Viscoacoustic reverse time migration in tilted TI media with attenuation compensation

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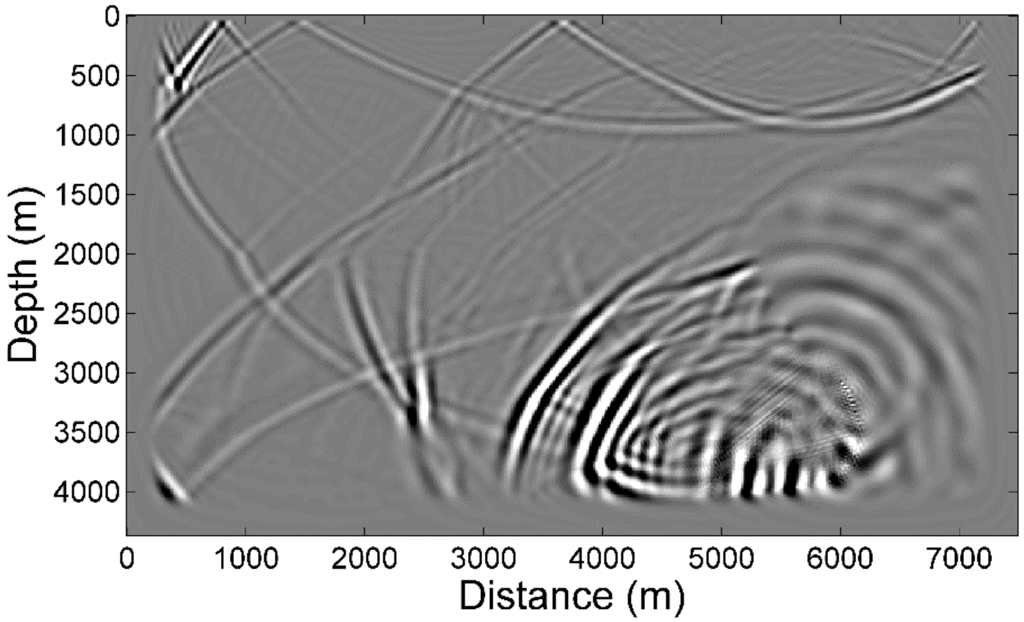
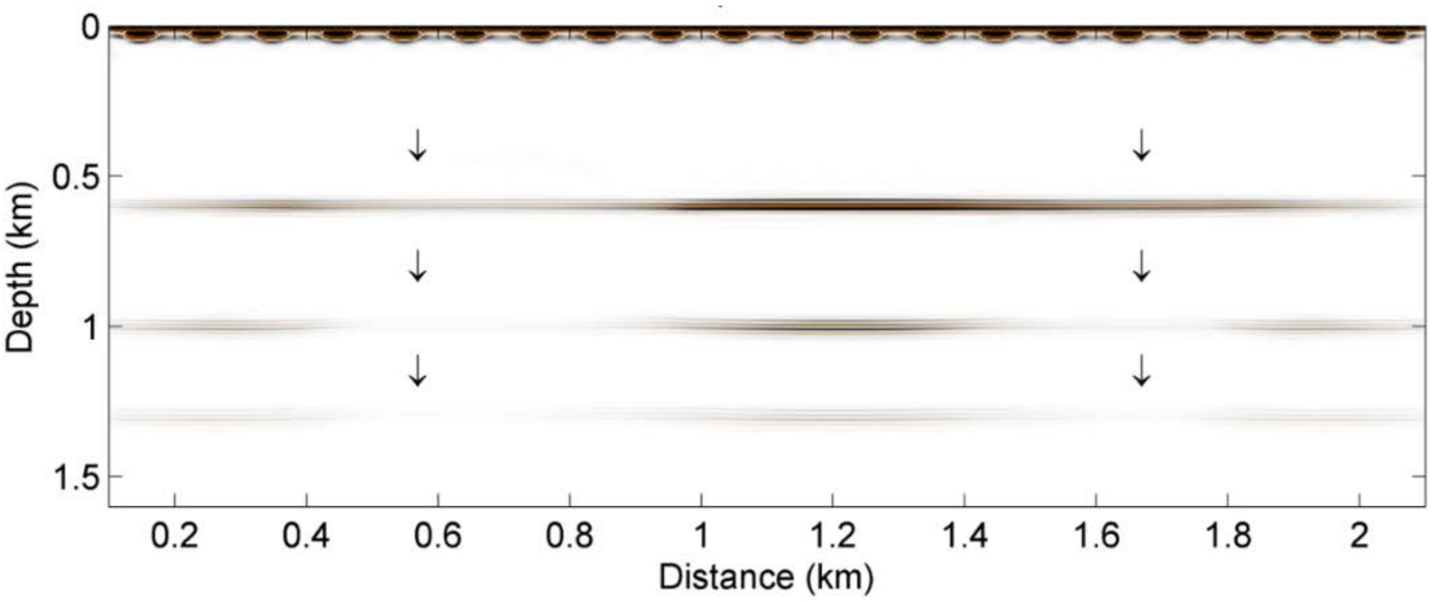
Motivation

Anisotropy and viscosity degraded waveform in amplitude, resulting in which reducing of image resolution

Anisotropy



Viscosity



Fathalian and Innanen 2017

Theory of viscoacoustic RTM in TTI

- ✓ Derive the new approach of the viscoacoustic wave equation in TTI media
- ✓ Explain the viscoacoustic RTM based on the constant-Q model
- ✓ Apply a suitable imaging condition for Q-compensated RTM

Viscoacoustic wave equation in TTI media

Viscoacoustic wave equation in TTI media

There are two way to consider the anisotropic medium

1) Pseudo-acoustic wave equation (Alkhalifah 1998)

2) Pure acoustic wave equation (Etgen and Brandsberg-Dahl 2009)

↓

Acoustic wave equation in VTI media (Duveneck et al., 2008)

$$\partial_t \sigma_H = \rho V_P^2 \left[(1 + 2\varepsilon) \partial_x u_x + \sqrt{1 + 2\delta}\, \partial_z u_z \right]$$

$$\partial_t \sigma_V = \rho V_P^2 \left[\sqrt{1 + 2\delta}\, \partial_x u_x + \partial_z u_z \right]$$

Viscoacoustic wave equation in TTI media

2D viscoacoustic wave equations in TTI media(Fathalian et al. 2017)

$$\partial_t \sigma_H = \rho V_P^2 \left[(1+2\varepsilon) \left[(\frac{\tau_\varepsilon}{\tau_\sigma})[(cos\theta cos\varphi \partial_x - sin\theta \partial_z)u_x] - r_H \right] + \sqrt{1+2\delta}[(cos\varphi sin\theta \partial_x + cos\theta \partial_z)u_z] \right]$$

$$\partial_t \sigma_V = \rho V_P^2 \left[\sqrt{1+2\delta}[(cos\theta cos\varphi \partial_x - sin\theta \partial_z)u_x] + (\frac{\tau_\varepsilon}{\tau_\sigma})[(cos\varphi sin\theta \partial_x + cos\theta \partial_z)u_z] - r_V \right]$$

$$\partial_t r_H = -\frac{1}{\tau_\sigma} r_H + \rho V_P^2((cos\theta cos\varphi \partial_x - sin\theta \partial_z)u_x)\frac{1}{\tau_\sigma}\left(1 - \frac{\tau_\varepsilon}{\tau_\sigma}\right)$$

$$\partial_t r_V = -\frac{1}{\tau_\sigma} r_V + \rho V_P^2((cos\varphi sin\theta \partial_x + cos\theta \partial_z)u_z)\frac{1}{\tau_\sigma}\left(1 - \frac{\tau_\varepsilon}{\tau_\sigma}\right)$$

Viscoacoustic wave equation in TTI media

Fourier transform to the frequency domain

Memory variable

$$\tilde{r}_H = \rho V_P^2((cos\theta cos\varphi \partial_x - sin\theta \partial_z)\tilde{u}_x)\frac{\tau_\sigma^{-1}(1-\tau_\varepsilon\tau_\sigma^{-1})}{(i\omega+\tau_\sigma^{-1})}$$

$$\tilde{r}_V = \rho V_P^2((cos\varphi sin\theta \partial_x + cos\theta \partial_z)\tilde{u}_z)\frac{\tau_\sigma^{-1}(1-\tau_\varepsilon\tau_\sigma^{-1})}{(i\omega+\tau_\sigma^{-1})}$$

After removing memory variable equations and some algebra manipulation

$$i\omega\tilde{\sigma}_H = \rho V_P^2\left[(1+2\varepsilon)\left[\left(\frac{(\omega^2\tau_\varepsilon\tau_\sigma+1)}{\omega^2\tau_\sigma^2+1} + i\frac{(\omega\tau_\varepsilon - \omega\tau_\sigma)}{\omega^2\tau_\sigma^2+1}\right)[cos\theta cos\varphi\partial_x - sin\theta\partial_z)\tilde{u}_x]\right] + \sqrt{1+2\delta}[(cos\varphi sin\theta\partial_x + cos\theta\partial_z)\tilde{u}_z]\right]$$

$$i\omega\tilde{\sigma}_V = \rho V_P^2\left[\sqrt{1+2\delta}[cos\theta cos\varphi\partial_x - sin\theta\partial_z)\tilde{u}_x] + \left(\frac{(\omega^2\tau_\varepsilon\tau_\sigma+1)}{\omega^2\tau_\sigma^2+1} + i\frac{(\omega\tau_\varepsilon - \omega\tau_\sigma)}{\omega^2\tau_\sigma^2+1}\right)[(cos\varphi sin\theta\partial_x + cos\theta\partial_z)\tilde{u}_z]\right]$$

Transformed back to the time domain

$$\partial_t \sigma_H = \rho V_P^2 \left[(1+2\varepsilon)\left[(a_1(2/A) + a_2(2/AQ))[cos\theta cos\varphi \partial_x - sin\theta \partial_z)u_x]\right] + \sqrt{1+2\delta}[(cos\varphi sin\theta \partial_x + cos\theta \partial_z)u_z\ ]\right]$$

$$\partial_t \sigma_V = \rho V_P^2 \left[\sqrt{1+2\delta}[cos\theta cos\varphi \partial_x - sin\theta \partial_z)u_x] + (a_1(2/A) + a_2(2/AQ))[(cos\varphi sin\theta \partial_x + cos\theta \partial_z)u_z]\right]$$

$$A = \left(\sqrt{1+\frac{1}{Q^2}} - \frac{1}{Q}\right)^2 + 1$$

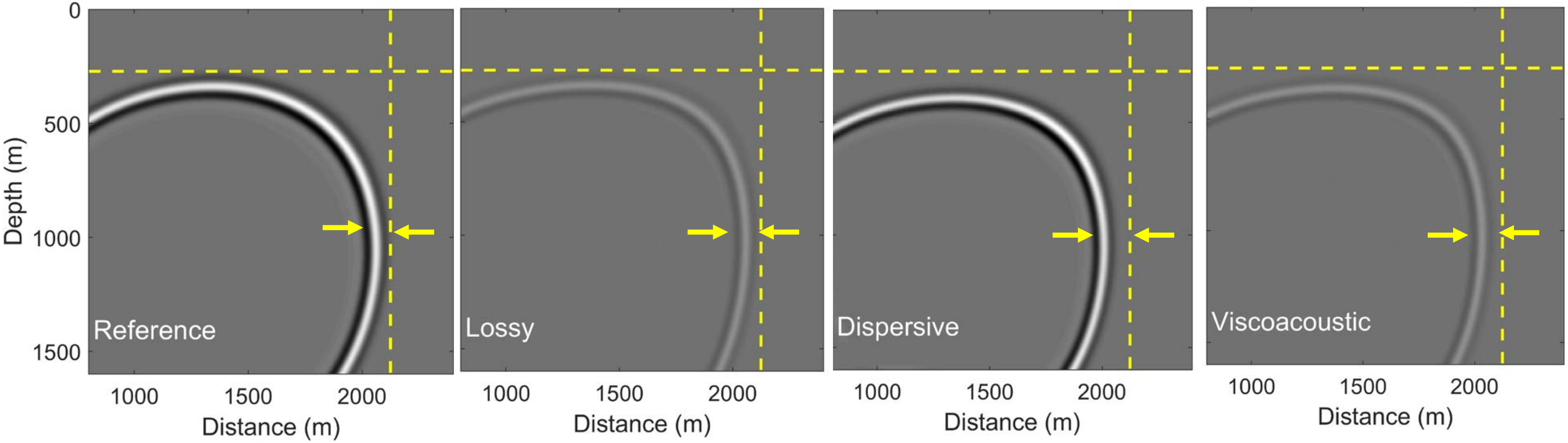
2/A: *Dispersion – dominated operator*

2/AQ: *Amplitude attenuation – dominated operator*

$$a_1, a_2 = 0\ \ or \pm 1$$

Viscoacoustic wave equation in TTI media

2D wavefield snapshots

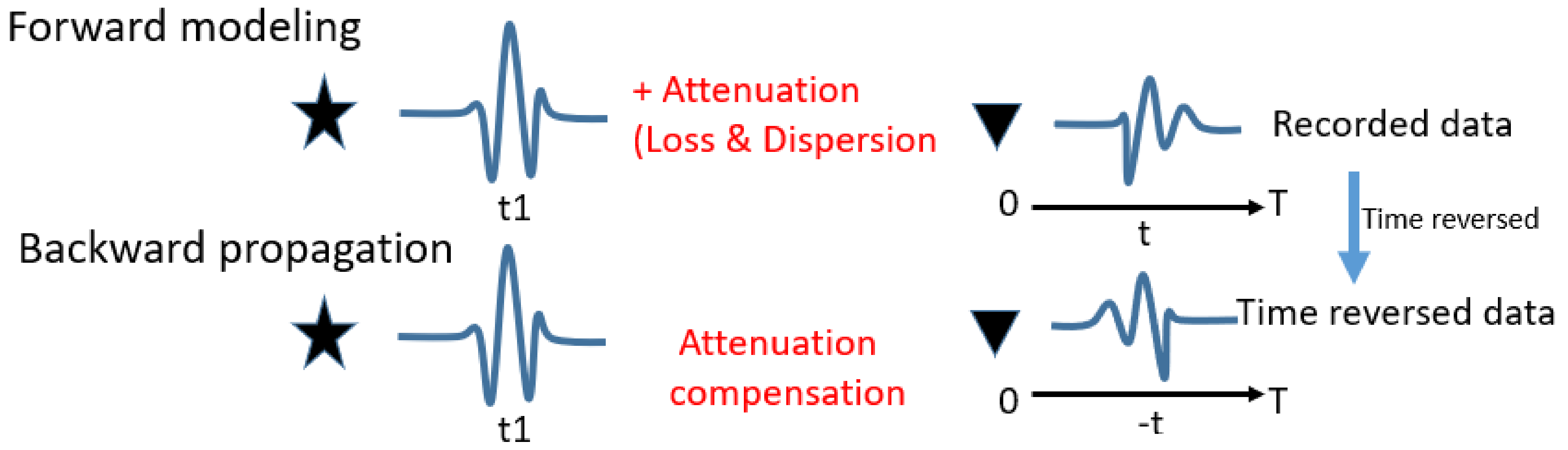


Viscoacoustic reverse time propagation in TTI media

Viscoacoustic reverse time propagation

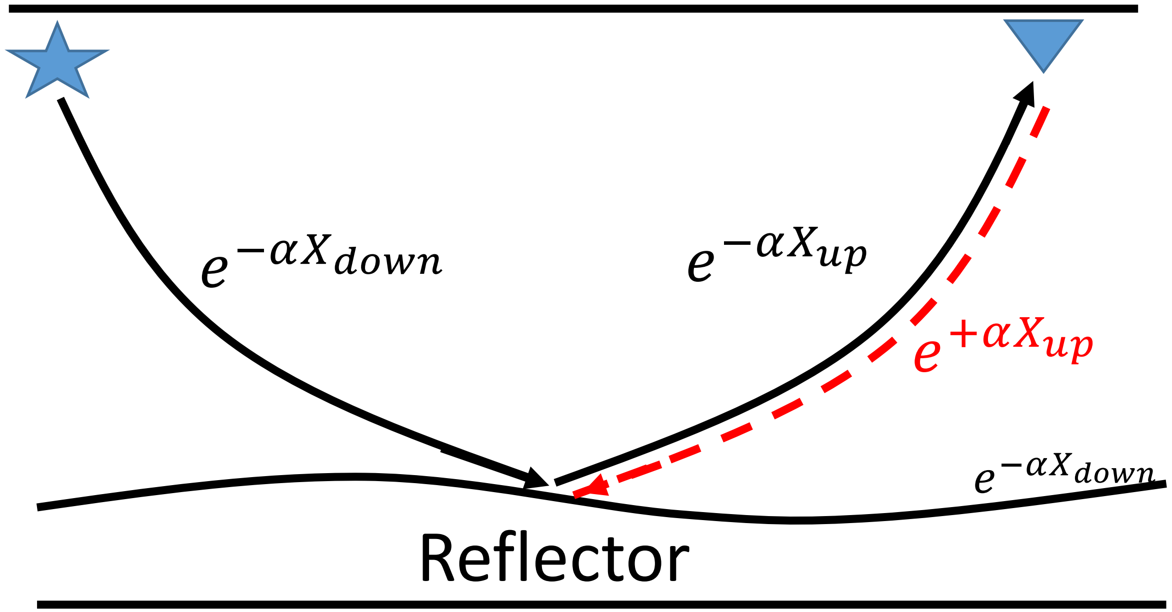
$$\partial_t \sigma_H = \rho V_P^2 [(1+2\varepsilon)\Big[\big((2/A) - (2/AQ)\big)[cos\theta cos\varphi \partial_x - sin\theta \partial_z)u_x]\Big]$$
$$+\sqrt{1+2\delta}[(cos\varphi sin\theta \partial_x + cos\theta \partial_z)u_z \,]]$$

$$\partial_t \sigma_V = \rho V_P^2 [\sqrt{1+2\delta}[cos\theta cos\varphi \partial_x - sin\theta \partial_z)u_x]$$
$$+\big((2/A) - (2/AQ)\big)[(cos\varphi sin\theta \partial_x + cos\theta \partial_z)u_z]$$



Imaging condition for Q-compensated RTM

Imaging condition



$$S^A(x,z,t) = S(x,z,t)e^{-\alpha X_{down}}$$

$$R^A(x,z,t) = R(x,z,t)e^{-\alpha X_{up}}e^{-\alpha X_{down}}$$

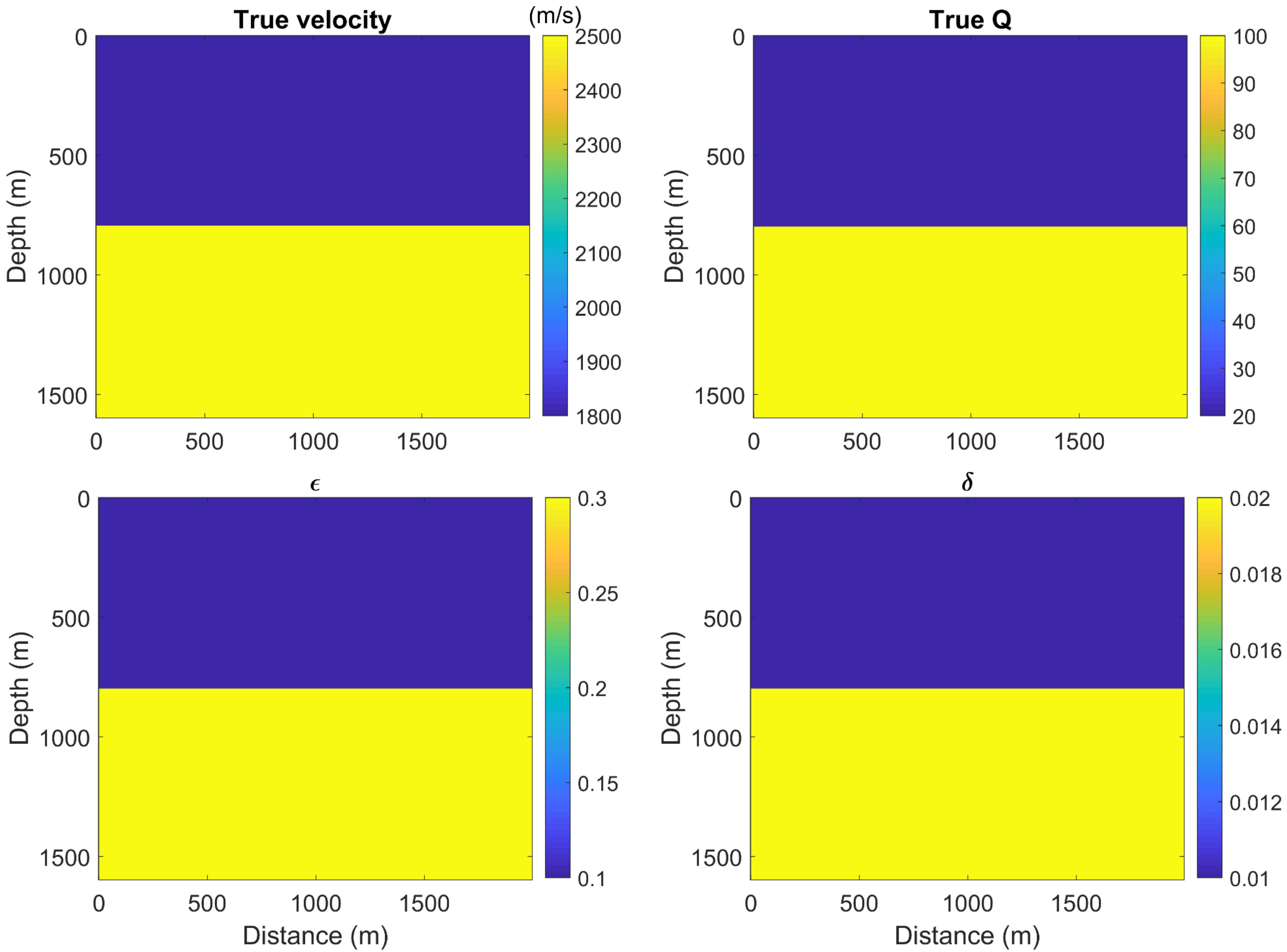
$$R^C(x,z,t) = R^A(x,z,t)e^{+\alpha X_{up}}$$

Source normalized cross-correlation imaging condition

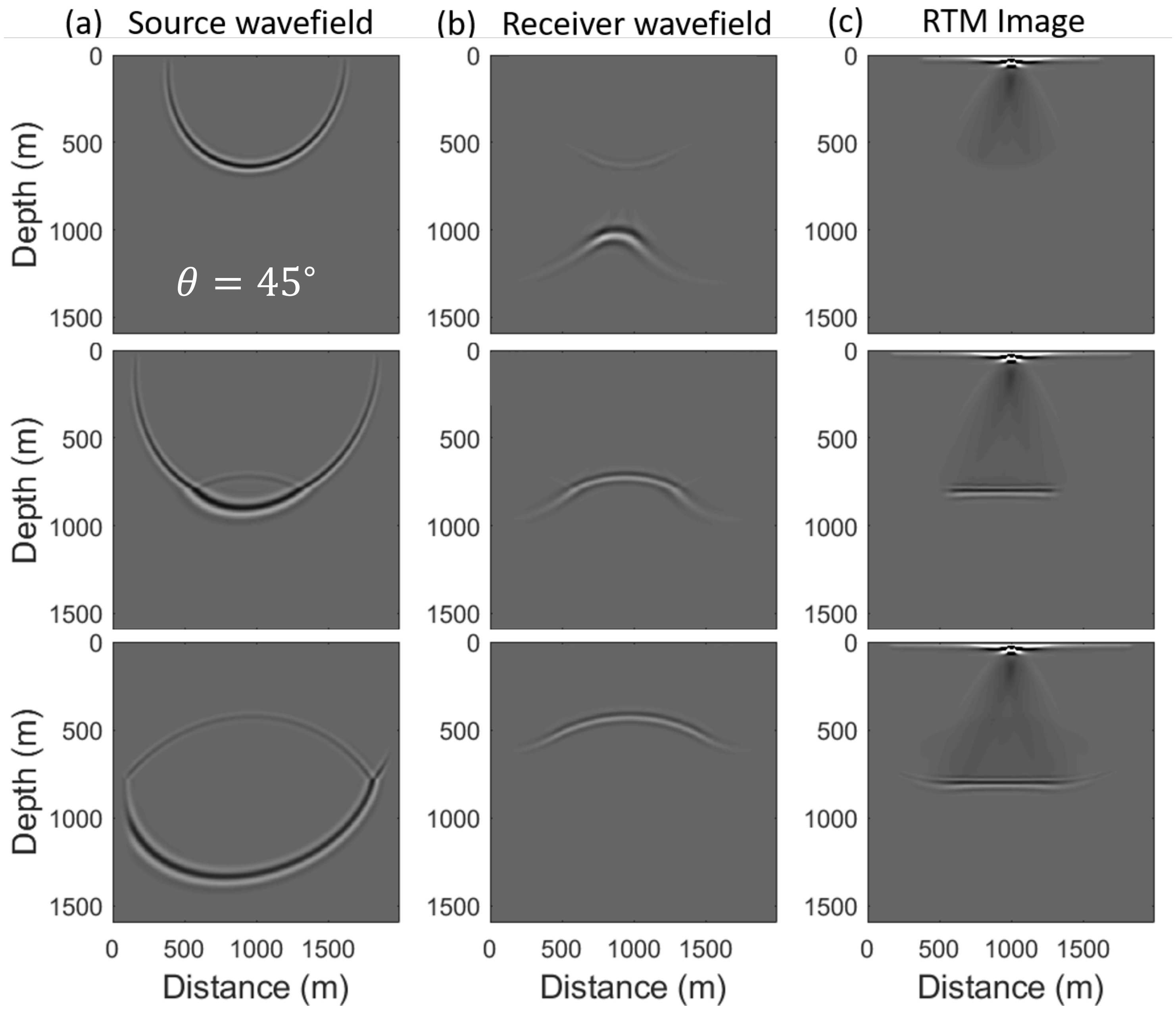
$$I^C(x,z) = \frac{\int S^A(x,z,t)R^C(x,z,t)dt}{\int S^{A2}(x,z,t)\,dt} = \frac{\int [S(x,z,t)e^{-\alpha X_{down}}][e^{+\alpha X_{up}}R(x,z,t)e^{-\alpha X_{up}}e^{-\alpha X_{down}}]\,dt}{\int e^{-2\alpha X_{down}}S^{A2}(x,z,t)\,dt} = \frac{\int S(x,z,t)R(x,z,t)dt}{\int S^2(x,z,t)dt}$$

Synthetic examples

2D synthetic example (Layered model)

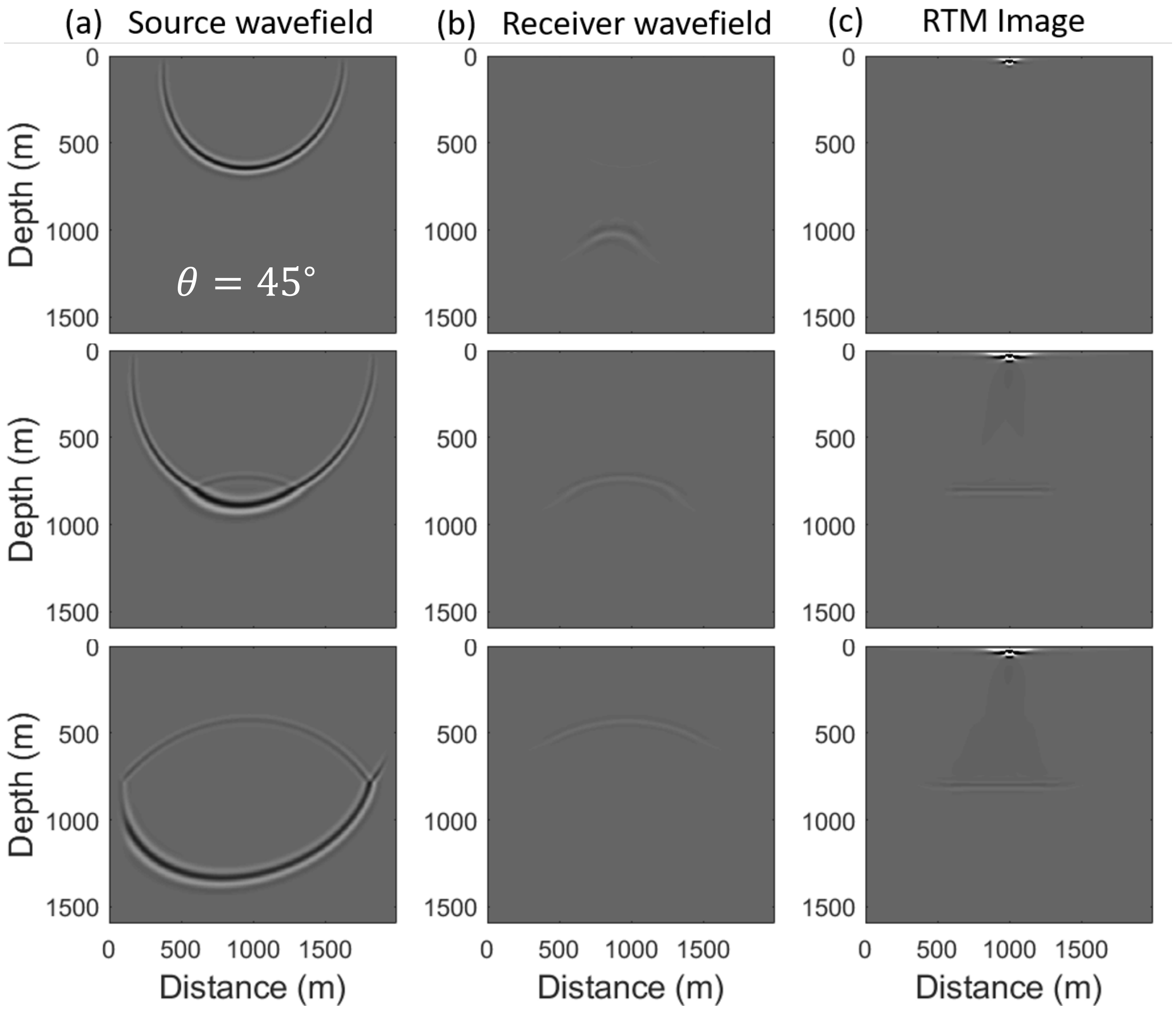


2D synthetic example (Layered model)



Reference snapshot results using acoustic RTM at different time step

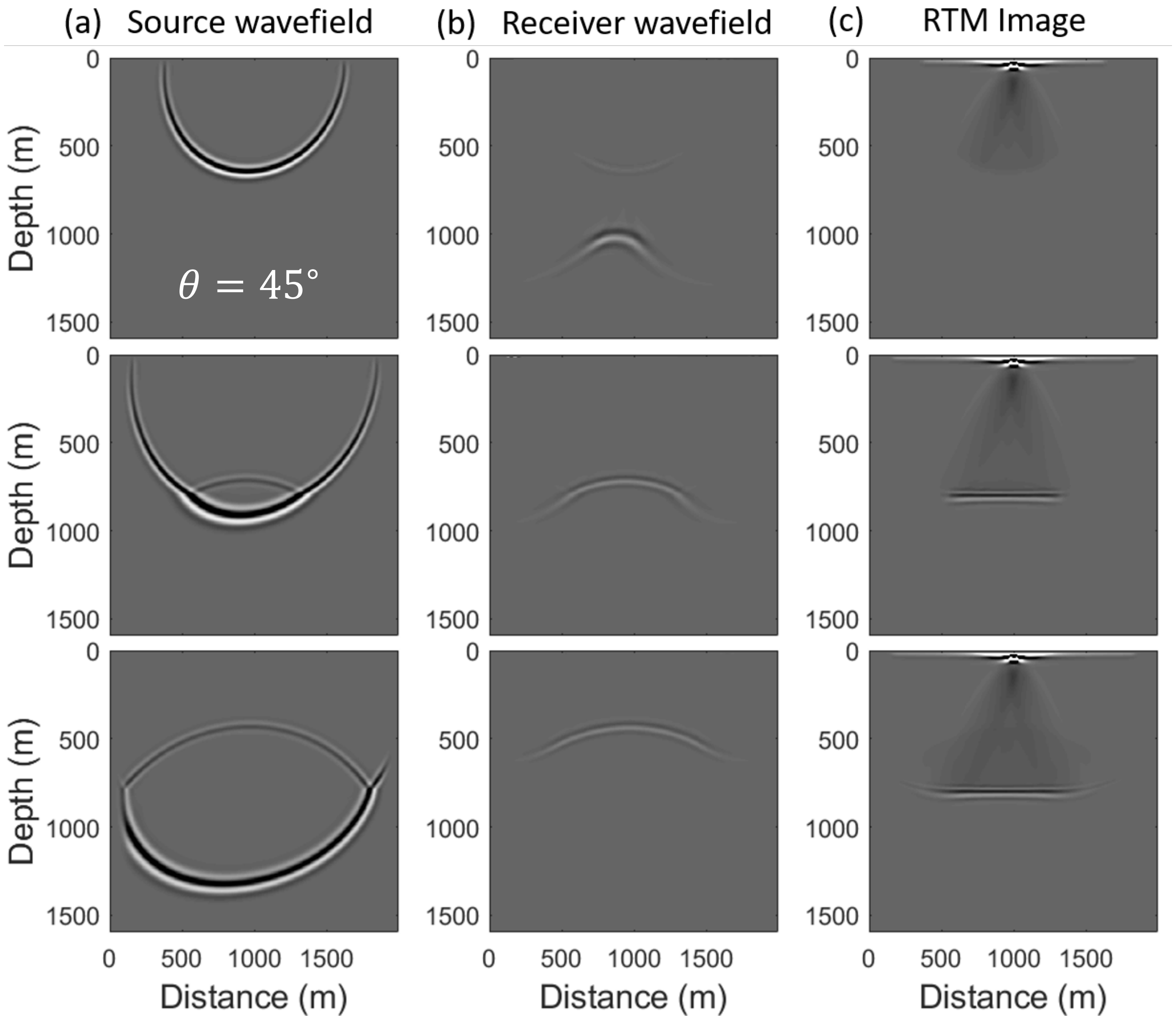
2D synthetic example (Layered model)



Non-compensated snapshot results using acoustic RTM with viscoacoustic data at different time step

- ✓ The receiver wavefield shows reduced wave amplitude while the source wavefield is comparable to the reference result
- ✓ Resulting images at three time slices are underestimated

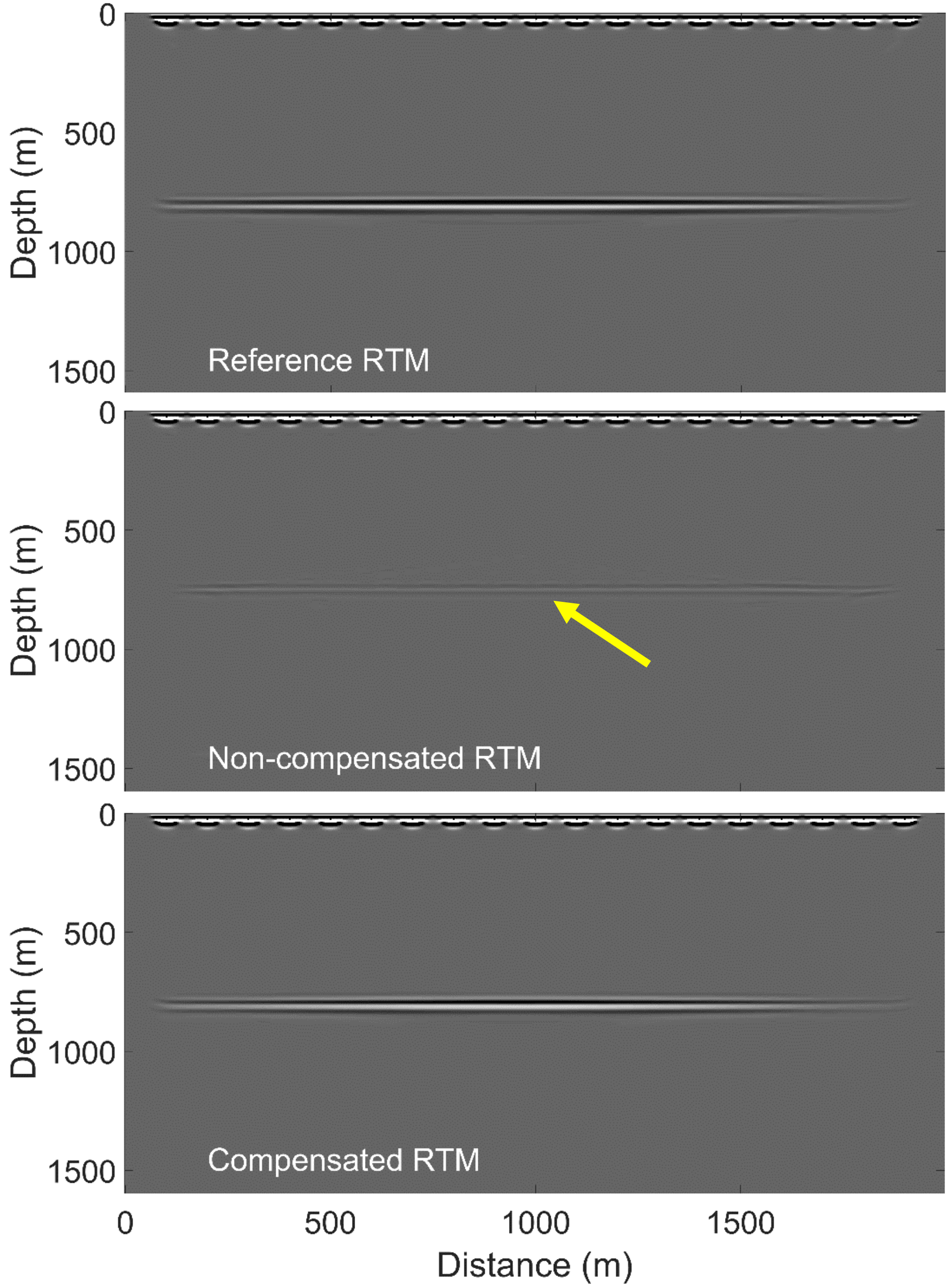
2D synthetic example (Layered model)



To improve image resolution, we test the new approach of Q-RTM on viscoacoustic data

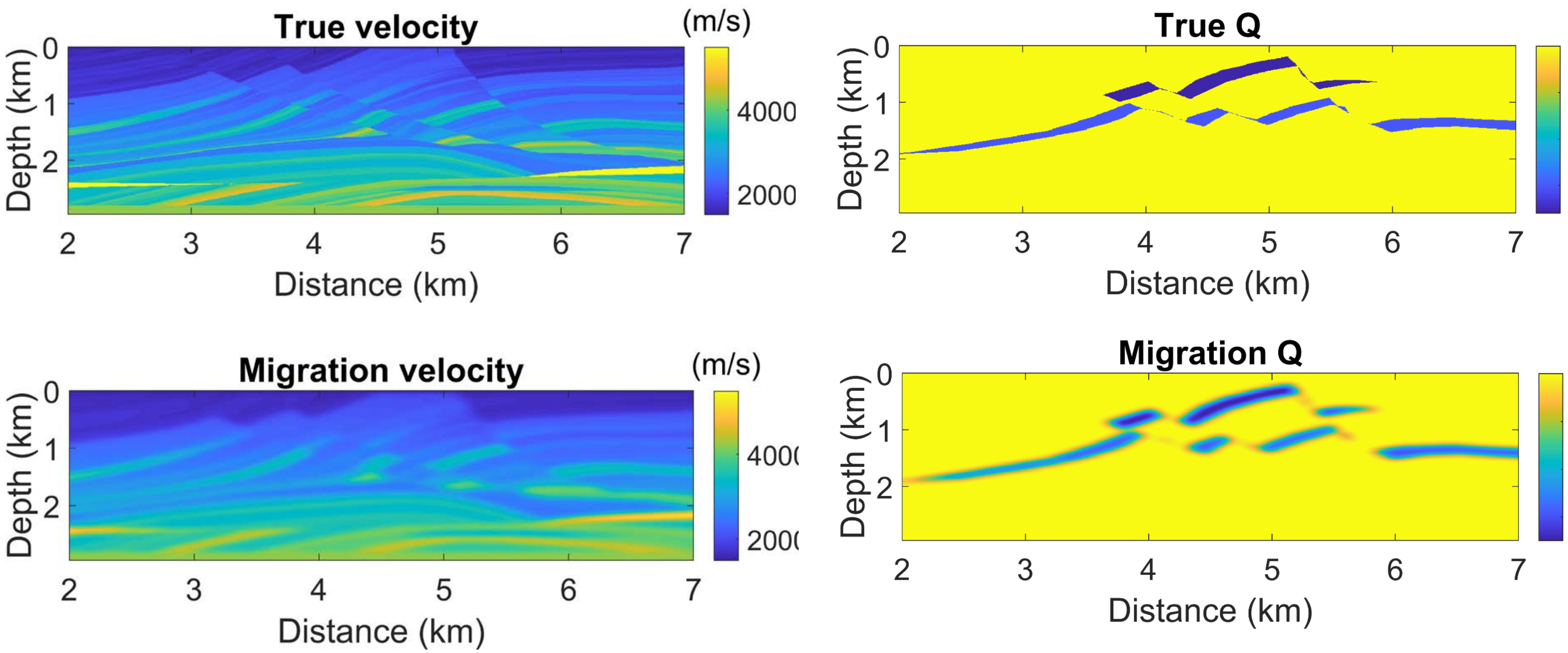
- ✓ Interestingly, such a balanced -attenuation compensation procedure leads to the crosscorrelated Q-RTM images that have comparable amplitude to the corresponding reference images

2D synthetic example (Layered model)

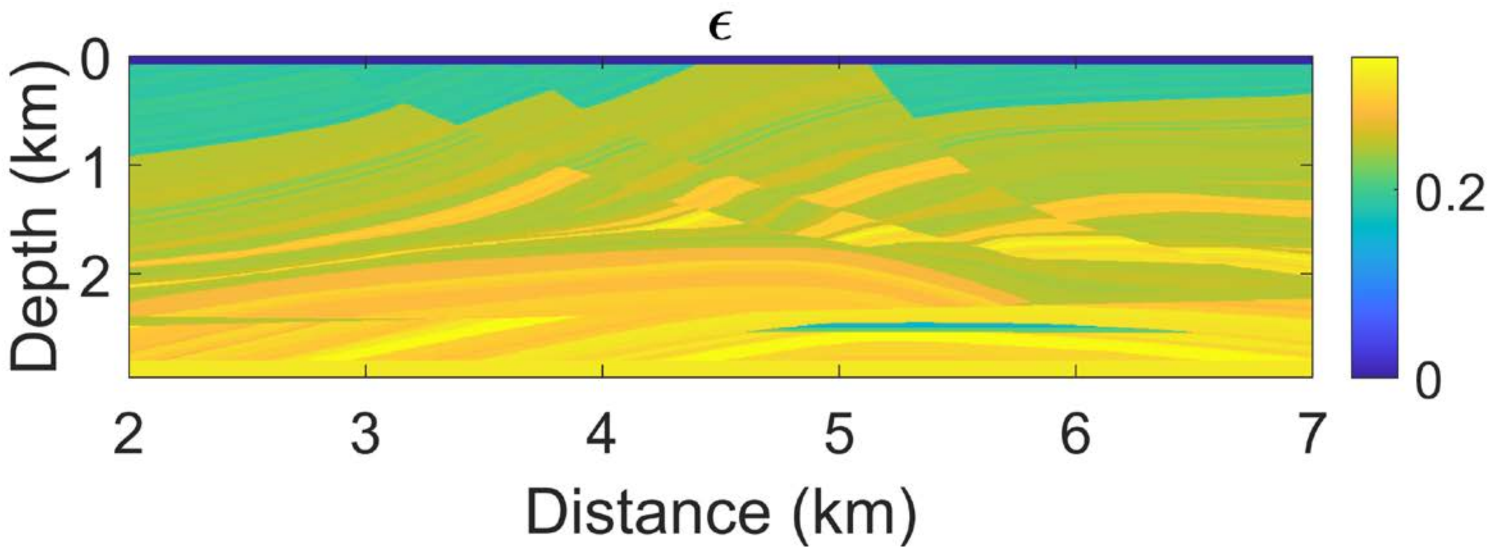


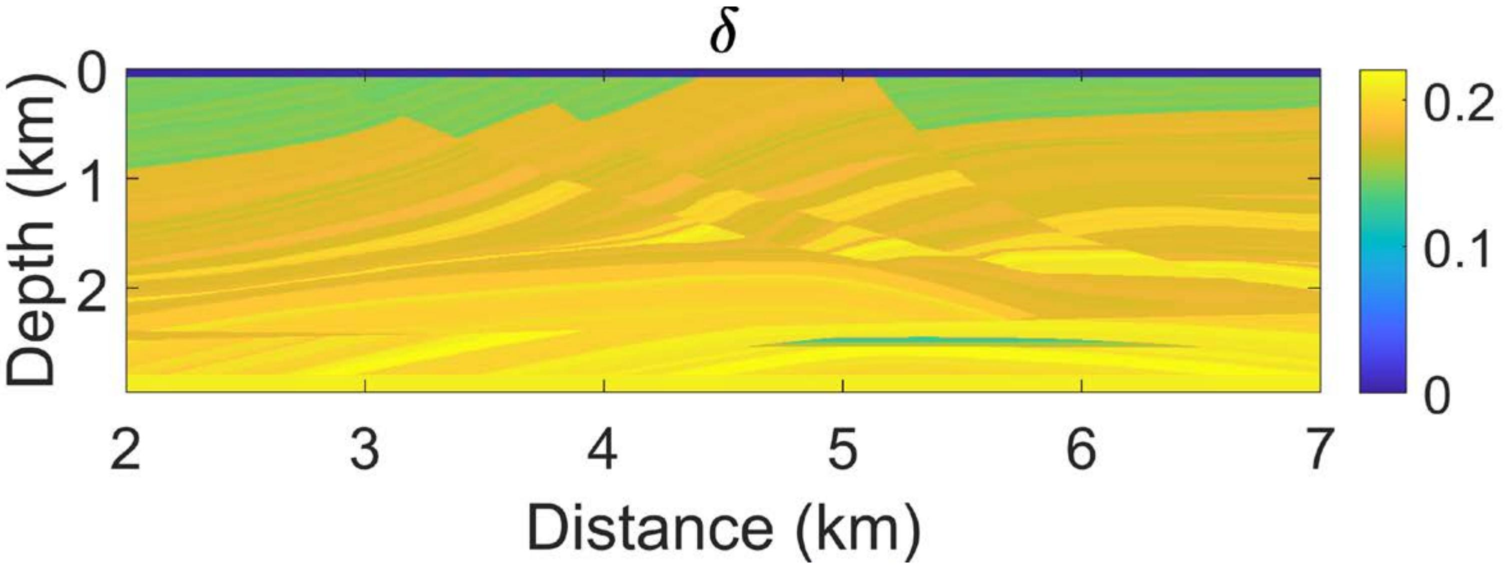
- ✓ In acoustic RTM with viscoacoustic data (non-compensated RTM), there is one reflector in the RTM-image with amplitude loss
- ✓ The result indicates improved RTM image with recovered amplitudes of the reflectors at the dip depths compared with the reference image

2D synthetic example (Marmousi model)



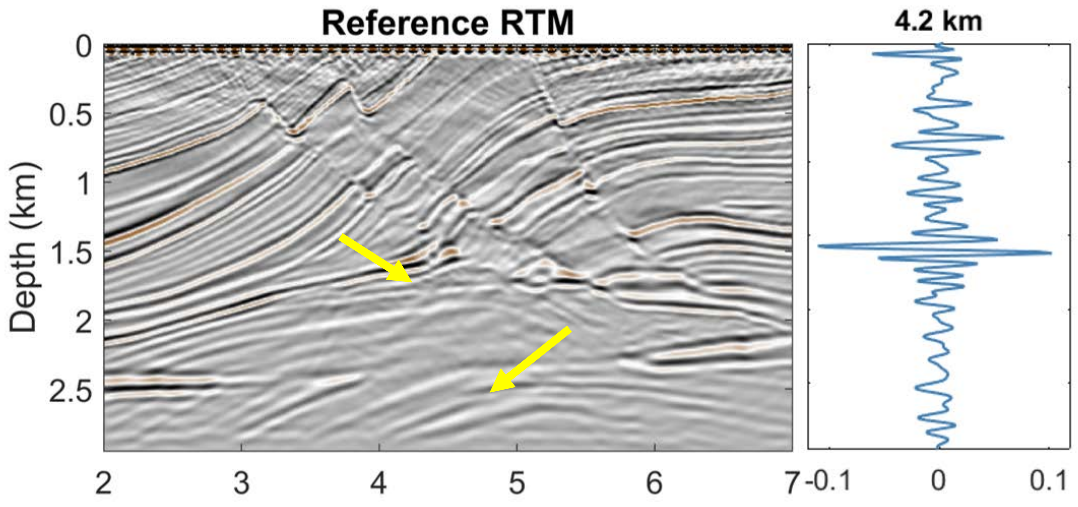
2D synthetic example (Marmousi model)

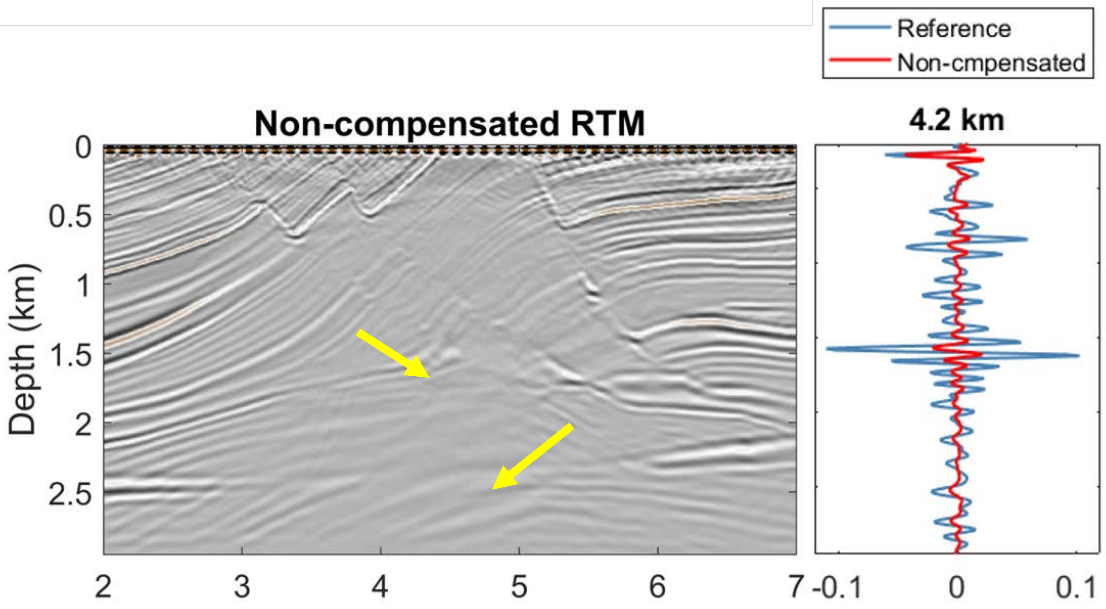


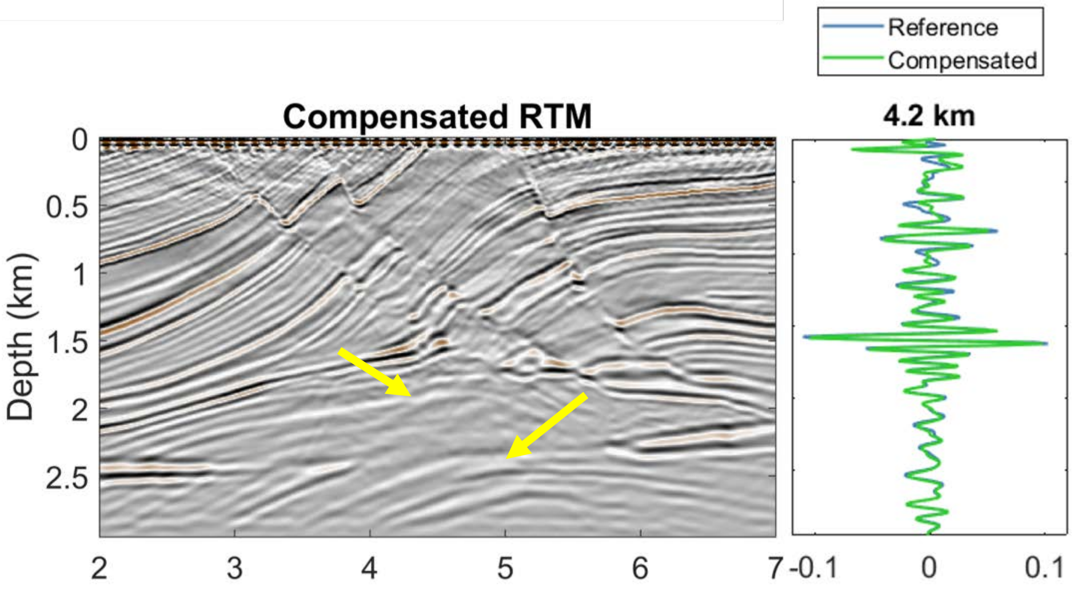


- ✓ some spots of high symmetry axis gradient produce large instabilities and blows up the amplitudes of the wavefield
- ✓ In area with instability, the anisotropy can be taken off around the selected high gradient points which set $\varepsilon = \delta$ to suppress artifacts from the source point in an anisotropic medium

2D synthetic example (Marmousi model)







Conclusions

- We have presented a viscoacoustic RTM imaging algorithm based on a decoupled viscoelastic wave equation that is able to mitigate attenuating and dispersion effects in the migrated images.
- The phase dispersion and amplitude attenuation operators in Q-RTM approach are separated, and the compensation operators are constructed by reversing the sign of the attenuation operator without changing the sign of the dispersion operator.
- We found that source normalized cross-correlation imaging condition more suitable, and only backward receiver wavefield is needed to compensated.

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