Least Squares Reverse Time Migration in time and frequency domain

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Outline

- Introduction
- Time domain LSRTM
- Frequency domain LSRTM
- Numerical Examples
- Conclusions
- Acknowledgment

Introduction

- Reverse time migration
- Least-squares migration
- Full waveform inversion

Forward Modelling

- Time domain LSRTM

Scalar acoustic waves equation in 2D case

$$\frac{\partial^2 p(x,z,t)}{\partial x^2} + \frac{\partial^2 p(x,z,t)}{\partial z^2} - \frac{1}{v^2(x,z)}\frac{\partial^2 p(x,z,t)}{\partial t^2} = f(t)\delta(x - x_s) \qquad (1)$$

4th order FD in spatial and 2nd order in time

$$p^{n+1} = 2p^n - p^{n-1} + \Delta t^2 v^2 L_1 p^n + \Delta t^2 f^n \qquad (2)$$

$$p^n = 0, n \leq 0$$

p^n is the pressure at time step n, L_1 is the 9-point Laplacian operator and f^n is the source term.

Imaging Condition

The imaging condition in the LSRTM in 2D case can be expressed as follows:

$$I(x,z) = \sum_{n_s} \sum_{t} U^n(x,z,t,n_s) V^n(x,z,t,n_s) \qquad (3)$$

The $U^n(x,z,t,n_s)$ and $V^n(x,z,t,n_s)$ is the source wavefield and receiver wavefield at time step n and $I(x,z)$ is the image of the model.

Conjugate Gradient Method

Conjugate gradient (CG) is applied to the linear unconstrained optimization problem:

$$min\{f(x):\ x \in R^n\}$$

A linear conjugate gradient method generates a sequence x_k, starting from an initial guess $x_0 \in R^n$, using

$$x_{k+1} = x_k + \alpha_k d_k \qquad (4)$$

where α_k is the step length, and the searching directions d_k are generated by the rule:

$$d_{k+1} = -g_{k+1} + \beta_k d_k, \qquad d_0 = -g_0 \qquad (5)$$

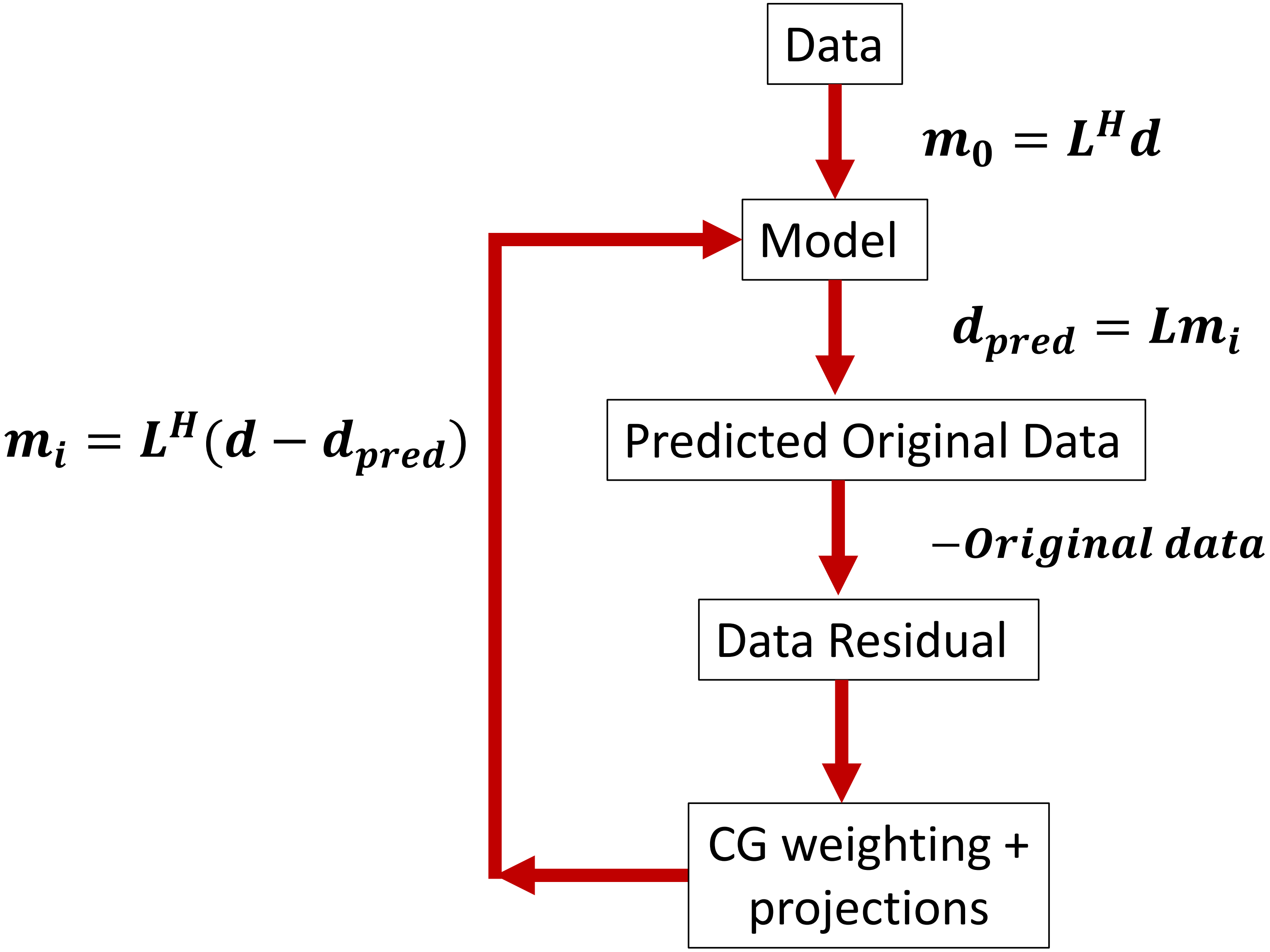
Here β_k is the CG update parameter and $g_k = \nabla f(x_k)$. Different CG methods corresponding to different choices for the scalar β_k.

$$\beta_k^{FR} = \frac{||g_{k+1}||^2}{||g_k||^2}$$ (Flecher and Reeves, 1964)

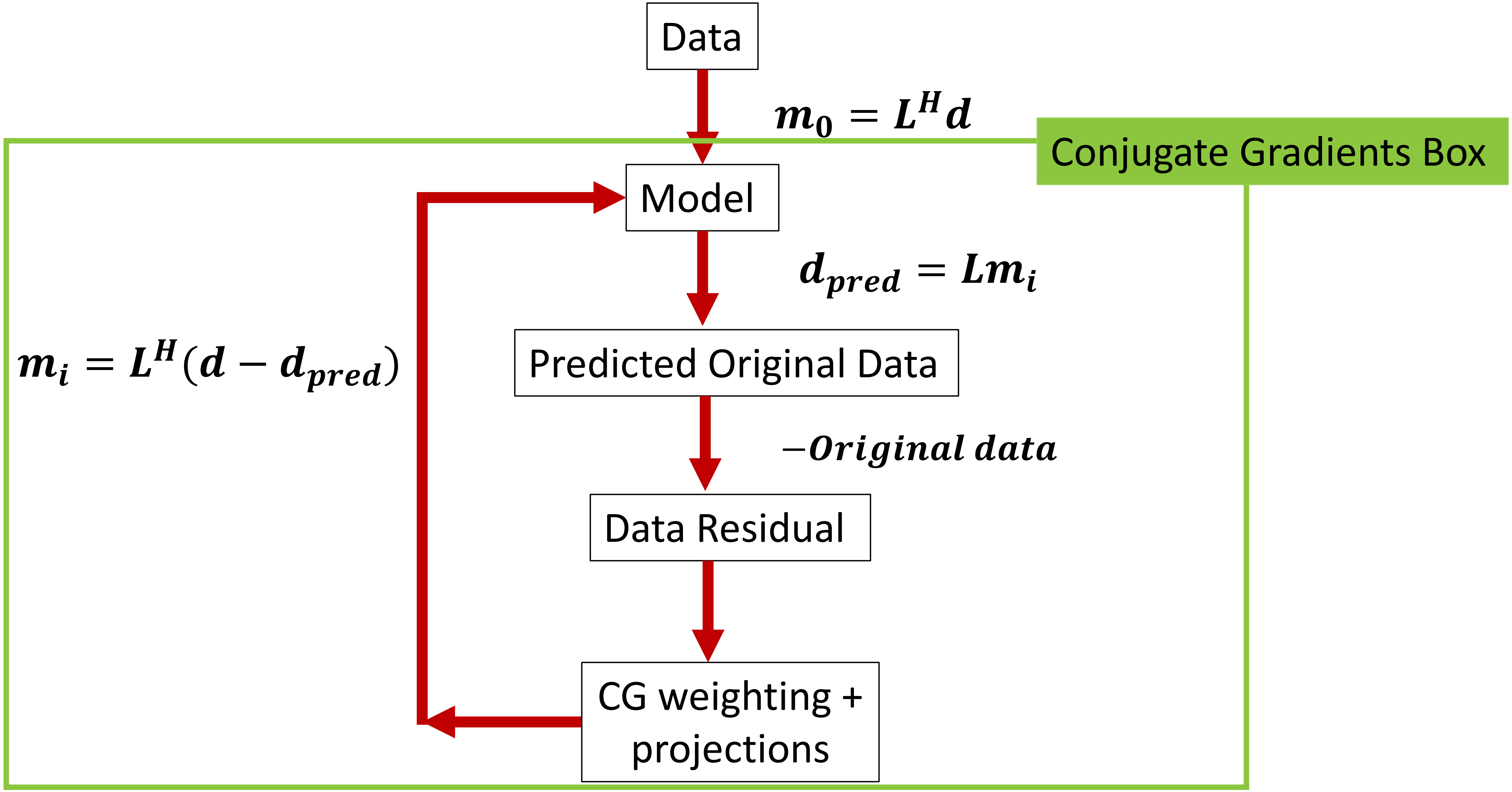
$$\beta_k^{HS} = \frac{g_{k+1}^T y_k}{d_k^T y_k}$$ (Hestenes and Stiefel, 1952)

$$\beta_k^{DY} = \frac{||g_{k+1}||^2}{d_k^T y_k}$$ (Dai and Yuan, 1999)

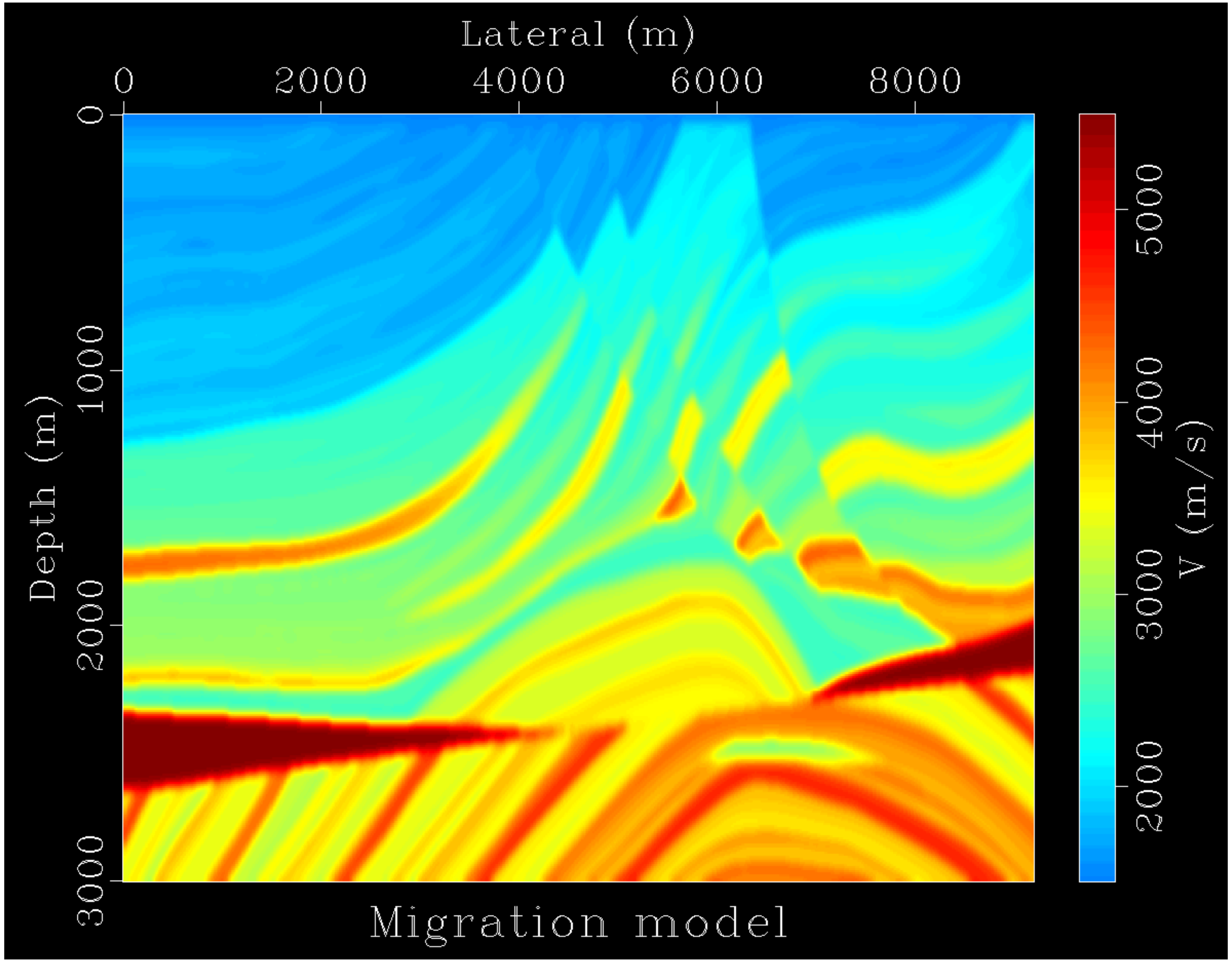
Conjugate Gradient Method



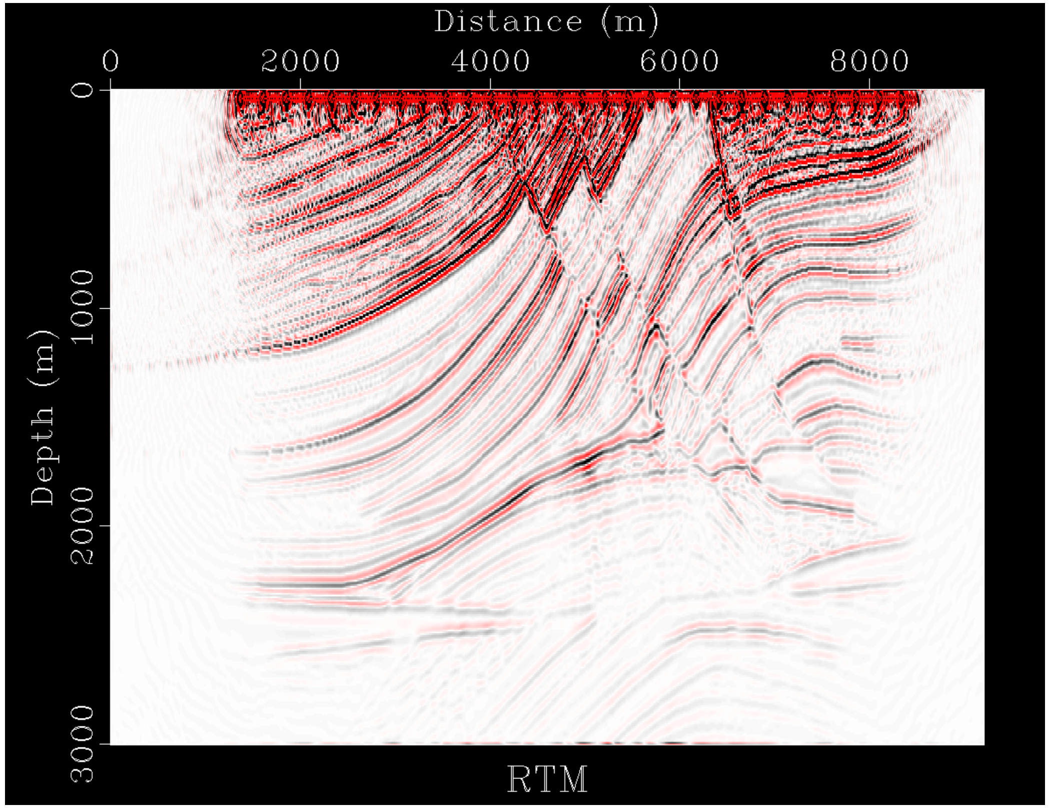
Conjugate Gradient Method

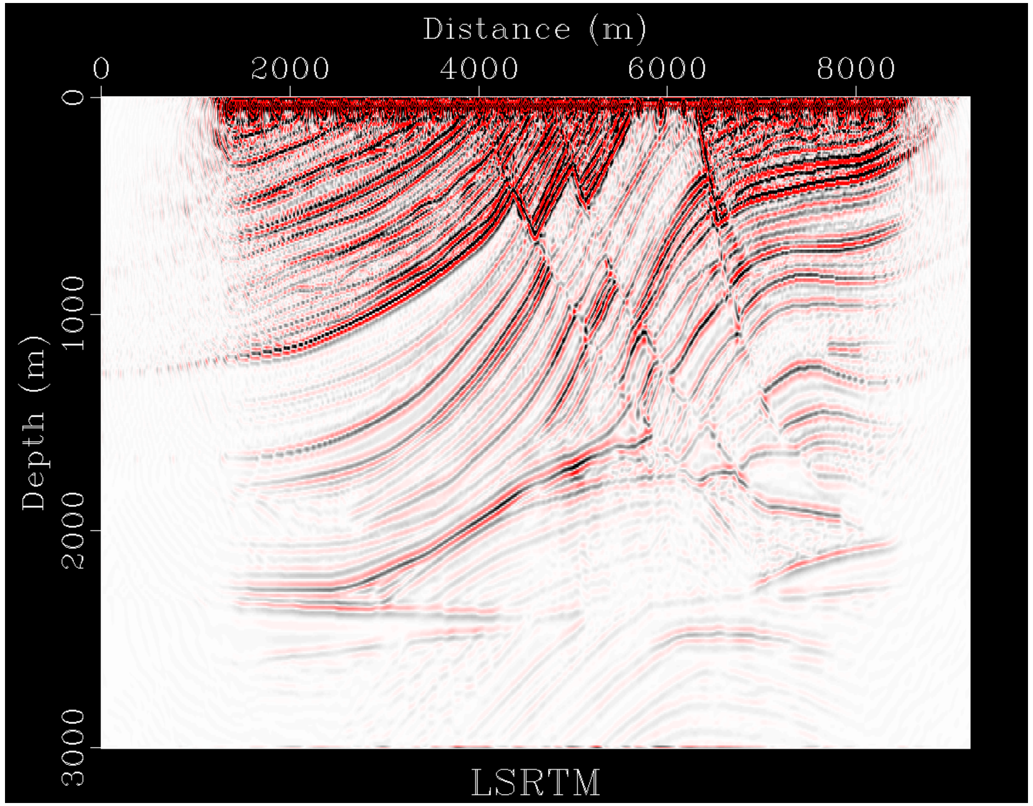


Examples


Lateral (m)
0
2000
4000
6000
8000
Depth (m)
0
1000
2000
3000
5000
4000
3000
2000
V (m/s)
Migration model

Examples





9 iterations

33 shots

Forward Modelling

- Frequency domain LSRTM

Scalar acoustic wave equation in frequency domain

$$\left(\frac{\omega^2}{v^2(x)} + \nabla^2\right) u(x, \omega, v) = f(\omega)\delta(x - x_s), \qquad (6)$$

We can rewrite this equation in matrix form

$$L(x, \omega; v)u(x, x_s, \omega) = f(\omega)\delta(x - x_s), \qquad (7)$$

where $L(x, \omega; v) = \left(\frac{\omega^2}{v^2(x)} + \nabla^2\right)$ is the discretized impedance matrix.

Imaging Condition

Using the adjoint state method (Plessix, 2006), the gradient can be calculated as

$$g(x) = -\sum_{x_g}\sum_{x_s}\sum_{\omega} Re\left(\omega^2 f(\omega) G(x_s, x, \omega) G\left(x, x_g, \omega\right) \Delta d^*\left(x_g, x_s, \omega\right)\right), \qquad (8)$$

In matrix form, this can be rewritten as (Virieux and Operto, 2009)

$$g(x) = -\sum_{x_g}\sum_{x_s}\sum_{\omega} u^T\left(x_s, x_g, \omega\right) \nabla_m L^T(m, \omega) L(m, \omega)^{-1} R^T \Delta d^*\left(x_s, x_g, \omega\right), \qquad (9)$$

Imaging Condition

Using the adjoint state method (Plessix, 2006), the gradient can be calculated as

$$g(x) = -\sum_{x_g}\sum_{x_s}\sum_{\omega} Re\left(\omega^2 f(\omega) G(x_s, x, \omega) G(x, x_g, \omega) \Delta d^*(x_g, x_s, \omega)\right), \qquad (8)$$

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Source wavefields

Back propagated receiver wavefields

Data space vs model space

The forward modelling process is

$$Am = d \qquad (10)$$

Usually the objective function is defined in the data domain:

$$J(m) = \frac{1}{2}\left|\left|d - Am\right|\right|^2, \qquad (11)$$

However, we have an image domain objective function:

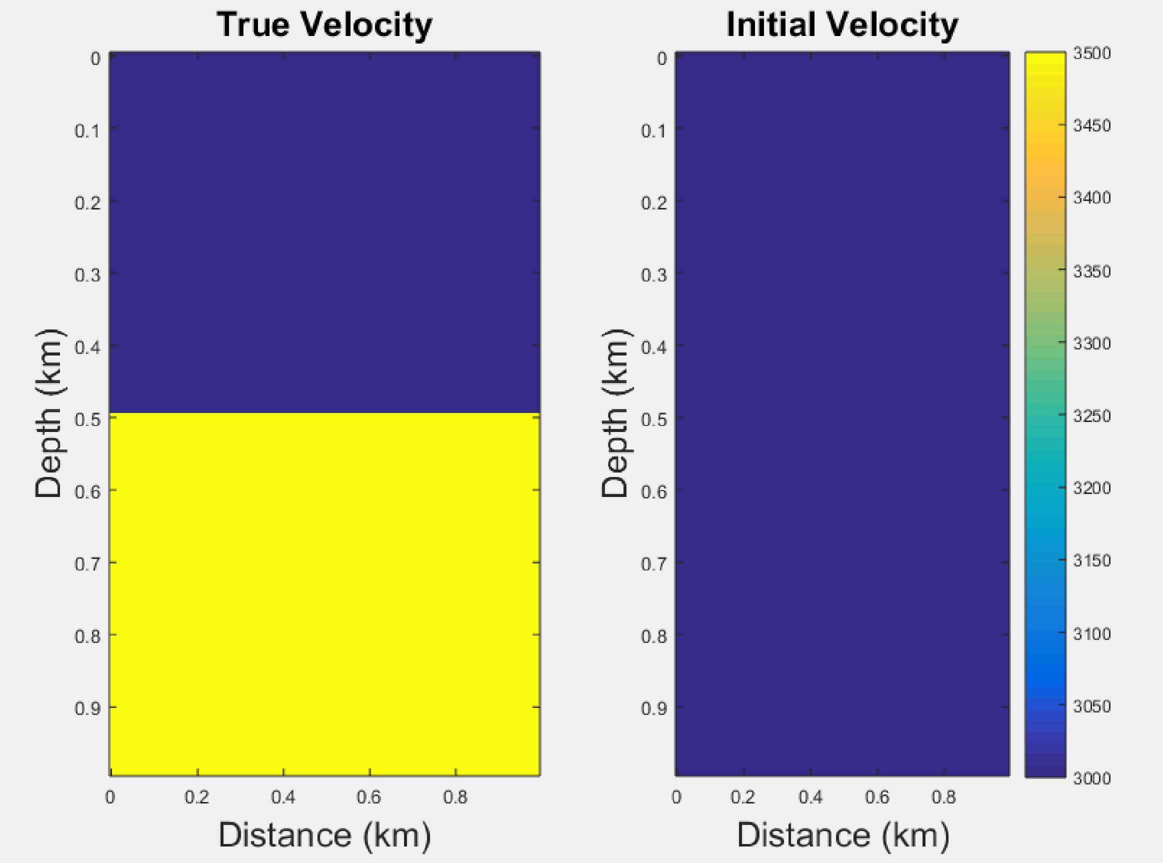
$$J(m) = \frac{1}{2}||A^T Am - A^T d||^2, \qquad (12)$$

The Hessian vector product (Métivier, 2013)

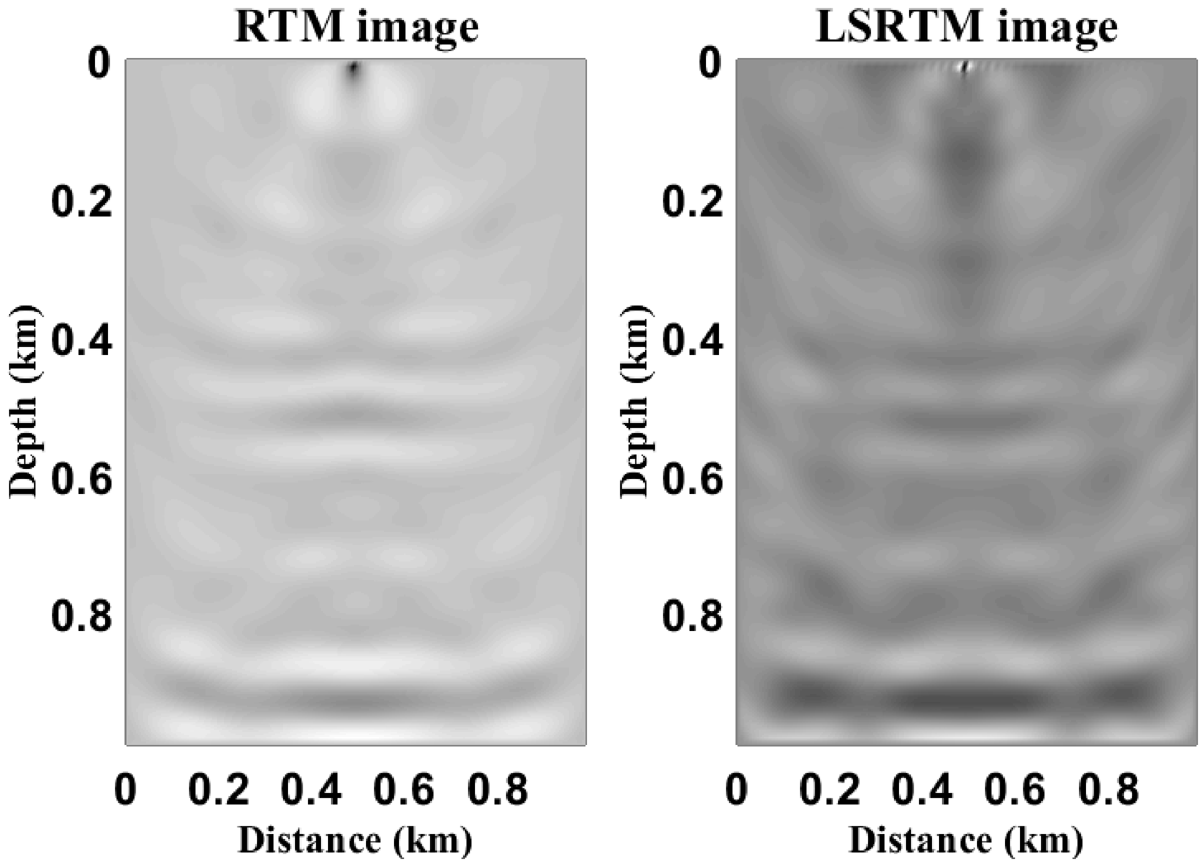
$$H_a v = u^T \nabla_m L^T (L^T)^{-1} R^T R (L^*)^{-1} \nabla_m L^* u^* v, \qquad (13)$$

where v is an arbitrary vector, usually setting as zero vector as the initial guess of the conjugate gradient method and R is the receiver coordinates.

Examples

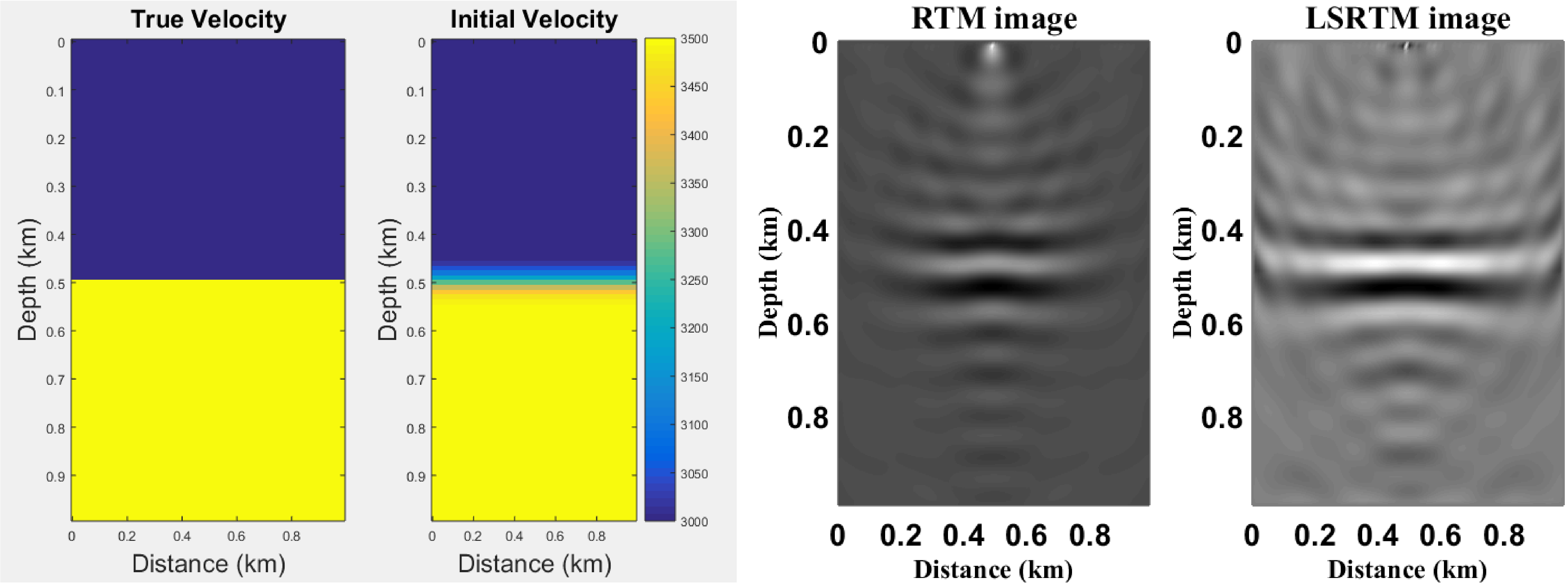


True Velocity
Initial Velocity
Depth (km)
Distance (km)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
3500
3450
3400
3350
3300
3250
3200
3150
3100
3050
3000

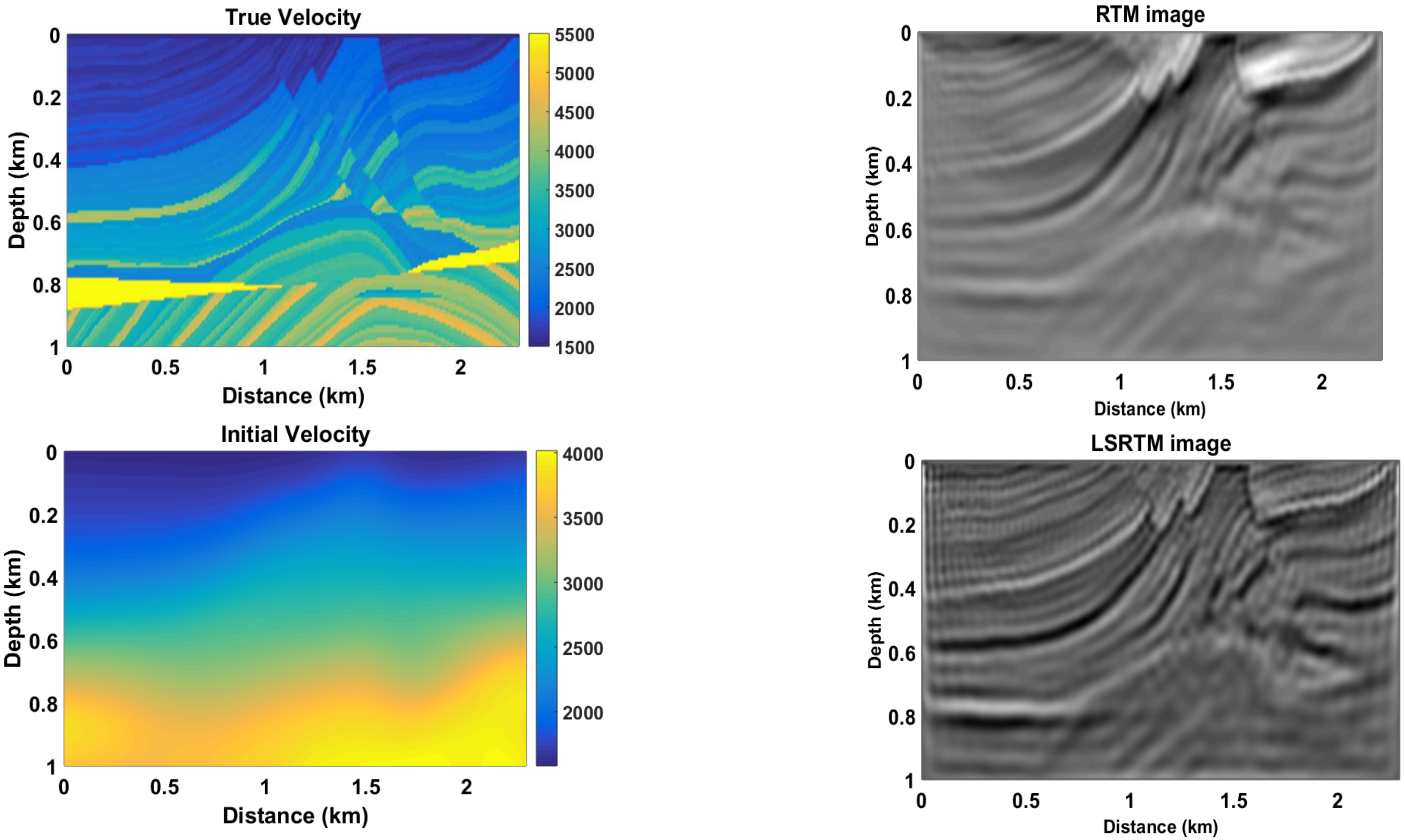


RTM image
LSRTM image
Depth (km)
Distance (km)
0
0.2
0.4
0.6
0.8

Examples


True Velocity
Initial Velocity
Depth (km)
Distance (km)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0.2
0.4
0.6
0.8
3500
3450
3400
3350
3300
3250
3200
3150
3100
3050
3000
RTM image
LSRTM image

Examples


True Velocity
Depth (km)
0
0.2
0.4
0.6
0.8
1
0
0.5
1
1.5
2
Distance (km)
5500
5000
4500
4000
3500
3000
2500
2000
1500
Initial Velocity
Depth (km)
Distance (km)
4000
3500
3000
2500
2000
RTM image
Depth (km)
Distance (km)
LSRTM image
Depth (km)
Distance (km)

Conclusions

- LSRTM need correct background velocity model to produce decent result
- The objective function for time domain LSRTM is in data domain while for frequency domain LSRTM, the objective function is in imaging domain. Both methods can improve the image
- Future work: The function of Hessian in the algorithm and compare the Hessian vector product with Born modelling operator

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Acknowledgment

Thank you!