Porosity prediction:
Cokriging with multiple secondary datasets

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Outline

- Introduction
- Theory of cokriging with multiple secondary datasets
- Case study – Blackfoot dataset
- Conclusion and Future work
- Acknowledgements

Introduction

- Traditional geostatistics uses the kriging method to optimally produce a map from a number of well log values such as porosity.

- Doyen (1988) used cokriging to predict porosity using well logs as the primary variable and inverted seismic data as the secondary variable.

- Babak and Deutsch (1992) extended the result of Doyen (1988) by merging a number of secondary seismic attributes into one dataset to improve the cokriging model, using a linear combination of attributes.

- Russell et al. (2002) extended the method of Babak and Deutsch by creating a merged dataset using an improved multi-attribute analysis, which involved cross-validation to find the optimum set of seismic attributes.

- In this study, I show how to extend the method proposed by Doyen (1988) by cokriging with two seismic attributes rather than a single merged attribute.

Introduction

Method	Merit	Shortcoming
Kriging	Honors well log values	Less accuracy of lateral resolution
Cokriging with single attribute	Improved lateral resolution, especially with merged dataset	Limited to single secondary attribute
Cokriging with multiple attributes	Better spatial resolution	How many attributes are optimum?

Kriging

- In kriging, we estimate a value at every point on a map from a set of *n* well values u_i using the weighted sum:

$$\hat{u}_0 = \sum_{i=1}^{n} a_i u_i$$

- The kriging weights are computed using the matrix equation:

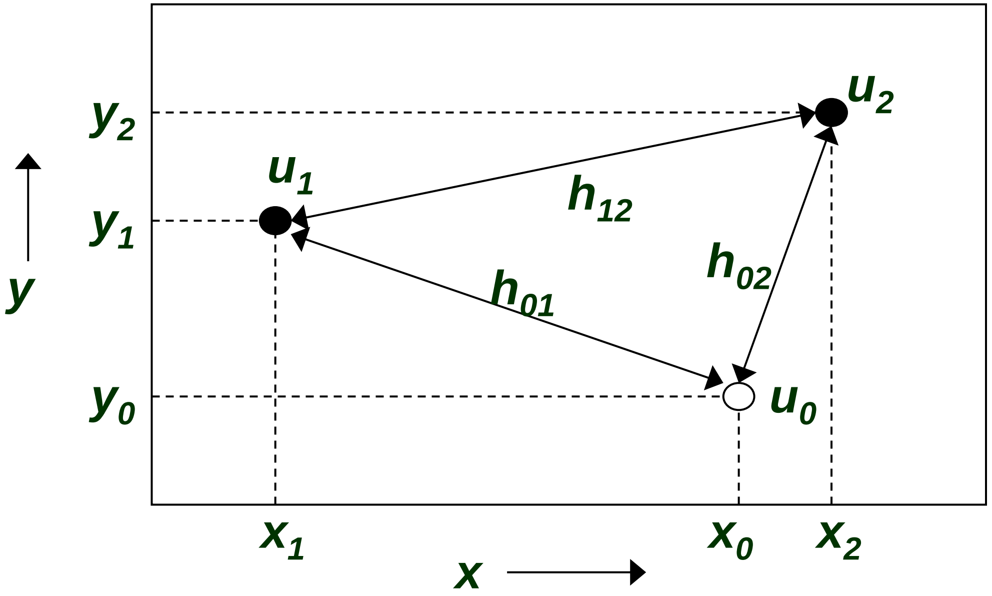
$$\begin{bmatrix} \boldsymbol{a} \\ \mu \end{bmatrix} = \begin{bmatrix} C_{uu} & \boldsymbol{1} \\ \boldsymbol{1}^T & 0 \end{bmatrix}^{-1} \begin{bmatrix} C_{u_0u} \\ 1 \end{bmatrix}, \boldsymbol{a} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}, \boldsymbol{1} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}, C_{uu} = \begin{bmatrix} C_{11} & \cdots & C_{1n} \\ \vdots & \ddots & \vdots \\ C_{n1} & \cdots & C_{nn} \end{bmatrix}, C_{u_0u} = \begin{bmatrix} C_{01} \\ \vdots \\ C_{0n} \end{bmatrix},$$

where :

C_{uu} = known well covariance, C_{u_0u} = known - to - unknown well covariance.

Kriging

- For two input values this can be easily understood:

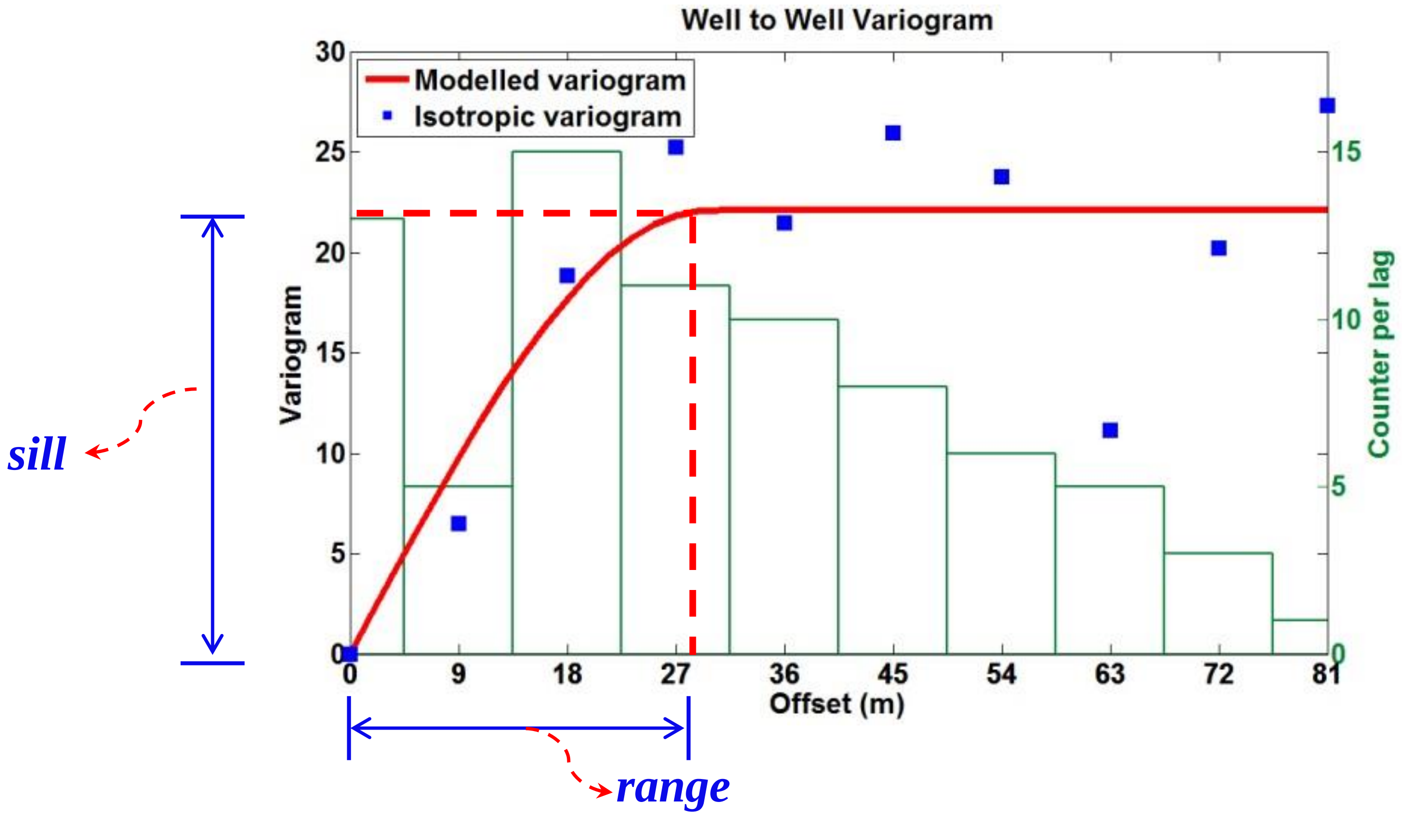


$$\begin{bmatrix} a_1 \\ a_2 \\ \mu \end{bmatrix} = \begin{bmatrix} C(0) & C(h_{12}) & 1 \\ C(h_{12}) & C(0) & 1 \\ 1 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} C(h_{01}) \\ C(h_{02}) \\ 1 \end{bmatrix}$$

Variogram

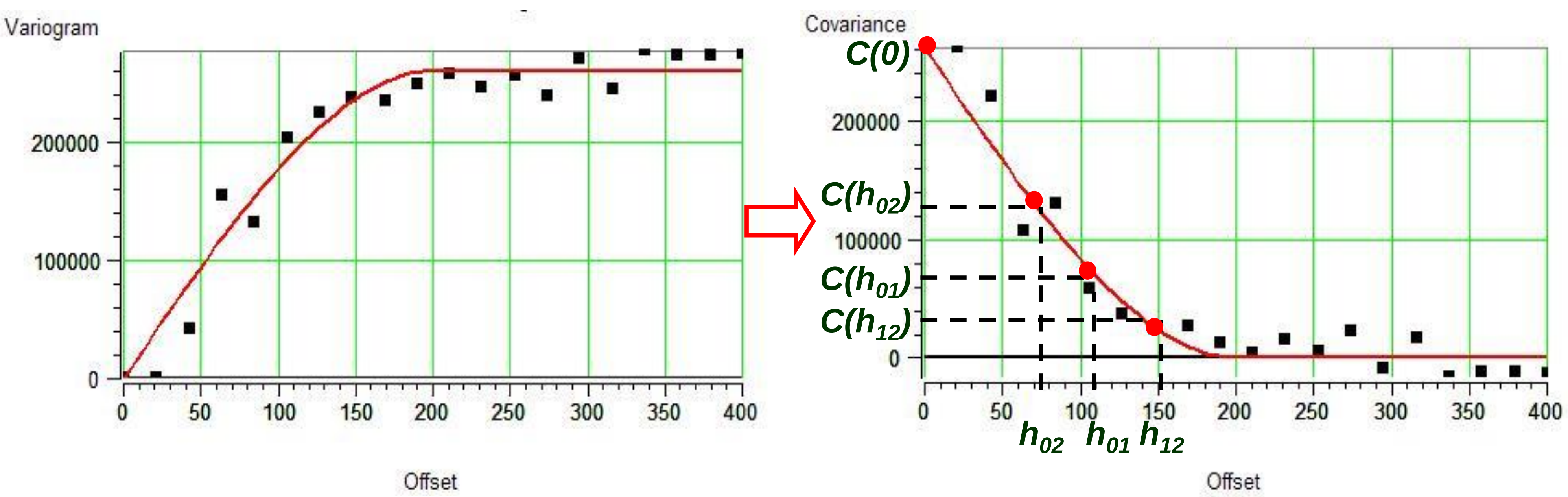
- A variogram is a way to describe the degree of spatial dependence between input data.

- We calculate covariance from a variogram $\mathbf{Cov}(h) = \gamma(\infty) - \gamma(h)$



We find the covariance values as follows:

- First, model the variogram, as shown in the left figure.
- Then, transform to covariance. $\mathbf{Cov}(h) = \gamma(\infty) - \gamma(h)$
- Finally, read the covariance values from the modeled covariance (the red line on the right figure) at the given offsets h_{ij}.



Cokriging with a single secondary dataset

- In traditional cokriging with a single secondary dataset we extend the computation using *m* secondary data values v_j:

$$\hat{u}_0 = \sum_{i=1}^{n} a_i u_i + \sum_{j=1}^{m} b_j v_j$$

- The cokriging weights are computed using the equation:

$$\begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \\ -\mu_1 \\ -\mu_2 \end{bmatrix} = \begin{bmatrix} C_{uu} & C_{uv} & \boldsymbol{1} & 0 \\ C_{vu} & C_{vv} & 0 & \boldsymbol{1} \\ \boldsymbol{1}^{\boldsymbol{T}} & 0 & 0 & 0 \\ 0 & \boldsymbol{1}^{\boldsymbol{T}} & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} C_{u_0 u} \\ C_{u_0 v} \\ 1 \\ 0 \end{bmatrix}, \text{ where:}$$

C_{uv} = well to seismic covariance, C_{vv} = seismic to seismic covariance, and $C_{u_0 v}$ = unknown well to seismic covariance.

New Method --- Cokriging with two secondary datasets

- We can extend cokriging from one to two secondary datasets as follows.

- Estimated values: $\hat{u}_0 = \sum_{i=1}^{n} a_i \cdot u_i + \sum_{j=1}^{m} b_j \cdot v_j + \sum_{k=1}^{p} c_k \cdot x_k$

- The cokriging weights are computed using the equation:

$$\begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \\ \boldsymbol{c} \\ \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix} = \begin{bmatrix} C_{uu} & C_{vu} & \boxed{C_{xu}} & \boldsymbol{1} & 0 & 0 \\ C_{uv} & C_{vv} & \boxed{C_{xv}} & 0 & \boldsymbol{1} & 0 \\ \boxed{C_{ux}} & \boxed{C_{vx}} & \boxed{C_{xx}} & 0 & 0 & \boldsymbol{1} \\ \boldsymbol{1}^T & 0 & 0 & 0 & 0 & 0 \\ 0 & \boldsymbol{1}^T & 0 & 0 & 0 & 0 \\ 0 & 0 & \boldsymbol{1}^T & 0 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} C_{u_0 u} \\ C_{u_0 v} \\ C_{u_0 x} \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

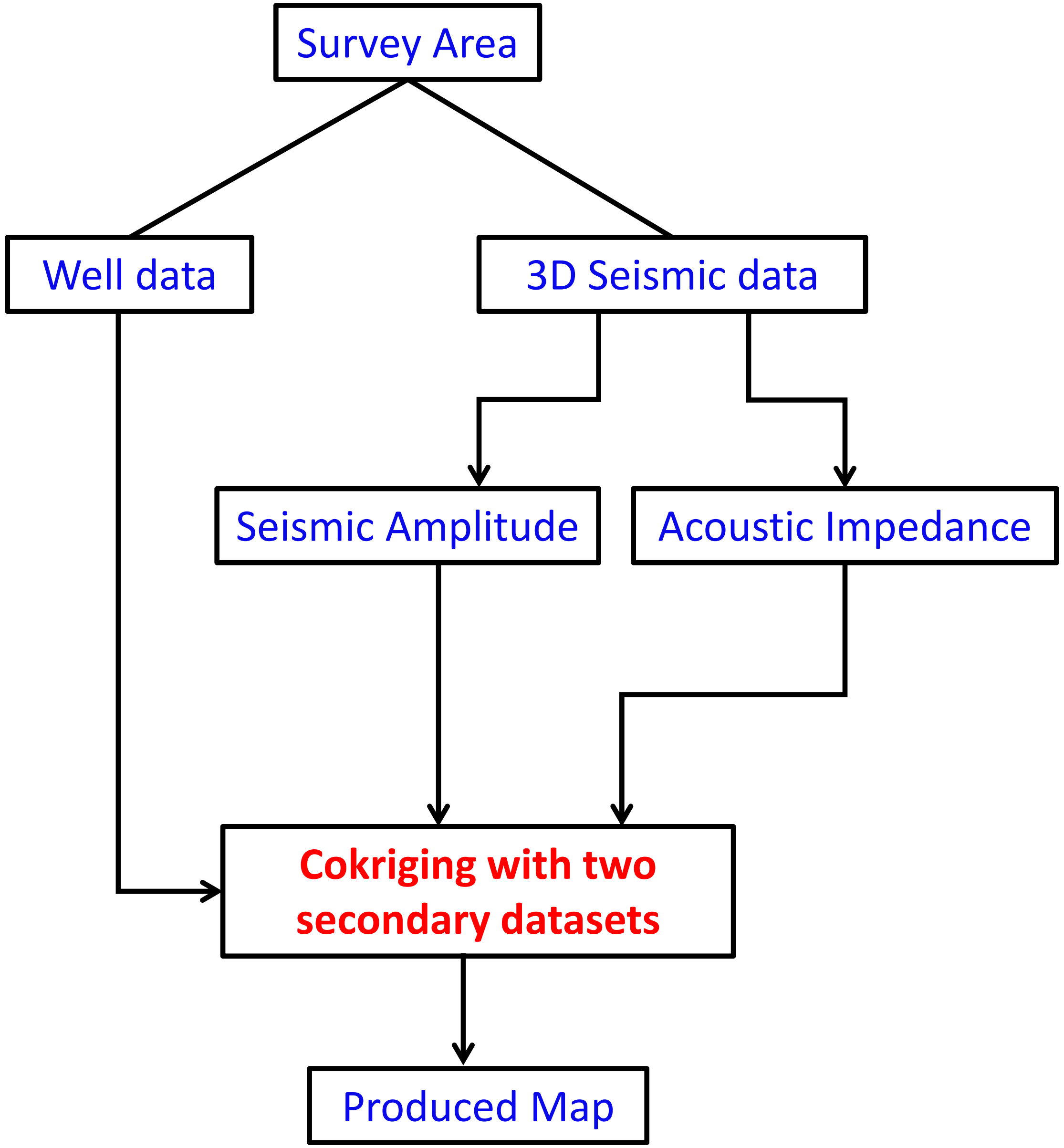
where C_{MN} represents covariance of lengths m and n, $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ are weighted vectors; and μ_1, μ_2, μ_3 are Lagrange parameters.

Cokriging with multiple secondary datasets

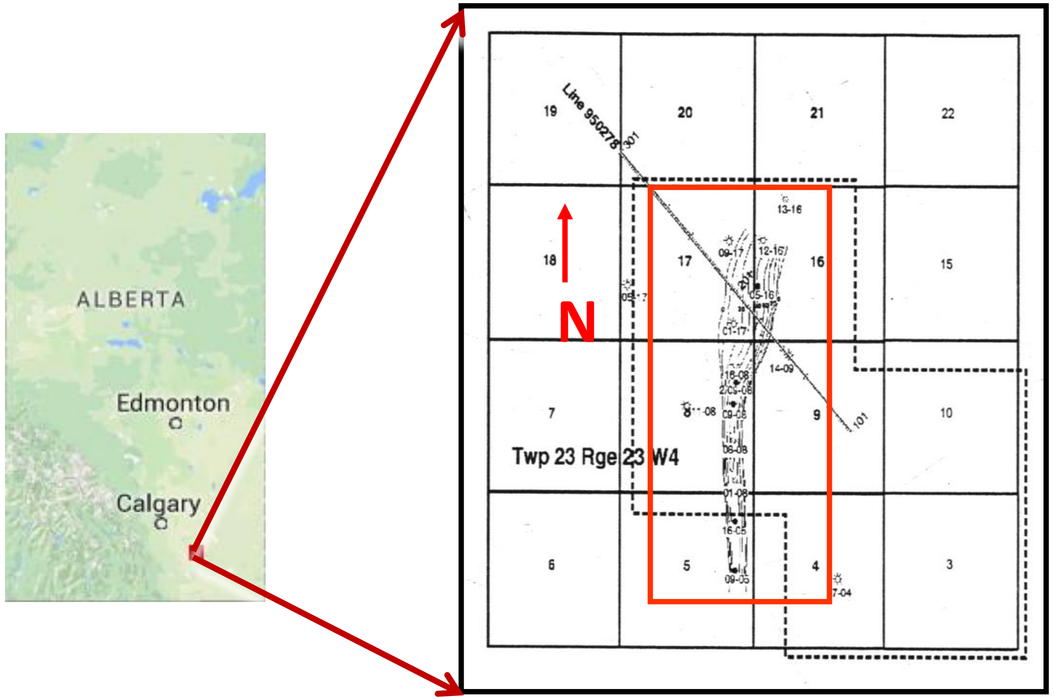
➢ Cokriging with n secondary datasets:

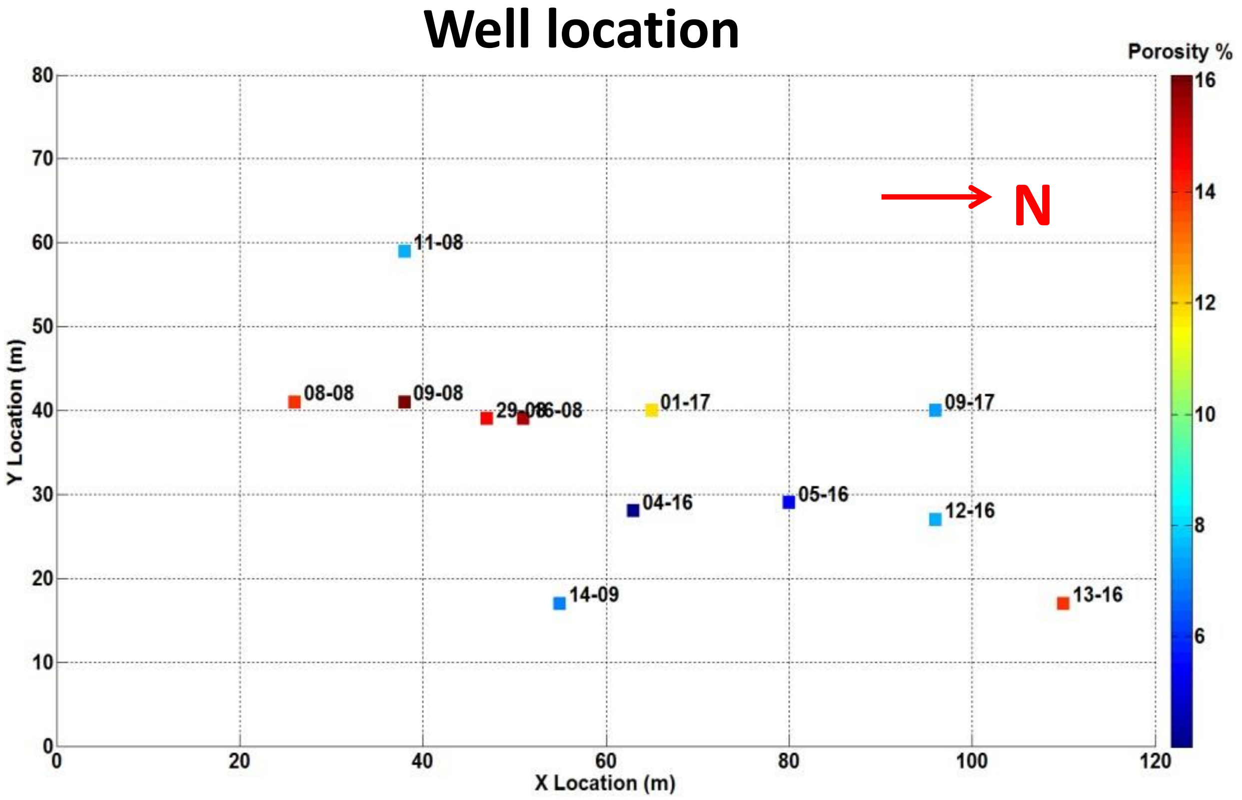
$$
\begin{bmatrix} \boldsymbol{w}_1 \\ \boldsymbol{w}_2 \\ \boldsymbol{w}_3 \\ \vdots \\ \boldsymbol{w}_n \\ \mu_1 \\ \vdots \\ \mu_n \end{bmatrix}
=
\begin{bmatrix}
C_{uu} & C_{v_1u} & C_{v_2u} & \cdots & C_{v_nu} & \boldsymbol{1} & \cdots & 0 \\
C_{uv_1} & C_{v_1v_1} & C_{v_2v_1} & & C_{v_nv_1} & 0 & & 0 \\
C_{uv_2} & C_{v_1v_2} & C_{v_2v_2} & & C_{v_nv_2} & 0 & & 0 \\
\vdots & & & \ddots & \vdots & \vdots & \ddots & \vdots \\
C_{uv_n} & C_{v_1v_n} & C_{v_2v_n} & \cdots & C_{v_nv_n} & 0 & \cdots & \boldsymbol{1} \\
\boldsymbol{1}^T & 0 & 0 & & 0 & 0 & & 0 \\
\vdots & & & \ddots & \vdots & \vdots & & \vdots \\
0 & 0 & 0 & \cdots & \boldsymbol{1}^T & 0 & \cdots & 0
\end{bmatrix}^{-1}
\begin{bmatrix} C_{u_0u} \\ C_{u_0v_1} \\ C_{u_0v_2} \\ \vdots \\ C_{u_0v_n} \\ 1 \\ \vdots \\ 0 \end{bmatrix}
$$

Case Study - Blackfoot



Case Study - Blackfoot

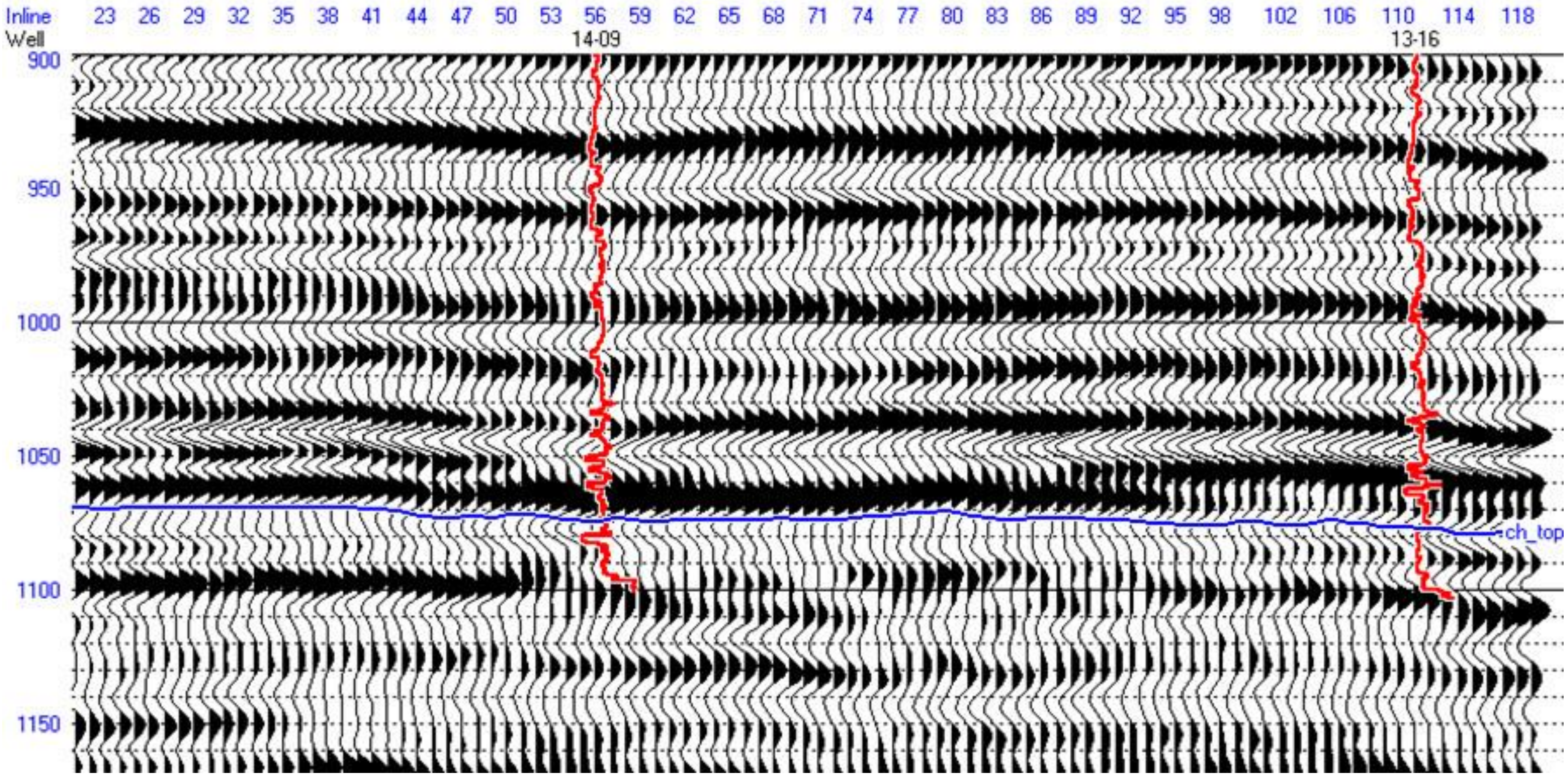




The survey was recorded in south of Alberta in 1995 for PanCanadian Petroleum.

12 Wells are located within the seismic survey area. The color indicates the average porosity value of each well.

Case Study --- Cross-line 18

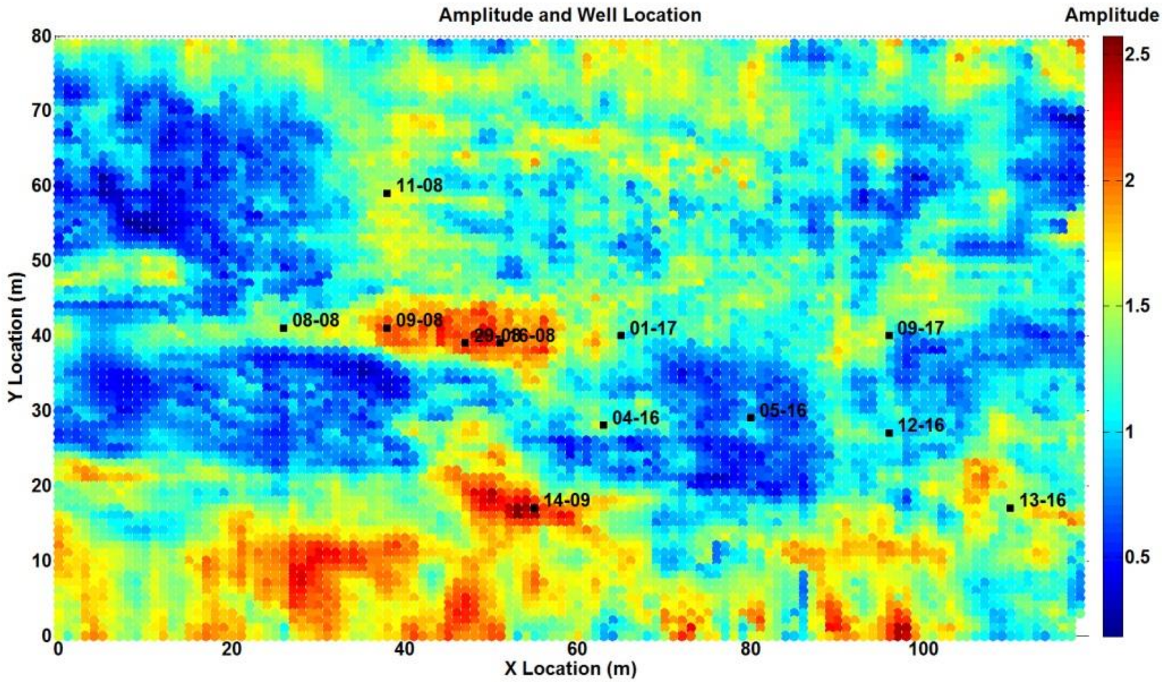


Here is the seismic data showing the picked channel top at around 1070ms.

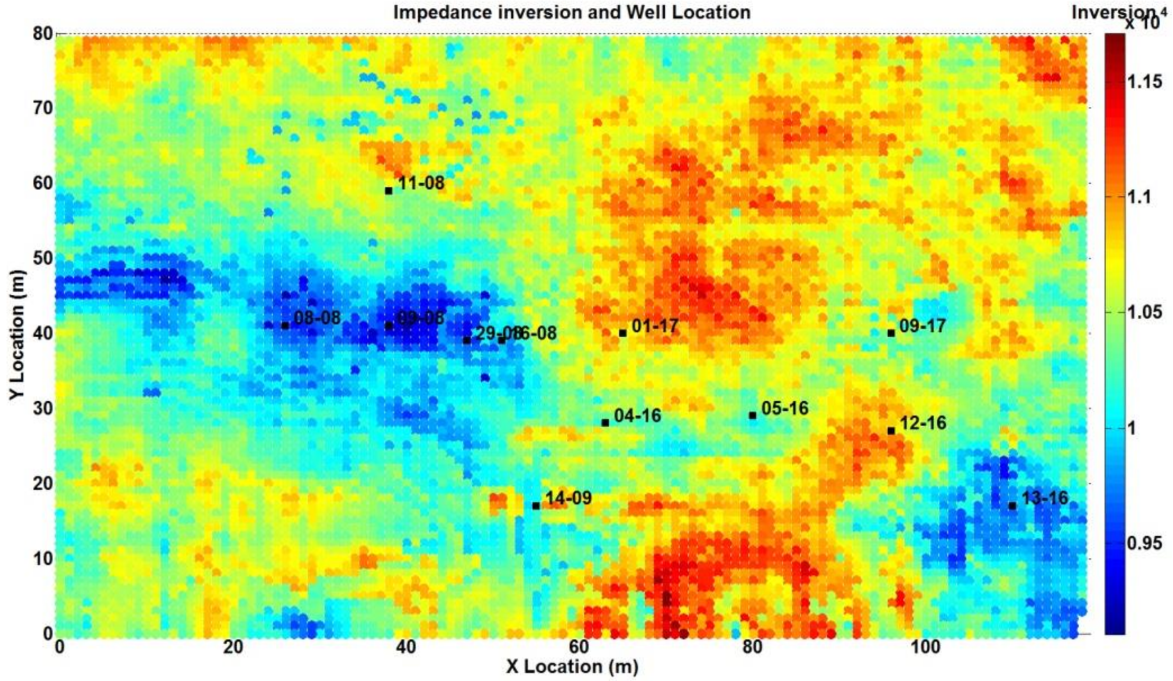
Case Study --- Seismic attributes

➢ Extracted two attribute slices

Seismic amplitude slice



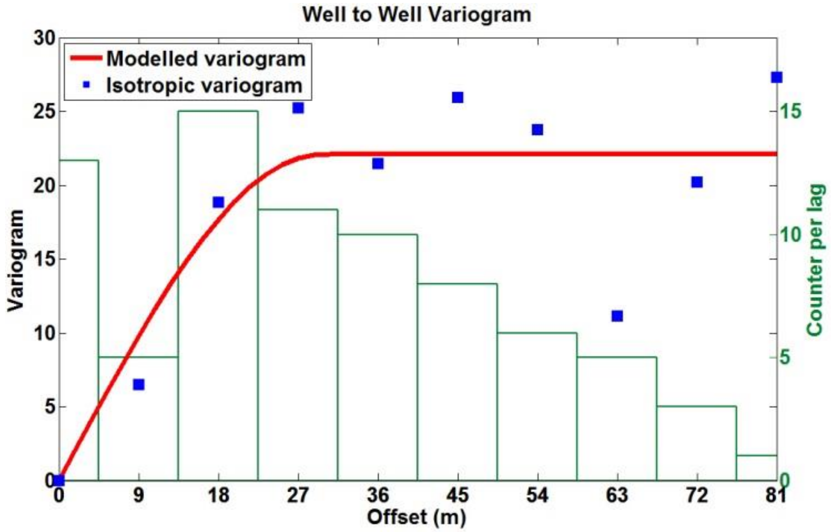
Inverted acoustic impedance slice



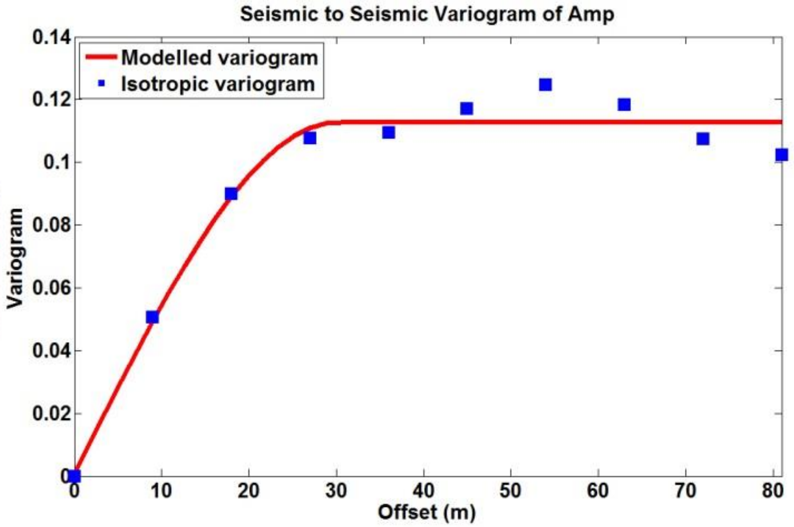
Data slices	Correlation
Acoustic Impedance	-0.65
Seismic amplitude	0.41

Case Study --- Variograms

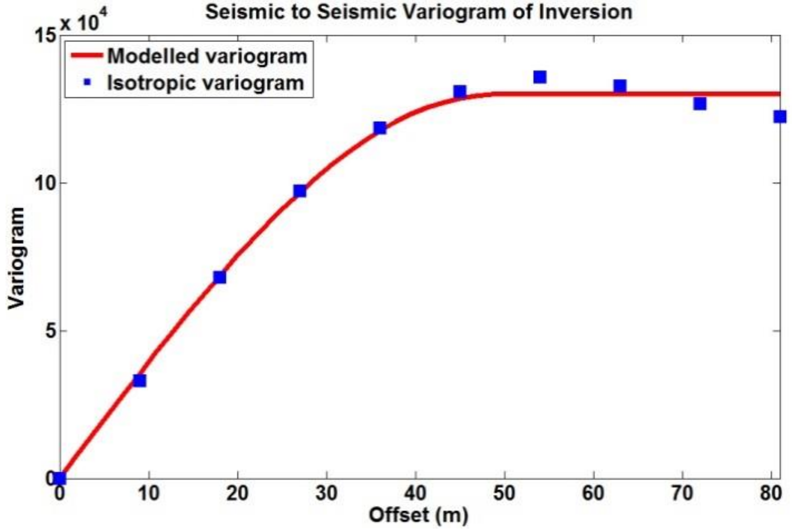
Well to Well



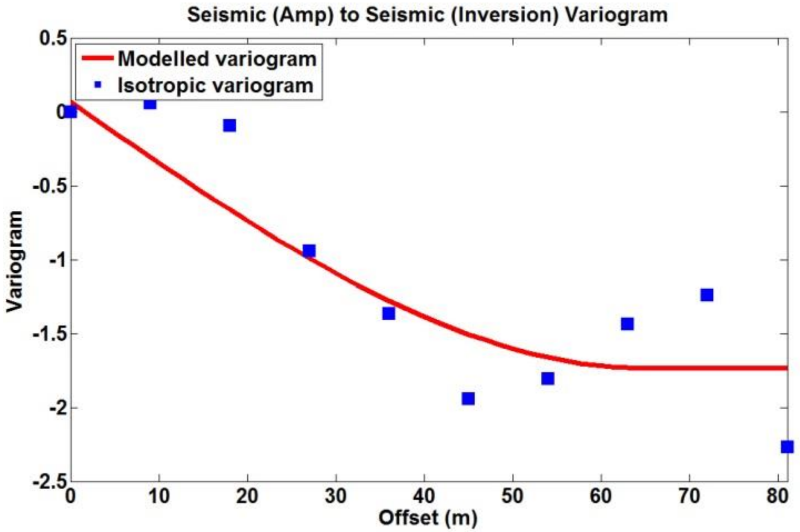
Seis Amp to Seis Amp



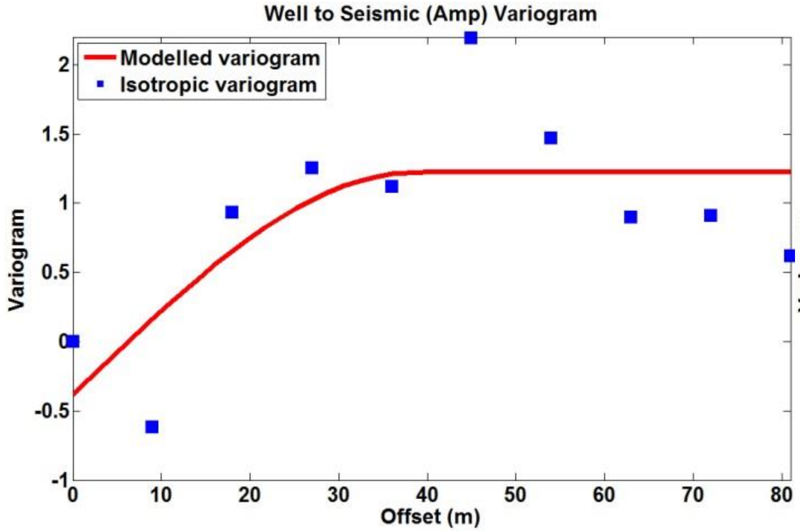
Impedance to Impedance



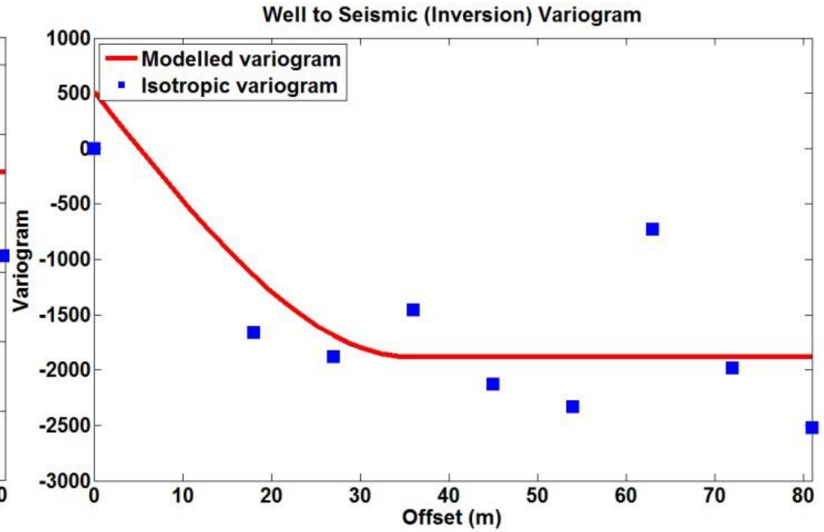
Seis Amp to Impedance



Well to Seis Amp

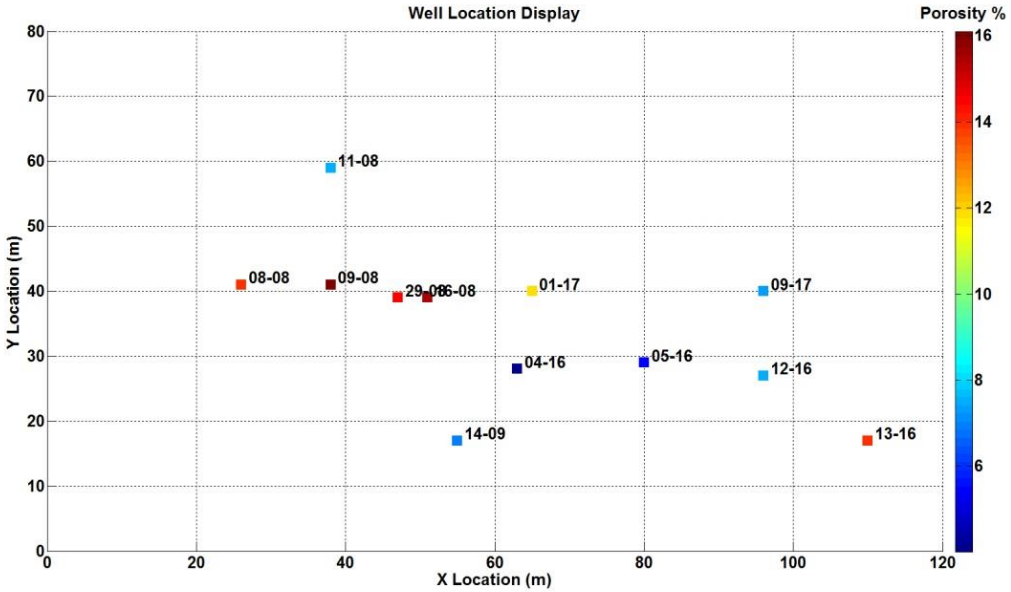


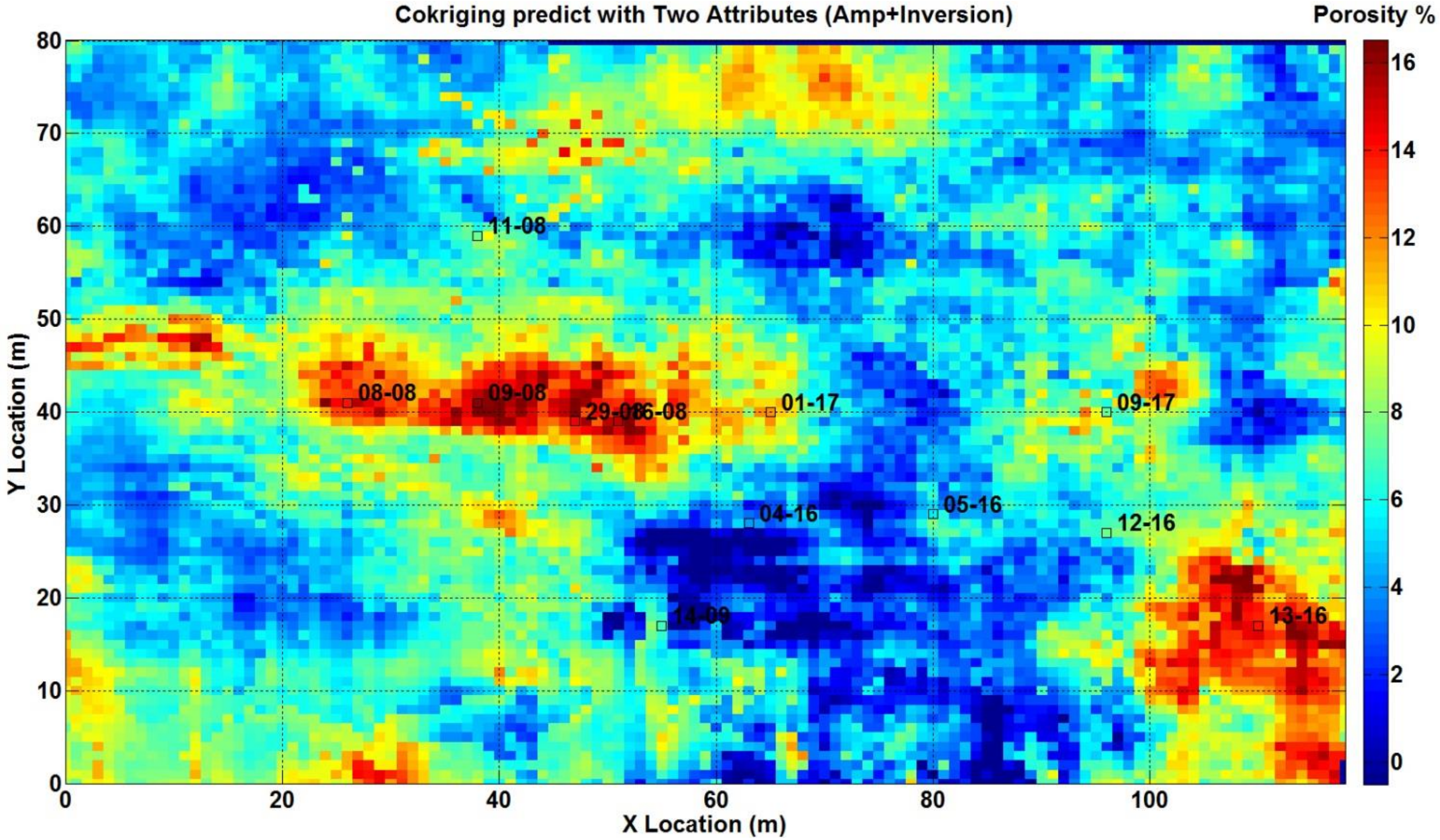
Well to Impedance



Resulting map using two attributes

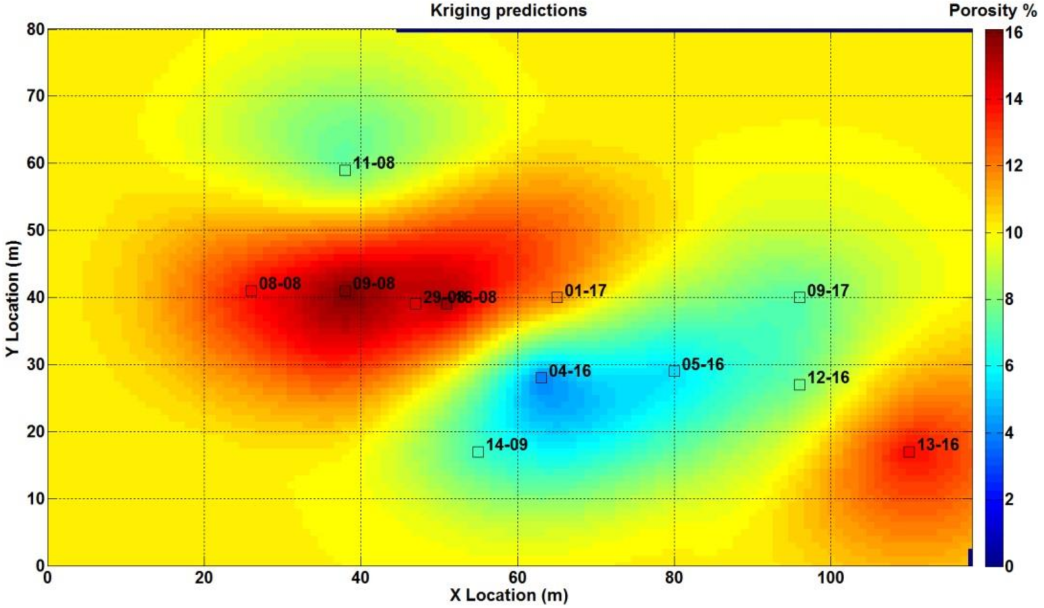
- ➢New cokriging estimate:



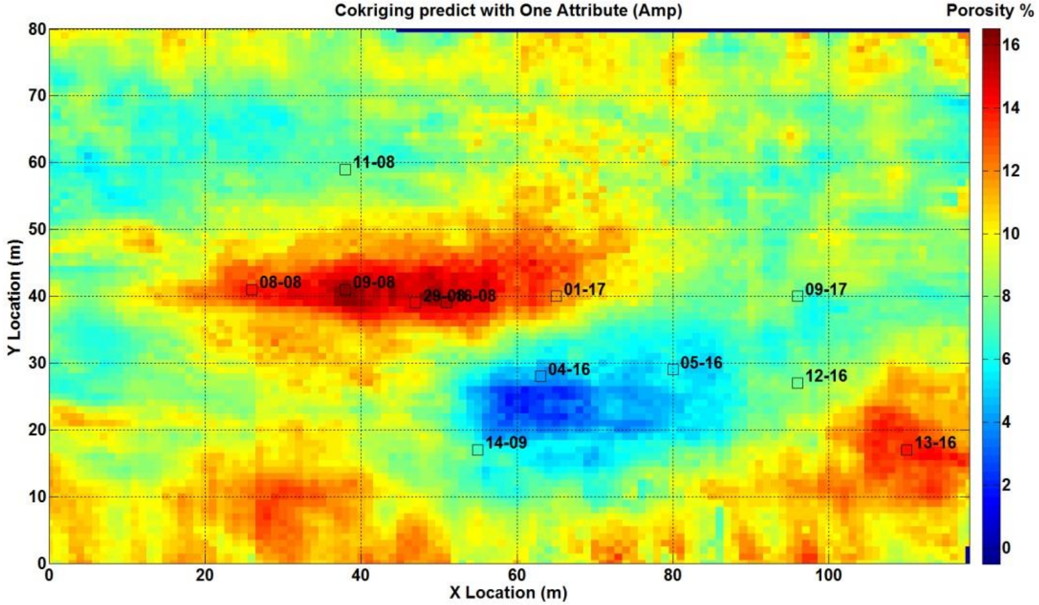


Comparison of all methods

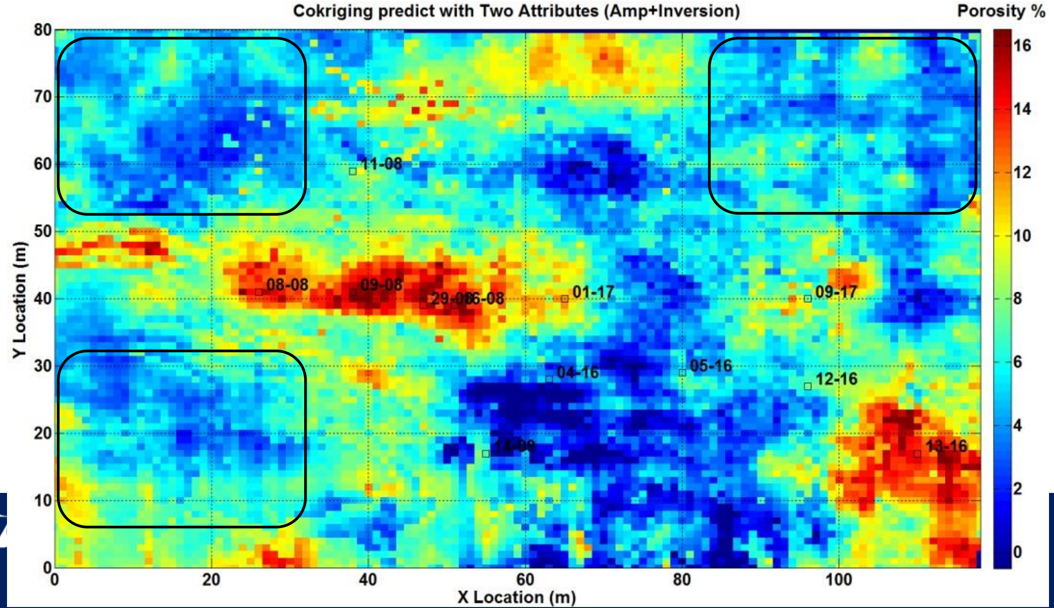
Kriging



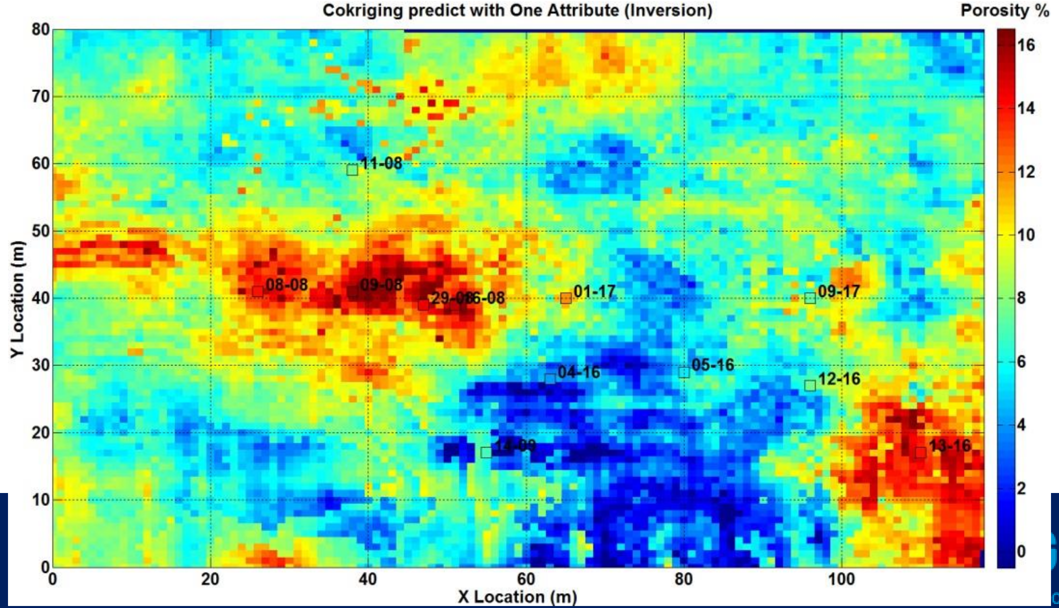
Cokriging with Amplitude



Cokriging with Impedance + Amplitude



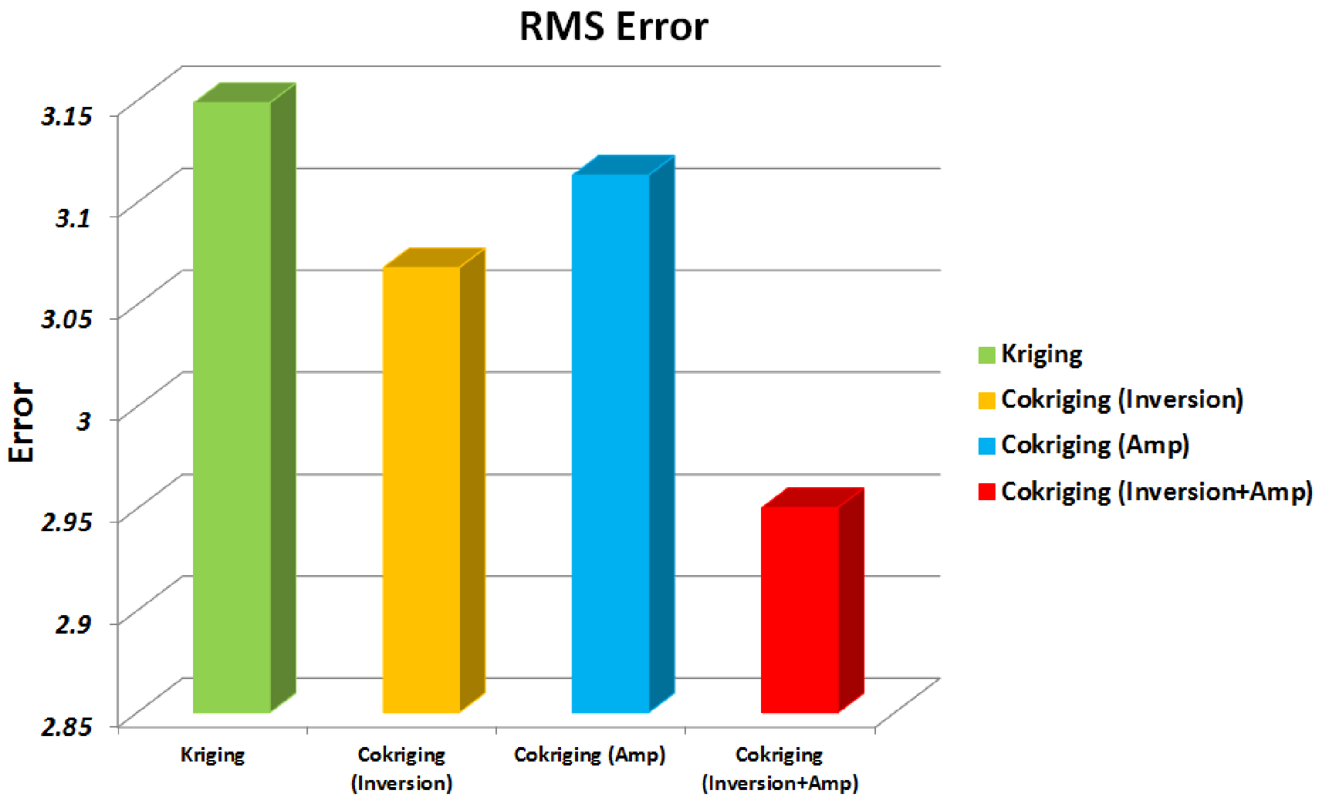
Cokriging with Impedance



Case Study --- Validation

- **Leave-one-out cross-validation**: Calculating the difference between the predicted and observed values by removing one well at a time.

- **RMS error**: $$\mathbf{E}_{RMS} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left\{z(x_i) - \hat{z}(x_i)\right\}^2}$$



Conclusion

- We presented a new cokriging system using one primary and two secondary variables.
- The "Leave-one-out" cross-validation method was applied to validate the accuracy of the new cokriging results.
- Two improvements resulted:
  - Increased lateral resolution
  - Reduced estimated error

Future work

- Compare with traditional cokriging using one super secondary data
- Cokriging test with more than two secondary inputs

Acknowledgements

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Thank You!