Elastic internal multiple prediction
—Theory and application

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Outline

- Born series decomposition in an elastic media
- Derivation of elastic internal multiple prediction (EIMP) using ISS
- Monotonicity condition in pseudo-depth and intercept time
- Synthetic example
- Conclusion and future work
- Acknowledgements

Decomposed Born series

Elastic medium: $$\mathfrak{L}(\mathbf{r},\omega)\mathcal{G}(\mathbf{r},\mathbf{r}_s,\omega) = -\delta(\mathbf{r}-\mathbf{r}_s)$$

Born series: $$\mathcal{G} = \mathcal{G}_0 + \mathcal{G}_0\mathcal{V}\mathcal{G}_0 + \mathcal{G}_0\mathcal{V}\mathcal{G}_0\mathcal{V}\mathcal{G}_0 + \mathcal{G}_0\mathcal{V}\mathcal{G}_0\mathcal{V}\mathcal{G}_0\mathcal{V}\mathcal{G}_0 + \ldots$$

Background: $$\mathfrak{L}_0(\mathbf{r},\omega)\mathcal{G}_0(\mathbf{r},\mathbf{r}_s,\omega) = -\delta(\mathbf{r}-\mathbf{r}_s)$$

Scattering potential: $$\mathcal{V} = \mathfrak{L} - \mathfrak{L}_0 = \begin{pmatrix} \mathcal{V}_{xx} & \mathcal{V}_{xy} & \mathcal{V}_{xz} \\ \mathcal{V}_{yx} & \mathcal{V}_{yy} & \mathcal{V}_{yz} \\ \mathcal{V}_{zx} & \mathcal{V}_{zy} & \mathcal{V}_{zz} \end{pmatrix}$$

Decomposed Born series

Derivative matrix: $\boldsymbol{\Pi} = \begin{pmatrix} \nabla \cdot \\ \nabla \times \end{pmatrix} = \begin{pmatrix} \partial_x & \partial_y & \partial_z \\ 0 & -\partial_z & \partial_y \\ \partial_z & 0 & -\partial_x \\ -\partial_y & \partial_x & 0 \end{pmatrix}$

Elastic medium:

$$\mathfrak{L}_D(\mathbf{r}, \omega)\mathcal{G}_D(\mathbf{r}, \mathbf{r}_s, \omega) = -\delta(\mathbf{r} - \mathbf{r}_s)$$

$$\mathfrak{L}_D = \boldsymbol{\Pi}\mathfrak{L}\boldsymbol{\Pi}^{-1}$$

$$\mathcal{G}_D = \boldsymbol{\Pi}\mathcal{G}\boldsymbol{\Pi}^{-1} = \begin{pmatrix} \mathcal{G}_{PP} & \mathcal{G}_{PS_x} & \mathcal{G}_{PS_y} & \mathcal{G}_{PS_z} \\ \mathcal{G}_{S_xP} & \mathcal{G}_{S_xS_x} & \mathcal{G}_{S_xS_y} & \mathcal{G}_{S_xS_z} \\ \mathcal{G}_{S_yP} & \mathcal{G}_{S_yS_x} & \mathcal{G}_{S_yS_y} & \mathcal{G}_{S_yS_z} \\ \mathcal{G}_{S_zP} & \mathcal{G}_{S_zS_x} & \mathcal{G}_{S_zS_y} & \mathcal{G}_{S_zS_z} \end{pmatrix}$$

Background:

$$\mathfrak{L}_{0D}(\mathbf{r}, \omega)\mathcal{G}_{0D}(\mathbf{r}, \mathbf{r}_s, \omega) = -\delta(\mathbf{r} - \mathbf{r}_s)$$

$$\mathfrak{L}_{0D} = \boldsymbol{\Pi}\mathfrak{L}_0\boldsymbol{\Pi}^{-1}$$

$$\mathcal{G}_{0D} = \boldsymbol{\Pi}\mathcal{G}_0\boldsymbol{\Pi}^{-1} = \begin{pmatrix} \mathcal{G}_{0P} & 0 & 0 & 0 \\ 0 & \mathcal{G}_{0S_x} & 0 & 0 \\ 0 & 0 & \mathcal{G}_{0S_y} & 0 \\ 0 & 0 & 0 & \mathcal{G}_{0S_z} \end{pmatrix}$$

Decomposed Born series

Wave operator:

$$\mathcal{L} = \boldsymbol{E}_r \mathfrak{L}_D \boldsymbol{E}_i^{-1} = \boldsymbol{E}_r \boldsymbol{\Pi}_r \mathfrak{L} (\boldsymbol{\Pi}^{-1})_i \boldsymbol{E}_i^{-1}$$

$$\mathcal{L}_0 = \boldsymbol{E}_r \mathfrak{L}_{0D} \boldsymbol{E}_i^{-1} = \boldsymbol{E}_r \boldsymbol{\Pi}_r \mathfrak{L}_0 (\boldsymbol{\Pi}^{-1})_i \boldsymbol{E}_i^{-1}$$

Green function:

$$\mathbf{G} = \boldsymbol{E}_r \mathcal{G}_D \boldsymbol{E}_i^{-1} = \boldsymbol{E}_r \boldsymbol{\Pi}_r \mathcal{G} (\boldsymbol{\Pi}^{-1})_i \boldsymbol{E}_i^{-1} = \begin{pmatrix} \mathbf{G}_{PP} & \mathbf{G}_{PSH} & \mathbf{G}_{PSV} \\ \mathbf{G}_{SHP} & \mathbf{G}_{SHSH} & \mathbf{G}_{SHSV} \\ \mathbf{G}_{SVP} & \mathbf{G}_{SVSH} & \mathbf{G}_{SVSV} \end{pmatrix}$$

$$\mathbf{G}_0 = \boldsymbol{E}_r \mathcal{G}_{0D} \boldsymbol{E}_i^{-1} = \boldsymbol{E}_r \boldsymbol{\Pi}_r \mathcal{G}_0 (\boldsymbol{\Pi}^{-1})_i \boldsymbol{E}_i^{-1} = \begin{pmatrix} \mathbf{G}_{0P} & 0 & 0 \\ 0 & \mathbf{G}_{0S} & 0 \\ 0 & 0 & \mathbf{G}_{0S} \end{pmatrix}$$

Wave equation:

$$\mathcal{L}(\mathbf{r}, \omega) \boldsymbol{G}(\mathbf{r}, \mathbf{r}_s, \omega) = -\delta(\mathbf{r} - \mathbf{r}_s)$$

$$\mathcal{L}_0(\mathbf{r}, \omega) \boldsymbol{G}_0(\mathbf{r}, \mathbf{r}_s, \omega) = -\delta(\mathbf{r} - \mathbf{r}_s)$$

Decomposed Born series

$$\mathcal{G} = \mathcal{G}_0 + \mathcal{G}_0\mathcal{V}\mathcal{G}_0 + \mathcal{G}_0\mathcal{V}\mathcal{G}_0\mathcal{V}\mathcal{G}_0 + \mathcal{G}_0\mathcal{V}\mathcal{G}_0\mathcal{V}\mathcal{G}_0\mathcal{V}\mathcal{G}_0 + \dots$$

$$\begin{aligned}(\boldsymbol{\Pi}^{-1})_i\boldsymbol{E}_i^{-1}\mathbf{G}\boldsymbol{E}_r\boldsymbol{\Pi}_r &= (\boldsymbol{\Pi}^{-1})_i\boldsymbol{E}_i^{-1}\mathbf{G}_0\boldsymbol{E}_r\boldsymbol{\Pi}_r + (\boldsymbol{\Pi}^{-1})_i\boldsymbol{E}_i^{-1}\mathbf{G}_0\boldsymbol{E}_r\boldsymbol{\Pi}_r\boldsymbol{\mathcal{V}}(\boldsymbol{\Pi}^{-1})_i\boldsymbol{E}_i^{-1}\mathbf{G}_0\boldsymbol{E}_r\boldsymbol{\Pi}_r \\ &+ (\boldsymbol{\Pi}^{-1})_i\boldsymbol{E}_i^{-1}\mathbf{G}_0\boldsymbol{E}_r\boldsymbol{\Pi}_r\boldsymbol{\mathcal{V}}(\boldsymbol{\Pi}^{-1})_i\boldsymbol{E}_i^{-1}\mathbf{G}_0\boldsymbol{E}_r\boldsymbol{\Pi}_r\boldsymbol{\mathcal{V}}(\boldsymbol{\Pi}^{-1})_i\boldsymbol{E}_i^{-1}\mathbf{G}_0\boldsymbol{E}_r\boldsymbol{\Pi}_r \\ &+ \dots\end{aligned}$$

$$\mathbf{G} = \mathbf{G}_0 + \mathbf{G}_0\mathbf{V}\mathbf{G}_0 + \mathbf{G}_0\mathbf{V}\mathbf{G}_0\mathbf{V}\mathbf{G}_0 + \mathbf{G}_0\mathbf{V}\mathbf{G}_0\mathbf{V}\mathbf{G}_0\mathbf{V}\mathbf{G}_0 + \dots$$

Scatter potential:

$$\mathbf{V} = \boldsymbol{E}_r\boldsymbol{\Pi}_r\boldsymbol{\mathcal{V}}(\boldsymbol{\Pi}^{-1})_i\boldsymbol{E}_i^{-1} = \begin{pmatrix} \mathbf{V}_{PP} & \mathbf{V}_{PSH} & \mathbf{V}_{PSV} \\ \mathbf{V}_{SHP} & \mathbf{V}_{SHSH} & \mathbf{V}_{SHSV} \\ \mathbf{V}_{SVP} & \mathbf{V}_{SVSH} & \mathbf{V}_{SVSV} \end{pmatrix}$$

Decomposed Born series

$$\begin{bmatrix} \varphi_P \\ \varphi_{SH} \\ \varphi_{SV} \end{bmatrix} = \boldsymbol{E}_r \boldsymbol{\Pi}_r \mathbf{u} = \boldsymbol{E}_r \boldsymbol{\Pi}_r \boldsymbol{\mathcal{G}} \mathbf{f} = \mathbf{G} \boldsymbol{E}_r \boldsymbol{\Pi}_r \mathbf{f} = \mathbf{G} \mathbf{F}$$

$$\boldsymbol{D}\mathbf{F} = (\mathbf{G} - \mathbf{G}_0)\mathbf{F} = \mathbf{G}_0\mathbf{V}\mathbf{G}_0\mathbf{F} + \mathbf{G}_0\mathbf{V}\mathbf{G}_0\mathbf{V}\mathbf{G}_0\mathbf{F} + \mathbf{G}_0\mathbf{V}\mathbf{G}_0\mathbf{V}\mathbf{G}_0\mathbf{V}\mathbf{G}_0\mathbf{F} + \dots$$

$$\begin{bmatrix} \boldsymbol{D}_{PP} & \boldsymbol{D}_{PSH} & \boldsymbol{D}_{PSV} \\ \boldsymbol{D}_{SHP} & \boldsymbol{D}_{SHSH} & \boldsymbol{D}_{SHSV} \\ \boldsymbol{D}_{SVP} & \boldsymbol{D}_{SVSH} & \boldsymbol{D}_{SVSV} \end{bmatrix} \mathbf{F} = \begin{bmatrix} \mathbf{G}_{0P} & 0 & 0 \\ 0 & \mathbf{G}_{0S} & 0 \\ 0 & 0 & \mathbf{G}_{0S} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{PP} & \mathbf{V}_{PSH} & \mathbf{V}_{PSV} \\ \mathbf{V}_{SHP} & \mathbf{V}_{SHSH} & \mathbf{V}_{SHSV} \\ \mathbf{V}_{SVP} & \mathbf{V}_{SVSH} & \mathbf{V}_{SVSV} \end{bmatrix} \begin{bmatrix} \mathbf{G}_{0P} & 0 & 0 \\ 0 & \mathbf{G}_{0S} & 0 \\ 0 & 0 & \mathbf{G}_{0S} \end{bmatrix} \mathbf{F}$$

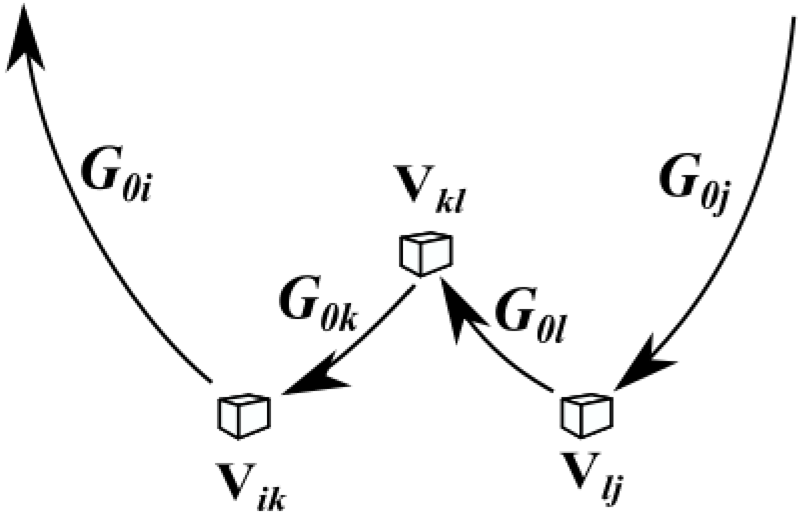
$$+ \begin{bmatrix} \mathbf{G}_{0P} & 0 & 0 \\ 0 & \mathbf{G}_{0S} & 0 \\ 0 & 0 & \mathbf{G}_{0S} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{PP} & \mathbf{V}_{PSH} & \mathbf{V}_{PSV} \\ \mathbf{V}_{SHP} & \mathbf{V}_{SHSH} & \mathbf{V}_{SHSV} \\ \mathbf{V}_{SVP} & \mathbf{V}_{SVSH} & \mathbf{V}_{SVSV} \end{bmatrix} \begin{bmatrix} \mathbf{G}_{0P} & 0 & 0 \\ 0 & \mathbf{G}_{0S} & 0 \\ 0 & 0 & \mathbf{G}_{0S} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{V}_{PP} & \mathbf{V}_{PSH} & \mathbf{V}_{PSV} \\ \mathbf{V}_{SHP} & \mathbf{V}_{SHSH} & \mathbf{V}_{SHSV} \\ \mathbf{V}_{SVP} & \mathbf{V}_{SVSH} & \mathbf{V}_{SVSV} \end{bmatrix} \begin{bmatrix} \mathbf{G}_{0P} & 0 & 0 \\ 0 & \mathbf{G}_{0S} & 0 \\ 0 & 0 & \mathbf{G}_{0S} \end{bmatrix} \mathbf{F} + \dots$$

Derivation of EIMP using ISS

$$D_{ij} = G_{0i}V_{ij}G_{0j} + G_{0i}V_{ik}G_{0k}V_{kj}G_{0j} + G_{0i}V_{ik}G_{0k}V_{kl}G_{0l}V_{lj}G_{0j} + \ldots \qquad i,j,k,l \in \{P, SH, SV\}$$

$$\begin{aligned}
D_{ij} &= G_{0i}V_{ij}^{(1)}G_{0j}, \\
0 &= G_{0i}V_{ij}^{(2)}G_{0j} + G_{0i}V_{ik}^{(1)}G_{0k}V_{kj}^{(1)}G_{0j}, \\
0 &= G_{0i}V_{ij}^{(3)}G_{0j} + G_{0i}V_{ik}^{(2)}G_{0k}V_{kj}^{(1)}G_{0j} + G_{0i}V_{ik}^{(1)}G_{0k}V_{kj}^{(2)}G_{0j} + \boxed{G_{0i}V_{ik}^{(1)}G_{0k}V_{kl}^{(1)}G_{0l}V_{lj}^{(1)}G_{0j}}, \\
&\vdots
\end{aligned}$$



Derivation of EIMP using ISS

Leading 1st-order EIMP algorithm:

$$
\begin{aligned}
&b_{3ij}(k_{ix_g}, k_{iy_g}, k_{jx_s}, k_{jy_s}, \omega) \\
&= -\frac{1}{(2\pi)^4} \iiiint_{-\infty}^{+\infty} \mathrm{d}k_{kx_1} \mathrm{d}k_{ky_1} \mathrm{d}k_{lx_2} \mathrm{d}k_{ly_2} e^{\mathrm{i}\nu_{k1}(z_s - z_g)} e^{-\mathrm{i}\nu_{l2}(z_s - z_g)} \\
&\quad \times \int_{-\infty}^{+\infty} \mathrm{d}z_1 e^{\mathrm{i}(\nu_{k1}+\nu_{ig})z_1} b_{1ik}(k_{ix_g}, k_{iy_g}, k_{kx_1}, k_{ky_1}, z_1) \\
&\quad \times \int_{-\infty}^{z_1} \mathrm{d}z_2 e^{-\mathrm{i}(\nu_{l2}+\nu_{k1})z_2} b_{1kl}(k_{kx_1}, k_{ky_1}, k_{lx_2}, k_{ly_2}, z_2) \\
&\quad \times \int_{z_2}^{+\infty} \mathrm{d}z_3 e^{\mathrm{i}(\nu_{js}+\nu_{l2})z_2} b_{1lj}(k_{lx_2}, k_{ly_2}, k_{jx_s}, k_{jy_s}, z_3)
\end{aligned}
$$

$$
b_{1ij}(k_{ix_g}, k_{iy_g}, k_{jx_s}, k_{jy_s}, \omega) = \mathrm{i}2\nu_{js} D_{ij}(k_{ix_g}, k_{iy_g}, k_{jx_s}, k_{jy_s}, \omega)
$$

Monotonicity condition

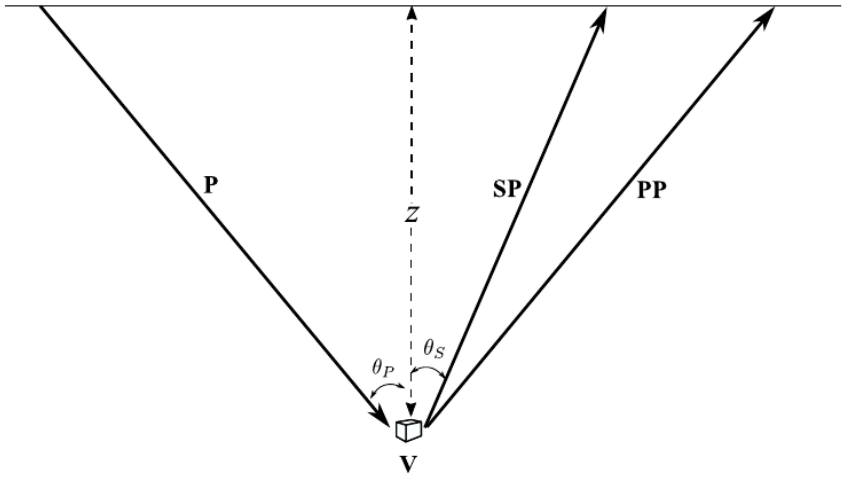
$$
\begin{aligned}
b_3^{ij}(k_g^i,\omega) = -\int_{-\infty}^{+\infty} \mathrm{d}z_1 e^{\mathrm{i}(\nu^m+\nu^i)z_1} b_1^{im}(k_g^i, z_1) \int_{-\infty}^{z_1-\epsilon} \mathrm{d}z_2 e^{-\mathrm{i}(\nu^n+\nu^m)z_2} b_1^{mn}(k_g^m, z_2) \\
\times \int_{z_2+\epsilon}^{+\infty} \mathrm{d}z_3 e^{\mathrm{i}(\nu^j+\nu^n)z_3} b_1^{nj}(k_g^n, z_3)
\end{aligned}
$$

Acoustic:

$z_1 < z_2$

$\Updownarrow$

$\tau_1 < \tau_2$



Elastic:

$z_1 < z_2$

$\not\Updownarrow$

$\tau_1 < \tau_2$

Monotonicity condition

$$b_3^{ij}(k_g^i,\omega) = -\int_{-\infty}^{+\infty} \mathrm{d}z_1 e^{\mathrm{i}(\nu^m+\nu^i)z_1} b_1^{im}(k_g^i,z_1) \int_{-\infty}^{z_1-\epsilon} \mathrm{d}z_2 e^{-\mathrm{i}(\nu^n+\nu^m)z_2} b_1^{mn}(k_g^m,z_2)$$
$$\times \int_{z_2+\epsilon}^{+\infty} \mathrm{d}z_3 e^{\mathrm{i}(\nu^j+\nu^n)z_3} b_1^{nj}(k_g^n,z_3)$$

For P-wave source only:

$$b_3^{\acute{P}\grave{P}} = \Theta_1(b_1^{\acute{P}\grave{P}})\Theta_2(b_1^{\acute{P}\grave{P}})\Theta_3(b_1^{\acute{P}\grave{P}}) + \Theta_1(b_1^{\acute{P}\grave{S}})\Theta_2(b_1^{\acute{S}\grave{P}})\Theta_3(b_1^{\acute{P}\grave{P}}) + \Theta_1(b_1^{\acute{P}\grave{P}})\Theta_2(b_1^{\acute{P}\grave{S}})\Theta_3(b_1^{\acute{P}\grave{P}})$$

$$b_3^{\acute{S}\grave{P}} = \Theta_1(b_1^{\acute{S}\grave{P}})\Theta_2(b_1^{\acute{P}\grave{P}})\Theta_3(b_1^{\acute{P}\grave{P}}) + \Theta_1(b_1^{\acute{S}\grave{P}})\Theta_2(b_1^{\acute{P}\grave{S}})\Theta_3(b_1^{\acute{S}\grave{P}})$$

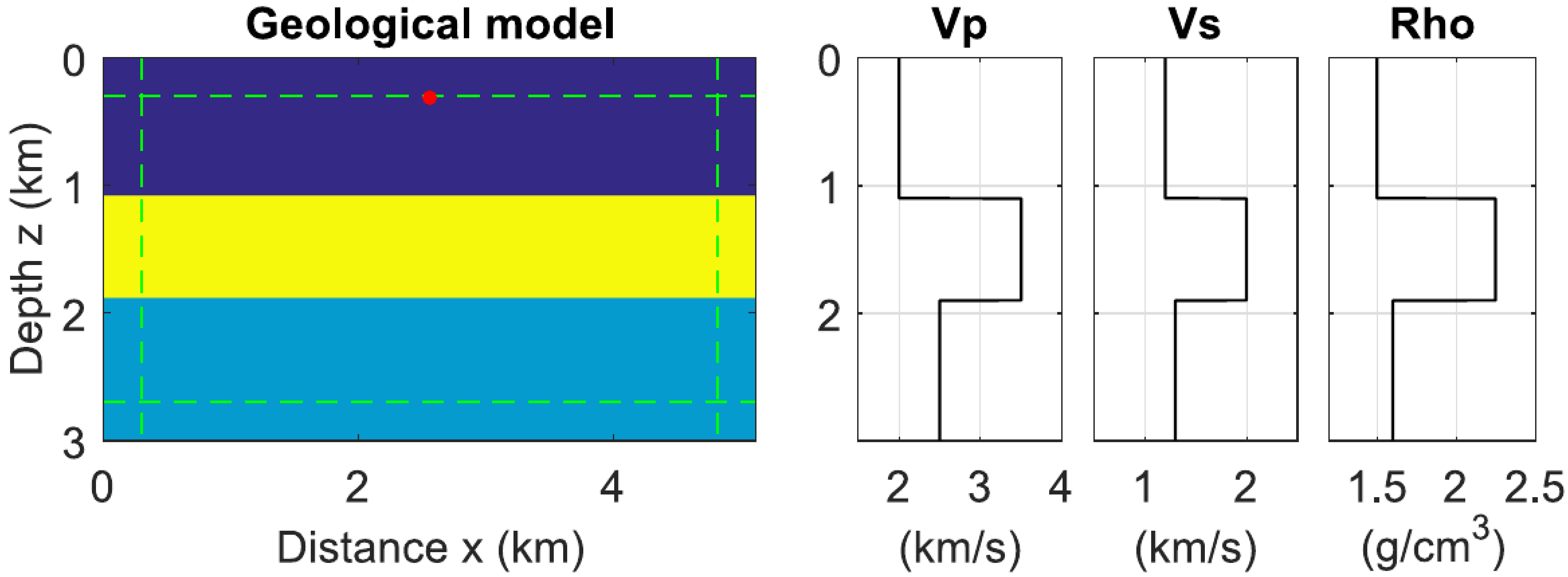
Monotonicity condition

$$b_3^{ij}(p_g,\omega) = -\int_{-\infty}^{+\infty} \mathrm{d}\tau_2^{mn} e^{-\mathrm{i}\omega\tau_2^{mn}} b_1^{mn}(p_g,\tau_2^{mn}) \int_{\Upsilon(\tau_2^{mn}|\tau_3^{nj})+\epsilon}^{+\infty} \mathrm{d}\tau_3^{nj} e^{\mathrm{i}\omega\tau_3^{nj}} b_1^{nj}(p_g,\tau_3^{nj}) \times \int_{\Upsilon(\tau_2^{mn}|\tau_1^{im})+\epsilon}^{+\infty} \mathrm{d}\tau_1^{im} e^{\mathrm{i}\omega\tau_1^{im}} b_1^{im}(p_g,\tau_1^{im})$$

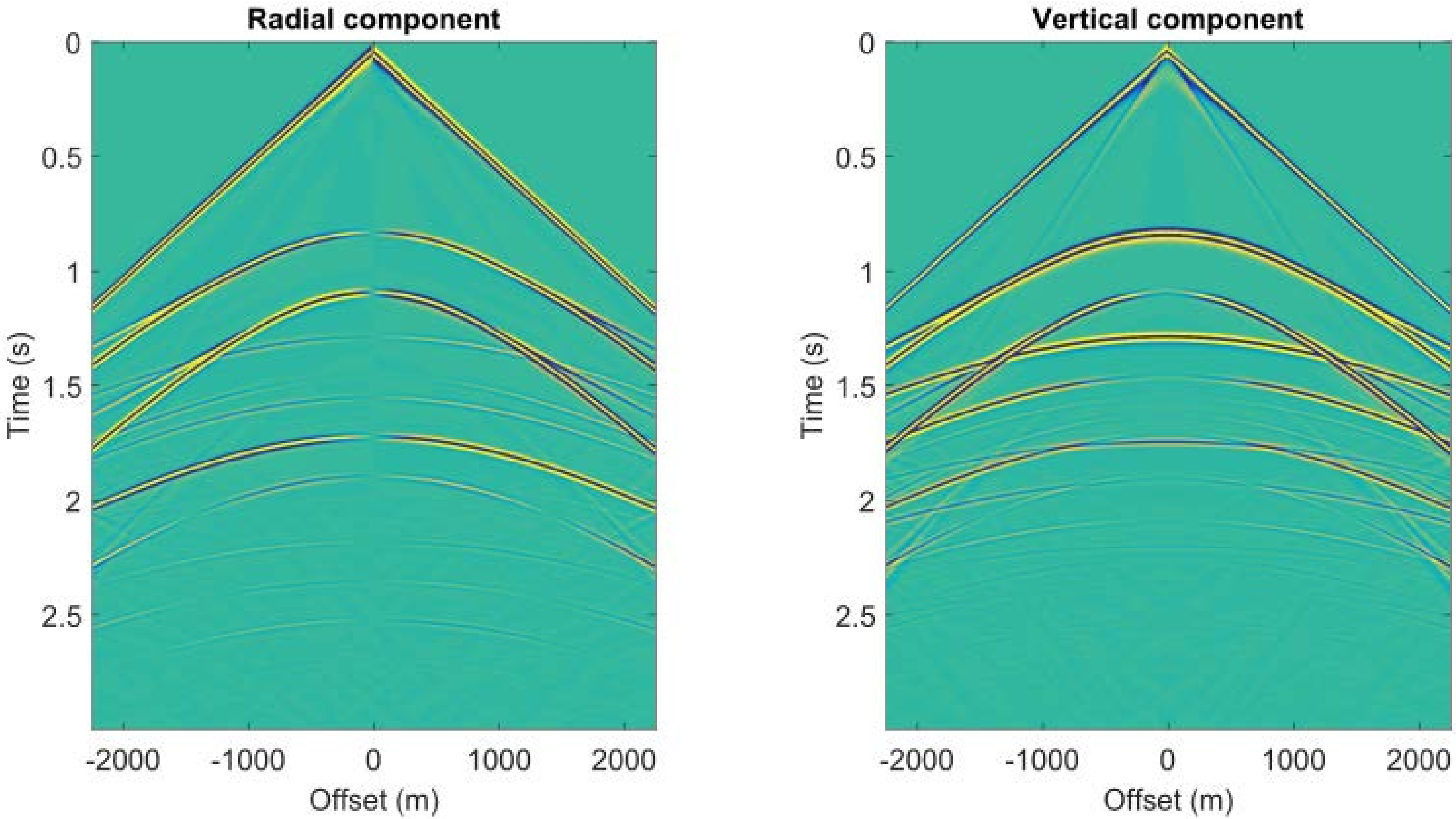
$$\Upsilon(\tau_2^{mn}|\tau_1^{nj}) = \begin{cases} \tau_2^{mn}, & j=m; \\ \frac{\alpha+\beta}{2\beta}\tau_2^{mn}, & j=S \;\&\; m=P; \\ \frac{2\beta}{\alpha+\beta}\tau_2^{mn}, & j=P \;\&\; m=S; \end{cases}$$

$$\Gamma(\tau_2^{mn}|\tau_1^{nj}) = \begin{cases} \tau_2^{mn}, & j=m \; or \; j=P \;\&\; m=S; \\ \frac{\alpha+\beta}{2\beta}\tau_2^{mn}, & j=S \;\&\; m=P; \end{cases}$$

Synthetic example



Synthetic example



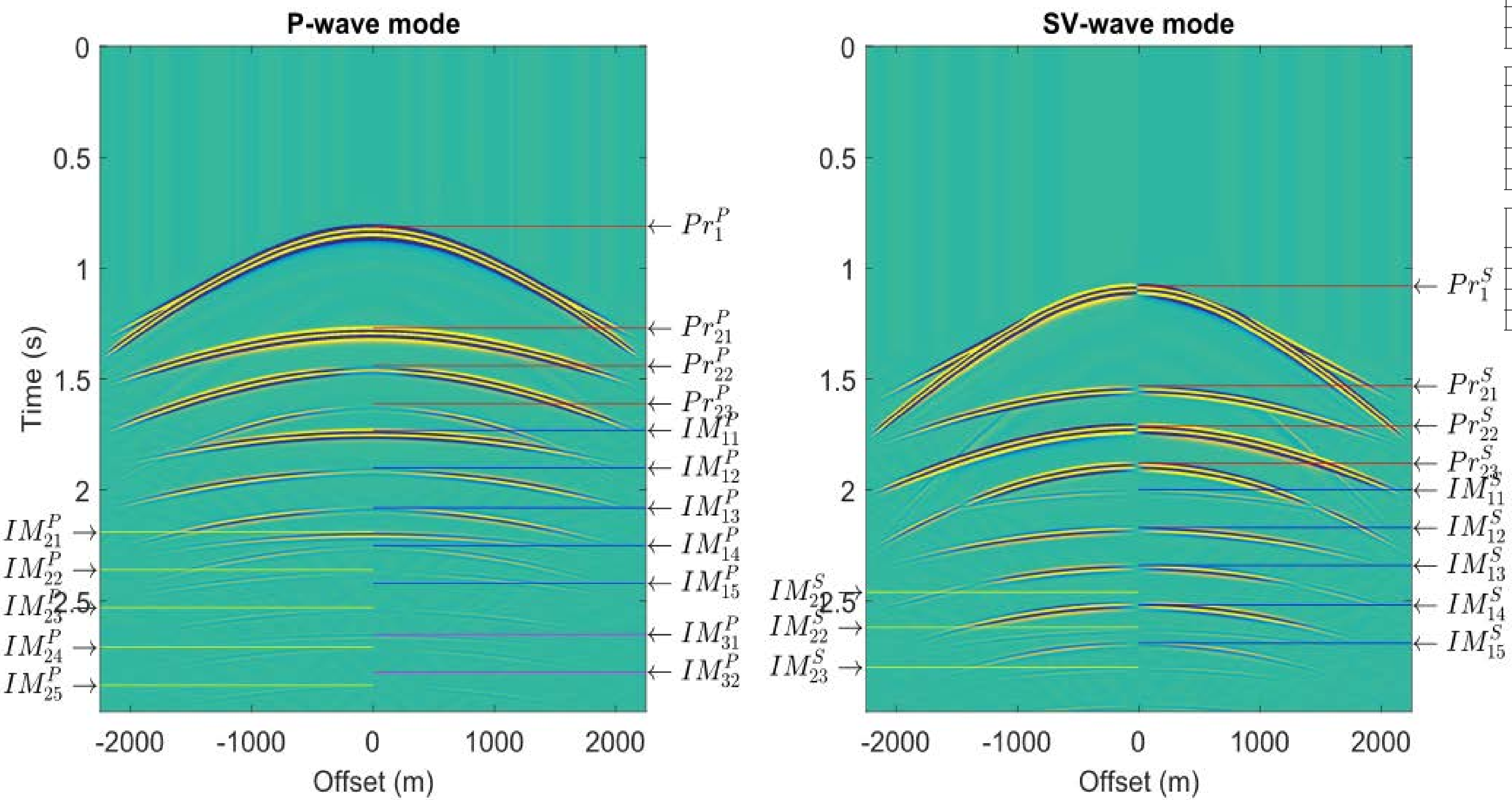
Synthetic example

Label-P	Primaries in P-mode
Pr_1^P	$\grave{P}\acute{P}$
Pr_{21}^P	$\grave{P}\grave{P}\acute{P}\acute{P}$
Pr_{22}^P	$\grave{P}\grave{P}\acute{S}\acute{P}$ & $\grave{P}\grave{S}\acute{P}\acute{P}$
Pr_{23}^P	$\grave{P}\grave{S}\acute{S}\acute{P}$

Label-P	1st-order IMs in P-mode
IM_{11}^P	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{P}\acute{P}$
IM_{12}^P	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{S}\acute{P}$
IM_{13}^P	$\grave{P}\grave{P}\acute{P}\grave{S}\acute{S}\acute{P}$
IM_{14}^P	$\grave{P}\grave{P}\acute{S}\grave{S}\acute{S}\acute{P}$
IM_{15}^P	$\grave{P}\grave{S}\acute{S}\grave{S}\acute{S}\acute{P}$

Label-P	2nd-order IMs in P-mode
IM_{21}^P	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{P}\grave{P}\acute{P}\acute{P}$
IM_{22}^P	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{P}\grave{P}\acute{S}\acute{P}$
IM_{23}^P	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{P}\grave{S}\acute{S}\acute{P}$
IM_{24}^P	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{S}\grave{S}\acute{S}\acute{P}$
IM_{25}^P	$\grave{P}\grave{P}\acute{P}\grave{S}\acute{S}\grave{S}\acute{S}\acute{P}$

Label-P	3rd-order IMs in P-mode
IM_{31}^P	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{P}\grave{P}\acute{P}\grave{P}\acute{P}\acute{P}$
IM_{32}^P	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{P}\grave{P}\acute{P}\grave{P}\acute{S}\acute{P}$

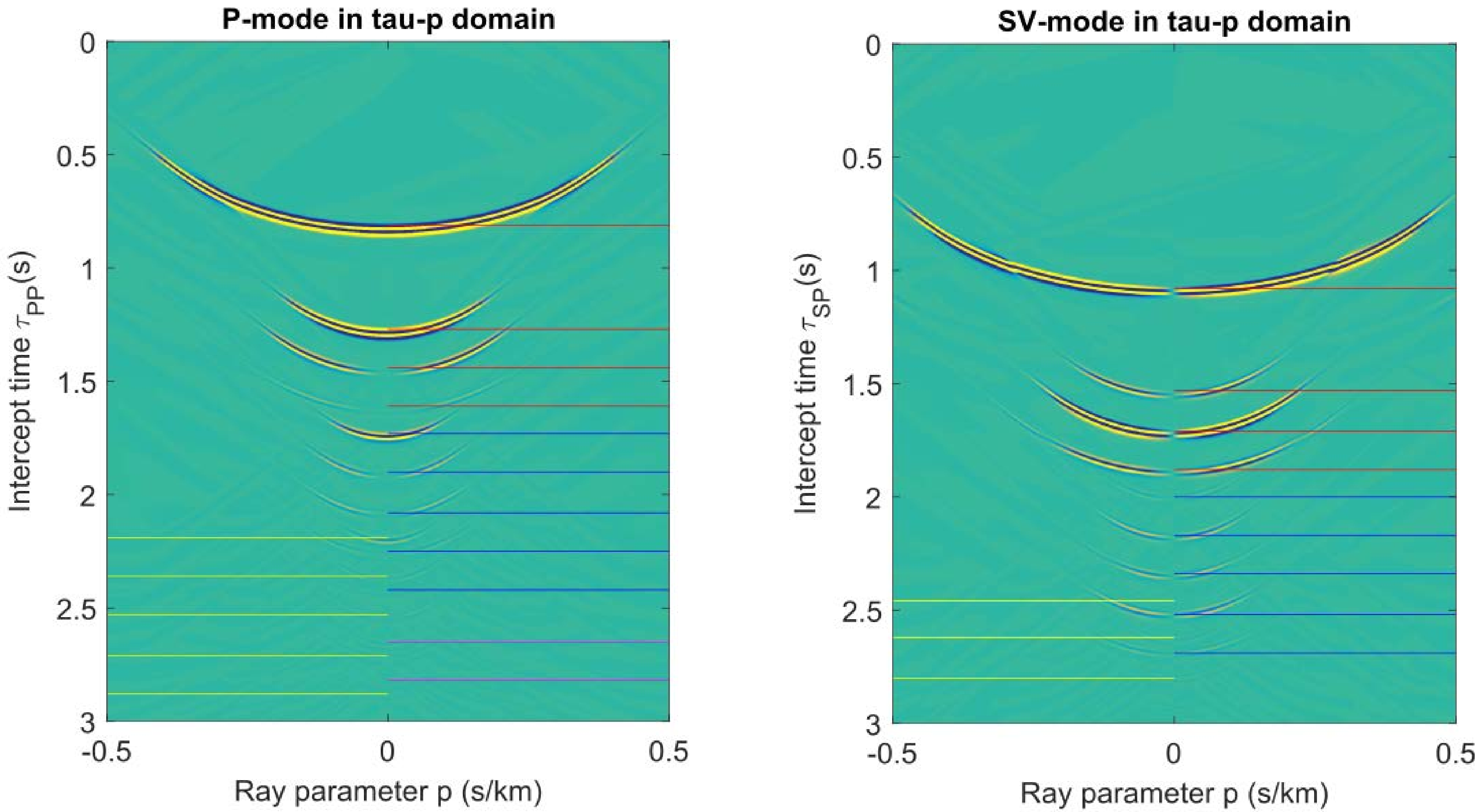


Label-S	Primaries S-mode
Pr_1^S	$\grave{P}\acute{S}$
Pr_{21}^S	$\grave{P}\grave{P}\acute{P}\acute{S}$
Pr_{22}^S	$\grave{P}\grave{P}\acute{S}\acute{S}$ & $\grave{P}\grave{S}\acute{P}\acute{S}$
Pr_{23}^S	$\grave{P}\grave{S}\acute{S}\acute{S}$

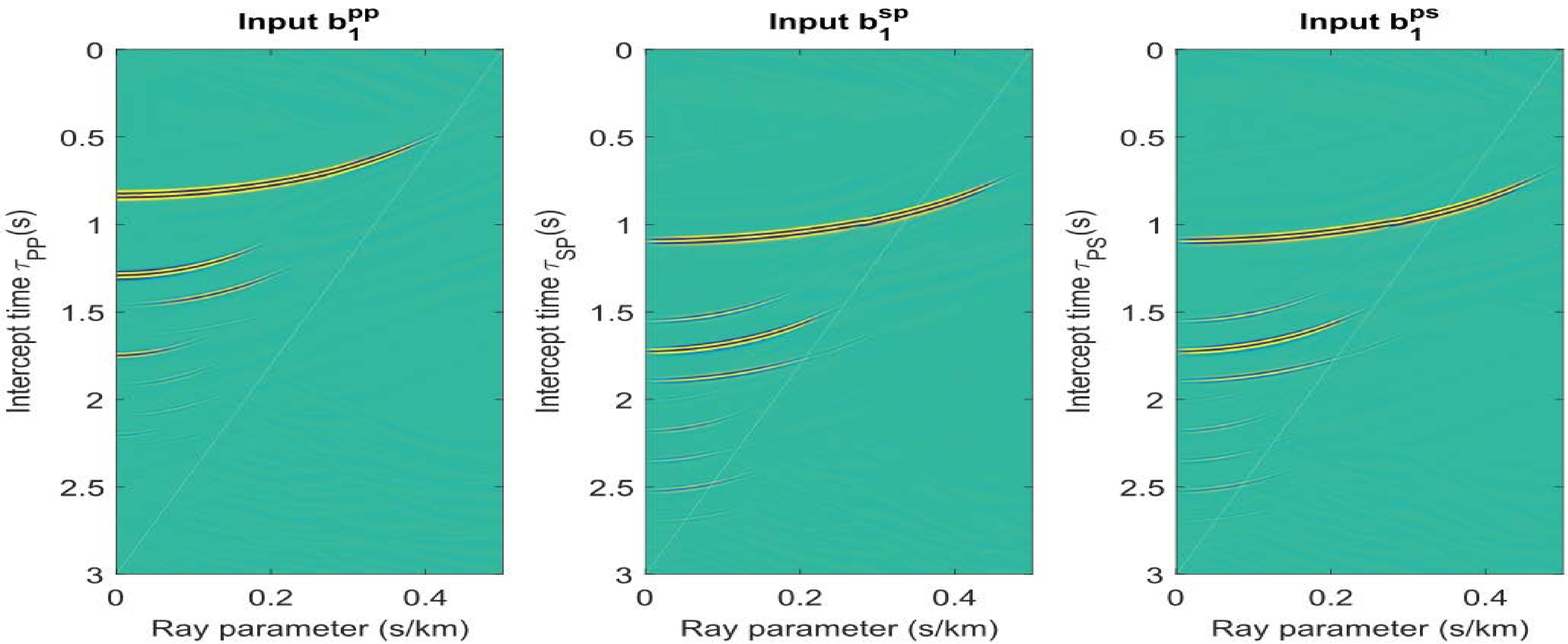
Label-S	1st-order IMs in S-mode
IM_{11}^S	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{P}\acute{S}$
IM_{12}^S	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{S}\acute{S}$
IM_{13}^S	$\grave{P}\grave{P}\acute{P}\grave{S}\acute{S}\acute{S}$
IM_{14}^S	$\grave{P}\grave{P}\acute{S}\grave{S}\acute{S}\acute{S}$
IM_{15}^S	$\grave{P}\grave{S}\acute{S}\grave{S}\acute{S}\acute{S}$

Label-S	2nd-order IMs in S-mode
IM_{21}^S	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{P}\grave{P}\acute{P}\acute{S}$
IM_{22}^S	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{P}\grave{P}\acute{S}\acute{S}$
IM_{23}^S	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{P}\grave{S}\acute{S}\acute{S}$

Synthetic example

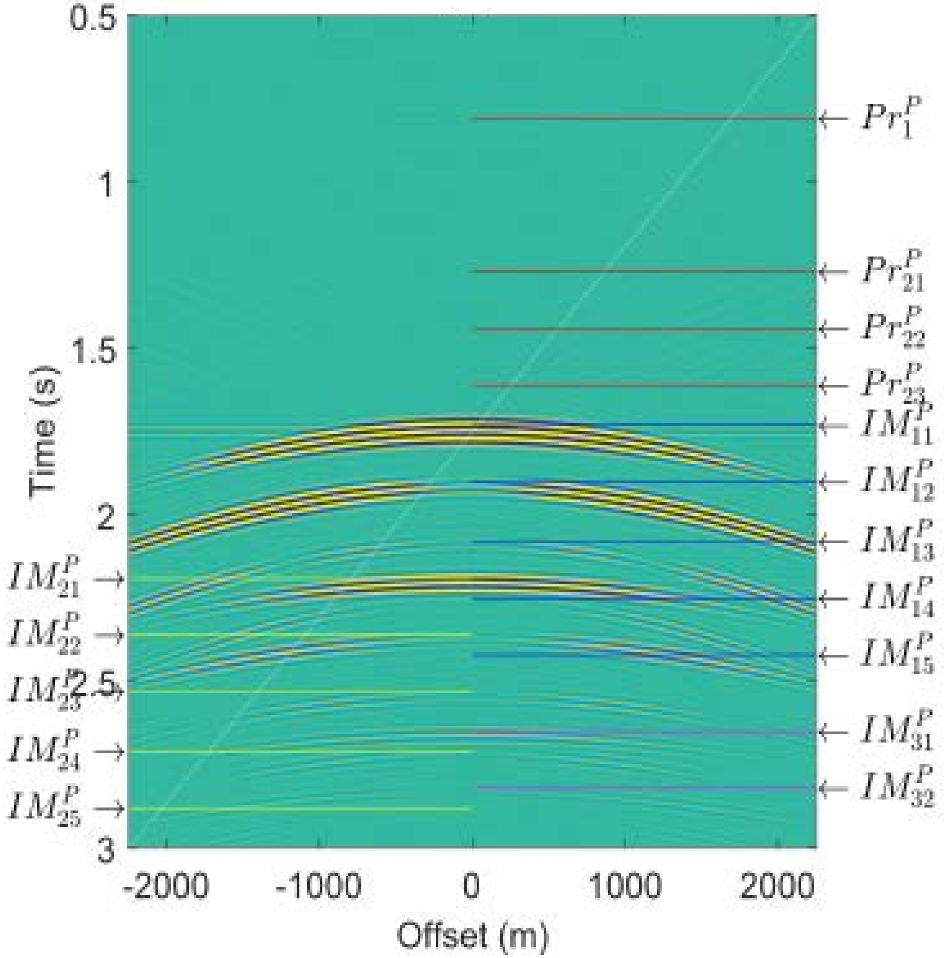


Synthetic example

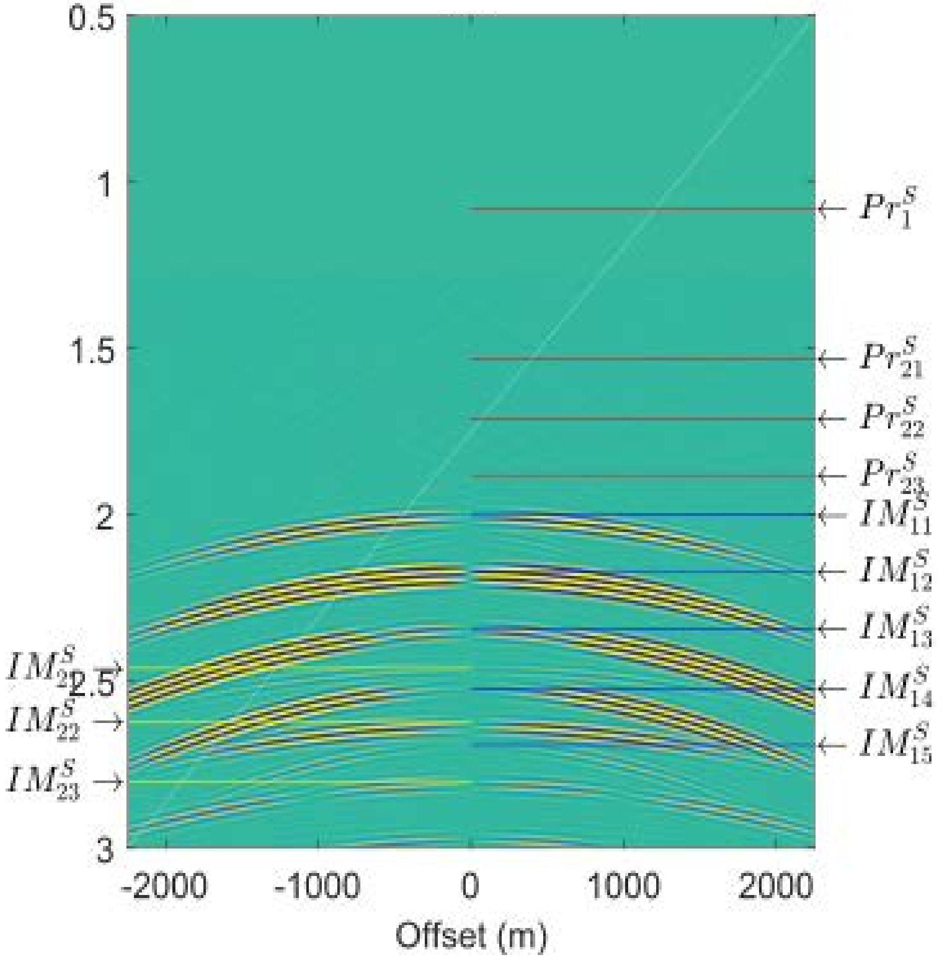


Synthetic example

EIMP in P-mode



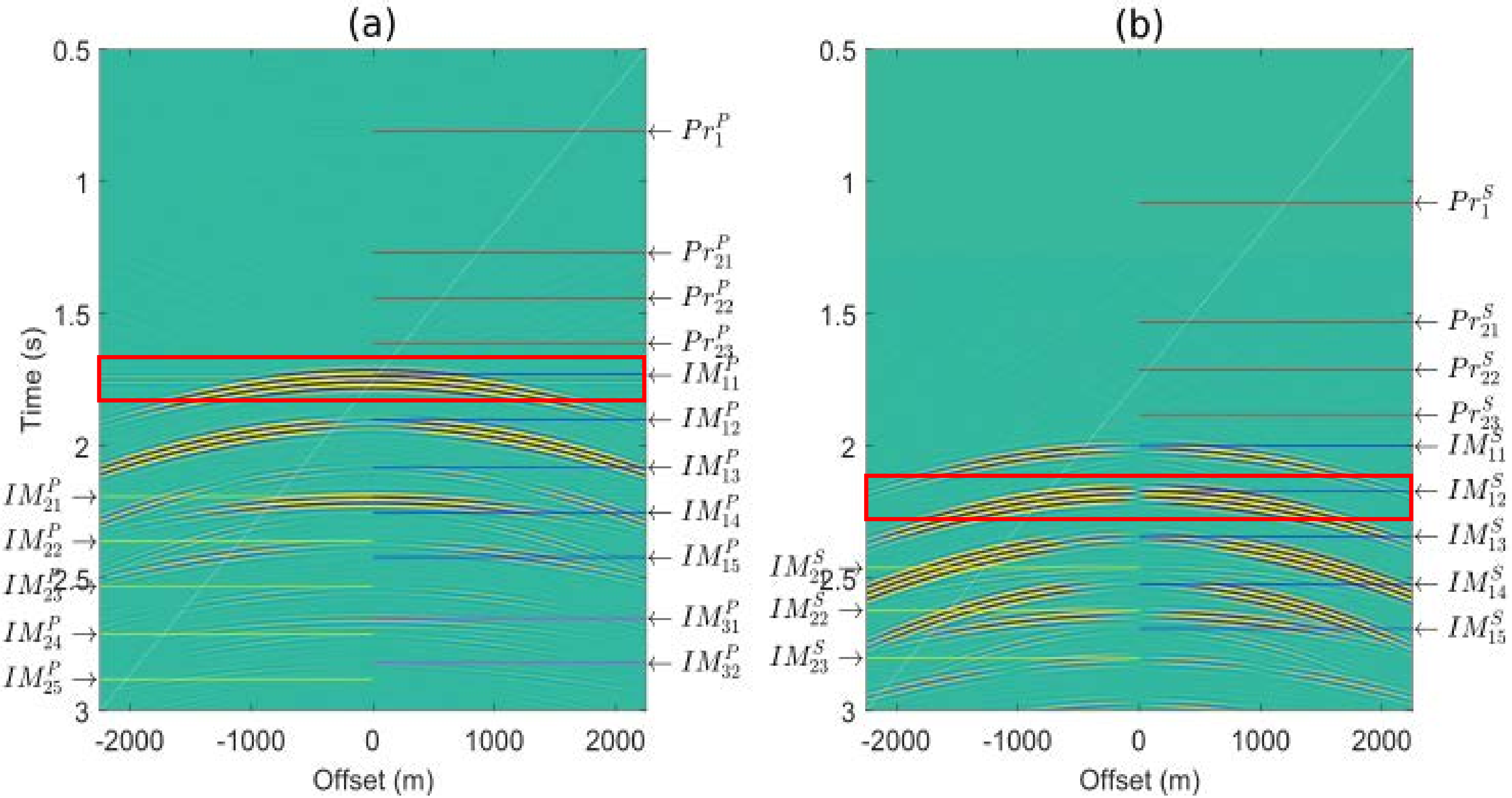
EIMP in SV-mode



Conclusion

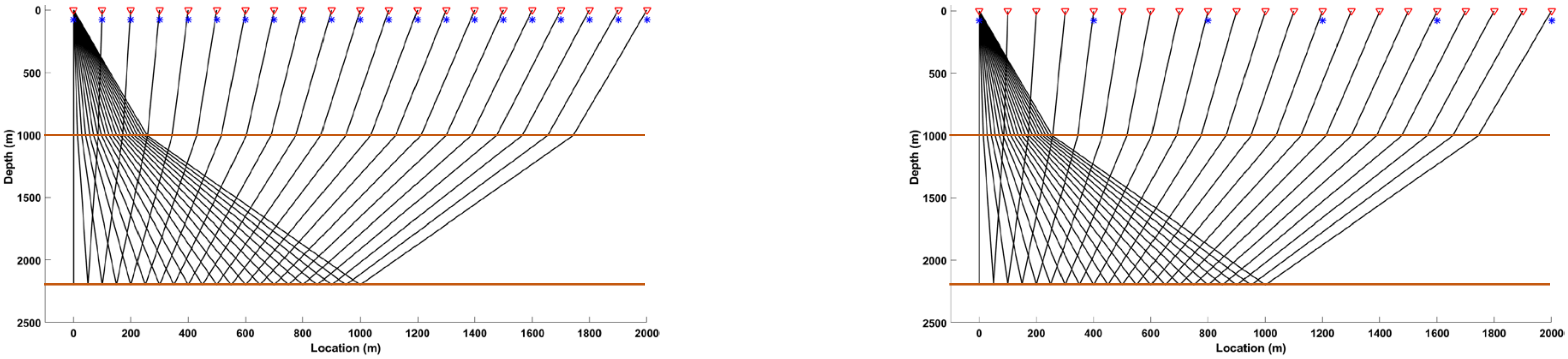
- Elastic internal multiple prediction algorithm is presented using inverse scattering series.
- We also discussed the monotonicity condition between pseudo-depth and intercept time.
- The algorithm can be implemented in several domains: $(\boldsymbol{k_g}, \boldsymbol{k_s}, \boldsymbol{z})$, $(\boldsymbol{k_g}, \boldsymbol{k_s}, \boldsymbol{t})$, $(\boldsymbol{p_g}, \boldsymbol{p_s}, \boldsymbol{z})$, $(\boldsymbol{p_g}, \boldsymbol{p_s}, \boldsymbol{\tau})$.
- Synthetic example is performed in $(\boldsymbol{p_g}, \boldsymbol{p_s}, \boldsymbol{\tau})$ domain, with several advantages:
  - ✓ More tractable computation with highly sparse input
  - ✓ Straight forward search parameter selection (relative stationary epsilon)
  - ✓ Reduced numerical noise at large offset (See in the companion paper)
  - ✓ Merging with high resolution radon transform to create input
  - ✓ Support time/tau domain formulation (Innanen, SEG 2016)

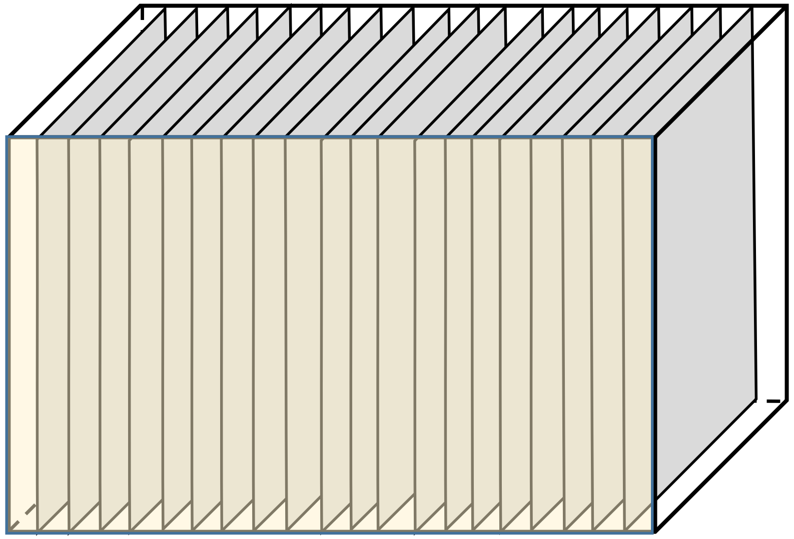
Future work

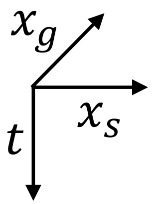


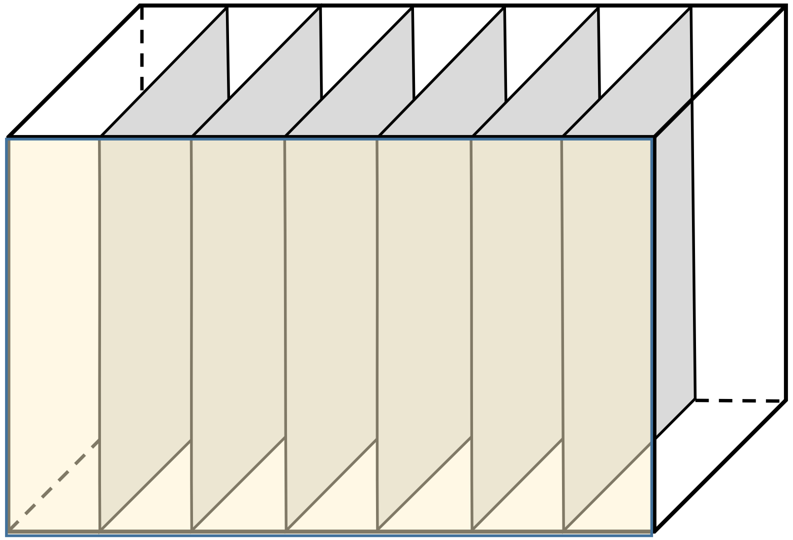
(a)
(b)
Time (s)
Offset (m)
0.5
1
1.5
2
2.5
3
-2000
-1000
0
1000
2000
Pr_1^P
Pr_{21}^P
Pr_{22}^P
Pr_{23}^P
IM_{11}^P
IM_{12}^P
IM_{13}^P
IM_{14}^P
IM_{15}^P
IM_{31}^P
IM_{32}^P
IM_{21}^P
IM_{22}^P
IM_{23}^P
IM_{24}^P
IM_{25}^P
Pr_1^S
Pr_{21}^S
Pr_{22}^S
Pr_{23}^S
IM_{11}^S
IM_{12}^S
IM_{13}^S
IM_{14}^S
IM_{15}^S
IM_{21}^S
IM_{22}^S
IM_{23}^S

Future work

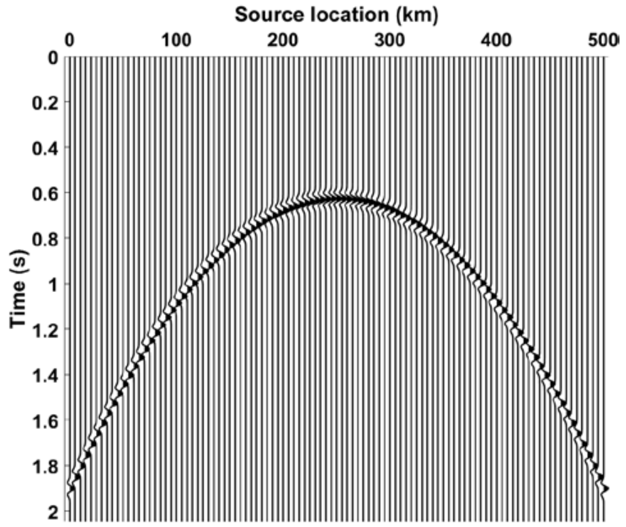

Depth (m)
Location (m)
0
500
1000
1500
2000
2500
0
200
400
600
800
1000
1200
1400
1600
1800
2000

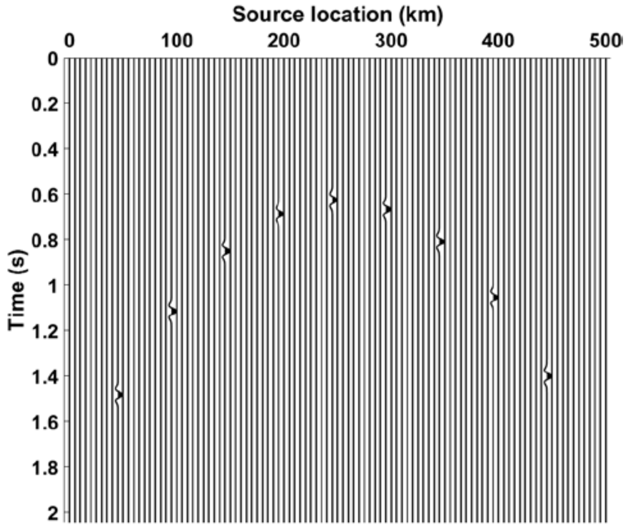


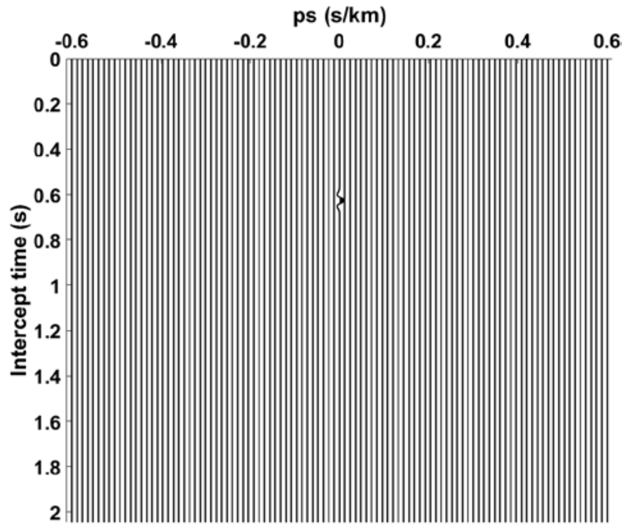

x_g
x_s
t



Future work


Source location (km)
0 100 200 300 400 500
Time (s)
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2


Source location (km)
0 100 200 300 400 500
Time (s)
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2


ps (s/km)
-0.6 -0.4 -0.2 0 0.2 0.4 0.6
Intercept time (s)
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2

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