Inversion based deblending using Stolt-based Radon operators

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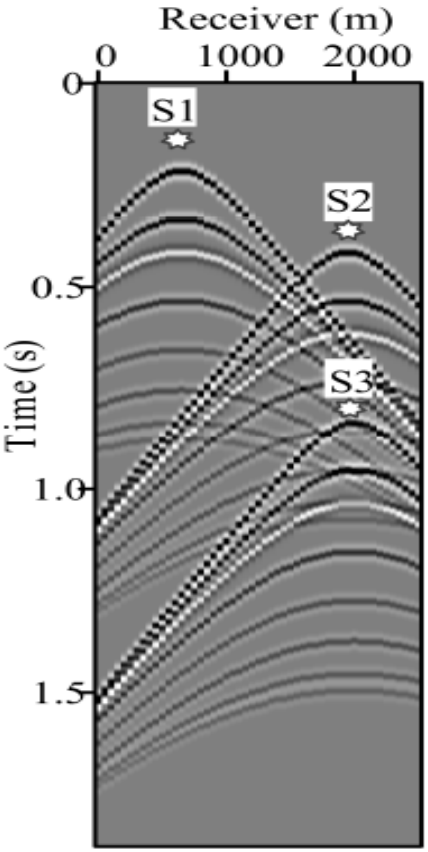
Outline

1. Blended Sources.
2. Deblending Methods.
3. Challenges.
   a) Strong interferences.
   b) Computational speed.
4. Stolt-based Radon Transforms.
5. Examples.

1) Blended Sources

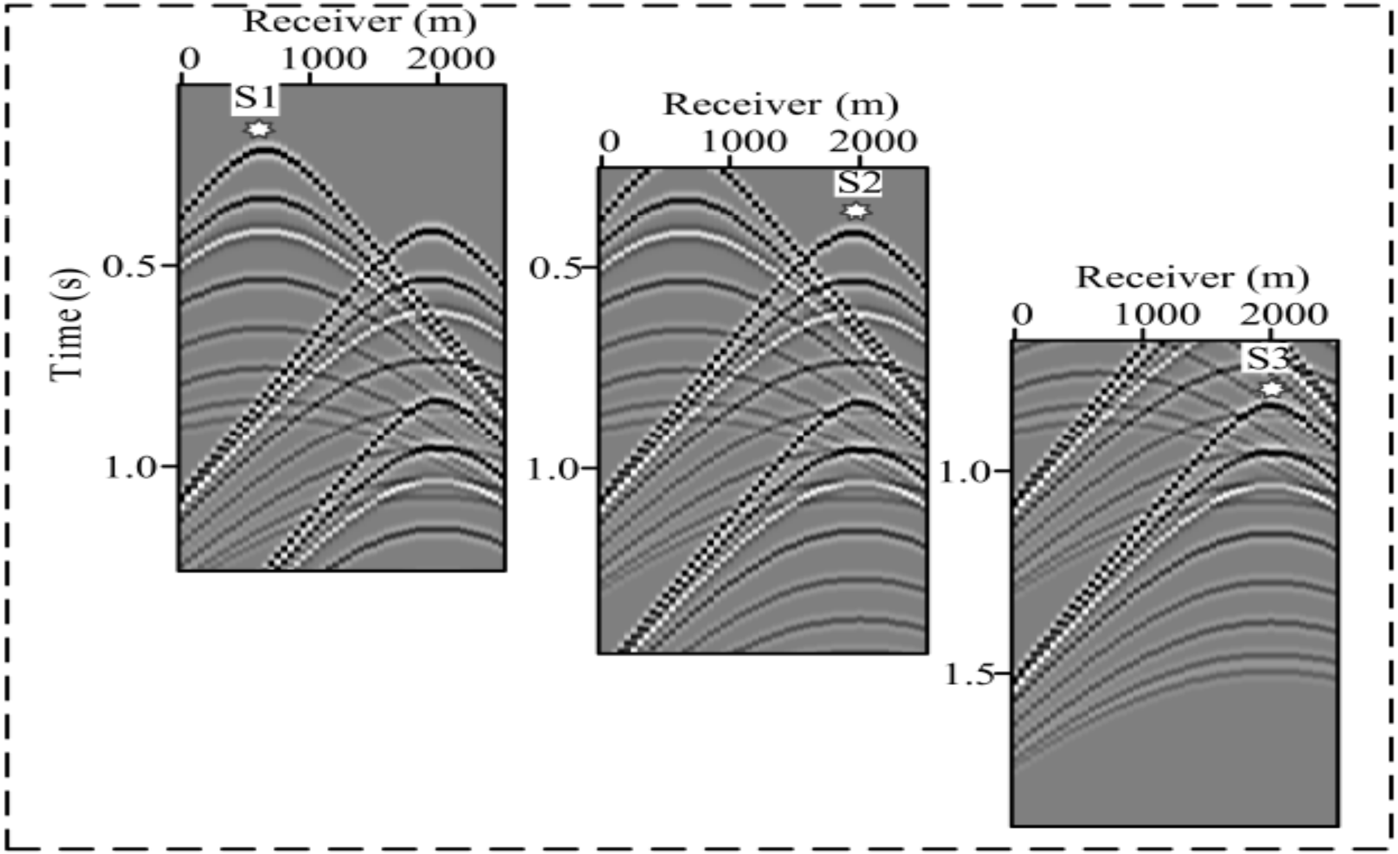
blending

$$\mathbf{b} = \Gamma\ \mathbf{D}$$

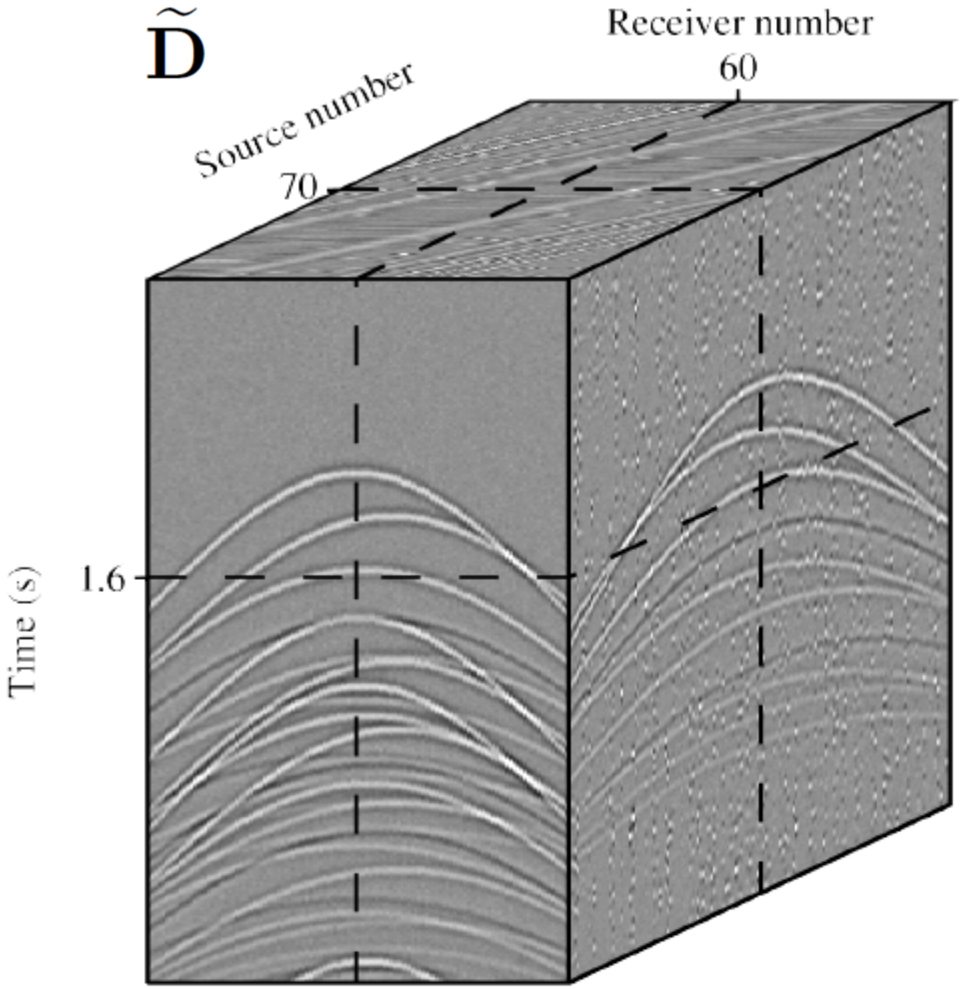


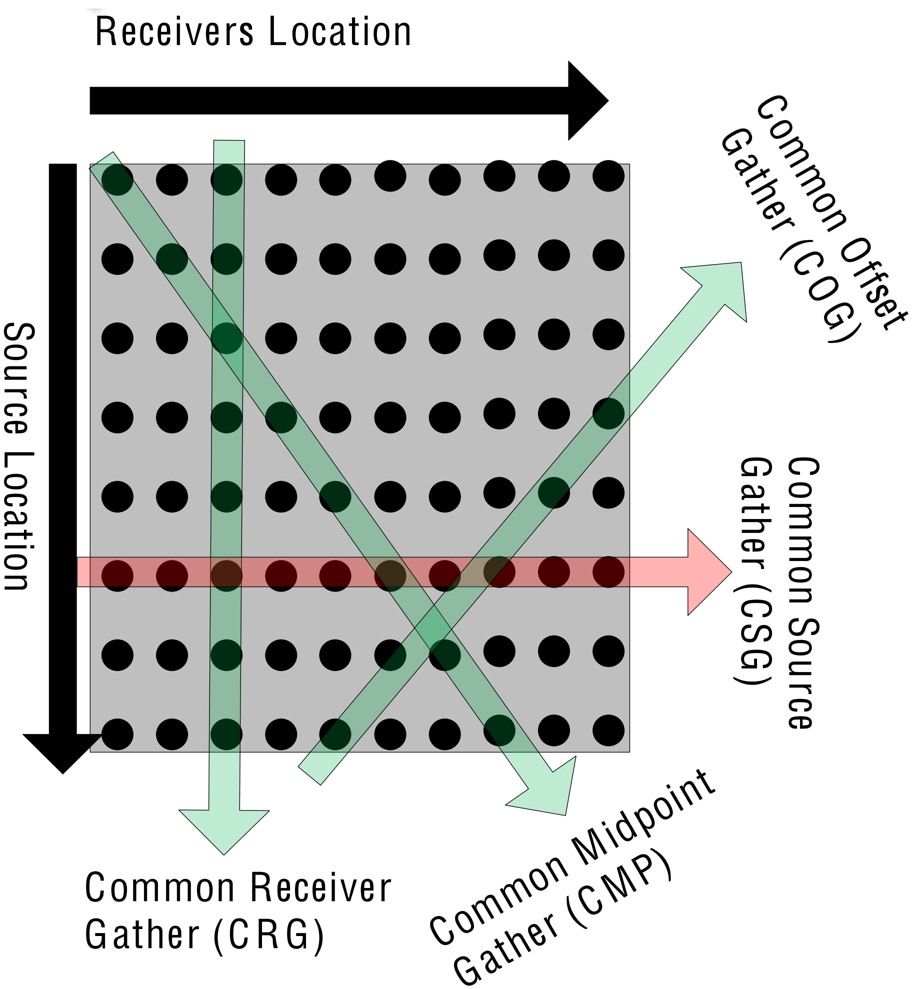
Pseudo de-blending

$$\widetilde{\mathbf{D}} = \Gamma^T \mathbf{b}$$



1) Blended Sources





2) Deblending Methods

Deblending (separation) methods

Denoising-based

$$\widetilde{\mathbf{D}} = \Gamma^T \mathbf{b}$$

$$J = \|\widetilde{\mathbf{D}} - \mathbf{L}\,\mathbf{m}\|_2^2 + \mu\|\mathbf{m}\|_1^1$$

Examples:

- Dip filtering (Beasley et al., 1998; Beasley, 2008)
- Adaptive subtraction (Kim et al., 2009)
- Apex Shifted Radon (Trad et al. 2012)
- Median filter (Huo et al., 2012)
- Robust Radon (Ibrahim and Sacchi 2014).
- Migration operators (Ibrahim and Sacchi 2015)

Inversion-based

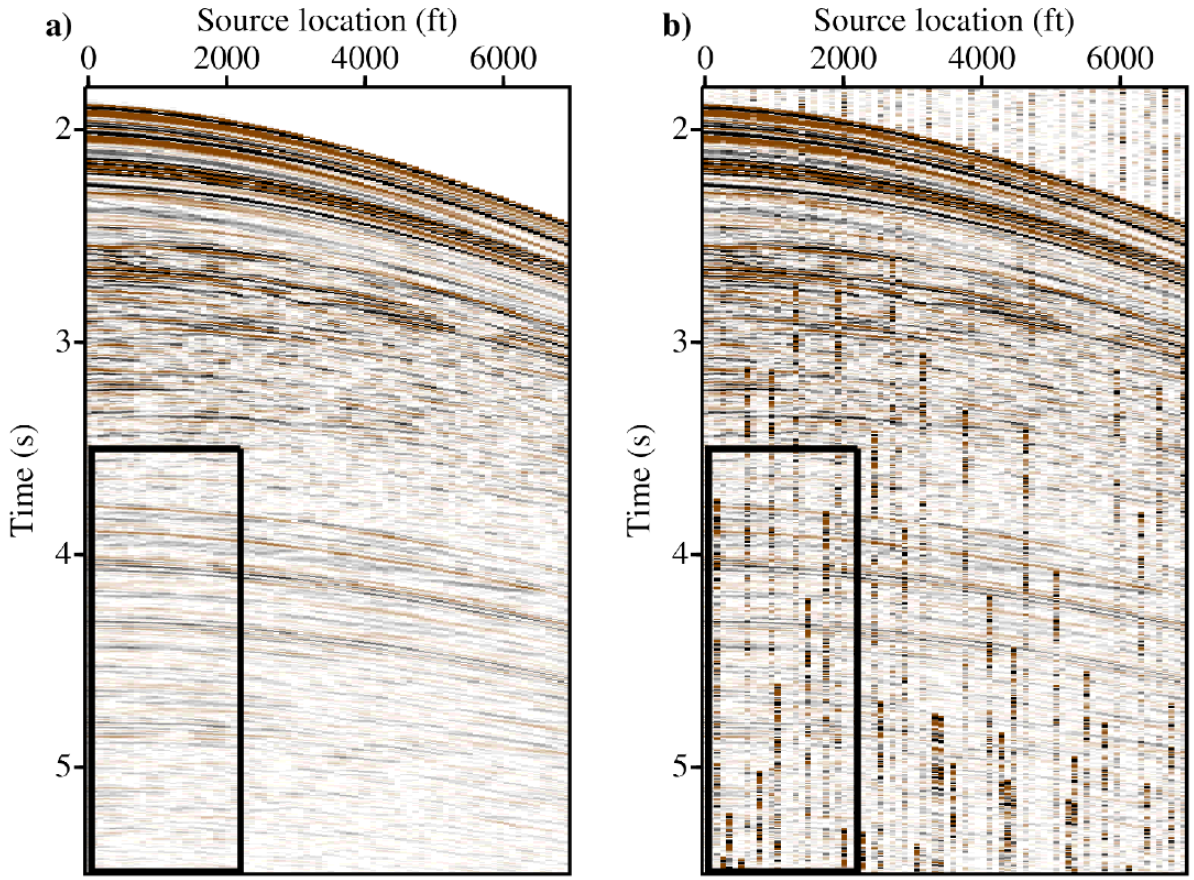
$$\mathbf{D} \quad = \mathbf{L}\mathbf{m}$$

$$J = \|\mathbf{b} - \Gamma\mathbf{L}\mathbf{m}\|_2^2 + \mu\|\mathbf{m}\|_1^1$$

Examples:

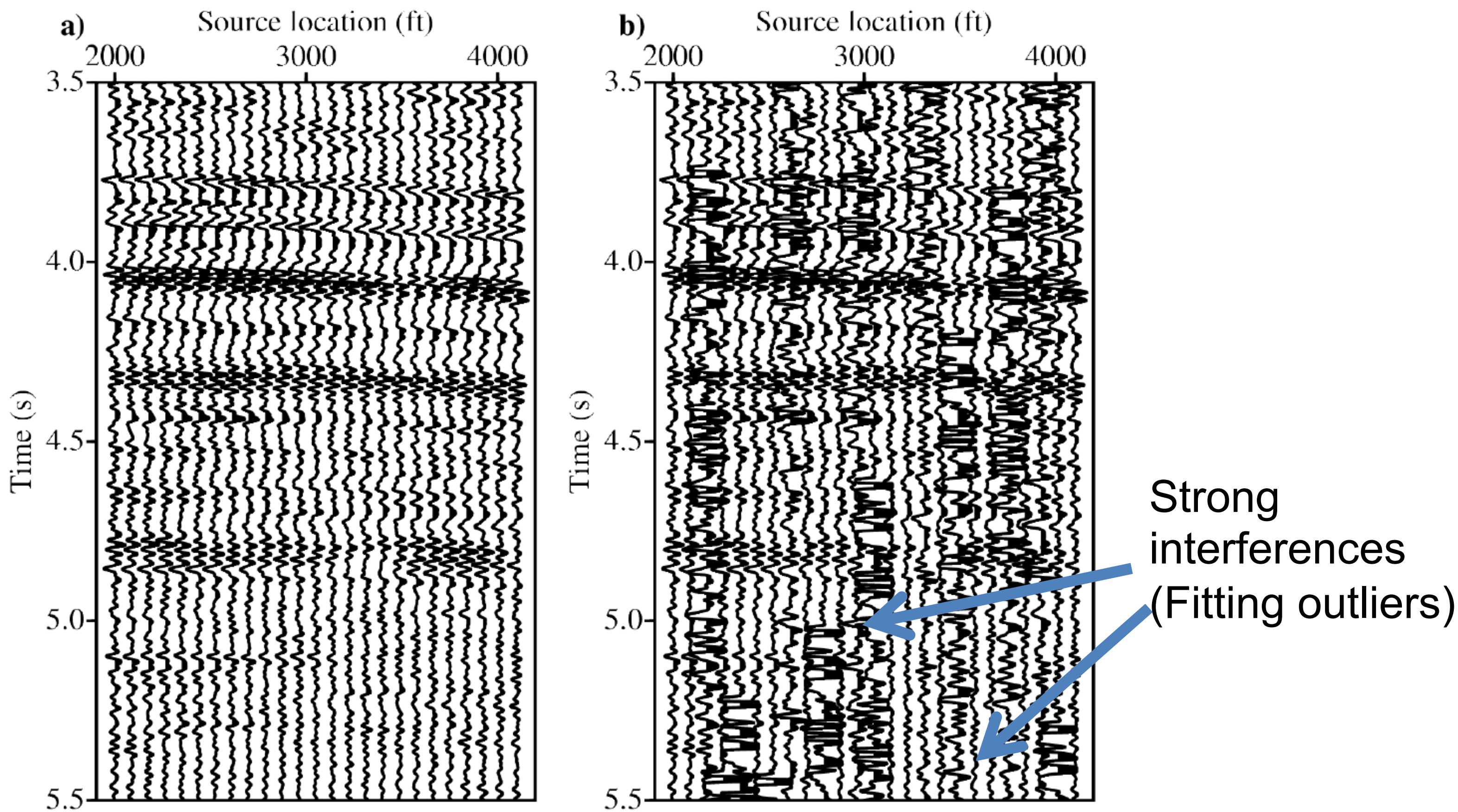
- Sparse Radon inversion (Moore et al., 2008;Akerberg et al., 2008)
- Iterative f -k filtering (Mahdad et al., 2011)
- Curvelet-based (Wason et al., 2011)
- Focal transform (Kontakis and Verschuur 2015)

3) Challenges: a) Strong Source Interferences



Numerically blended Gulf of Mexico data

3) Challenges: a) Strong Source Interferences



Numerically blended Gulf of Mexico data

3) Challenges: a) Strong Source Interferences

$$J = \|\mathbf{r}\|_p^p + \mu\|\mathbf{m}\|_q^q$$
$$= \|\mathbf{d} - \mathbf{Lm}\|_p^p + \mu\|\mathbf{m}\|_q^q$$

Misfit

Regularization (penalty)

$\|\mathbf{r}\|_1^1$

Robust

Claerbout&Muir 1973;
Guitton&Symes, 2003;
Ji, 2006, 2012;
Ibrahim and Sacchi 2014,2015

$\|\mathbf{r}\|_2^2$

Non-robust

$\|\mathbf{m}\|_2^2$

Least Squares

Hampson 1986

$\|\mathbf{m}\|_1^1$

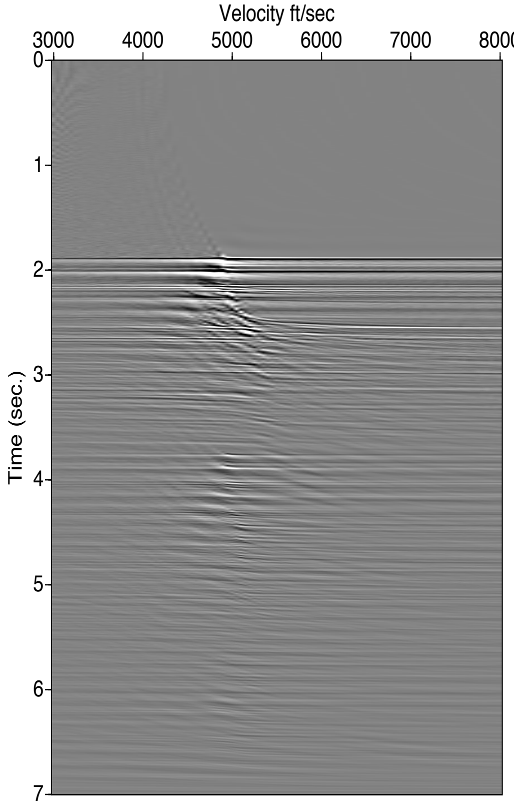
Sparse (High resolution)

Thorson&Claerbout 1985;
Sacchi&Ulrych 1995;
Trad et al. 2003

3) Challenges: b) Computational cost.

m (1751X101)
176851

d (1751X183)
320433

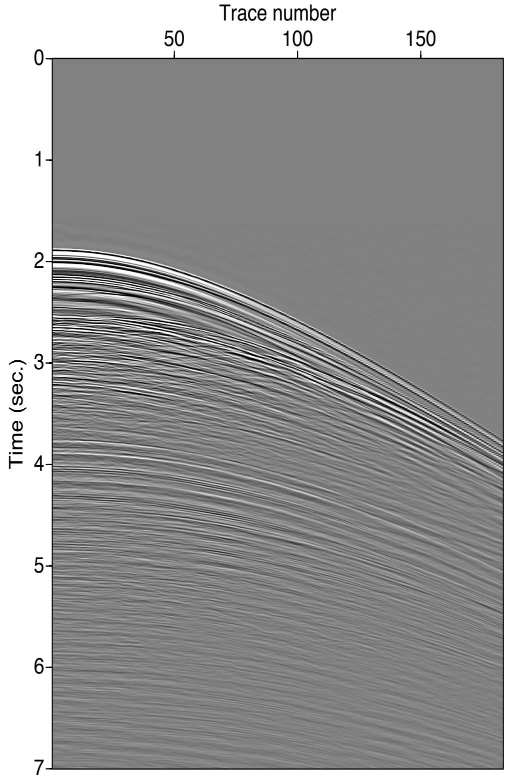


L^T

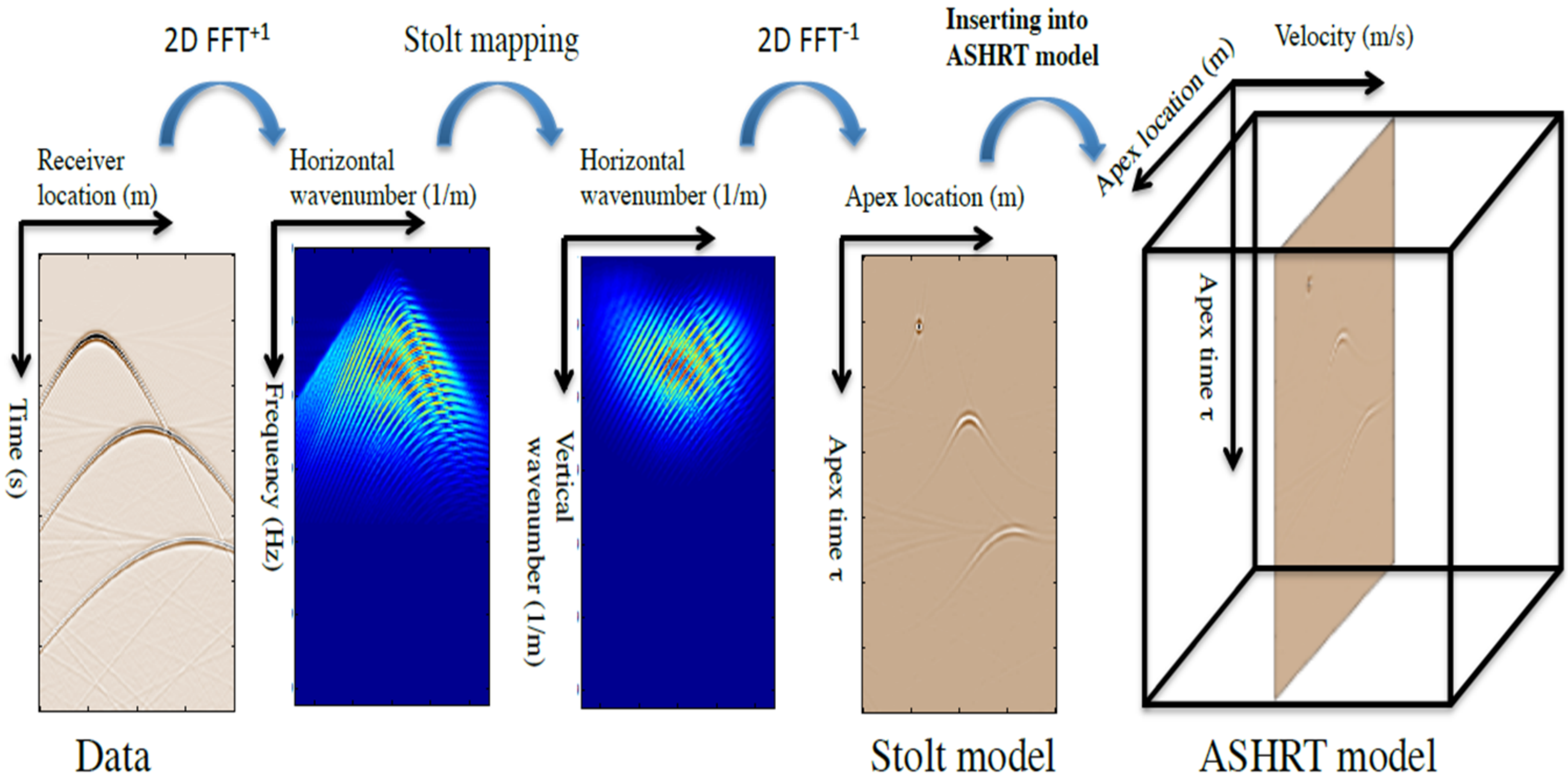
(176851 X 320433)

Too Big

We have to use fast operators



4) Stolt-based Radon Transform



4) Stolt-based Radon Transform

Apex Shifted Hyperbolic Radon (ASHRT) Transform

$$\mathbf{d}(t,h) = \sum_{a_{min}}^{a_{max}} \sum_{v_{min}}^{v_{max}} \mathbf{m}\left(\tau = \sqrt{t^2 - \frac{(h-a)^2}{v^2}}, v, a\right)$$

$$\widetilde{\mathbf{m}}(\tau, v, a) = \sum_{h_{min}}^{h_{max}} \mathbf{d}\left(t = \sqrt{\tau^2 + \frac{(h-a)^2}{v^2}}, h\right)$$

Stolt-based ASHRT Transform

$$\omega_\tau = \sqrt{\omega^2 - (vk_x)^2}$$

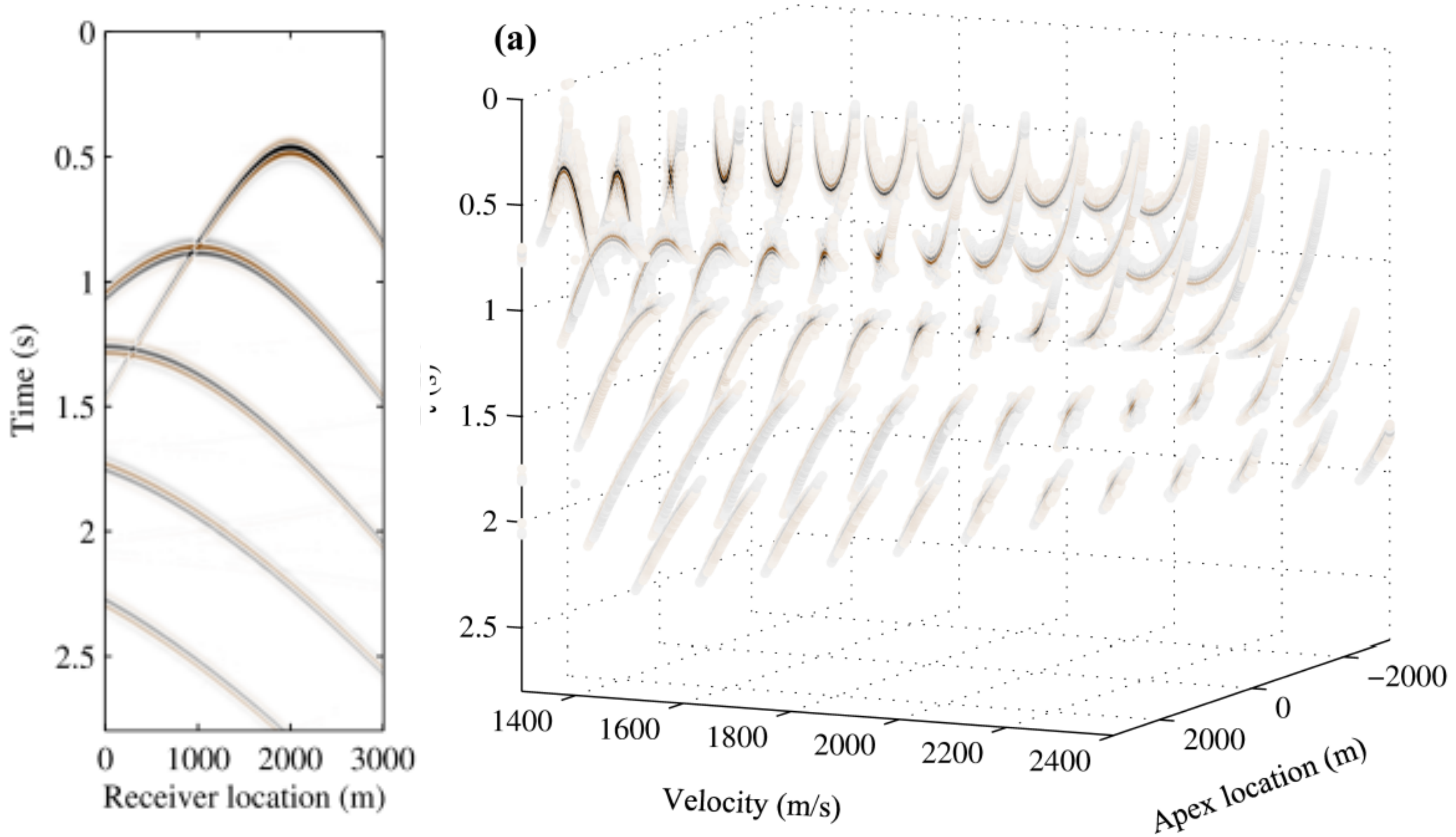
$$\mathbf{d}(t,x) = \int\int\int \mathbf{m}(\omega_\tau = \sqrt{\omega^2 - (vk_x)^2}, v, k_x)\ e^{ik_x x + i\omega t}\ d\omega\ dk_x\ dv$$

$$\widetilde{\mathbf{m}}(\tau, v, x) = S \int\int \mathbf{d}(\omega = \sqrt{\omega_\tau^2 + (vk_x)^2}, k_x)\ e^{-ik_x x - i\omega_\tau(v)\tau}\ d\omega_\tau\ dk_x$$

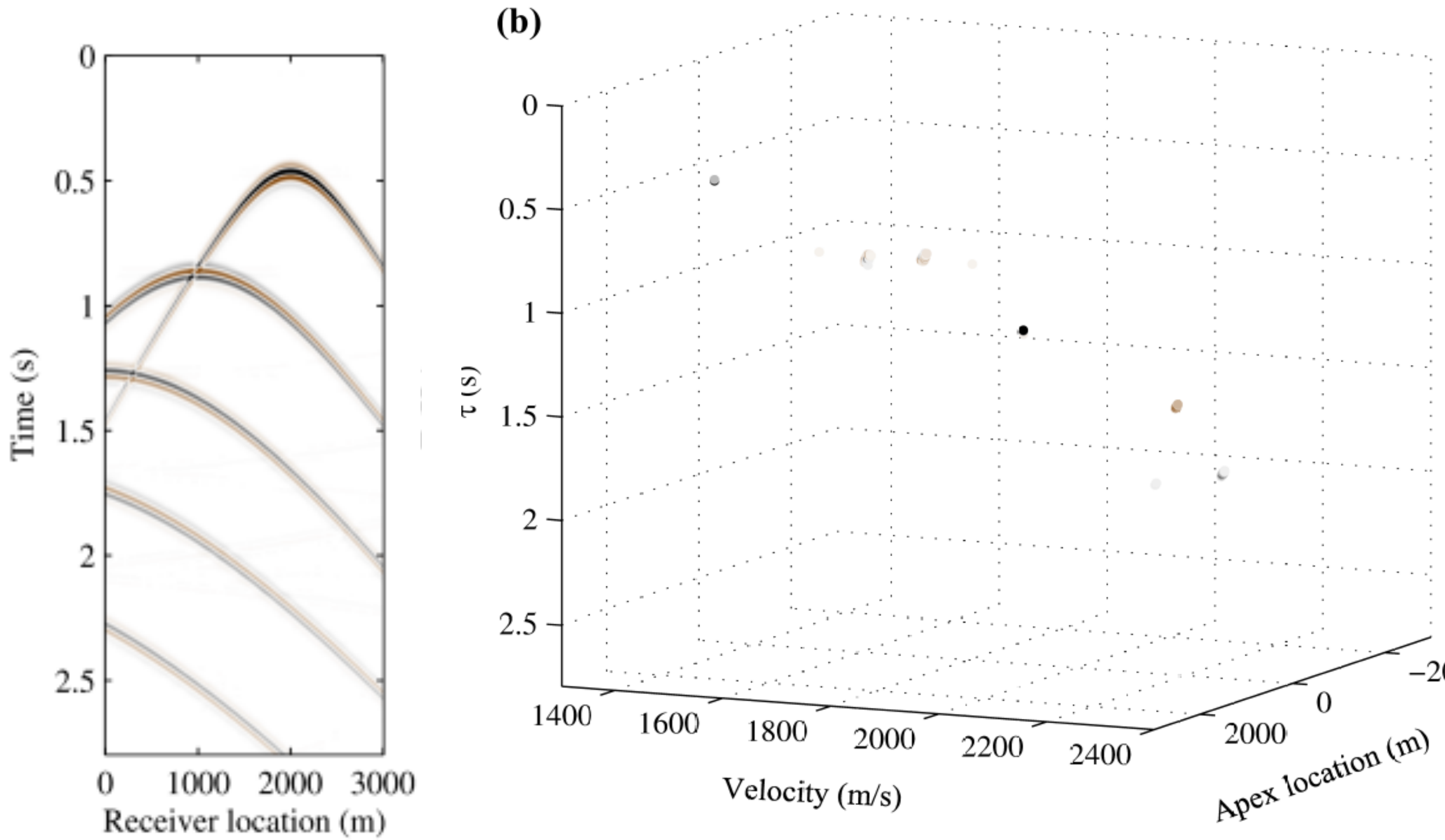
Trad, D. 2003, Interpolation and multiple attenuation with migration operators, Geophysics 68 (6), P. 2043–2054

Ibrahim and Sacchi, 2014, Simultaneous source separation using a robust Radon transform, Geophysics 79(1): V1-V11.

4) Stolt-based Radon Transform



4) Stolt-based Radon Transform



4) Stolt-based Radon Transform: Diffractions

The double square root equation for diffractions travel-time

$$t = \sqrt{t_d^2 + (x_s - x_d)^2/v^2} + \sqrt{t_d^2 + (x_d - x_r)^2/v^2}$$

$$\tau_0 = \sqrt{t_d^2 + (x_s - x_d)^2/v^2}$$

We can use this equation to define the new Asymptote and Apex Shifted Radon (AASHRT)

$$t = \tau_0 + \sqrt{t_d^2 + \frac{(x_d - x_r)^2}{v^2}}$$

Ibrahim , A, Trenghi, P. and Sacchi, M. D. 2018, Simultaneous reconstruction of seismic reflections and diffractions using a global hyperbolic Radon dictionary, Geophysics 83 (6), V315-V323

4) Stolt-based Radon Transform: Diffractions

The time domain AASHRT operators are

$$d(t,x_r) = \sum_{\tau_0}\sum_{x_a}\sum_{v} m\left(\tau = \sqrt{t^2 - \frac{(x_r - x_a)^2}{v^2}} - \tau_0, v, x_a, \tau_0\right)$$

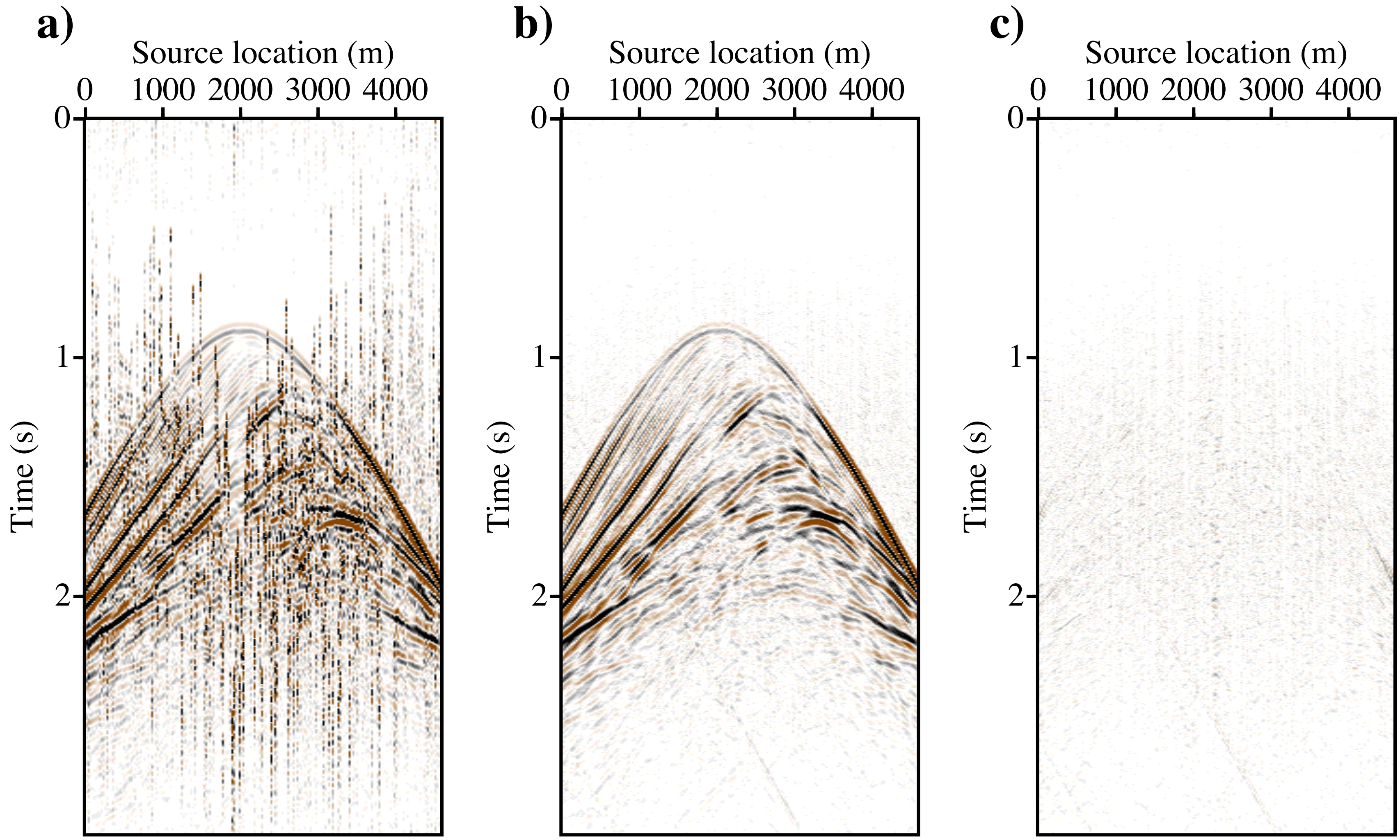
$$\widetilde{m}(\tau, v, x_a, \tau_0) = \sum_{x_r} d\left(t = \tau_0 + \sqrt{\tau^2 + \frac{(x_r - x_a)^2}{v^2}}, x_r\right)$$

The Stolt-based AASHRT operators are

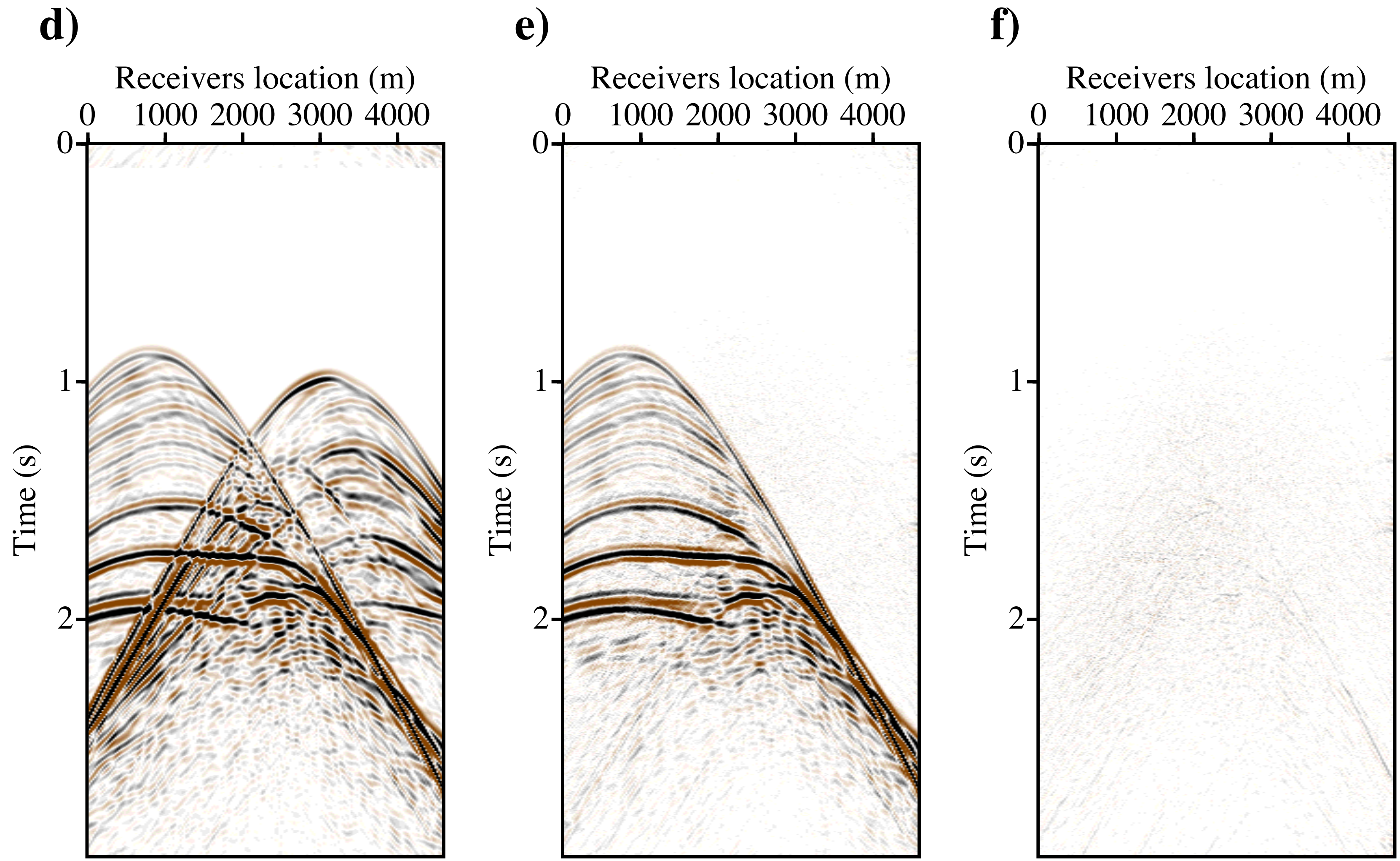
$$d(t,x) = \int\int\int\int m\left(\omega_\tau = \sqrt{\omega^2 - (vk_x)^2}, v, k_x\right) \times \exp[-i\omega_\tau \tau_0] \exp[ik_x x + i\omega t]\, d\omega\, dk_x\, dv\, d\tau_0$$

$$\widetilde{m}(\tau, v, x_a, \tau_0) = C \int\int \exp[i\omega_\tau \tau_0] d\left(\omega = \sqrt{\omega_\tau^2 + (vk_x)^2}, k_x\right) \times \exp[-ik_x x - i\omega_\tau(v)\tau]\, d\omega_\tau\, dk_x$$

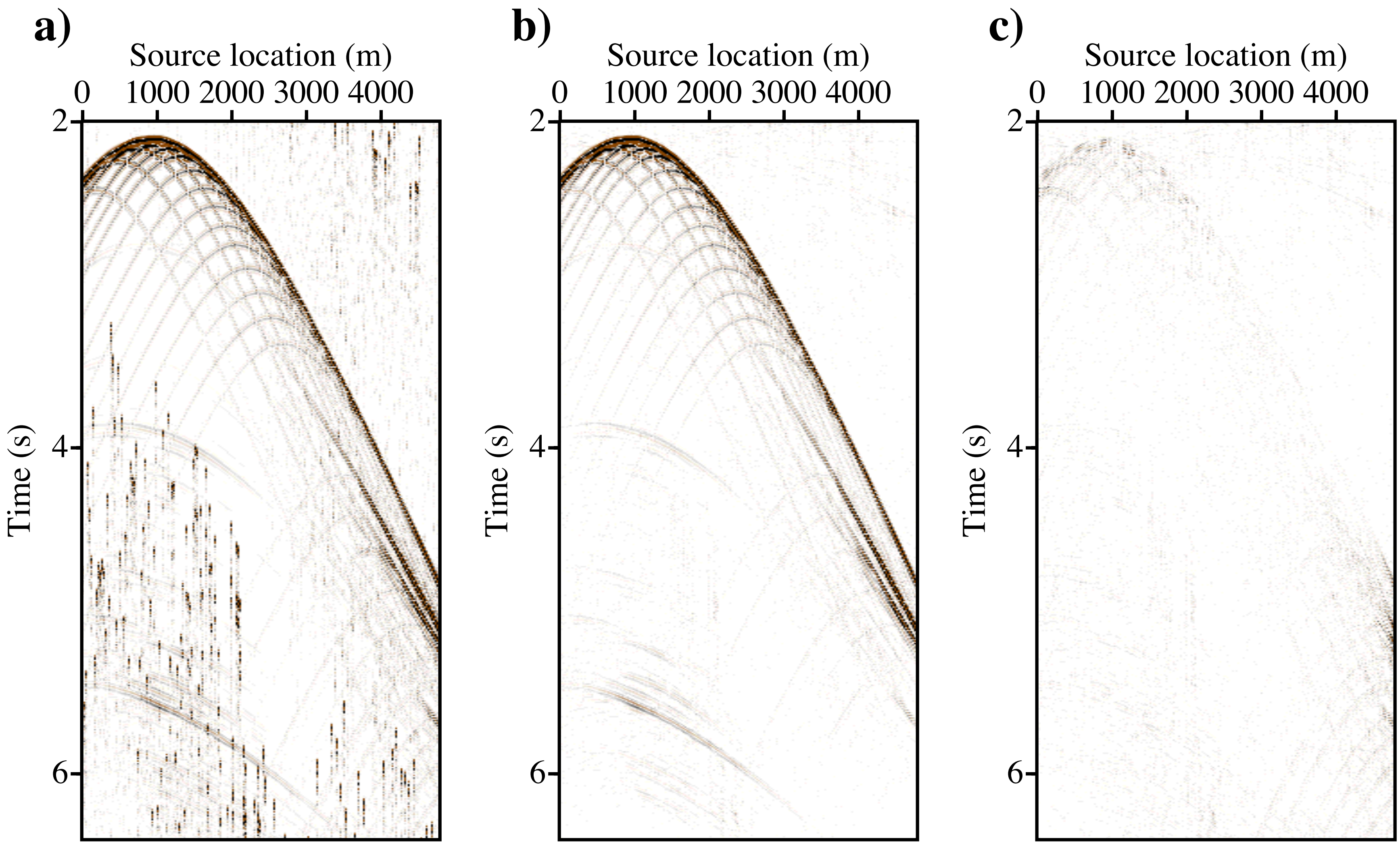
5) Examples: Marmousi – Common Receiver Gather



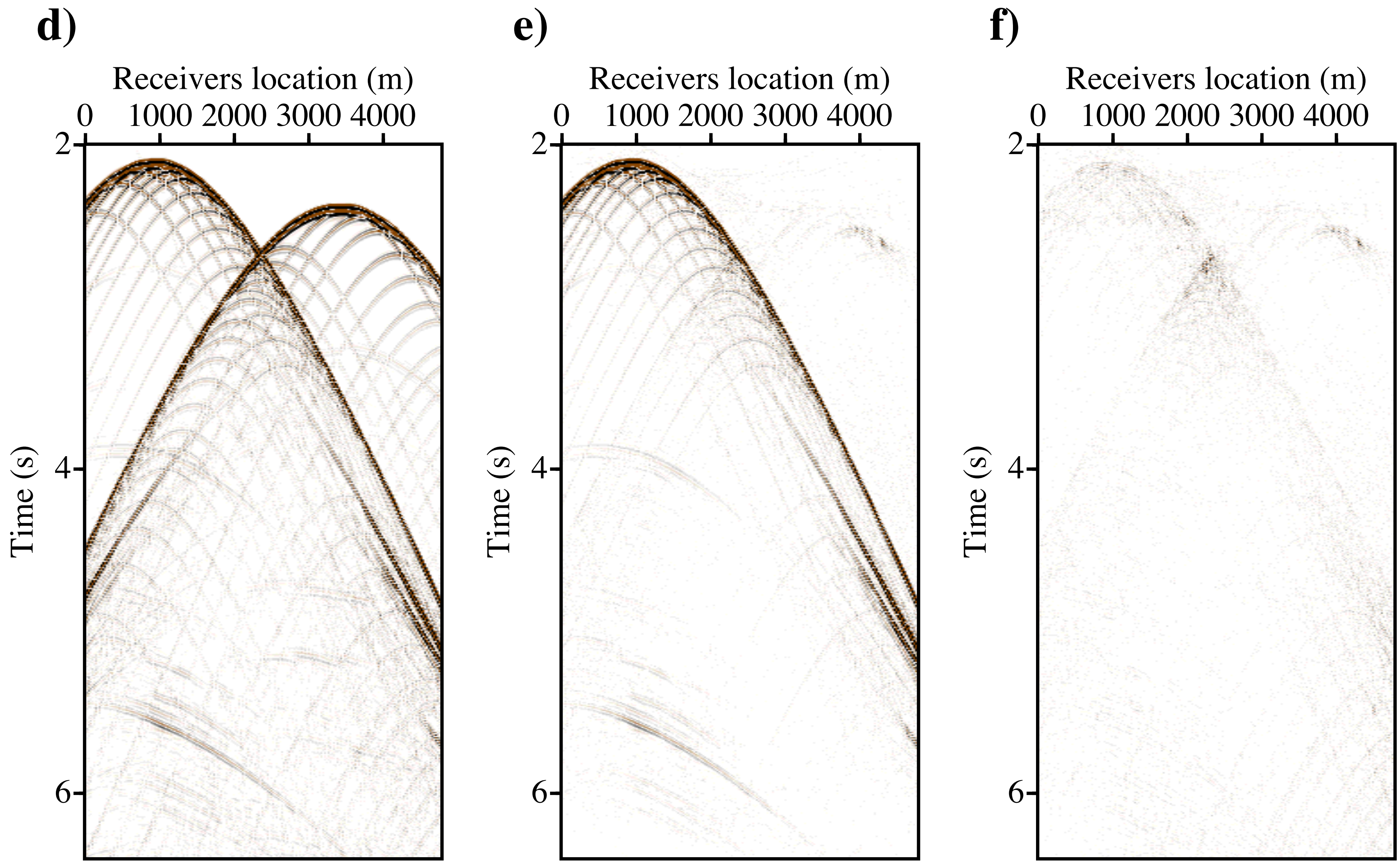
5) Examples: Marmousi – Common Shot Gather



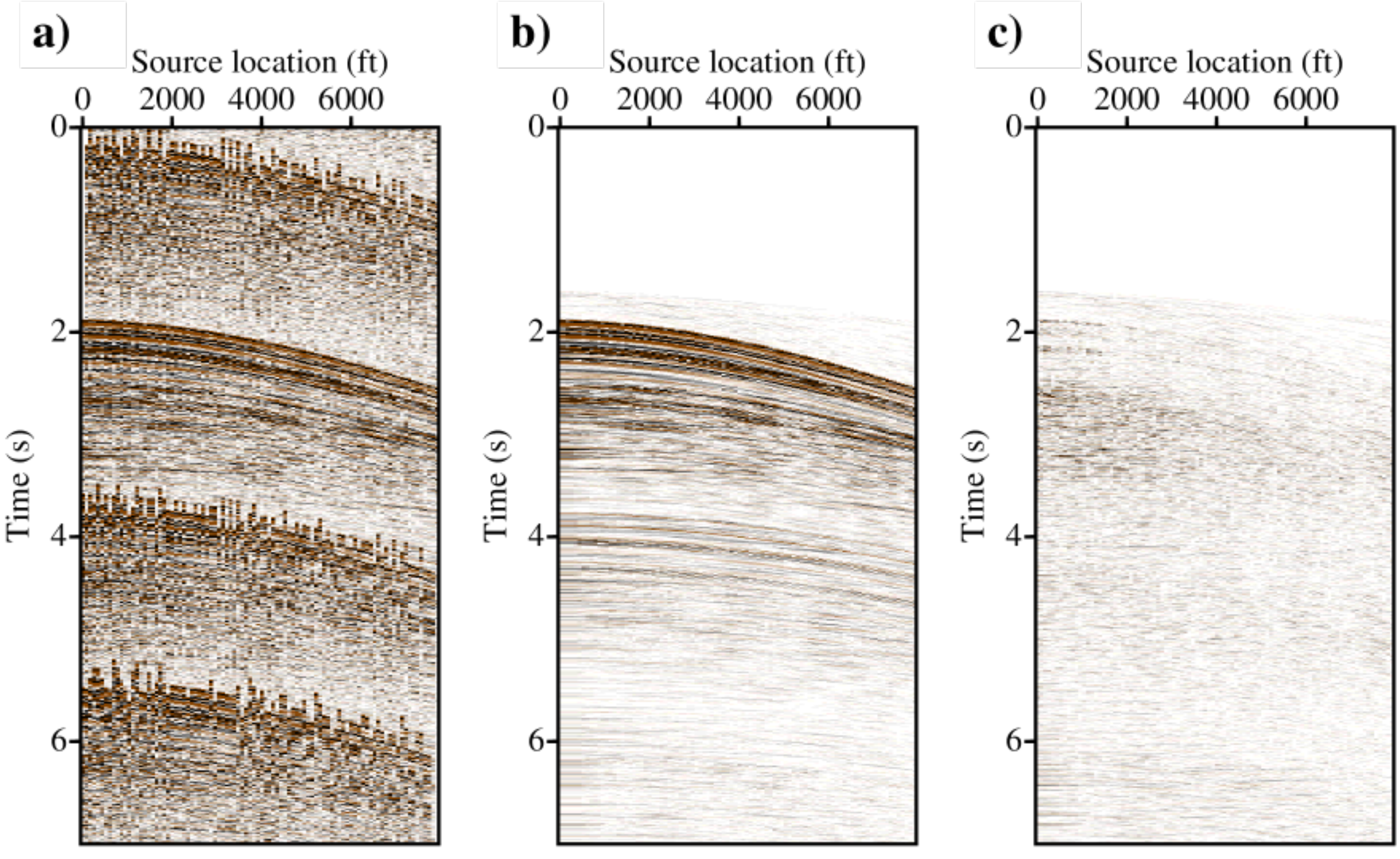
5) Examples: SEAM – Common Receiver Gather



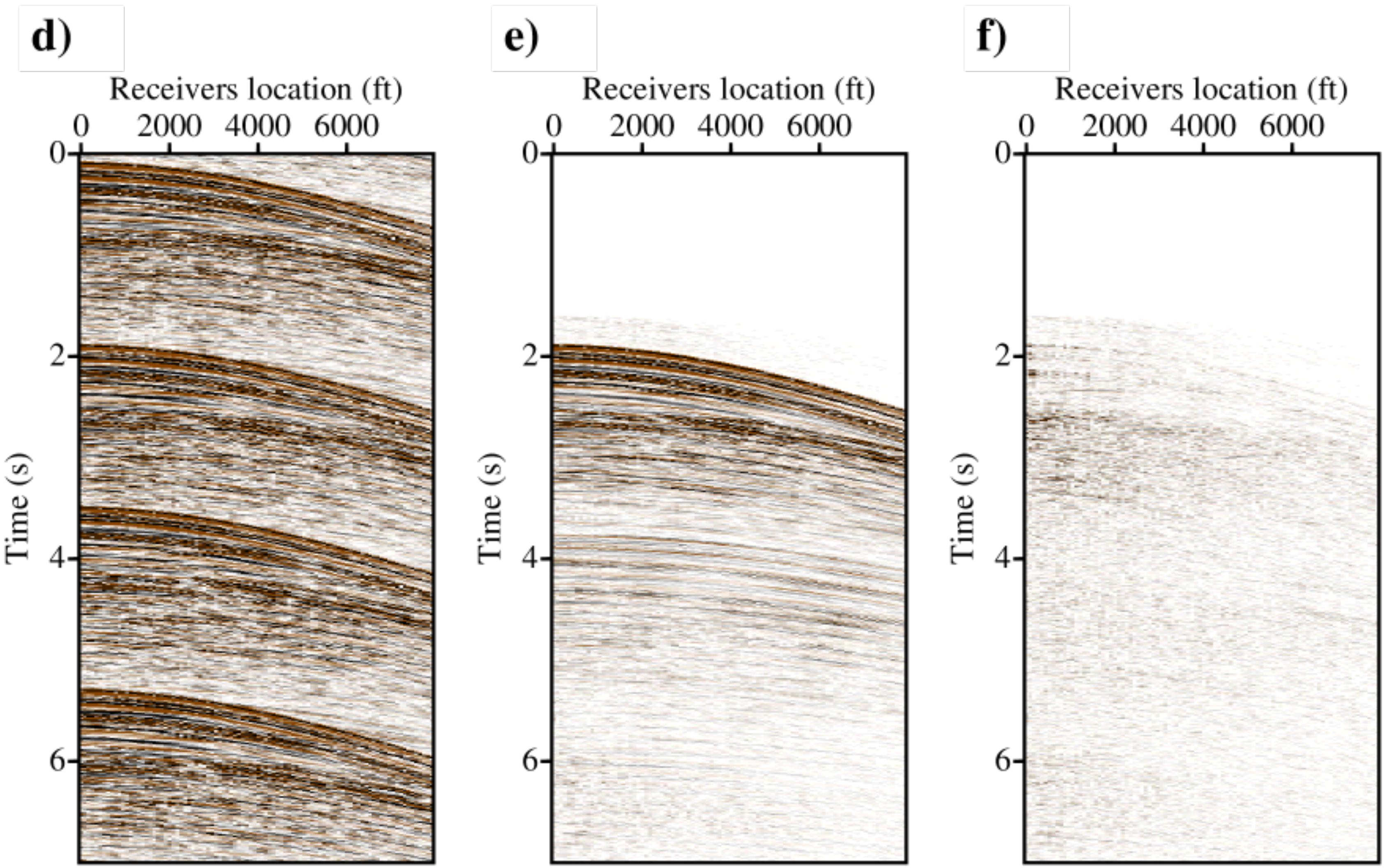
5) Examples: SEAM – Common Shot Gather



5) Examples: Gulf of Mexico – CRG



5) Examples: Gulf of Mexico - CSG



6) Conclusion

- We have implemented an asymptote and apex shifted hyperbolic Radon transform with a Stolt migration/demigration operator as its kernel to speed up computation.
- The new transform dictionary is designed to closely match both reflections and diffractions.
- Synthetic and field data results show that the inversion based deblending using Stolt operators can significantly attenuate source interferences.
- The main challenge is a trade-off between completeness of the dictionary (range and sampling density of the operator parameters) and the convergence rate of inversion.
- Future work: Hybrid transforms and Local transforms.