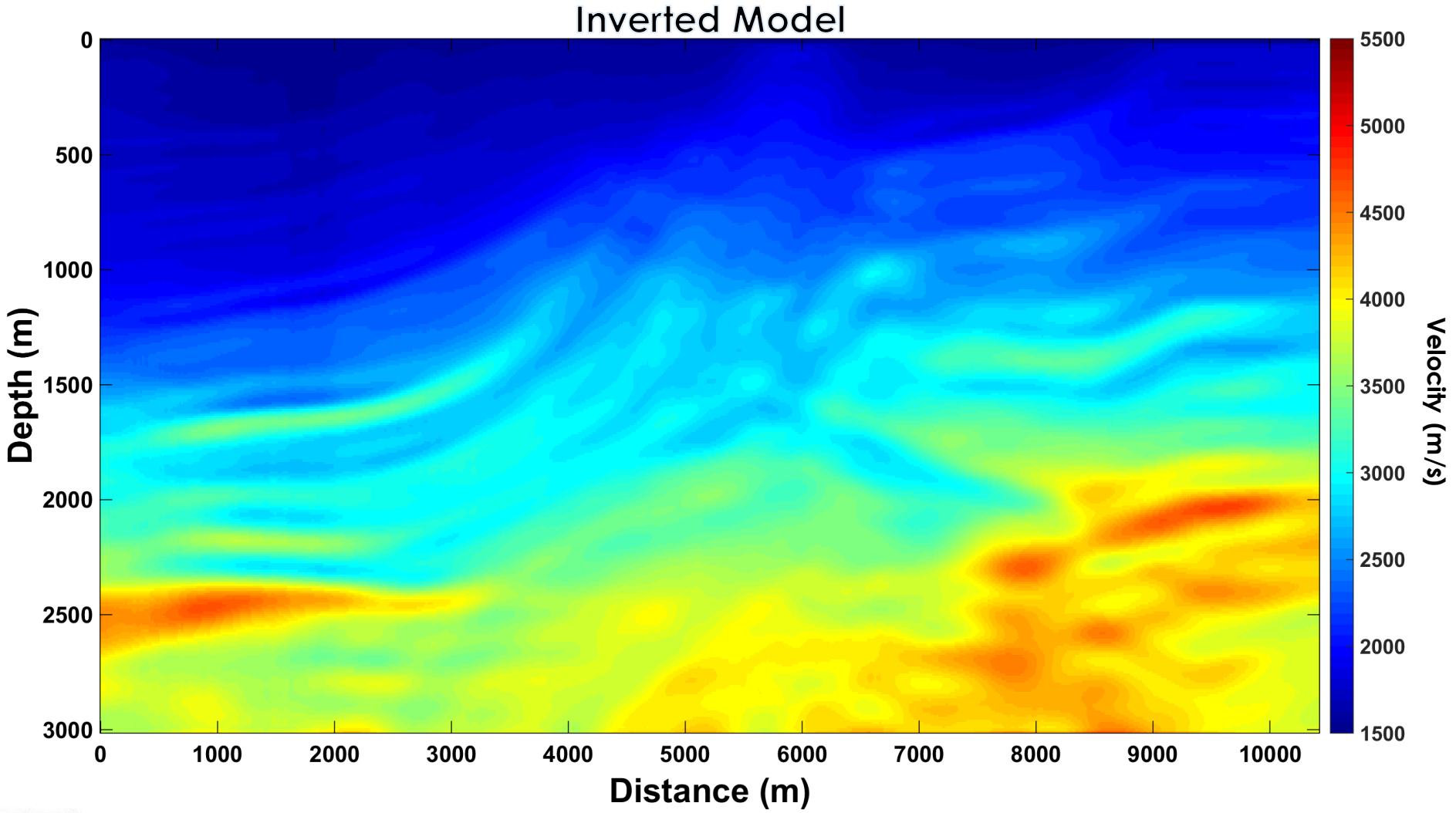
$$V(t) = V_0 e^{2\int_{t_1}^{t} R(\tau)d\tau}$$

- Preliminary results

Impedance inversion



Impedance inversion



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Conclusions

- Least squares step length: cheaper with same quality
- Averaged monochromatic gradients: stable
- Initial model: FWI requires an initial model closer to the Earth's model
- Conjugate gradient: improved inversion
- Strong quality increment around dominant frequency
- Impedance inversion: promising

Future work

- Improve impedance inversion – try well tie inversion
- Wavelet estimation/inversion
- Apply on real data

Acknowledgements

- CREWES sponsors
- NSERC
- Gary Margrave
- Babatunde Arenrin
- CREWES staff and friends

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