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# Azimuthal Seismic Difference Inversion for Tilted Fracture Weaknesses

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## Summary

Tilted transverse isotropy (TTI) provides a useful model for analyzing how tilted fractures affect seismic wave propagation in subsurface layers. To determine the TTI properties of a medium, we propose an approach of employing azimuthal differencing of seismic amplitude data to estimate tilted fracture weaknesses. We first derive a linearized P-to-P reflection coefficient expression in terms of tilted fracture weaknesses, and then we formulate a Bayesian inversion approach in which amplitude differences between seismic data along two azimuths are used to determine tilted fracture weaknesses. Tests with simulated data confirm that the unknown parameter vector involving tilted fracture weaknesses is stably estimated from seismic data containing a moderate degree of additive Gaussian noise. Applying the inversion approach to real data, we obtain interpretable tilted fracture weaknesses, which are consistent with expected reservoir geology.



#### Introduction

Transversely isotropic (TI) models play an important role in the characterization of wave propagation in such media. Rocks containing sets of parallel vertical fractures respond as horizontal transversely isotropic (HTI) media (Schoenberg and Sayers, 1995); finely-layered rocks with a vertical symmetry axis are described in terms of vertical transversely isotropic (VTI) media (Carcione, 2000). TI media with a tilted symmetry axis are referred to as TTI media, and are usually formed by rotating HTI or VTI media. Here we focus on TTI media formed by rotating vertically fractured rocks. The Zoeppritz equations describe how seismic wave energy partitions at a reflecting interface, and can be applied to calculate exact displacement reflection and transmission coefficients for plane seismic waves (Aki and Richards, 2002). These equations have been extended to apply to weakly anisotropic media, by introducing polarization vectors and azimuthal angles. However, the complexity and nonlinearity of the Zoeppritz equations limit their applicability in the inversion of observed seismic data. For rocks containing vertical and rotationally invariant fractures, a linear slip model was established to characterize the influence of fractures on the properties of rocks in terms of normal and tangential fracture weaknesses (Schoenberg and Sayers, 1995). Bakulin et al. (2000) related the normal and tangential fracture weaknesses to fracture properties (fracture density and aspect ratio). To set up a inversion procedure for estimating fracture weaknesses and tilt angles from seismic data, we derive a linearized expression for the PP-wave reflection coefficient in terms of tilted fracture weaknesses. We rotate the HTI stiffness matrix arising from the linear slip model, obtaining a TTI matrix in terms of tilted fracture weaknesses. Using perturbation in these stiffness parameters, we obtain the linearized P-to-P reflection coefficient approximation. Employing the seismic differences across azimuth angles, an inversion approach is established to determine tilted fracture weaknesses. The stability and accuracy of the proposed inversion is examined using synthetic seismic data with random noise. Tilted fracture models with interpretable and geologically-reasonable fracture properties are shown from the example of testifying our apporach using a field data set acquired over a fractured reservoir.

#### **Method and Theory**

The TTI stiffness matrix  $C_{TTI}$  is straightforwardly expressed by rotating either  $C_{VTI}$ , the stiffness matrix of the corresponding VTI medium through the tilt angle v (Figure 1), or the HTI equivalent through its complement. For instance,

$$C_{\text{TTI}} = M_{\nu} C_{\text{VTI}} M_{\nu}^{\dagger}, \qquad (1)$$

where  $M_{\nu}$  is the matrix of Bond transform, and  $M_{\nu}^{\dagger}$  is the transpose of  $M_{\nu}$ . Within the linear slip model,



*Figure 1 P*-wave propagation in TTI media,  $\theta$  and  $\phi$  are incidence angle and azimuth.



the stiffness matrix of an HTI medium is given by (Schoenberg and Sayers, 1995)

$$C_{\rm HTI} = \begin{bmatrix} M(1-\delta_N) & \lambda(1-\delta_N) & \lambda(1-\delta_N) & 0 & 0 & 0\\ \lambda(1-\delta_N) & M(1-\chi^2\delta_N) & \lambda(1-\chi\delta_N) & 0 & 0 & 0\\ \lambda(1-\delta_N) & \lambda(1-\chi\delta_N) & M(1-\chi^2\delta_N) & 0 & 0 & 0\\ 0 & 0 & 0 & \mu & 0 & 0\\ 0 & 0 & 0 & 0 & \mu(1-\delta_T) & 0\\ 0 & 0 & 0 & 0 & 0 & \mu(1-\delta_T) \end{bmatrix},$$
(2)

where  $M = \lambda + 2\mu$ ,  $\lambda$  and  $\mu$  are the Lamé constants of the homogeneous isotropic host rock,  $\chi = \lambda/M$ , and  $\delta_N$  and  $\delta_T$  are the normal and tangential fracture weaknesses. We express the stiffness matrix of a TTI medium in terms of fracture weaknesses as

$$C_{\text{TTI}} = M_{\nu} (M_{\nu=90^{\circ}})^{-1} C_{\text{HTI}} (M^{\dagger}_{\nu=90^{\circ}})^{-1} M_{\nu}^{\dagger}.$$
 (3)

Using perturbations in stiffness matrix, we derive the P-to-P reflection coefficient as the sum of isotropic part  $R_{PP}^{iso}(\theta)$  and anisotropic term  $R_{PP}^{ani}(\theta, \phi, v)$ :

$$R_{\rm PP}(\theta, \phi, \nu) = R_{\rm PP}^{\rm iso}(\theta) + R_{\rm PP}^{\rm ani}(\theta, \phi, \nu).$$
(4)

Fracture properties can be determined from amplitude differencing of seismic data at different azimuthal angles (Chen and Innanen, 2018). Evaluating equation 4 at two azimuthal angles ( $\phi_1$  and  $\phi_2$ ) and forming the difference, we obtain

$$R_{\rm PP}(\theta,\phi_1,\phi_2) \approx A'(\theta,\phi_1,\phi_2)\beta_N + B'(\theta,\phi_1,\phi_2)\delta_N + C'(\theta,\phi_1,\phi_2)\beta_T + D'(\theta,\phi_1,\phi_2)\delta_T, \tag{5}$$

where

$$\beta_N = \delta_N \sin^2 \nu, \ \beta_T = \delta_T \sin^2 \nu, \tag{6}$$

and A', B', C' and D' are parameters related to incidence and azimuthal angles. Following Buland and Omre (2003), we write the forward model for generating azimuthal seismic amplitude difference

$$\mathbf{d} = \mathbf{G}\mathbf{x},\tag{7}$$

where **x** is vector of unknown parameters involving fracture weaknesses and tilted fracture weaknesses, **d** is vector of azimuthal seismic amplitude differences, and **G** is vector related to wavelet and parameters A', B', C' and D'. We develop a Bayesian inversion approach of employing azimuthal seismic amplitude differences to estimate tilted fracture weaknesses. Assuming the random noise/error to be Gaussian and the priors to be Cauchy, we construct the objective function  $J(\mathbf{x})$  as

$$J(\mathbf{x}) = (\mathbf{d} - \mathbf{G}\mathbf{x})^{\dagger} (\mathbf{d} - \mathbf{G}\mathbf{x}) / (2\sigma_{\rm e}^2) + \sum_{k=1}^{4n} \ln\left(1 + \frac{x_k^2}{\sigma_{\mathbf{x}}^2}\right),\tag{8}$$

and  $\sigma_x^2$  and  $\sigma_e^2$  represent the variances of x and random noise/errors, respectively. With these quantities in hand the condition  $\frac{\partial J(x)}{\partial x} = 0$  leads to the system of equations

$$\left(\mathbf{G}^{\dagger}\mathbf{G}/\sigma_{\mathrm{e}}^{2}+2\mathbf{Q}/\sigma_{\mathrm{x}}^{2}\right)\mathbf{x}=\mathbf{G}^{\dagger}\mathbf{d}/\sigma_{\mathrm{e}}^{2}.$$
(9)

To solve for the unknown parameter vector  $\mathbf{x}$ , an iterative inversion appraoch is employed.

#### Results

To examine the basic numerical response and stability of the approach described in the previous section, we apply it to simulated data generated using an overthrust model. We generate synthetic seismic amplitude differences for azimuthal angles ( $\phi_1$  and  $\phi_2$ ) of 0° and 90°, utilizing a 25Hz Ricker wavelet. We add Gaussian random noise into the synthetic seismic data to generate noisy seismic data of signal-to-noise ratio being 2. These values were next used as input to the inverse procedure, and the estimated tilted fracture weaknesses are plotted in Figures 2. Figure 3 plots comparisons between true values and  $82^{nd}$  EAGE Conference & Exhibition 2020





Figure 2 Inversion results of tilted fracture weaknesses.



Figure 3 Comparisons between true values and estimated results of tilted fracture weaknesses.

inversion results of tilted fracture weaknesses at CDP 500. We observe a close match between inversion results and true values of tilted fracture weaknesses given data with a moderate noise.

Finally, we apply the proposed approach to a real data set to further confirm its feasibility. The data are acquired over a fractured carbonate rock reservoir, and they have been sorted to common azimuth gathers and transferred from offset domain to incidence angle domain for each azimuth sector. In Figure 4, we show stacked seismic profiles along azimuths  $\phi_1$  and  $\phi_2$ . Figure 5 presents the final inversion results for tilted fracture weaknesses. We observe that at the location of fractured reservoir marked by the ellipse, the tilted fracture weaknesses exhibit relatively large values. Using the inverted fracture weaknesses and tilted fracture weaknesses, we may compute the tilt angle, which may help to indicate tilt in the fractures in the reservoir.



*Figure 4* Stacked seismic profiles and differences. a)  $\phi_1 = 0^\circ$ ; and b)  $\phi_2 = 90^\circ$ .





Figure 5 Inversion results of tilted fracture weaknesses.

### Conclusion

We first express the stiffness matrix of a tilted transversely isotropic (TTI) medium in terms of tilted fracture weaknesses. Using perturbations in stiffness parameters, we derive a linearized reflection coefficient in terms of tilted fracture weaknesses. We set up an inversion approach based on differences in these amplitudes to estimate both fracture weakness and tilted fracture weakness. Applying the inversion approach and workflow to noisy synthetic data generated using an overthrust model, we conclude that the fracture weaknesses and tilted fracture weaknesses are estimated stably in the presence of a moderate level of noise/error (SNR $\geq 2$ ). A test on a real data set acquired over a fractured reservoir reveals that the proposed inversion method appears to provide interpretable and geologically consistent estimates of tilted fracture weaknesses. Combining the estimated normal fracture weakness and tilted normal fracture weakness, we may compute the tilt angle of fractures.

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