A PML absorbing boundary condition for 2D viscoacoustic wave equation in time domain: modeling and imaging

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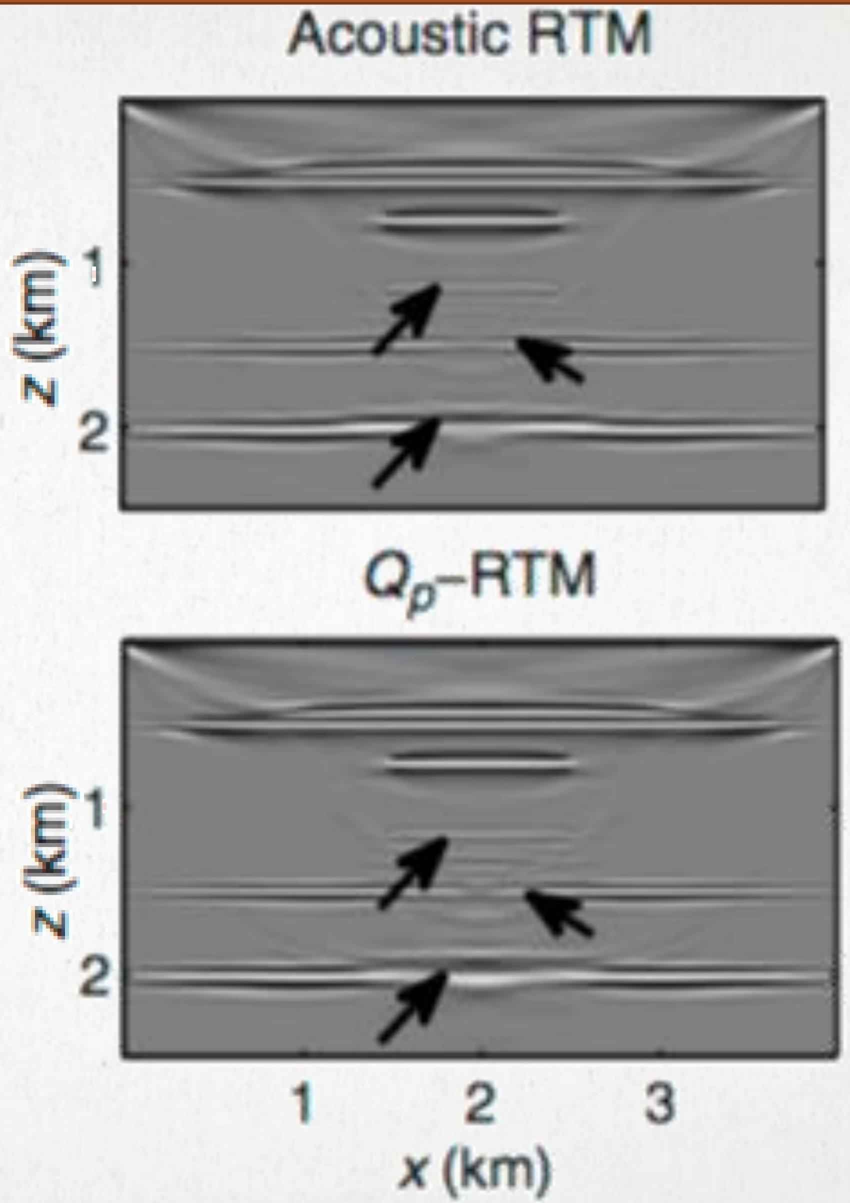
Outline

- Motivation
- Introduction
    - -Viscoacoustic wave equations
    - -Perfectly matched layers absorbing boundary condition (PML)
- Unsplit-PML formulation for 2D viscoacoustic wave equation in time domain
- FD implementation of PML
- Numerical results
- Conclusions

Motivation



(Gaurav Dutta et al. 2014)

Viscoacoustic wave equation

The 2D viscoacoustic medium can be expressed as a system of first-order differential equations in terms of the particle velocities and stresses.

Newton's second law equations:

$$\frac{\partial u_x}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial u_z}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

Stress-strain relations:

$$\frac{\partial p}{\partial t} = -\rho c_p^2\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}\right)\left(1 - \sum_{\iota=1}^{L}\left(1 - \frac{\tau_{\varepsilon\iota}}{\tau_{\varepsilon\iota}}\right)\right) + \sum_{\iota=1}^{L} r_\iota$$

$$\frac{\partial r_\iota}{\partial t} = -\frac{1}{\tau_{\sigma\iota}} r_\iota + \rho c_p^2\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}\right)\frac{1}{\tau_{\sigma\iota}}\left(1 - \frac{\tau_{\varepsilon\iota}}{\tau_{\sigma\iota}}\right), 1 \leq \iota \leq L$$

u : Particle velocity
p : Compressional stress
τ : Relaxation time
r : Memory variable

(Carcions et al., 1988)

Viscoacoustic wave equation

For one memory variable, L = 1, the first-order linear differential equations of viscoacoustic wave propagation are

$$\frac{\partial u_x}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial u_z}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

$$\frac{\partial p}{\partial t} = -\rho c_p^2 \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}\right)\left(\frac{\tau_\varepsilon}{\tau_\sigma}\right) - r$$

$$\frac{\partial r}{\partial t} = -\frac{1}{\tau_\sigma} r + \rho c_p^2 \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}\right)\frac{1}{\tau_\sigma}\left(1 - \frac{\tau_\varepsilon}{\tau_\sigma}\right)$$

where τ_ε and τ_σ are the stress and strain relaxation times (Robertsson et al., 1994)

$$\tau_\sigma = \frac{\sqrt{1 + 1/Q^2} - 1/Q}{f_0} \quad , \quad \tau_\varepsilon = \frac{1}{f_0^2 \tau_\sigma}$$

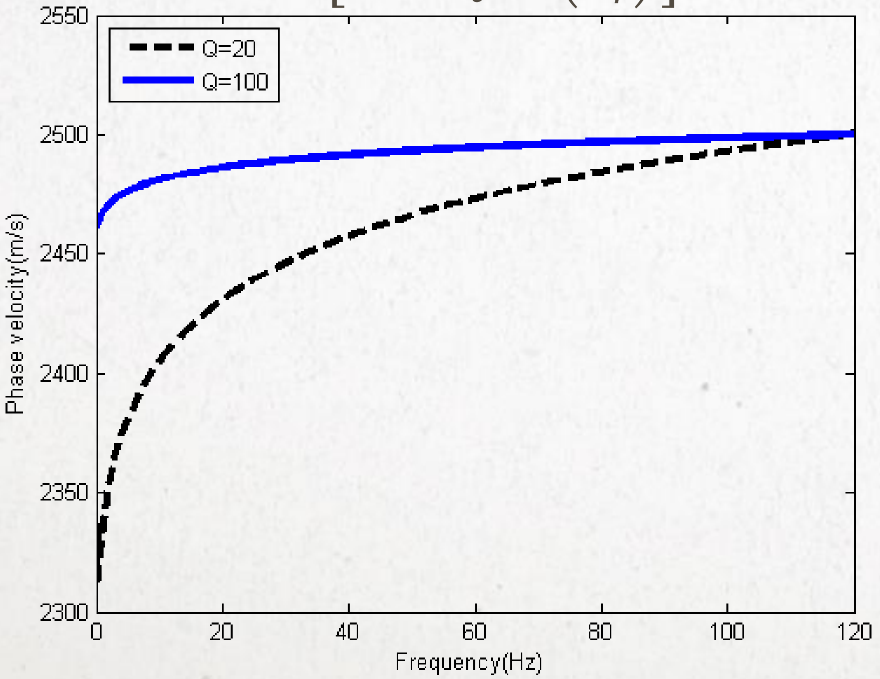
Absorption and attenuation

Attenuation is defined as the ratio of maximum amplitude of a wave field for a particular frequency to the change of amplitude per cycle

$$\alpha(\omega) = \frac{\omega}{2c_0Q}$$

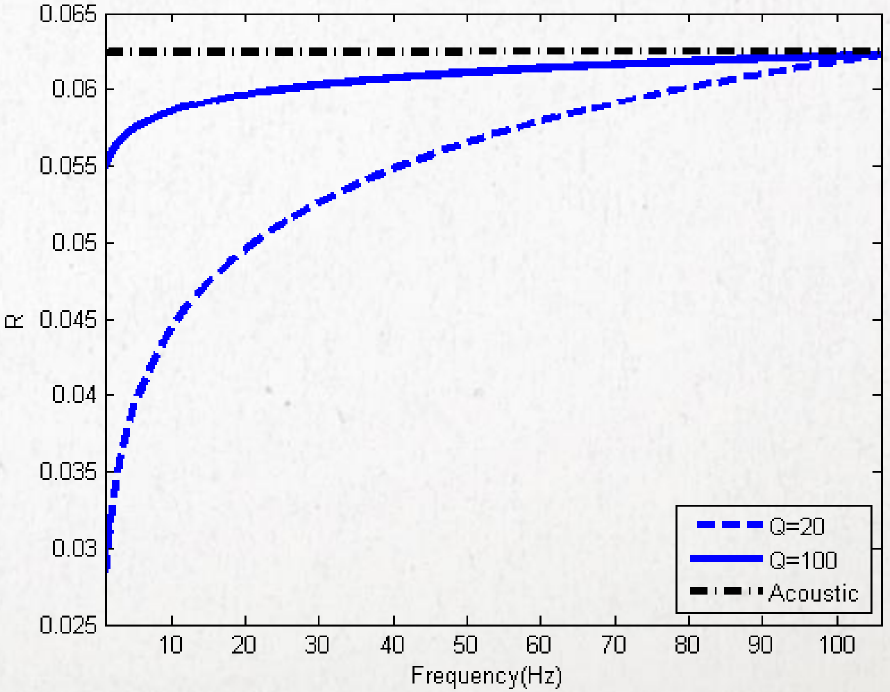
The real part of complex velocity is attenuated phase velocity (Aki and Richards, 2002)

$$v_p(\omega) = c_0\left[1 + \frac{1}{\pi Q} ln\left(\frac{\omega}{\omega_r}\right)\right]$$



The reflection coefficient of viscoacoustic media is complex and frequency dependent and can be written as function of scattering potential (Fathalian and Innanen, 2015).

$$R_{va} \cong \frac{i\omega}{2c_0 \cos\theta_0}\big(\alpha(z) - 2\zeta(z)F(k) + \beta(z)(1 + \cos\sigma)\big)$$



Perfectly matched layers absorbing boundary condition (PML)

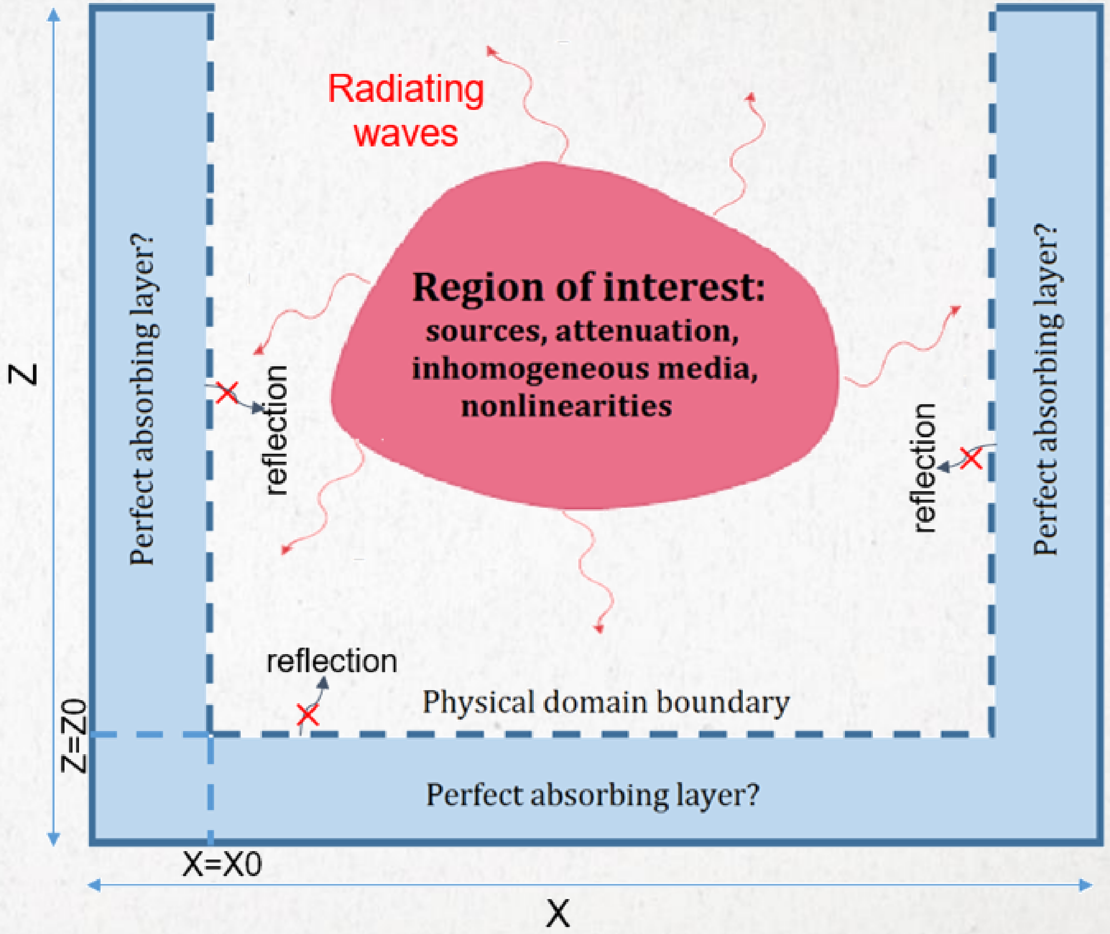
There are three different PML absorbing regions:

PML for x direction (d(z) = 0)

PML for y direction (d(x) = 0)

PML in the corners(d(x) > 0, d(z) > 0)

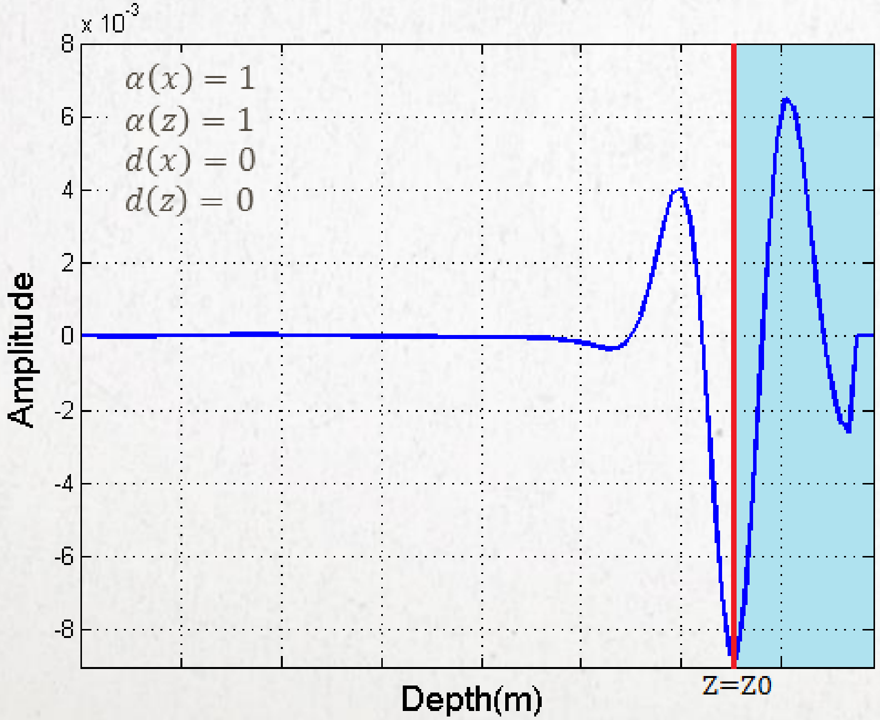
Both damping parameters d(x) and d(z) are positive, and either d(z) = 0 or d(x) = 0 inside the PML region in the x and z directions.

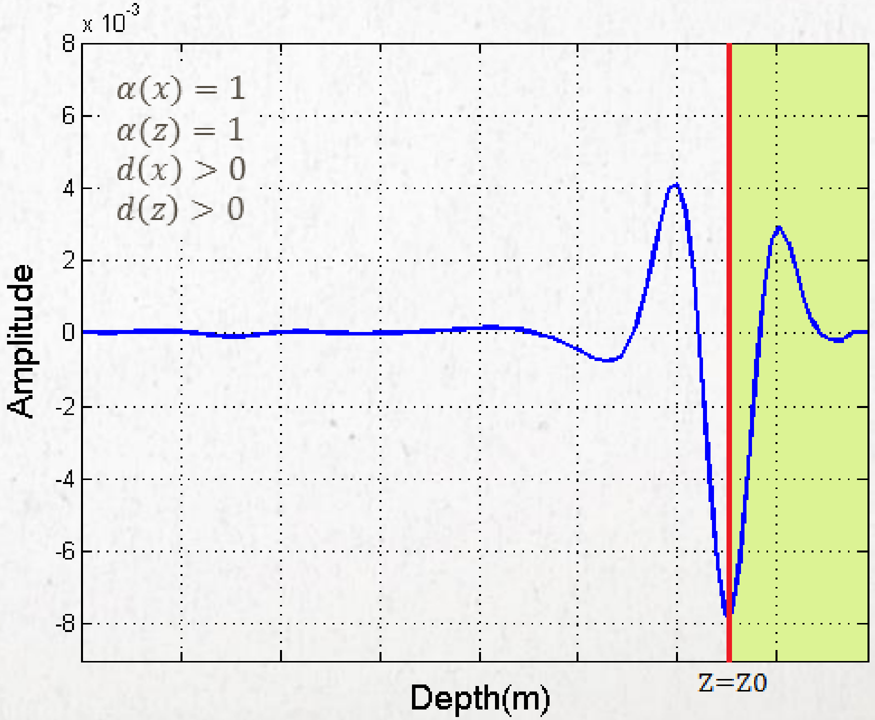


Unsplit-PML for 2D viscoacoustic wave equation

In order to introduce the PML for visco-acoustic wave, the first-order linear differential equations will be modified using the a complex coordinate stretching approach. In the frequency domain, the PML formulations can be derived as

$$\partial x \rightarrow \alpha(x)\left(1+\frac{id(x)}{\omega}\right)\partial x \quad , \quad \partial z \rightarrow \alpha(z)\left(1+\frac{id(z)}{\omega}\right)\partial z$$





Unsplit-PML for 2D viscoacoustic wave equation

By applying the complex coordinate stretching expressed to the linearized equation of motion and equation of deformation in the frequency domain and transforming back to time domain, the unsplit-field PML formulations for viscoacoustic can be obtain as:

$$\frac{\partial u_x}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - d(x)u_x$$

$$\frac{\partial u_z}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - d(z)u_z$$

$$\frac{\partial p}{\partial t} = -\rho c_p^2\left(\frac{\partial(u_x + d(z)\boldsymbol{u_X})}{\partial x} + \frac{\partial(u_z + d(x)\boldsymbol{u_Z})}{\partial z}\right)\left(\frac{\tau_\varepsilon}{\tau_\sigma}\right) - \big(d(x) + d(z)\big)p - d(x)d(z)\boldsymbol{p} - r$$

$$\frac{\partial r}{\partial t} = -\frac{1}{\tau_\sigma}r + \rho c_p^2\left(\frac{\partial(u_x + d(z)\boldsymbol{u_X})}{\partial x} + \frac{\partial(u_z + d(x)\boldsymbol{u_Z})}{\partial z}\right)\frac{1}{\tau_\sigma}\left(1 - \frac{\tau_\varepsilon}{\tau_\sigma}\right) - \big(d(x) + d(z)\big)r - d(x)d(z)\boldsymbol{r}$$

The auxiliary variables (the time-integrated components for velocity, pressure and memory variable fields):

$$\boldsymbol{u_X}(X,t) = \int_{-\infty}^{t} u_x(X,t')dt' \qquad \boldsymbol{p}(X,t) = \int_{-\infty}^{t} p(X,t')dt'$$

$$\boldsymbol{u_Z}(X,t) = \int_{-\infty}^{t} u_x(X,t')dt' \qquad \boldsymbol{r}(X,t) = \int_{-\infty}^{t} r(X,t')dt'$$

FD implementation of PML

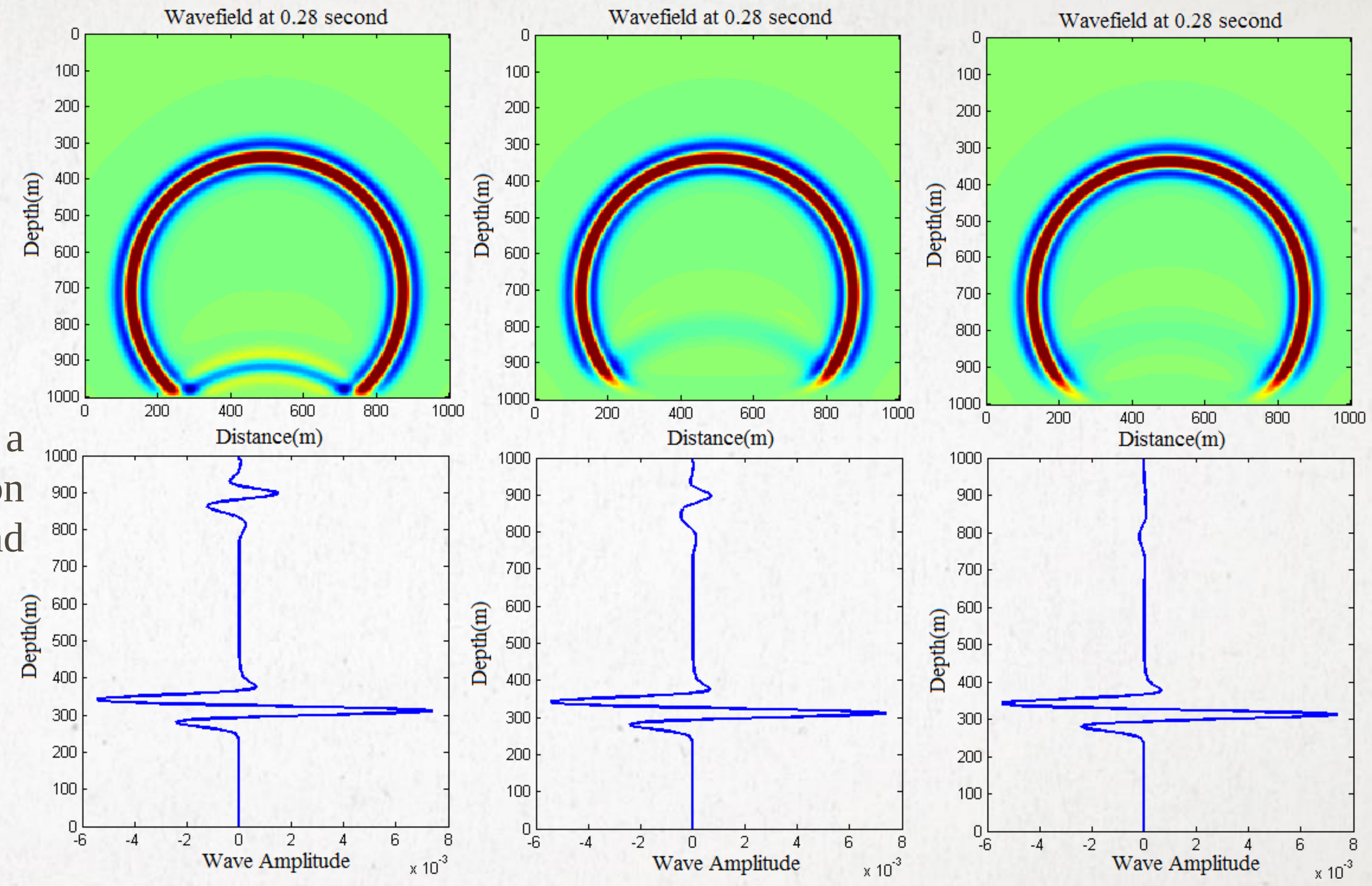
A 2-4 FD scheme is used in this work and the stability condition is

$$\frac{\Delta x}{\Delta t} \geq \frac{7\sqrt{n}}{6} V_{max}$$

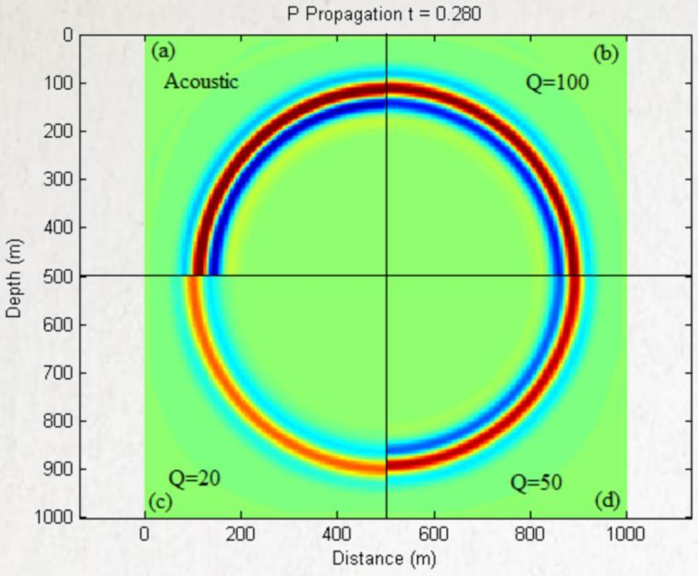
Collino and Tsogka (2001) presented a relation based on a theoretical reflection coefficient, where the PML thickness and the *p*-wave velocity

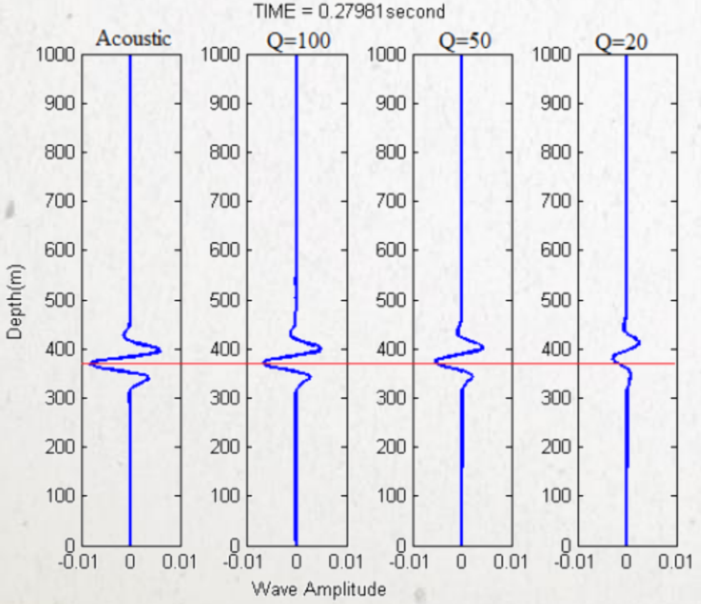
$$d(x) = d_0 \left(\frac{x}{\delta}\right)^2$$

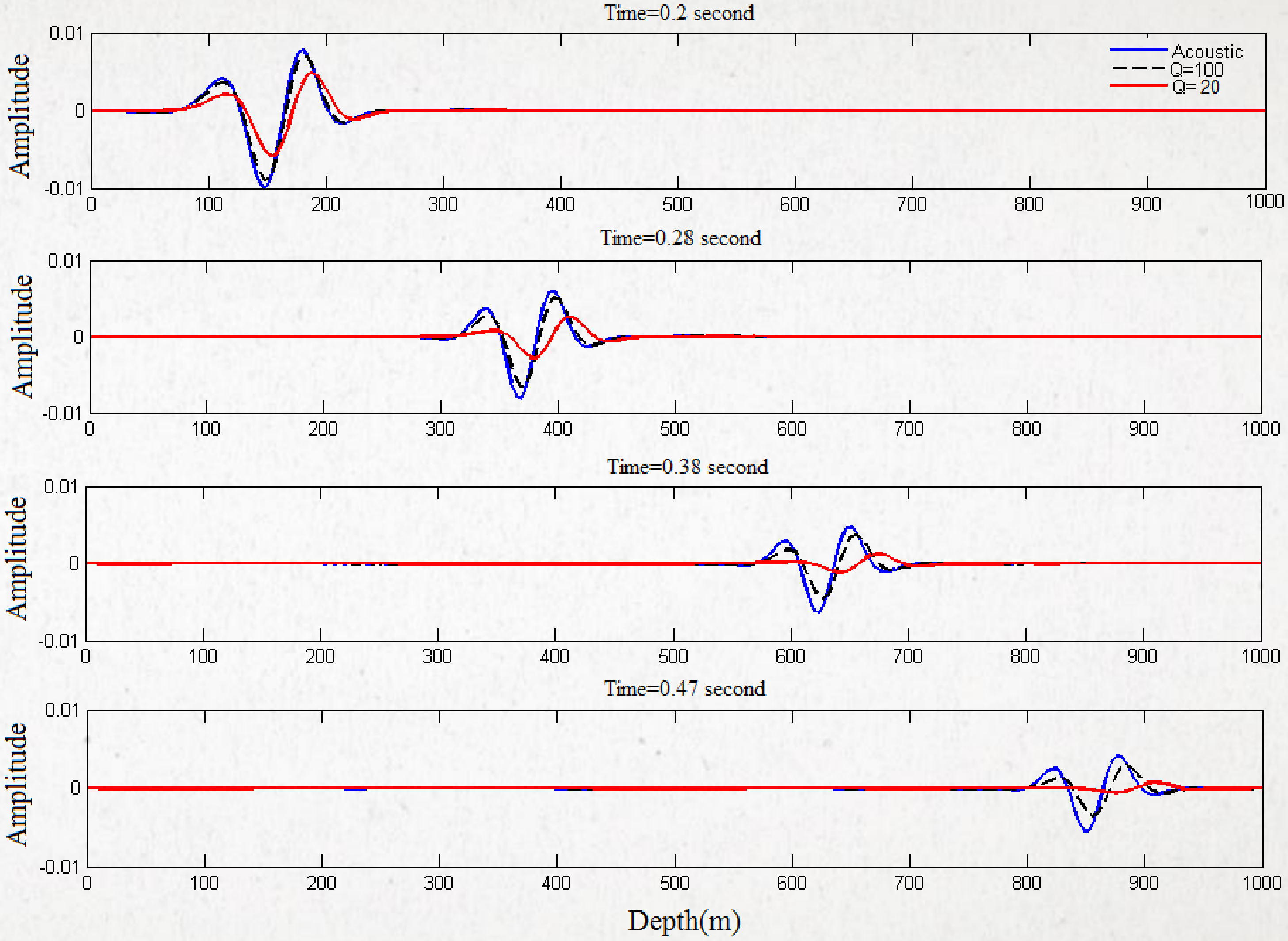
$$d_0 = log\left(\frac{1}{R}\right)\frac{3V_P}{2\delta}$$



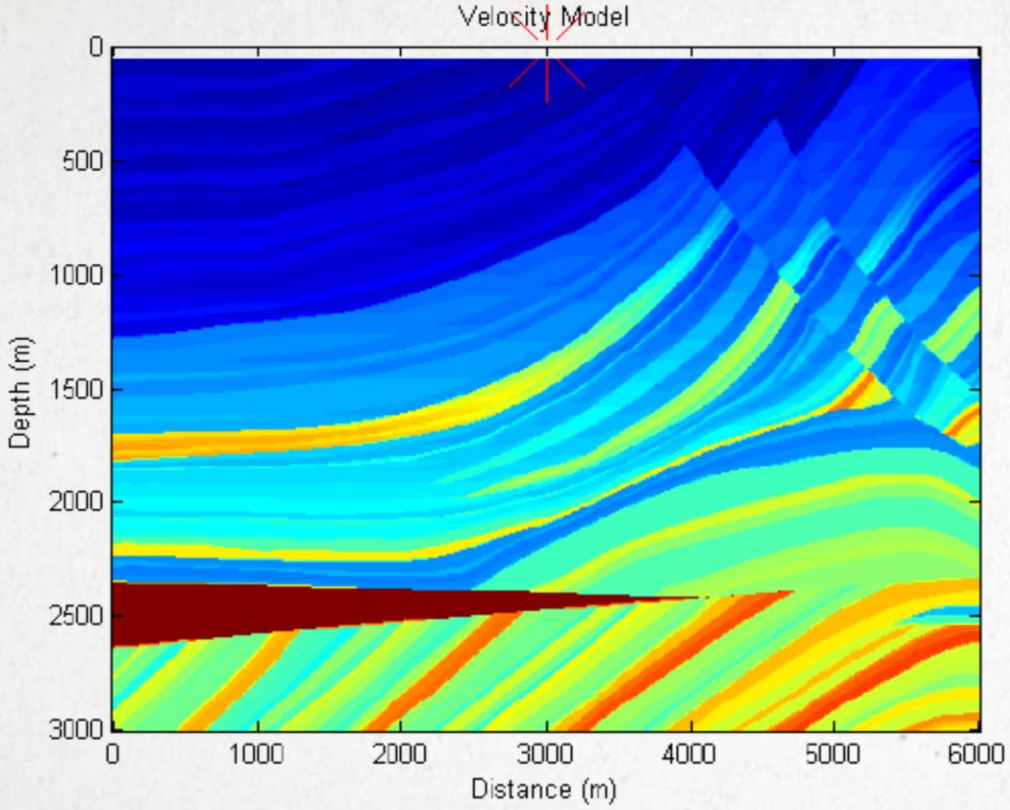
Numerical results

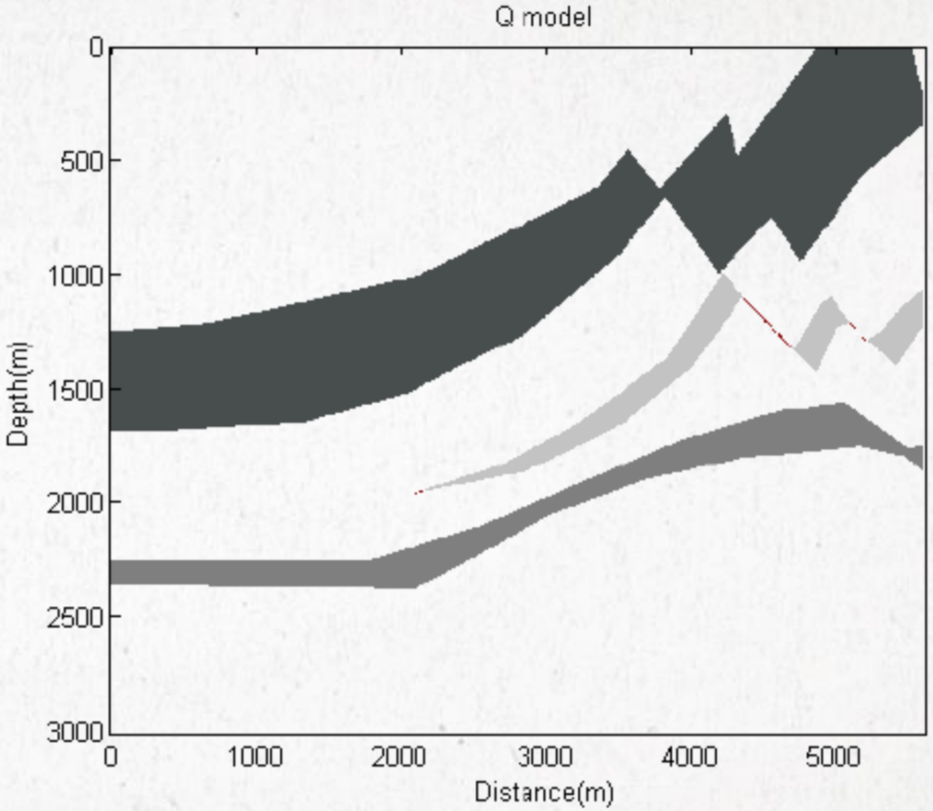




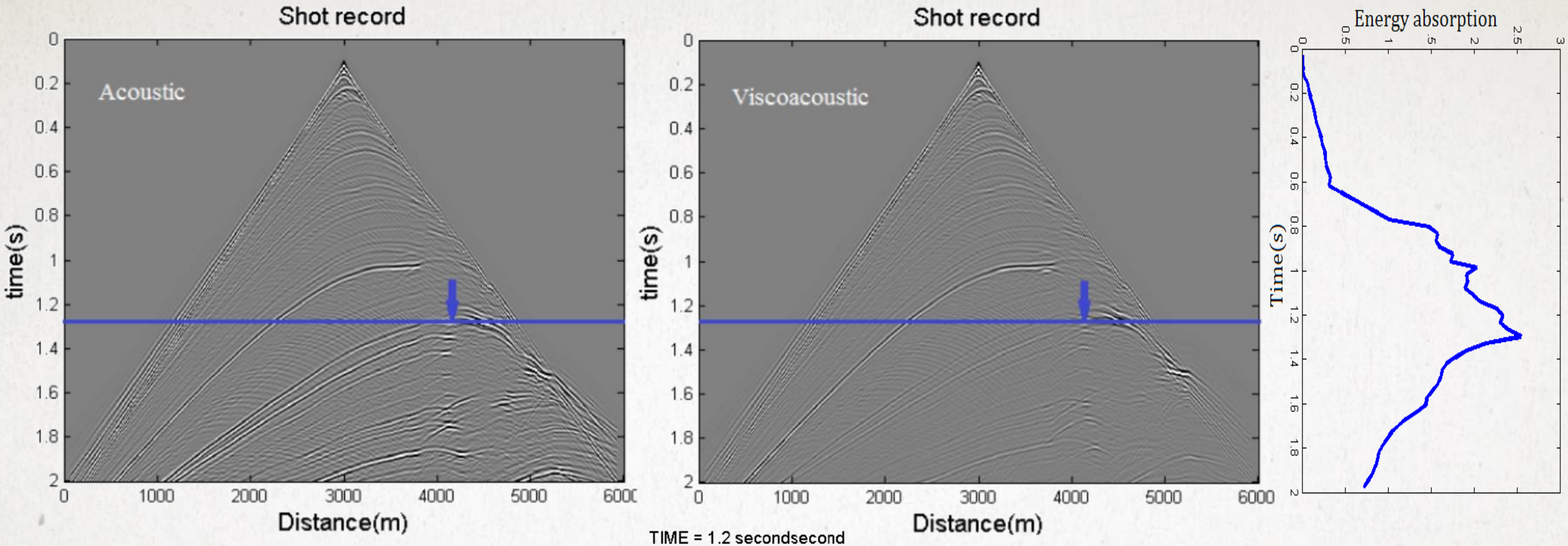


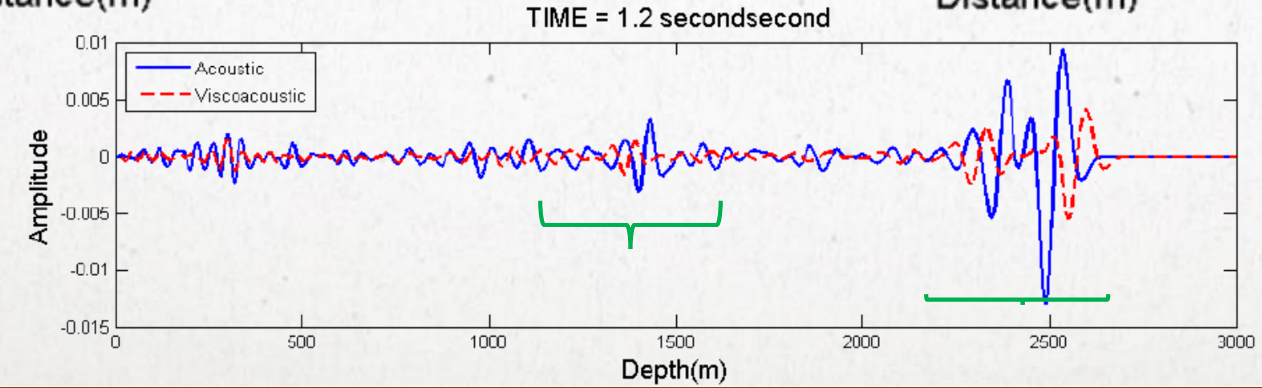
Numerical results



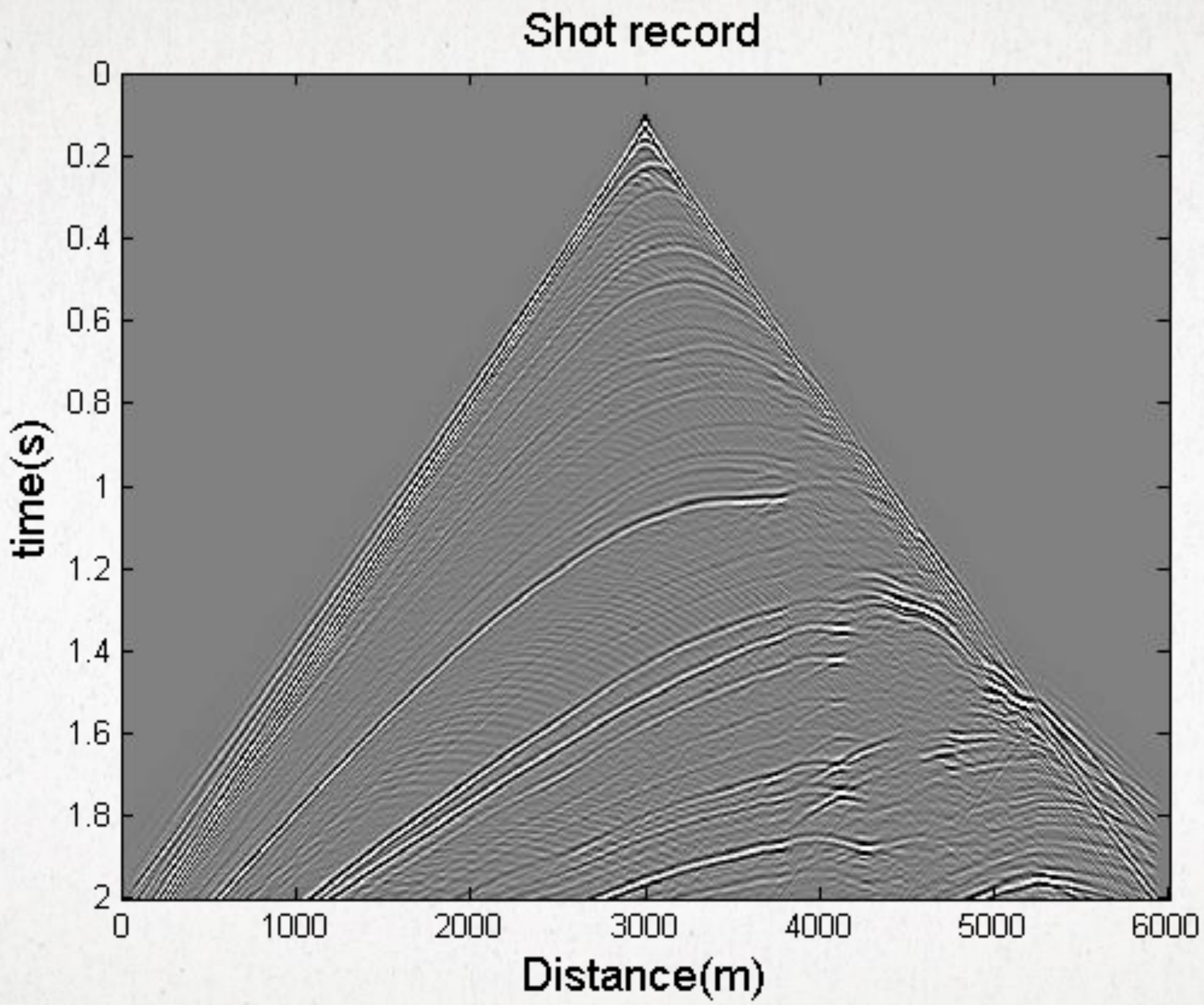


Numerical results

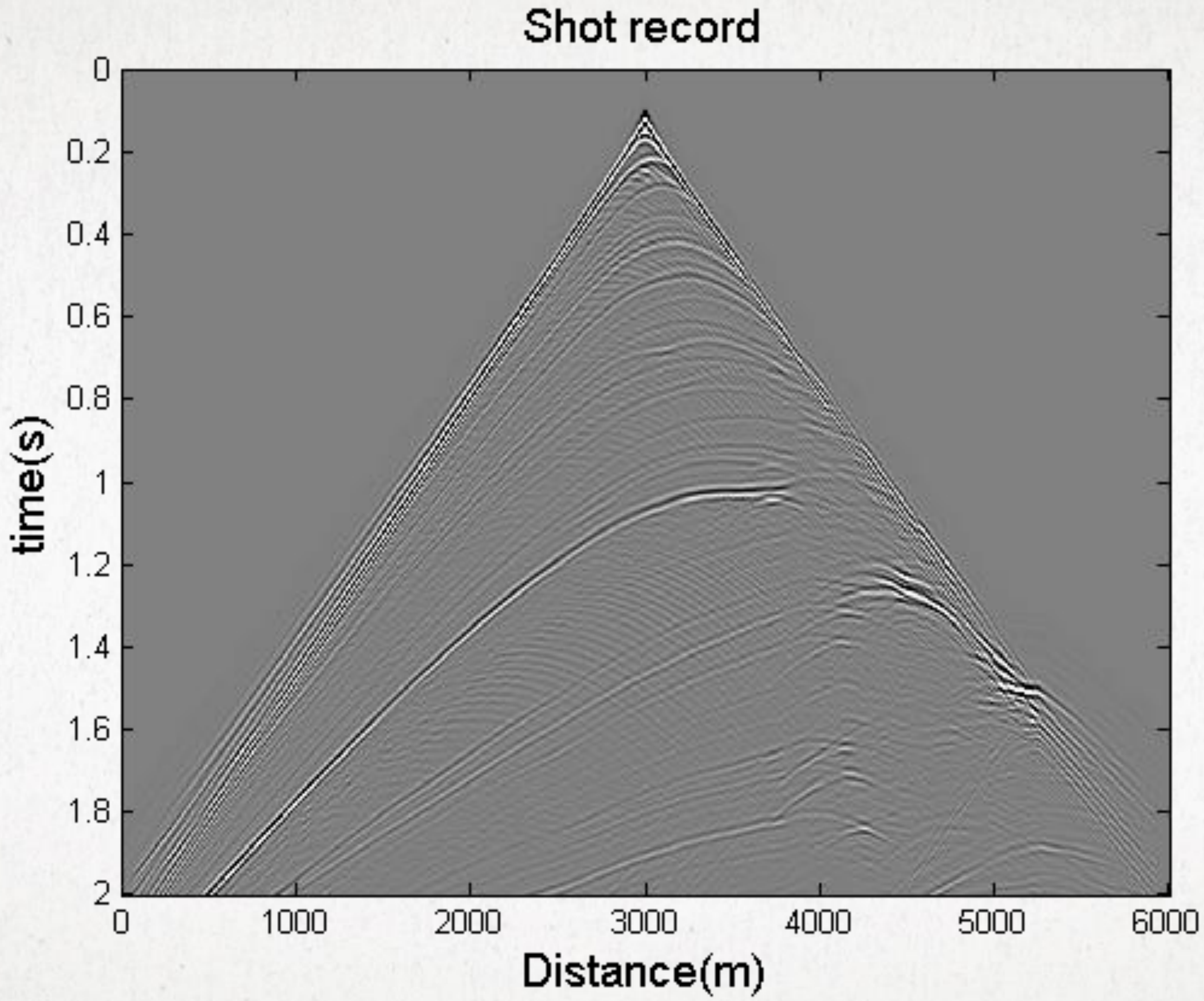




Numerical results

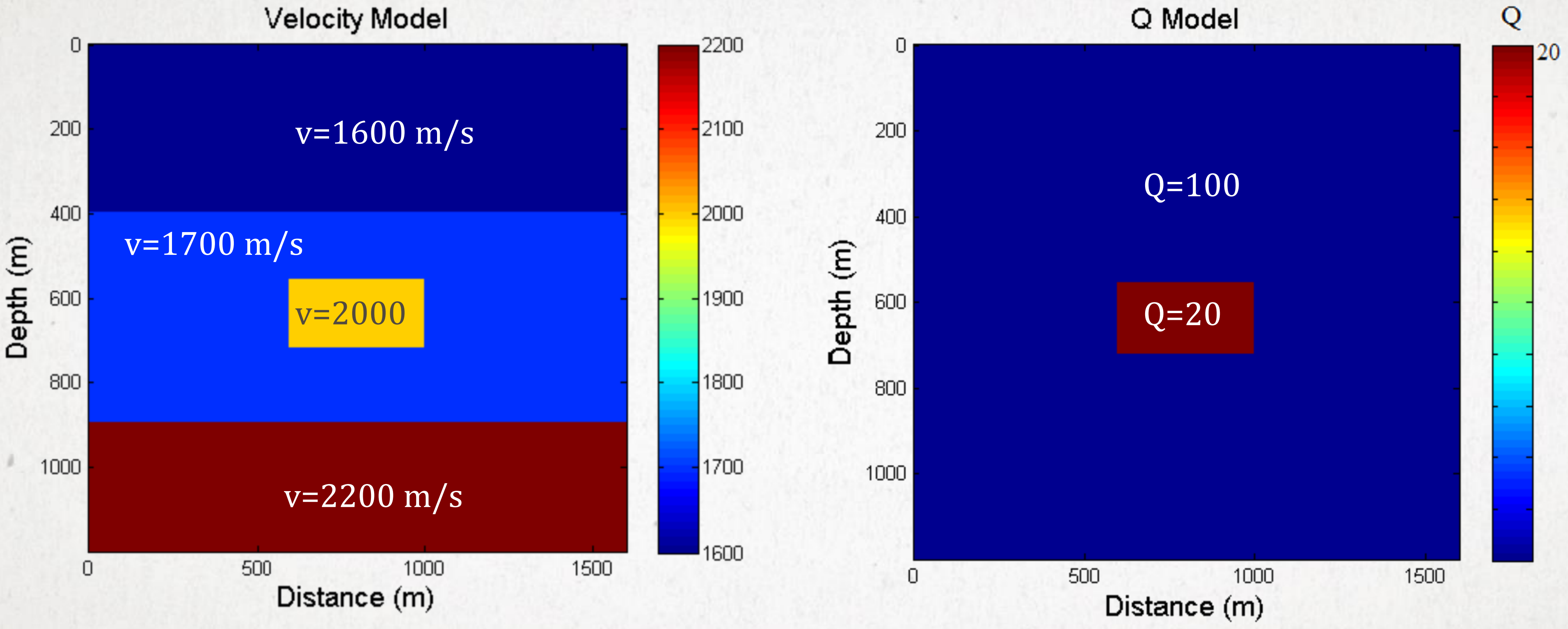


Numerical results



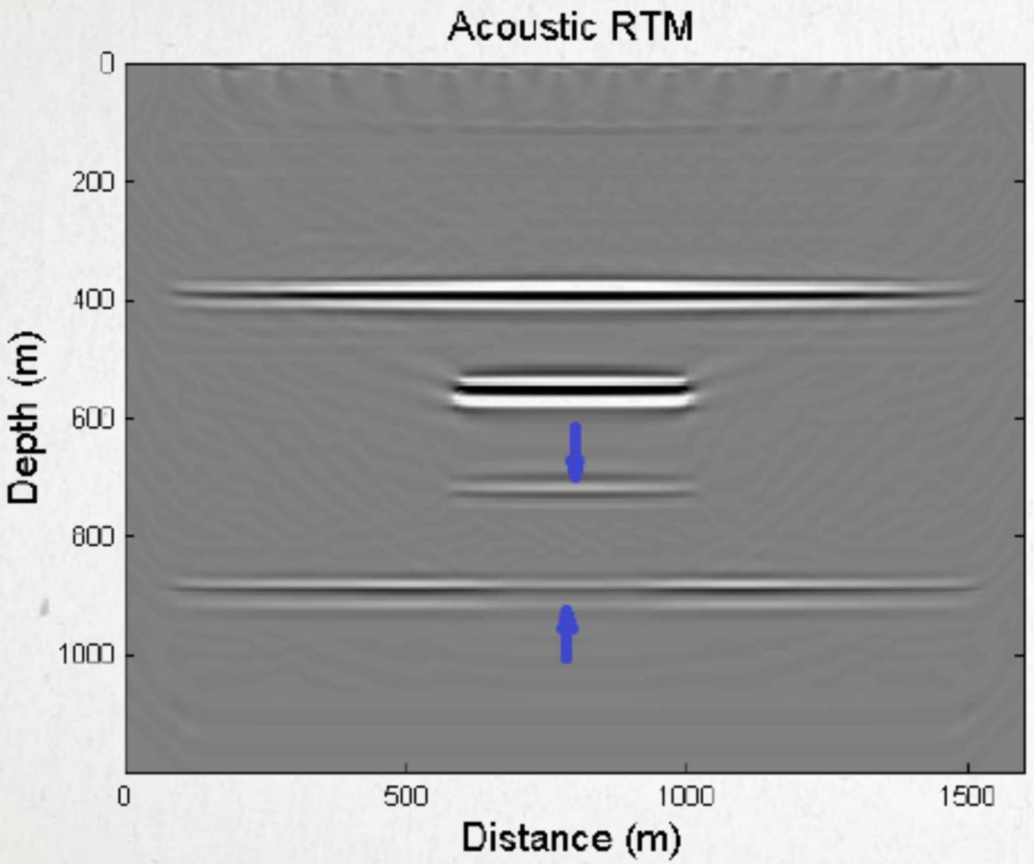
Numerical results

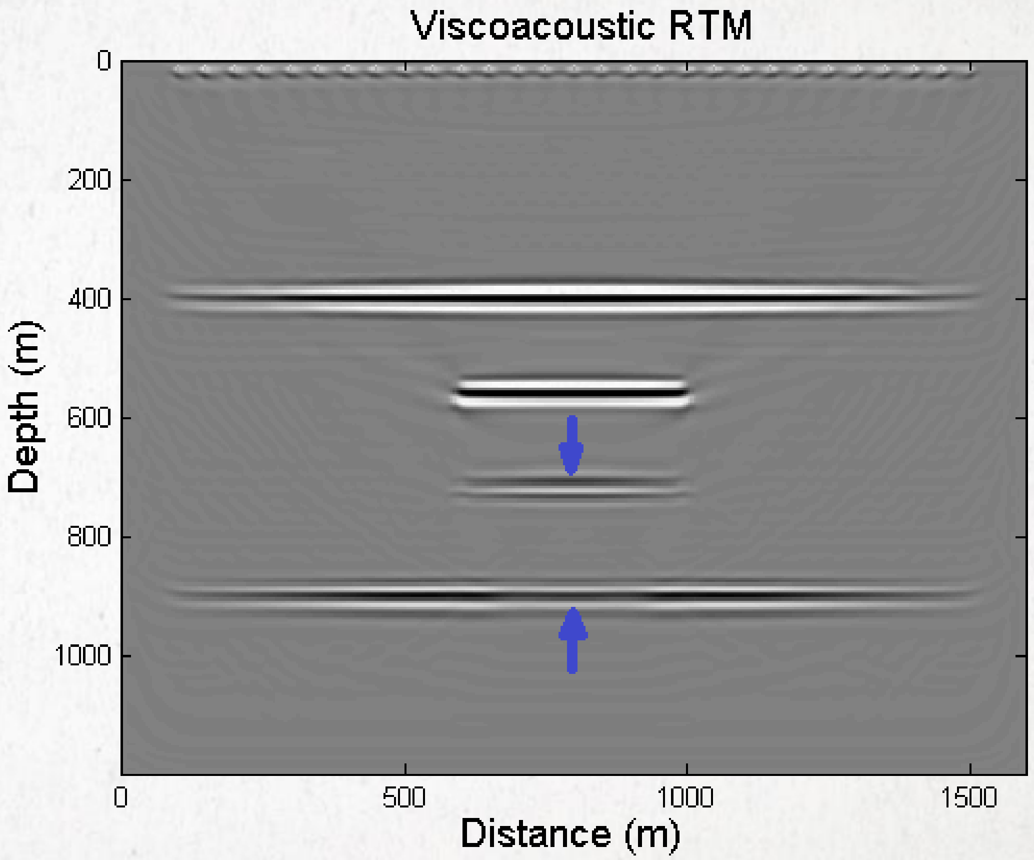
Reverse time migration



Numerical results

Compares the RTM images for acoustic and viscoacoustic approximations with the attenuation



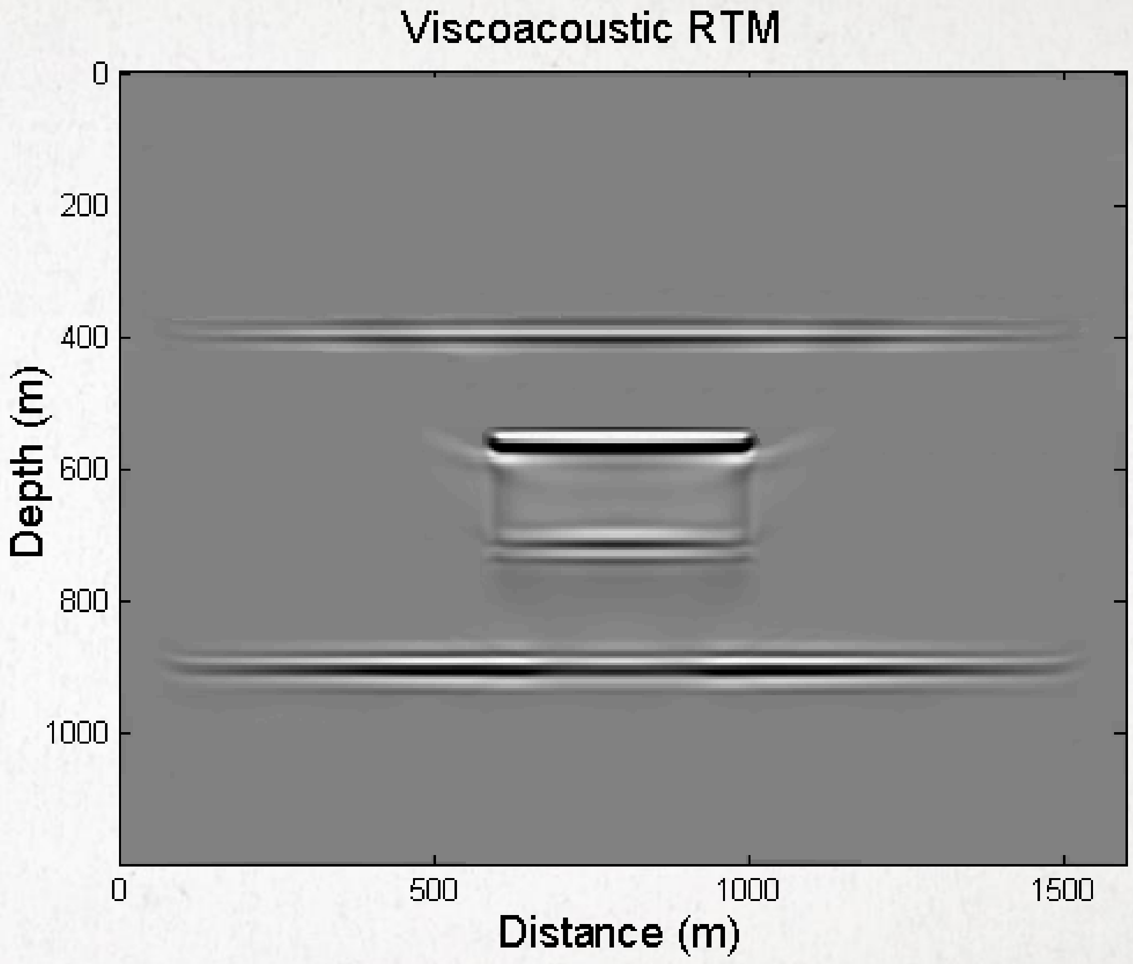


Numerical results

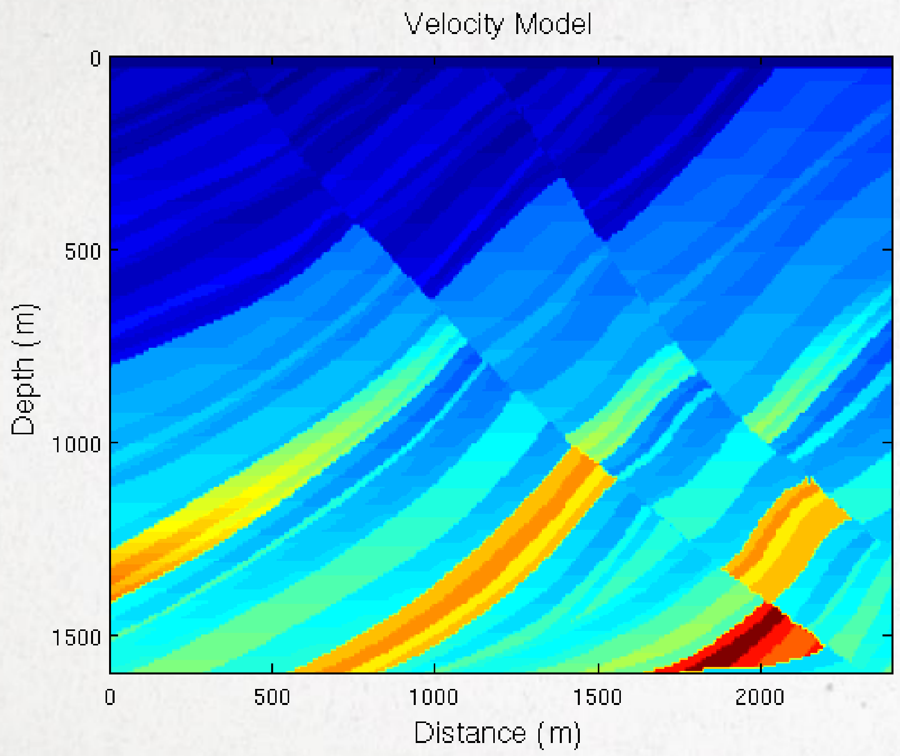
To eliminate source signatures and low frequency noises we used the imaging condition and highpass filter respectively

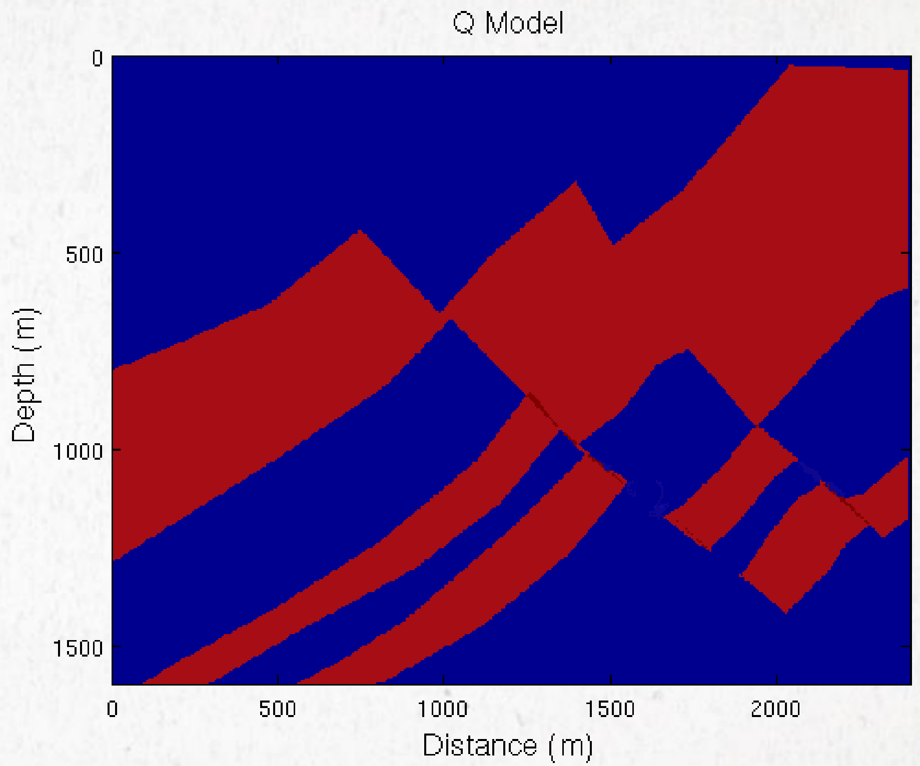
Imaging conditions are used to correlate the source and receiver wavefield snapshots to get the subsurface images(Whitmore and Lines, 1986)

$$I(x,z) = \frac{\int_t S(x,z,t)R(x,z,t)}{\int_t S^2(x,z,t)}$$

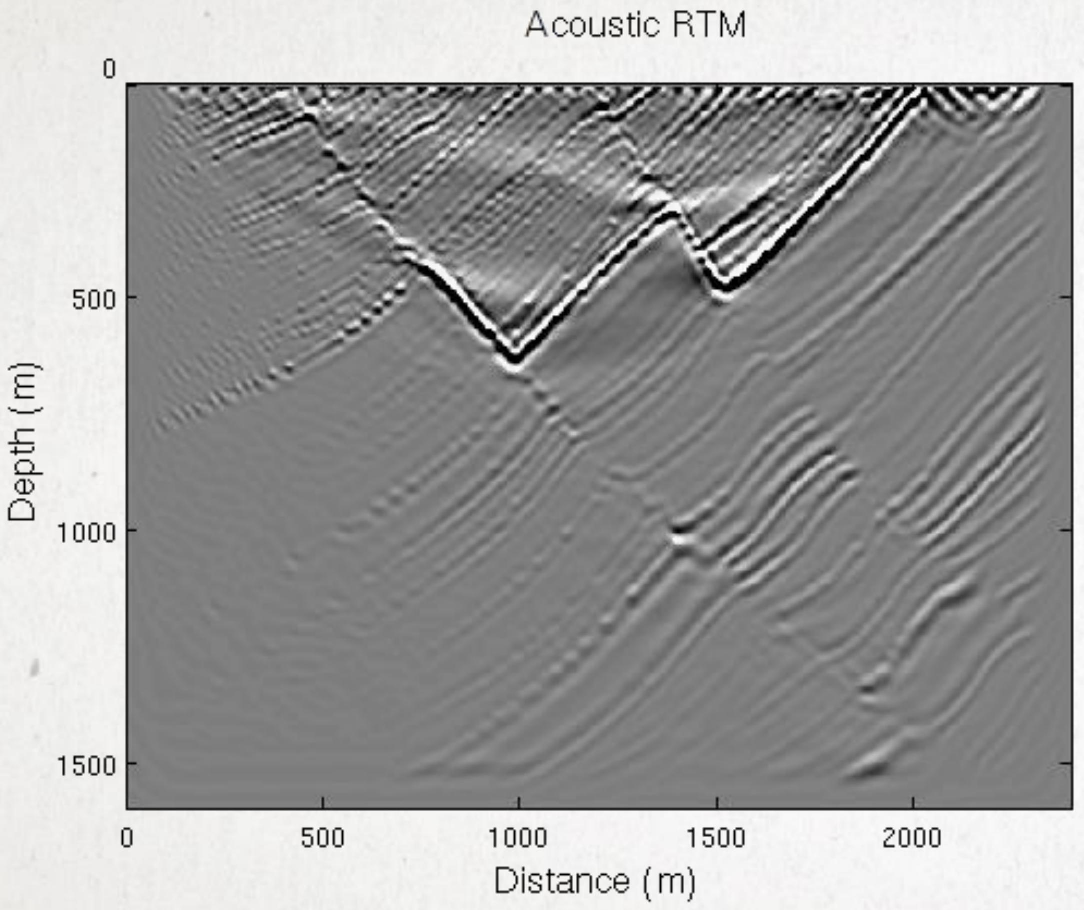


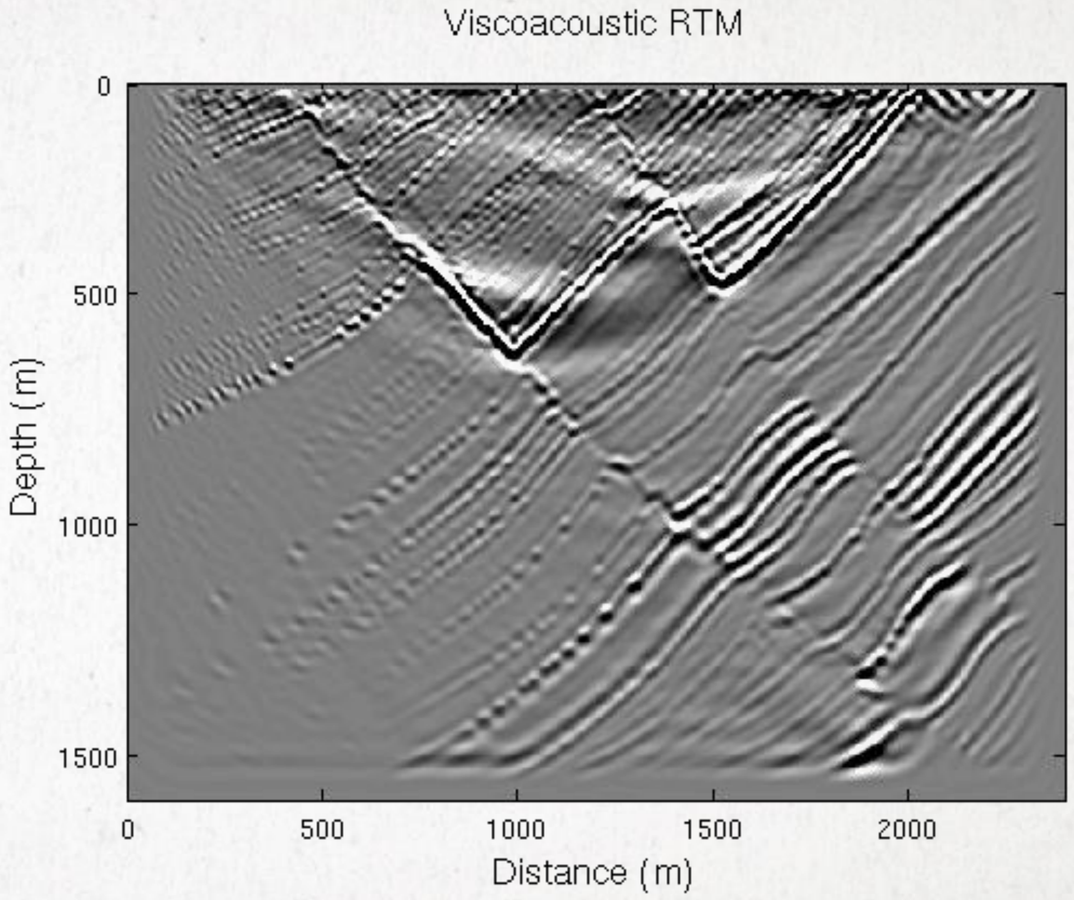
Numerical results





Numerical results





Conclusions

- The difference waveforms between the acoustic and viscoacoustic data show that the energy loss of wavefiled when wave propagated in the attenuative media.
- Numerical synthetic data illustrated for strong attenuation the acoustic RTM cannot correct for the attenuation loss, while the unsplit viscoacoustic wave equations can compensate the attenuation loss during the iterations.
- Comparing the synthetic data results for unsplit viscoacoustic and acoustic RTMs show that the migration amplitudes of layers are more accurate than the acoustic RTM and the reflectors are imaged at the correct locations in strong attenuative media.

Acknowledgments

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