Quantum computing for seismic problems

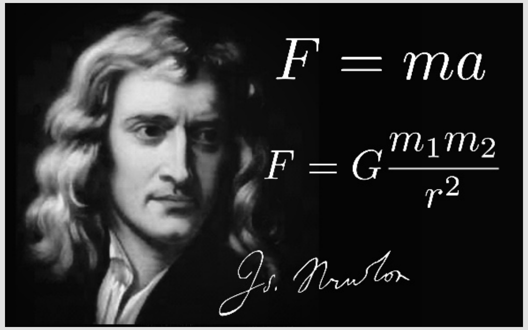
Shahpoor Moradi
Daniel Trad

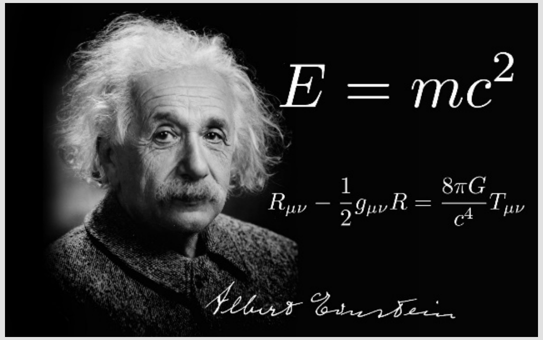
Overview

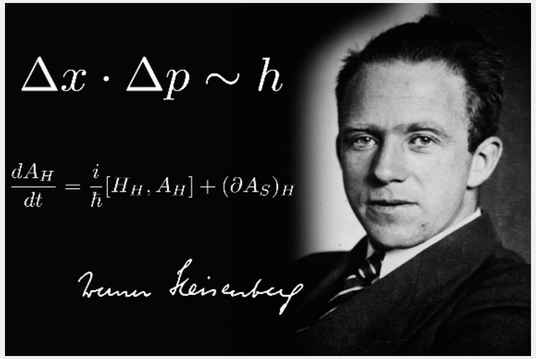
- Quantum computing in a nutshell
- MATLAB toolbox for quantum computation
- Ongoing research and future directions for seismic application

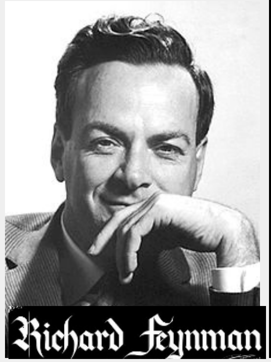
Background

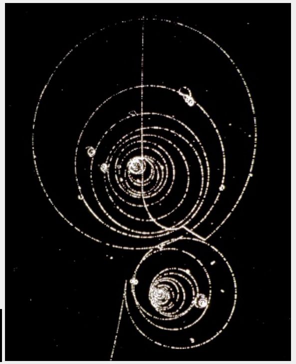
	Low speed $\ll 3 \times 10^8$ m/s	High speed $\sim 3 \times 10^8$ m/s
Large size $\gg 10^{-9}$m	**Classical Physics**	**Relativistic Physics**
Small size $\ll 10^{-9}$m	**Quantum Physics**	**Quantum Field Theory**









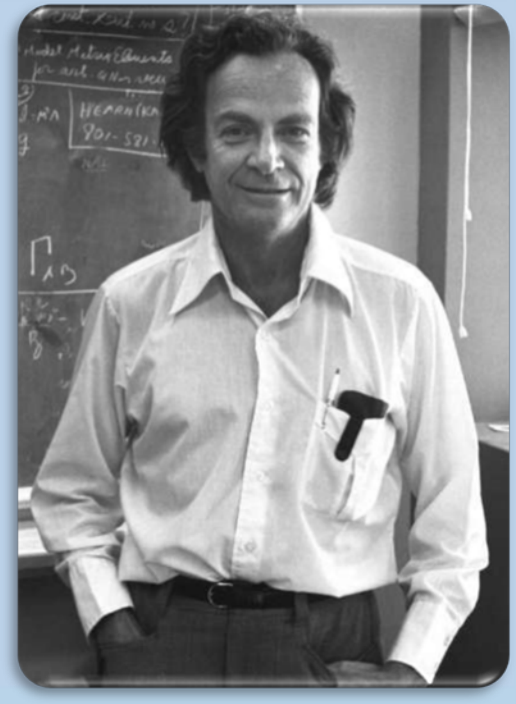


Background

Simulating physics with computers-1982

Richard P. Feynman (Nobel Prize in Physics 1965)

"Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."

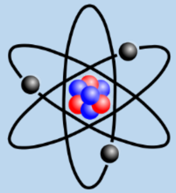


Background

- **Biological computer:** DNA-based computation
- **Optical computer:** Laser-based computation

Classical computers: Information is represented and manipulated in classical way, they compute using bits.

Quantum computers: Information is represented and manipulated in **quantum** way, they compute using **qubits**.

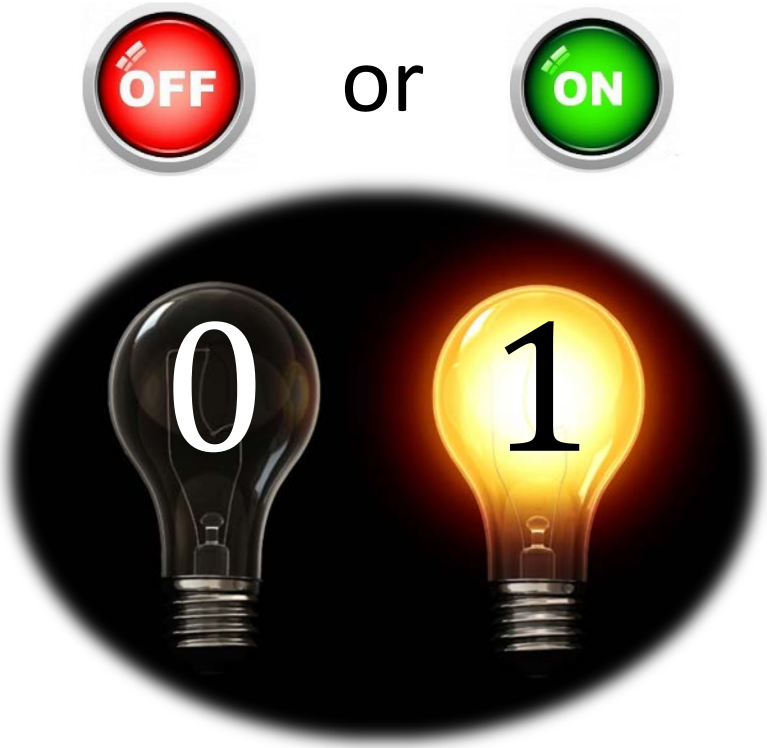


"With a quantum computer, you could quickly do a calculation that a traditional computer the size of the Universe wouldn't be able to carry out before the death of the Sun." -Gilles Brassard

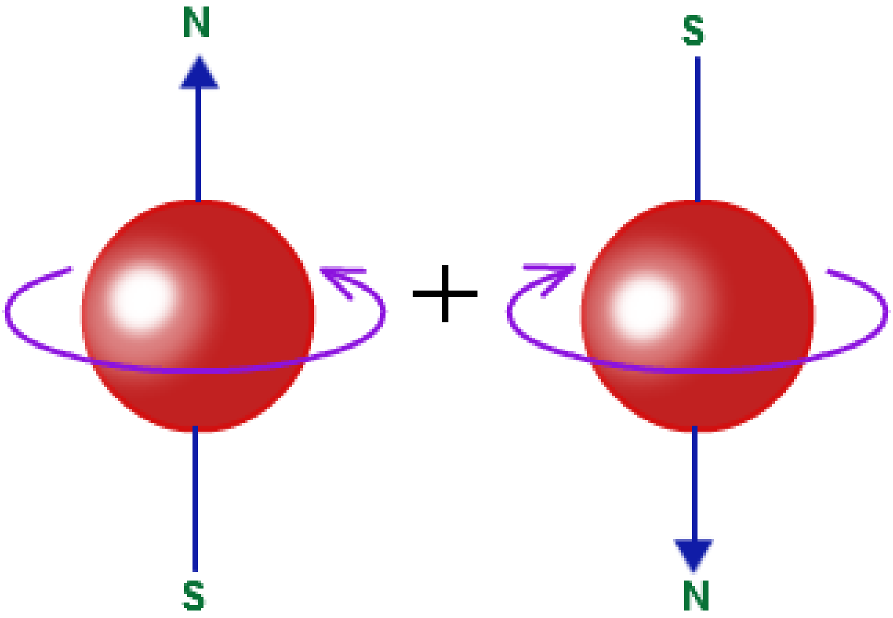
Quantum bit (Qubit)

Unit of quantum information

Classical bit



Quantum bit



$|0\rangle \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $|1\rangle \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Quantum bit (Qubit)

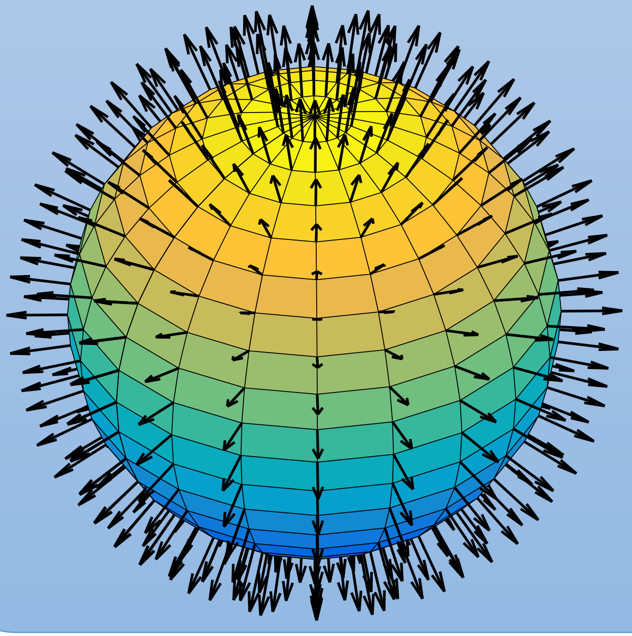
Mathematics of 1-Qubit

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$|\alpha|^2$

Probability that qubit to be in $|0\rangle$

$|\alpha|^2 + |\beta|^2 = 1$



$|\beta|^2$

Probability that qubit to be in $|1\rangle$

```
[X,Y,Z] = sphere(20);
[U,V,W] = surfnorm(X,Y,Z);
figure
quiver3(X,Y,Z,U,V]);
hold on
surf(X,Y,Z)
```

Infinite number of possibility

Quantum bit (Qubit)

Power of qubit: Quantum Parallelism

Two qubit state representing a system of two electron, can be in four different states at once (00, 01, 10, and 11),

$$|\Psi_4\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

$|\alpha|^2$ probability of outcome $|00\rangle$

$|\beta|^2$ probability of outcome $|01\rangle$

$|\gamma|^2$ probability of outcome $|10\rangle$

$|\delta|^2$ probability of outcome $|11\rangle$

n-qubit state representing a system of n electron, can be in 2^n different states at once.

The exponential increase!

It's possible to perform multiple calculations on 2^n numbers simultaneously.

Quantum bit (Qubit)

Computational basis generator

```
n=3; %number of qubit
N=2^n;%number of computational basis
I = eye(N);%identity matrix
for k=0:N-1;
h=dec2bin(k,n);%decimal to binary
disp(['|',eval(['num2str(k)']),...
'>=','|',eval(['num2str(h)']),'>='])
disp([eval(['I(:,k+1)'])])%binary number to vector
end
```

Two qubit- 4 basis

|0>=|00>=	|1>=|01>=	|2>=|10>=	|3>=|11>=
1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Three qubit- 8 basis

|0>=|000>=	|1>=|001>=	|2>=|010>=	|3>=|011>=
1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

|4>=|100>=	|5>=|101>=	|6>=|110>=	|7>=|111>=
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Quantum bit (Qubit)

Random qubit generator

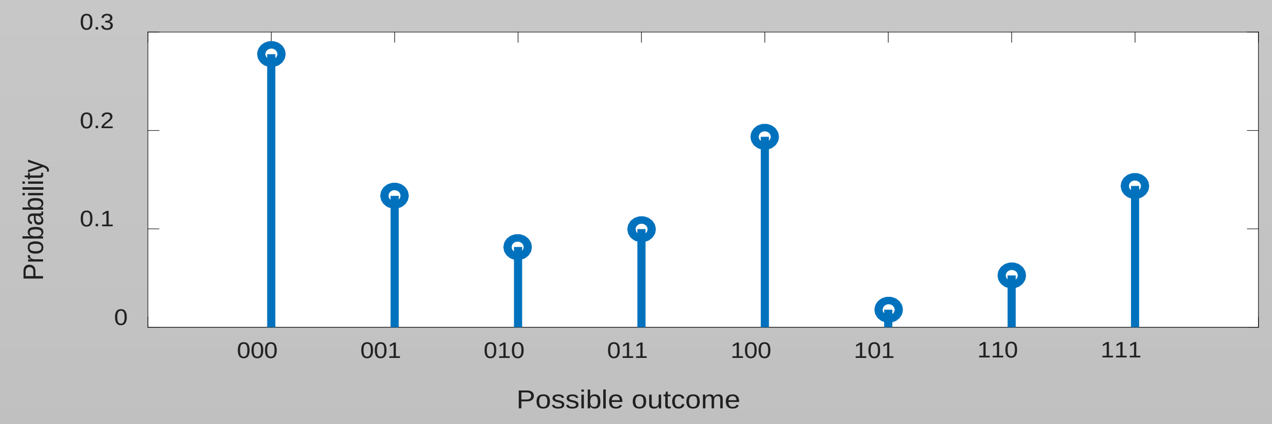
```
n=3;%number of qubits
N=2.^n;%number of registers
a=rand(N,1);%random vector
b=rand(N,1);%random vector
complex=a+1i*b;%complex random vector
qubitN=complex./norm(complex,2)%n-qubit state
prob=abs(qubitN).^2;% probability of each state
stem(prob,'LineWidth',6,'Markersize',15);hold on
xlim([0 N+1]);set(gca,'fontsize',30);
Y=char(blanks(1),dec2bin(0:N-1,n));%qubit x-axis label
set(gca,'XTickLabel',Y);
```

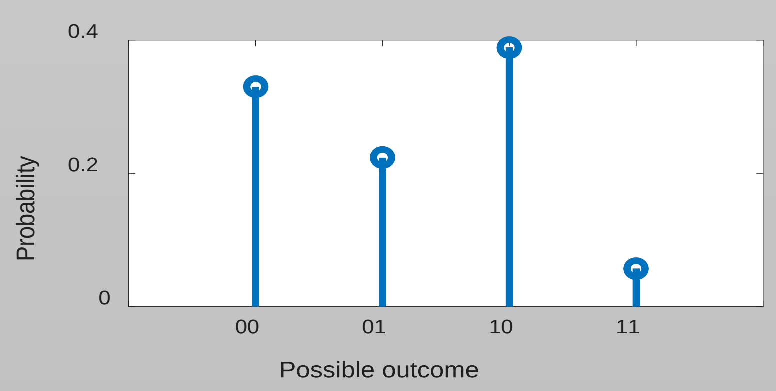
qubit3 =

(0.3920 + 0.1079i)|000>
+(0.0500 + 0.3045i)|001>
+(0.1888 + 0.3629i)|010>
+(0.2653 + 0.2943i)|011>
+(0.1172 + 0.1644i)|100>
+(0.3048 + 0.3791i)|101>
+(0.2255 + 0.1032i)|110>
+(0.1728 + 0.2154i)|111>

qubit2 =

(0.5506 + 0.1419i) |00>
+(0.2962 + 0.3240i) |01>
+(0.0087 + 0.4453i) |10>
+(0.3826 + 0.3731i) |11>

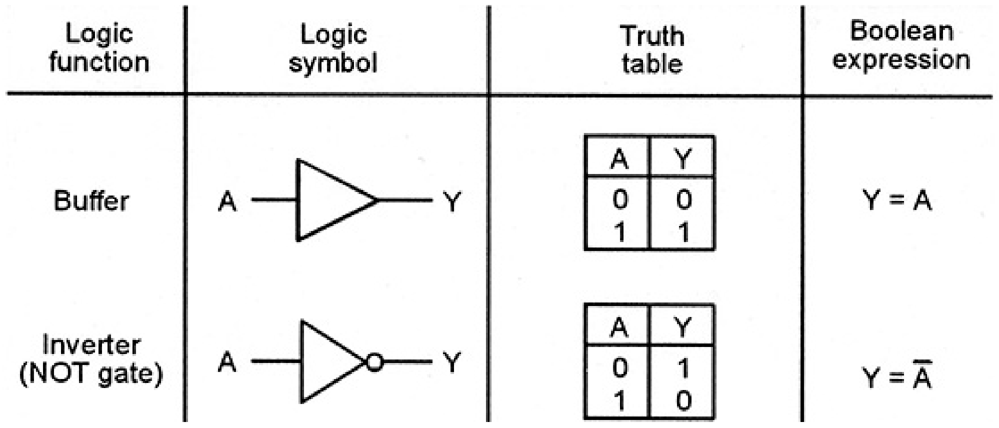




Quantum gates

Building block of a quantum circuit

- Classical 1-bit gate



Logic function	Logic symbol	Truth table	Boolean expression
Buffer	A → Y	A: 0, 1; Y: 0, 1	Y = A
Inverter (NOT gate)	A → Y	A: 0, 1; Y: 1, 0	$Y = \overline{A}$

There are only two 1-bit gates

Since the output can be only 0 or 1.

- Quantum 1-bit gate

$$|x\rangle \text{ — } \boxed{U} \text{ — } \alpha|0\rangle + \beta|1\rangle$$

Example: Hadamard gate

$$|0\rangle \text{ — } \boxed{H} \text{ — } \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$$

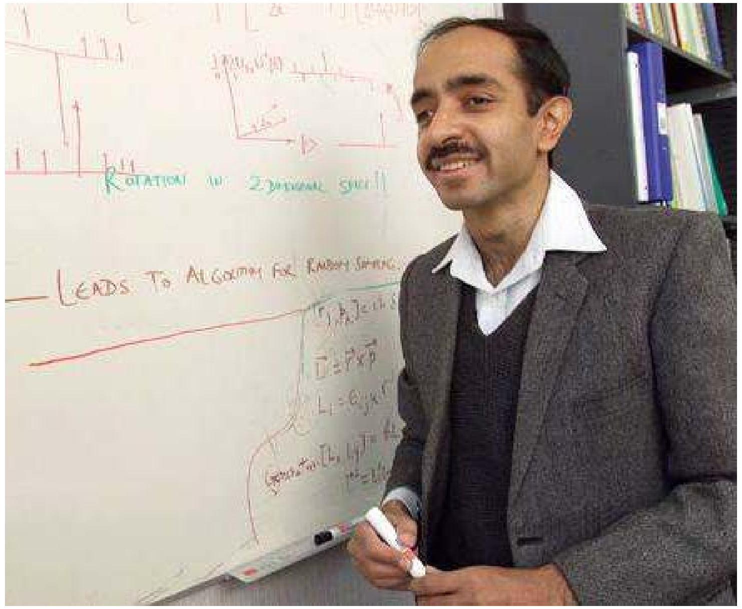
$$|1\rangle \text{ — } \boxed{H} \text{ — } \frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$$

<u>Uncountable number of 1-qubit gates!!</u>

Quantum Database Search

Quantum mechanics helps in searching for a needle in a haystack-
Lov Grover-PRL, 1997

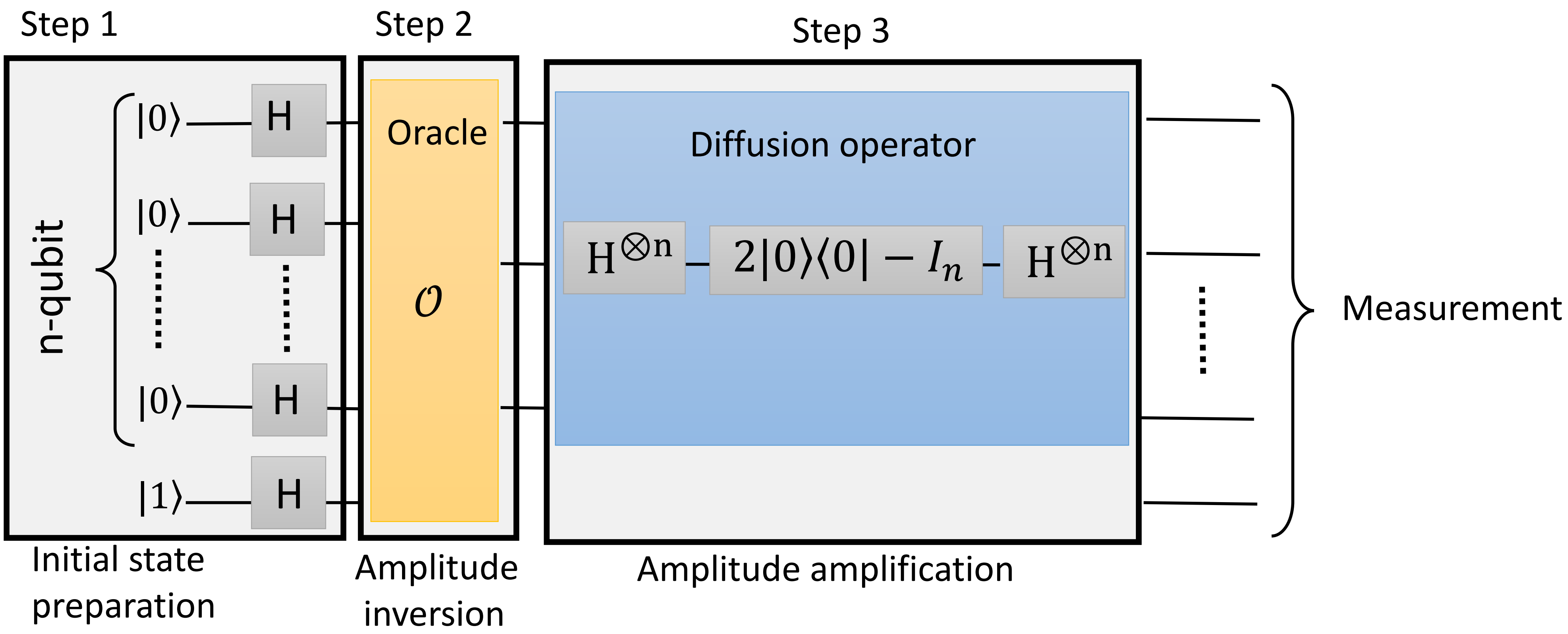
Performs a search over an unordered set of $N=2^n$ items to find the unique element that satisfies some condition.



The probability of the desired answer is increased by repeating the algorithm. In this way the probability of the failure answer is decreased.

Quantum Database Search

Quantum algorithm for search space of size $N=2^n$

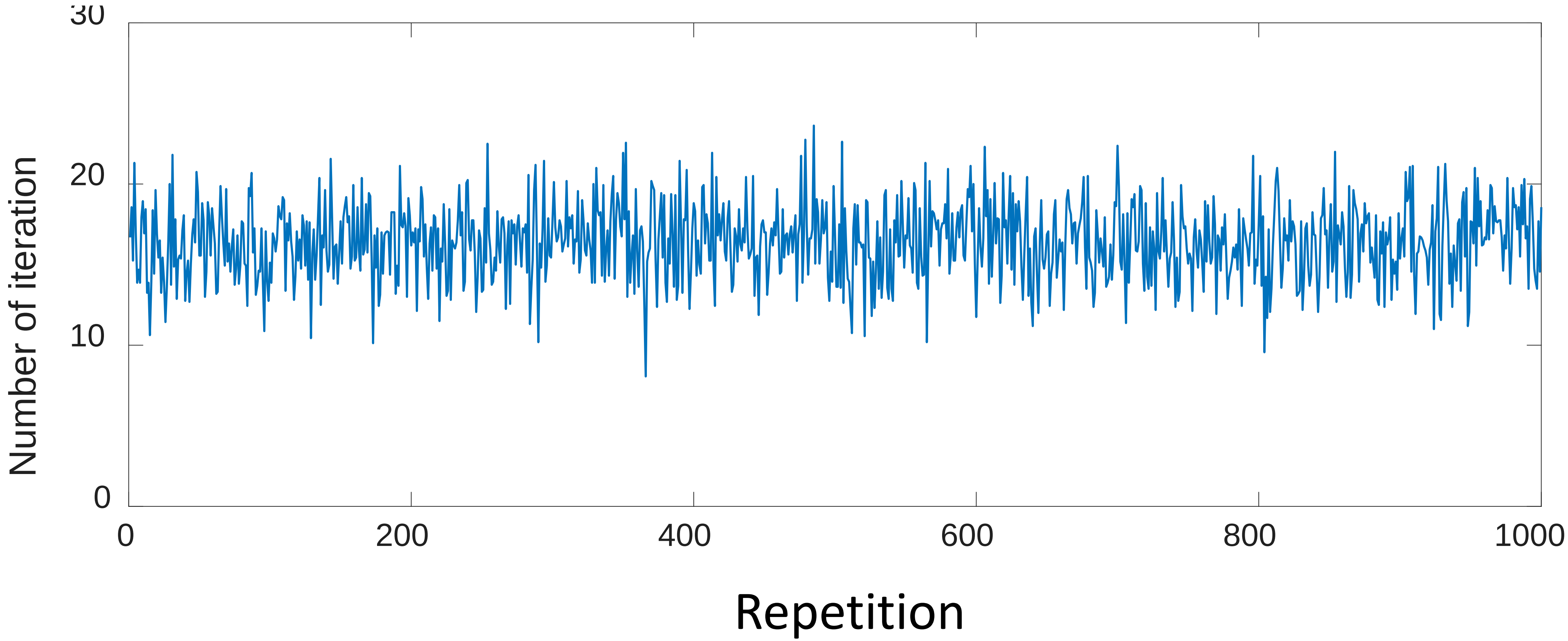


Quantum Database Search

Classical algorithm

Number of iteration to complete the search on average is N/2

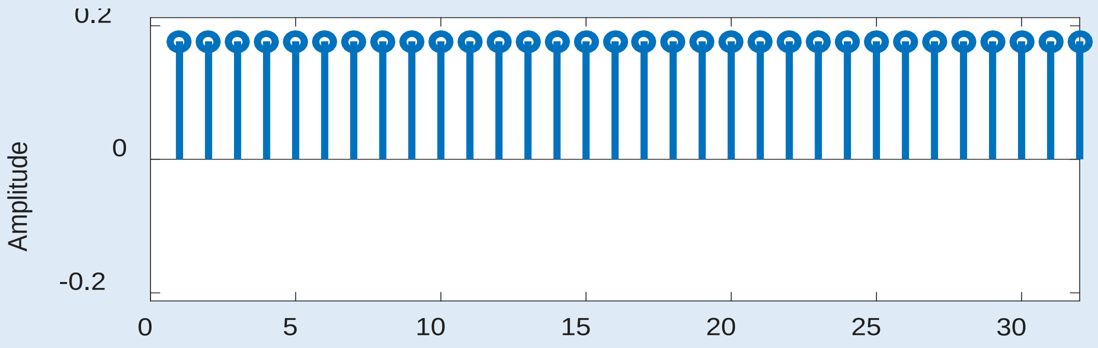
No classical algorithm can do better than this!



Quantum Database Search

Numerical modeling N=$32=2^5$

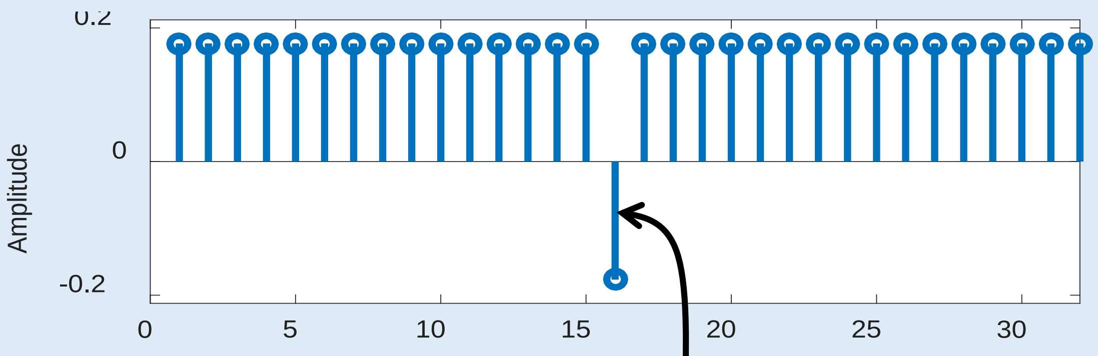
- Step 1 Initial state preparation



$$|\Psi\rangle = \frac{1}{\sqrt{32}}\left(|0\rangle + |1\rangle + \cdots + |16\rangle + \cdots |31\rangle\right)$$

Measurement of the initial state results the same probability for all items, $P=\left(\frac{1}{\sqrt{32}}\right)^2 \approx 0.1768$

- Step 2 Amplitude inversion

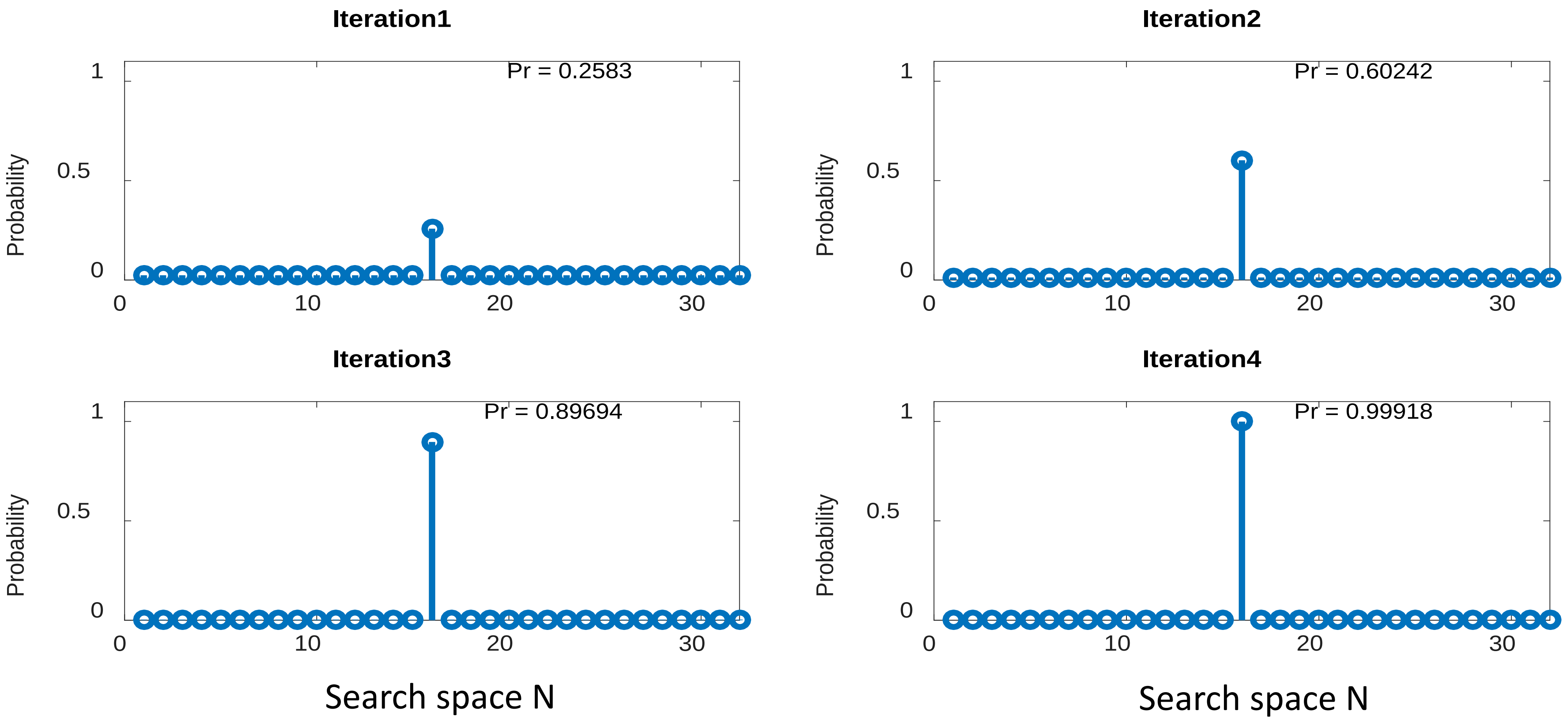


$$|\Psi\rangle = \frac{1}{\sqrt{32}}\left(|0\rangle + |1\rangle + \cdots - |16\rangle + \cdots |31\rangle\right)$$

Quantum Database Search

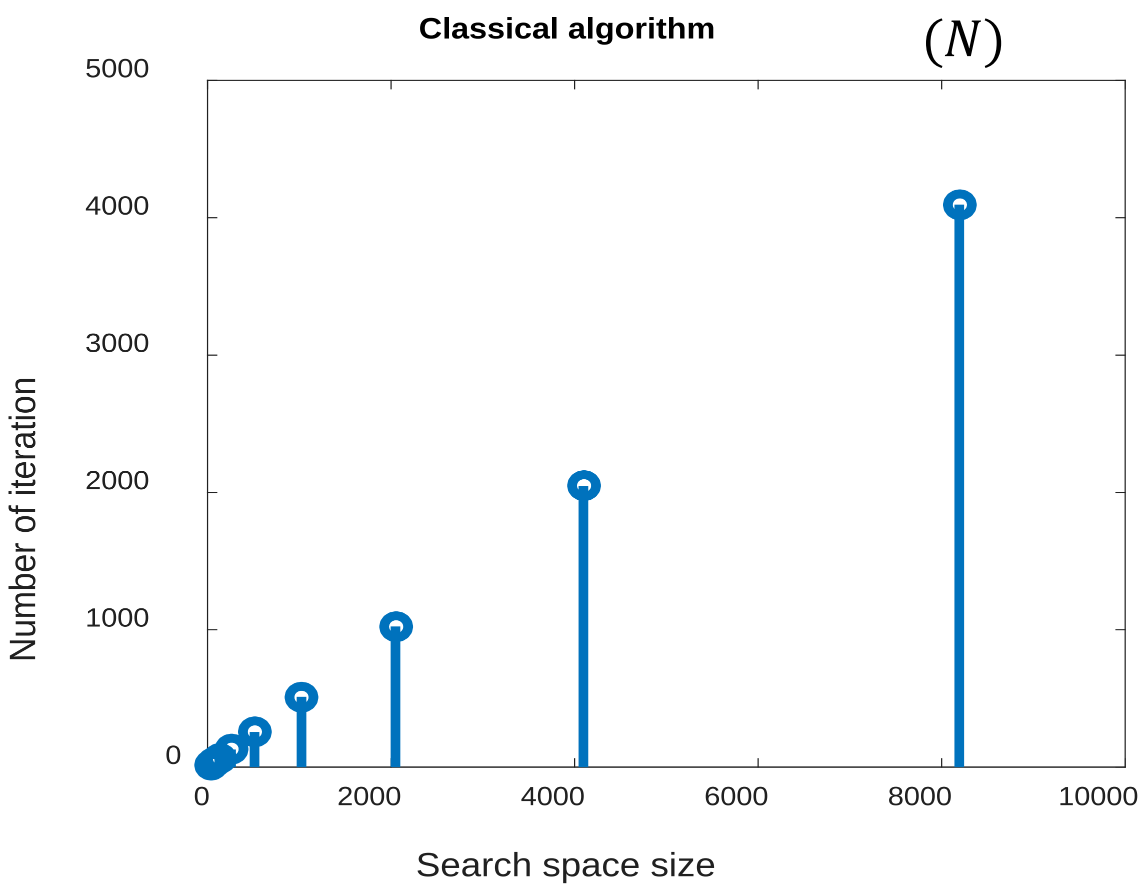
Numerical modeling N=32=2^5

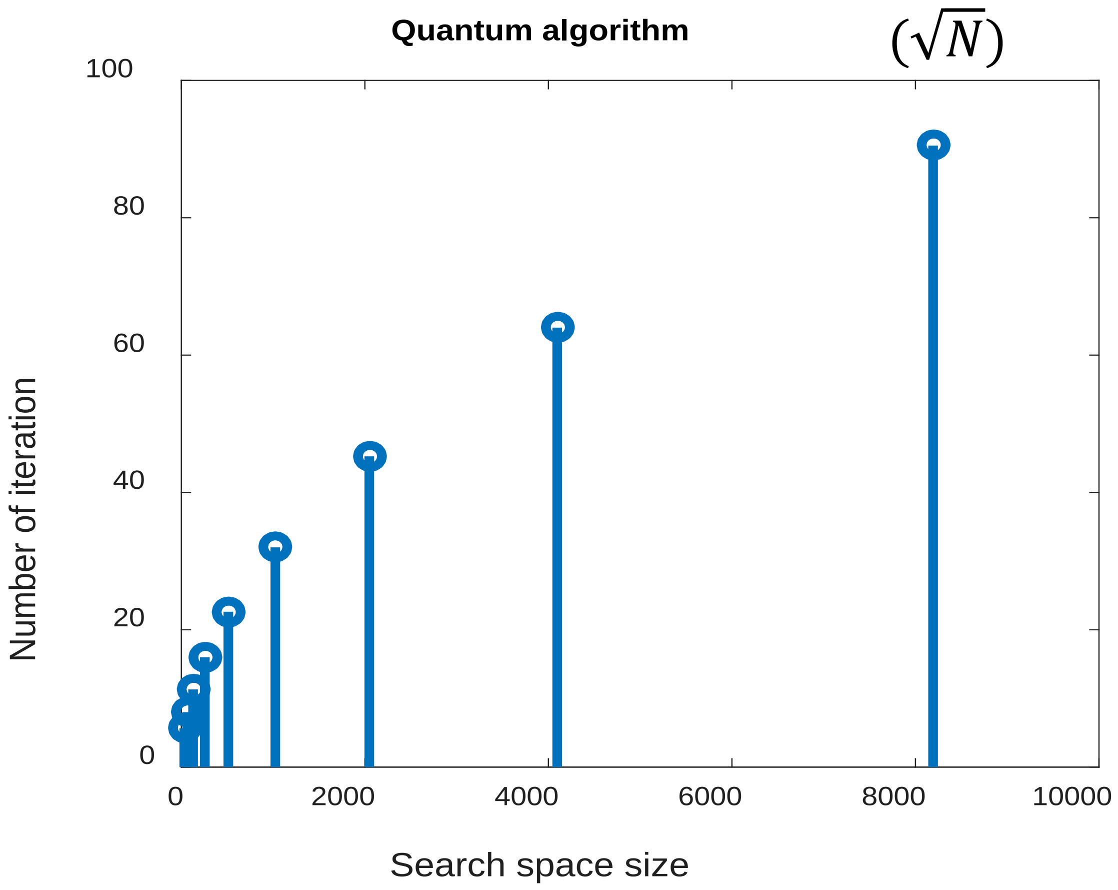
- Step3: Amplitude amplification



Quantum Database Search

Quadratic speed up in computational time



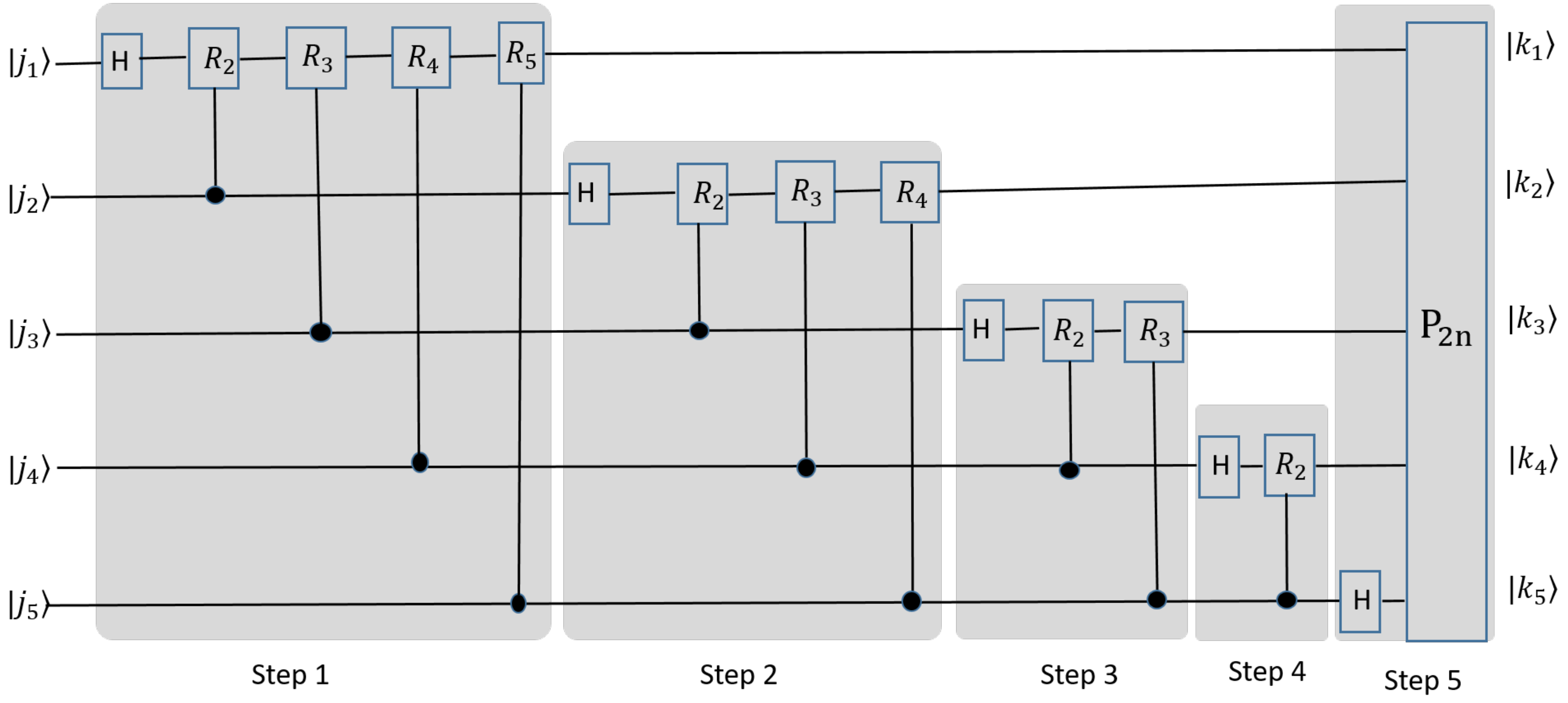


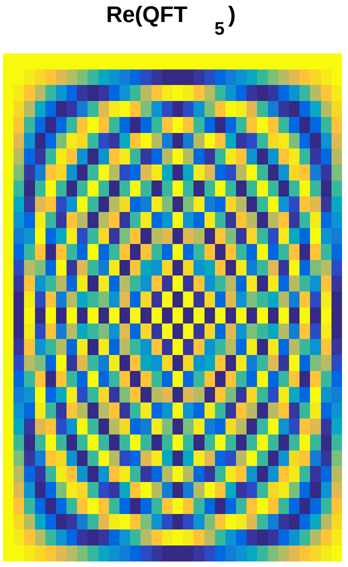
Quantum Fourier Transform

DFT	QFT
DFT: $(x_0, \ldots, x_{N-1}) \mapsto (y_0, \ldots, y_{N-1})$	QFT: $\sum_{j=1}^{N-1} x_j \vert j\rangle \mapsto \sum_{j=1}^{N-1} y_k \vert k\rangle$
$y_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} x_j e^{i2\pi jk/N}$	$\vert j\rangle \mapsto \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} x_j e^{i2\pi jk/N} \vert k\rangle$
DFT on $N = 2^n$ elements requires $O(n2^n)$ gates	QFT on $N = 2^n$ elements requires only $O(n^2)$ gates (Exponential speed up!)

Quantum Fourier Transform

Simulation of QFT algorithm for 5-qubit
(32-element vector)





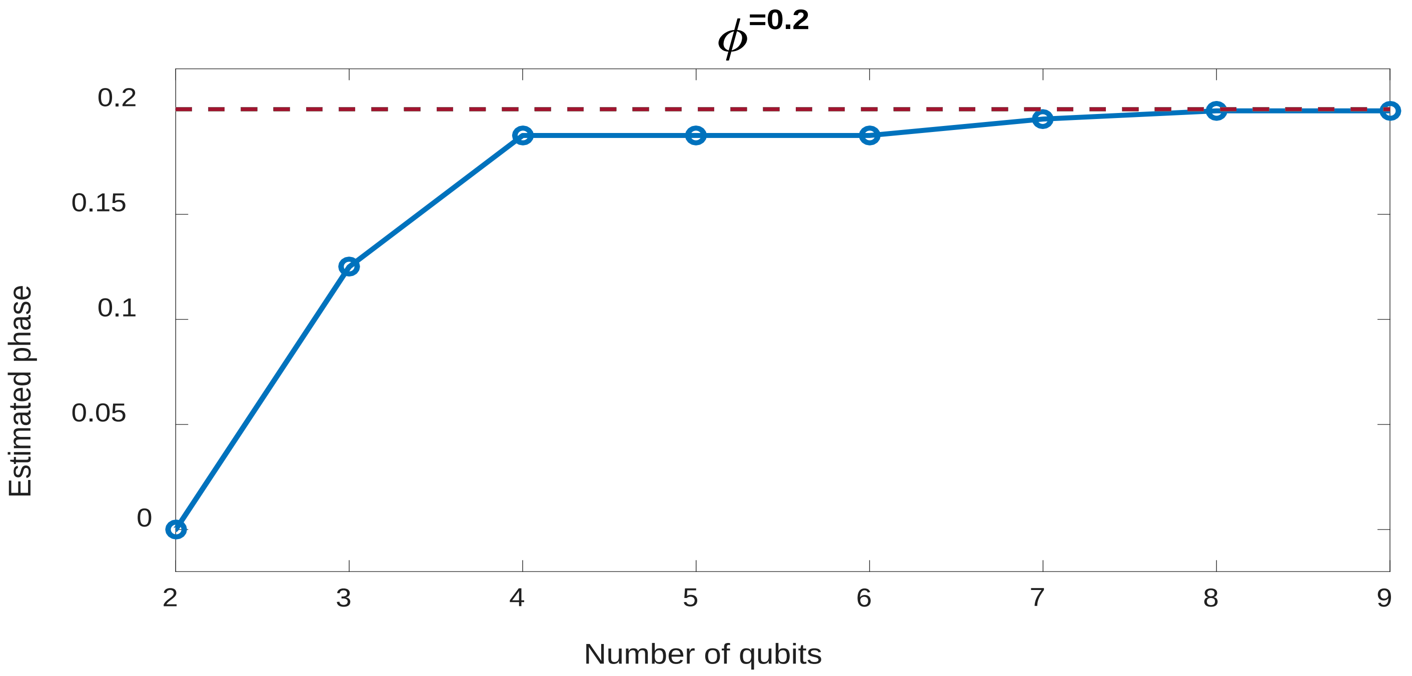
Quantum Phase Estimation

Eigenvalue-eigenvector estimation

U is a unitary operator

$$U|0\rangle = e^{i\varphi} \quad |0\rangle$$
$$U|1\rangle = e^{-i\varphi} |1\rangle$$

Find the eigenvalue
is equivalent to estimating
the phase φ



Quantum Finite Difference Modeling

Strategy

Modeling in the frequency domain

$$\nabla^2 \tilde{p}(\vec{r}, \omega) + \frac{\omega^2}{c^2} \tilde{p}(\vec{r}, \omega) = \tilde{f}(\vec{r}, \omega)$$

Classical algorithm

$$O(n_s n_t N^3)$$

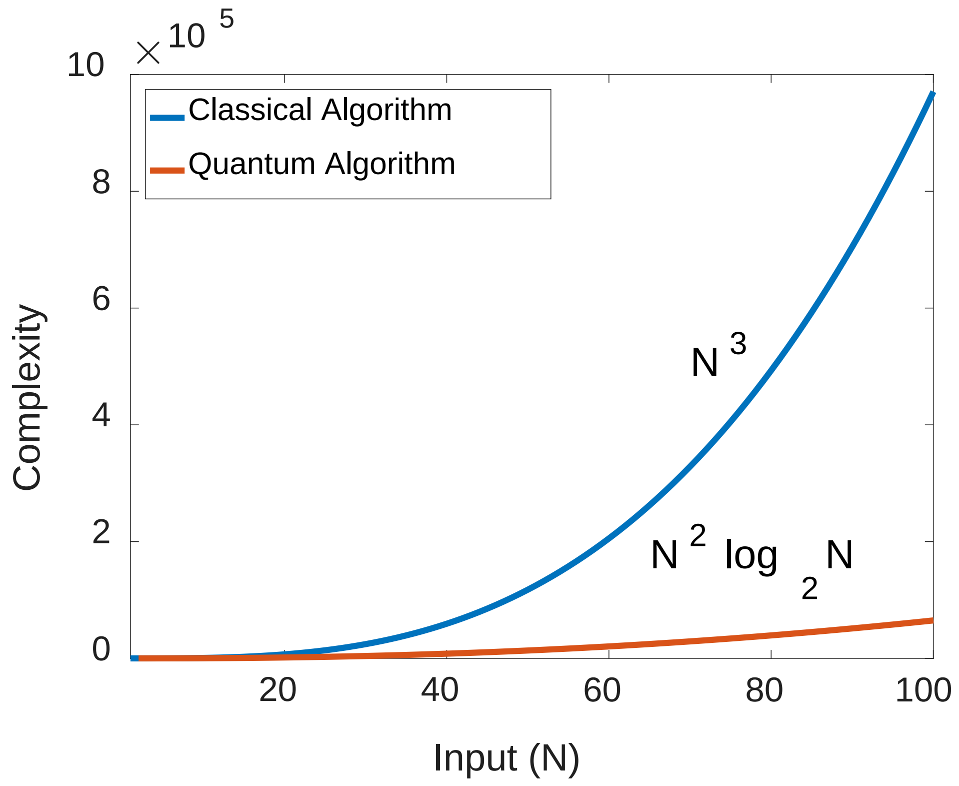
N Number of grids

n_s Number of shots

n_t Number of time steps

Quantum Algorithm

$$O(\text{poly}[\log(n_s), \log(n_t)]\, N^2 \log N\,)$$



Quantum Finite Difference Modeling

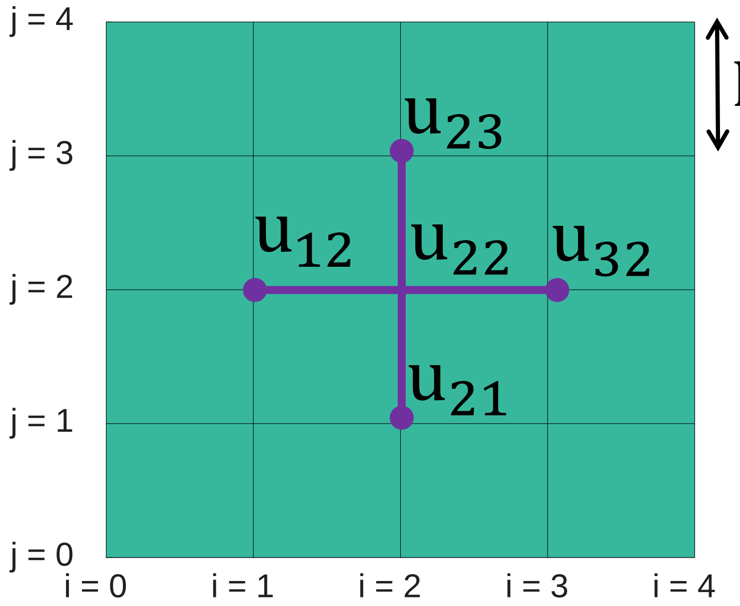
Strategy

2D Poisson equation

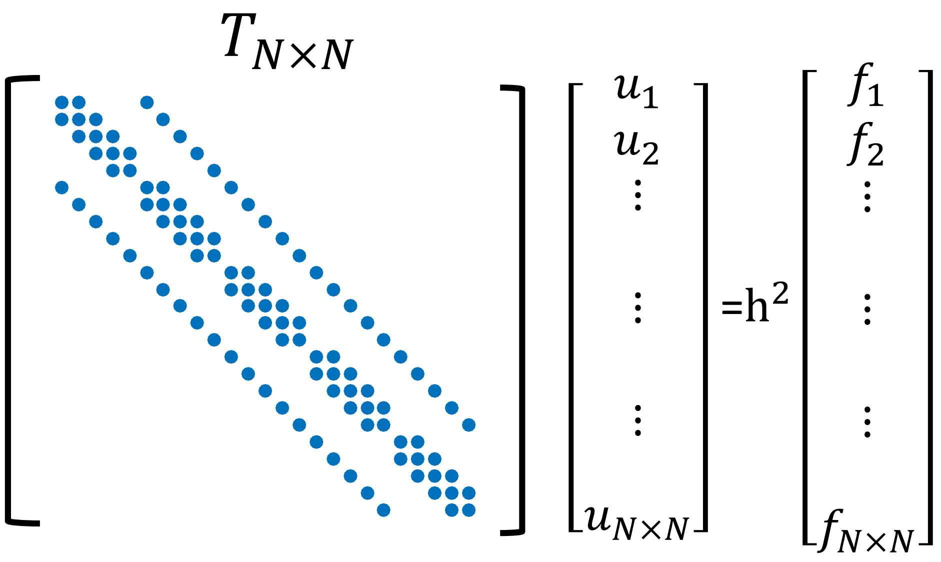
$$\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} = -f(x,y)$$

Discretization

$$u_{ij} = u(ih, jh)$$
$$f_{ij} = f(ih, jh)$$



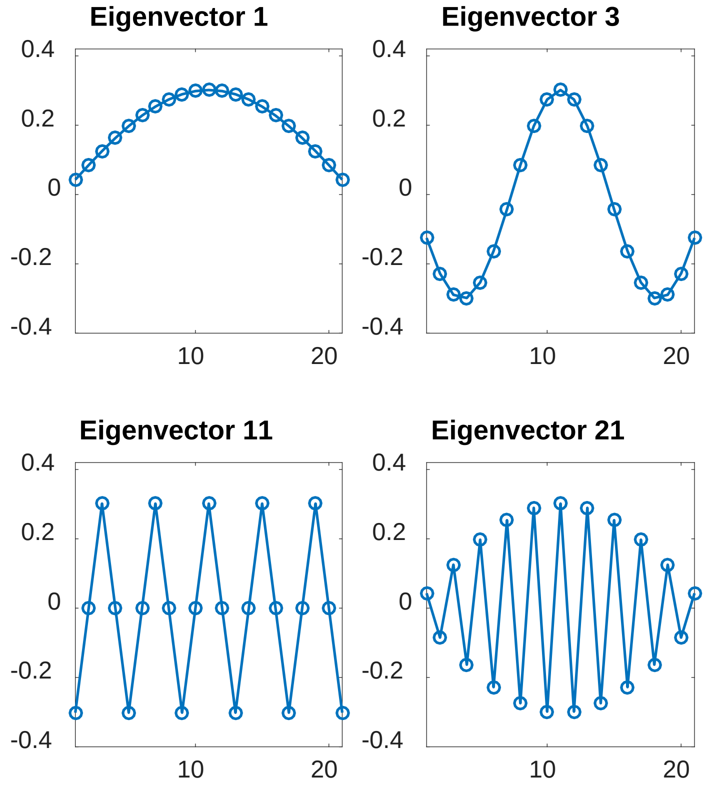
$$h = \frac{1}{N+1}$$



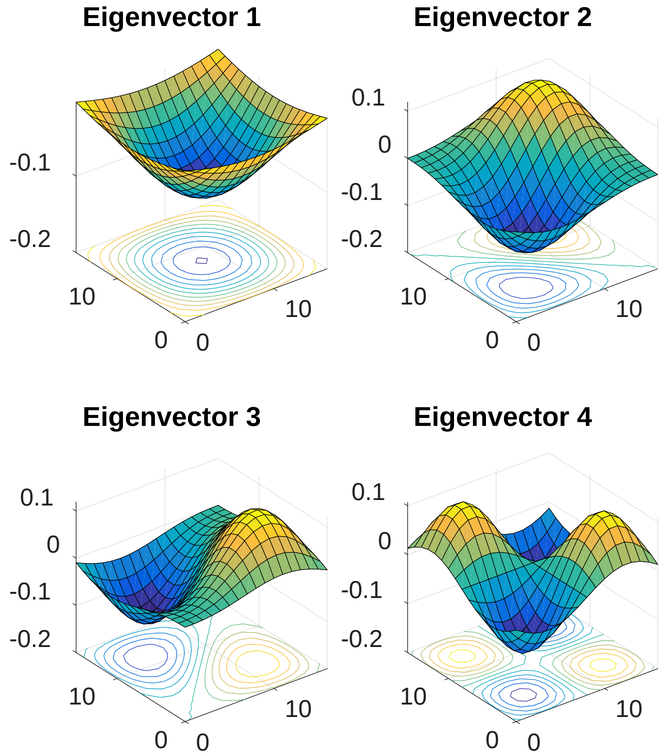
Quantum Finite Difference Modeling

Strategy

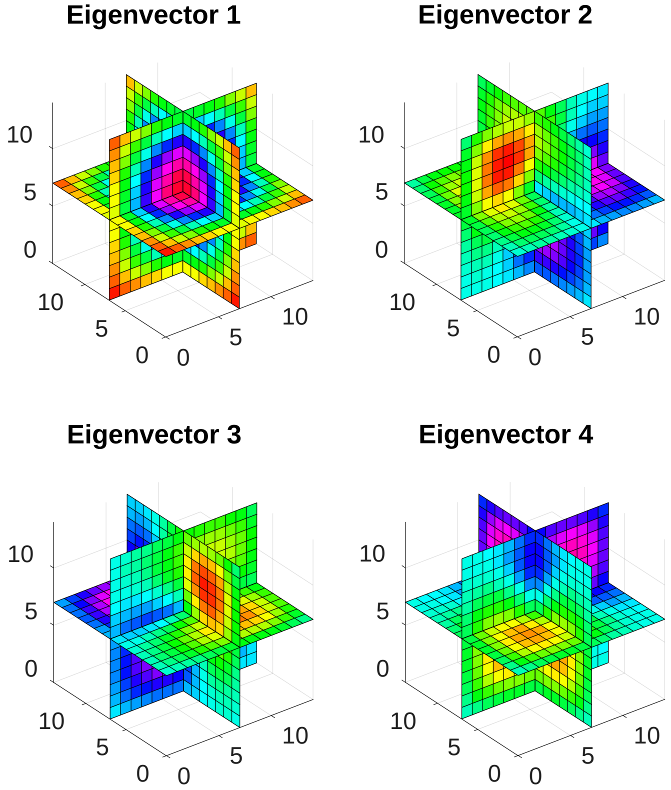
1 D Poisson operator T_N



2D Poisson operator $T_{N \times N}$



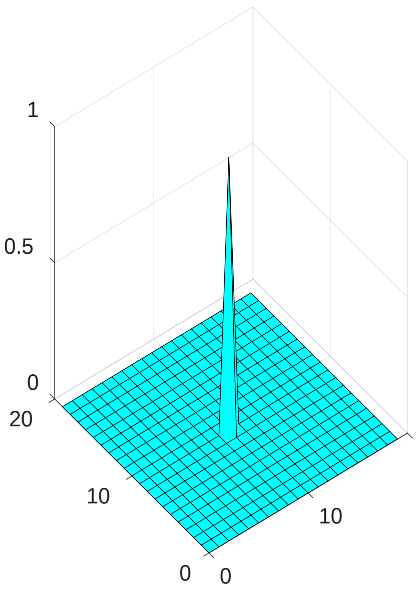
3D Poisson operator $T_{N \times N \times N}$



Quantum Finite Difference Modeling

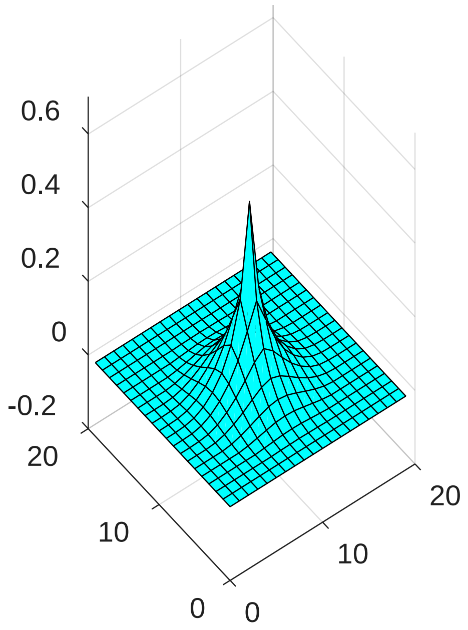
Strategy

Source: right hand side



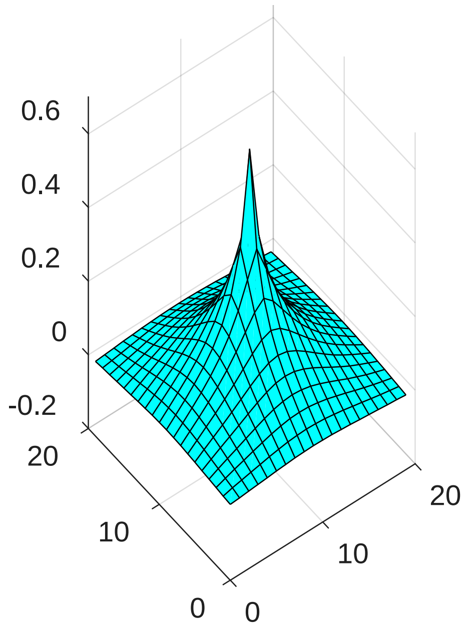
Generalized minimum residual method

iteration 10
relative residual 0.074



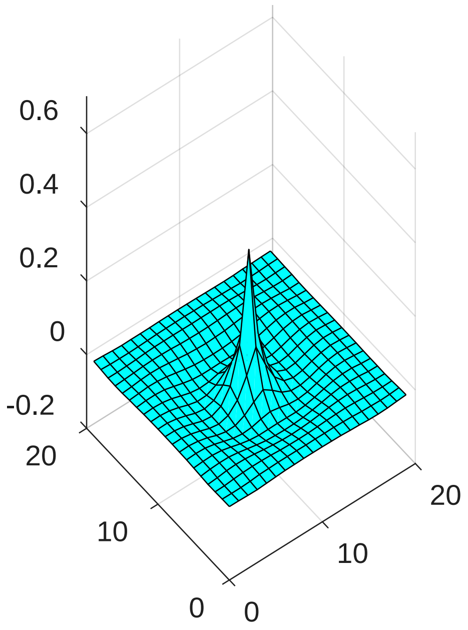
Preconditioned conjugate gradients method

iteration 20
relative residual 0.022

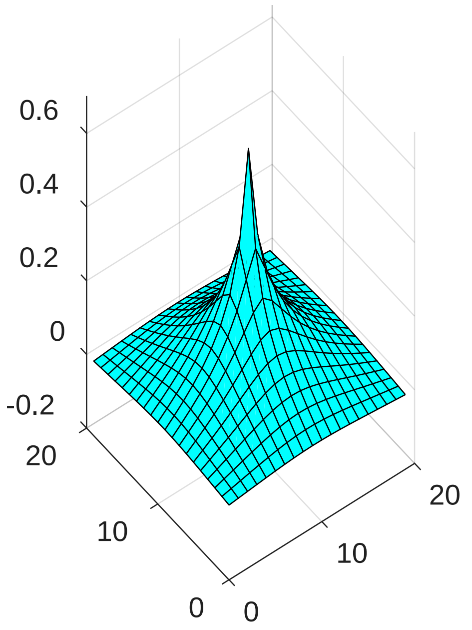


LSQR method

iteration 20
relative residual 0.16



Exact solution



Summary

- **Development**
  Develop a MATALAB toolbox for quantum computing

- **Ongoing research**
  Quantum algorithms for finite difference modeling
  (time domain and frequency domain)

- **Future work**
  Quantum algorithms for seismic migration and inversion (imaging)

Acknowledgement

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Thank you