Log-validated and data-validated waveform inversion with one-way wave equation migration

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Outline

- Introduction
  - ✓ Inversion process
- Data validation
- Log validation
- Application
  - ✓ Synthetic examples
    - Anticline model
    - Marmousi model
  - ✓ Hussar dataset
- Conclusions

Waveform inversion

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Waveform inversion

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2) Reflectivity residual $\delta R = Stk[Mig(\delta d)]$

$$\delta R = \int \sum_{s,r} \frac{\delta U_r(x,z,\omega) D_s^*(x,z,\omega)}{D_s(x,z,\omega) D_s^*(x,z,\omega) + \mu I_{max}(z)} d\omega$$

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Data validation

$$\delta m = \mu g$$

- Line search
- Minimize data residuals

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Data validation

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$$\alpha_k = 0.05\sqrt{\frac{< m_k|m_k >}{< g_k|g_k >}}$$

$$m_{k+1} = m_k + \mu_k Imp(\delta R)_k$$

$$\mu_k = \beta \alpha_k$$

↓ minimizes

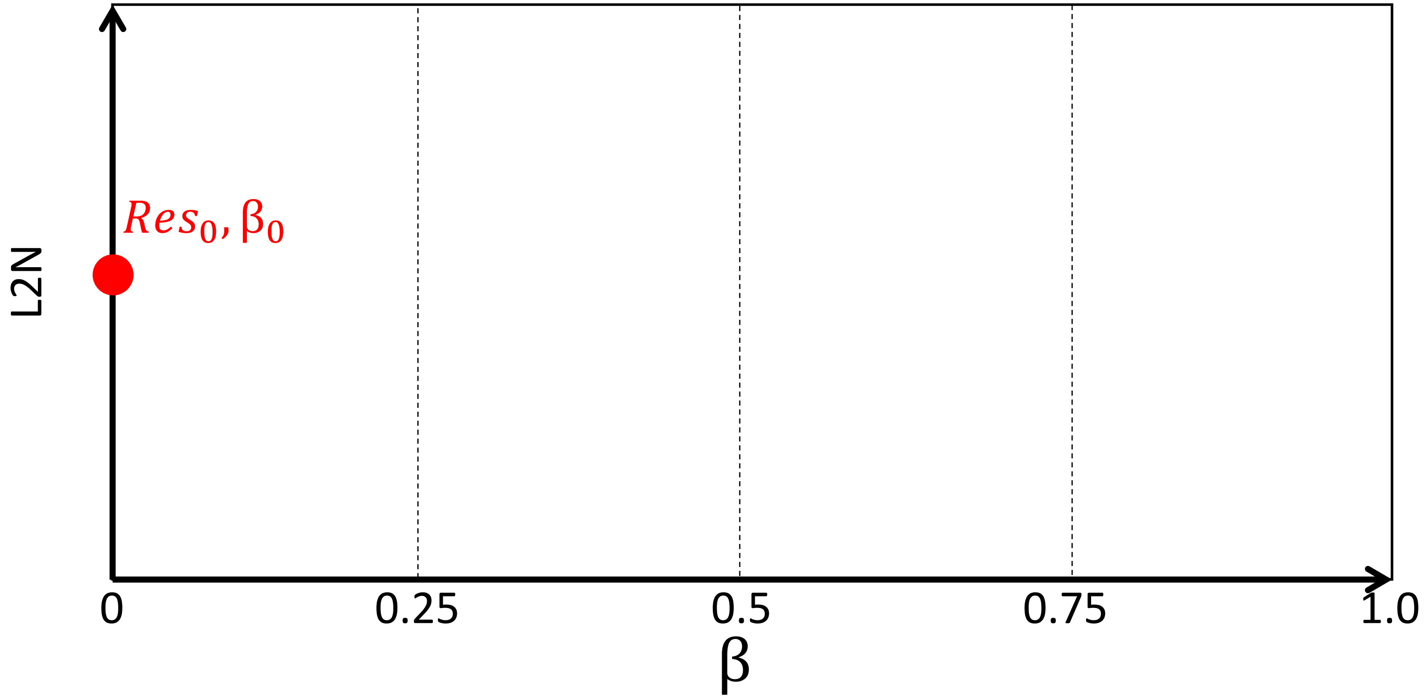
$$Res = sum[(d_0 - d_m)^2]$$

Data validation

$$m_{k+1} = m_k + \mu_k g_k$$

$$\mu_k = \beta \alpha_k$$

$Res_0(m_k), \beta_0 = 0$



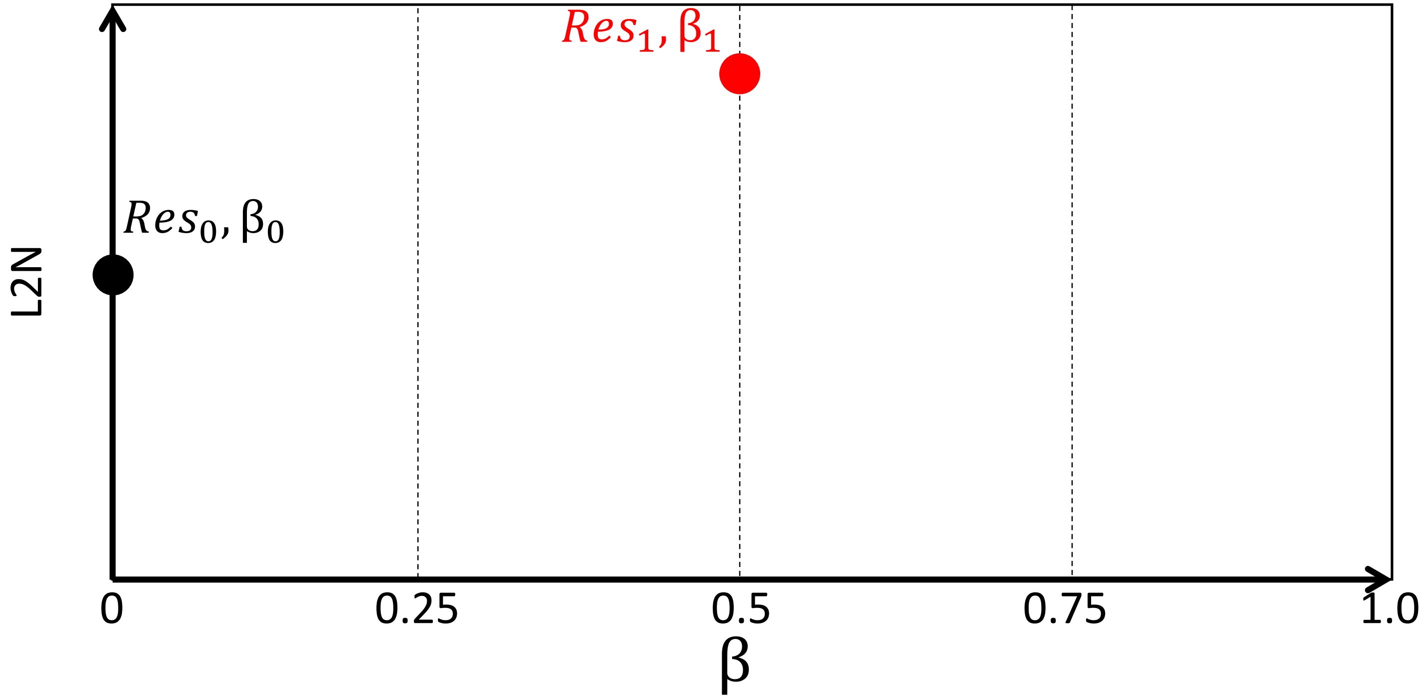
Case 1

$$m_{k+1} = m_k + \mu_k g_k$$

$$\mu_k = \beta\alpha_k$$

$Res_0(m_k), \beta_0 = 0$

$Res_1(m_{k+1}), \beta_1 = 0.5$



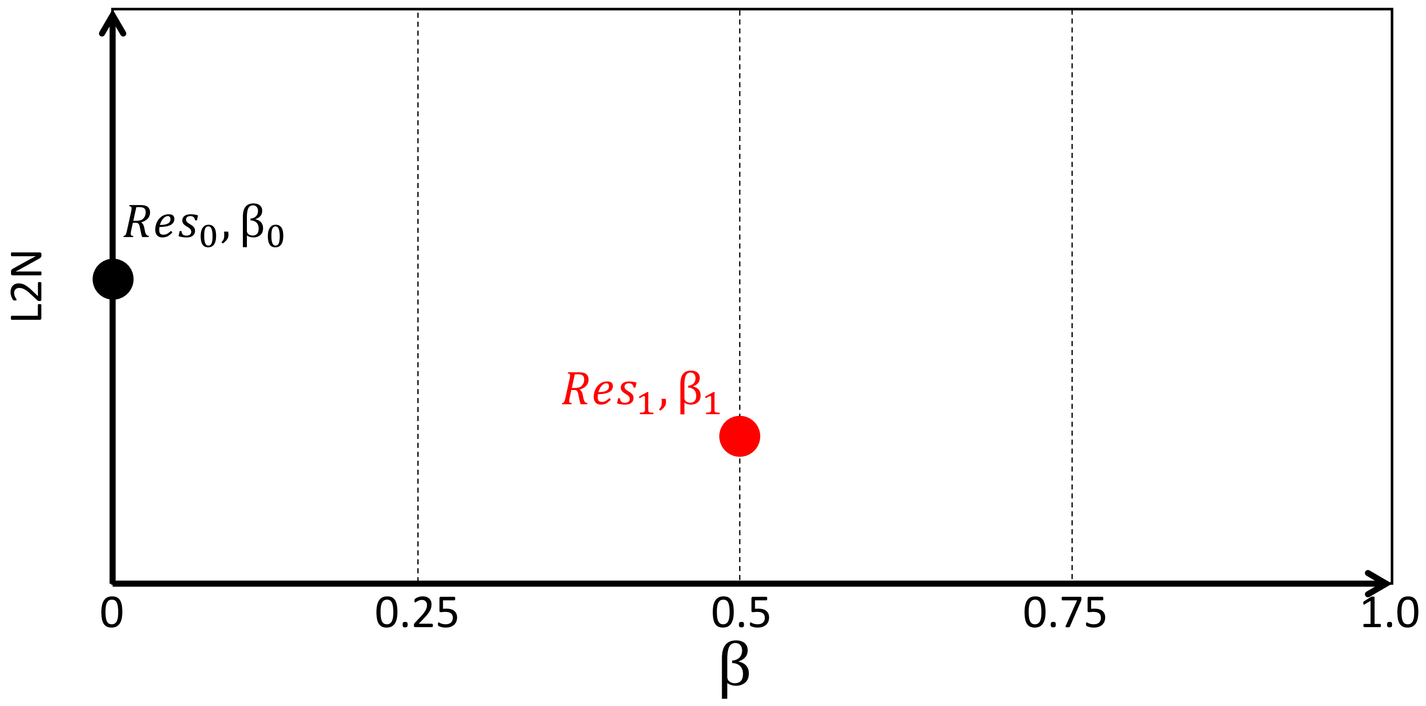
Case 2

$$m_{k+1} = m_k + \mu_k g_k$$

$$\mu_k = \beta\alpha_k$$

$Res_0(m_k), \beta_0 = 0$

$Res_1(m_{k+1}), \beta_1 = 0.5$



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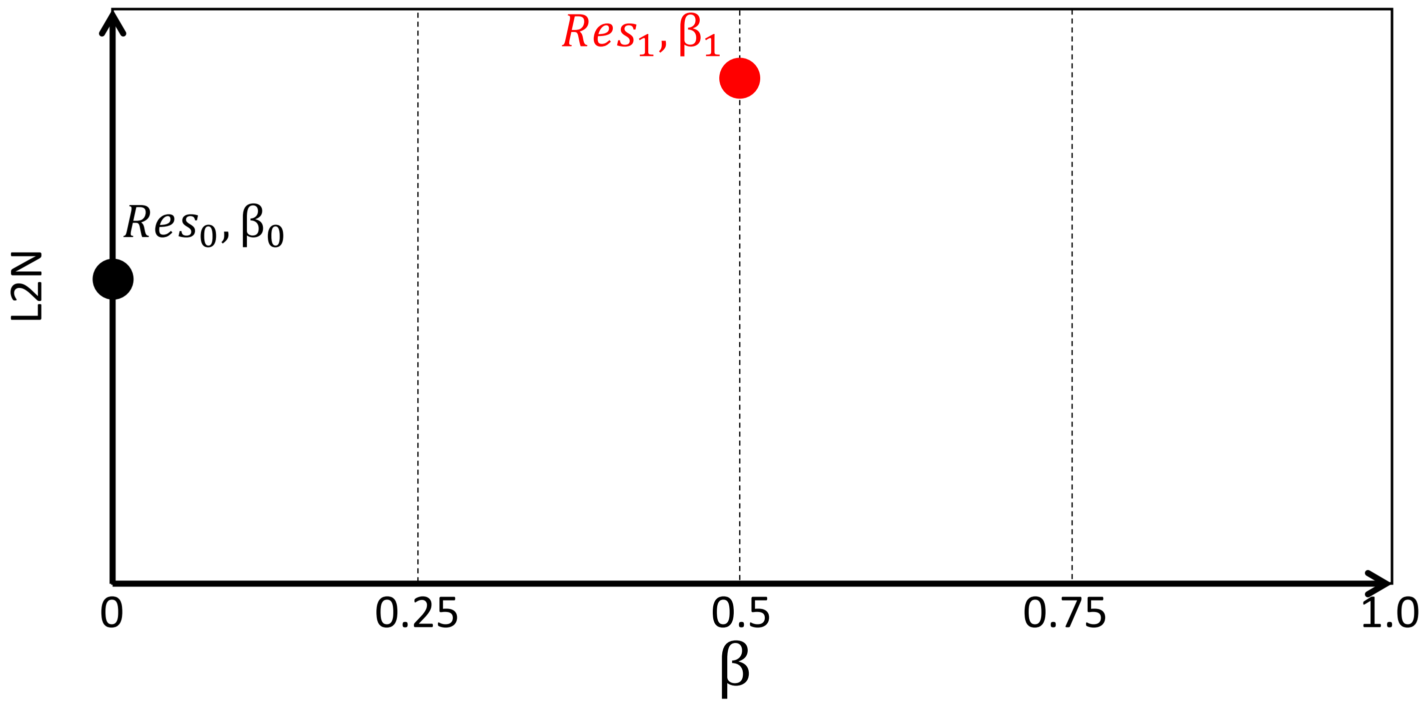

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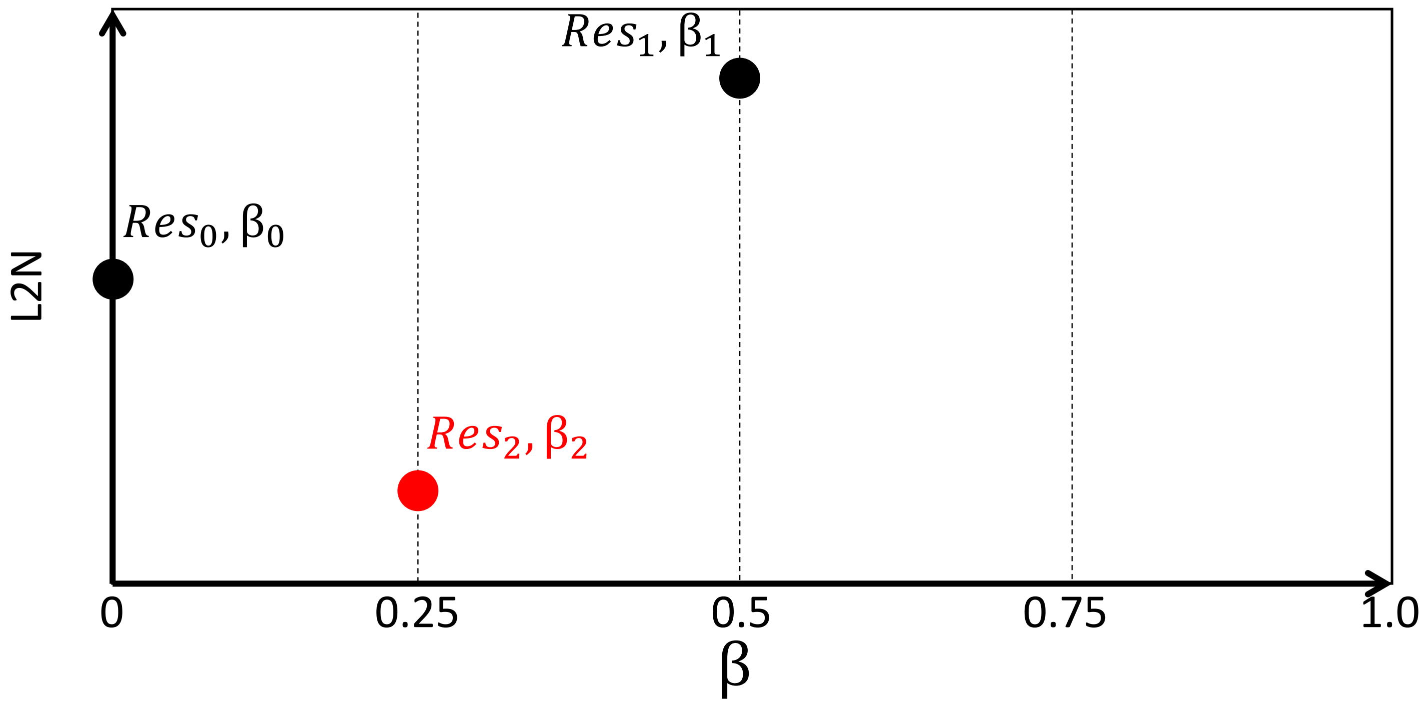
Backtracking



$$m_{k+1} = m_k + \mu_k g_k$$

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Quadratic interpolation

$Res_0(m_k), \beta_0 = 0$

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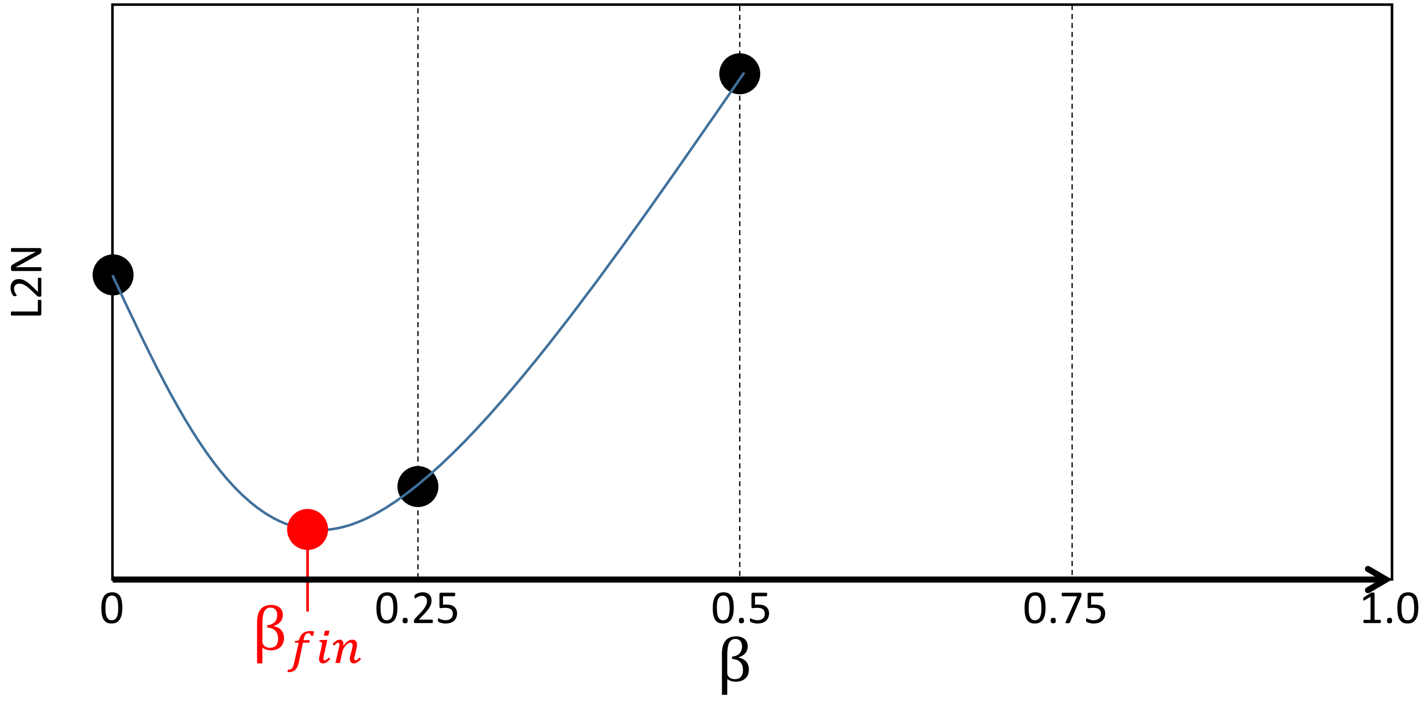

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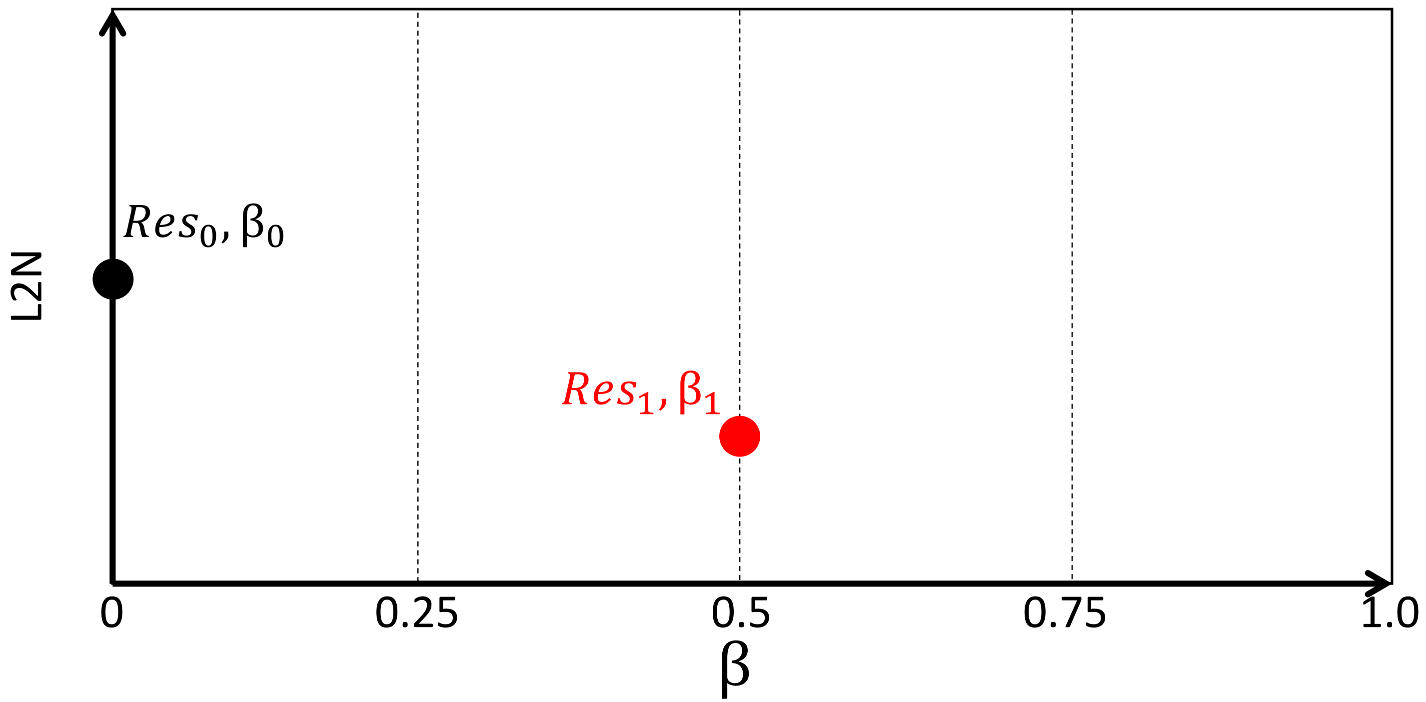

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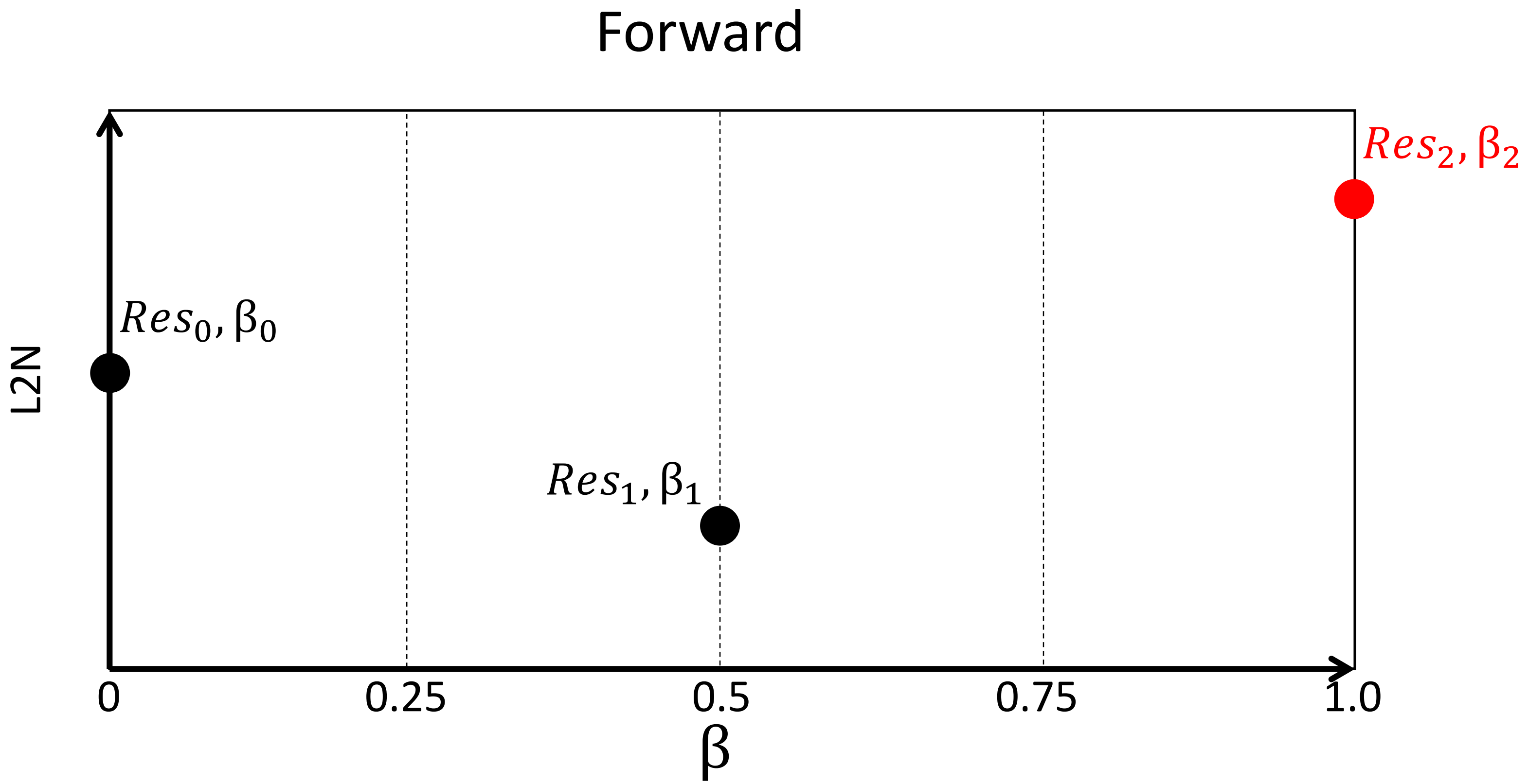

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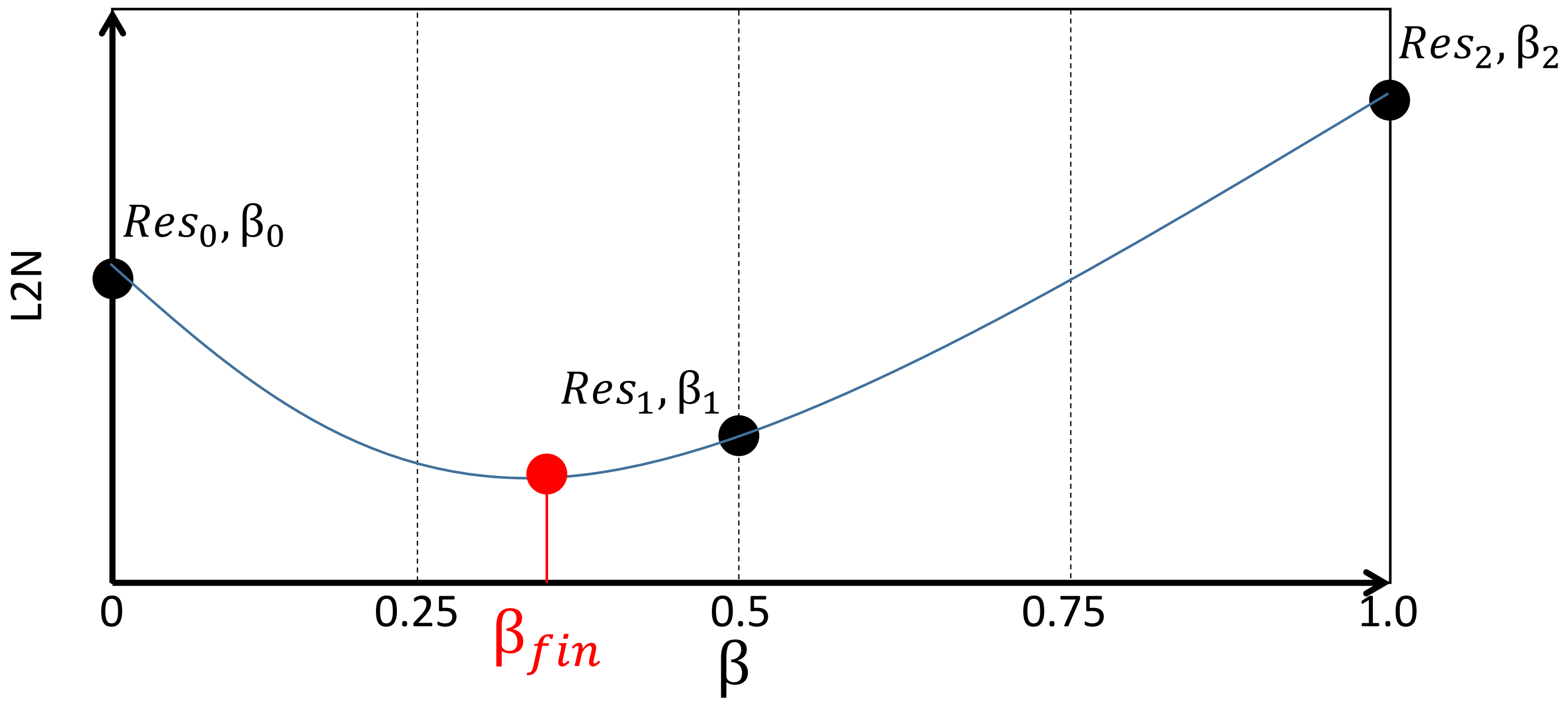


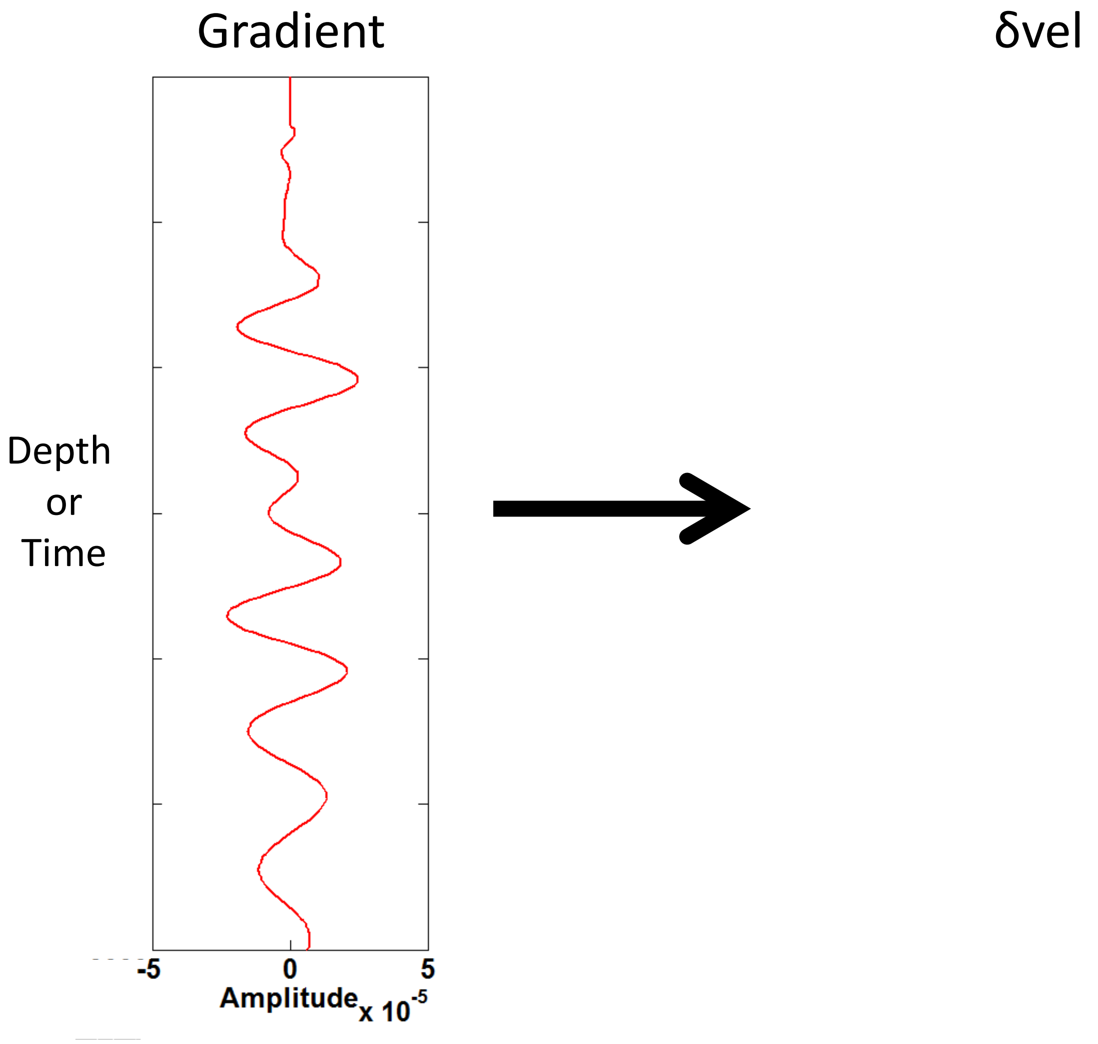
Log validation

Log validation

Velocity Log − Initial velocity = δvel

Depth
or
Time

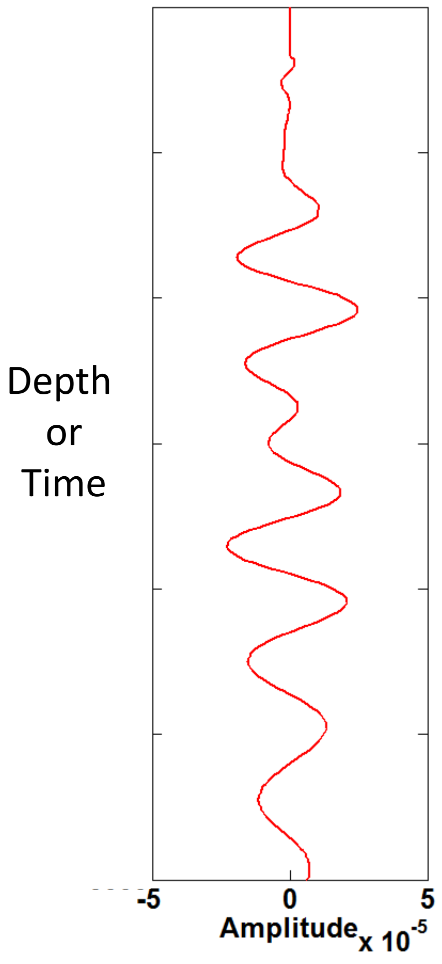
Log validation



Log validation

Estimate the amplitude scalar and phase rotation angle that make the gradient similar to δvel

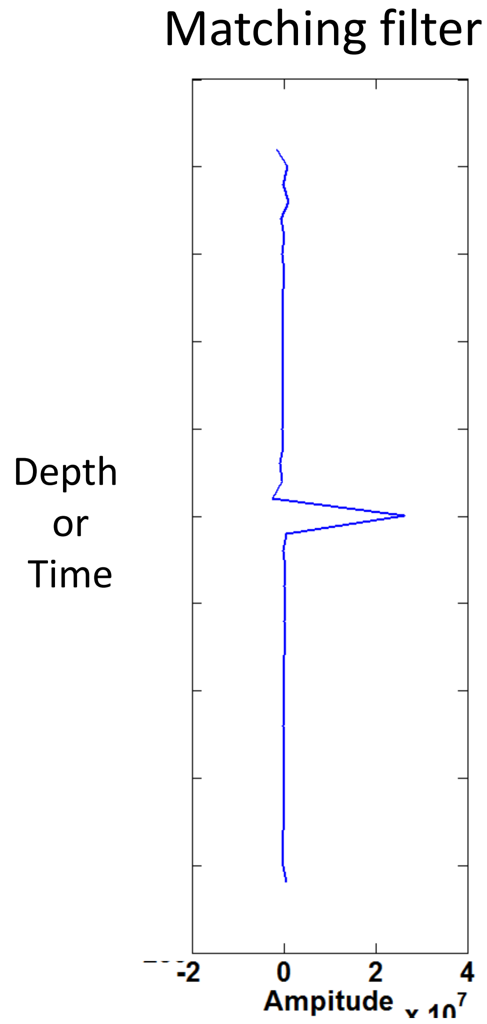
Gradient

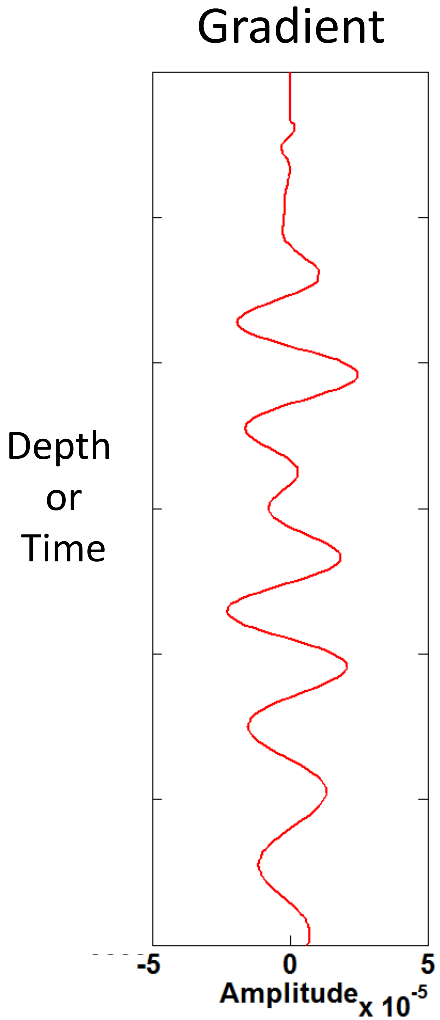


δvel

Log validation

λ

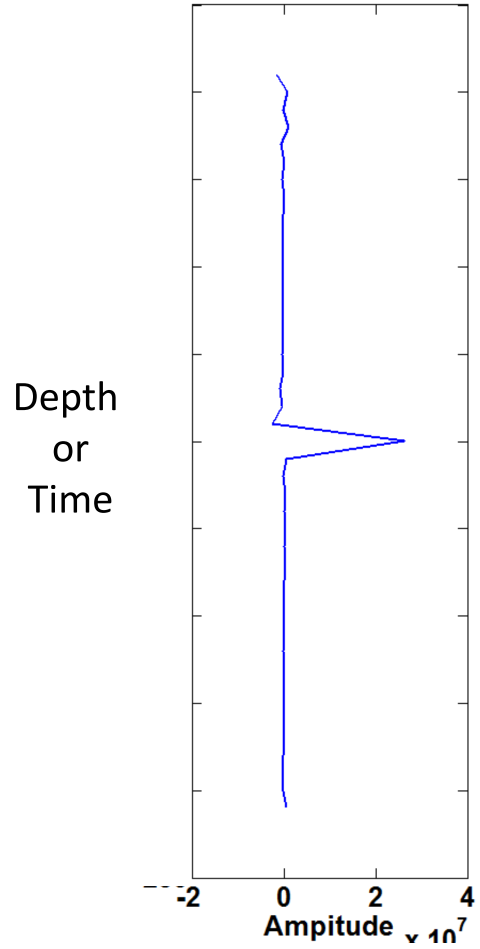




δvel

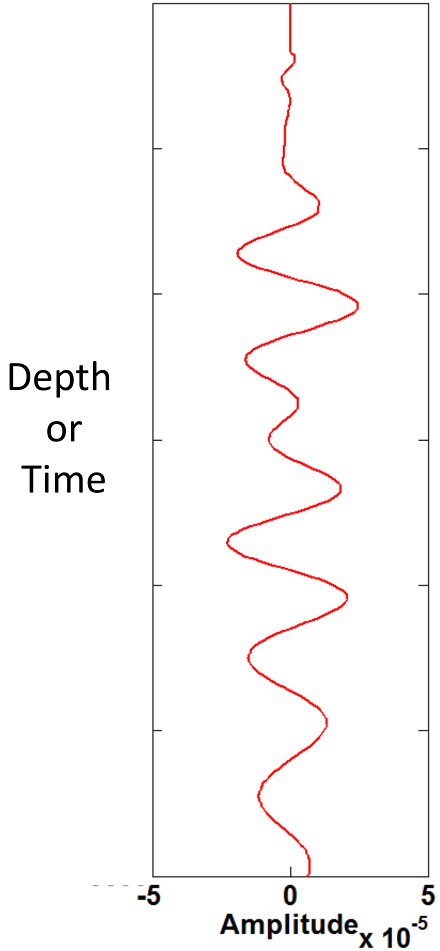
Log validation

Matching filter



*

Gradient



→

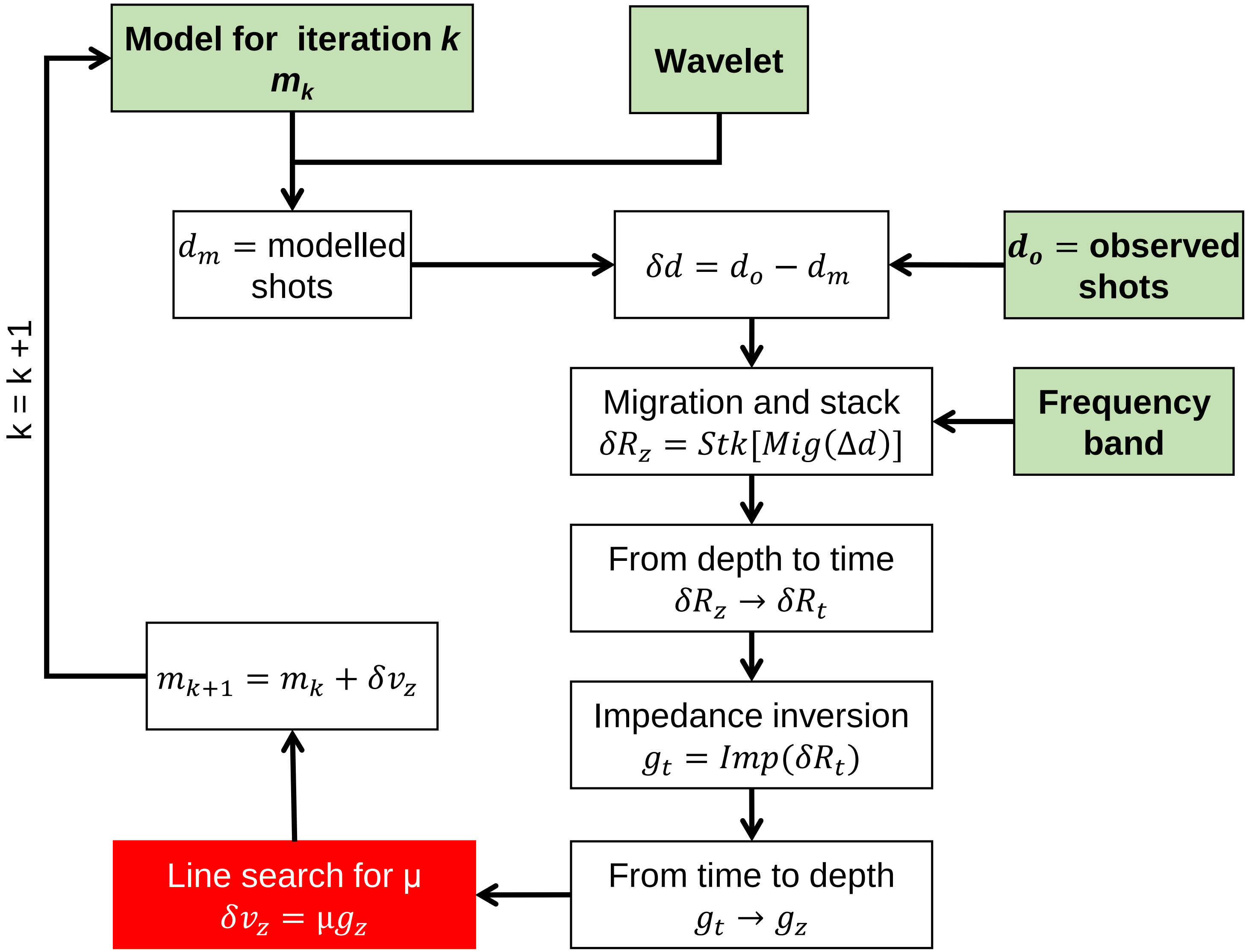
δvel

— δvel

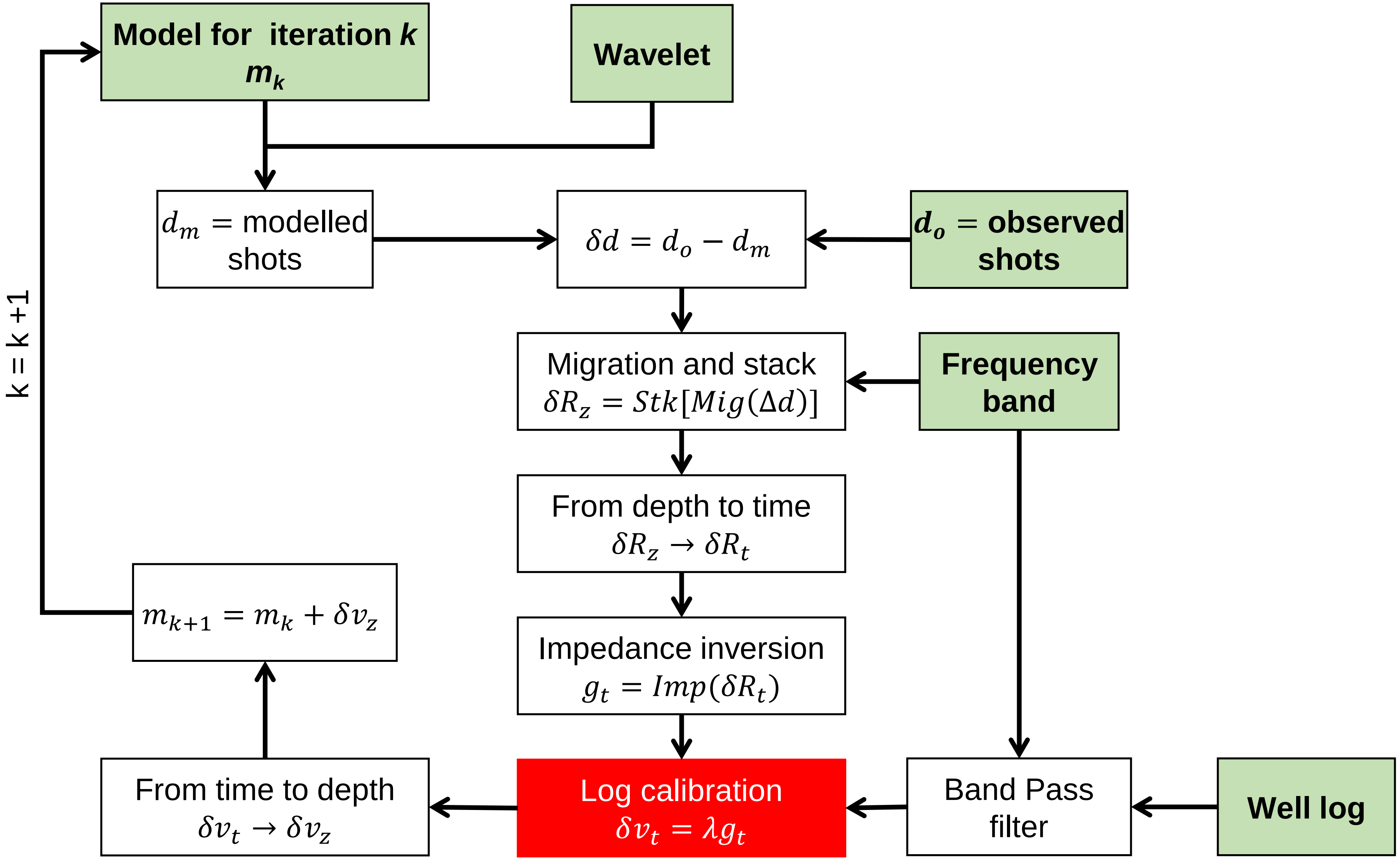
— Calibrated gradient

Workflows

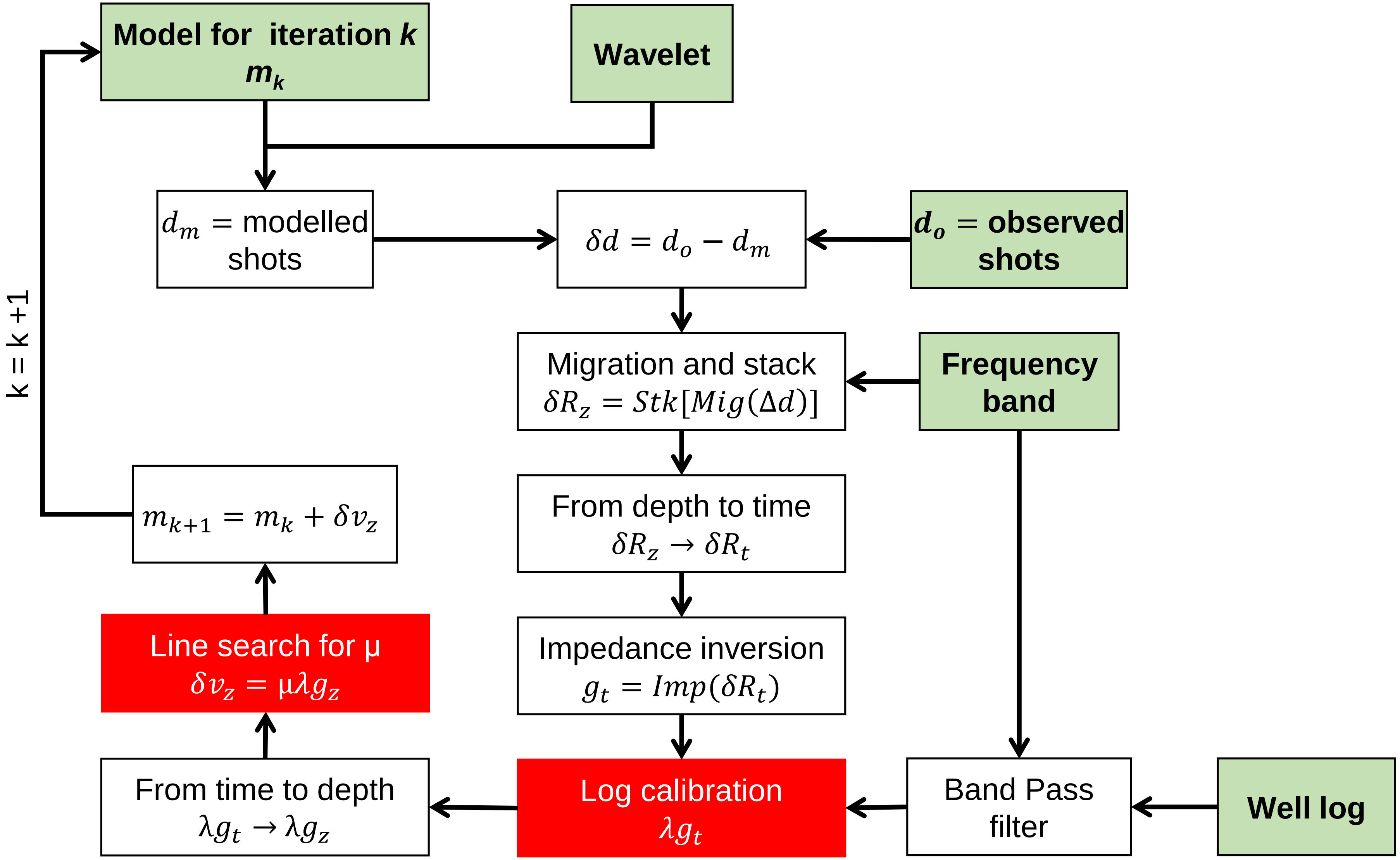
Data validation



Log validation

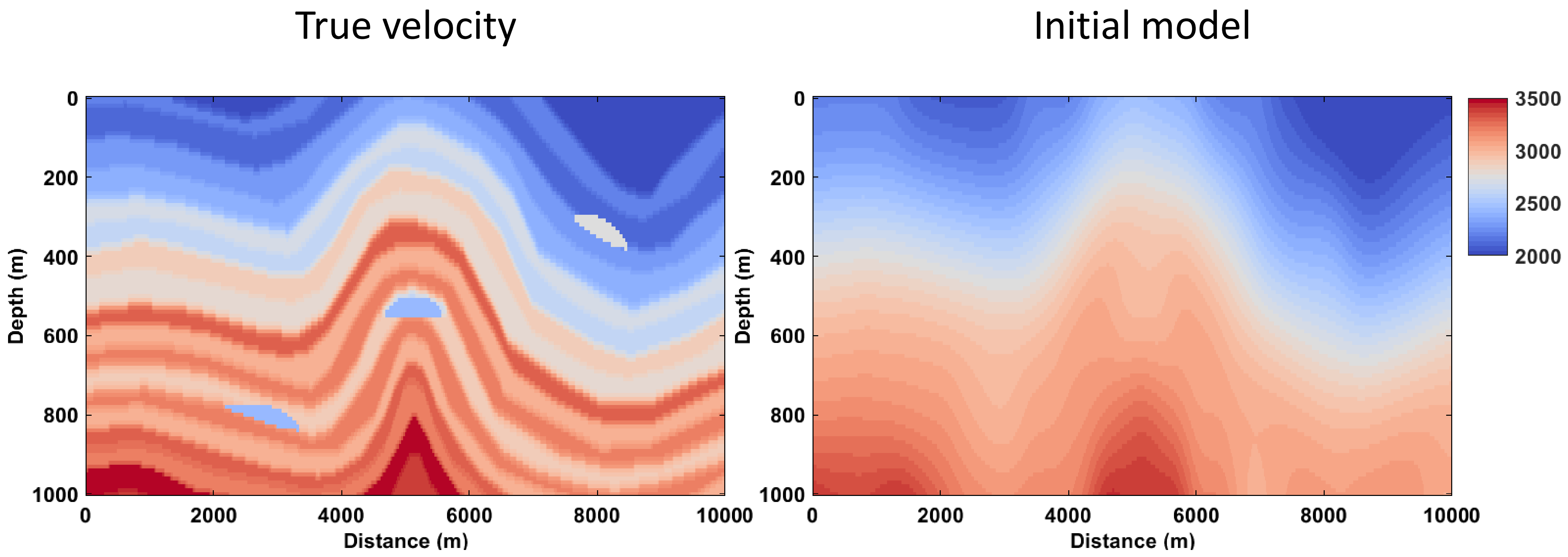


Log-&-data validation

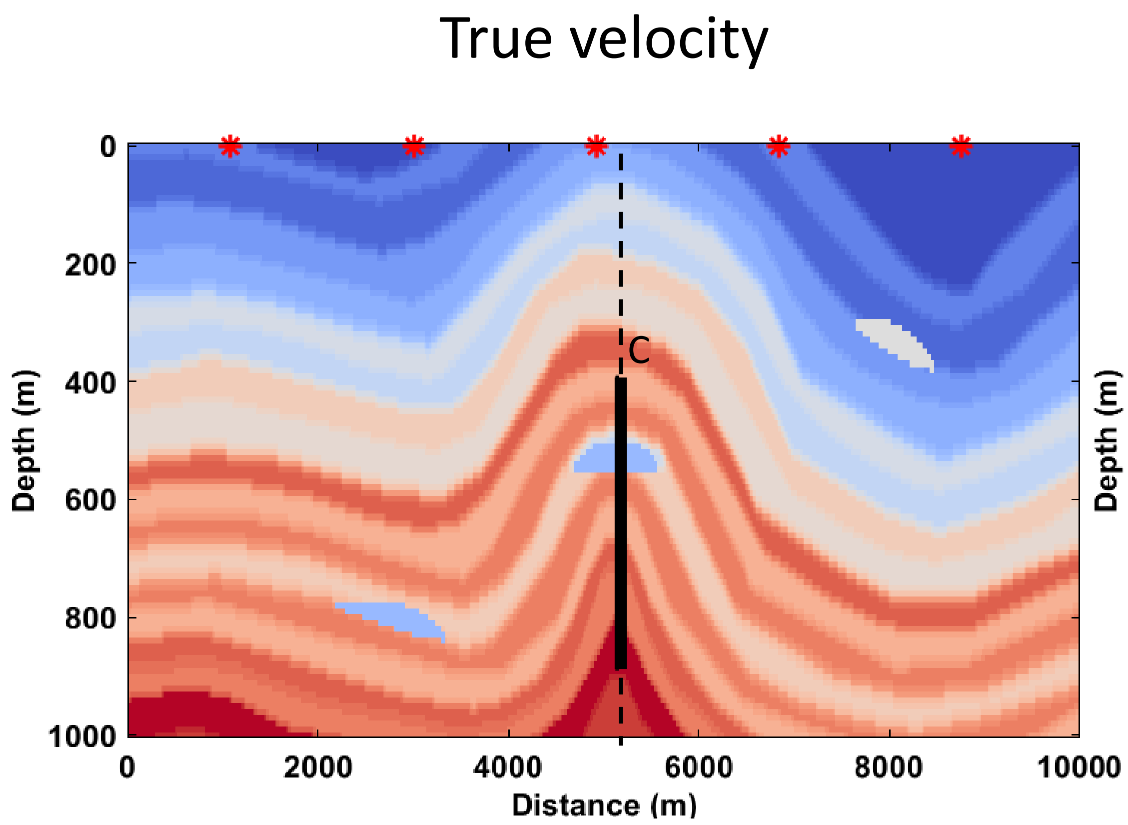


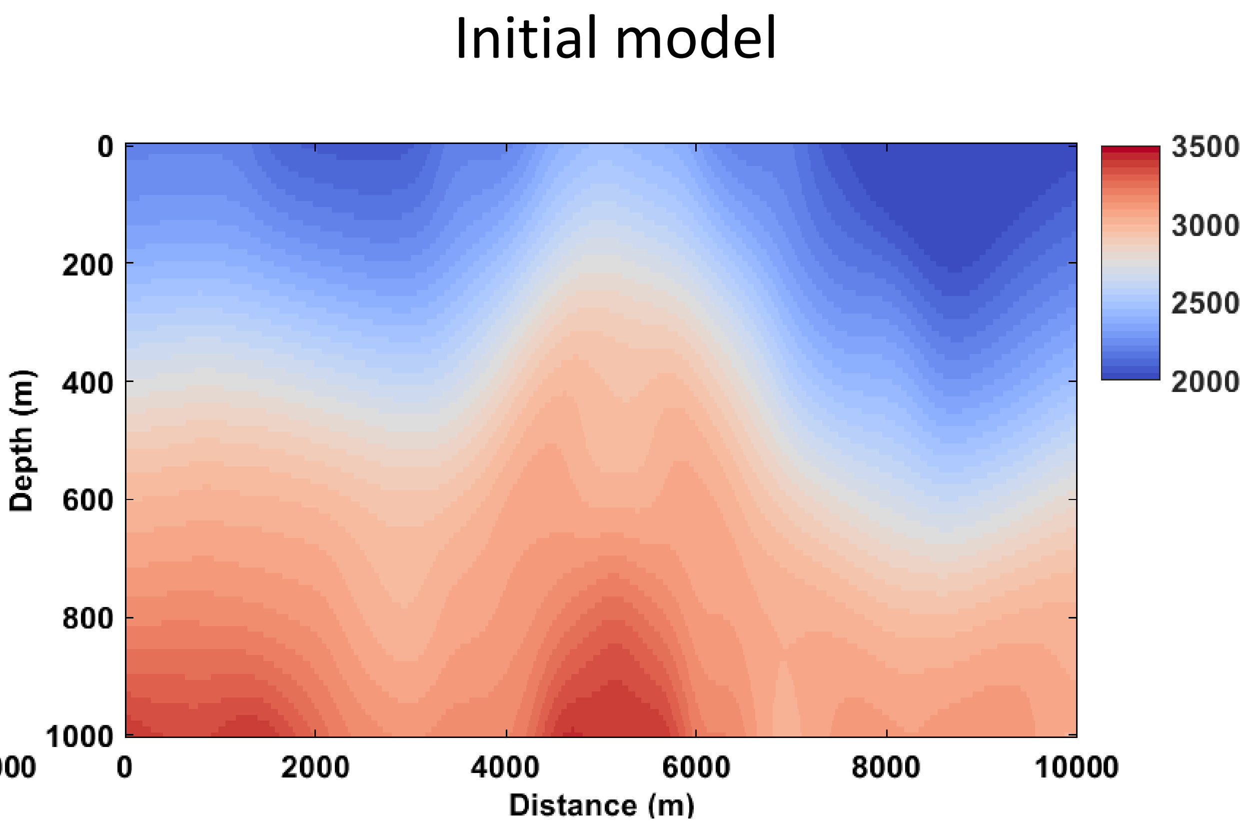
Numerical examples

Anticline model

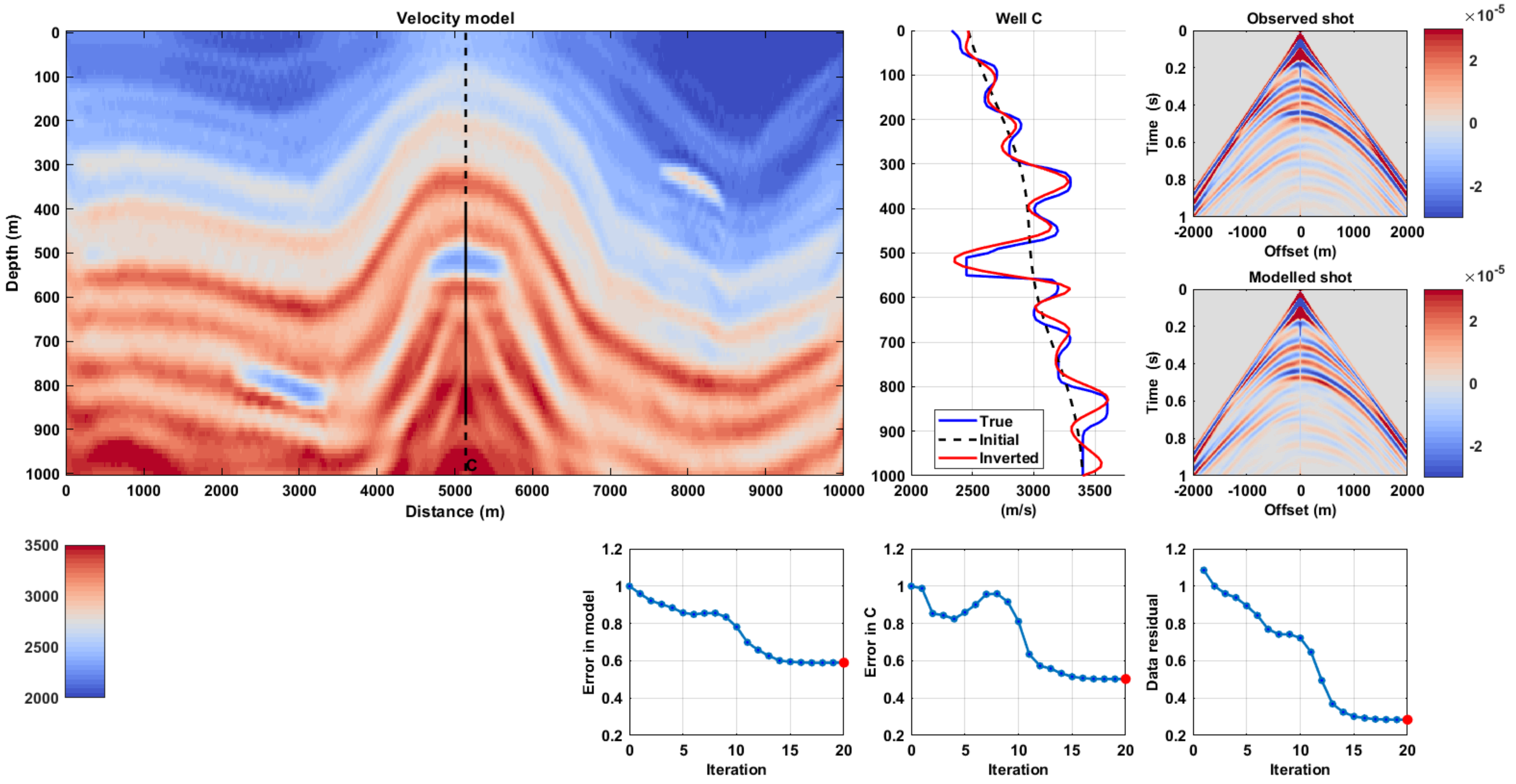


Anticline model

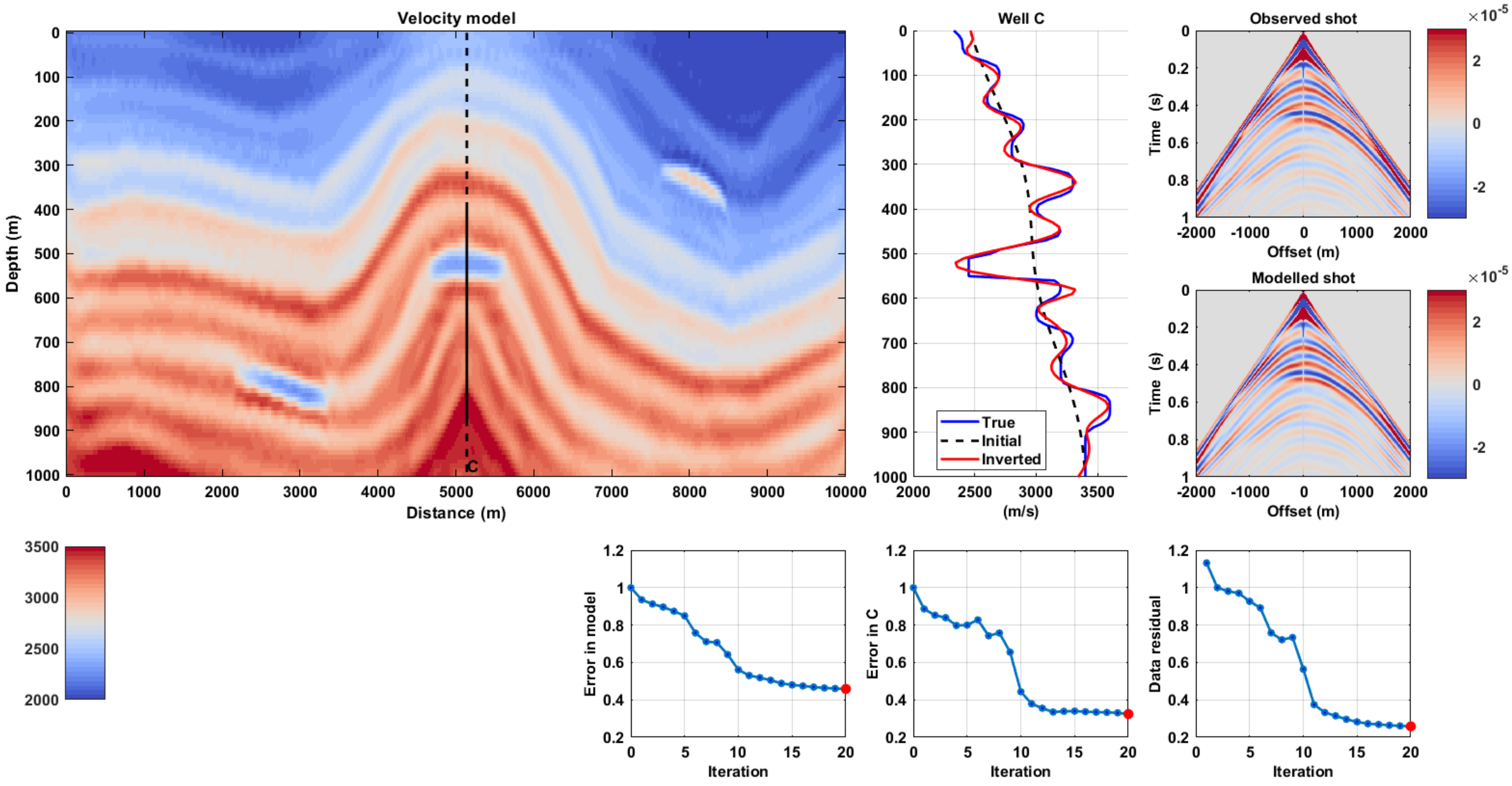




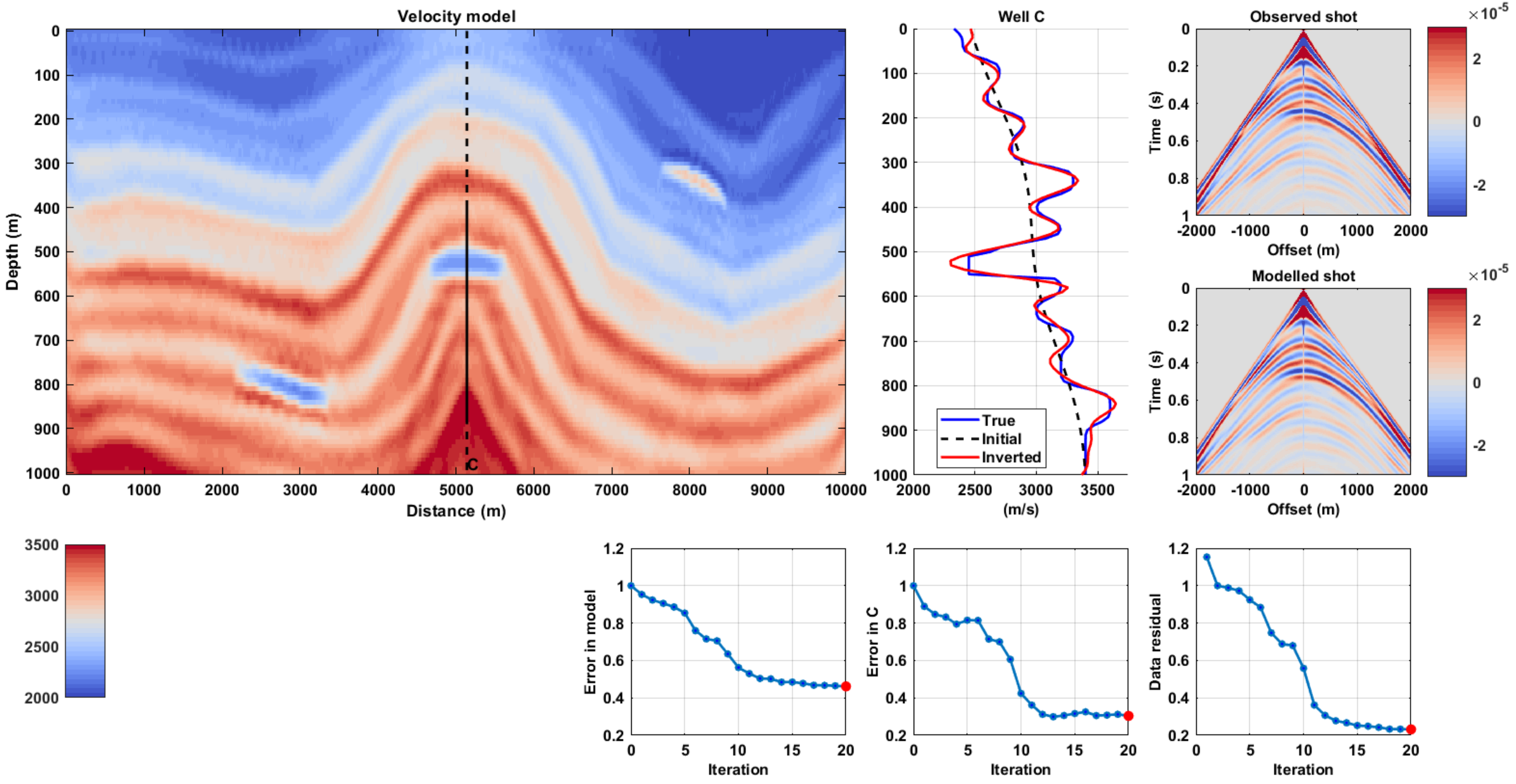
Data validation



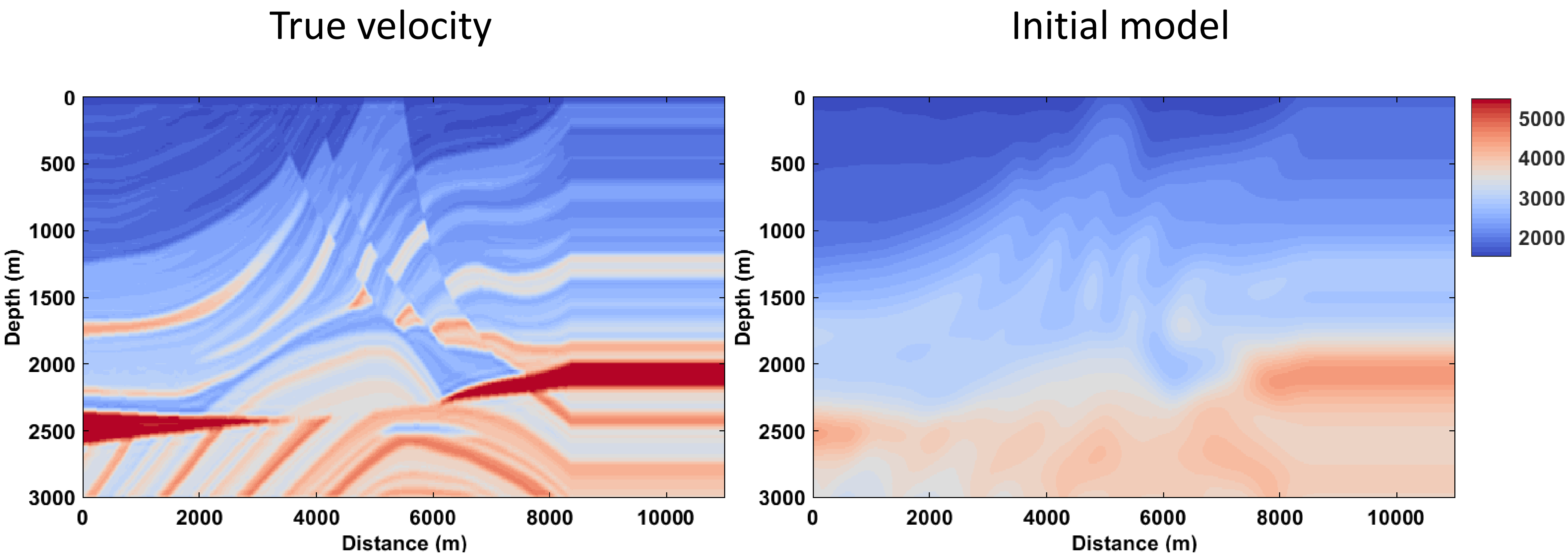
Log validation



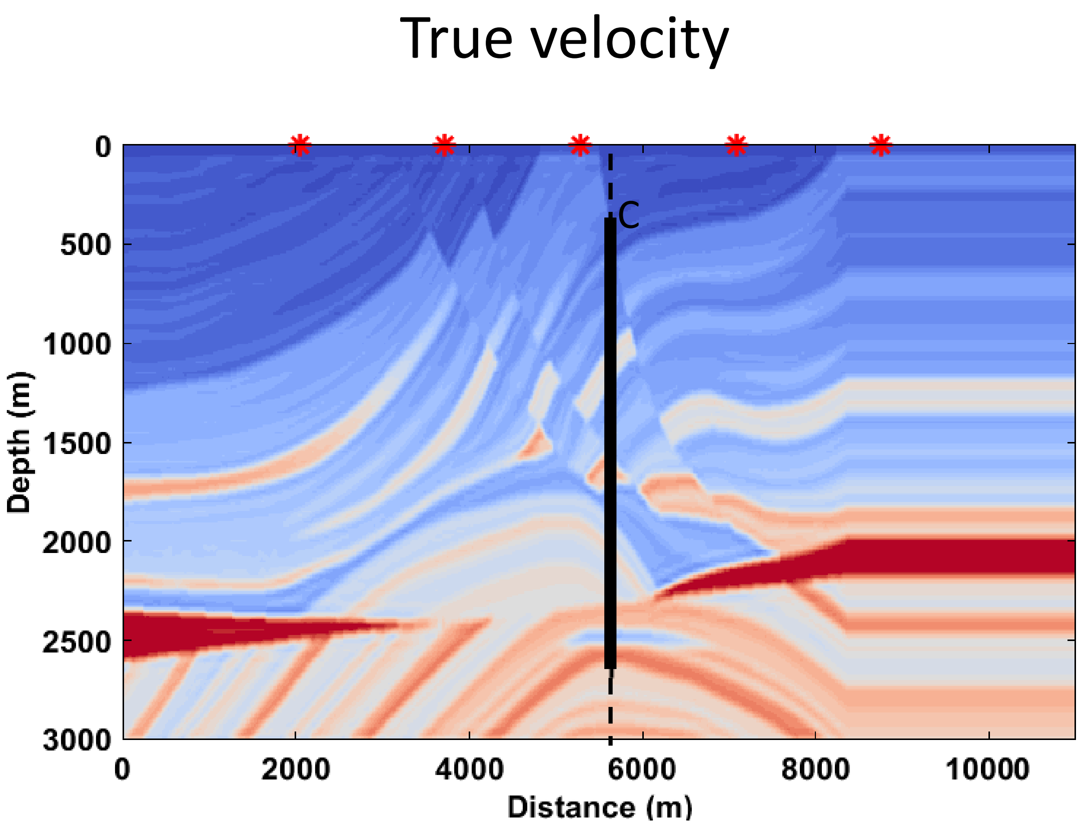
Log-&-data validation

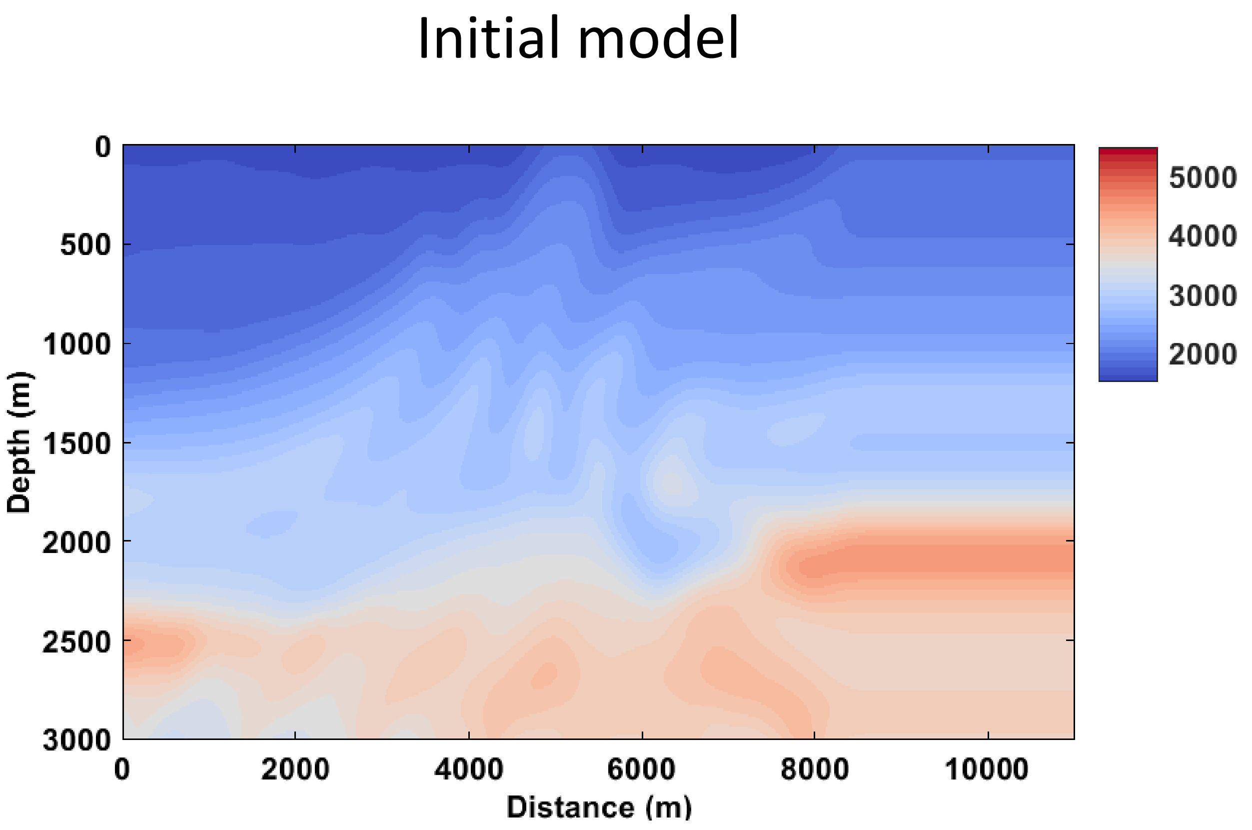


Marmousi model



Marmousi model





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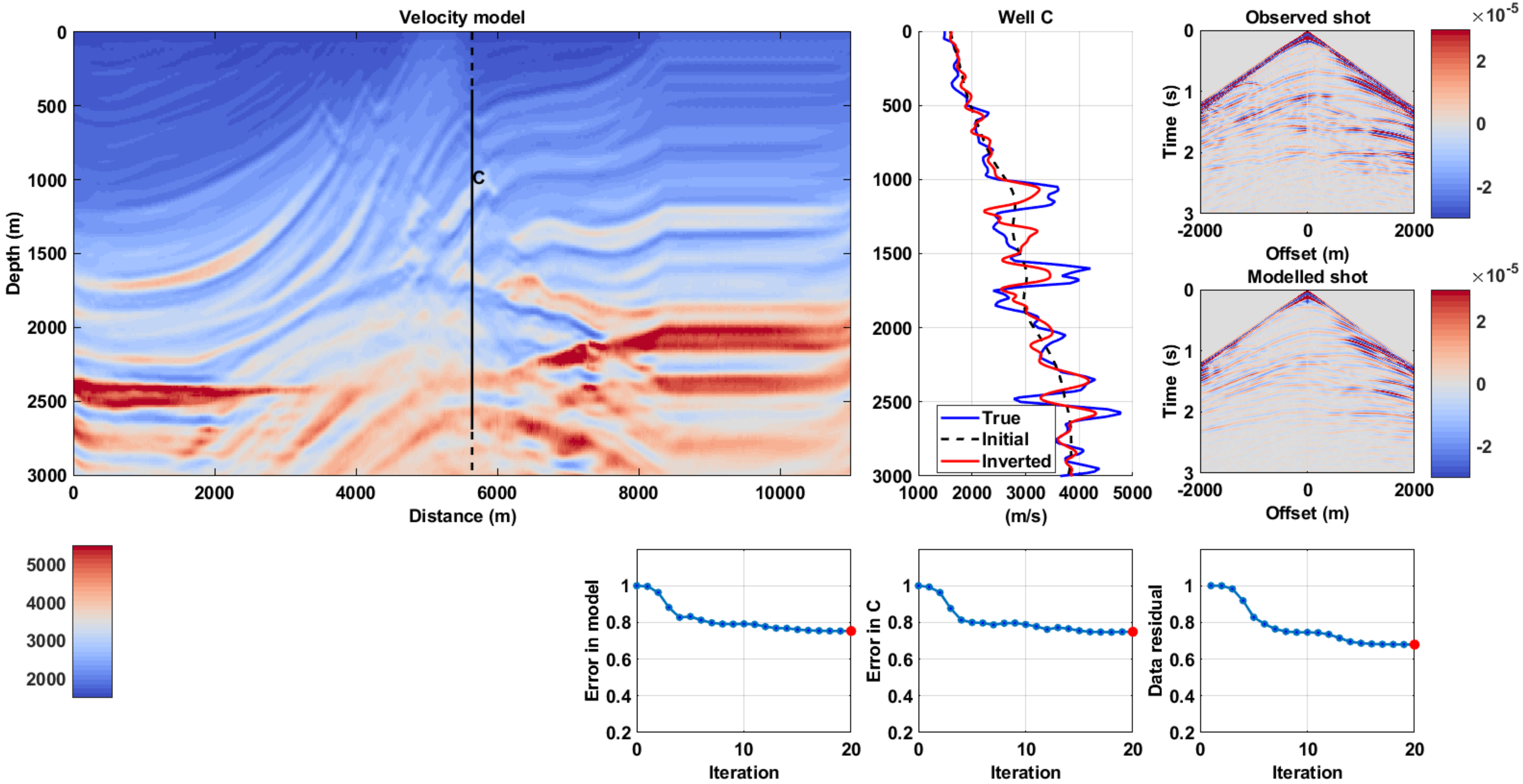

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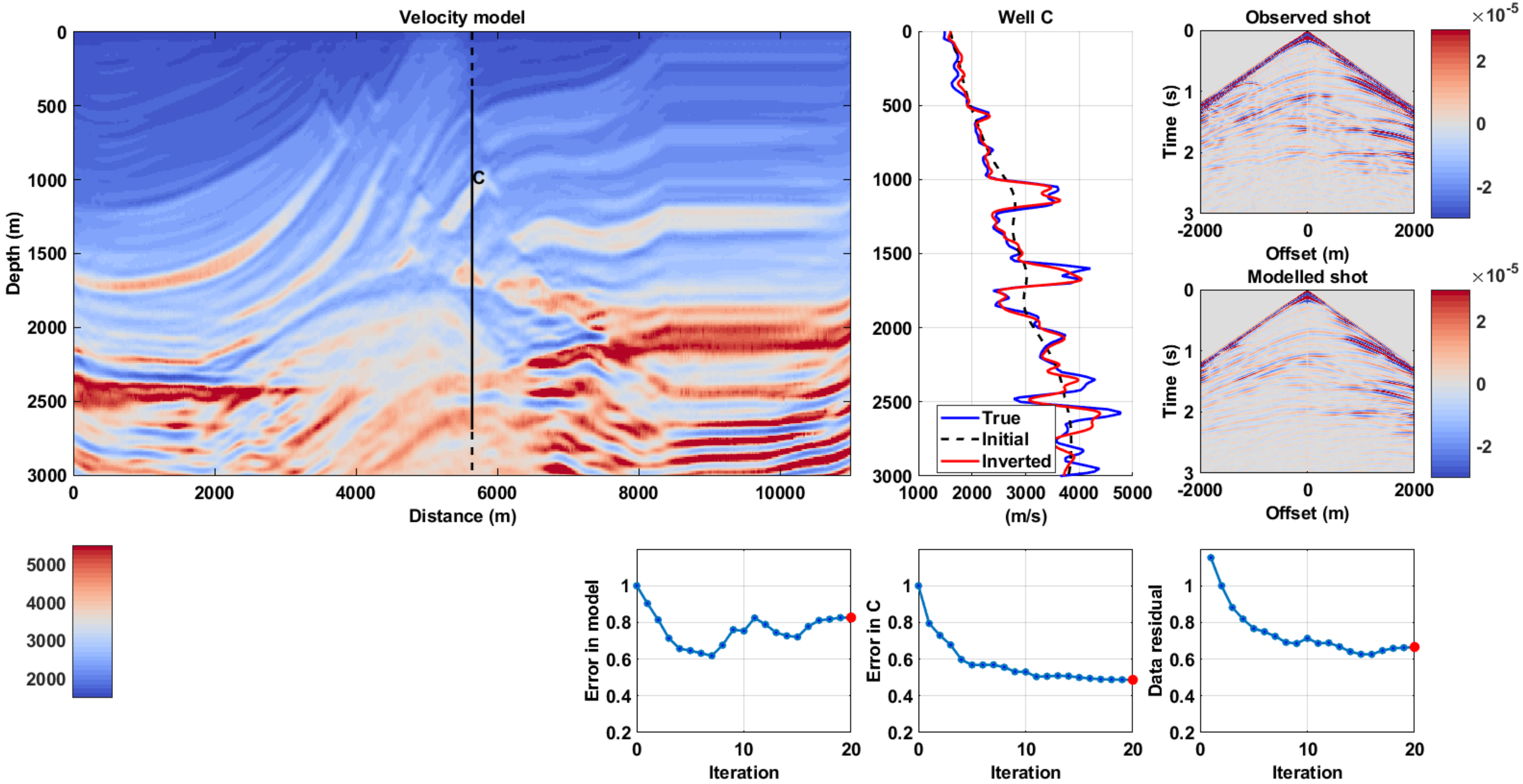
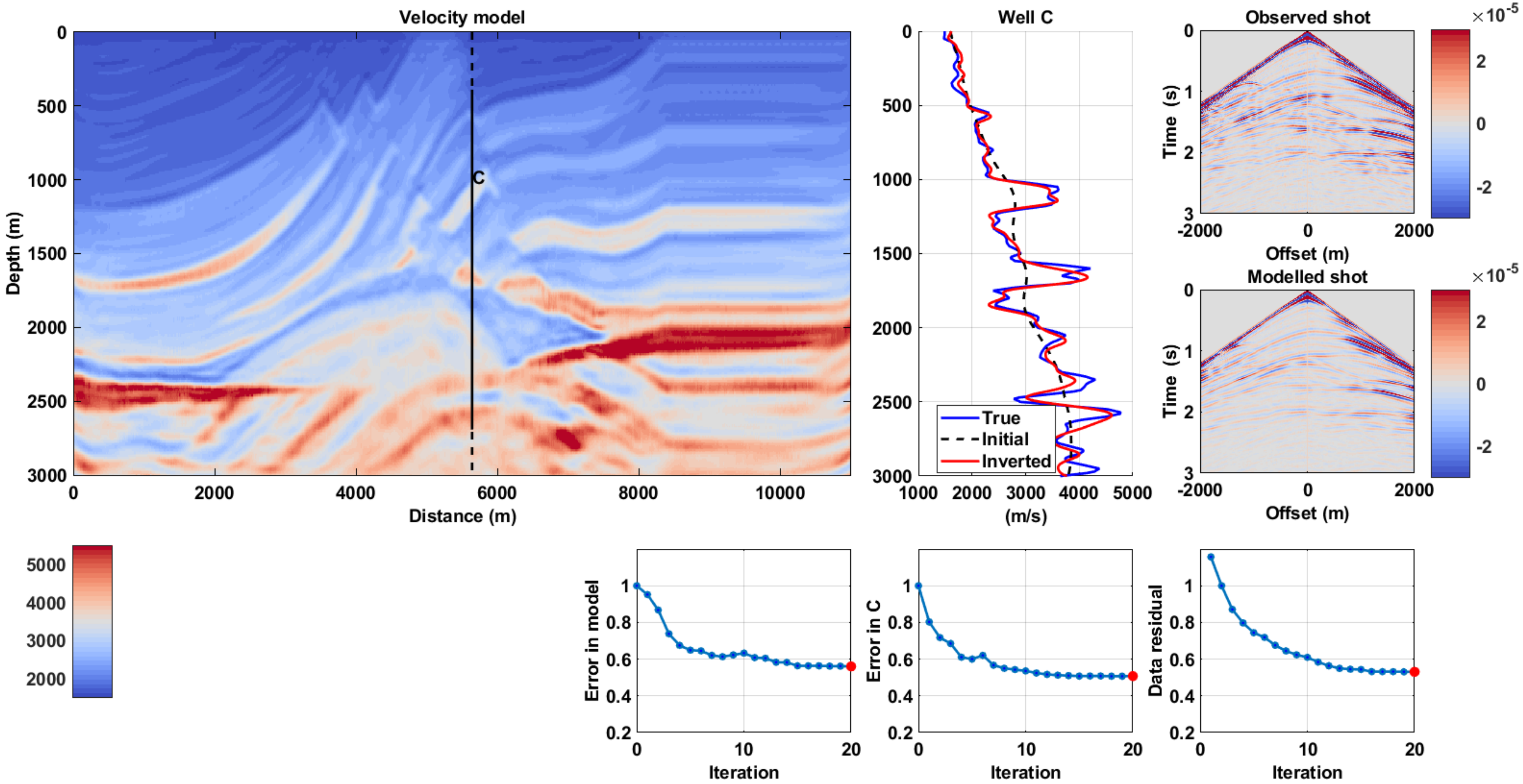


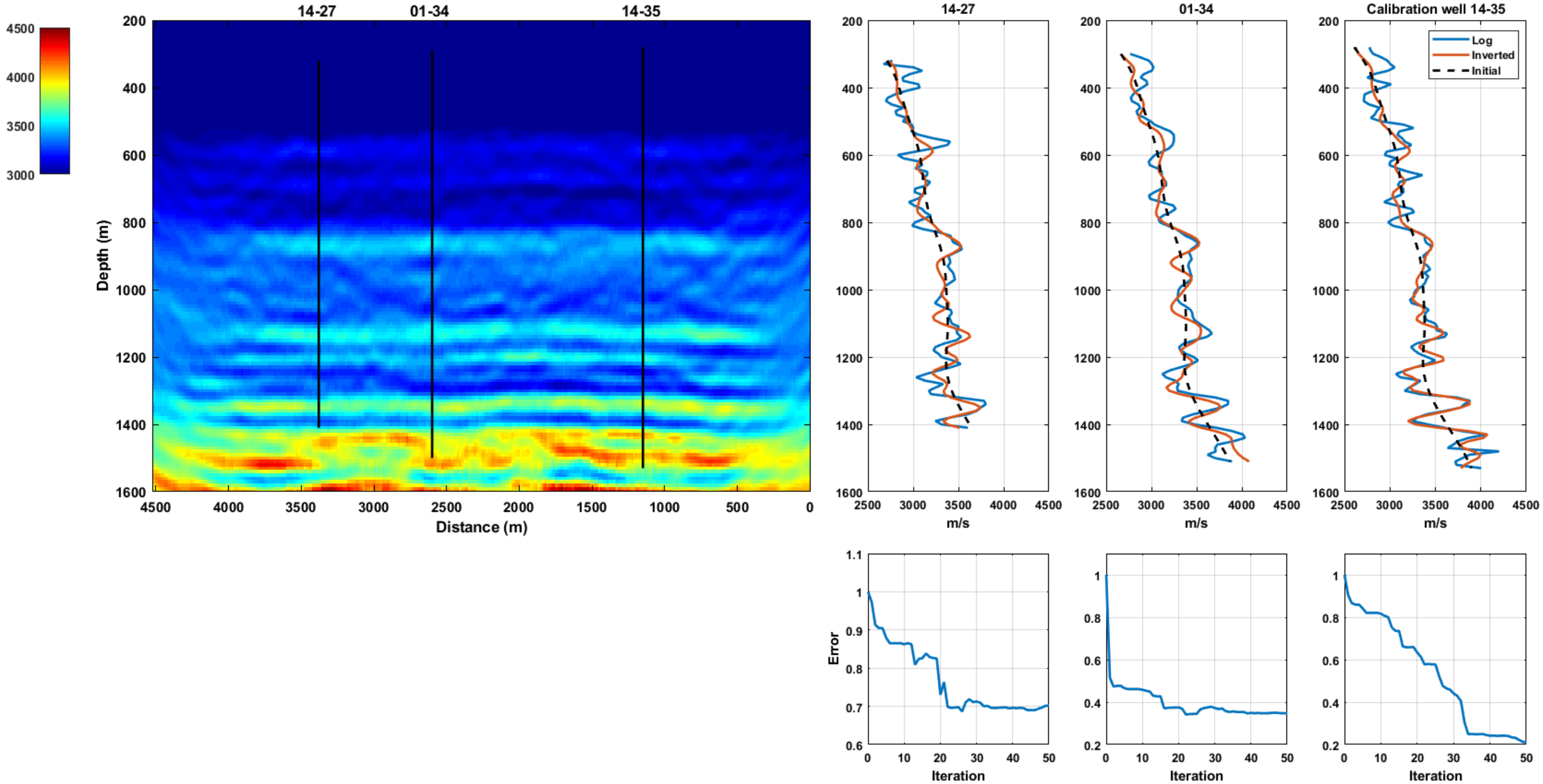
Log-&-data validation



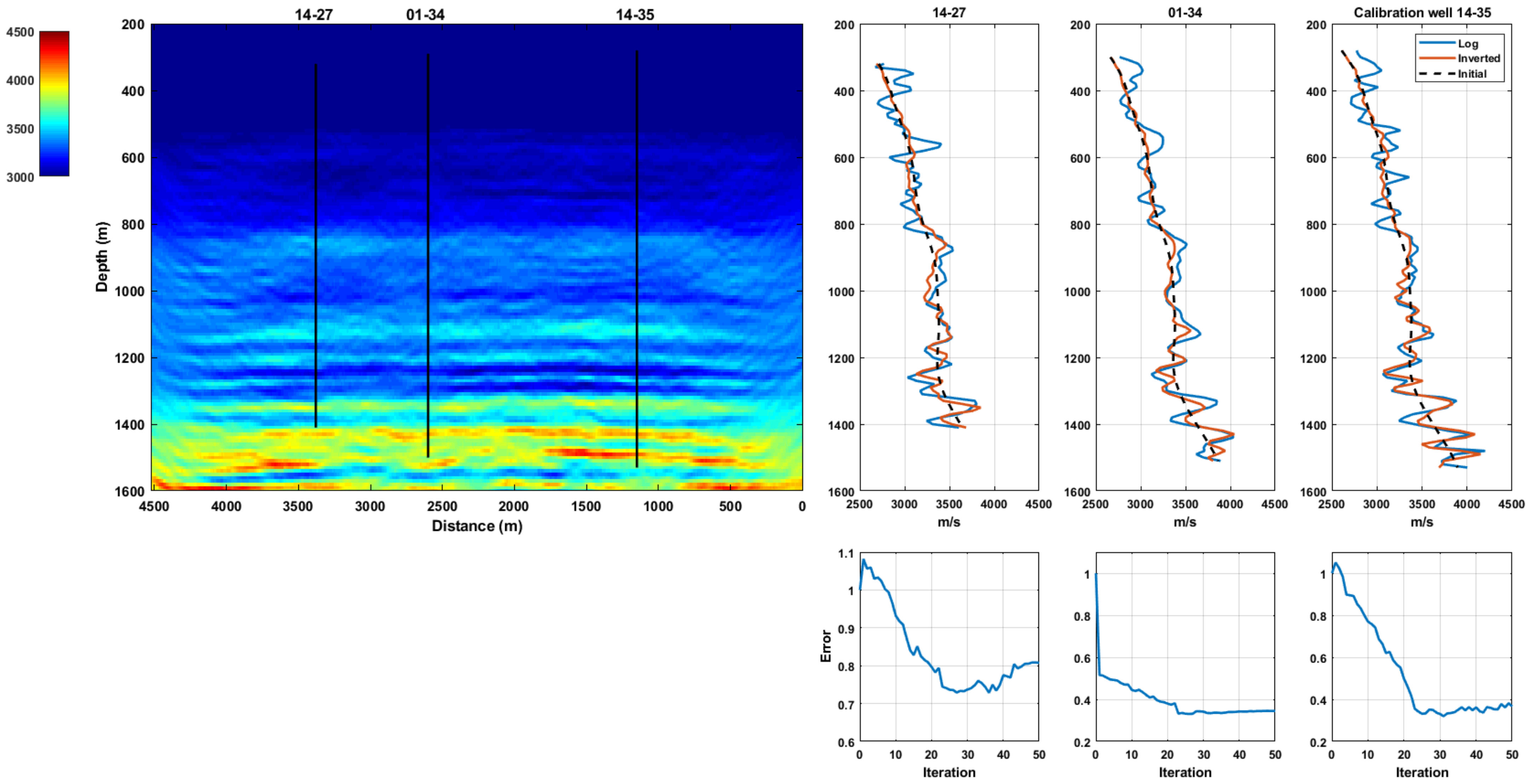
Hussar dataset

Romahn, S., and K. A. Innanen, 2018, Log-validated FWI with wavelet phase and amplitude updating applied on Hussar data: CREWES Research Report, 30.

Log validation



Log-&-data valid



Conclusions

- We compared two ways to scale or calibrate the gradient: data validation and log validation
- Combination of log-and-data validation
- Log validation is cheaper
- We recommend the use of log validation for simple models
- For more complex models a combination of log-and-data validation may be a better option
- Application to Hussar dataset

Acknowledgments

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Thank you!