PSTD wave field simulation and gradient calculations for anisotropic FWI

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- 2 3D staggered grid Fourier pseudospectral time-domain (PSTD) for SH simulation
- 3 Gradient calculation in TTI media
- 4 3D staggered-grid FD forward simulation with random boundary condition
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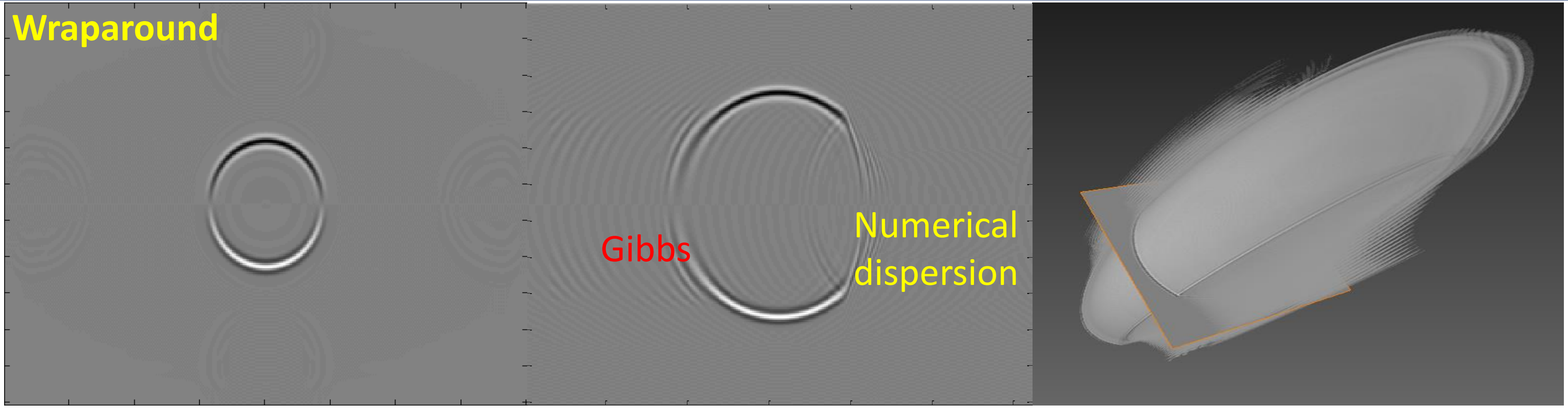
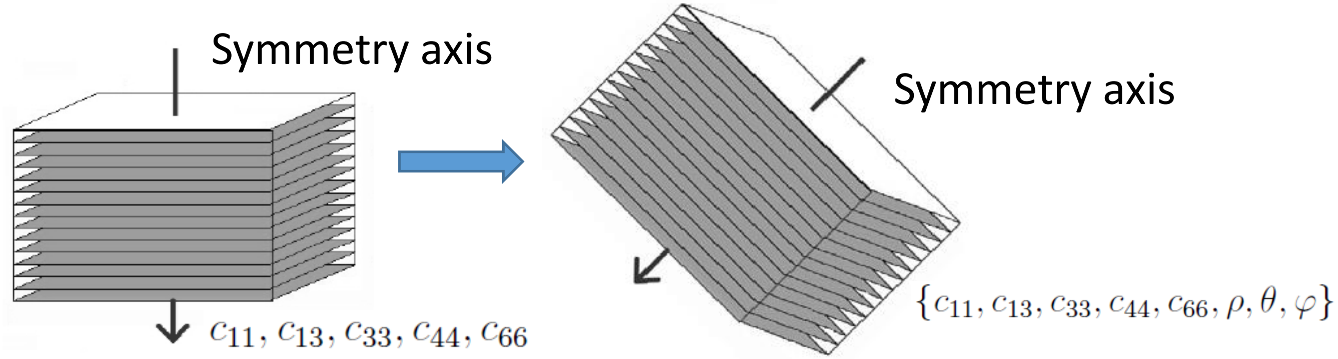
Introduction

- Conventional PSTD:



- Solution: Staggered-grid, first order SH
- Gradient calculation in TTI:



- Random boundary layer for staggered-grid FD

First Order SH equation

Second order SH equation:

$$c_{66}\nabla_2^2\chi + c_{44}\frac{\partial^2\chi}{\partial z^2} + \rho\omega^2\chi = 0$$

~~PML~~

Suppose: $\mathbf{v} = (v_x, v_y, v_z)$, $\mathbf{X} = (\chi_x, \chi_y, \chi_z)$

First order SH:

$$\frac{\partial \mathbf{v}}{\partial t} = \mathbf{AX}$$

$$\frac{\partial \mathbf{X}}{\partial t} = \mathbf{Bv}$$

$$\mathbf{A} = \begin{bmatrix} -\frac{c_{66}}{\rho}\frac{\partial}{\partial x} & -\frac{c_{66}}{\rho}\frac{\partial}{\partial x} & -\frac{c_{66}}{\rho}\frac{\partial}{\partial x} \\ -\frac{c_{66}}{\rho}\frac{\partial}{\partial y} & -\frac{c_{66}}{\rho}\frac{\partial}{\partial y} & -\frac{c_{66}}{\rho}\frac{\partial}{\partial y} \\ -\frac{c_{44}}{\rho}\frac{\partial}{\partial z} & -\frac{c_{44}}{\rho}\frac{\partial}{\partial z} & -\frac{c_{44}}{\rho}\frac{\partial}{\partial z} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} -\frac{\partial}{\partial x} & 0 & 0 \\ 0 & -\frac{\partial}{\partial y} & 0 \\ 0 & 0 & -\frac{\partial}{\partial z} \end{bmatrix}$$

Staggered-grid Fourier pseudospectral derivatives

First-order Fourier derivative:

$$\mathcal{D}_x u(x_i) = \mathcal{DFT}^{-1}\left[-jk_x \mathcal{DFT}\left(u(x_i)\right)\right]$$

heterogeneity

Özdenvar and McMechan (1996)

Staggered grid first-order derivative:

$$\mathcal{D}_x^{\pm} u(x_{i\pm\frac{1}{2}}) = \mathcal{DFT}^{-1}\left[-jk_x exp\left(\frac{\mp jk_x \triangle x}{2}\right)\mathcal{DFT}\left(u(x_i)\right)\right]$$

$$\mathcal{D}_{xm}^{\pm} u(x_{i\pm\frac{1}{2}}) = \mathcal{DFT}^{-1}\left[(-jk_x)^m exp\left(\frac{\mp jk_x \triangle x}{2}\right)\mathcal{DFT}\left(u(x_i)\right)\right]$$

high-order

$$\mathcal{D}_{xm} u(x_i) = \mathcal{DFT}^{-1}\left[(-jk_x)^m \mathcal{DFT}\left(u(x_i)\right)\right]$$

$$\frac{\partial \mathbf{v}}{\partial t} = \mathbf{AX}$$

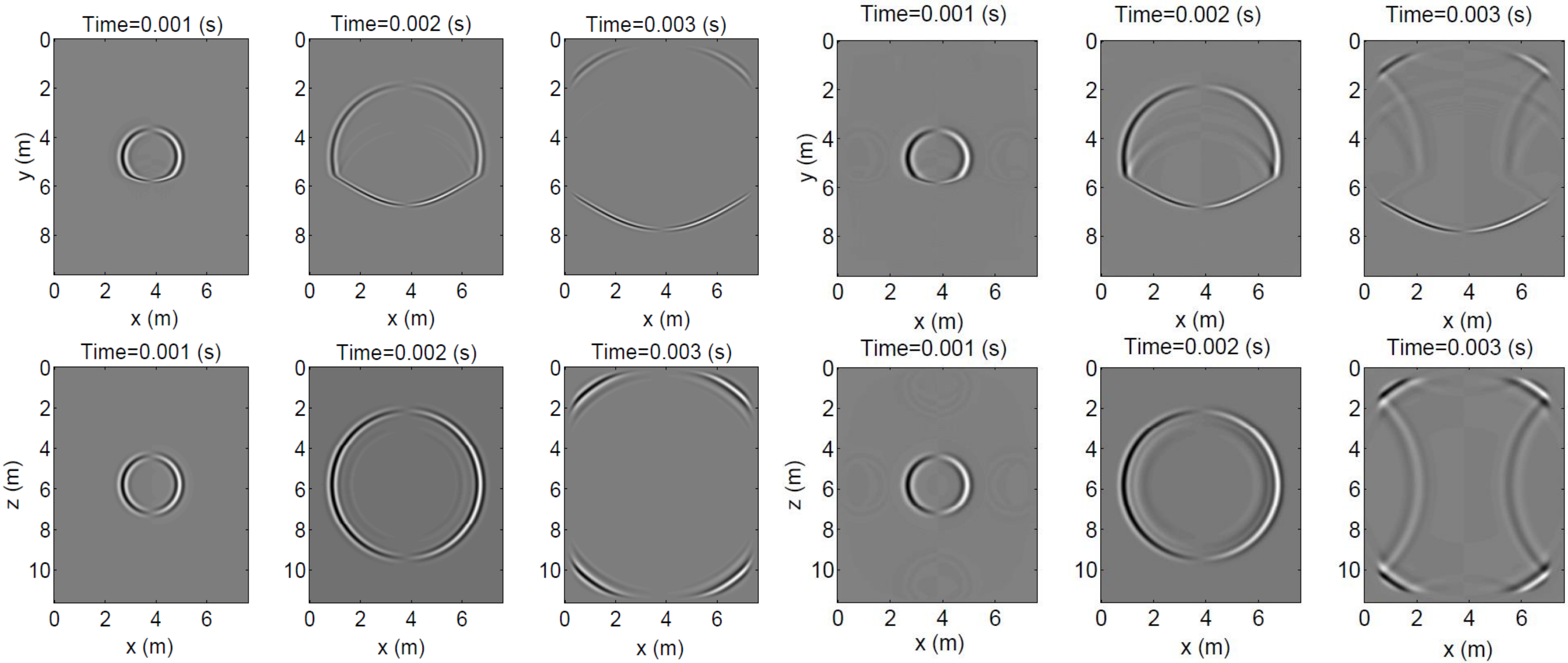
$$\frac{\partial \mathbf{X}}{\partial t} = \mathbf{Bv}$$

$$\mathbf{v}\left(t + \tfrac{1}{2}\triangle t\right) - \mathbf{v}\left(t - \tfrac{1}{2}\triangle t\right) \approx \left(\Delta t\mathbf{A} + \tfrac{1}{24}\Delta t^3\mathbf{ABA}\right)\mathbf{X}(t)$$

$$\mathbf{X}(t+\Delta t) - \mathbf{X}(t) \approx \left(\Delta t\mathbf{B} + \tfrac{1}{24}\Delta t^3\mathbf{BAB}\right)\mathbf{v}\left(t + \tfrac{1}{2}\triangle t\right)$$

Stability Relation: $\Delta t v_{so}/\Delta x \leq \sqrt{3}/\pi$

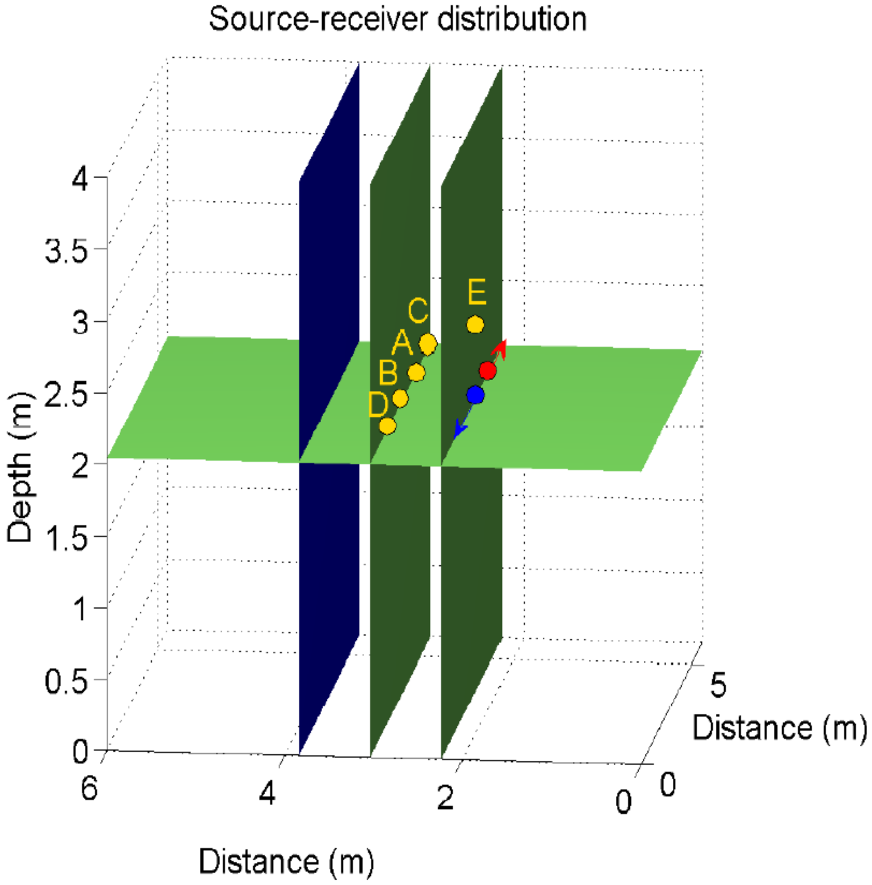
Snapshots for SH propagation

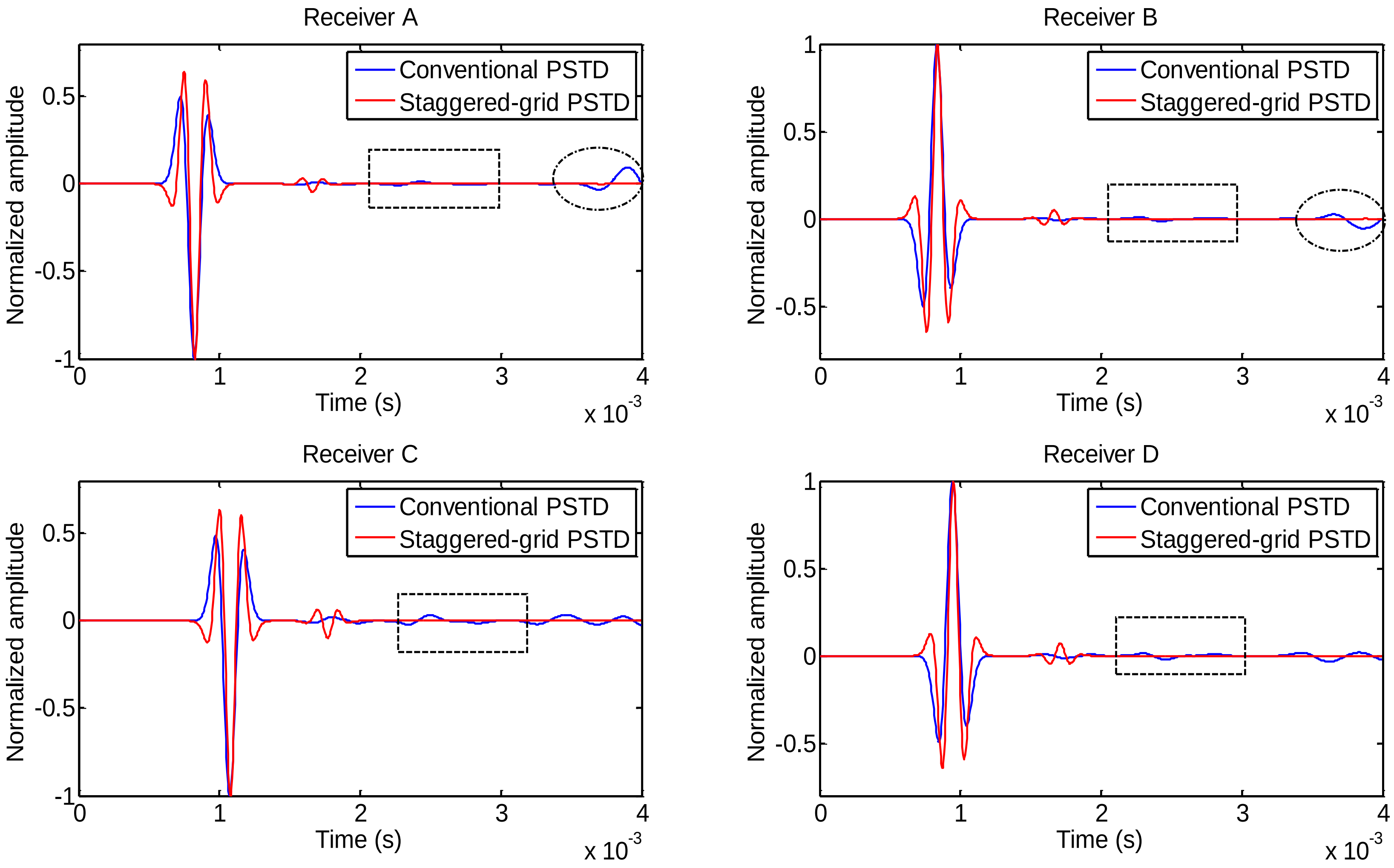


PSTD for First-order SH

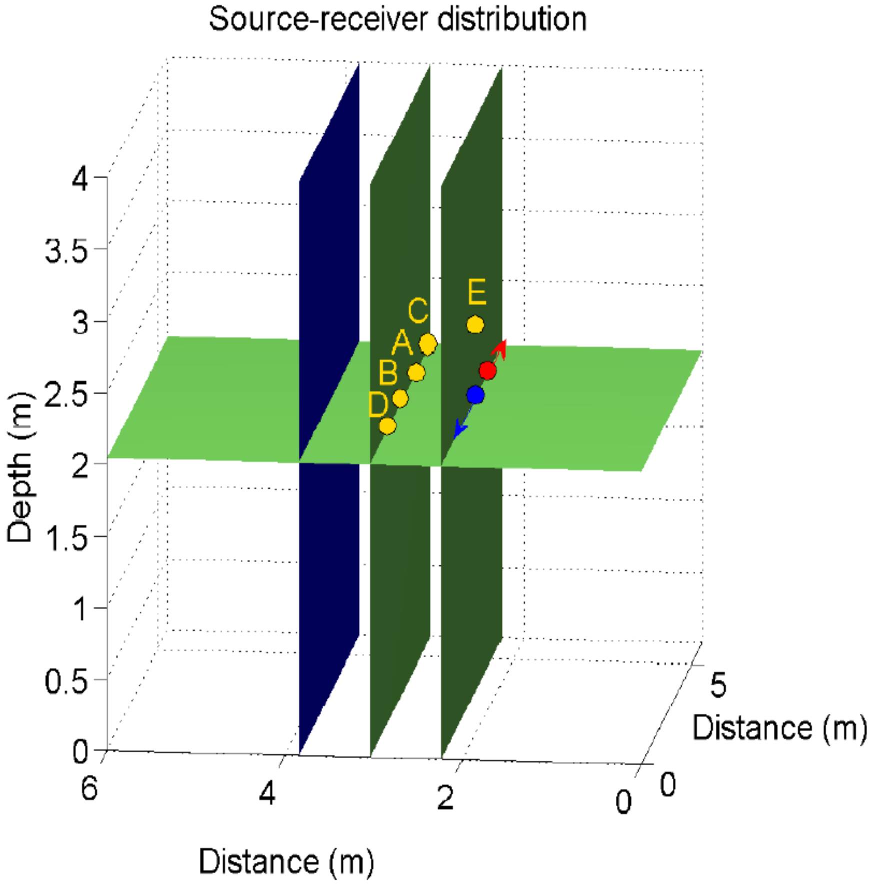
Conventional PSTD for second-order SH

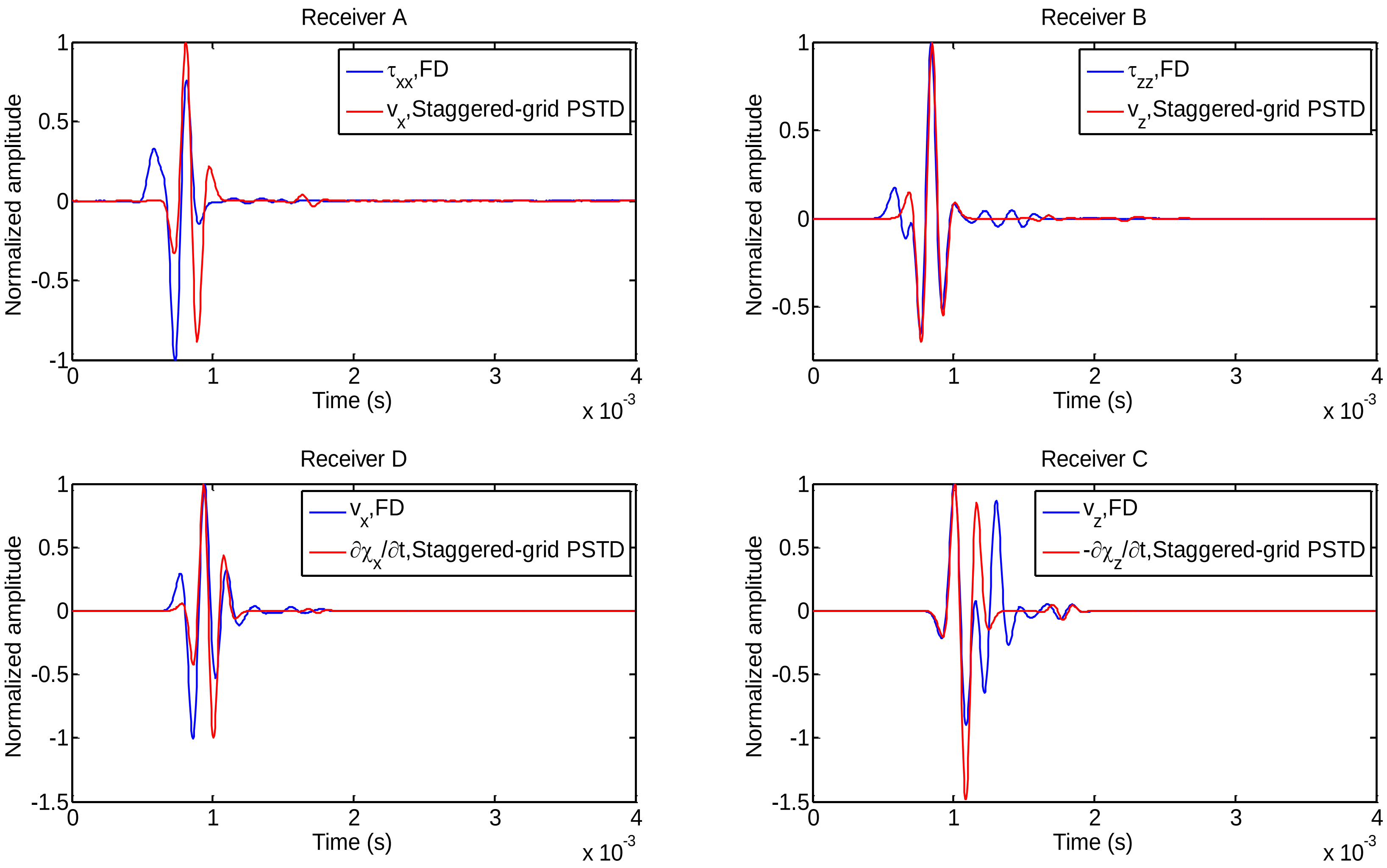
Comparisons with FD and conventional PSTD



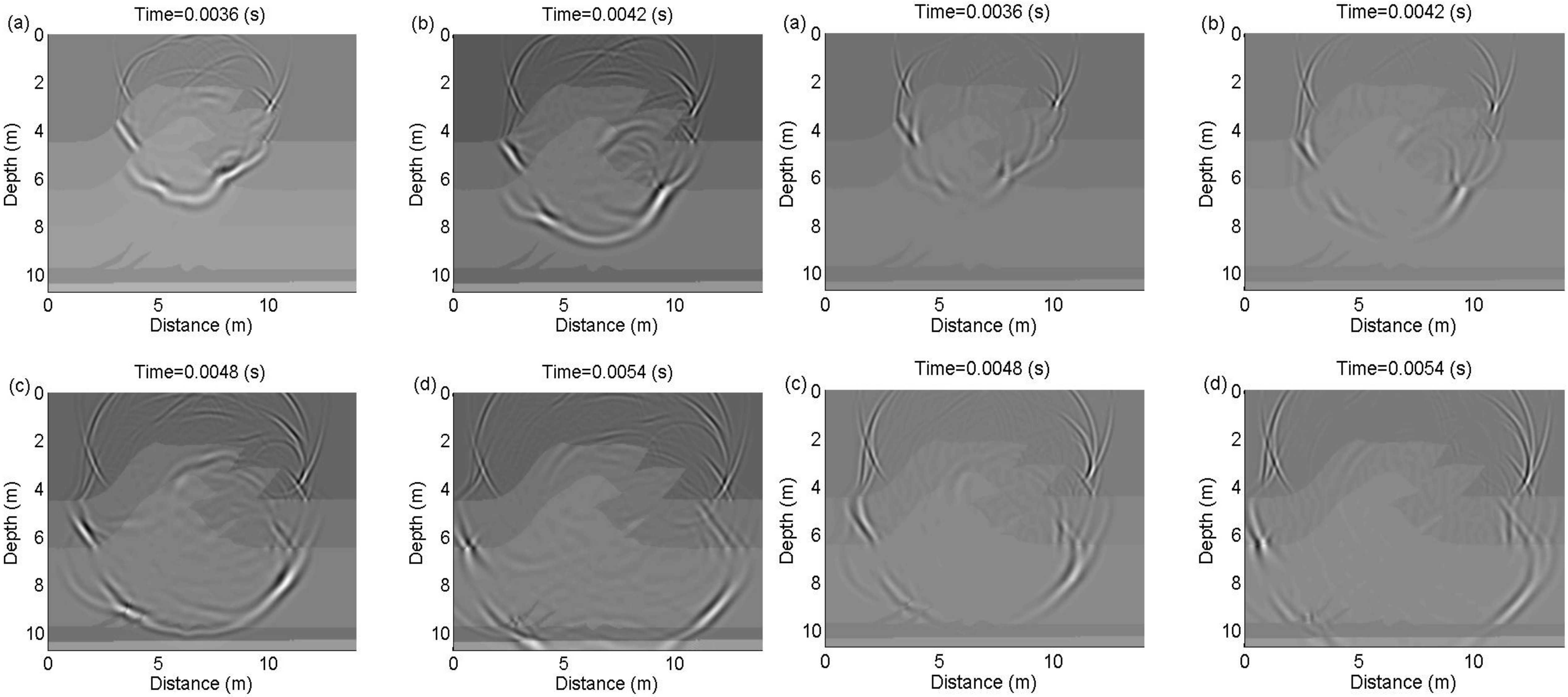


Comparisons with FD and conventional PSTD





Snapshots for SH propagation in thrust fault model



Advantages for PSTD in FWI

$$\Delta t v_{so}/\Delta x \leq \sqrt{3}/\pi$$

Larger space and time interval than FD

$$\frac{\partial \mathbf{v}}{\partial t} = \mathbf{AX}$$

$$\frac{\partial \mathbf{X}}{\partial t} = \mathbf{Bv}$$

Displacement component can be directly obtained

By using separated qP-, qSV- and SH- wavefield simulations, the nonlinearity of inversion and the crosstalk between parameters can be greatly reduced.

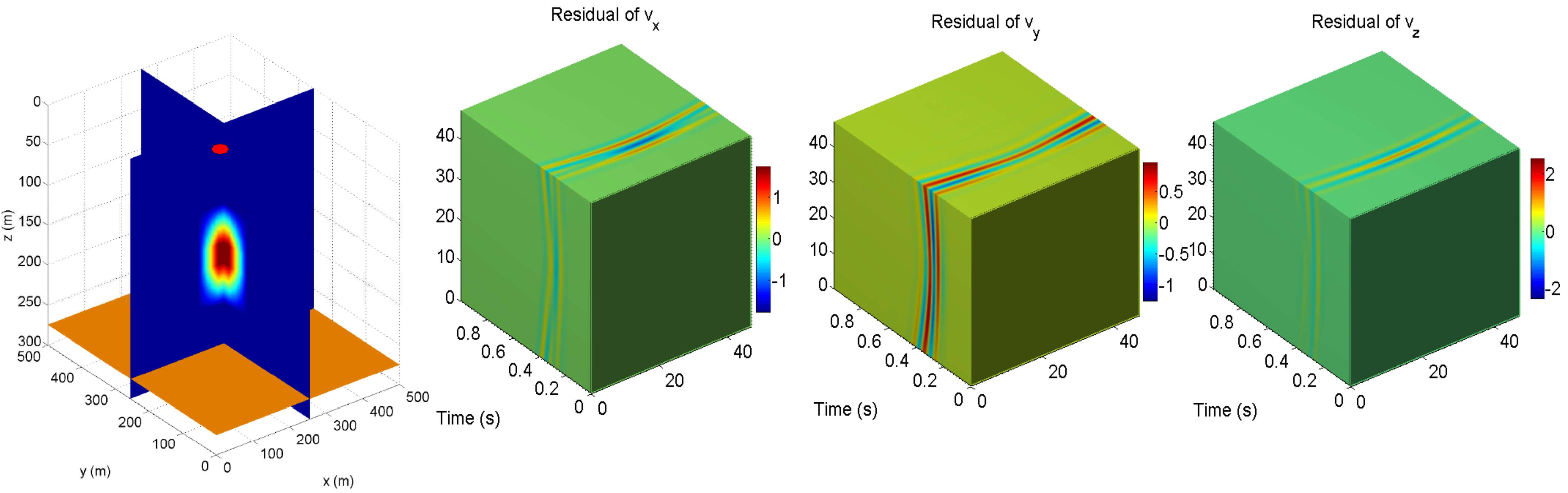
Gradient calculation for anisotropic (TTI) FWI

$$K_{m_n} = -\sum_{ijkl} K_{ijkl} \frac{\partial c_{ijkl}}{\partial m_n} dt \qquad K_{c_{ijkl}} = -\sum_{s=1}^{S}\sum_{r=1}^{R}\int \frac{\partial \delta u_i}{\partial x_j}\frac{\partial G_k}{\partial x_l} dt$$

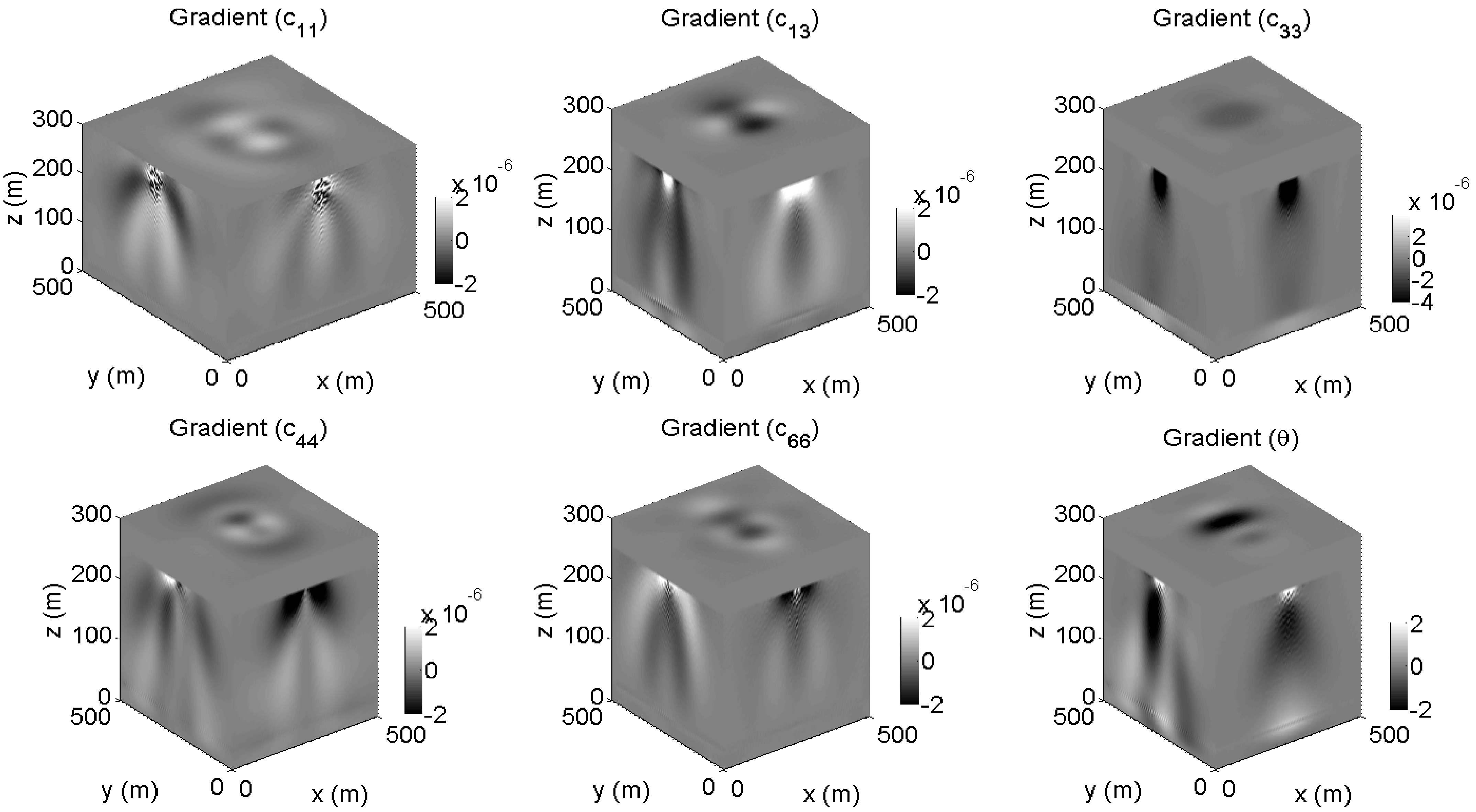
$$\begin{bmatrix} c_{11} & c_{11}-2c_{66} & c_{13} & & & \\ c_{11}-2c_{66} & c_{11} & c_{13} & & & \\ c_{13} & c_{13} & c_{33} & & & \\ & & & c_{44} & & \\ & & & & c_{44} & \\ & & & & & c_{66} \end{bmatrix} \longrightarrow \begin{bmatrix} c_{11}' & c_{12}' & c_{13}' & & c_{15}' & \\ c_{12}' & c_{22}' & c_{23}' & & c_{25}' & \\ c_{13}' & c_{23}' & c_{33}' & & c_{35}' & \\ & & & c_{44}' & & c_{46}' \\ c_{15}' & c_{25}' & c_{35}' & & c_{55}' & \\ & & & c_{46}' & & c_{66}' \end{bmatrix}$$

$$\boxed{C' = M_{\theta^0} C^0 M_{\theta^0}{}^T}$$

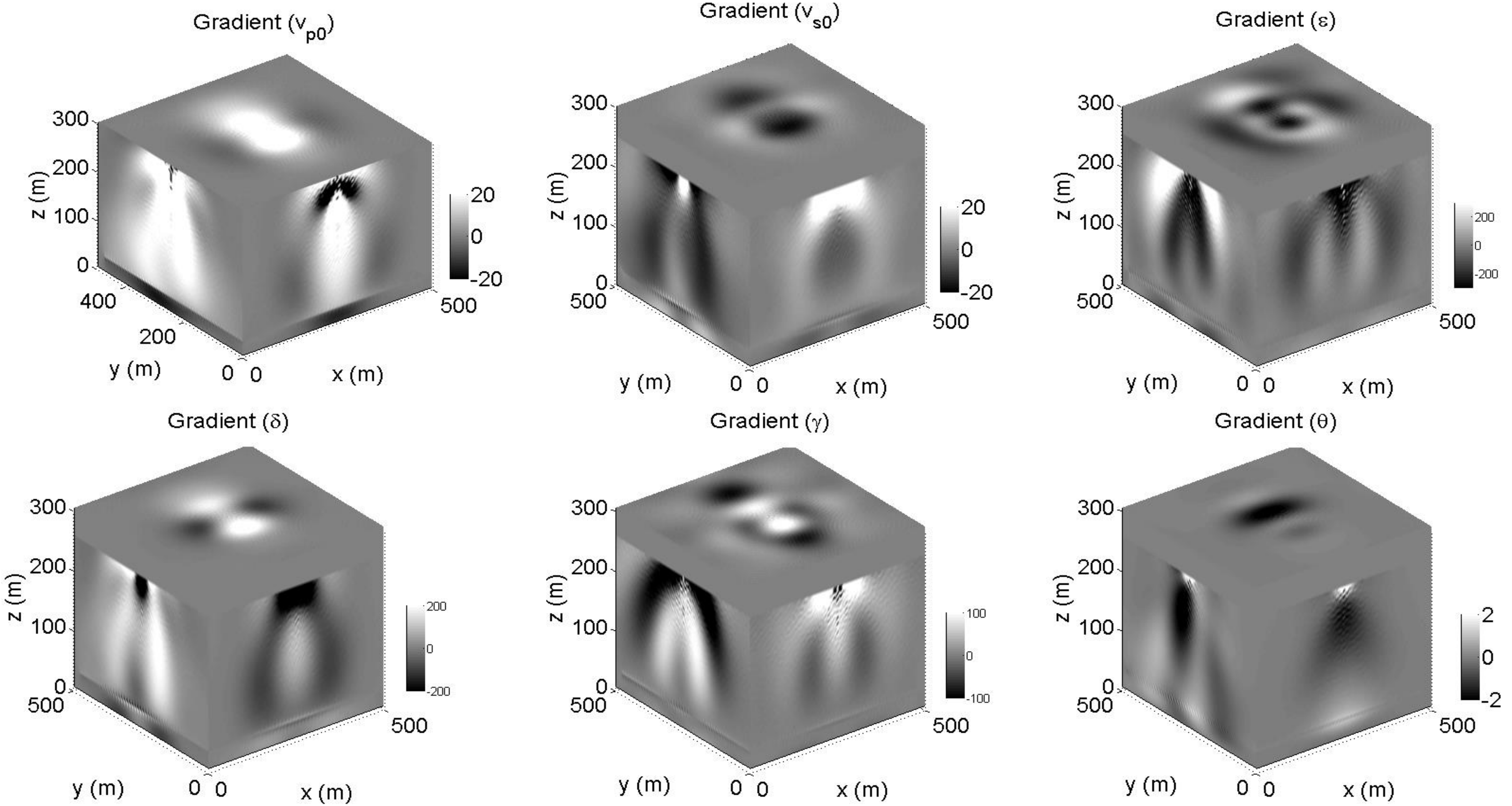
Residuals of Gaussian anomaly model



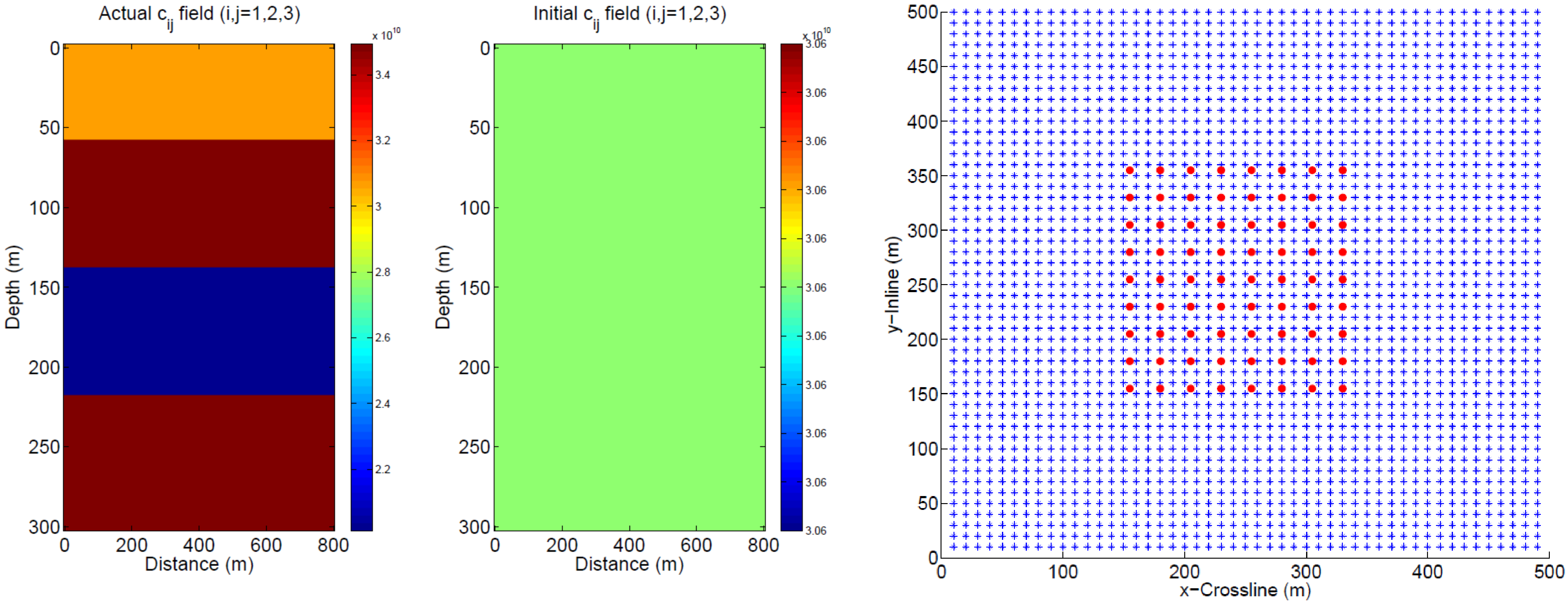
Gradients of constitutive elastic moduli and polar angle



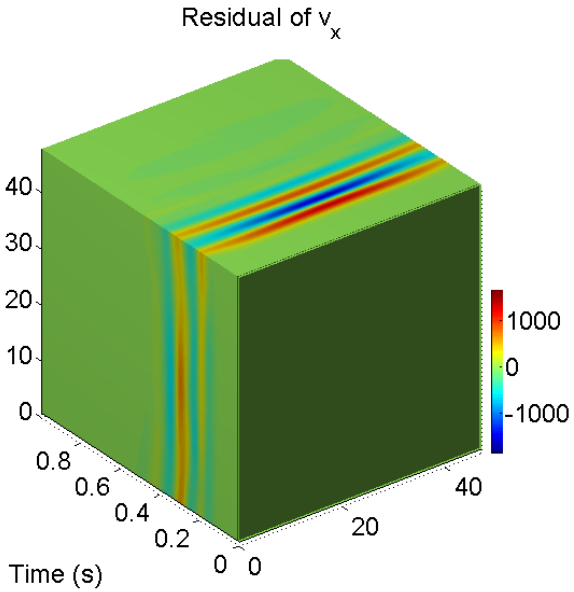
Gradients of Thomsen parameters and polar angle

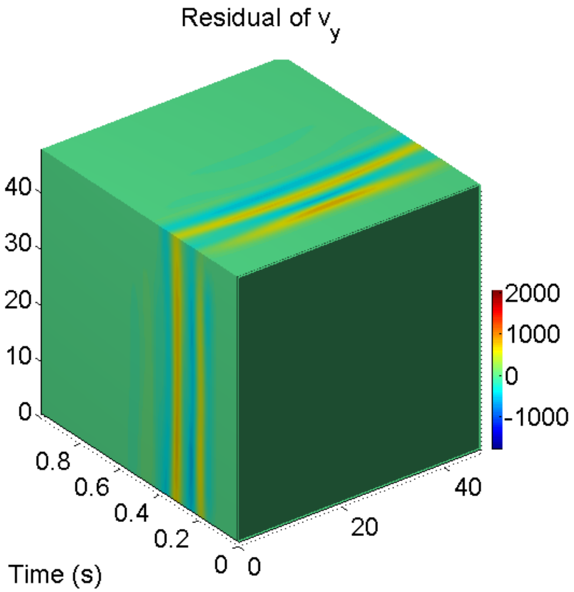


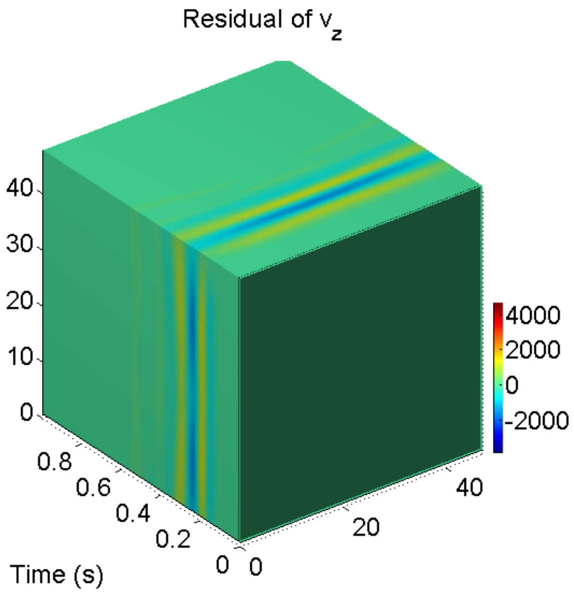
Cross-section and source receiver distribution



Residuals and elastic moduli of 3D layered model

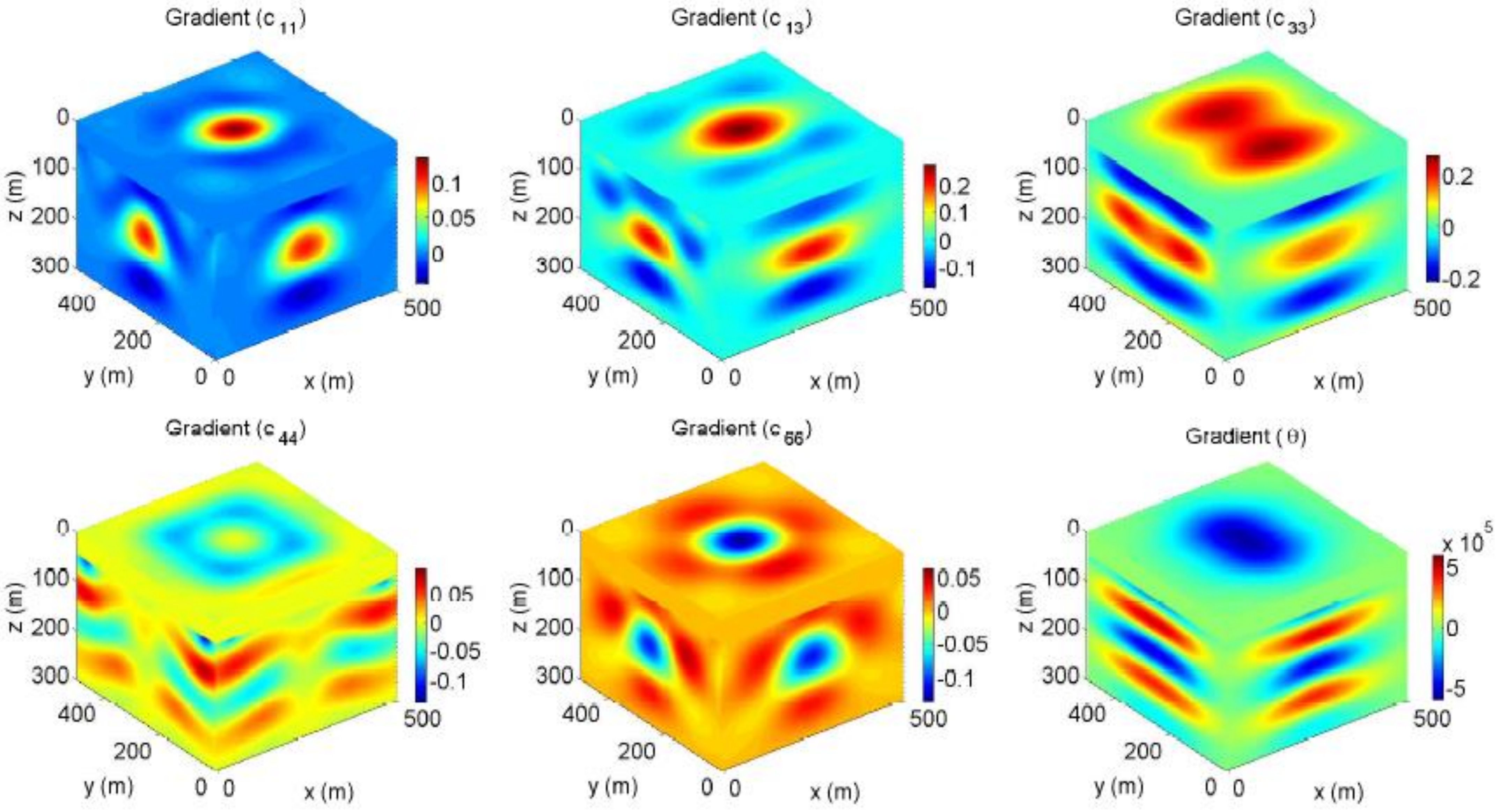




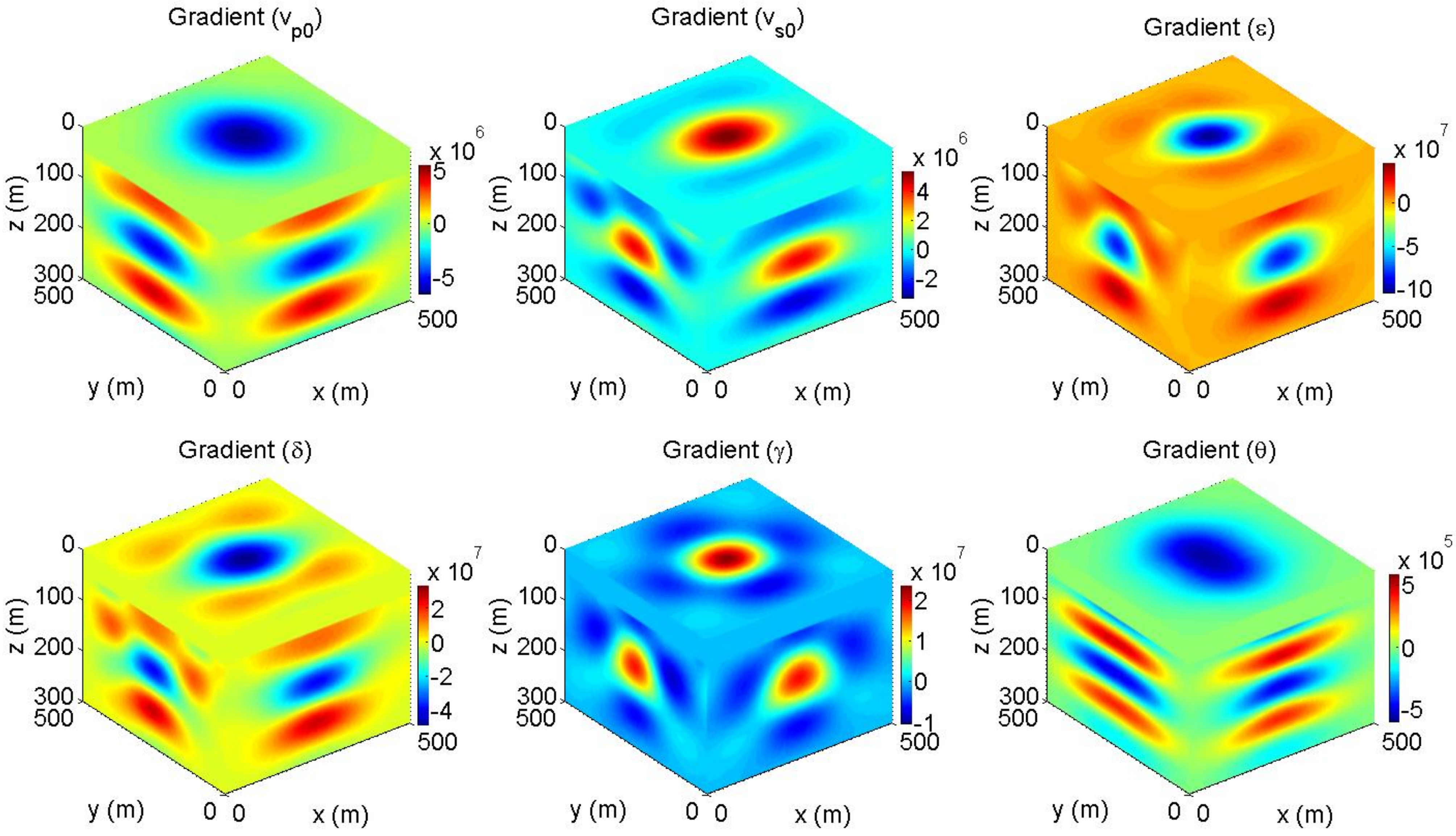


	First layer	Second layer	Third layer	Fourth layer
c_{13}	10.625	5.2	11.39	12.81
c_{22}	30.6	18	23.87	40
c_{23}	10.625	8	11.18	21.25
c_{33}	30.6	16.2	15.86	38.4
c_{44}	10	4.85	3.145	9
c_{55}	10	4.3	4.371	11.81
c_{66}	10	4.85	3.895	11
c_{15}	0	1.5	2.77	1.62
c_{25}	0	1.6	2.40	4.76
c_{35}	0	1.5	0.925	1.41
c_{46}	0	0.65	0.650	-1.73

Gradients of constitutive elastic moduli and polar angle



Gradients of Thomsen parameters and polar angle



Discussion

Gradient calculation using random boundary

Forward wavefield propagation for model data generation without wavefield recording

Forward wavefield propagation using random boundary, recording last tow time slices of source wavefield

Reverse time propagation of residual data and source wavefield using random boundary

Gradient calculation using PML

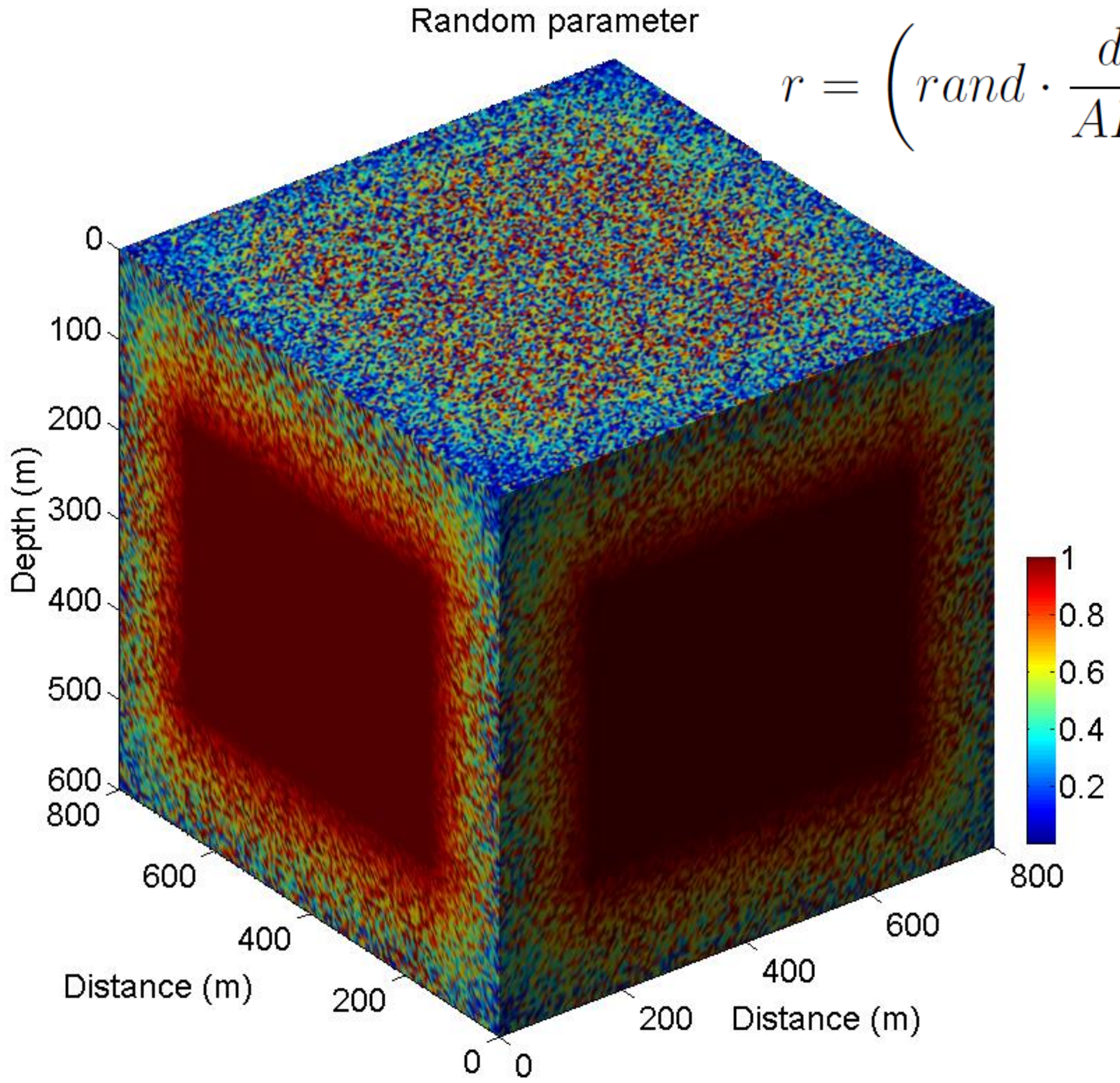
Forward wavefield propagation for model data generation

Record source wavefield

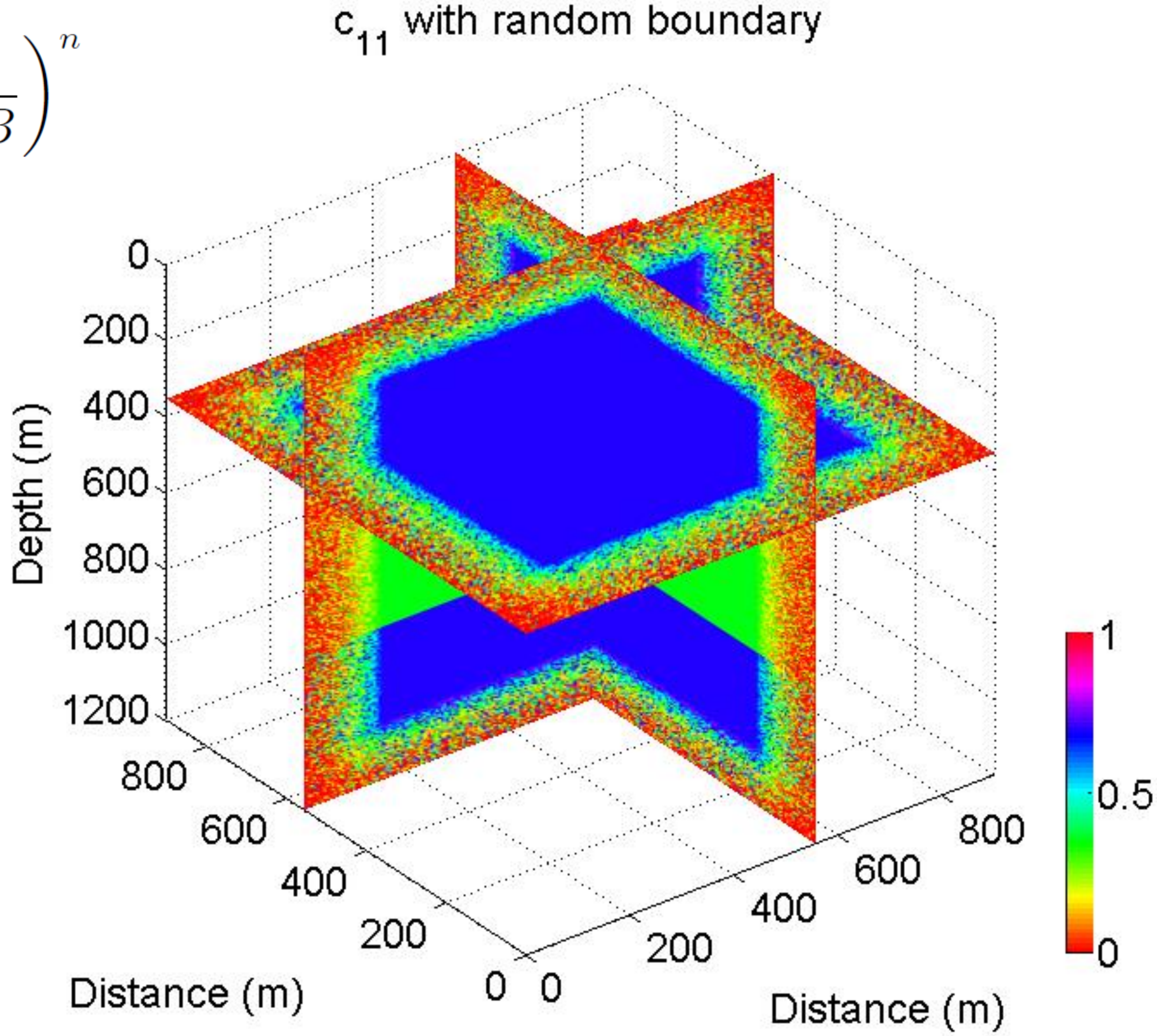
Reverse-time propagation of residual data

Read in recorded source wavefield on the disk and correlate with the residual wavefield

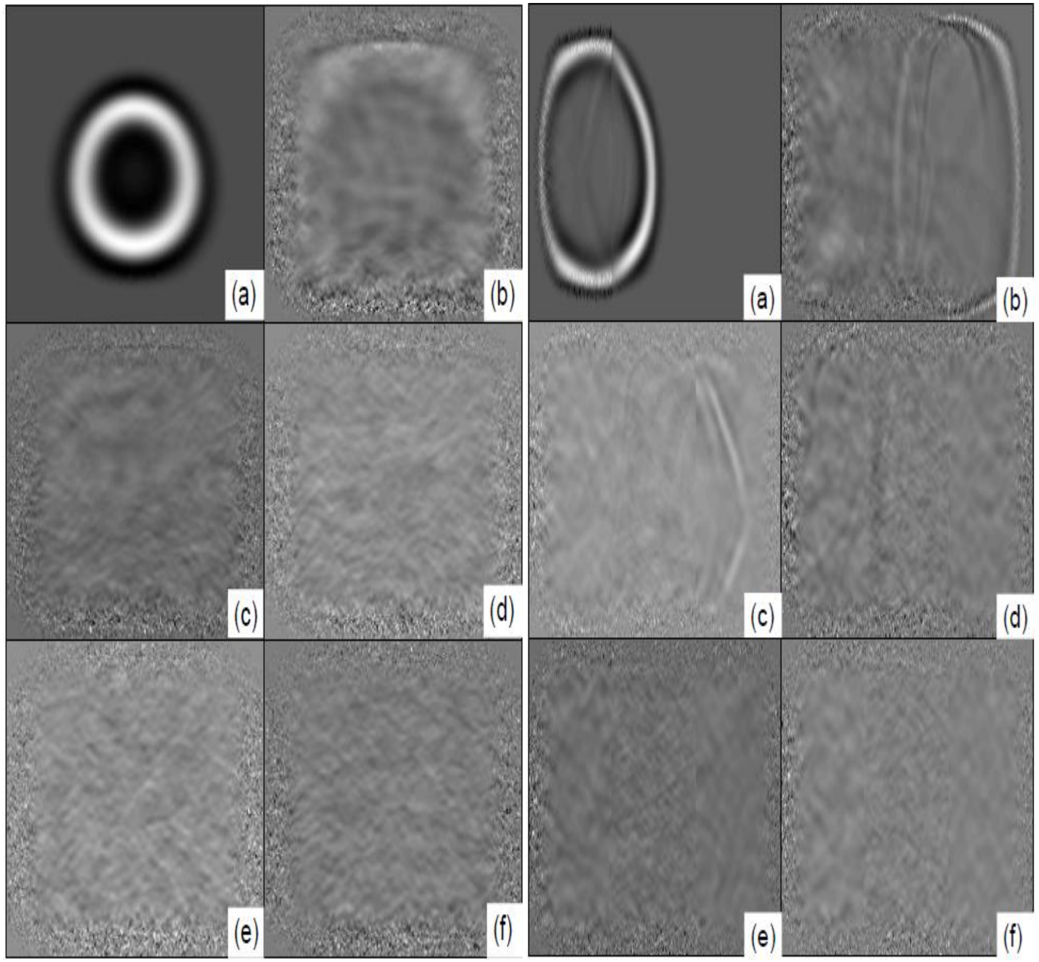
Random parameter and randomized normal stress component

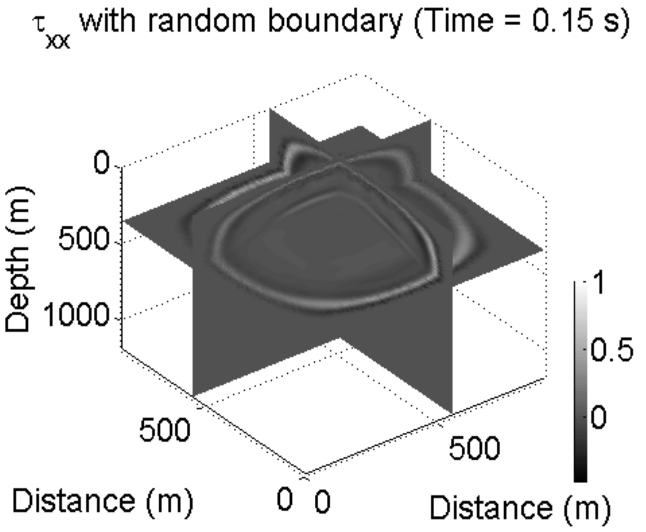


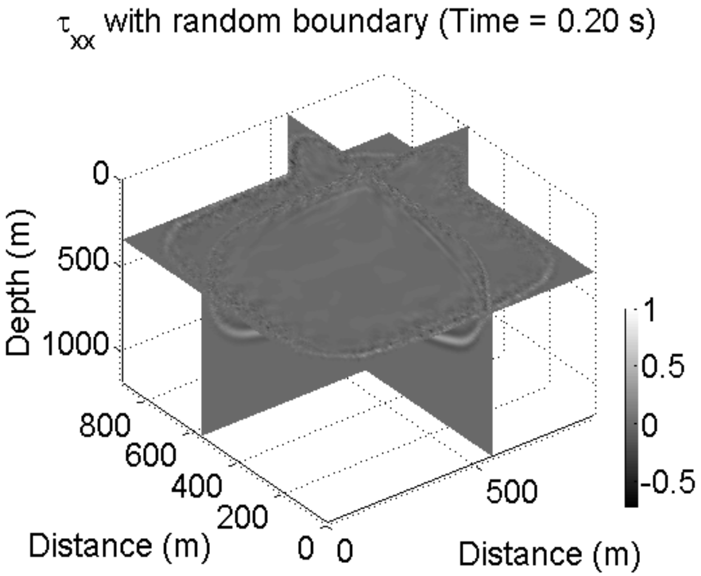
$$r = \left(rand \cdot \frac{d}{AB}\right)^n$$

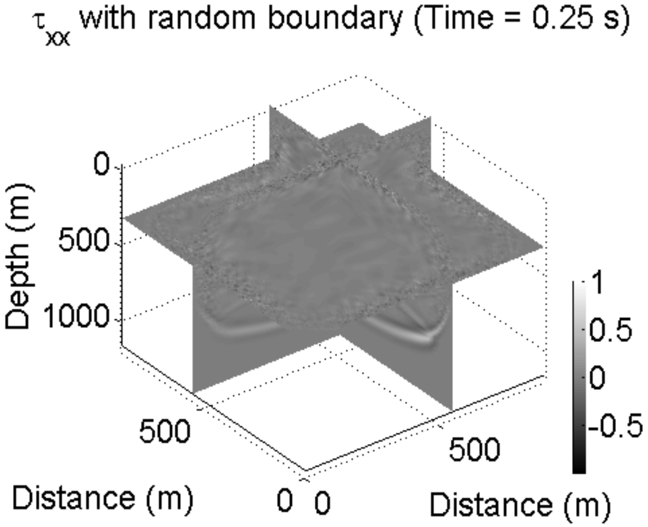


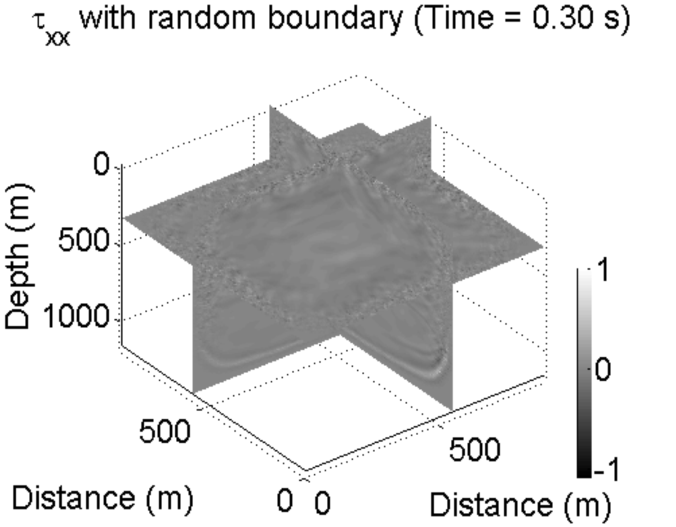
Wavefield propagation with random boundary layers



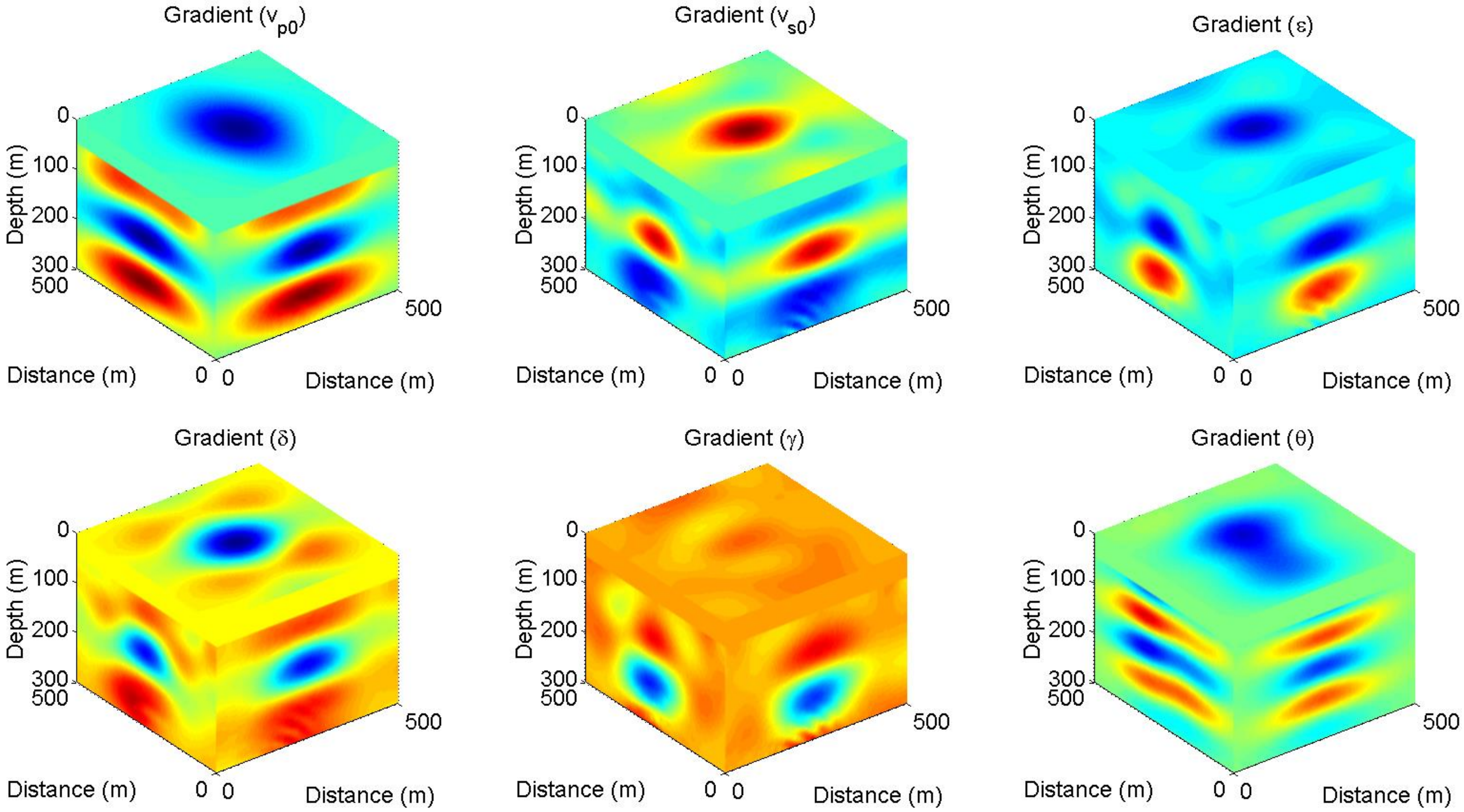








Gradients of Thomsen parameters and polar angle （Random boundary）



Conclusions

- A temporal fourth-order PSTD for SH wave propagation in VTI media in conjunction with the HPML method is capable of solving wraparound effect, Gibbs phenomenon and frequency dispersion.

- The time domain FWI basically uses first-order velocity-stress staggered-grid finite difference method for wavefield simulation, the velocity components should be transferred into displacement components; the displacement components can be acquired directly from PSTD wavefield simulation.

- The gradients with respect to both Thomsen parameters and constitutive elastic moduli as well as the polar angle are calculated. The use of staggered-grid FD with PML, overburden the memory cost and computational efficiency. The random boundary layer is one of the solutions to relief the I/O stream and memory storage.

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Questions & Comments