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UNIVERSITY OF CALGARY

Borehole wave field modeling, reflection extraction and reverse time migration in acoustic
reflection imaging logging

by

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A THESIS

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Abstract

This thesis systematically examines, develops and refines the basic procedures of acoustic reflection imaging logging. The reflections derived from the energy that leaks away from the borehole and is reflected back to the receivers after interacting with the structures outside the borehole, can be extracted from the full waveforms recorded by the downhole receivers. A three-dimensional finite difference method based on the first order velocity and stress hyperbolic equations is then developed to simulate wave propagation in both isotropic and anisotropic media. A hybrid-perfectly matched layer absorbing boundary condition is proposed to mitigate the artificial reflections. Finally, the borehole reverse time migration is developed to image the near borehole structures.

The reflection extraction from the borehole full waveforms is not straightforward. Under acoustic well logging conditions, reflected wave signals used in sonic reflection logging are generally submerged in the full waveform records, hidden by the dominant direct waves (direct P- and S-waves, and the Stoneley wave). It is critical, therefore, to effectively extract the reflection signals from the acoustic full waveforms in acoustic reflection well logging data processing. The Karhunen-Loève transformations combined with a band limiting filter is used to extract reflections of interest out of dominant direct waves. Under the assumption that large energy (squared-amplitude) differences exist between each wave component, the direct Stoneley wave, S-wave and the P-wave are eliminated sequentially by subtracting the most significant principle components, after which the remaining signal is seen to be dominated by reflected events. The extracted reflections can then be used in migration so as to get a clear image of the structures outside of the borehole.

During wavefield modeling, an issue faced by finite difference methods, which has particular importance in borehole applications, is the mitigation of artificial reflections from computational boundaries. This computational boundary artificial reflection problem has been a persistent topic in the literature of wave modelling, no complete solution has yet been found. To address this, a

hybrid perfectly matched layer methodology is introduced and discussed in the context of standard perfectly matched layer, convolutional perfectly matched layer, and multiaxial perfectly matched layer methods, and their abilities relative to the suppression of artificial reflections are compared. The method is a hybrid in the sense of combining aspects of the convolutional perfectly matched layer and the multiaxial perfectly matched layer schemes.

The fact that waves can impinge on the borehole instrument from all azimuths is an important source of ambiguity. This azimuthal ambiguity has been an issue ever since the beginning of borehole acoustic reflection imaging. The data (which actually may be from every possible direction of underneath formations) is considered in standard imaging and processing to have come from one direction. The 4-component dipole acoustic well logging technique is designed to solve the azimuth ambiguity problem by analyzing the azimuthal information contained in the recorded shear wave signals. Thereafter, standard migration procedure can be applied to get the imaging result. In this thesis, the 3D reverse time migration in the borehole environment is proposed and applied in the simulated data set with a similar source and receiver system as sonic scanner tool developed by Schlumberger. The staggered-grid finite difference RTM performs perfectly in fluid-solid boundary with a source located in the fluid-filled borehole. The imaging result shows the directional information of the structures outside the borehole can be directly obtained.

Preface

The PhD thesis is written in manuscript-based format based on two published papers and two manuscript papers that are ready to be submitted. I am the first author of these papers. All of these work is carried out under the supervision of Dr. Kris Innanen of CREWES project at University of Calgary and Dr. Tao of Petroleum Institute (UAE). Dr. Kris Innanen was involved in all of my research projects as the supervisory author. These papers are republished in this thesis with the permission from the co-authors.

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After finishing the last undergraduate course of Physics when I was a sophomore, I sent a message to all of my friends and relatives to celebrate that Physics would never appear in my life again. Guess what happened in the next semester? I have to choose one as my major from applied geophysics and petrophysics! I had no idea but to choose applied geophysics. And now, I am in the last year as a PhD candidate in geophysics, which becomes the one that I will dedicate the rest of my whole life to.

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Dedication

To my parents on the other side of the ocean and to my wife Cindy Wang.

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List of Symbols, Abbreviations and Nomenclature

Symbols

Symbol	Definition
ρ	Formation density
u_j	Displacement vector in j_{th} direction
$\sigma_{ij}, \epsilon_{kl}$	Stress and strain tensor, i,j,k,l=1,2,3(or, x,y,z)
$\sigma_{ij,j}$	derivative of stress tensor in j_{th} direction
c_{ijkl}	elastic stiffness tensor
λ, μ	Lamé constants
$C_{ISO}, C_{VTI}, C_{HTI}$	Elastic stiffness matrix in ISO, VTI and HTI media
V_x, V_y, V_z	Particle velocity component in x,y,z direction
$f(x)$	Univariate function
$\Delta x, \Delta y, \Delta z$ and Δt	increments in X, Y and Z directions and Δt is time step
δ_{ij}	Kronecker delta, which equals 1 as $i = j$; otherwise it equals zero
d_x, d_y, d_z	Damping profile in x,y and z direction
x, y, z	Cartesian spatial coordinates
$\tilde{x}$	Stretched x direction
N	N^{th} iteration step
$x(x)$	The complex stretching function in x direction
κ_x, α_x	Frequency dependent coefficients
ω	Angular frequency
$m_{x/z}$	Weighting factor of damping profile d_z in x direction
V_P, V_S and V_f	P-, S-wave and borehole fluid Velocity
$\mathbf{u}_{in}, \mathbf{u}_{out}$	Wavefield displacements in borehole and outside formation
φ_f	Displacement potential in the borehole fluid

φ	Compressional wave potential
Ψ	S-wave potential
∇	Gradient operator
χ, η	SH- and SV- wave displacement potentials
$S(\omega), s(t)$	Source spectrum and its time-domain form
ϕ	Angle between particle polarization direction and source orientation
$E(\omega, k)$ and $F(\omega, k)$	Undetermined functions which depend on boundary conditions
$K_1(sr)$	Modified Bessel function with radial wave number s in r direction
k_z	Wavenumber in z direction
R	Distance between field point and the source
$RD(\omega), RC(\omega)$	The borehole radiation and borehole receiver response
$RF(\omega)$	The acoustic reflectivity at the reflector
D, Q_β	The total propagation distance and shear wave attenuation
RD_{SH} and RD_{SV}	The far-field radiation for SH – and SV – waves
$R_{(SH)}, r_{(SH)}$	The isotropic SH reflection coefficient in frequency and time domain
$H(\omega)$	The Fourier transform of the time function $h(t)$
β_1, β_2	Shear wave velocity in the first and second layer
ρ_1, ρ_2	Formation density in the first and second layer
φ_1, φ_2	Shear wave incident and transmitted angle
c_{44}^I, c_{66}^I and c_{44}^{II}, c_{66}^{II}	Elastic constants of the media
C_X, C_Y	The $m \times m$ covariance matrix of X and Y
$\lambda_j, (j = 1, 2, \dots, m)$	The eigenvalue spectrum of the covariance matrix C_X
X	An $m \times N$ matrix
$e_j, (j = 1, 2, \dots, m)$	Eigenvectors of λ_j
A	An $m \times m$ orthogonal matrix composed of eigenvectors
Y, y_i	$m \times N$ matrix of X after KL transform and each component of Y

$\hat{A}, \hat{X}, \hat{Y}$	The approximation of A, X, Y , respectively
$\varepsilon(k)$	The mean square error between X and $\hat{X}$
P_1, S_1	The refracted P- and S-waves in the formation respectively
P_f	The compressional wave in the borehole fluid
R_{PP}, T_{PP}, T_{PS}	P reflection coefficient and the P- and S transmission coefficients
A_I, A_R and A_T	The amplitudes of the incident, reflected and refracted P-waves
B_T	The amplitude of the refracted S-wave
$\mathcal{R}, \mathcal{T}$	The energy reflection coefficient and energy transmission coefficient
∇_2	$\nabla_2 = \hat{\mathbf{x}} \frac{\partial}{\partial x} + \hat{\mathbf{y}} \frac{\partial}{\partial y}$
$\mathbf{u}_2$	The projection of $\mathbf{u}$ in the x - y plane
ε, δ , and γ	The Thomsen parameters
v_{P0}, v_{S0}	P- and SV-wave velocities along the VTI symmetry axis
f	$f = 1 - \left(\frac{v_{S0}}{v_{P0}} \right)^2$
$\mathbf{v}$	$\mathbf{v} = [v_x, v_y, v_z]^T$ is the particle velocity
$\mathbf{X}$	The displacement in accordance with the split-field technique
$\mathbf{A}_1, \mathbf{B}_1$	Derivative matrix for first order system of SH wave equation
$\mathcal{D}_x u(x_i)$	The first-order Fourier derivative of a function $u(x)$
DFT, DFT ⁻¹	The forward and inverse discrete Fourier transforms

Abbreviations

Abbreviation	Definition
BARS	Borehole Acoustic Reflection Survey
EVA	Evaluation of Velocity and Attenuation
2D	two dimension
4-C	four-component
3D	three dimension
f-k	frequency-wavenumber
RTM	reverse time migration
FFT	fast Fourier transform
PSM	fast Fourier transform
FD/FDM	finite difference/finite difference method
PSM/PSTD	pseudo-spectral method/pseudo-spectral time domain
FWI	full waveform inversion
PML	perfectly matched layer
C-PML	convolutional perfectly matched layer
M-PML	multiaxial perfectly matched layer
H-PML	hybrid perfectly matched layer
S- wave	secondary wave, shear or transverse wave
P- wave	primary wave, compressional wave or longitudinal wave
SH wave	shear-horizontal wave
SV wave	shear-vertical wave
K-L	Karhunen-Loève
CFABC	continued fraction absorbing boundary condition
VTI	vertical transverse isotropic
HTI	horizontal transverse isotropic

R1/R2	receiver 1/ receiver 2
BSS	blind signal separation
MSTC	multi-scale slowness-time-coherence
NUFFT	non-uniform fast Fourier transform
DFT	discrete Fourier transforms

Chapter 1

Introduction

1.1 Background

The target of oil and gas exploration has now been transferred from the conventional, large oil and gas reservoirs to unconventional small and subtle fractured reservoirs. However, due to the limited resolution, surface seismic methods have difficulty delineating these targets. New geophysical methods that can characterize these reservoirs with higher resolution are therefore of significant interest. The lateral depth of investigation by conventional acoustic well logging can only reach up to 0.6m outside the borehole wall. Recent studies have revealed the potential of acoustic reflection imaging logging in detecting near borehole geology structures, whose lateral detection depth can reach up to 10-20 m away from the borehole. Nevertheless, most research on the subject of acoustic reflection imaging logging are still based on the assumption of isotropy and are framed for two-dimensional media. The main objective of this thesis is to develop a theory of 3D anisotropic acoustic reflection imaging techniques, from 3D borehole wavefield forward modeling, wavefield extraction to migration and imaging.

1.1.1 Acoustic reflection imaging

Acoustic reflection imaging logging, originally described by Hornby (1989), who presented data processing and imaging methods for the sonic tool BARS (Borehole Acoustic Reflection Survey), interrogates reflections from near borehole fractures and microstructures caused by incident leaky waves. By analyzing received waveforms, the structure information of nearby subtle and fractured reservoirs can be obtained. In the ensuing years, monopole acoustic imaging has been reported to successfully delineate near-borehole structures(Fortin et al., 1991; Coates et al., 2000; Li et al., 2002). However, the omni-directional monopole acoustic prototype only measures acoustic pres-

sure and is insensitive to reflector azimuth. To resolve this directional ambiguity, dipole acoustic reflection imaging has been developed (Tang et al., 2003; Tang, 2004; Tang and Patterson, 2009; Bolshakov et al., 2011). In dipole methods, dispersive flexure waves, whose velocity at the cutoff frequency equals the shear wave velocity, are analyzed. These data, given the deviation angle of well bore and tool azimuthal angle, can determine the azimuth of the structures outside the bore hole after migration (Tang et al., 2003).

In research involving acoustic reflection imaging tools, the acoustic reflection logging prototype EVA (Evaluation of Velocity and Attenuation) (Fortin et al., 1991) was designed in 1982. The borehole Acoustic Reflection Survey (BARS) (Esmersoy et al., 1998) and Sonic Scanner (Pistre et al., 2005) were then developed by Schlumberger. In 2007, Bohai Drilling Corporation of China Petroleum developed an acoustic reflection imaging tool with a closest spacing of 10.6m and a certain degree of phase steering (Chai et al., 2009). The new design makes it easier to extract the reflection signals from the borehole mode waves.

1.1.2 Borehole wavefield simulation

For wavefield simulation in fluid-filled borehole environment, the finite difference (FD) method is widely used. The staggered-grid FD method was first proposed for seismology by Madariaga (1976), and further developed by Virieux (1986) who used the methodology to simulate P-SV wave modes, and by Levander (1988), who introduced a fourth-order staggered grid formulation. According to Virieux (1986), staggered-grid finite difference methods are suitable for numerical wave propagation in media with fluid-solid interfaces without any special treatment of the discontinuity. De Basabe and Sen (2015) illustrate detailed comparisons between standard grid FD method and staggered-grid FD method. In their paper, grid-dispersion analysis shows the standard-grid FD method can yield results almost as accurate as staggered-grid FD method, but at the cost of twice as many nodes in each direction. The numerical experiments show that the standard-grid FD method yields the largest errors in most cases considered, whereas, the 4th order staggered-grid FD method reproduces waveforms with good or excellent accuracy. Therefore, this approach has been

applied to borehole acoustic well logging in which the source is deployed in a fluid-filled borehole (Mallan et al., 2009). An issue faced by FD methods, which has particular importance in borehole applications, is the mitigation of artificial reflections from computational boundaries. Mitigation approaches include sponge zones (Cerjan et al., 1985; Sochacki et al., 1987), optimized conditions (Peng and Toksöz, 1995), eigenvalue decomposition (Dong et al., 2005), continued-fraction absorbing conditions (Guddati and Lim, 2006), absorbing conditions on spherical contours (Grote, 2000), and asymptotic local or nonlocal operators (Clayton and Engquist, 1980; Givoli, 1991; Hagstrom and Hariharan, 1998). No complete solution has yet been found, however, and certain artifacts, e.g., boundary reflections at grazing incidence, persist to some degree in every available technique. Furthermore, each of these methods introduces significant additional computational expense to the processing flow. For wavefield simulation in deviated wells, a 2.5-D FD code to model monopole and dipole acoustic logs in a deviated borehole penetrating an anisotropic formation is proposed by Leslie and Randall (1992), which greatly saves the cost of time and memory yet requires the media to be homogeneous in the direction of borehole extending. Sinha et al. (2006) used a 3-D cylindrical FD method to study the influence of a sonic tool on wave propagation in anisotropic formations. However, the detailed wavefield simulation is not included in his paper. In 3D wavefield simulation, in order to make it practical for the FD staggered-grid method to be applied in the high deviated acoustic reflection well logging, the influence of the stair-like grids around the borehole due to the model grid dividing should also be carefully tackled with. Especially when the grid size is comparatively large enough, the waveform simulation results will greatly suffer from these corresponding bogus waves derived from the stair-like grids.

1.1.3 Borehole reflection extraction

One issue of acoustic reflection imaging logging is the reflection wave extraction. Under acoustic well logging conditions, the borehole constitutes a wave guide which traps the energy transmitted by the acoustic source in the borehole and generates the so-called mode waves such as P and S head waves, pseudo Rayleigh wave, and Stoneley wave for a monopole tool. Only a small

portion of the emitted energy can leak from the borehole and propagate in the outside formation, which can be reflected back to receivers when formation structures such as faults and fractures are present. These reflected wave signals are difficult to detect in the full waveform records because of the much larger amplitudes of the dominant mode waves. It is critical, therefore, to effectively extract the reflection signals from the acoustic full waveforms in acoustic reflection well logging data processing. Hornby (1989) used f-k filter to extract reflection signals from waveforms. Tang (2004); Zheng and Tang (2005) used the parametric prediction method to extract reflection waves from waveforms. We have processed a number of data sets of acoustic reflection logs obtained from Chinese oil fields recently. Our own conclusion is that the above methods do not lead to satisfactory results.

1.1.4 Borehole reverse time migration

In the last decade, attempts have been made to utilize borehole acoustic measurements to obtain an image of geological structures away from the borehole (Hornby, 1989; Li et al., 2002). Recent studies have revealed the great potential of acoustic reflection logging in detecting near-borehole fractures and vugs (Tang, 2004; Tang and Patterson, 2009; Tao et al., 2008b).

One of the main issues concerning acoustic reflection logging is migration and imaging. Several migration techniques have been applied to the problem of acoustic reflection data processing. Among others, Hornby (1989) employed a back projection algorithm with a generalized Radon transform. Li et al. (2002) used a Kirchhoff depth migration and Zheng and Tang (2005) adapted the pre-stack frequency-wavenumber (f-k) migration for acoustic log configurations. Zhang and Zhang (2009) utilized the equivalent offset migration for the acoustic log configurations imaging. Reverse time migration (RTM), first proposed by Whitmore et al. (1983) in the 53rd SEG conference, has the ability to migrate any type of multiples (surface and internal) to their correct location in the subsurface, can handle multi-pathing, image turning waves and steep dips. Although the application of RTM in acoustic reflection imaging logging has been presented previously (Li et al., 2013), its validity in anisotropic media is yet to be discussed. Wave propagation is greatly influ-

enced by anisotropy, which negatively impacts the precision of borehole RTM in isotropic medium.

1.2 Outline

This thesis is organized as follows.

In Chapter 2, I briefly introduce finite difference method (FDM) in three dimensional space. Modeling of wavefield propagation in underground formation is based on the first order velocity and stress equations. The staggered-grid finite difference scheme is used to get the discretized version of these first order velocity and stress equations in both isotropic and anisotropic media. The perfectly matched layer (PML) scheme is used to eliminate boundary reflections. According to comparisons among conventional PML, Convolutional PML (C-PML) and multiaxial PML (M-PML), I introduce a hybrid PML (H-PML) that combines the C-PML and M-PML. The new PML scheme performs perfectly in both isotropic and anisotropic media. The original research contribution contained in this chapter is the introduction and validation of the new PML scheme.

In Chapter 3, I talk about numerical simulations of radiation, reflection and multipole reception of elastic waves, excited by a dipole source. In order to optimize azimuthal detection, the relationships between S- wave polarization and both source-receiver offset and source-reflector angle are analyzed. Results indicate that the S-S reflection is most sensitive to the angle between the incident ray and the normal to the reflector. Its maximum amplitude occurs as the incident angle reaches critical, a fact that can be used to calculate the total propagation distance of the S-S wave. The critical angle as well as the SH wave velocities of geological structures outside the borehole can thus be determined. The original research contribution contained in this chapter is the determination of S-wave velocity outside of the borehole.

Although, the most widely-applied numerical approach for modeling the propagation of seismic waves is the FDM, different wave modes (P-, SV- and SH- wave) are to be simulated simultaneously, which causes the crosstalk from the interference of different modes. This crosstalk reduces the precision of imaging condition during migration and impedes the determination of for-

mation parameter gradients during time domain full waveform simulation (FWI). The independent simulation of decoupled wave modes is expected to reduce this kind of crosstalk. Because under idealized circumstances the SH-mode of the full elastic wavefield propagates independently of the P-SV modes, it can be sensed approximately independently in multicomponent experiments. The three-dimensional (3D) simulation of this component of the full elastic response is simple, but also meaningful from the point of view of data simulation. In chapter 4, a 3D temporal fourth-order pseudo-spectral method (PSM) for solving the elastic SH wave equations in VTI media has been proposed, which appears to suppress the wrap-around and Gibbs' artifacts that have been observed in other methodologies when waves propagate through heterogeneous formations—especially in the presence of large and abrupt changes in the medium properties.

Under acoustic well logging conditions, reflected wave signals used in sonic reflection logging are generally submerged in the full waveform records, hidden by the dominant direct waves (direct P- and S- waves, and the Stoneley wave). Chapter 5 discusses methods on effectively extracting reflection signals from acoustic full waveforms in acoustic reflection well logging data processing. The Karhunen-Loève (K-L) transformations combined with a band limiting filter is used to extract reflections of interest from dominant direct waves. Under the assumption that large energy (squared-amplitude) differences exist between each wave component, the direct Stoneley wave, S-wave and the P-wave are eliminated sequentially by subtracting the most significant principle components, after which the remaining signal is seen to be dominated by reflected events. Thereafter, the extracted reflections can be used in migration to provide interpretable images of the structures outside the borehole. Synthetic data are used to develop and justify our procedure for subtraction of appropriate KL principle components; laboratory data are used to demonstrate a decrease in unwanted residuals in comparison to a common multiscale approach for separation. The procedure is exemplified on a field data case with attention paid in particular to the consequences to imaging of near-borehole structures.

The azimuth ambiguity has been an issue ever since the beginning of borehole acoustic reflec-

tion imaging. This imaging authenticity indistinguishability, occurring not only in the borehole reflection imaging but also in seismic imaging, is due to the intrinsic defect of the 2D data processing which treats recorded 3D data as a 2D data set. A 3D migration scheme should be applied to eliminate the azimuth ambiguity. Reverse time migration (RTM) was first introduced in the late 1970s (Hemon, 1978) and shows promising imaging capabilities (Baysal et al., 1983; Whitmore et al., 1983; McMechan, 1983; Loewenthal and Mufti, 1983). In the application of borehole acoustic reflection imaging, the 2-D borehole RTM in isotropic medium is first introduced in 2014 (Li et al., 2014b). In chapter 6, a 3-D anisotropic RTM in borehole environment is developed for migration of the reflected signals extracted from the simulated waveforms. To make a comparison, the 2-D synthetic data from two horizontal wells is also simulated and migrated by a 2-D borehole RTM scheme.

In Chapter 7, I summarize the results obtained by the schemes and workflow proposed in this thesis. Plans for further studies are also given at the end.

Chapter 2

Finite difference method

2.1 Introduction

The application of the finite difference method (FDM) to geophysical wavefield simulation can be dated back to roughly half a century, when early applications were based on the displacement formulation of wave equations (Alterman and Karal, 1968; Boore, 1972; Alford et al., 1974). The staggered-grid FDM, which uses the first order velocity-stress formulation to simulate wavefield propagation, was originally proposed by Yee et al. (1966) for Maxwell's equations. It was then used in seismic forward modeling by Madariaga (1976) and further developed by Virieux (1986) who used the methodology to simulate P-SV wave modes, and by Levander (1988), who introduced a fourth-order staggered grid formulation. According to Virieux (1986), the staggered-grid FDM is suitable for numerical wave propagation in media with fluid-solid interfaces without any special treatment of the discontinuity. Cheng et al. (1995) also points out the advantages of staggered-grid scheme over the conventional schemes: 1) stable for any Poisson ratio and 2) small grid dispersion. Dougherty and Stephen (1991) used this two dimensional staggered-grid FDM in research on scattering of seismoacoustic energy from rough water-solid interfaces. Graves (1996) applied the staggered-grid FDM to solve wave propagation problems in 3D elastic media.

Van Vossen et al. (2002) addressed the importance of using harmonic averaging of the shear modulus at fluid-solid surfaces. De Basabe and Sen (2015) make detailed comparisons between standard grid FD method and staggered-grid FD method. In their paper, grid-dispersion analysis shows that standard-grid FD methods can yield results almost as accurate as staggered-grid FD method, but they require twice as many nodes in each direction. Numerical experiments show that the standard-grid FD method yields the largest errors in most of the considered experiments, whereas, the 4th order staggered-grid FD method reproduces waveforms with good or excellent

accuracy. Therefore, it has been extended to borehole acoustic well logging applications where the source is positioned in a fluid-filled borehole (Mallan et al., 2009).

An issue faced by FDMs, which has particular importance in borehole applications, is the mitigation of artificial reflections from computational boundaries. In order to eliminate these artificial boundary reflections, Lindman (1975) proposed an absorbing boundary condition for the acoustic wave equation, which is capable of generating reflection coefficients less than 1 percent for a wide range of frequencies and incident angles. Randall (1988, 1989) extended Lindman's idea to the elastic wave case and the staggered grid scheme. Yet, it doesn't specify how to treat the grid corners. A very popular absorbing boundary condition was discussed by Engquist and Majda (1977), which is based on paraxial approximation for the wave equations. Reynolds (1978) used a technique similar to that of Clayton and Engquist, although less rigorous, and obtained a local boundary condition in Cartesian coordinates. His scheme works well at boundaries with lateral inhomogeneities. Cerjan et al. (1985) proposed using damping layers or sponge zones, which introduces a gradual reduction of displacement amplitudes in a strip of nodes along the boundaries, to simulate a non-reflecting boundary condition. Israeli and Orszag (1981) combined both of the paraxial conditions and damping layers. Peng and Toksöz (1995) designed a class of optimal absorbing boundary conditions for a given operator length, which yields reflection coefficients with a smaller magnitude than Higdon absorbing condition (Higdon, 1986) and Reynolds absorbing condition (Reynolds, 1978). Guddati and Lim (2006) implemented continued fraction absorbing boundary conditions (CFABCs) in the standard finite element setting. But these authors report the need for additional care in choosing the parameters (e.g., time step) are required if this method is extended to time-domain calculation of elastic wave propagation. Most of these early implementations of the finite difference method use the second order wave equations, which results in the difficulty in dealing with the artificial boundary reflections. So, a need remains to develop and refine simulation tools for the borehole dipole imaging problem that balance numerical accuracy, artifact minimization, and computational efficiency.

2.2 Formulation

Wave propagation in an elastic medium is governed by the equation:

$$\rho \partial_t^2 u_j = \sigma_{ij,j}, \quad (2.1)$$

where ρ is the density, u_i is the displacement vector and σ_{ij} is stress tensor, and where $\sigma_{ij,j}$ represent spatial derivatives of the stress tensor. The comma between subscripts is used for spatial derivatives. The Einstein summation convention for repeated subscripts is assumed. According to Hooke's law, the relationship between the stress and strain tensors is,

$$\sigma_{ij} = c_{ijkl} \epsilon_{kl}, \quad (2.2)$$

where c_{ijkl} are the elastic stiffness coefficients. The strain tensor ϵ_{kl} is

$$\epsilon_{kl} = \frac{1}{2}(u_{k,l} + u_{l,k}), \quad (2.3)$$

Equation (2.2) can be expressed in matrix form as

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix} \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ 2\epsilon_{23} \\ 2\epsilon_{13} \\ 2\epsilon_{12} \end{bmatrix}. \quad (2.4)$$

If the medium is isotropic, the elastic constant tensor c_{ijkl} becomes a fourth-order isotropic tensor given by

$$c_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}), \quad (2.5)$$

or, in the two-index notation,

$$c_{ISO} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}, \quad (2.6)$$

where δ_{ij} is the Kronecker delta, which equals 1 as $i = j$; otherwise it equals zero. λ and μ are the Lamé constants. The P wave velocity V_P in isotropic medium is given by $\sqrt{\frac{\lambda+2\mu}{\rho}}$ and the S wave velocity V_S is given by $\sqrt{\frac{\mu}{\rho}}$.

The elastic constant tensor simplifies in a vertical transverse isotropic (VTI) medium to

$$c_{VTI} = \begin{bmatrix} c_{11} & c_{11} - 2c_{66} & c_{13} & 0 & 0 & 0 \\ c_{11} - 2c_{66} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}, \quad (2.7)$$

and in a horizontal transverse isotropic (HTI) to

$$c_{HTI} = \begin{bmatrix} c_{11} & c_{13} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{11} & c_{33} - 2c_{44} & 0 & 0 & 0 \\ c_{13} & c_{33} - 2c_{44} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}. \quad (2.8)$$

Taking the VTI case in equation (2.7) as an example, equation (2.1) leads to nine coupled first

order velocity and stress equations:

$$\begin{aligned}
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} &= \rho \frac{\partial V_x}{\partial t} \\
\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} &= \rho \frac{\partial V_y}{\partial t} , \\
\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} &= \rho \frac{\partial V_z}{\partial t}
\end{aligned} \tag{2.9}$$

and

$$\begin{aligned}
\frac{\partial \sigma_{xx}}{\partial t} &= c_{11} \frac{\partial V_x}{\partial x} + (c_{11} - 2c_{66}) \frac{\partial V_y}{\partial y} + c_{13} \frac{\partial V_z}{\partial z} \\
\frac{\partial \sigma_{yy}}{\partial t} &= (c_{11} - 2c_{66}) \frac{\partial V_x}{\partial x} + c_{11} \frac{\partial V_y}{\partial y} + c_{13} \frac{\partial V_z}{\partial z} \\
\frac{\partial \sigma_{zz}}{\partial t} &= c_{13} \frac{\partial V_x}{\partial x} + c_{13} \frac{\partial V_y}{\partial y} + c_{33} \frac{\partial V_z}{\partial z} \\
\frac{\partial \sigma_{yz}}{\partial t} &= c_{44} \left(\frac{\partial V_y}{\partial z} + \frac{\partial V_z}{\partial y} \right) \\
\frac{\partial \sigma_{xz}}{\partial t} &= c_{44} \left(\frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x} \right) \\
\frac{\partial \sigma_{xy}}{\partial t} &= c_{66} \left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x} \right),
\end{aligned} \tag{2.10}$$

where $[V_x, V_y, V_z]^T$ is the particle velocity vector.

2.3 Staggered-grid finite difference approximation

The finite difference modeling of wave equations is extensively discussed by Moczo et al. (2014). In this section we focus on the staggered-grid finite difference method for wavefield simulation. For a univariate function $f(x)$, the $2N^{th}$ staggered-grid FD formulation makes use of differencing rule

$$\delta_{2N} f = \frac{1}{\Delta x} \sum_{m=0}^{N-1} a_m \left[f \left(x + \frac{2m+1}{2} \Delta x \right) - f \left(x - \frac{2m-1}{2} \Delta x \right) \right], \tag{2.11}$$

where a_m is the coefficient of the $2N^{th}$ -order FD approximation to the first-order derivative. The quantity $\delta_{2N}f$ is applied on a 3-D grid in Cartesian coordinates $(l_x\Delta x, l_y\Delta y, l_z\Delta z)$ at time $n\Delta t$, where $\Delta x, \Delta y, \Delta z$ are the increments in X, Y and Z directions and Δt is time step. Figure 2.1 schematically illustrates the staggered-grid discretization for different parameters (e.g., velocity and stress components). For example, the shear stress component σ_{xy}^n is expressed as $\sigma_{xy}^n(l_x + \frac{1}{2}, l_y + \frac{1}{2}, l_z)$; the velocity component V_x is expressed as $V_x^{n+\frac{1}{2}}(l_x + \frac{1}{2}, l_y, l_z)$, with n as time index, etc. According to Moczo et al. (2014), equation (2.9) can be discretized as,

$$\begin{aligned}
V_x^{n+\frac{1}{2}}(l_x + \frac{1}{2}, l_y, l_z) &= V_x^{n-\frac{1}{2}}(l_x + \frac{1}{2}, l_y, l_z) + \frac{\Delta t}{\rho(l_x + \frac{1}{2}, l_y, l_z)} (\delta_x \sigma_{xx}^n(l_x + \frac{1}{2}, l_y, l_z) \\
&\quad + \delta_y \sigma_{xy}^n(l_x + \frac{1}{2}, l_y + \frac{1}{2}, l_z) + \delta_z \sigma_{xz}^n(l_x + \frac{1}{2}, l_y, l_z + \frac{1}{2})) \\
V_y^{n+\frac{1}{2}}(l_x, l_y + \frac{1}{2}, l_z) &= V_y^{n-\frac{1}{2}}(l_x, l_y + \frac{1}{2}, l_z) + \frac{\Delta t}{\rho(l_x, l_y + \frac{1}{2}, l_z)} (\delta_x \sigma_{xy}^n(l_x + \frac{1}{2}, l_y + \frac{1}{2}, l_z) \\
&\quad + \delta_y \sigma_{yy}^n(l_x, l_y + \frac{1}{2}, l_z) + \delta_z \sigma_{yz}^n(l_x, l_y + \frac{1}{2}, l_z + \frac{1}{2})) \\
V_z^{n+\frac{1}{2}}(l_x, l_y, l_z + \frac{1}{2}) &= V_z^{n-\frac{1}{2}}(l_x, l_y, l_z + \frac{1}{2}) + \frac{\Delta t}{\rho(l_x, l_y, l_z + \frac{1}{2})} (\delta_x \sigma_{xz}^n(l_x + \frac{1}{2}, l_y, l_z + \frac{1}{2}) \\
&\quad + \delta_y \sigma_{yz}^n(l_x, l_y + \frac{1}{2}, l_z + \frac{1}{2}) + \delta_z \sigma_{zz}^n(l_x, l_y, l_z + \frac{1}{2})),
\end{aligned} \tag{2.12}$$

where $\delta_x \sigma_{xx}^n$, for example, is given by

$$\delta_x \sigma_{xx}^n(l_x + \frac{1}{2}, l_y, l_z) = \frac{1}{\Delta x} \sum_{m=0}^{N-1} a_m [\sigma_{xx}^n(l_x + m + 1, l_y, l_z) - \sigma_{xx}^n(l_x - m, l_y, l_z)]. \tag{2.13}$$

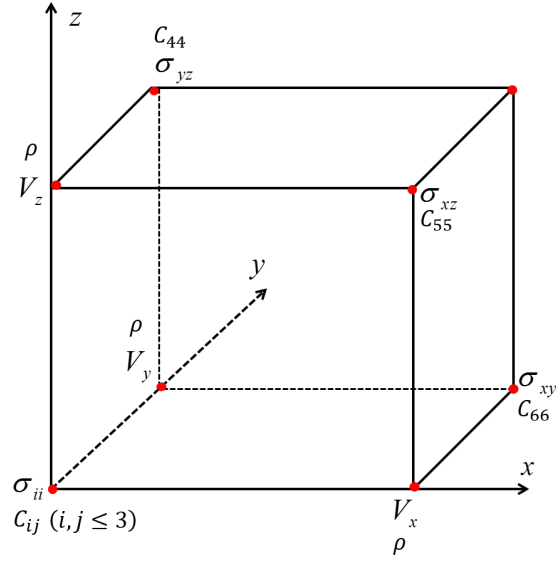


Figure 2.1: Schematic illustration of the staggered-grid discretization for different components. The normal stress components are located in the middle of the grid; velocity components are set in 1/2 of the grid points towards corresponding directions; and shear stress components are located in their corresponding 1/2 grid points.

Likewise, equation (2.10) is discretized as

$$\begin{aligned}
 \sigma_{xx}^{n+1} &= \sigma_{xx}^n + \Delta t [c_{11} \delta_x V_x^{n+1/2} + (c_{11} - 2c_{66}) \delta_y V_y^{n+1/2} + c_{13} \delta_z V_z^{n+1/2}] \\
 \sigma_{yy}^{n+1} &= \sigma_{yy}^n + \Delta t [(c_{11} - 2c_{66}) \delta_x V_x^{n+1/2} + c_{11} \delta_y V_y^{n+1/2} + c_{13} \delta_z V_z^{n+1/2}] \\
 \sigma_{zz}^{n+1} &= \sigma_{zz}^n + \Delta t [c_{13} \delta_x V_x^{n+1/2} + c_{13} \delta_y V_y^{n+1/2} + c_{33} \delta_z V_z^{n+1/2}] \\
 \sigma_{yz}^{n+1} &= \sigma_{yz}^n + \Delta t c_{44} [\delta_y V_z^{n+1/2} + \delta_z V_y^{n+1/2}] \\
 \sigma_{xz}^{n+1} &= \sigma_{xz}^n + \Delta t c_{44} [\delta_x V_z^{n+1/2} + \delta_z V_x^{n+1/2}] \\
 \sigma_{xy}^{n+1} &= \sigma_{xy}^n + \Delta t c_{66} [\delta_x V_y^{n+1/2} + \delta_y V_x^{n+1/2}].
 \end{aligned} \tag{2.14}$$

During the velocity components update, the densities are acquired from the average of the two assigned densities nearby, which can be written as

$$\begin{aligned}\rho(l_x, l_y + \frac{1}{2}, l_z) &= \frac{\rho(l_x + \frac{1}{2}, l_y + \frac{1}{2}, l_z) + \rho(l_x - \frac{1}{2}, l_y + \frac{1}{2}, l_z)}{2} \\ \rho(l_x + \frac{1}{2}, l_y, l_z) &= \frac{\rho(l_x + \frac{1}{2}, l_y + \frac{1}{2}, l_z) + \rho(l_x + \frac{1}{2}, l_y - \frac{1}{2}, l_z)}{2} \\ \rho(l_x + \frac{1}{2}, l_y + \frac{1}{2}, l_z + \frac{1}{2}) &= \frac{\rho(l_x + \frac{1}{2}, l_y + \frac{1}{2}, l_z + 1) + \rho(l_x + \frac{1}{2}, l_y + \frac{1}{2}, l_z)}{2}.\end{aligned}\tag{2.15}$$

During the stress components update, the elastic moduli related to shear stress tensors (such as c_{44} and c_{66} in VTI medium) are determined by the harmonic average. Taking c_{44} as an example, its harmonic average form can be written as

$$\begin{aligned}\frac{4}{c_{44}(l_x, l_y, l_z)} &= \frac{1}{c_{44}(l_x + \frac{1}{2}, l_y + \frac{1}{2}, l_z)} + \frac{1}{c_{44}(l_x - \frac{1}{2}, l_y + \frac{1}{2}, l_z)} + \frac{1}{c_{44}(l_x + \frac{1}{2}, l_y - \frac{1}{2}, l_z)} + \frac{1}{c_{44}(l_x - \frac{1}{2}, l_y - \frac{1}{2}, l_z)} \\ \frac{4}{c_{44}(l_x, l_y + \frac{1}{2}, l_z + \frac{1}{2})} &= \frac{1}{c_{44}(l_x + \frac{1}{2}, l_y + \frac{1}{2}, l_z)} + \frac{1}{c_{44}(l_x + \frac{1}{2}, l_y + \frac{1}{2}, l_z + 1)} + \frac{1}{c_{44}(l_x - \frac{1}{2}, l_y + \frac{1}{2}, l_z)} + \frac{1}{c_{44}(l_x - \frac{1}{2}, l_y + \frac{1}{2}, l_z + 1)} \\ \frac{4}{c_{44}(l_x + \frac{1}{2}, l_y, l_z + \frac{1}{2})} &= \frac{1}{c_{44}(l_x + \frac{1}{2}, l_y + \frac{1}{2}, l_z)} + \frac{1}{c_{44}(l_x + \frac{1}{2}, l_y + \frac{1}{2}, l_z + 1)} + \frac{1}{c_{44}(l_x + \frac{1}{2}, l_y - \frac{1}{2}, l_z)} + \frac{1}{c_{44}(l_x + \frac{1}{2}, l_y - \frac{1}{2}, l_z + 1)}.\end{aligned}\tag{2.16}$$

2.4 Absorbing boundary conditions

The absorbing boundary condition is used to eliminate reflections from computational boundaries. The perfectly matched layer (PML) is one of the most widely-used approaches in staggered-grid FDM. In this section, the basic principals of PML scheme as well as some improved PML approaches will be discussed, based on which, a new PML scheme is proposed to improve the effectiveness of PML in anisotropic media.

2.4.1 Perfectly matched layer (PML)

Wavefield simulation boundaries tend to produce artificial reflections which must be suppressed or absorbed. The PML approach to boundary absorption, introduced by Berenger (1994), has proven to be very efficient compared with previously developed methods (Collino and Tsogka, 2001; Komatitsch and Tromp, 2003; Festa and Vilotte, 2005). As illustrated in Figure 2.2, the PML concept involves splitting velocity and stress fields in terms of components perpendicular and parallel to the interface (Chew and Weedon, 1994; Collino and Monk, 1998; Collino and Tsogka, 2001). Taking the x direction as an example, a damping profile d_x is created, with $d_x = 0$ in the physical domain and $d_x > 0$ in the defined PML layer. The new operator $\nabla_{\tilde{x}} = [\frac{\partial}{\partial \tilde{x}}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}]$ is thus introduced, where $\frac{\partial}{\partial \tilde{x}} = \frac{1}{s_x} \frac{\partial}{\partial x}$, $s_x = 1 + \frac{d_x}{i\omega}$.

The split-field PML has two main imperfections: 1) the velocity and stress fields are required to be split into two subfields respectively; and 2) its efficiency decreases at grazing incidence after discretization, because the damping coefficient is inversely proportional to the angular frequency and thus depends on the direction of propagation of the wave.

2.4.2 Convolutional perfectly matched layer (C-PML)

In order to improve the response of the discrete PML at grazing incidence, the convolutional PML (or C-PML) method (Kuzuoglu and Mittra, 1996) or the complex frequency shifted-PML (CFS-PML) method (Bérenger, 2002) can be invoked. The CFS-PML method introduces a frequency-dependent term which eliminates the requirement that the velocity-stress equation be split into separate terms. The C-PML scheme involves adding not only the damping profile, but two other real variables, such that:

$$s_x = \kappa_x + \frac{d_x}{\alpha_x + i\omega}. \quad (2.17)$$

When $\kappa_x = 1$ and $\alpha_x = 0$, the C-PML degenerates to the classic PML case. Snapshots of the wavefield in a 2D medium are illustrated in the following discussion to make detailed comparisons among different PML layers. The top row of Figure 2.3 contains snapshots of a monopole-source

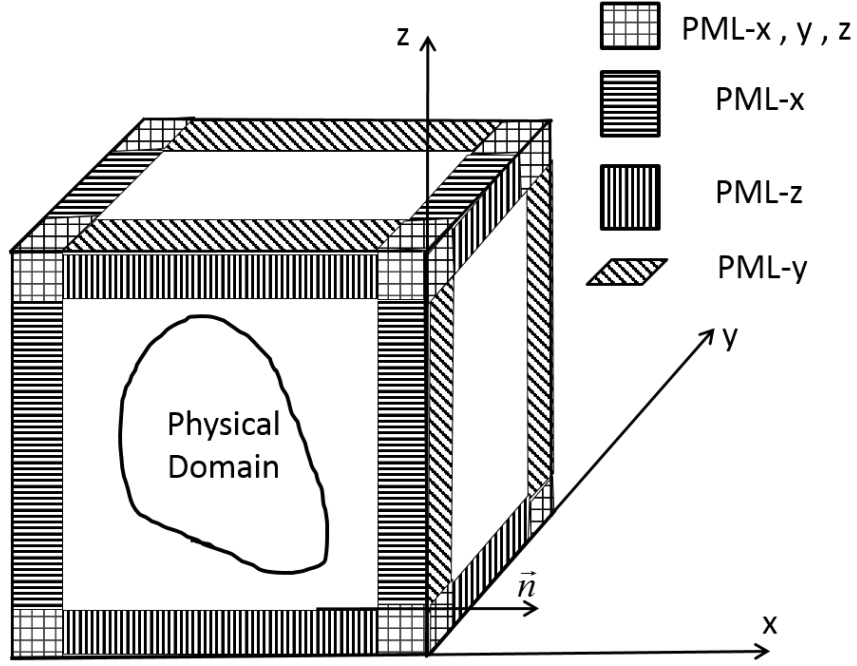


Figure 2.2: Perfectly-matched layers (PML) outside the physical domain.

wavefield propagating in an isotropic medium as calculated with C-PML absorbing layers. The artificial boundary reflections are effectively suppressed. The C-PML is, however less stable than the split PML scheme because of its frequency-dependent term and/or the convolution operations. Abarbanel et al. (2002) report that C-PML solutions for Maxwell's equations experience slowly growing spurious solutions, which eventually spread throughout the physical domain. Furthermore, Komatitsch and Martin (2007) observe that the C-PML approach exhibits strong instabilities when applied to waves in elastic anisotropic media. The bottom row of Figure 2.3 contains snapshots of a monopole source wavefield propagating in a VTI medium with C-PML absorbing layers. At times greater than 1.2 ms, artificial reflections emerge at the edges of the snapshots. The stiffness matrices in isotropic (C_{iso}) and anisotropic (C_{ani}) media used in this section are,

$$C_{iso} = \begin{bmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 13.55 \end{bmatrix}, \quad \text{and}, \quad C_{ani} = \begin{bmatrix} 4 & 7.5 & 0 \\ 7.5 & 20 & 0 \\ 0 & 0 & 2 \end{bmatrix}. \quad (2.18)$$

where, the parameters of stiffness matrix in anisotropic case are followed by Kristel C. Meza-Fajardo's paper (Meza-Fajardo and Papageorgiou, 2008).

2.4.3 Multiaxial perfectly matched layer (M-PML)

The multiaxial perfectly matched layer (M-PML) method has been found to be stable even for media exhibiting very large degrees of anisotropy (Meza-Fajardo and Papageorgiou, 2008). In an M-PML application, in contrast to equation (2.17), the s_x term is

$$s_x = \kappa_x + \frac{d_x + m_{x/z}d_z}{i\omega}, \quad (2.19)$$

where $m_{x/z}$ is a weighting factor. The top row of Figure 2.4 contains snapshots of a monopole source wavefield propagating in an isotropic medium with M-PML absorbing layers. While most of the boundary artificial reflections are effectively suppressed, small artificial reflections are noticeable at some incidence angles (compare with the top row of Figure 2.3). However, the accuracy of the M-PML method for waves propagating in anisotropic media is relatively high. The bottom row of Figure 2.4 contains snapshots of the monopole source wavefield propagating in the VTI medium with M-PML absorbing layers. In comparison with the C-PML result in the bottom row of Figure 2.3, a significant performance up-tick is observed. Some minor artificial reflections still exist at some incidence angles, but these can be reduced (though at higher computational cost) by increasing the size of the PML layers.

2.4.4 Hybrid perfectly matched layer (H-PML)

To maximize both accuracy and stability I construct a hybrid PML (H-PML) method, that combines the advantages of both the C-PML and the M-PML through the optimization of the damping profile. Because the C-PML and M-PML are independent of one another, the two can be straightforwardly hybridized by introducing

$$s_x = \kappa_x + \frac{d_x + m_{x/z}d_z}{\alpha_x + i\omega}. \quad (2.20)$$

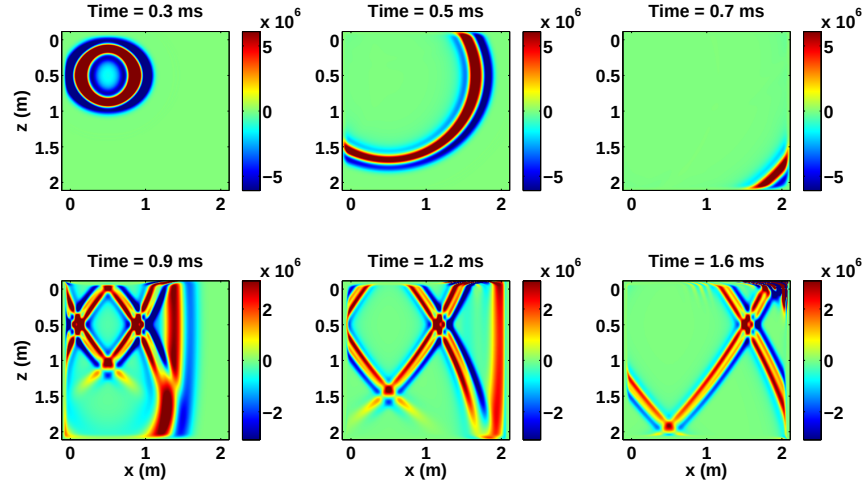


Figure 2.3: Snapshots of the monopole source wavefield propagation in both isotropic media (top row) and anisotropic media (bottom row) with C-PML absorbing layers. For the isotropic medium, the elastic constants are in matrix C_{iso} , shown in equation (2.18). The recording time is 0.7 ms. For the anisotropic medium, the elastic constants are in matrix C_{ani} , shown in equation (2.18). The recording time is 1.6 ms.

In order to invoke the above equation in a time domain implementation, convolution or auxiliary variables are required.

Figure 2.5 contains snapshots of a monopole source wavefield propagating in both isotropic and anisotropic media with H-PML absorbing layers. The artificial boundary reflections are effectively suppressed for both isotropic and anisotropic media.

We wish to further study the difference among these three PML implementations on seismograms in both isotropic and anisotropic media. Figure 2.6 shows the profiles of a single-layer model with different stiffness parameters (The above three seismograms are obtained in an isotropic medium, with V_P, V_S velocities and density of 2000 m/s, 1200 m/s and 1500 kg/m³ respectively; the lower three seismograms are obtained in anisotropic medium, with $C_{11}, C_{12}, C_{22}, C_{16}, C_{26}, C_{66}$ and a density of 17.47e9 N/m², 11.93e9 N/m², 17.47e9 N/m², -2.14e9 N/m², -2.14e9 N/m², 4.91e9 N/m² and 2250 kg/m³ respectively.). The spatial interval is 5 m, and the grid number is 1025 × 601. The source is a Ricker wavelet with a dominant frequency of 20 Hz. When the

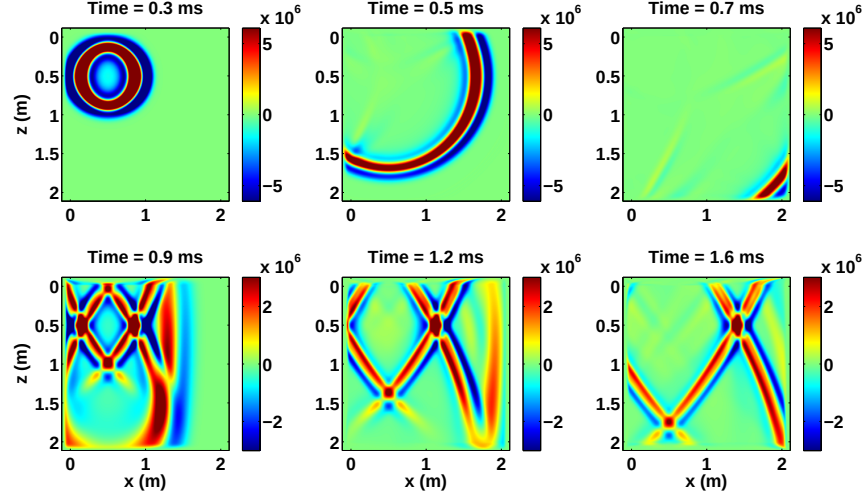


Figure 2.4: Snapshots of the monopole source wavefield propagation in both isotropic media (top row) and anisotropic media (bottom row) with M-PML absorbing layers. For the isotropic medium, the elastic constants are in matrix C_{iso} , shown in equation (2.18). The recording time is 0.7 ms. For the anisotropic medium, the elastic constants are in matrix C_{ani} , shown in equation (2.18). The recording time is 1.6 ms.

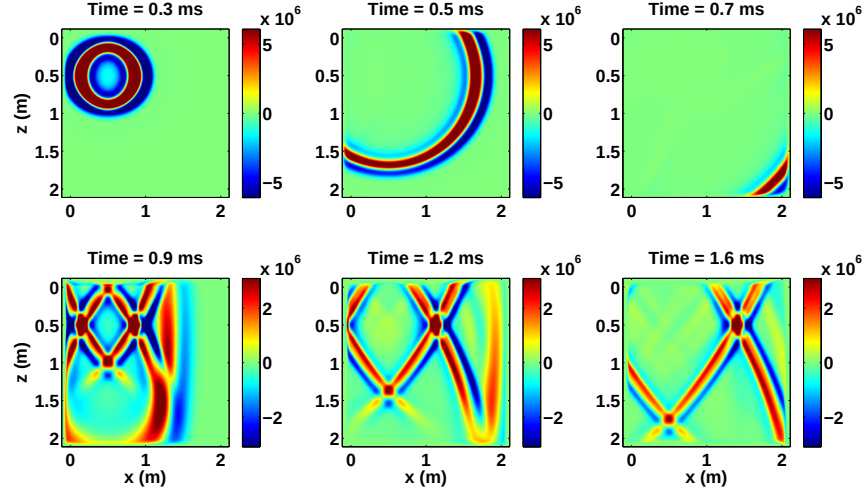


Figure 2.5: Snapshots of the monopole source wavefield propagation in both isotropic media (top row) and anisotropic media (bottom row) with H-PML absorbing layers. For the isotropic medium, the elastic constants are in matrix C_{iso} , shown in equation (2.18). The recording time is 0.7 ms. For the anisotropic medium, the elastic constants are in matrix C_{ani} , shown in equation (2.18). The recording time is 1.6 ms.

medium is isotropic (Figure 2.6a, b and c), the seismograms indicate that the C-PML and H-PML perform better than the M-PML, as some artificial signals from the boundaries can be detected in Figure 2.6b, where the events pointed out by red dashed arrows denote artifacts caused by M-PML. When the medium is anisotropic (Figure 2.6d, e and f), the recorded profiles indicate that the M-PML and H-PML perform better than the C-PML, whose seismic seismograms from 3 to 5 seconds suffer severely from artificial boundary reflections. H-PML provides satisfactory results in both isotropic and anisotropic formations.

The presence of direct waves whose energy dominates over other arrivals, as in Figure 2.6 will tend to mask the influence of different PML implementations on true reflected signals from the underground interfaces or structures. To study the difference on reflection profiles using different PML methods, Figure 2.7 shows a two-layer model and the reflections recorded by the receivers when C-PML, M-PML and H-PML methods are applied, respectively. The basic parameters of the model (such as the grid size, dominant frequency of the Ricker wavelet and the total recording time) are the same as the above single-layer model when the medium is isotropic, except that the formation under 1.2 km is anisotropic, whose elastic parameters are the same as the above single-layer model when the medium is anisotropic. The reflection seismograms contain significant artificial reflections when C-PML are used (the artificial boundary reflections are present in red dashed square area). The M-PML results are also subject to some degree of artificial reflections on both sides of the seismograms during 1.5 to 3.5 seconds (red dashed areas). Seismograms with the fewest artificial reflections are obtained when H-PML is applied.

Figure 2.8 shows comparisons of full recorded waveform data at three different receiver offsets calculated using C-PML (blue curve), M-PML (black curve) and H-PML (red curve) for (a)-(a1) isotropic, (b)-(b2) anisotropic and (c)-(c2) two-layer media, respectively. In isotropic media, waveforms acquired by C-PML and H-PML overlap with each other quite well. However, some minor artifacts can be detected in waveforms obtained using M-PML, as illustrated in black dashed areas in Figure 2.8a1. In anisotropic media, waveforms acquired by M-PML and H-PML match with

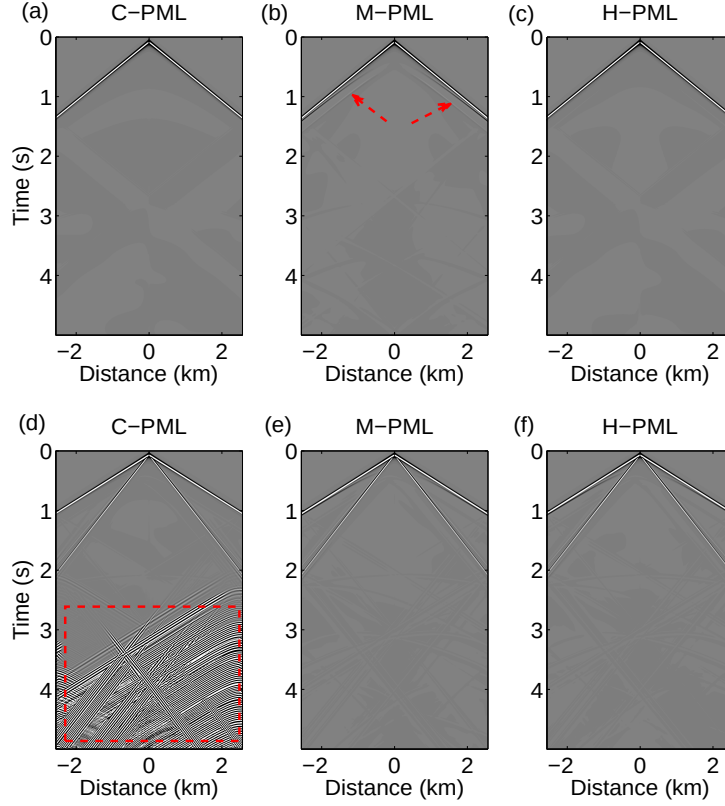


Figure 2.6: Synthetic seismograms in both isotropic((a), (b), (c)) and anisotropic((d), (e), (f)) media using C-PML, M-PML and H-PML. The seismograms indicate that the C-PML and H-PML perform better than the M-PML in isotropic medium. However, the M-PML and H-PML perform better than the C-PML. The seismograms of C-PML suffer severely from artificial boundary reflections, shown in red area of (d).

each other perfectly (Figure 2.8b1), yet the waveforms obtained by C-PML in both anisotropic and two-layer media are strongly affected by artificial boundary reflections when propagation time is greater than 4 seconds (black dashed areas in Figure 2.8b and b2, Figure 2.8c and c2). The M-PML method provides satisfactory results in two-layer media, except in some areas minor artifacts are persistently present, shown in Figure 2.8c1 and Figure 2.7.

In Figure 2.9, 3D aspects of the wavefield propagating in an isotropic medium are illustrated; Table 2.1 contains the medium properties used. This example has been set up to resemble a borehole logging experiment; as time advances in Figures 2.9a–d, the wavefield can be seen prop-

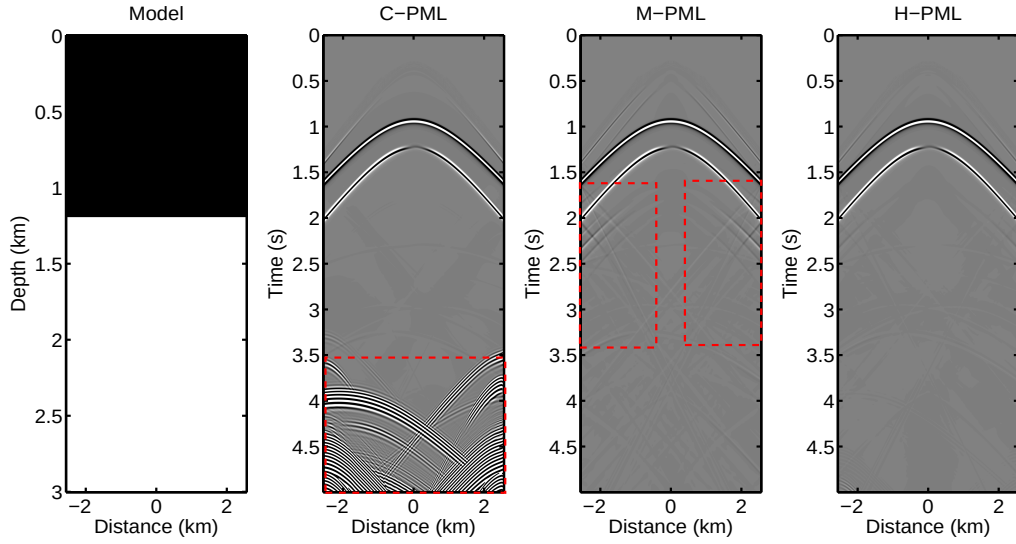


Figure 2.7: A two-layer model and its reflection profiles when C-PML, M-PML and H-PML are implemented, respectively. The seismograms indicate that both the C-PML and M-PML suffer from boundary reflections, shown in red areas of C-PML and M-PML results.

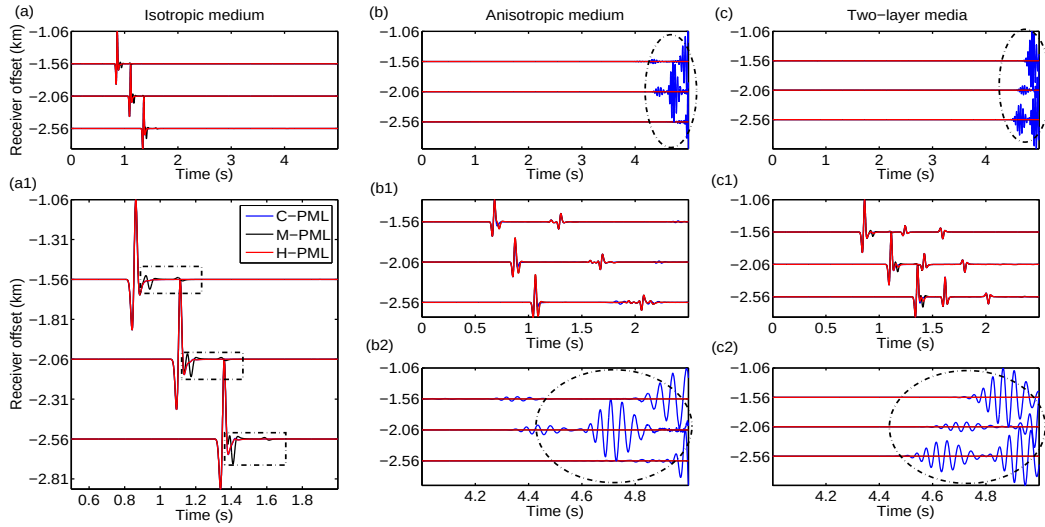


Figure 2.8: Comparisons of full recorded waveform data calculated using C-PML (blue curve), M-PML (black curve) and H-PML (red curve) for (a)-(a1) isotropic, (b)-(b2) anisotropic and (c)-(c2) two-layer media, respectively. The artificial reflections are dominant in both anisotropic and two-layer medium when C-PML is used. The M-PML performs better than C-PML in both media, however, yet some minor artifacts are still detected in isotropic medium. Hardly can artifacts be detected when H-PML is applied for both isotropic and anisotropic media.

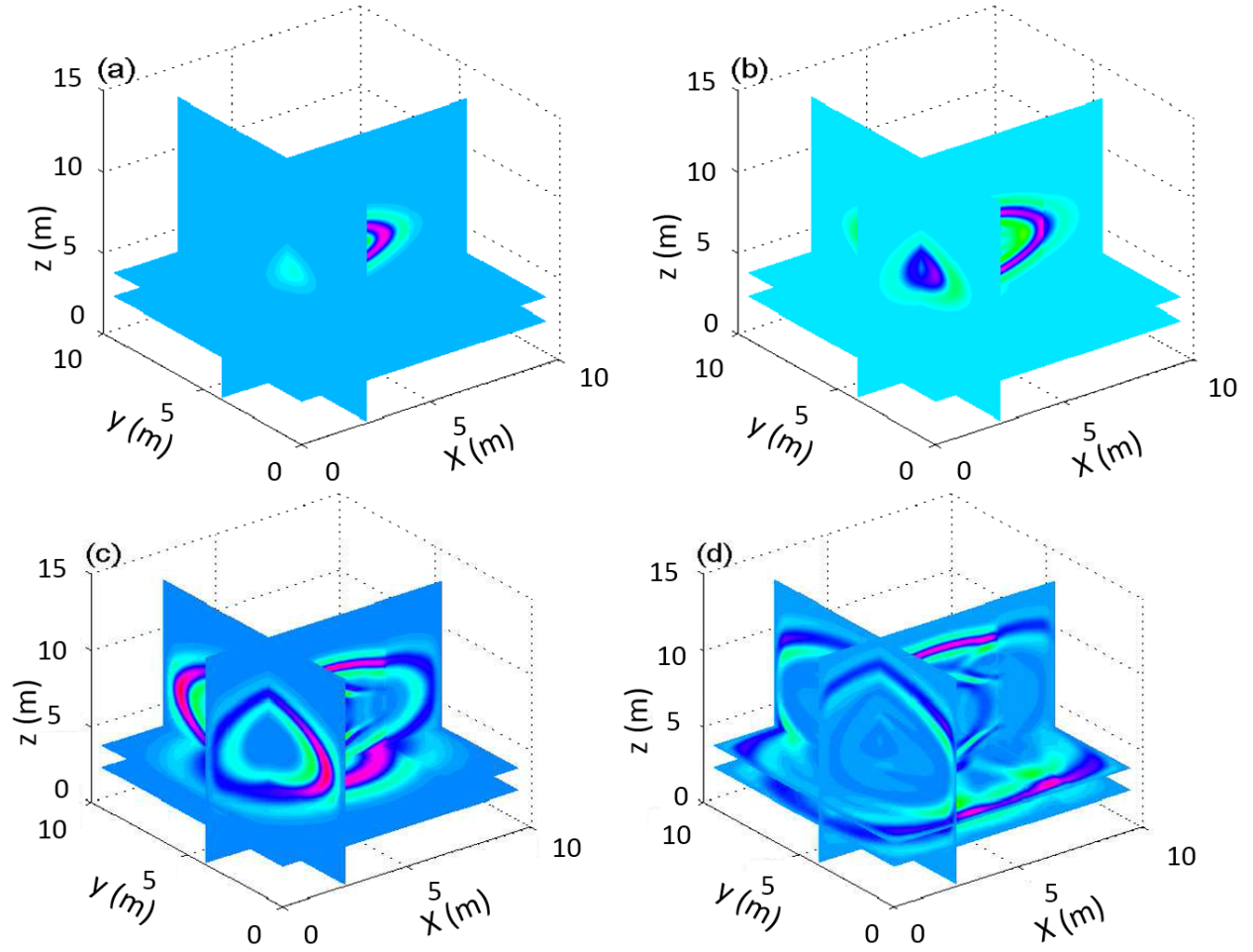


Figure 2.9: Snapshots of wave propagation in 3D medium.

agating from the borehole to the outside formation and interacting with an interface, after which reflected and transmitted wave energy can be seen propagating back to and away from the borehole.

Table 2.1: The parameters for 3D modeling of wavefield propagation.

	$V_f(m/s)$	$V_P(m/s)$	$V_S(m/s)$	$\rho(g/cm^3)$
Borehole	1500	-	-	1.0
Near Borehole formation	-	3000	1200	2.0
Second layer	-	4000	2300	2.5

2.5 Conclusions

The elastic medium staggered-grid finite difference method is discussed for borehole acoustic wave simulation. For suppression of the artificial boundary reflections, standard PML, C-PML and M-PML methods can be employed. The C-PML approach involves a general boundary formula designed for the grazing incidence. However, in some cases, instability can be observed either because of the frequency-dependent term or the convolution operations. The M-PML, though of higher efficiency especially in grazing incidence, nevertheless produces spurious reflections if the damping parameter is not optimized and the thickness of the PML is not large enough. A hybrid PML based on the C-PML and M-PML is demonstrated to produce a significantly reduced set of artificial boundary reflections.

Chapter 3

Wavefield simulation for a dipole source

3.1 Introduction

Simulation of elastic waves in borehole environments is of great importance in developing our understanding of the characteristics of wave propagation in acoustic well logging. The FD method has been extensively applied into two-dimensional wavefield simulation in acoustic well logging (Stephen et al., 1985; Randall et al., 1991; Leslie and Randall, 1992). With the development of parallel computers, the FD method has also been applied to 3D borehole wavefield propagation problems (Daube and Randall, 1991; Eppstein and Dougherty, 1998; Yoon and McMechan, 1992; Chen, 1994; Chen et al., 1998; Tao et al., 2008a; Wang et al., 2015). In this chapter, the numerical simulations of radiation, reflection and multipole reception of elastic waves excited by a dipole source will be discussed by implementing the 3D borehole wavefield propagation in anisotropic media.

3.2 Numerical simulation of waves from a dipole source in isotropic media

Seismic S-waves excited by a monopole source, when propagating in slow formations (i.e., those for which the S-wave velocity in the formation is lower than the acoustic wave velocity in the borehole), are not received by instruments in the borehole. This impediment contributes to the requirement for dipole or even multipole acoustic well logging. Dipole transmitters are, essentially, pistons which create a pressure increase on one side of the borehole and a decrease on the other (Close et al., 2009). In this section I consider the numerical modeling of the elastic wave response to the excitation of a dipole source. We begin by developing some analytical expressions to be used for benchmarking.

For sonic waveforms with a dipole source, let the displacement wavefield inside the borehole and in the formation outside the borehole be

$$\begin{aligned}\mathbf{u}_{in} &= \nabla \varphi_f \\ \mathbf{u}_{out} &= \nabla \varphi + \nabla \times \Psi,\end{aligned}\tag{3.1}$$

where $\mathbf{u}_{in}$ and $\mathbf{u}_{out}$ are the wavefield displacements in borehole fluid and formation outside the borehole, respectively. φ_f denotes its potential in the borehole fluid, φ is the compressional wave potential within the formation. The S-wave potential Ψ can be expressed as

$$\Psi = \chi \hat{\mathbf{z}} + \nabla \times (\eta \hat{\mathbf{z}}),\tag{3.2}$$

where χ and η are the SH- and SV- wave displacement potentials, respectively. In homogeneous media, they have the integral representations (Tang and Patterson, 2009)

$$\chi(\omega; r, z) = \frac{S(\omega) \cos(\phi)}{4\pi} \int_{-\infty}^{+\infty} E(\omega, k) K_1(sr) e^{ikz} dk,\tag{3.3}$$

and

$$\eta(\omega; r, z) = \frac{S(\omega) \sin(\phi)}{4\pi} \int_{-\infty}^{+\infty} F(\omega, k) K_1(sr) e^{ikz} dk,\tag{3.4}$$

where $S(\omega)$ is the source spectrum, ϕ is the angle between the direction of particle polarization of the wave and the source orientation, $E(\omega, k)$ and $F(\omega, k)$ are undetermined functions which depend on boundary conditions, $K_1(sr)$ is a modified Bessel function with radial wave number s .

The far field solution for the SH- and SV-wave displacement components can then be expressed as (Tang et al., 2014)

$$u_\phi \sim [i\rho\beta\omega E(\omega, k_0) \sin\theta \cos\phi] \frac{e^{i\omega R/\beta}}{4\pi\mu R} S(\omega),\tag{3.5}$$

and

$$u_\theta \sim [\rho\omega^2 F(\omega, k_0) \sin\theta \sin\phi] \frac{e^{i\omega R/\beta}}{4\pi\mu R} S(\omega),\tag{3.6}$$

where ρ and μ are the formation density and shear modulus respectively. θ is the tilt angle ($\theta = 90^\circ$ in this thesis). Let us use these results to characterize wave amplitudes in the borehole environment.

Waveforms recorded by borehole receivers are affected by the source radiation from borehole to formation, reflection processes occurring within the formation, and responses of the borehole to the reflected waves; also, wavefield amplitude attenuation occurs during propagation. This can be described with a convolution equation in the frequency domain as (Tang and Patterson, 2009)

$$RWV(\omega) = S(\omega) * RD(\omega) * RF(\omega) * RC(\omega) \frac{e^{i\omega D/\beta}}{D} e^{-\frac{\omega D}{2Q\beta}}, \quad (3.7)$$

in which RWV denotes the received reflections; $S(\omega)$ is the source spectrum; $RD(\omega)$ stands for the borehole radiation (Meredith, 1990); $RC(\omega)$ is related to borehole receiver response (Peng et al., 1993), and $RF(\omega)$ is the acoustic reflectivity at the reflector. D is the total propagation distance from a source to a receiver; Q_β denotes the shear wave attenuation. "*" used in this thesis denotes convolution. For example, the convolution of $S(\omega)$ and $RD(\omega)$ can be expressed as

$$S(\omega) * RD(\omega) = \int_{-\infty}^{+\infty} S(\eta) RD(\omega - \eta) d\eta. \quad (3.8)$$

According to Tang et al. (2014), the far-field radiation for SH - and SV - waves (RD_{SH} and RD_{SV}) can be expressed as

$$RD_{SH} = i\rho\beta\omega E(\omega, k_0) \sin\theta \cos\phi, \quad (3.9)$$

$$RD_{SV} = \rho\beta\omega F(\omega, k_0) \sin\theta \sin\phi.$$

Tang et al. (2014) point out that, in accordance with elastic reciprocity (Achenbach, 2003), the radiation and receiver patterns both for SH - and SV - waves are equal:

$$RC(\omega, \theta) = RD(\omega, \theta). \quad (3.10)$$

For simplicity, the wave field attenuation factor $e^{-\frac{\omega D}{2Q\beta}}$ in equation (3.7) will not be discussed here and only the SH component will be discussed in the following section. Let $R_{(SH)}$ be the isotropic SH reflection coefficient. By combining equations (3.9)–(3.10), the received SH reflection signal $RWV_{SH}(\omega)$ becomes

$$RWV_{SH}(\omega) = S(\omega) * RD_{(SH)} * R_{(SH)} * RD_{SH} \frac{e^{i\omega D/\beta}}{D}. \quad (3.11)$$

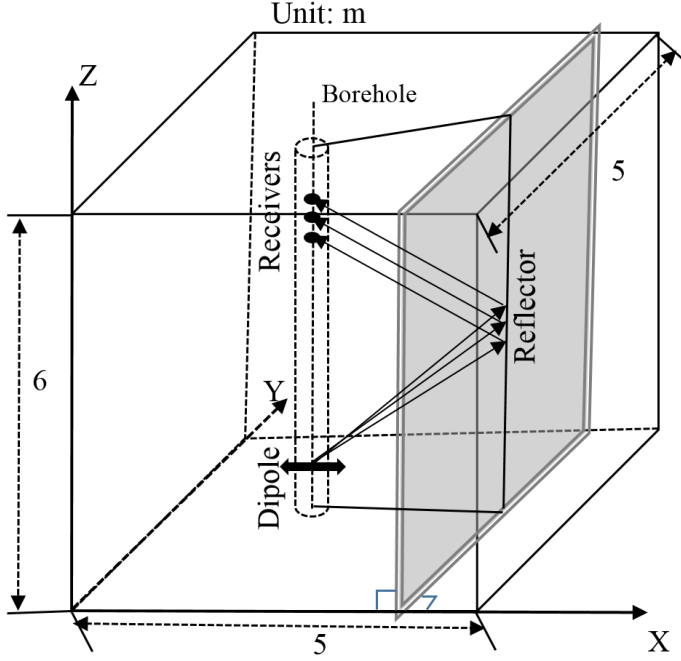


Figure 3.1: The 3D profile of the isotropic model with dipole source and quadrupole receivers. A dipole source is oriented along the x-axis of the coordinate system defined by the tool. The reflector is parallel to and 3m away from the borehole.

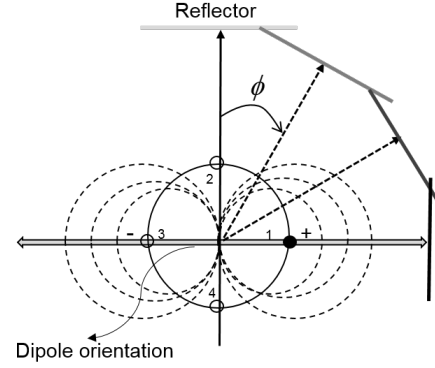


Figure 3.2: Displacement of receivers around the tool. The angle ϕ between the dipole and the reflector interface varies between 0° - 90° with an interval of 15° .

The low frequencies used in dipole acoustic logging makes available wider lateral detection. Let us consider propagation in isotropic media. In Figure 3.1, a dipole source is oriented along the x-axis of the coordinate system defined by the tool. The angle ϕ between the dipole and the reflector interface, as illustrated in Figure 3.2, varies between 0° - 90° with an interval of 15° . The reflector is 3 m away and parallel to the borehole, which means the dip angle θ in equation (3.9) is 90° .

The dipole source is located within the water-filled borehole, which has a diameter of 0.21 m. 30 receiver stations are deployed at 0.15 m intervals along the axis of the tool, with a distance of 0.15 m from the first receiver to the source. Each receiver station is populated by four azimuthal sensors, evenly spaced around the tool. We refer to the two sub-receivers parallel to the dipole source as receivers 1 and 3, and the two perpendicular to the dipole source as receivers 2 and 4 (see Figure 3.2). The dominant frequency of the source field is 3 kHz.

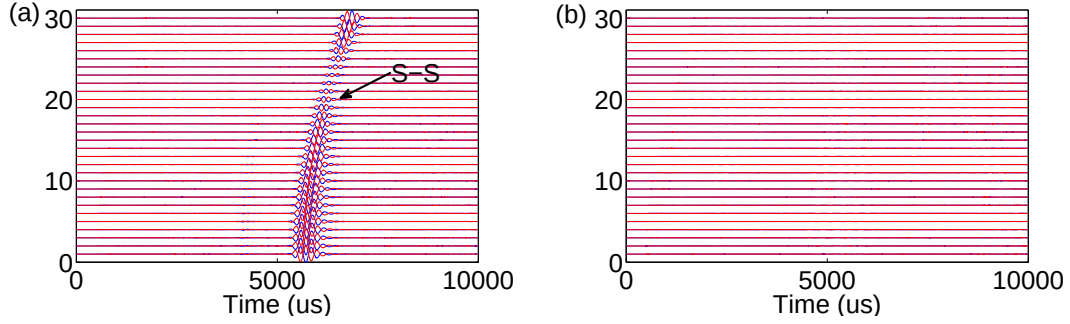


Figure 3.3: (a) Received reflections of receiver 1 (red) and receiver 3 (blue); and (b) Received reflections of receiver 2 (red) and receiver 4 (blue). The value in Y axis denotes the receiver number.

We next carry out a set of numerical experiments to validate the wave field simulation framework. When the source is oriented along the reflector strike, in accordance with equation (3.6) a pure SH wave and its corresponding SH reflection are generated (see Figure 3.3). This is illustrated in Figure 3.3a in which data from receivers 1 (red) and 3 (blue) are plotted. SH signals received by receivers 1 and 3 should have a phase difference resulting from differing path lengths, which is confirmed by Figure 3.3a. Furthermore, the dipole character of the source should according to theory cause the energy received by receivers 2 and 4 to be cancelled out, and this too is confirmed in Figure 3.3b.

We next examine the variability of the various modes measured in the presence of the dipole source. For situations when $90^\circ > \phi > 0^\circ$, reflected SH, SV and P modes should be received by all four receivers around the tool. Figure 3.4 shows the results when ϕ is 30° (see Figure 3.4a-b) and 60° (see Figure 3.4 c-d). In Figure 3.4a, for example, the reflections recorded by receivers 1 and 3 consist of a leaky P wave reflection, a leaky P-SV reflection and an S-S reflection. This conforms with the statements of Tang and Cheng (2004), who assert that a leaky P wave will be excited by a dipole source. The P-P and P-SV reflections measured by receivers 1 and 3 appear to be identical to those measured by receivers 2 and 4 (see Figure 3.4). Whereas, in the case of the S-S reflections, for both receiver pairs 1-3 and 2-4, the reflection signals show a distinctive phase difference when

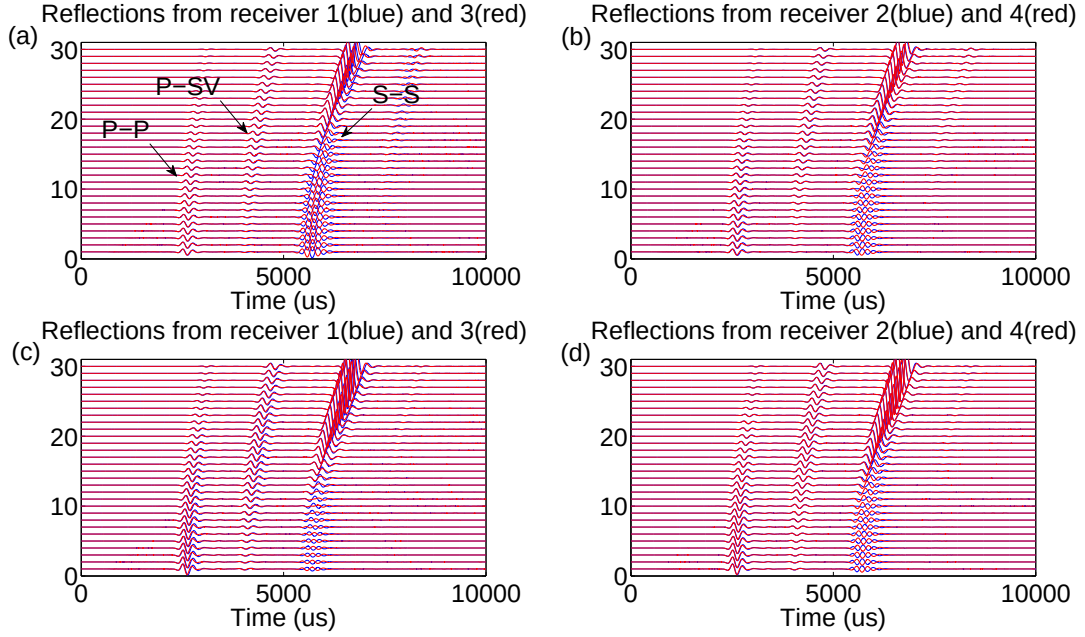


Figure 3.4: (a) and (b) Received reflections when ϕ is 30° ; (c) and (d) Received reflections when ϕ is 60° . The value in Y axis denotes the receiver number.

the offsets are relatively small. As offset increases, they gradually merge with each other. This means that the recorded signals are dominated by SH modes at small offsets, and SV modes at large offsets.

When the reflector azimuth is perpendicular to the orientation of the dipole source (ϕ is 90°), a pure SV wave and its corresponding SV reflection are generated, as shown in Figure 3.5. Figure 3.5a shows the reflections observed by receiver 1 (red) and receiver 3 (blue). The reflections recorded by receivers 1 and 3 include a leaky P wave reflection, a leaky P-SV reflection and pure SV-SV reflection. Figure 3.5b is a plot of the reflections measured by receivers 2 (red) and 4 (blue); because receivers 2 and 4 are the same distance from the reflector as 1 and 3, the reflections are identical. Through Figures 3.3-3.5, we observe that when ϕ increases from 0° to 90° , the merging of S-S reflections, i.e., the change from the case of a dominant SH mode to the case of a dominant SV mode begins at closer offsets.

We observe from the above results that the leaky P-P reflected wave amplitude as well as the

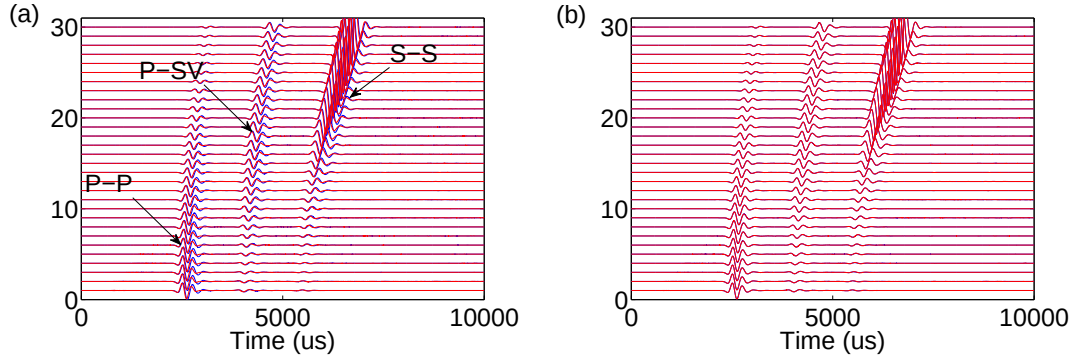


Figure 3.5: (a) Received reflections of receiver 1 (red) and receiver 3 (blue); (b) Received reflections of receiver 2 (red) and receiver 4 (blue) . The value in Y axis denotes the receiver number.

leaky P-SV wave amplitudes vary with the azimuth angle of the reflector: the amplitudes increase with its azimuth angle. Whereas, the P-SV wave amplitude variation versus offset, from near to far, is opposite to that of the P-P wave. For the S-S reflection, more detailed analysis will be presented in the following discussion.

Figure 3.6 is a plot of the signals measured by receiver 1 from $\phi = 0^\circ$ to $\phi = 90^\circ$ with offsets ranging from 1.5 m to 3.75 m. In Figure 3.6, when the offset is 1.5 m, P- and S- head waves, P-P, P-SV reflected waves and the S-S reflected wave are all visible. Focusing on the rectangular area, as the azimuth angle between the source and reflector increases, the SV wave energy increases while the SH wave energy decreases; this is consistent with the source-reflector angle and S-wave polarization relationship in equation (3.9). The response at receiver 3 (not plotted) is similar.

In Figure 3.7 normalized reflection signals observed at receiver 1, from $\phi = 0^\circ$ to $\phi = 90^\circ$, are plotted with offset ranging from 1.5 m to 3.75 m. The P-P reflection amplitude (highlighted by a dashed red rectangle) increases with the azimuth angle of the reflector, and decreases with offset. In contrast, there is no such trend observed for the S-S reflection signals (dashed black rectangle), because of the transformation between SH- to SV- wave signals, a result which differs from that described by Wei and Tang (2012). This is because the amplitude of S-S reflection depends on the azimuth angle and also source-receiver offset.

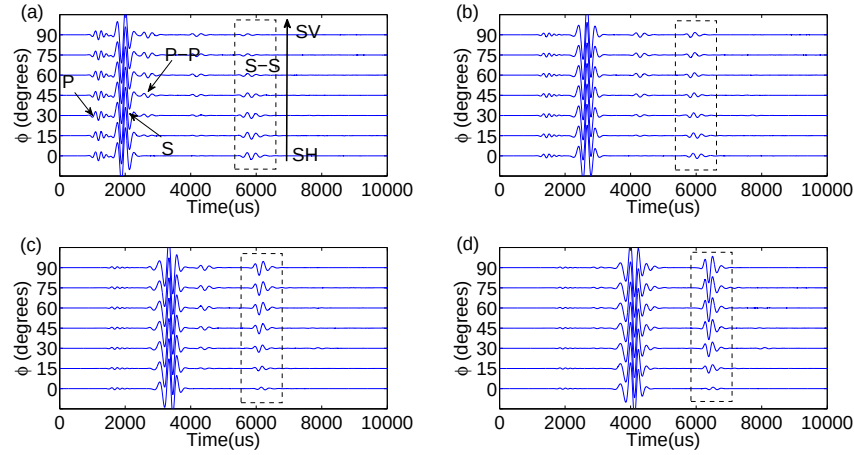


Figure 3.6: Full waveforms of R1 with an offset of (a) 1.5 m, (b) 2.25 m, (c) 3.0 m and (d) 3.75 m.

In Figure 3.8 signals measured by receiver 2 from $\phi = 0^\circ$ to $\phi = 90^\circ$, with offset ranging from 1.5 m to 3.75 m, are plotted. No P- and S- head waves appear; only the P-P, P-SV and S-S modes are visible. This is because the P- and S- direct waves destructively interfere between the x -directional dipole source and the y -directional receiver 2. As the azimuth angle between the source and reflector increases, the SV wave energy increases, as emphasized in the black dashed rectangular area in Figure 3.8. Signals received by receiver 4 are similar and are not shown.

In Figure 3.9, reflection signals measured by receiver 2 from $\phi = 0^\circ$ to $\phi = 90^\circ$, with offset ranging from 1.5 m to 3.75 m, are plotted. The reflected signal here is the same as the full waveforms discussed above; receivers which are orthogonal to the radiation direction of a dipole source record pure reflections only.

We have discussed the relationship between the offset and the S-wave polarization in the presence of different reflector azimuth angles. We next consider in greater detail the relationship between the S-S reflection amplitude and the offset. We do so by extracting the S-S reflection for different reflector azimuth angles. In Figure 3.10 the normalized S-S reflected amplitude with different reflector azimuth angles versus receiver offset is plotted.

Analysis of Figures 3.6-3.9 revealed that the SH reflection changes into an SV reflection as

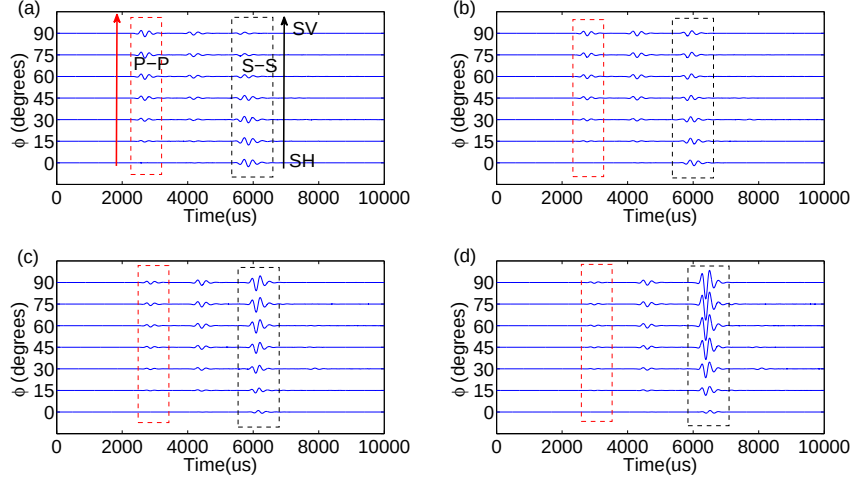


Figure 3.7: (a) Reflected waves of R1 with an offset of 1.5 m; (b) Reflected waves received by R1 with an offset of 2.25 m; (c) Reflected waves received by R1 with an offset of 3.0 m; (d) Reflected waves received by R1 with an offset of 3.75 m.

offset increases. The amplitude of the SV reflection is larger than that of the SH reflection, as seen in Figures 3.10b-d, where the maximum normalized S-S reflected amplitude happens to appear at a far and fixed offset (in this case, the offset is 4.125 m). However, it does not happen in the example in Figure 3.10a, i.e., when $\phi = 15^\circ$. This is because the transition from SH to SV reflection is not complete, and there is no pure SV reflection recorded even at the maximum offset.

Next we consider the reason why the maximum normalized S-S reflected amplitude occurs at an offset of 4.125 m. In equation 3.11, let

$$H(\omega) = S(\omega) * RD_{SH} * R_{(SH)} * RD_{SH}, \quad (3.12)$$

be the Fourier transform of the time function $h(t)$:

$$H(\omega) = \int_{-\infty}^{+\infty} h(t) e^{i\omega t} dt. \quad (3.13)$$

By multiplying $\frac{e^{i\omega D/\beta}}{D}$ with $H(\omega)$, we obtain

$$H(\omega) \frac{e^{i\omega D/\beta}}{D} = \frac{1}{D} \int_{-\infty}^{+\infty} h(t) e^{i\omega(t-D/\beta)} dt = RWV(\omega), \quad (3.14)$$

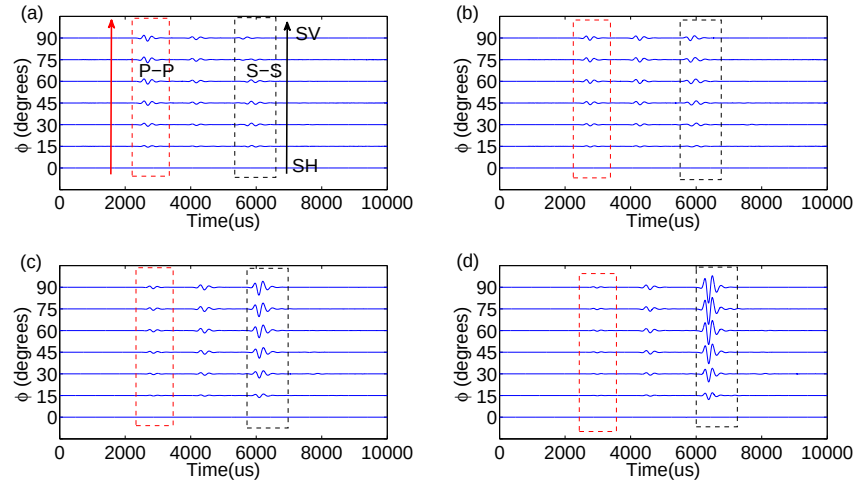


Figure 3.8: (a) Full waveforms of R2 with an offset of 1.5 m; (b) Full waveforms received by R2 with an offset of 2.25 m; (c) Full waveforms received by R2 with an offset of 3.0 m; (d) Full waveforms received by R2 with an offset of 3.75 m.

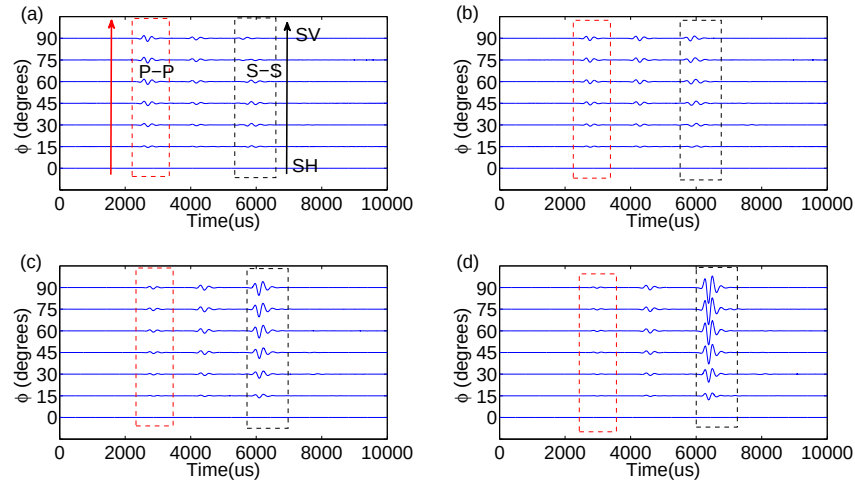


Figure 3.9: (a) Reflected waves of R2 with an offset of 1.5 m; (b) Reflected waves received by R2 with an offset of 2.25 m; (c) Reflected waves received by R2 with an offset of 3.0 m; (d) Reflected waves received by R2 with an offset of 3.75 m.

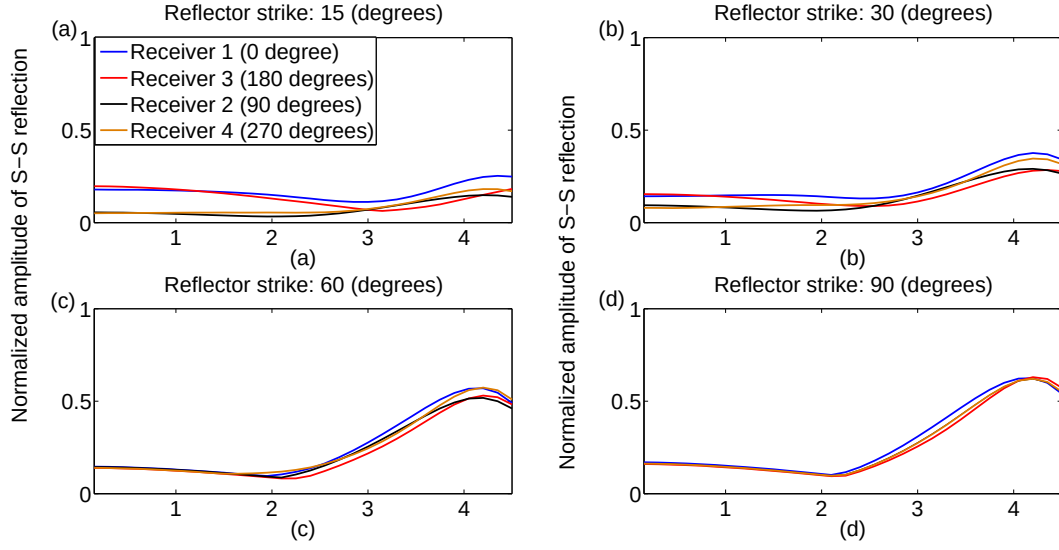


Figure 3.10: Cross-plots of maximum amplitude versus receiver offset. The value in X axis denotes the offset.

where

$$h(t) = s(t)rd_{(SH)}^2 r_{(SH)}, \quad (3.15)$$

and where $s(t)$ is the time-domain form of $S(\omega)$. Based on equations (3.7), (3.9) and (3.10), the time domain form of $RD(\omega)$, or $RC(\omega)$, is found to be

$$rd_{(SH)} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} i\rho\beta\omega E(\omega, k_0) \cos\phi e^{i\omega t} d\omega. \quad (3.16)$$

The SH reflection response from equation (3.14) is,

$$RWV(\omega) = r_{(SH)} \frac{-\frac{1}{4\pi^2} \rho^2 \beta^2 \cos^2 \phi}{D} \int_{-\infty}^{+\infty} s(t) e^{i\omega(t-D/\beta)} dt \left(\int_{-\infty}^{+\infty} \omega E(\omega, k_0) e^{i\omega t} d\omega \right)^2, \quad (3.17)$$

where, $r_{(SH)}$ is the isotropic SH reflection coefficient

$$r_{(SH)} = \frac{\rho_1 \beta_1 \cos \phi_1 - \rho_2 \beta_2 \cos \phi_2}{\rho_1 \beta_1 \cos \phi_1 + \rho_2 \beta_2 \cos \phi_2}. \quad (3.18)$$

The amplitude part of equation (3.17) is

$$A = r_{(SH)} \frac{-\frac{1}{4\pi^2} \rho^2 \beta^2 \cos^2 \phi}{D}. \quad (3.19)$$

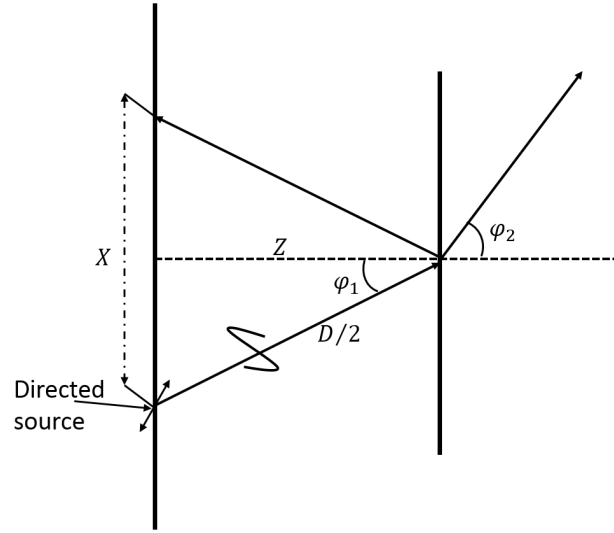


Figure 3.11: S wave emission, reflection and transmission.

Therefore, determining the extreme value of A is equivalent to determining the extreme value of the reflection coefficient. In this case, the maximum value of the above equation occurs when $\cos(\varphi_2) = 0$, where φ_2 is as shown in Figure 3.11. We have,

$$\cos \varphi_1 = \frac{2Z}{D} , \quad (3.20)$$

$$\cos \varphi_2 = \sqrt{1 - \frac{\beta_2^2}{\beta_1^2} \left(1 - \frac{4Z^2}{D^2}\right)} .$$

Letting the reflection angle change from 0° to 60° with a sample rate of 0.5° in equation (3.11), the amplitude difference versus different reflection angles can be obtained, as shown in Figure 3.12. In Figure 3.12, the normalized amplitude of S wave reflection is observed to reach its maximum value when the incident angle is 36° . Given the geometrical relationship in Figure 3.11, the calculated offset with an incident angle of 36° is about 4.125 m, which is consistent with the results in Figure 3.10.

Based on the cross-plot of maximum amplitude versus receiver offsets, the offset of maximum amplitude can be found, and used to determine the total propagation distance of D (see Figure 3.11). Both the distance between the borehole and the reflector and the critical angle can therefore

be calculated. As a result, the shear wave velocity of the second layer outside a borehole can be obtained using Snell's law.

3.3 Numerical simulation of waves from a dipole source in anisotropic media

Waveforms excited by a dipole directional source may also be simulated in an anisotropic medium, the model that we are using is similar from Figure 3.1, except the size of the model is $5 \times 5 \times 8$ (m^3). The distance between the source and the first receiver is 1 m and the distance between receivers is 0.16 m, with altogether 30 receivers ranging from 1 m to 5.64 m. The layer close to the borehole is a VTI medium, whose elastic parameters are

$$c_{VTI} = \begin{bmatrix} 23.87 & 15.33 & 9.79 & 0 & 0 & 0 \\ 15.33 & 23.87 & 9.79 & 0 & 0 & 0 \\ 9.79 & 9.79 & 15.33 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.77 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.77 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.27 \end{bmatrix} \quad (3.21)$$

Borehole parameters and the parameters of the second layer are identical to those of the isotropic case, as shown in Table 2.1. 8 sub-receivers of each receiver are evenly spaced around the borehole, as shown in Figure 3.13.

With the source orientation along the reflector strike, the received reflections with offsets ranging from 1 m to 5.64 m are shown in Figure 3.14; a pure SH reflection is generated (see Figure 3.14a, c and d). Because of the geometrical difference of the receivers in terms of the directional dipole source and reflector, the SH reflections received by pair 4 and 8 as well as pair 2 and 6 do not have exactly opposite phases compared with the receiver pair 1 and 5. Receiver pair 3 and 7 do not receive reflections because of the source directionality.

When ϕ is 30° , the qP-qP, qP-qSV and the SH reflections are present at each receiver, as

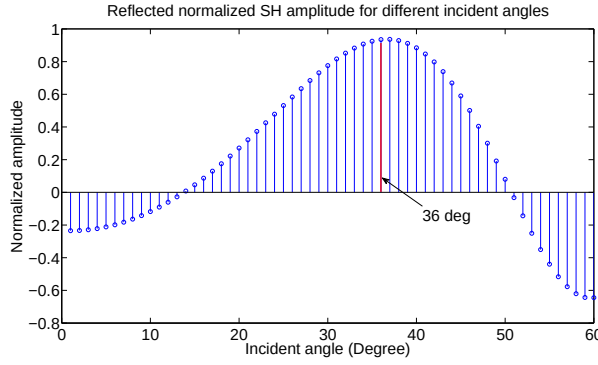


Figure 3.12: Cross-plot of normalized amplitude versus different reflection angles.

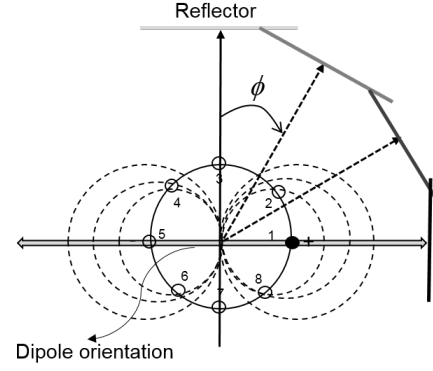


Figure 3.13: Displacement of 8 receivers around tool.

shown in Figure 3.15. The SH reflection amplitude reaches its peak at the near offset, where the qSV reflection emerges. The amplitude of the qP-qP reflection increases with offset, which is the opposite of that observed in the isotropic medium case (Figure 3.4a and b).

Figure 3.16 and Figure 3.17 show the received reflections for the 8 receivers when $\phi = 60^\circ$ and $\phi = 90^\circ$, respectively. According to Figures 3.14-3.17, with the increase of the angle between source orientation and the reflector strike, the amplitudes of the qP-qP, qP-qSV reflections increase, as does the qSV-qSV reflection. However, the amplitude of the SH-SH reflection is decreasing and it reaches to its minimum value when $\phi = 90^\circ$. The SH-SH reflection from different reflector azimuth angles is then extracted, and the normalized SH-SH reflected amplitude with different reflector azimuth angles versus receiver offset is plotted in Figure 3.18. The SH-SH amplitude of each receiver reaches its peak at an offset of 2.76 m in spite of different reflector strikes. In a VTI medium, the received waveforms recorded by the receivers will behave differently than the prediction of equation (3.11). However, theoretically, the maximum value of the received SH amplitude will occur when the wave propagates to the interface at the critical angle. From Figure 3.18, the relationship between the maximum amplitude and the receiver offset is observed to be unrelated to the azimuth angle of the reflector. In fact, the $RF(\omega)$ term plays a dominant role in the change of the SH reflection amplitude.

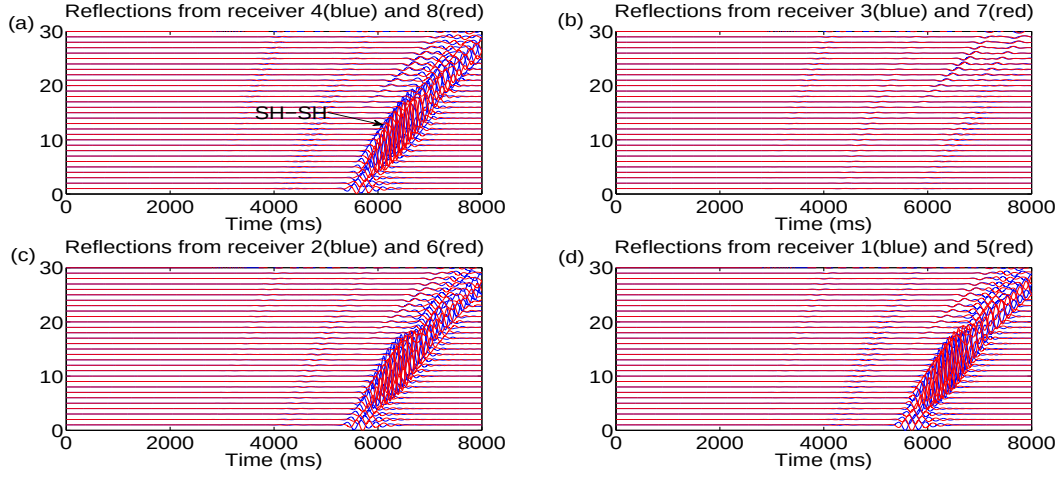


Figure 3.14: Received reflections for 8 evenly spaced receivers when the strike of reflector is parallel to the radiation of the directional source. The value in Y axis denotes the receiver number.

The SH reflection coefficient r_{SH} , according to Slawinski (2003), can be expressed as,

$$r_{SH} = \frac{\frac{\sqrt{\rho_1} c_{44}^I \cos \varphi_1}{\sqrt{c_{66}^I \sin^2 \varphi_1 + c_{44}^I \cos^2 \varphi_1}} - \frac{\sqrt{\rho_2} c_{44}^{II} \cos \varphi_2}{\sqrt{c_{66}^{II} \sin^2 \varphi_2 + c_{44}^{II} \cos^2 \varphi_2}}}{\frac{\sqrt{\rho_1} c_{44}^I \cos \varphi_1}{\sqrt{c_{66}^I \sin^2 \varphi_1 + c_{44}^I \cos^2 \varphi_1}} + \frac{\sqrt{\rho_2} c_{44}^{II} \cos \varphi_2}{\sqrt{c_{66}^{II} \sin^2 \varphi_2 + c_{44}^{II} \cos^2 \varphi_2}}}, \quad (3.22)$$

where c_{44}^I, c_{66}^I and c_{44}^{II}, c_{66}^{II} are the elastic constants of the media supporting the incident and the refracted waves, φ_1, φ_2 are the incident and transmitted angles, respectively, and ρ_1 and ρ_2 denote the density of the incident and transmitted layers, respectively. In order to eliminate φ_2 in the above equation, we make use of Snell's law in VTI media as

$$\frac{\sin \varphi_1}{\sqrt{\frac{c_{66}^I \sin^2 \varphi_1 + c_{44}^I \cos^2 \varphi_1}{\rho_1}}} = \frac{\sin \varphi_2}{\sqrt{\frac{c_{66}^{II} \sin^2 \varphi_2 + c_{44}^{II} \cos^2 \varphi_2}{\rho_2}}}. \quad (3.23)$$

Solving equation (3.23), we obtain

$$\varphi_2 = \arcsin \sqrt{\frac{\rho_1 c_{44}^{II} \sin^2 \varphi_1}{[\rho_2 (c_{66}^I - c_{44}^I) - \rho_1 (c_{66}^{II} - c_{44}^{II})] \sin^2 \varphi_1 + \rho_2 c_{44}^I}}. \quad (3.24)$$

Then we change the incident angle from 0° to 90° with a sample rate of 0.5° in equation (3.22) to obtain the amplitude difference for various reflection angles, which is shown in Figure 3.19.

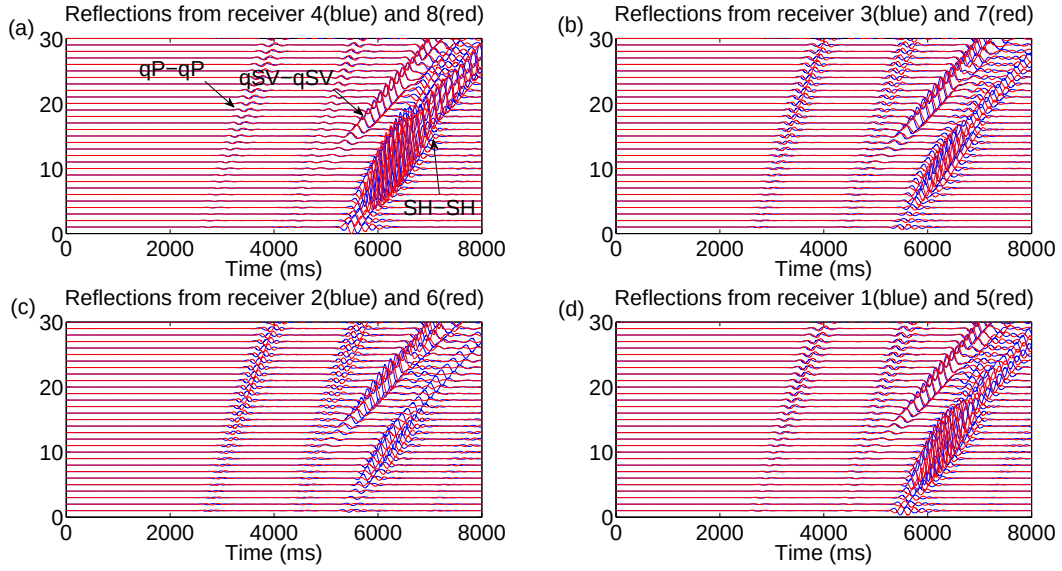


Figure 3.15: Received reflections for 8 evenly spaced receivers when there is a 30° angle difference between the strike of reflector and the radiation of the directional source. The value in Y axis denotes the receiver number.

The normalized amplitude of the SH reflection is observed to reach its maximum value when the reflection angle is 50.5° . When the incident angle is 50.5° , the calculated offset according to Figure 3.18 is about 2.76 m, which is consistent with the result in Figure 3.19.

3.4 Conclusions

The 3D elastic staggered-grid finite difference method is applied to the investigation of wavefield simulation for a directional dipole source, illuminating a parallel reflector. The reflector is rotated around the source, both in isotropic and anisotropic media. In the isotropic medium, the reflections observed at the four evenly spaced receivers around the borehole show an angular dependence related to the geometry of the reflector. Furthermore, a transition is detected between the SH-SH reflection and SV-SV reflection with the increase of the offset. Analysis of the relationships between the borehole wavefield reception, radiation and reflection of S-S reflected signals show that the maximum S-S reflected amplitude occurs when the incident angle of S wave reaches its

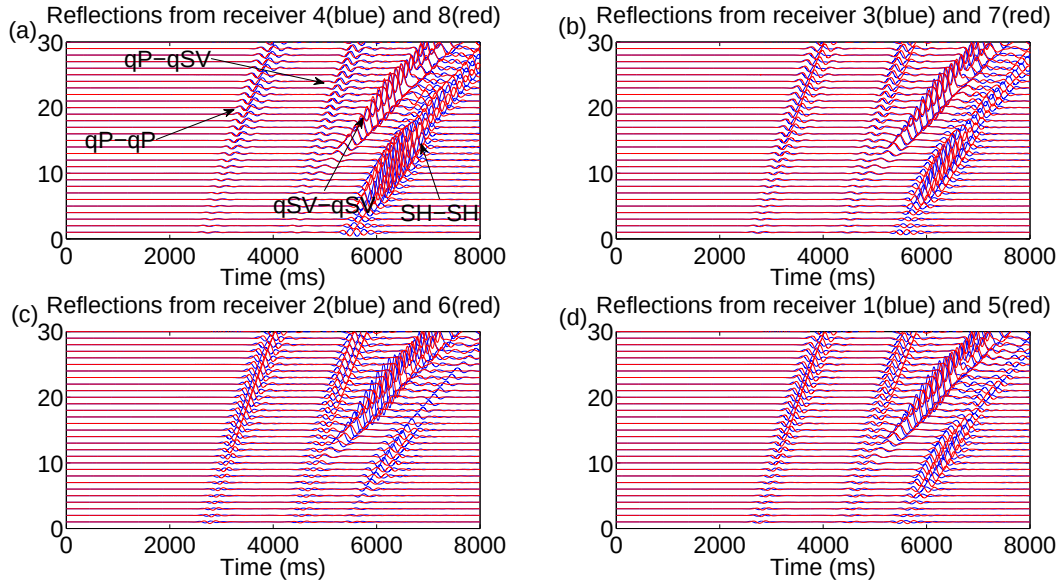


Figure 3.16: Received reflections for 8 evenly spaced receivers when there is a 60° angle difference between the strike of reflector and the radiation of the directional source. The value in Y axis denotes the receiver number.

critical value (when total reflection occurs). Based on the cross-plot of maximum amplitude versus receiver offsets, the offset of maximum amplitude can be found, and used to determine the total travel distance. Both the distance between the borehole and the reflector and the critical angle can therefore be calculated. As a result, the shear wave velocity of the second layer outside a borehole can be obtained according to Snell's law.

In the VTI medium, the received waveforms recorded by the receivers is different from the isotropic medium. In theory, the maximum value of the received SH amplitude will occur when the wave propagates to the interface with a critical angle. The SH-SH reflection coefficient in the VTI medium is introduced and used to calculate the relationship between the incident angle and reflected amplitude. As a result, our expectation is confirmed through simulation of the relationship between offset and the reflected amplitude in conjunction of the calculated change of the SH-SH reflection coefficient with the incident angle.

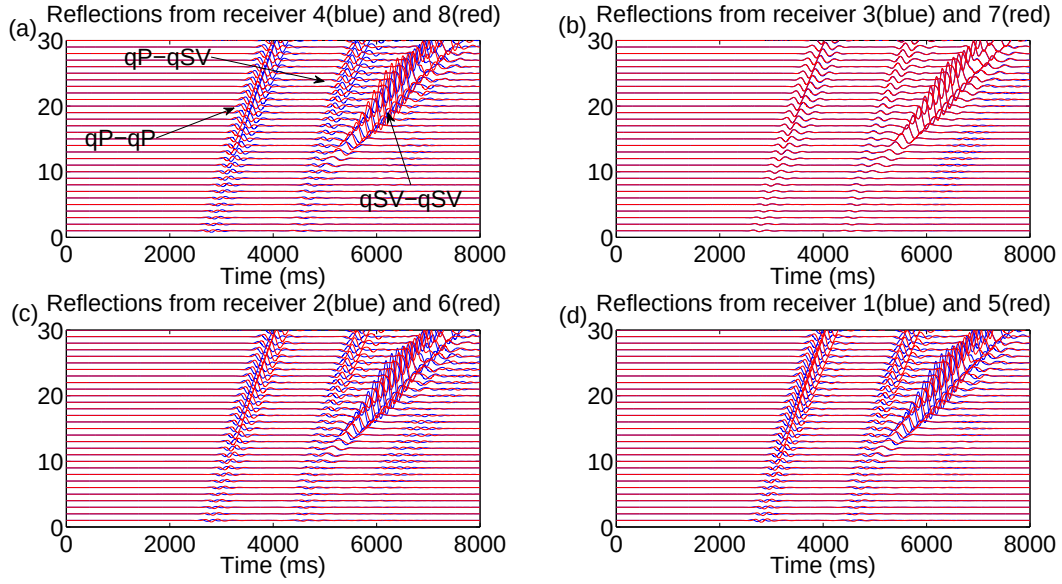


Figure 3.17: Received reflections for 8 evenly spaced receivers when there is a 90° angle difference between the strike of reflector and the radiation of the directional source. The value in Y axis denotes the receiver number.

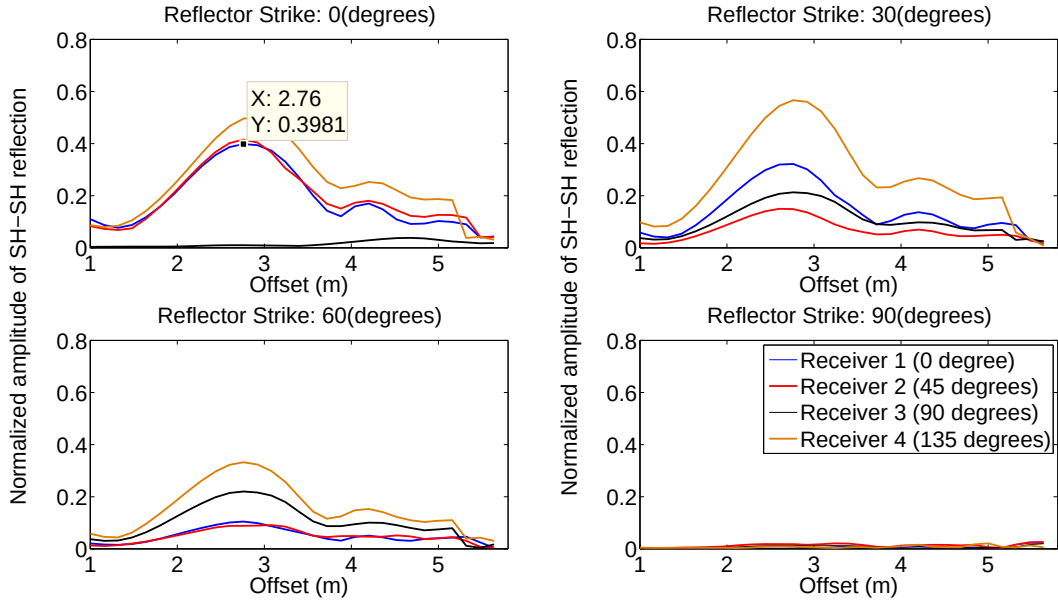


Figure 3.18: Cross-plot of normalized amplitude versus different receiver offsets in VTI medium.

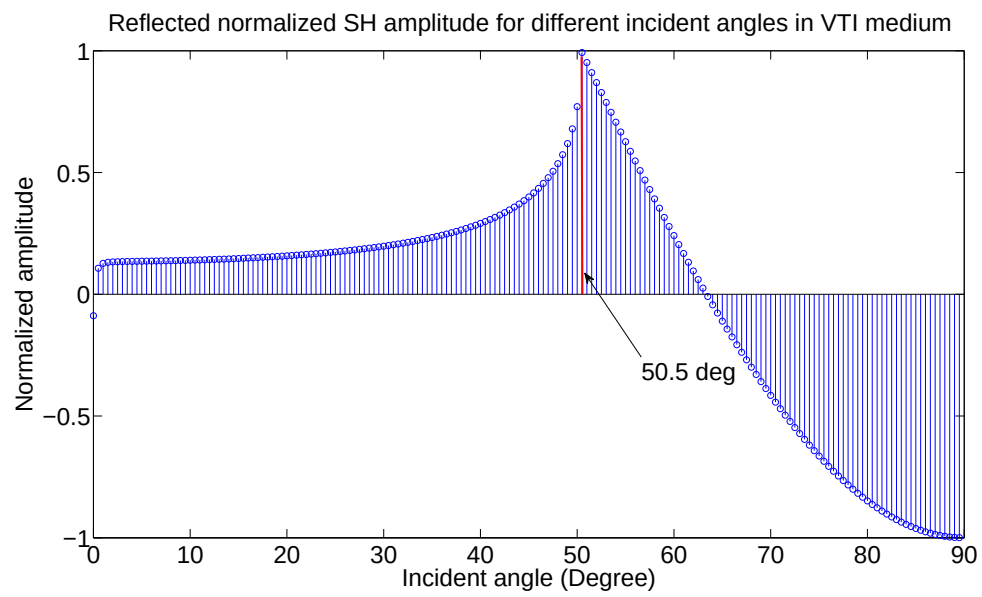


Figure 3.19: Cross-plot of normalized amplitude versus different reflection angles in VTI medium.

Chapter 4

A 3D pseudo-spectral method for SH wave simulation

Accurate and efficient numerical tools for modeling of seismic wave propagation in reservoir rocks are becoming increasingly indispensable, in both research and industry settings, as full-waveform processing and inversion methods are developed and refined. Representations of rocks that have spatially-varying fracture orientations and densities, stress distributions, complex bedding, etc., via anisotropic models, are of particular importance. From previous chapter, the amplitude of the SH- reflection wave indicates that in borehole acoustic logging environments, the SH energy is strongest among the others. The SH- reflection can be used in the migration and imaging step to obtain structure information such as azimuth. And the crosstalk caused by the interference from other modes is expected to be significantly less in any subsequent SH-mode based full waveform inversion (FWI) procedure. Furthermore, under idealized circumstances the SH-mode of the full elastic wavefield propagates independently of the P-SV modes, and can be sensed approximately independently in multicomponent experiments. For all of these reasons, 3D simulation of this component of the full elastic response is simple, but also meaningful from the point of view of data simulation.

4.1 Introduction

The most widely-applied numerical approach for modeling the propagation of seismic waves is the FD method (Alterman and Karal, 1968; Alford et al., 1974; Kelly and Iversen, 1976; Madariaga, 1976; Virieux, 1986). SH-wave modeling using FD started as early as 1970, when it was applied to SH-wave propagation in laterally inhomogeneous media Boore (1970a). This FD representation, however, does not satisfy the equations of motion (Boore, 1970b); Kummer and Behle (1982) progressed matters with a second-order finite-difference modeling of SH-wave propagation in lat-

erally inhomogeneous media. Virieux (1984) re-arranged the second-order hyperbolic SH-wave equation into a first-order velocity-stress hyperbolic system in a generally heterogeneous medium. However, he pointed out that a corner wave as well as a head wave would appear, which could have severe consequences in applications such as migration. Moczo (1989) developed an explicit finite-difference scheme using irregular rectangular grids for SH-waves in 2D media, which reduces staircase diffractions and the number of grid points. Igel and Weber (1995) derived an axisymmetric formulation to model SH-wave propagation in spherical coordinates to calculate seismograms for global earth models. Slawinski and Krebes (2002) developed a finite difference scheme for modeling SH-wave propagation in fractured media, in which fractures were modeled as internal interfaces in nonwelded contact. The computational complexity and cost of this approach, because dense meshes are required when dealing with non-planar fractures, and when the distances between fractures are smaller than the seismic wave length, impede its practical implementation.

In spite of major developments in, for instance, parallel computing technology, memory and computational time are still the dominant limitations faced by users of FD methods. The Fourier pseudo-spectral method (PSM) (Kosloff et al., 1984) generates numerical solutions with the same accuracy as FD methods but with significantly fewer grid points, making it an attractive alternative. PSM makes use of an accurate differentiation scheme, in which the fast Fourier transform (FFT) is used for calculating spatial derivatives and finite differences are used for calculating the time derivatives, which reduces memory usage and computation time (Fornberg, 1987; Daudt et al., 1989).

The periodicity condition implied by the discrete Fourier transform causes the periodically extended wavefield on either side of the computational domain to propagate back from computational boundaries, which results in wraparound artifacts. To avoid this, Fornberg (1996) suggested the Chebyshev PSM be employed, which increases the grid density requirement to π nodes per minimum wavelength. Alternatively, absorbing boundaries (Cerjan et al., 1985), or perfectly matched layers (or PML, Collino and Tsogka, 2001) can be used to damp the wraparound phases through

a gradual reduction of the wavefield amplitude in the vicinity of the grid boundary. Liu (1998) combined the conventional Fourier PSM with perfectly matched layers (PML) to effectively eliminate the wraparound effect. Furumura and Takenaka (1995) pointed out that improper selection of absorption parameters can result in reflected waves of relatively large amplitude. He proposed a solution for this problem, involving an anti-periodic extension technique based on a simple modification of the wavefield, however, this method only partially eliminates wraparound.

PSM solutions also tend to exhibit non-causal ringing artifacts (Gibbs' phenomena), particularly in the presence of large and/or abrupt changes in the medium. This occurs because the Fourier transform is a global rather than a local operator: each wavenumber contributes to all space. Smoothing is recommended by Pan and Wang (2000), and by Mast et al. (2001) and Tabei et al. (2002), to alleviate this issue, but, because we are often specifically interested in the wave response to rapid medium property variations, this solution is for many users incomplete. A variable grid-density PSM (Liu, 1999; Liu et al., 2000) has been proposed, in which discontinuities in the medium are better resolved. But, because this method employs a non-uniform fast Fourier transform (NUFFT) algorithm, which includes an interpolation step, the improved resolution comes at a cost to computational efficiency. An elegant mapping method to obtain spatial derivatives was introduced by Bayliss and Turkel (1992); Gao et al. (2004) further established a general procedure to construct mapping curves. The choice of initial grid points is not straightforward, but some rules for deciding on the positions of grid points were presented. Witte et al. (1987) proposed a pseudo-spectral calculation on a staggered grid to suppress Nyquist errors (Özdenvar and McMechan, 1996), and Bale (2002) developed a 3D, fully anisotropic scheme based on a decomposition orthorhombic and non-orthorhombic stiffnesses. Yet, ringing artifacts were not completely mitigated.

Classical finite difference approximations of time derivatives are subject to numerical dispersion (Özdenvar and McMechan, 1996). To mitigate this, the rapid expansion method proposed by Kosloff et al. (1989) can be employed, after which a more accurate time integration is obtained.

Similarly to Tal-Ezer et al. (1987), Bessel functions and modified Chebyshev polynomials are incorporated in the method, expanding the cosine operator so that it is accurate and numerically stable with large time steps. However, most of these approaches are designed for the solution of the second-order acoustic wave equation, which impedes the direct use of PML boundary conditions.

Long et al. (2013) proposed a temporal fourth-order scheme to solve 2D first-order acoustic wave equations with perfectly matched layers in the time domain. In this chapter, we extend this temporal fourth-order scheme to 3D SH-wavefield simulation in heterogeneous VTI media. A Hybrid-PML (Li et al., 2016) boundary condition (as described in Chapter 2) is combined with a Fourier pseudo-spectral time-domain (PSTD) method to eliminate wraparound effects. A staggered-grid based PSM is applied to spatial derivatives to eliminate the Gibbs' phenomenon. To make comparative conclusions, in this chapter, we also implement an SH-wavefield simulation based on second order PSTD, with a sponge absorbing boundary condition (as described in Chapter 2) (Israeli and Orszag, 1981). The SH wavefield simulation results are also compared with those obtained using the staggered-grid FD scheme. Finally, SH wavefield simulation in a 3D over-thrust model is illustrated.

4.2 First-order SH wave equations in VTI media

In vertical transverse-isotropic (VTI) media (Thomsen, 1986), the elastic constants in matrix form can be expressed as (2.7). Substituting equation (2.2),(2.3) and equation (2.7) into equation (2.1), and making use of the equations of motion

$$\begin{aligned}\rho \frac{\partial^2 u_x}{\partial t^2} &= \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z}, \\ \rho \frac{\partial^2 u_y}{\partial t^2} &= \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z}, \\ \rho \frac{\partial^2 u_z}{\partial t^2} &= \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z},\end{aligned}\tag{4.1}$$

we obtain the following VTI system:

$$\begin{aligned}
c_{11} \frac{\partial \nabla_2 \cdot \mathbf{u}_2}{\partial x} + c_{13} \frac{\partial^2 u_z}{\partial x \partial z} + c_{66} \frac{\partial}{\partial y} \left(\frac{\partial u_x}{\partial y} - \frac{\partial u_y}{\partial x} \right) + c_{44} \frac{\partial}{\partial z} \left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x} \right) + \rho \omega^2 u_x &= 0 \\
c_{11} \frac{\partial \nabla_2 \cdot \mathbf{u}_2}{\partial y} + c_{13} \frac{\partial^2 u_z}{\partial y \partial z} + c_{66} \frac{\partial}{\partial x} \left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} \right) + c_{44} \frac{\partial}{\partial z} \left(\frac{\partial u_y}{\partial z} - \frac{\partial u_z}{\partial y} \right) + \rho \omega^2 u_y &= 0, \\
(c_{13} + c_{44}) \frac{\partial \nabla_2 \cdot \mathbf{u}_2}{\partial z} + c_{33} \frac{\partial^2 u_z}{\partial z^2} + c_{44} \nabla_2^2 u_z + \rho \omega^2 u_z &= 0
\end{aligned} \tag{4.2}$$

where, ∇_2 is the operator in the x - y plane, $\nabla_2 = \hat{\mathbf{x}} \frac{\partial}{\partial x} + \hat{\mathbf{y}} \frac{\partial}{\partial y}$, and $\mathbf{u}_2$ is the projection of $\mathbf{u}$ in the x - y plane, $\mathbf{u}_2 = \hat{\mathbf{x}} u_x + \hat{\mathbf{y}} u_y$. The displacement can be decomposed into

$$\mathbf{u} = \nabla \varphi + \nabla \times (\chi \hat{\mathbf{z}}) + \nabla \times \nabla \times (\eta \hat{\mathbf{z}}), \tag{4.3}$$

where, φ , η and χ are the scalar displacement potentials. Expanding equation (4.3) we have

$$\begin{aligned}
u_x &= \frac{\partial \varphi}{\partial x} + \frac{\partial \chi}{\partial y} + \frac{\partial^2 \eta}{\partial x \partial z} \\
u_y &= \frac{\partial \varphi}{\partial y} - \frac{\partial \chi}{\partial x} + \frac{\partial^2 \eta}{\partial y \partial z} \\
u_z &= \frac{\partial \varphi}{\partial z} - \frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2}
\end{aligned} \tag{4.4}$$

Substitution of equation (4.4) into (4.2) yields

$$\begin{aligned}
c_{11} \nabla_2^2 \varphi + (c_{13} + 2c_{44}) \frac{\partial^2 \varphi}{\partial z^2} \\
+ \rho \omega^2 \varphi + \frac{\partial}{\partial z} \left[(c_{11} - c_{13} - c_{44}) \nabla_2^2 \eta + c_{44} \frac{\partial^2 \eta}{\partial z^2} + \rho \omega^2 \eta \right] &= 0,
\end{aligned} \tag{4.5}$$

$$\begin{aligned}
\frac{\partial}{\partial z} \left[(c_{13} + 2c_{44}) \nabla_2^2 \varphi + c_{33} \frac{\partial^2 \varphi}{\partial z^2} + \rho \omega^2 \varphi \right] \\
- \nabla_2^2 \left[c_{44} \nabla_2^2 \eta + (c_{33} - c_{13} - c_{44}) \frac{\partial^2 \eta}{\partial z^2} + \rho \omega^2 \eta \right] &= 0,
\end{aligned} \tag{4.6}$$

and

$$c_{66} \nabla_2^2 \chi + c_{44} \frac{\partial^2 \chi}{\partial z^2} + \rho \omega^2 \chi = 0. \tag{4.7}$$

Equations (4.5)-(4.6) are coupled equations for φ and η (i.e., the P- and SV-wave potentials respectively). The exact dispersion relation for P and SV waves in VTI media (derived by Tsvankin, 1996) is

$$\frac{v^2(\theta)}{v_{p0}^2} = 1 + \varepsilon \sin^2 \theta - \frac{f}{2} \pm \frac{f}{2} \left[1 + \frac{2\varepsilon \sin^2 \theta}{f} \right] \left[1 - \frac{2(\varepsilon - \delta) \sin^2 2\theta}{f(1 + \frac{2\varepsilon \sin^2 \theta}{f})^2} \right]^{1/2}, \tag{4.8}$$

where θ is the phase angle measured from the symmetry axis, $v(\theta)$ is the phase velocity of the coupled wave modes; ε , δ , and γ are the Thomsen parameters Thomsen (1986),

$$\begin{aligned}\varepsilon &= \frac{c_{11} - c_{33}}{2c_{33}}, \\ \gamma &= \frac{c_{66} - c_{44}}{2c_{44}}, \\ \delta &= \frac{(c_{13} + c_{44})^2 - (c_{33} - c_{44})^2}{2c_{33}(c_{33} - c_{44})},\end{aligned}\tag{4.9}$$

and $f = 1 - \left(\frac{v_{S0}}{v_{P0}}\right)^2$, where v_{P0} and v_{S0} are the P- and SV-wave velocities $v_{P0} = \sqrt{c_{33}/\rho}$ and $v_{S0} = \sqrt{c_{44}/\rho}$ along the VTI symmetry axis. The plus and minus signs correspond to the P and SV-waves, respectively. Equation (4.7) is the SH-wave equation in a VTI medium.

Let the vector $\mathbf{v} = [v_x, v_y, v_z]^T$ be the particle velocity and $\mathbf{X} = [\chi_x, \chi_y, \chi_z]^T$ be the displacement in accordance with the split-field technique (Chew and Weedon, 1994; Collino and Tsogka, 2001).

The SH-wave equation can be described with the following first-order system

$$\begin{aligned}\frac{\partial v_x}{\partial t} &= -\frac{c_{66}}{\rho} \left(\frac{\partial \chi_x}{\partial x} + \frac{\partial \chi_y}{\partial x} + \frac{\partial \chi_z}{\partial x} \right) \\ \frac{\partial v_y}{\partial t} &= -\frac{c_{66}}{\rho} \left(\frac{\partial \chi_x}{\partial y} + \frac{\partial \chi_y}{\partial y} + \frac{\partial \chi_z}{\partial y} \right) \\ \frac{\partial v_z}{\partial t} &= -\frac{c_{44}}{\rho} \left(\frac{\partial \chi_x}{\partial z} + \frac{\partial \chi_y}{\partial z} + \frac{\partial \chi_z}{\partial z} \right) \\ \frac{\partial \chi_x}{\partial t} &= -\frac{\partial v_x}{\partial x} \\ \frac{\partial \chi_y}{\partial t} &= -\frac{\partial v_y}{\partial y} \\ \frac{\partial \chi_z}{\partial t} &= -\frac{\partial v_z}{\partial z}\end{aligned}\tag{4.10}$$

The system in equation (4.10) can be expressed in matrix form:

$$\begin{aligned}\frac{\partial \mathbf{v}}{\partial t} &= \mathbf{A}_1 \mathbf{X} \\ \frac{\partial \mathbf{X}}{\partial t} &= \mathbf{B}_1 \mathbf{v}\end{aligned},\tag{4.11}$$

where $\mathbf{A}_1$ and $\mathbf{B}_1$ are

$$\mathbf{A}_1 = \begin{bmatrix} -\frac{c_{66}}{\rho} \frac{\partial}{\partial x} & -\frac{c_{66}}{\rho} \frac{\partial}{\partial x} & -\frac{c_{66}}{\rho} \frac{\partial}{\partial x} \\ -\frac{c_{66}}{\rho} \frac{\partial}{\partial y} & -\frac{c_{66}}{\rho} \frac{\partial}{\partial y} & -\frac{c_{66}}{\rho} \frac{\partial}{\partial y} \\ -\frac{c_{44}}{\rho} \frac{\partial}{\partial z} & -\frac{c_{44}}{\rho} \frac{\partial}{\partial z} & -\frac{c_{44}}{\rho} \frac{\partial}{\partial z} \end{bmatrix},\tag{4.12}$$

and

$$\mathbf{B}_1 = \begin{bmatrix} -\frac{\partial}{\partial x} & 0 & 0 \\ 0 & -\frac{\partial}{\partial y} & 0 \\ 0 & 0 & -\frac{\partial}{\partial z} \end{bmatrix}. \quad (4.13)$$

4.3 Staggered-grid Fourier pseudo-spectral derivatives

Although Luo (1994); Luo and Yedlin (1997) proposed using a pseudospectral time marching method is used in second order equations, in this thesis, we will introduce another method. The first-order Fourier derivative of a function $u(x)$ can be discretized over a finite grid of N points by (Witte and Richards, 1990)

$$\mathcal{D}_x u(x_i) = \text{DFT}^{-1} \left[-jk_x \text{DFT}[u(x_i)] \right], \quad (4.14)$$

where $j = \sqrt{-1}$, and $x_i = i\Delta x$, and $i = 1, \dots, N-1$, with Δx being the sampling interval. The quantity $k_x = 2n\pi/(N\Delta x)$ is the discrete wavenumber in the x direction. For even values of N , n should be chosen as $-N/2 \leq n \leq N/2$, where $n = -N/2$ corresponds to the Nyquist wavenumber. For odd values of N , we choose $-N/2 < n < N/2$. In this case the Nyquist wavenumber does not correspond to one of the grid points. The operators DFT and DFT^{-1} are the forward and inverse discrete Fourier transforms, respectively. In a homogeneous medium, the conventional Fourier derivative in equation (4.14) is adequate, but Özdenvar and McMechan (1996) point out that instances of Gibbs' phenomenon emerge when the waveform propagates through heterogeneous regions. Compared with the conventional Fourier transforms, staggered-grid Fourier pseudo-spectral differentiation reduces Gibbs' errors caused by phase jumps at the Nyquist wavenumber. The staggered-grid version of the first-order derivative of $u(x)$ can be expressed, in terms of the half-grid-spacing phase-shift of the standard Fourier derivative, as

$$\mathcal{D}_x^\pm u(x_{i \pm \frac{1}{2}}) = \text{DFT}^{-1} \left[-jk_x \exp\left(\frac{\mp jk_x \Delta x}{2}\right) \text{DFT}(u(x_i)) \right], \quad (4.15)$$

in which $\pm$ implies forward versus backward differentiations. Similarly, high-order derivatives at mid-points $x_{i\pm\frac{1}{2}}$ are given by

$$\mathcal{D}_{x^m}^\pm u(x_{i\pm\frac{1}{2}}) = \text{DFT}^{-1} \left[(-jk_x)^m \exp\left(\frac{\mp jk_x \Delta x}{2}\right) \text{DFT}(u(x_i)) \right], \quad (4.16)$$

for odd m . When m is even, the m^{th} derivative at the point x_i is

$$\mathcal{D}_{x^m} u(x_i) = \text{DFT}^{-1} [(-jk_x)^m \text{DFT}(u(x_i))]. \quad (4.17)$$

Equation (4.11) can be approximated by

$$\begin{aligned} \mathbf{v}(t + \tfrac{1}{2}\Delta t) - \mathbf{v}(t - \tfrac{1}{2}\Delta t) &\approx (\Delta t \mathbf{A}_1 + \tfrac{1}{24} \Delta t^3 \mathbf{A}_1 \mathbf{B}_1 \mathbf{A}_1) \mathbf{X}(t) \\ \mathbf{X}(t + \Delta t) - \mathbf{X}(t) &\approx (\Delta t \mathbf{B}_1 + \tfrac{1}{24} \Delta t^3 \mathbf{B}_1 \mathbf{A}_1 \mathbf{B}_1) \mathbf{v}(t + \tfrac{1}{2}\Delta t) \end{aligned} \quad (4.18)$$

The spatial derivatives in the above equations can then be approximated by the Fourier derivatives as

$$\begin{aligned} v_x^{t+\frac{1}{2}\Delta t} &= v_x^{t-\frac{1}{2}\Delta t} - \left[\Delta t \frac{c_{66}}{\rho} \mathcal{D}_x^+ + \frac{1}{24} \Delta t^3 \frac{c_{66}^2}{\rho^2} \left[\mathcal{D}_x^+ + \mathcal{D}_x^+ \mathcal{D}_y^2 + \mathcal{D}_x^+ \mathcal{D}_z^2 \right] \right] (\chi_x + \chi_y + \chi_z) \\ v_y^{t+\frac{1}{2}\Delta t} &= v_y^{t-\frac{1}{2}\Delta t} - \left[\Delta t \frac{c_{66}}{\rho} \mathcal{D}_y^+ + \frac{1}{24} \Delta t^3 \frac{c_{66}^2}{\rho^2} \left[\mathcal{D}_y^+ + \mathcal{D}_y^+ \mathcal{D}_x^2 + \mathcal{D}_y^+ \mathcal{D}_z^2 \right] \right] (\chi_x + \chi_y + \chi_z) \\ v_z^{t+\frac{1}{2}\Delta t} &= v_z^{t-\frac{1}{2}\Delta t} - \left[\Delta t \frac{c_{44}}{\rho} \mathcal{D}_z^+ + \frac{1}{24} \Delta t^3 \frac{c_{44}^2}{\rho^2} \left[\mathcal{D}_z^+ + \mathcal{D}_z^+ \mathcal{D}_x^2 + \mathcal{D}_z^+ \mathcal{D}_y^2 \right] \right] (\chi_x + \chi_y + \chi_z) \\ \chi_x^{t+\Delta t} &= \chi_x^t - \left[\left(\Delta t \mathcal{D}_x^- + \frac{c_{66}}{\rho} \frac{1}{24} \Delta t^3 \mathcal{D}_x^- \right) + \frac{c_{66}}{\rho} \frac{1}{24} \Delta t^3 \mathcal{D}_x^2 \mathcal{D}_y^- + \frac{c_{66}}{\rho} \frac{1}{24} \Delta t^3 \mathcal{D}_x^2 \mathcal{D}_z^- \right] v_x^{t+\frac{1}{2}\Delta t}, \\ \chi_y^{t+\Delta t} &= \chi_y^t - \left[\left(\Delta t \mathcal{D}_y^- + \frac{c_{66}}{\rho} \frac{1}{24} \Delta t^3 \mathcal{D}_y^- \right) + \frac{c_{66}}{\rho} \frac{1}{24} \Delta t^3 \mathcal{D}_y^2 \mathcal{D}_x^- + \frac{c_{66}}{\rho} \frac{1}{24} \Delta t^3 \mathcal{D}_y^2 \mathcal{D}_z^- \right] v_y^{t+\frac{1}{2}\Delta t} \\ \chi_z^{t+\Delta t} &= \chi_z^t - \left[\left(\Delta t \mathcal{D}_z^- + \frac{c_{44}}{\rho} \frac{1}{24} \Delta t^3 \mathcal{D}_z^- \right) + \frac{c_{44}}{\rho} \frac{1}{24} \Delta t^3 \mathcal{D}_z^2 \mathcal{D}_x^- + \frac{c_{44}}{\rho} \frac{1}{24} \Delta t^3 \mathcal{D}_z^2 \mathcal{D}_y^- \right] v_z^{t+\frac{1}{2}\Delta t} \end{aligned} \quad (4.19)$$

where χ_x , χ_y and χ_z are the components of scalar displacement potential χ , and $\chi = \chi_x + \chi_y + \chi_z$.

Stability and dispersion relations associated with the above equations are arrived at by summing the split displacement potentials:

$$\begin{aligned} \chi(t + \Delta t) &= \chi(t) - \left[\left(\Delta t \frac{\partial}{\partial x} + \frac{1}{24} \Delta t^3 \frac{c_{66}}{\rho} \left(\frac{\partial^3}{\partial x^3} + \frac{\partial^2}{\partial x^2} \frac{\partial}{\partial y} + \frac{\partial^2}{\partial x^2} \frac{\partial}{\partial z} \right) \right) v_x \left(t + \frac{1}{2} \Delta t \right) \right] \\ &\quad - \left[\left(\Delta t \frac{\partial}{\partial y} + \frac{1}{24} \Delta t^3 \frac{c_{66}}{\rho} \left(\frac{\partial^3}{\partial y^3} + \frac{\partial^2}{\partial y^2} \frac{\partial}{\partial x} + \frac{\partial^2}{\partial y^2} \frac{\partial}{\partial z} \right) \right) v_y \left(t + \frac{1}{2} \Delta t \right) \right], \\ &\quad - \left[\left(\Delta t \frac{\partial}{\partial z} + \frac{1}{24} \Delta t^3 \frac{c_{44}}{\rho} \left(\frac{\partial^3}{\partial z^3} + \frac{\partial^2}{\partial z^2} \frac{\partial}{\partial x} + \frac{\partial^2}{\partial z^2} \frac{\partial}{\partial y} \right) \right) v_z \left(t + \frac{1}{2} \Delta t \right) \right] \end{aligned} \quad (4.20)$$

with $v_x(t + \frac{1}{2}\Delta t)$, $v_y(t + \frac{1}{2}\Delta t)$, and $v_z(t + \frac{1}{2}\Delta t)$ being given by

$$\begin{aligned} v_x(t + \frac{1}{2}\Delta t) &= v_x(t - \frac{1}{2}\Delta t) - \left[\Delta t \frac{c_{66}}{\rho} \frac{\partial}{\partial x} + \frac{1}{24} \Delta t^3 \frac{c_{66}^2}{\rho^2} \left[\frac{\partial^3}{\partial x^3} + \frac{\partial}{\partial x} \frac{\partial^2}{\partial y^2} + \frac{\partial}{\partial x} \frac{\partial^2}{\partial z^2} \right] \right] \chi \\ v_y(t + \frac{1}{2}\Delta t) &= v_y(t - \frac{1}{2}\Delta t) - \left[\Delta t \frac{c_{66}}{\rho} \frac{\partial}{\partial y} + \frac{1}{24} \Delta t^3 \frac{c_{66}^2}{\rho^2} \left[\frac{\partial^3}{\partial y^3} + \frac{\partial}{\partial y} \frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y} \frac{\partial^2}{\partial z^2} \right] \right] \chi \\ v_z(t + \frac{1}{2}\Delta t) &= v_z(t - \frac{1}{2}\Delta t) - \left[\Delta t \frac{c_{44}}{\rho} \frac{\partial}{\partial z} + \frac{1}{24} \Delta t^3 \frac{c_{44}^2}{\rho^2} \left[\frac{\partial^3}{\partial z^3} + \frac{\partial}{\partial z} \frac{\partial^2}{\partial y^2} + \frac{\partial}{\partial z} \frac{\partial^2}{\partial x^2} \right] \right] \chi \end{aligned} \quad (4.21)$$

By eliminating the velocity components in equation (4.20), the scalar displacement potential equation can be expressed with temporal fourth-order accuracy as

$$\chi(t + \Delta t) - 2\chi(t) + \chi(t - \Delta t) = (r_1 s_1 + r_2 s_2 + r_3 s_3) \chi(t), \quad (4.22)$$

where, r_1, r_2, r_3 and s_1, s_2, s_3 are expressed as

$$\begin{aligned} r_1 &= \Delta t \frac{\partial}{\partial x} + \frac{1}{24} \Delta t^3 \frac{c_{66}}{\rho} \left(\frac{\partial^3}{\partial x^3} + \frac{\partial^2}{\partial x^2} \frac{\partial}{\partial y} + \frac{\partial^2}{\partial x^2} \frac{\partial}{\partial z} \right) \\ r_2 &= \Delta t \frac{\partial}{\partial y} + \frac{1}{24} \Delta t^3 \frac{c_{66}}{\rho} \left(\frac{\partial^3}{\partial y^3} + \frac{\partial^2}{\partial y^2} \frac{\partial}{\partial x} + \frac{\partial^2}{\partial y^2} \frac{\partial}{\partial z} \right) \\ r_3 &= \Delta t \frac{\partial}{\partial z} + \frac{1}{24} \Delta t^3 \frac{c_{44}}{\rho} \left(\frac{\partial^3}{\partial z^3} + \frac{\partial^2}{\partial z^2} \frac{\partial}{\partial x} + \frac{\partial^2}{\partial z^2} \frac{\partial}{\partial y} \right) \\ s_1 &= \Delta t \frac{c_{66}}{\rho} \frac{\partial}{\partial x} + \frac{1}{24} \Delta t^3 \frac{c_{66}^2}{\rho^2} \left[\frac{\partial^3}{\partial x^3} + \frac{\partial}{\partial x} \frac{\partial^2}{\partial y^2} + \frac{\partial}{\partial x} \frac{\partial^2}{\partial z^2} \right] \\ s_2 &= \Delta t \frac{c_{66}}{\rho} \frac{\partial}{\partial y} + \frac{1}{24} \Delta t^3 \frac{c_{66}^2}{\rho^2} \left[\frac{\partial^3}{\partial y^3} + \frac{\partial}{\partial y} \frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y} \frac{\partial^2}{\partial z^2} \right] \\ s_3 &= \Delta t \frac{c_{44}}{\rho} \frac{\partial}{\partial z} + \frac{1}{24} \Delta t^3 \frac{c_{44}^2}{\rho^2} \left[\frac{\partial^3}{\partial z^3} + \frac{\partial}{\partial z} \frac{\partial^2}{\partial y^2} + \frac{\partial}{\partial z} \frac{\partial^2}{\partial x^2} \right] \end{aligned} \quad (4.23)$$

The dispersion relation arising from equation (4.22) is

$$-4 \sin^2 \left(\frac{\omega \Delta t}{2} \right) = R_1 S_1 + R_2 S_2 + R_3 S_3, \quad (4.24)$$

with r_1, r_2, r_3 and s_1, s_2, s_3 expressed in k -space as

$$\begin{aligned} R_1 &= ik_x \Delta t \left(1 - \frac{1}{24} \Delta t^2 k_x (kx + ky + kz) \right) \\ R_2 &= ik_y \Delta t \left(1 - \frac{1}{24} \Delta t^2 k_y (kx + ky + kz) \right) \\ R_3 &= ik_z \Delta t \left(1 - \frac{1}{24} \Delta t^2 k_z (kx + ky + kz) \right) \\ S_1 &= i \Delta t \frac{c_{66}}{\rho} k_x \left(1 - \frac{1}{24} \Delta t^2 \frac{c_{66}}{\rho} k^2 \right) \\ S_2 &= i \Delta t \frac{c_{66}}{\rho} k_y \left(1 - \frac{1}{24} \Delta t^2 \frac{c_{66}}{\rho} k^2 \right) \\ S_3 &= i \Delta t \frac{c_{44}}{\rho} k_z \left(1 - \frac{1}{24} \Delta t^2 \frac{c_{44}}{\rho} k^2 \right) \end{aligned} \quad (4.25)$$

where $k = \sqrt{k_x^2 + k_y^2 + k_z^2}$. Equation (4.24) allows us to write

$$0 \leq \sin^2 \left(\frac{\omega \Delta t}{2} \right) = \frac{1}{4} (R_1 S_1 + R_2 S_2 + R_3 S_3) \leq 1. \quad (4.26)$$

For a model with a uniform grid spacing in each direction ($\Delta x = \Delta y = \Delta z$), the Nyquist wave numbers for each k -space coordinate satisfy $k_x = k_y = k_z = \frac{\pi}{\Delta x}$. We define the horizontal S-wave velocity v_{sh} such that $v_{sh} = \sqrt{c_{66}/\rho}$; because the S-wave velocity along the symmetry axis is $v_{s0} = \sqrt{c_{44}/\rho}$ we have the relation

$$v_{sh}^2 = v_{s0}^2 (1 + 2\gamma). \quad (4.27)$$

According to Wang (2001), $\gamma < 0.2$ in most weak anisotropic media, and in sedimentary rocks the highest γ appears in shales ($\gamma_{max}=0.553$). Assuming that $\gamma \leq 0.5$, the stability condition is expressed by the inequality

$$0 \leq \frac{1}{4} \frac{\pi^2}{\Delta x^2} \Delta t^2 v_{s0}^2 \left[2 - \frac{3}{8} \frac{\pi^2}{\Delta x^2} \Delta t^2 v_{s0}^2 + \frac{5}{192} \frac{\pi^4}{\Delta x^4} \Delta t^4 v_{s0}^2 \right], \quad (4.28)$$

or, to second order in Δt ,

$$\frac{v_{s0} \Delta t}{\Delta x} \leq \frac{4\sqrt{3}}{3\pi}. \quad (4.29)$$

4.4 H-PML boundary conditions

When a PSM is applied to wavefield simulation, computational boundaries tend to produce wrap-around artifacts which must be suppressed. The perfectly matched layer (PML) approach to boundary absorption, introduced by Berenger (1994), has been proven to be very efficient. Taking the x direction as an example, a damping profile $d_x(x)$ is created, with $d_x = 0$ in the physical domain and $d_x > 0$ in the PML layer. The new operator $\nabla_{\tilde{x}} = [\frac{\partial}{\partial \tilde{x}}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}]$ is introduced, where $\frac{\partial}{\partial \tilde{x}} = \frac{1}{s_x} \frac{\partial}{\partial x}$, $s_x = 1 + \frac{d_x}{i\omega}$.

The convolutional PML (or C-PML) method (Kuzuoglu and Mittra, 1996) and the complex frequency shifted-PML (CFS-PML) method (Bérenger, 2002) introduce frequency-dependent terms,

eliminating the requirement that the velocity-stress equation be split into separate terms. The C-PML scheme involves adding not only the damping profile, but two other real variables, such that:

$$s_x = \kappa_x + \frac{d_x}{\alpha_x + i\omega}. \quad (4.30)$$

When $\kappa_x = 1$ and $\alpha_x = 0$, the C-PML reduces to the classic PML form.

The multiaxial perfectly matched layer (M-PML) method has been found to be stable even for media exhibiting very large degrees of anisotropy (Meza-Fajardo and Papageorgiou, 2008). In an M-PML application, in contrast to equation (4.30), the s_x term is

$$s_x = \kappa_x + \frac{d_x + m_{x/y}d_y + m_{x/z}d_z}{i\omega}, \quad (4.31)$$

where $m_{x/y}d_y$ and $m_{x/z}$ are weighting factors.

To maximize both accuracy and stability we construct a hybrid PML (H-PML) method, that combines the advantages of both the C-PML and the M-PML through the optimization of the damping profile (Li et al., 2016). Because the C-PML and M-PML are independent of one another, the two can be straightforwardly hybridized by introducing

$$s_x = \kappa_x + \frac{d_x + m_{x/y}d_y + m_{x/z}d_z}{\alpha_x + i\omega}, \quad (4.32)$$

so that a new differential operator in the x direction emerges:

$$\partial_{\bar{x}} = \bar{s}_x(t) * \partial_x, \quad (4.33)$$

where $*$ denotes convolution, and $\bar{s}_x(t)$ is the inverse Fourier transform of $1/s_x$ (Roden and Gedney, 2000; Komatitsch and Martin, 2007):

$$\bar{s}_x(t) = \frac{\delta t}{\kappa_x} - \frac{d_x}{\kappa_x^2} e^{-(d_x/\kappa_x + \alpha_x)t} H(t) = \frac{\delta t}{\kappa_x} + \zeta_x(t), \quad (4.34)$$

and where δt and $H(t)$ are Dirac delta and Heaviside distributions, respectively. The operator in equation (4.33) now becomes

$$\partial_{\bar{x}} = \frac{1}{\kappa_x} \partial_x + \zeta_x(t) * \partial_x. \quad (4.35)$$

Komatitsch and Martin (2007) replace equation (4.35) with

$$\partial_{\bar{x}} = \frac{1}{\kappa_x} \partial_x + \psi_x, \quad (4.36)$$

where ψ_x is a memory variable updated at each time step n :

$$\psi_x^n = b_x \psi_x^{n-1} + c_x (\partial_x)^{n-1/2}, \quad (4.37)$$

in which

$$\begin{aligned} b_x &= \exp \left[- \left(\frac{d_x + m_{x/y} d_y + m_{x/z} d_z}{\kappa_x + \alpha_x} \right) \Delta t \right] \\ c_x &= \left[\frac{d_x + m_{x/y} d_y + m_{x/z} d_z}{\kappa_x (d_x + m_{x/y} d_y + m_{x/z} d_z) + \kappa_x \alpha_x} \right] (b_x - 1). \end{aligned} \quad (4.38)$$

The coefficient matrices in equations (4.12) and (4.13) are therefore expressed as

$$\mathbf{A}_1 = - \begin{bmatrix} \frac{c_{66}}{\rho} & \frac{c_{66}}{\rho} & \frac{c_{66}}{\rho} \\ \frac{c_{66}}{\rho} & \frac{c_{66}}{\rho} & \frac{c_{66}}{\rho} \\ \frac{c_{44}}{\rho} & \frac{c_{44}}{\rho} & \frac{c_{44}}{\rho} \end{bmatrix} \begin{bmatrix} \frac{1}{\kappa_x} \partial_x + \psi_x \\ \frac{1}{\kappa_y} \partial_y + \psi_y \\ \frac{1}{\kappa_z} \partial_z + \psi_z \end{bmatrix}, \quad (4.39)$$

and

$$\mathbf{B}_1 = \begin{bmatrix} - \left(\frac{1}{\kappa_x} \partial_x + \psi_x \right) & 0 & 0 \\ 0 & - \left(\frac{1}{\kappa_y} \partial_y + \psi_y \right) & 0 \\ 0 & 0 & - \left(\frac{1}{\kappa_z} \partial_z + \psi_z \right) \end{bmatrix}. \quad (4.40)$$

If the elastic parameters and the density are spatially invariant, they can be directly incorporated in each Fourier derivative without introducing artifacts. In heterogeneous media (e.g., in layered media), they must be incorporated in the space domain in order to avoid Gibbs' artifacts.

4.5 Numerical examples

In this section, we present several numerical examples whose purpose is to validate and verify important features of the combination of H-PML and PSM used for SH wave simulation developed in the previous sections. We focus on stability, Gibbs' artifacts, and boundary reflections.

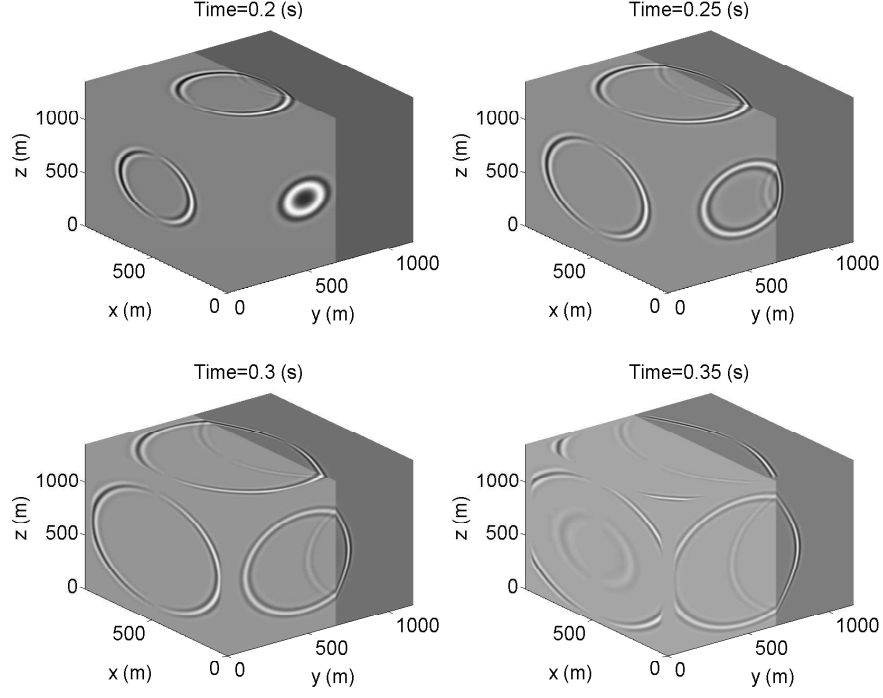


Figure 4.1: The 3D layered anisotropic model containing displacement wavefield snapshots in cross-sectional planes; in each panel propagation time progresses, starting at 0.2s and ending at 0.35s. The left-hand layer of the model ($y < 600\text{m}$) is a VTI medium. The right-hand layer ($y > 600\text{m}$) of this model is purely isotropic.

4.5.1 Two-layer model

We make use of a two-layer medium to carry out benchmarking. The computational grid is $291 \times 191 \times 241$ with grid spacing $\Delta x = \Delta y = \Delta z = 5\text{m}$, including one H-PML layer of 15 grid points beyond each computational boundary. Waves are initiated with a dipole source comprised of two Ricker wavelets with central frequency $f_0 = 30\text{Hz}$. Based on the stability condition in equation (4.29), we select the time step $\Delta t = 1 \times 10^{-3}\text{s}$. The structure of the model is illustrated in the four panels of Figure 4.1. The left-hand layer of the model ($y < 600\text{m}$) is a VTI medium with elastic

constants

$$C = \begin{bmatrix} 26.37 & 10.57 & 9.57 & 0 & 0 & 0 \\ 10.57 & 26.37 & 9.57 & 0 & 0 & 0 \\ 9.57 & 9.57 & 36.73 & 0 & 0 & 0 \\ 0 & 0 & 0 & 12.45 & 0 & 0 \\ 0 & 0 & 0 & 0 & 12.45 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8.08 \end{bmatrix} \times 10^9 \text{N/m}^2. \quad (4.41)$$

The right-hand layer ($y > 600\text{m}$) of this model is purely isotropic, with P- and S- wave velocities 2300m/s and 1000m/s respectively. The densities of the layers are 2500kg/m^3 (left) and 2000kg/m^3 (right). In the four panels of Figure 4.1, wavefield snapshots ($y = 400\text{m}$ for x - z plane snapshots; $x = 250\text{m}$ for y - z plane snapshots; $z = 700\text{m}$ for x - y plane snapshots) are plotted for propagation times $t = 0.2\text{s}$, 0.25s , 0.3s and 0.35s . The dipole source is oriented in the x -direction, and the SH wave is polarized in the horizontal plane.

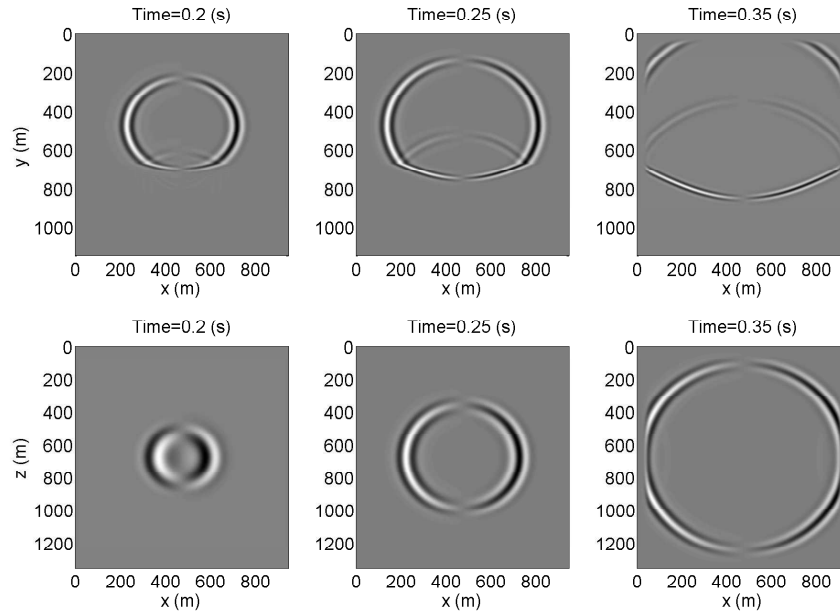


Figure 4.2: SH wavefield snapshots computed using the PSTD staggered-grid method with H-PML. Left to right: snapshot times 0.2s to 0.35s . Top row: x - y plane, lateral component; bottom row: x - z plane, vertical component.

In Figure 4.2, snapshots for SH propagation in x - y (lateral) and x - z (vertical) sections are plot-

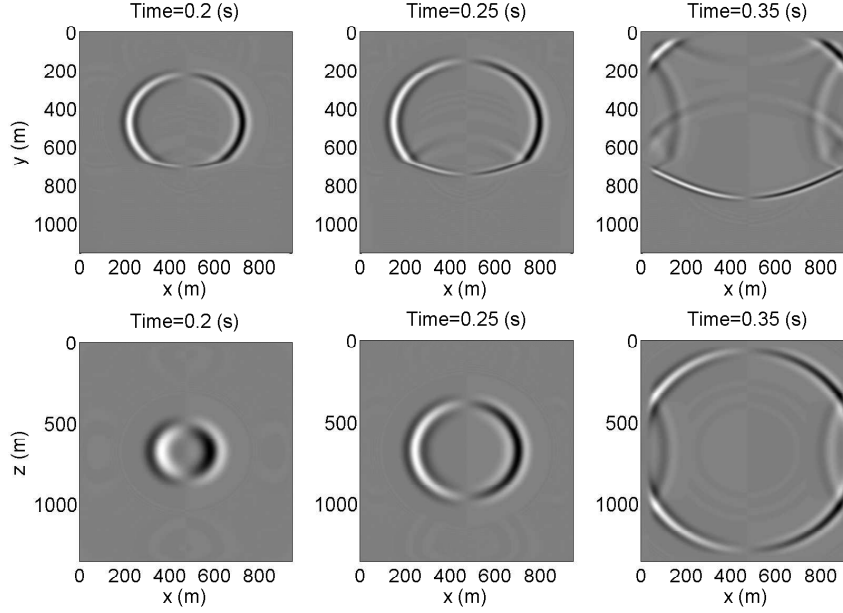


Figure 4.3: SH wavefield snapshots computed using the second order PSTD method with a sponge absorbing boundary. Left to right: snapshot times 0.2s to 0.35s. Top row: x - y plane, lateral component; bottom row: x - z plane, vertical component.

ted. For comparison, in Figure 4.3, snapshots for SH propagation in identical sections using a second-order PSTD method with a sponge absorbing boundary are illustrated. At $t = 0.2$ s, all snapshots obtained by the method we have introduced in this thesis contain waveforms without noticeable wrap-around at the computational boundaries; in contrast, those associated with the second order PSTD method at the same time, illustrated in Figure 4.3, exhibit significant wrap-around. At $t = 0.25$ s, the second order PSTD wavefield exhibits Gibbs' artifacts at and around the reflection model. At $t = 0.35$ s, the snapshots obtained by second order PSTD method may also be observed to exhibit boundary reflections. In contrast, neither wrap-around nor Gibbs' artifacts appear as the wavefield, computed using the new approach, evolves. We next illustrate individual traces as synthesized by the new staggered-grid PSTD and conventional PSTD methods. In Figure 4.4, the positions of four sensors within the velocity model are illustrated; the blue surface is the interface, and the blue-red circle pair illustrates the position and orientation of the dipole source, which radiates in the x -direction. In Figure 4.5, waveforms recorded at the four receivers are plotted for both the conventional and newly-proposed PSTD method (receivers A-D). Receiver pairs

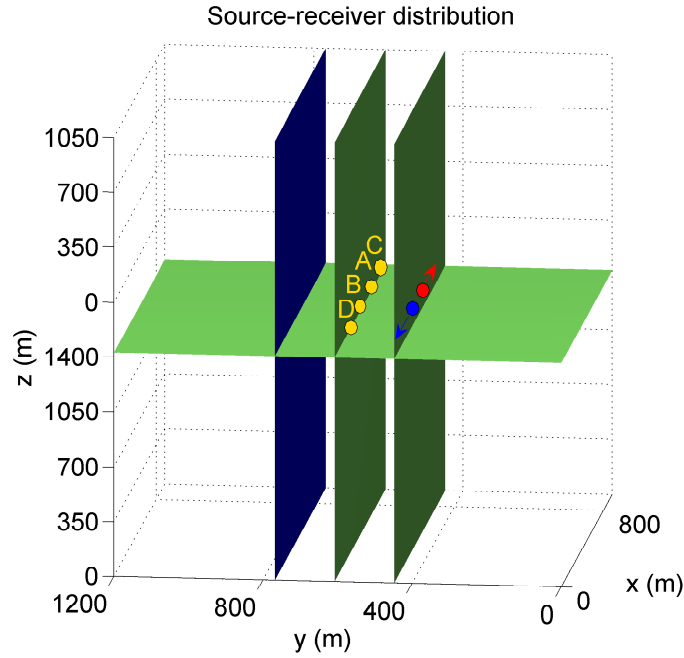


Figure 4.4: The 2-layered model with source-receiver positioning. The blue surface is the physical interface. A, B, C and D denote the receiver locations.

(A,B) and (C,D) are symmetrically located on either side of the dipole, so events appear in pairs with the same arrival times but opposing amplitudes. The large event appearing before 0.1s in each trace is the direct arrival. The two methods compare quite well for these arrivals, apart from some differences in the side-lobes, with oscillations dying down rapidly in both cases. The next event, occurring at about 0.18s in each trace, is the arrival associated with the reflection from the interface. The amplitudes of these reflected arrivals calculated by the conventional PSTD method are smaller than those obtained by the staggered-grid PSTD. Also, Gibbs' phenomena appear and persist for significant intervals (as indicated with dashed rectangles), i.e., non-negligible errors appear even far from the reflected event. The conventional PSTD method also exhibits significant boundary reflection artifacts (dashed ovals in Figure 4.5).

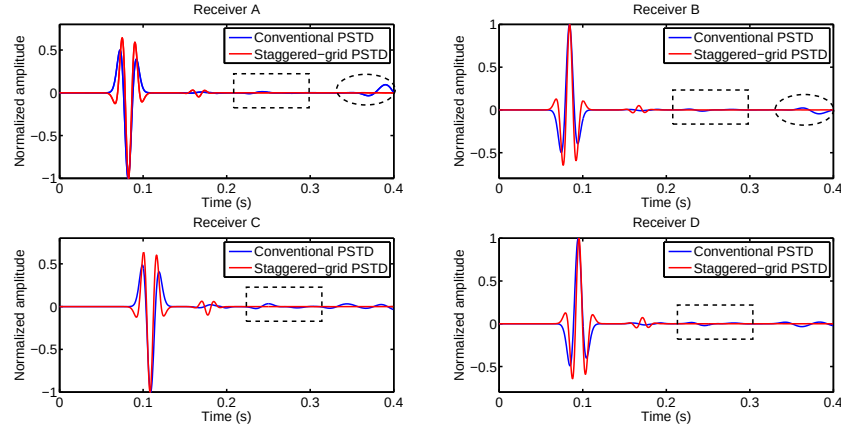


Figure 4.5: Trace comparisons between the conventional (blue) and proposed (red) PSTD methods, recorded at receiver positions A-D. Artifacts are indicated with dashed boxes/ovals.

In Figure 4.6, the recorded waveforms sensed at receiver positions A-D, computed by the staggered-grid FD method and the new proposed PSTD method, are compared. From equation (4.10) and the first order velocity-stress wave equation, $\partial v_x / \partial t$ and $\partial \tau_{xx} / \partial t$ are equivalent. Therefore, comparison between the normal stress component τ_{xx} obtained by the staggered grid FD method and v_x as calculated by our new PSTD is appropriate. The large arrival near 0.1s is again the direct arrival. The two methods match well near their peaks, but the FD method output produces sustained oscillations out to almost 0.2s, the time at which the reflected-wave signals arrive; notice that the reflection peak as generated by the FD approach is difficult to identify in the receiver A and B traces. In contrast, oscillations die down abruptly in the traces computed with the new PSTD method, and the reflections can be identified near 0.18s.

By equation (4.19), $\partial \chi_i / \partial t$ is equivalent to v_i , so for the waveforms received at C and D we compare the quantities v_x as obtained through the staggered grid FD method, and $\partial \chi_x / \partial t$ as calculated by the new PSTD method. The two methods match well in their computation of the direct arrivals at receiver D, but, at receiver C, the staggered grid FD method generates strong side lobes on the direct arrival; oscillations decay rapidly in the output of the staggered-grid PSTD method. Reflection signals for both methods overlap at about 0.18ms.

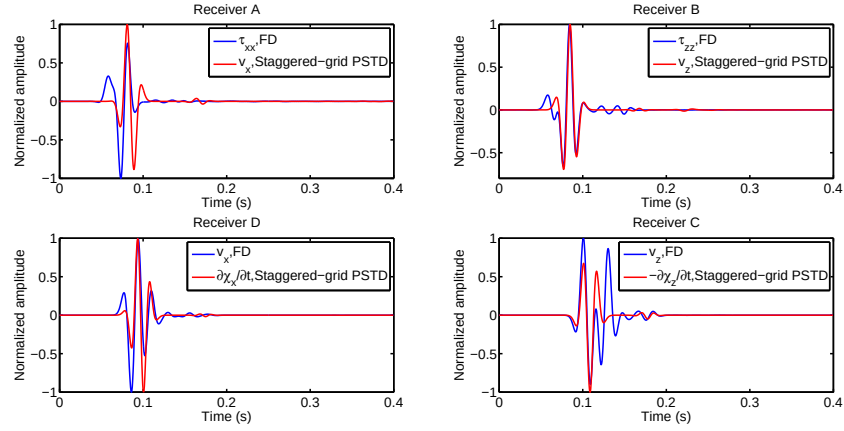


Figure 4.6: Trace comparisons between the staggered-grid FD and proposed PSTD methods, recorded at receiver positions A-D.

Displacement components, as computed by any finite difference methodology, including the new method discussed here, are important for certain applications, for instance simulation in full waveform inversion. The quantity $\partial\chi_i/\partial t$ is the velocity component v_i in the i th direction, which can be obtained during the updating of displacement components (as indicated in equation (4.19)). The velocity components are not available through the conventional PSTD method, and the displacement components for each direction can not be directly updated through a staggered-grid FD method. As is discussed in a companion paper (Li et al., 2017), the source implementing with different components in both forward simulation and time reversing procedure plays an important role in the gradient calculation, which in turn, influences the FWI results. Based on our results, different sources are sensitive to different formation parameters.

4.5.2 3D thrust fault anisotropic model

In this section, a heterogeneous anisotropic model with complicated thrust faults is used to examine the stability of the new scheme. The model is part of a thrust fault system. We extend a 2D thrust fault model (with variation in the x - z plane) in the y direction to create the 3D model.

The model is $1000m \times 800m \times 1400m$ with a grid of $200 \times 160 \times 280$. The first layer of the

model is isotropic, with P-wave and S-wave velocities at 2400m/s and 1280m/s respectively. The vertical P-wave velocity of the model ranges from 2400 m/s in first layer to 6000 m/s in the bottom layer. The source is an x -oriented dipole which emits a Ricker wavelet with dominant frequency of 30Hz. The space and time intervals used in this model are 5m and 1ms respectively. The maximum space and time intervals used for the second order in time/fourth-order in space finite difference method are 4m and 0.3ms respectively.

In Figure 4.7 the wavefield is plotted at various times during propagation. The dipole source is composed of two monopoles which are positioned at (120 m, 480 m, 675 m) and (130 m, 480 m, 675 m). In Figures 4.8-4.9, the SH wavefield snapshots in x - z plane for z - and x -components are plotted respectively. As time evolves, the SH wave passes from the isotropic regions of the model into the anisotropic layered regions without generating wrap-around errors. No evidence of Gibbs' phenomena appears.

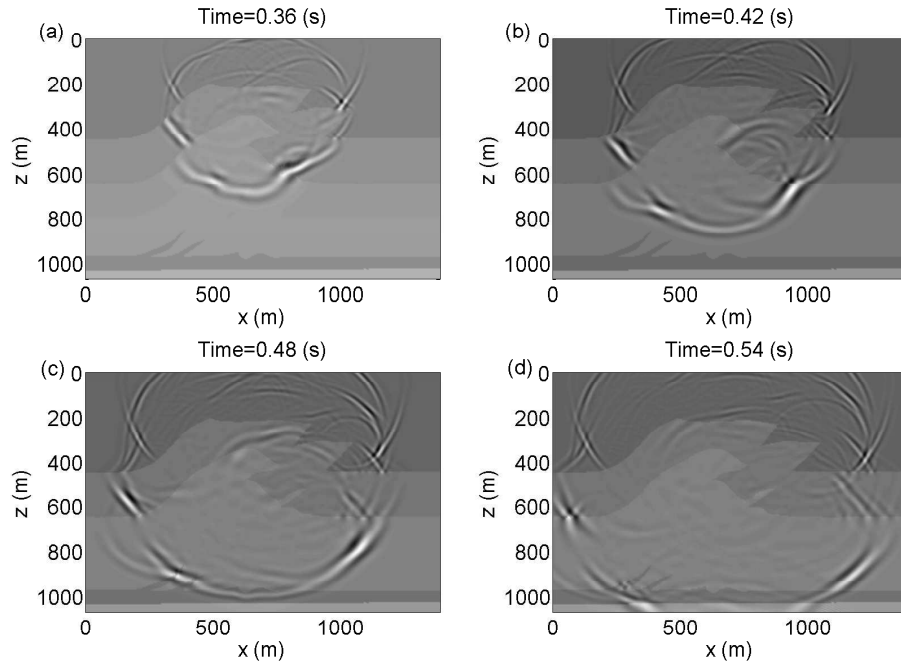


Figure 4.8: Snapshots for SH propagation in x - z plane (z -component).

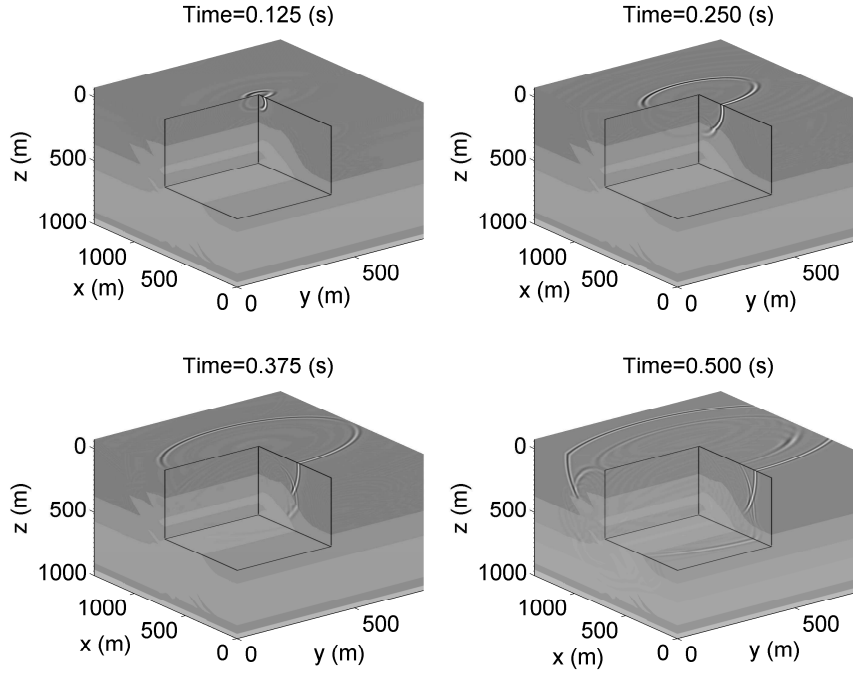


Figure 4.7: Snapshots for SH wave propagation in the thrust fault model with $\Delta t = 10\mu s$. The dipole source is composed of two monopoles which are positioned at (120 m, 480 m, 675 m) and (130 m, 480 m, 675 m).

4.6 Discussion and conclusions

A temporal fourth-order scheme for solving the elastic SH wave equations in VTI media has been proposed, which is designed to suppress the wrap-around and Gibbs' artifacts that have been observed in other methodologies when waves propagate through heterogeneous formations—especially in the presence of large and abrupt changes in the medium properties. The efficiency of the new method is slightly reduced in comparison with the conventional PSTD using second order centered-grid Fourier derivatives, because of the requirement for calculation of Fourier derivatives using high-order staggerer-grid method. H-PML can be successfully incorporated after the SH wave equation has been reduced into a set of first-order equations, which eliminates wrap-around artifacts. Numerical comparisons carried out within a two-layer model between the new approach and conventional second-order schemes illustrates significant reduction of both wrap-

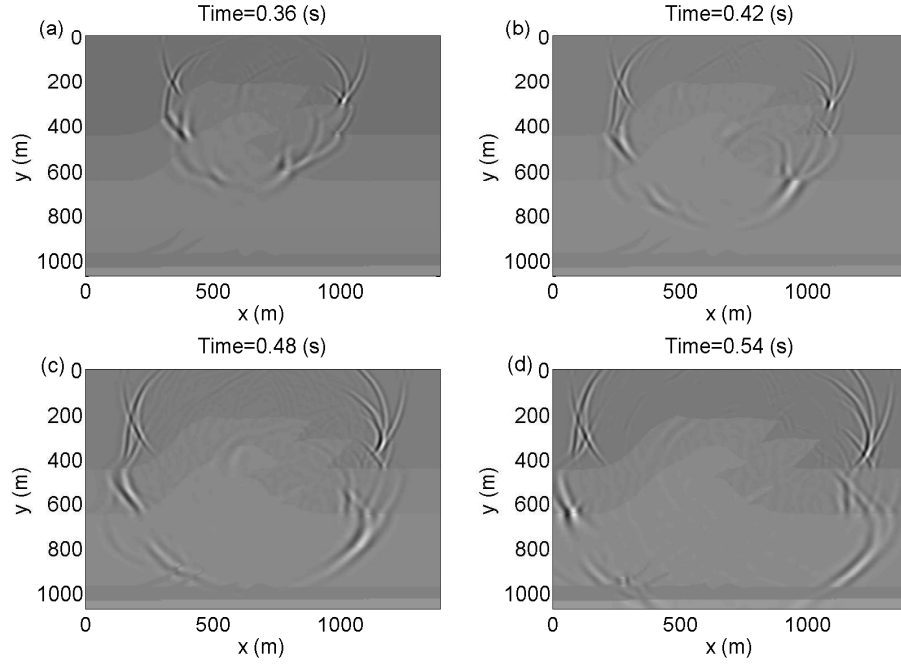


Figure 4.9: Snapshots for SH propagation in in x - z plane (x -component).

around and Gibbs' artifacts. Further experiments on a 3D anisotropic thrust fault model introduce no heterogeneity-based errors.

The stability condition for this scheme is also discussed. Given a specific spatial interval that meets the requirement for the PSTD, the maximum time step can be determined. In comparison to second-order in time/fourth order in space staggered grid finite difference methods, larger intervals in both space and time are available, which has a significant positive effect on computational expense. However, its efficiency is reduced by the requirement for the calculation of first-order Fourier derivatives rather than second order Fourier derivatives directly.

The new computational scheme was inspired by previous studies of SH amplitudes in borehole environments. Its promising features, and the fact that it is expressed conveniently in terms of displacement and velocity components, has led us to consider its use in migration and full waveform inversion, which is ongoing work.

Chapter 5

Reflection extraction from sonic log waveforms

5.1 Introduction

For acoustic reflection imaging, another critical procedure is to effectively extract the reflection signals from the acoustic full waveforms in acoustic reflection well logging data processing. Hornby (Hornby, 1989) presented data processing and imaging methods involving secondary compression arrivals to form an image of the formation structures, in which a frequency-wavenumber transform is used for elimination of the direct arrivals and other sources of noise. However, because the amplitudes of reflected waves are generally small compared to those of direct waves, it is difficult to extract them from the acoustic full waveforms. Several techniques have been used to extract reflections from acoustic data. Hornby (Hornby, 1989) proposed an f-k filtering method to extract reflections from direct waves. Li et al. (2002) used a combination of FK and median filtering techniques for reflection extraction in single-well imaging with acoustic reflection survey. The median filter is used to remove direct waves, and then the FK filter is applied to separate downgoing and upgoing reflections. Tang (2004) and Zheng and Tang (2005) used the parametric prediction method to extract reflection waves from waveforms. When this did not achieve satisfactory results, a geometric spreading factor was introduced to modify the parametric prediction method (Bing et al., 2011). The blind source separation (BSS) was also introduced to borehole geophysics (Li et al., 2014a) to extract reflections. However, amplitude information is missed using BSS, which makes it difficult to obtain ideal reflections by simply deducting the extracted head waves by applying BSS from the full waveforms.

The Karhunen-Loève (KL) transform has been widely used in data analysis such as image compression (Ahmed and Rao, 2012) and image coding (Andrews and Patterson III, 1976). The KL transform has also been used in seismic exploration for signal-to-noise improvement and separation

of diffractions from reflections (Yedlin et al., 1987). Hsu (1990) applied the KL transform to sonic logging waveforms to extract direct waves, combining a threshold detection scheme and the KL transform to exclude the unwanted signals. Here we discuss the application of the KL transform to the acoustic reflection well logging problem, to separate reflections from P- and S-head waves, and others such as the pseudo-Rayleigh wave, the water wave, the Stoneley wave, etc. We find that when extracting reflection signals using energy difference conditions (the direct waves have higher energy than do the reflections), the unwanted signals can be eliminated by choosing a wide processing window, within which a precise threshold is not necessary. The main obstacle we face is that, when borehole irregularities, permeable fractures and formations exist around a borehole, they will tend to excite the so-called Stoneley wave reflection (Paillet and White, 1982; Hornby et al., 1989). These Stoneley reflections cannot be suppressed along with the direct Stoneley wave by using the KL transform. We manage this problem in the simplest way we can, by applying a bandlimiting filter as a first step, to suppress the low frequency direct and reflected Stoneley waves.

In order to examine the reliability of reflection extraction using the KL transform in acoustic reflection logging data, synthetic well-logging data, simulated with a finite difference (FD) method, and laboratory sonic tool acquisition data acquired in a water tank are created as input and processed. The results are compared with those obtained by the multi-scale slowness-time-coherence (MSTC) method (Tao et al., 2008b). A field data case is also examined and images formed using the reflections are analyzed as a means to validate the approach.

5.2 Karhunen-Loeve(KL) Transform

A typical sonic log can be expressed as an $m \times N$ matrix $X = [x_1 \ x_2 \ \cdots \ x_N]$, in which each vector element $x_i = (x_{i1}, x_{i2}, \cdots, x_{im})^T$, $(i = 1, 2, \cdots, N)$ can be treated as a recorded waveform at a specific depth, where m is the total recording time of a waveform and N is the total number of

recorded waveforms. The mean vector of this $m \times N$ matrix is

$$\mu = \frac{1}{N} \sum_{i=1}^N x_i, \quad (5.1)$$

where, $\mu = (\mu_1, \mu_2, \dots, \mu_m)^T$.

Expanding the mean vector μ into an $m \times N$ matrix $\mu_e = [\mu \quad \mu \quad \dots \quad \mu]$, the $m \times m$ covariance matrix of X thus can be expressed as

$$C_X = \frac{1}{N} \sum_{i=1}^N (x_i - \mu)(x_i - \mu)^T, \quad (5.2)$$

where $\cdot^T$ denotes the transpose of a matrix. Assuming $\lambda_j, (j = 1, 2, \dots, m)$ to be the eigenvalue spectrum of the covariance matrix, each eigenvalue belongs to one of the system of eigenvectors $e_j, (j = 1, 2, \dots, m)$, where each e_j is a row vector. There is an $m \times m$ orthogonal matrix A such that:

$$A = [e_1^T \quad e_2^T \quad e_3^T \quad \dots \quad e_m^T]^T, \quad (5.3)$$

with $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \dots \geq \lambda_m$. The eigenvectors are ordered so that the first row of A is the eigenvector belonging to the largest eigenvalue, and the last row is the eigenvector belonging to the smallest eigenvalue. Therefore, we may write an $m \times N$ matrix equation embodying the forward definition of KL transform

$$Y = A(X - \mu_e), \quad (5.4)$$

and

$$y_i = A(x_i - \mu). \quad (5.5)$$

$y_i = (y_{i1}, y_{i2}, \dots, y_{im})^T$ denotes each component of $Y, i = 1, 2, \dots, N$. The covariance matrix of Y is

$$C_Y = AC_X A^T = \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_m \end{bmatrix}. \quad (5.6)$$

In this particular case, we find that the eigenvalues and eigenvectors of C_Y are the same as those of C_X . And since in this case, the off-diagonal elements are 0, which means the elements of Y are uncorrelated. This makes it possible to eliminate signals with small amount of energy to approximate the original signals. Because A is an orthogonal matrix ($A^{-1} = A^T$), multiplying both sides of equation (5.4) by A^T , the $m \times N$ matrix X can be described as

$$X = A^T Y + \mu_e. \quad (5.7)$$

The above equation is treated as the “inverse” KL transform. Next we consider approximations based on the first k largest eigenvectors

$$\hat{A} = [e_1^T \quad e_2^T \quad e_3^T \quad \cdots \quad e_k^T]^T, \quad (5.8)$$

with $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_k$, $k < m$.

Based on this approximation $\hat{A}$ of A , an approximation of matrix X with dimensions of $k \times N$ can be obtained with

$$\hat{X} = \hat{A}^T \hat{Y} + \mu_e, \quad (5.9)$$

and

$$\hat{x}_i = \hat{A}^T y_i + \mu. \quad (5.10)$$

where $\hat{x}_i$ are the vector columns of $\hat{X}$ and $\hat{Y}_i$ are the first $k \times N$ of Y . The mean square error between X and $\hat{X}$ is

$$\varepsilon(k) = E\{(X - \hat{X})^T (X - \hat{X})\} = \frac{1}{m-k} \sum_{i=k+1}^m x_i x_i^T. \quad (5.11)$$

In geophysical signal analysis, the received signal is usually considered as a zero mean value (Saggaf and Robinson, 2000), which means μ equals to 0. Therefore, the mean square error will be minimized when

$$C_X e_i = \lambda_i e_i, \quad i = k+1, \cdots, m. \quad (5.12)$$

Therefore, we have

$$\varepsilon(k) = \sum_{i=k+1}^m \lambda_i. \quad (5.13)$$

This equation provides a way to determine the confidence level while choosing the number of k , which will be discussed in detail in the later section of this chapter. The matrix as approximated in equation (5.9) can in this way be used to reconstruct the original signal, if and only if the mean square error approaches 0. This means, if the k most dominant eigenvectors are used to reconstruct the original information, the dominant energy will be preferentially included. This is the basis of KL transform methods. In acoustic reflection logging, most of the energy comes from the direct P-, S- and Stoneley waves, which we will assume can be extracted from measured waveforms by means of KL transform. The residuals should then be dominated by reflections, which can be used after extraction in acoustic reflection imaging.

5.3 Wave Separation Using KL Transformation

5.3.1 Synthetic testing

To examine the accuracy of the reflection extraction from sonic logging by KL transform, a synthetic data set is created with a numerical finite difference (FD) method. The model is illustrated in Figure 5.1. We make use of the term depth to indicate the standard up-down direction, but because the borehole reflection experiment is rotated 90° from a standard surface experiment, the lateral position will be referred to as *lateral depth*. The model is a fluid-filled borehole with a fault-like interface lying to one side of the borehole with a dip of 45° . The model is 15m in depth and 10m in lateral depth, and the borehole, whose diameter is 0.2m, is located at the lateral depth of 1 m (in blue in Figure 5.1). The formation in red will be referred to as medium I and that in yellow as medium II. The acoustic reflection logging tool is designed to have a source-receiver spacing of 3m, and a total of thirteen receivers, evenly spaced, with an interval of 0.15m. The borehole and formation parameters are given in Table 5.1.

Table 5.1: Parameters of fault-like model outside the borehole

	c_{11}	c_{13}	c_{33}	c_{44}	$\rho(g/cm^3)$
Yellow	23.87	9.79	15.33	2.77	2.5
Borehole	2.25	2.25	2.25	0	1
Red	40	13.55	40	13.225	2.5

Data are simulated for 60 source points, starting at a depth of 5m and ending at 13.85m with a shot interval of 0.15m. A 2D FD numerical modelling scheme is used to simulate full waveforms as measured by the acoustic array at each source point. Figure 5.2 (left panel) shows the measured waveform when the dip angle is 45° . The full waveforms shown in Figure 5.2 are signals recorded by the nearest receiver(3m away from the source). The total recording time is 15ms. Because of the slow formation outside the borehole, the first arrival is a leaky P-wave (Haldorsen et al., 2006); the water wave (whose velocity is roughly 1500 m/s) propagating in the borehole fluid arrives behind the P-wave; the signal behind the water wave arrival is the Stoneley wave. No shear wave propagates in the borehole in the slow formation, in accordance with Snell's law. The reflections in this simulated data set can be obtained by subtracting it from the full waveform computed without the interface (Figure 5.2, right panel).

If a KL transform is applied in a window from 0ms to 5ms, the first principal component is expected to be the Stoneley wave, the coherent arrival with the highest amplitude. Figure 5.3 shows the four dominant normalized eigenvectors of the covariance matrix calculated from the selected range of full waveforms (the normalized eigenvectors are obtained using equation (5.2)). Compared with the take-off time of each event as shown in Figure 5.2 (left panel), the first dominant eigenvector consists not only of the Stoneley component (i.e., the event at about 3ms), but also of the water wave arrival (the event at about 2.5ms). The eigenvalue of the first dominant eigenvector is 0.9884 (which represents over 98% of energy). The second largest eigenvalue is 0.0031 (we will see in the following discussion that the energy of this eigenvector comes from the lower interface 4m away from the borehole). The other eigenvectors contribute so little to the first principle component that they can be ignored.

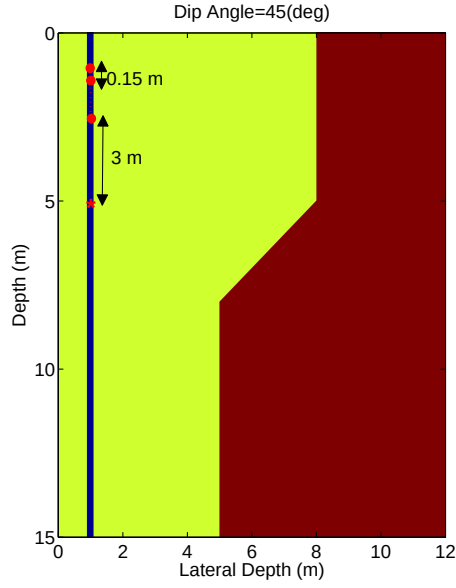


Figure 5.1: Synthetic model: Blue area is the borehole. Yellow and red areas are two formations with different properties.

Therefore, the first principal component can be largely reconstructed using the first dominant eigenvector, as shown in Figure 5.4 (left panel). The earliest linear arrival is the water wave; the second linear arrival is the Stoneley wave, compared with the arrival times of each event in Figure 5.2 (right panel). Figure 5.4 (right panel) shows the covariance residuals after the first principal component is subtracted. The reflected waves have become visible, arriving shortly after what will now be the new principal component (the direct P-wave arrival).

In order to suppress the new dominant component of the signal, the P-head wave, the same time window from 0ms to 5ms is applied to calculate the covariance matrix of the waveform. Its four dominant eigenvectors are plotted in Figure 5.5 (left panel).

The first eigenvector has a normalized eigenvalue of 0.9731 (which, with the stronger arrivals now removed, we associate with the P-wave energy). Therefore, the first dominant eigenvector is used to reconstruct the P-wave, as shown in Figure 5.5 (left panel). Figure 5.6 (right panel) shows the residuals after the P-wave component has been removed, which are identifiable as being the

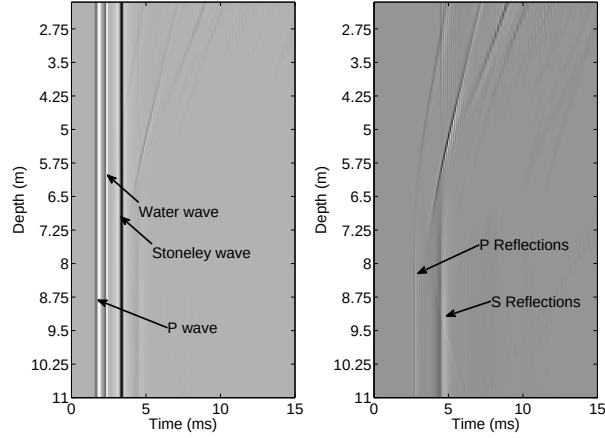


Figure 5.2: Received full waveform when the dip angle is 45 degree (Left) and theoretical reflection signals by subtracting the full waveform of this synthetic model from that of a model without the interface (Right). The data is received by the first receiver.

reflections with the direct P-wave, water wave and Stoneley wave largely removed. Compared with the theoretical reflections shown in Figure 5.2 (right panel), almost all the reflection details have been revealed.

We next compare these results with the reflections extracted using the multi-scale slowness-time-coherence (MSTC) approach (Tao et al., 2008b), which is also displayed in Figure 5.6 (left panel), where some details of the reflection signals are missing and low frequency noises appear during the whole recording time.

We next compare, trace-by-trace, the exactly-separated benchmark events in the simulated data against those of the KL-separation method. Figure 5.7 illustrates the direct waveforms (red), true reflections (black) and reflections from KL transform (blue) at different recording depths from (a) 3.5m, (b) 5.75m, (c) 8m and (d) 9.5m, respectively. In Figure 5.7a the direct events do not overlap with the reflections, because the upper interface is 7m away from the borehole, which makes it easy to differentiate the reflections from the direct waves. Here the KL reflections precisely match the true reflections. In Figures 5.7b-d the situation is more complicated, because the events to

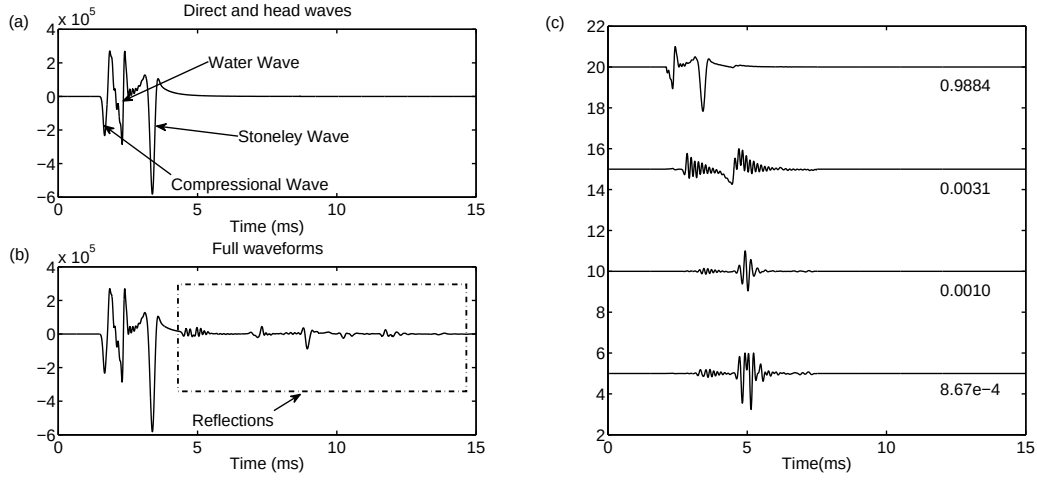


Figure 5.3: (a) Waveforms received by the receiver when there is no reflected signals. (b) Full waveforms recorded by the receiver when reflections are present. (c) Four dominant normalized eigenvectors of covariance matrix calculated from windowed waveforms. The processing time window is from 0ms to 5ms. The eigenvalues of each eigenvector are also shown.

be separated overlap significantly (between 2.5ms to 5ms). Nevertheless the extracted reflections using the KL transform match with the true reflections at all depths with no apparent flaws. The reason for this is also apparent: the first reflection wave packets in each of Figures 5.7c-d are essentially completely captured by the second eigenvector, as plotted in Figure 5.3.

Finally, the borehole reverse time migration(RTM)(Li et al., 2014b) is applied. The migration results using reflections extracted by both the KL transform method and MSTC method are illustrated in Figure 5.8. The velocity model used in the RTM is the same as in Table 5.1, except there's no red formation outside the borehole. Figure 5.8a is the RTM result using the reflections extracted by the KL transform method, the fault-like interface is clearly observed. Figure 5.8b shows the RTM result using the reflections extracted by the MSTC method. As is pointed out in the circle areas, some noise is detected.

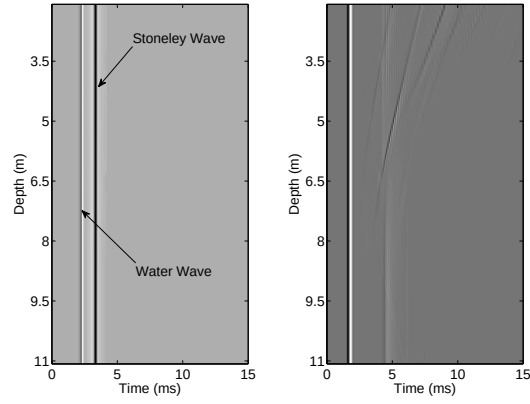


Figure 5.4: First principal component of the raw data (Left) and residuals after principal component has been removed (Right).

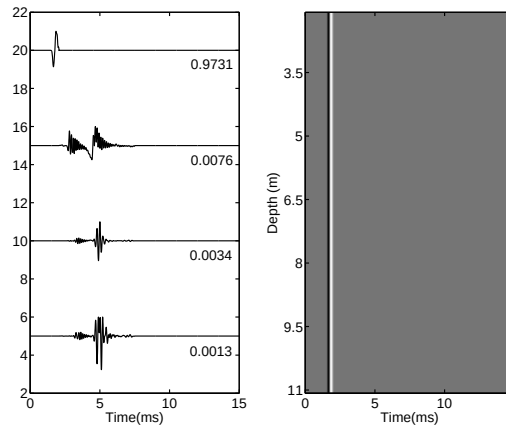


Figure 5.5: Four dominant eigenvectors of the residuals after principal component is removed(Left) and the reconstructed P-wave component extracted using the first eigenvector (Right)

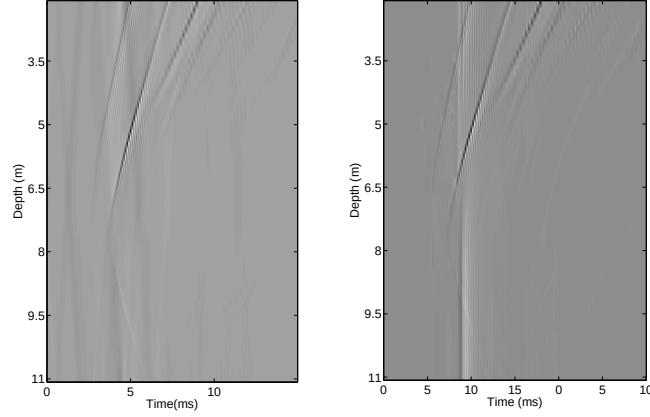


Figure 5.6: Reflections extracted using MSTC (Left) and KL transform (Right).

5.3.2 The determination of the k value in the KL transform

Based on the principles and synthetic tests in the previous two sections, a key issue for this method is how to choose the value of k with which to approximate the original data. In this section, we will analyze the relationship between the k value and the energy distribution of the signals transmitted by the source in the borehole.

When the transmitter is set off in a borehole, energy spreads out in all directions. The compressional wave is generated and emitted in the borehole fluid. It reflects back or transmits (or refracts) into the near borehole formation when it meets the borehole wall. When the incident angle θ reaches critical, critically-refracted waves propagate along the interface and compressional and shear head waves are created (Close et al., 2009). The P- and S-head waves travel with formation velocities and are received by the receivers in the borehole. A schematic of the wave modes is shown in Figure 5.9a. Figure 5.9b illustrates the generation of the P-head wave. P_f is the compressional wave in the borehole fluid, θ is the incident angle, P_1 and S_1 are the refracted P- and S-waves in the formation respectively. When the incident angle reaches to the critical angle, the refracted P-wave will travel along the interface with the formation P-wave velocity, which will further transformed into P-head wave ($P_f P_1 P_f$) received by the receiver.

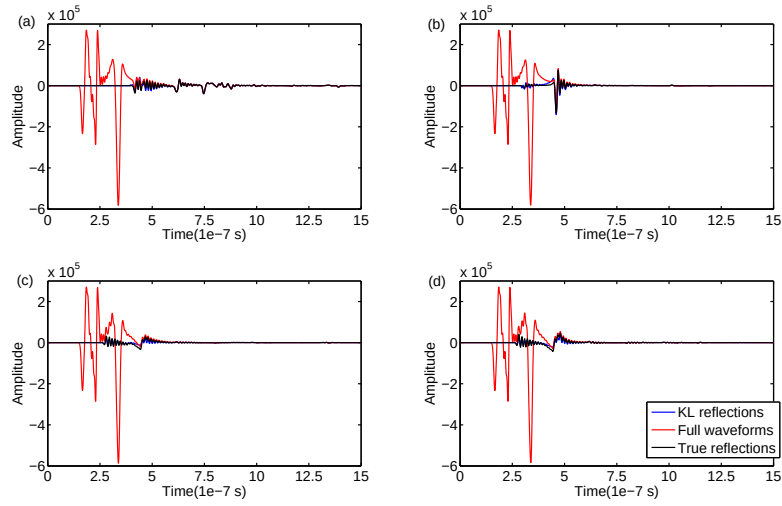


Figure 5.7: Comparison of the direct waveforms (red), true reflections (black) and reflections from KL transform (blue) at recording depths from 3.5 m (a), 5.75 m (b), 8 m (c) and 9.5 m (d).

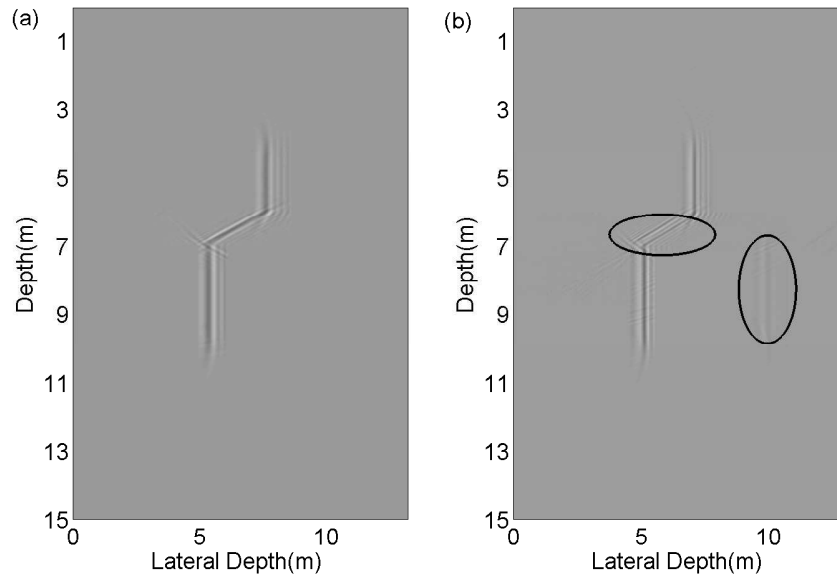


Figure 5.8: RTM results when (a) reflections extracted by the KL transform method are used and when (b) the reflections extracted by the MSTC method are used.

As illustrated in Figure 5.9, although most of the wave energy is trapped in the borehole, a small amount leaks into the formation in the forms of P- and S-transmitted(refracted) waves. The percentage of energy leaked is related to the formation parameters. Table (5.2) enumerates the parameters for borehole fluid, fast and slow formations, respectively.

Table 5.2: Parameters of the borehole fluid and formations outside borehole.

	$V_f(m/s)$	$V_P(m/s)$	$V_S(m/s)$	$\rho(g/cm^3)$
Borehole	1500	-	-	1.0
Fast formation	-	4500	2500	2.5
Slow formation	-	2300	1100	2.0

For a P-wave incident in the borehole fluid, and with particle displacement reflection and transmission coefficients expressed as R and T, we denote the P-wave reflection coefficient and the P- and S-wave transmission coefficients as R_{PP} , T_{PP} , and T_{PS} respectively. Let A_I , A_R and A_T be the amplitudes of the incident, reflected and refracted P-waves, and B_T be the amplitude of the refracted S-wave. The particle displacement reflection and transmission coefficients are:

$$R_{PP} = A_R/A_I, \quad T_{PP} = A_T/A_I, \quad T_{PS} = B_T/A_I. \quad (5.14)$$

The plane-wave reflection and transmission coefficients are plotted in Figure 5.10. For the fast formation (Figure 5.10a), the critical angle for the P-wave reflection is 19.47° ; for larger incident angles, the refracted P wave will travel along the fluid-solid interface in the form of a P-head wave (Cerveny and Ravindra, 1973). When the incident angle reaches 36.87° , the refracted S-wave will begin propagating as a head wave as well. In the slow formation (Figure 5.10b), the refracted P wave is converted to a P-head wave when the incident angle reaches 40.71° . There is no S-head wave.

To relate wave reflection and transmission to energy conservation, we introduce the energy reflection coefficient $\mathcal{R}$ and energy transmission coefficient $\mathcal{T}$ defined as the fraction of the incident

energy that is reflected and transmitted, respectively. The conservation of energy can be expressed as

$$\mathcal{R}_{PP} + \mathcal{T}_{PP} + \mathcal{T}_{PS} = 1. \quad (5.15)$$

where

$$\begin{aligned} \mathcal{R}_{PP} &= \left| \frac{A_R}{A_I} \right|^2, \\ \mathcal{T}_{PP} &= \left(\frac{\rho_2 V_P \cos \theta_2}{\rho_f V_f \cos \theta_1} \right) \left| \frac{A_T}{A_I} \right|^2, \\ \mathcal{T}_{PS} &= \left(\frac{\rho_2 V_S \cos \phi_2}{\rho_f V_f \cos \theta_1} \right) \left| \frac{B_T}{A_I} \right|^2. \end{aligned} \quad (5.16)$$

In Figure 5.11, we plot the energy reflection and transmission coefficients with respect to incident angle. For a liquid-solid interface, Donato (1964) calculates the amplitude of P-head waves. As is also pointed out by Heelan (1953), the energy of head waves is derived entirely from the refracted waves. Consider the P-head wave in the fast formation as an example (Figure 5.11a). When the incident angle reaches to the P-wave critical angle (19.4712°), the P-wave energy transmission coefficient decreases to 0.2 (normalized assuming unit incident energy). According to Heelan (1953), it is this part of the wave energy that induces the P-head wave and that is received by the receiver in the borehole. For incident angles larger than the critical angle, at which P-head waves make up the received signal, the energy is above 0.4. The transmitted P-wave, before the critical angle, has, according to Figure 5.11a, an average energy of 0.3; when this part of the energy travels outside the borehole, it will propagate back after impinging on external structures. Suppose the energy reflection coefficient is the same as that in 5.11a. Then, energy in the amount of 0.3×0.6 will be reflected back. Subsequently, the borehole wall acts as a third interface, reducing further the reflected energy. Within the incident angle range $0-20^\circ$ for effective transmitted P-wave, the estimated $P_f P_1 P_1 P_f$ reflection energy is less than 0.02. The ratio of P-reflected to head wave energy is about 0.95.

Based on this approximate calculation, we set our confidence level or k value at 0.95. When

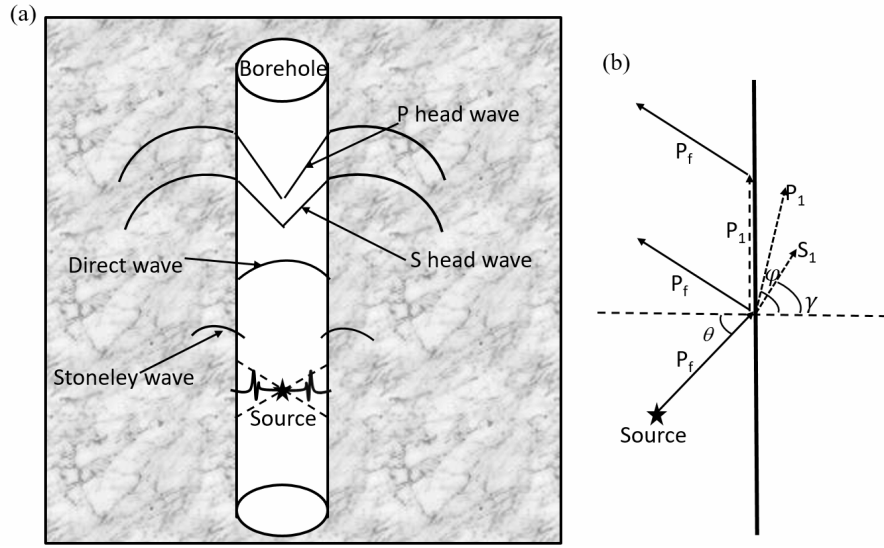


Figure 5.9: Figure (a) illustrates sonic waveform propagation in a borehole. In fast formation, the wave modes received by the receivers in the borehole are P-head wave, head-Shear wave, direct wave and Stoneley wave as a function of time. Figure (b) shows the generation of P-head wave, which is composed of $P_f P_1 P_f$

the sum of the first k eigenvalues reaches 0.95, we assume that k eigenvectors are sufficient to approximate the principle wave mode. In this thesis, the confidence level of 0.95 is used.

5.4 Examples

5.4.1 Laboratory data example

In this section, the laboratory data is acquired in a specially designed large water tank by the remote exploration acoustic reflection imaging (Chai et al., 2009). The reflector in the water tank is a steel pad placed at an angle of 20° relative to the measurement axis, a distance of 3m away from the tool. The depth interval is 0.1524m and the distance between the source to the first receiver is 5.3m. The schematic diagram of the water tank model is shown in Figure 5.12.

The waveforms recorded by the first sensor are plotted in Figure 5.13 (left panel). The 900m/s event at roughly 6ms has the highest energy. The compressional head-wave event occurring at

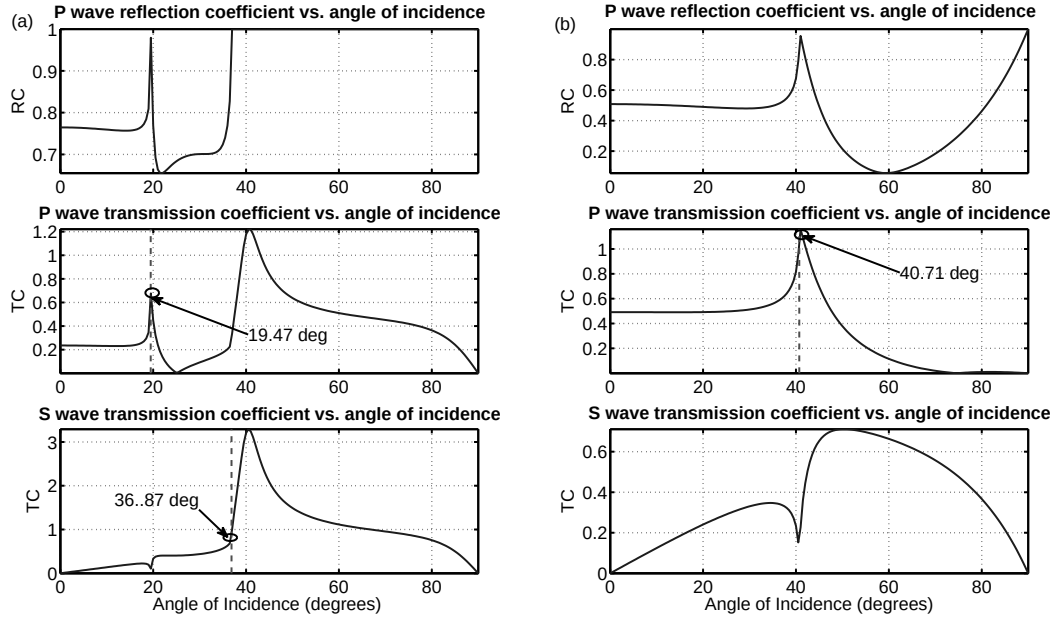


Figure 5.10: P- and S-wave reflection and transmission coefficients vs. angle of incidence for (a) fast formation and (b) slow formation.

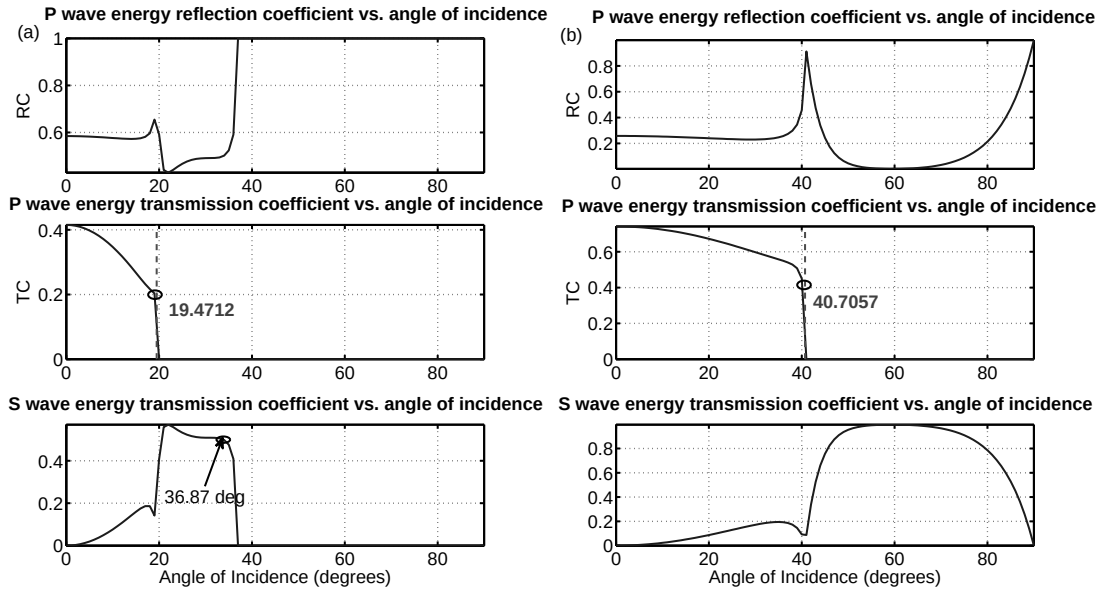


Figure 5.11: P- and S-wave reflection and transmission coefficients vs. angle of incidence for (a) fast formation and (b) slow formation.

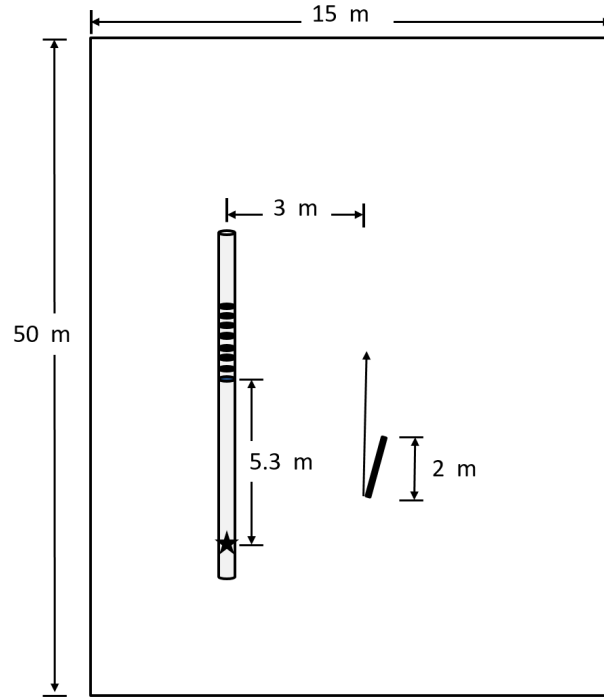


Figure 5.12: Water tank model with a size of $15\text{m} \times 50\text{m}$. The source-receiver spacing of the sonic tool in the tank is 5.3m. A roughly 2m long steel pad is located 3m away from the tool with a dip angle of 20° towards the vertical direction.

roughly 4ms has a velocity of 1400m/s. The data exhibit low-frequency noise in early part of the waveform. Figure 5.13 (right panel) shows the reflection extracted using the MSTC method. The reflections from the steel pad emerge, but are accompanied by significant residual direct-wave energy along the depth interval.

We next apply the two-step KL methodology to isolate reflections on the same data set. After setting the first window range to be 4-8 ms, the eigenvalue matrix is obtained. The largest four eigenvalues are 0.6938, 0.2486, 0.0296 and 0.0073, respectively. To satisfy the confidence level, we allow the first three eigenvalues to approximate the energy of the first principle component. In Figure 5.14 (left panel), the first principal component extracted by the KL transform is plotted. It matches very closely with the event at 6 ms in Figure 5.13. Figure 5.14 (right panel) shows the residuals after the principal component is removed. The weak reflection from the steel pad emerges

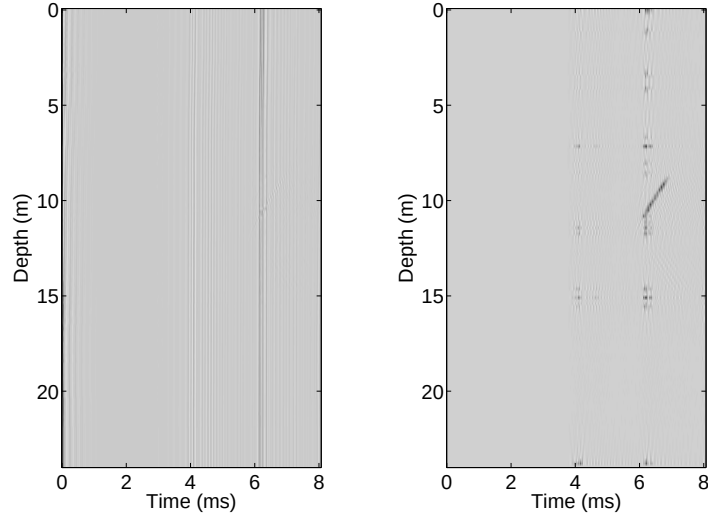


Figure 5.13: Received full waveform of the water tank data (Left) and reflection signals using MSTC (Right).

in the residual waveforms, yet the low-frequency noise at the beginning and the compressional head-wave still dominate.

Figure 5.15 (left panel) shows the secondary principal component extracted by the KL transform, which takes place after the low frequency noise has been removed by a high bandpass filter (the same high bandpass filter procedure is also applied in MSTC method). The eigenvalue matrix is obtained for 1-8ms window. The largest four eigenvalues are 0.7373, 0.1774, 0.0188 and 0.0172, respectively. These four eigenvalues are used to approximate the energy of the first principle component. This component is the fluid wave. Figure 5.15 (right panel) shows the reflection signals from the steel pad after the secondary component is removed. Compared with the result obtained using MSTC, there is no residual noise visible, and the reflection has been cleanly extracted.

When a well bore intersects geological structures, the measured full-wave acoustic-logging data can be used to image the formation structures. In this water tank data, the borehole RTM is applied, in which a staggered grid finite difference method is used for the forward and backward propagation operators. The imaging result using reflections extracted by KL transform is shown

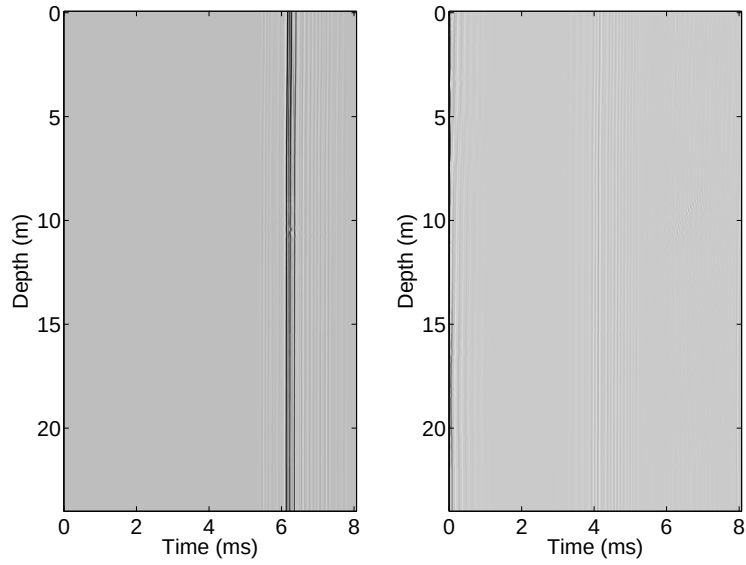


Figure 5.14: First principal component of the raw data (Left) and residuals after principal component has been removed (Right).

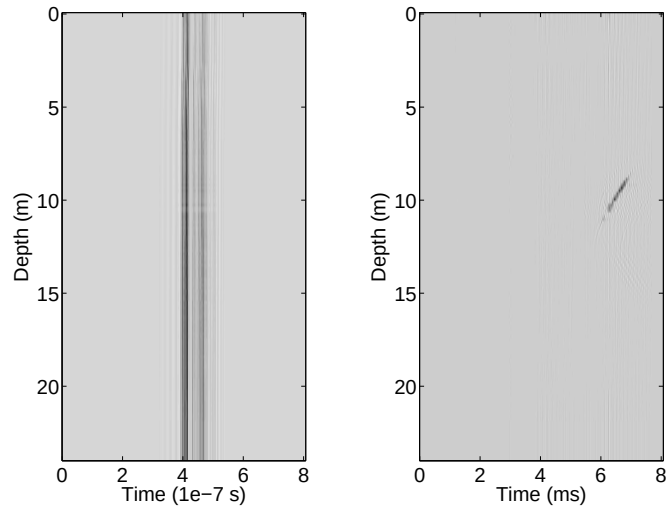


Figure 5.15: P wave component reconstructed using the first eigenvector (Left) and residuals after secondary component has been removed (Right).

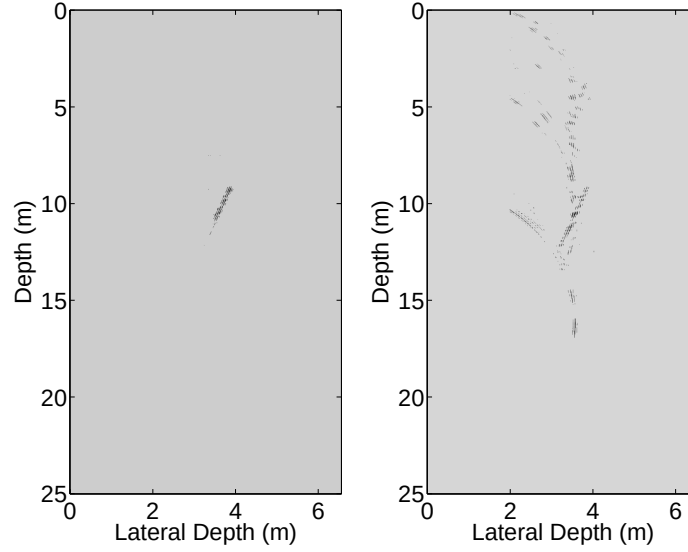


Figure 5.16: Imaging results of reflections from KL transform (Left) and from MSTC (Right).

in Figure 5.16 (left panel). Here, the steel pad is clearly shown at an angle of 20° and a distance of 3m away from the borehole. The shape of the pad can be clearly observed. In comparison, the image created using reflections obtained by the MSTC method is also displayed in Figure 5.16 (right panel). Here the imaging result is strongly affected by the noise left over after a suboptimal reflection signal extraction.

5.4.2 Field data example

The field data used in this thesis are acquired by an acoustic reflection imaging instrument from East Asia. The reservoir is well-developed with fractures and vugs. The source-receiver distance is 3.6576m and the 8 receivers are spaced at 0.1524m. With the source positions incremented at 0.1524m, the receivers record waveforms transmitted from the source with a recording sampling of $12\mu s$. In Figure 5.17 (left panel), only data from receiver 1 of an eight-receiver array are displayed. The single receiver data show the typical acoustic logging data set: P-direct wave with an early arrival (about 0.8ms, P-velocity is about $4900m/s$) at a high frequency; S-direct wave as a second arrival (about 1.5-2ms, S-velocity is about $2750m/s$) and Stoneley behind S- wave (at about 3 ms,

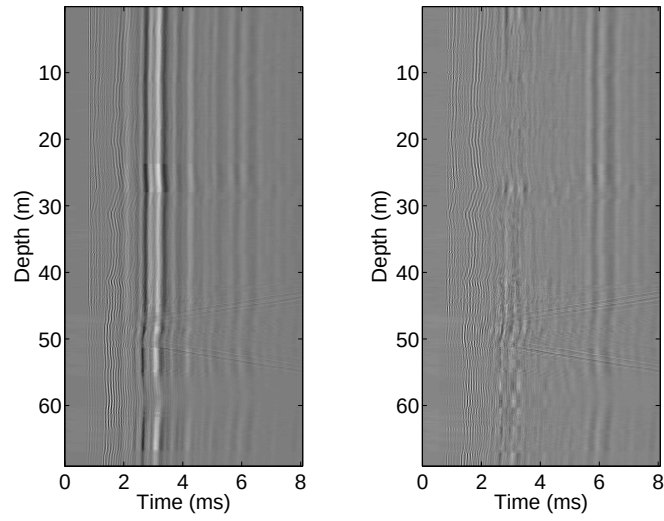


Figure 5.17: The raw waveforms recorded by a receiver (Left); the residuals after the Stoneley direct wave is removed (Right)

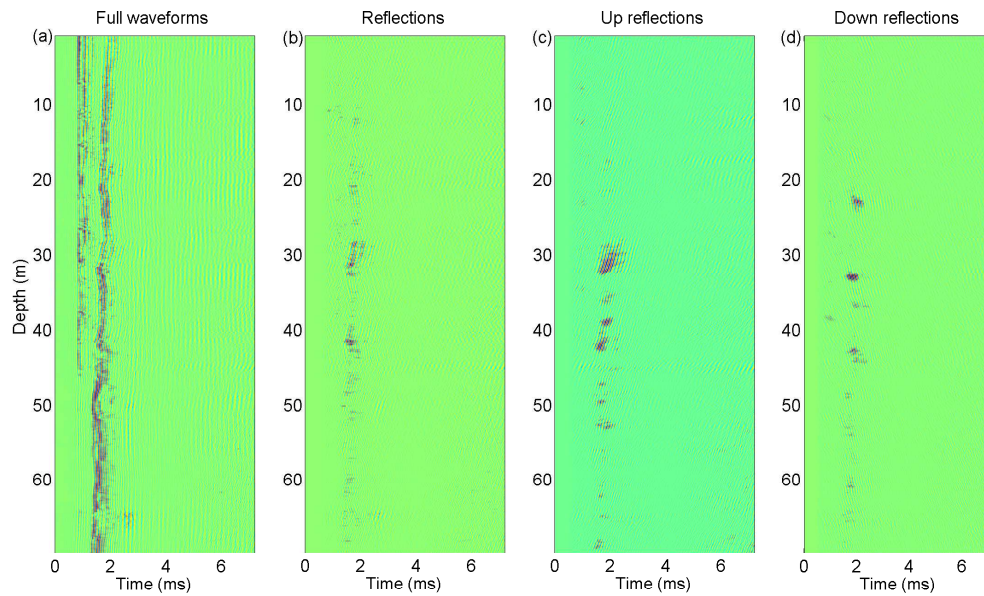


Figure 5.18: (a) Full waveforms after Stoneley signals (direct and reflected Stoneley) are mitigated; (b) reflections extracted using KL transform; (c) the upgoing reflections by the common-receiver gather and (d) the downgoing reflections by the common-source gather

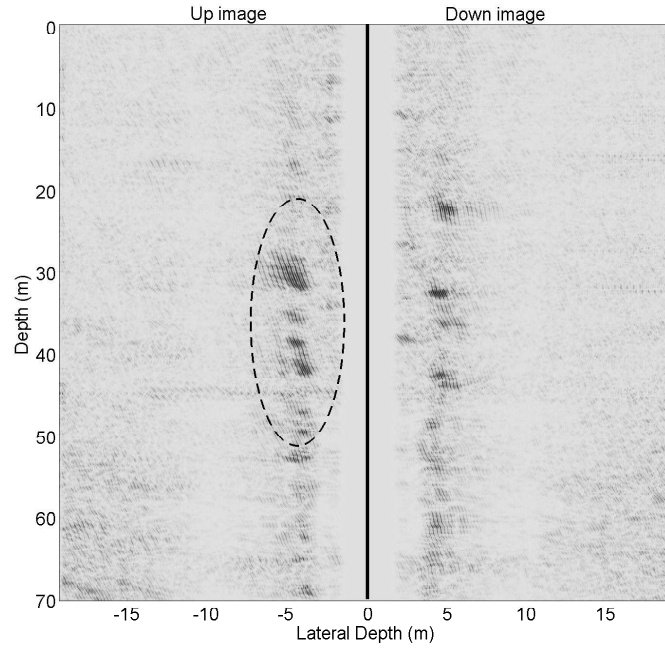


Figure 5.19: Up going image (Left) and down going image (Right)

P-velocity is about 1300m/s as a low-frequency event. The reflections are submerged within the direct waves. According to KL transform, the principal energy (Stoneley wave component) should be removed first. Figure 5.17 (right panel) shows the residuals after the Stoneley direct wave is removed. However, the Stoneley reflections are still present between 40m to 55m from 3ms to 8ms. Therefore, a high pass filter based on the frequency difference between Stoneley waves (Stoneley direct and reflected waves) and P- and S-waves is used to remove the Stoneley energy. Figure 5.18a shows the waveforms after the Stoneley waves have been successfully removed.

The KL transform approach next suppresses the direct P- and S-waves, as is shown in Figure 5.18b. To improve the image result and remove the ghost events caused by mixed reflections, Tang (Tang et al., 2007) used the receiver array data (common-source gather) to obtain down-going reflection waves that illuminate the lower side of a fractured structure or a bed boundary and the common-receiver gather to obtain the up-going reflections. In this thesis, the respective application of this wave separation method to receiver array and transmitter array data is applied to separate

reflection data into upgoing and downgoing waves, as shown in Figure 5.18c-d. Compared with the reflections in Figure 5.18b, the upgoing and downgoing are effectively separated.

We next apply borehole reverse time migration, during which the upgoing- and downgoing data are respectively migrated to the spatial domain to form updip and downdip images of formation reflectors. Figure 5.19 shows the imaging result for both upgoing and downgoing reflections. At depths between 20-40m, there is a bed boundary in the up image which pinches out in the down image. The comparison between reflection data and image results indicates that quality of the image is closely related to that of the reflection data.

5.5 Discussion

An important consideration in this approach concerns situations in which the relative amplitude of each mode changes with the depth. To address this, we start with a model in which the formation outside the borehole is homogeneous(each received primary mode will then be the same as the source moves downwards). Figure 5.20a is a schematic diagram of the model. The parameters of each formation are enumerated in Table 5.3.

Figure 5.20a illustrates the source and receiver distribution. Suppose we move the source downward from depth of 4.20m to 9.0m with an interval of 0.15m for each shot. In Figure 5.20b and c, the full waveforms and true reflections recorded by the receiver are plotted. Following our KL transform procedure, the reflections are obtained after the primary waves are eliminated. This is shown in Figure 5.20f.

Next, we consider a new model, in which formation 1 contains three-layers, with V_P , V_S vertically increasing from the top layer of 3350m/s, 1814m/s to the medium layer of 3650m/s, 2064m/s, to the bottom layer of 3850m/s, 2244m/s. This model is illustrated in Figure 5.21a. The source and receiver distribution is the same as in Figure 5.20a. In Figure 5.21b and c, the full waveforms and true reflections recorded by the receiver are plotted. We observe that the shapes of the primaries (between 1ms to 4ms) are no longer linear, whereas the true reflections are almost the same as in

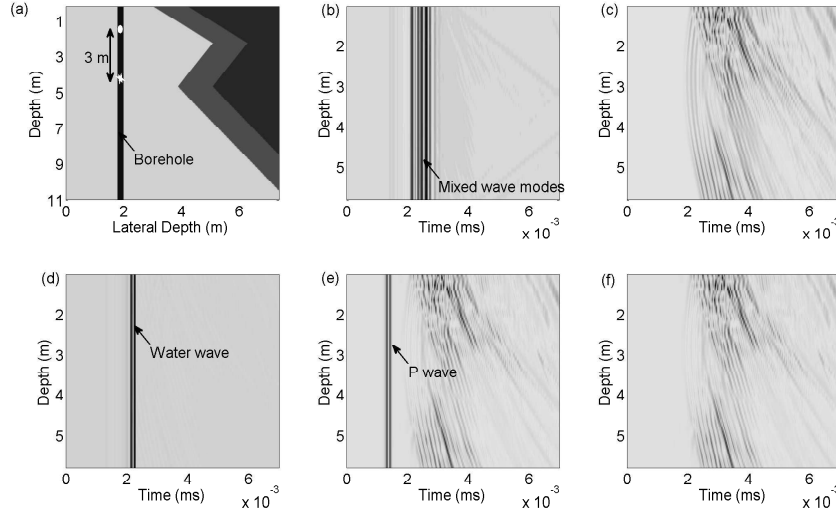


Figure 5.20: Sonic waveform extraction from a three-layer model, the adjacent layer outside the borehole is homogeneous. (a) Source-receiver distribution in a three-layer model. (b) and (c) are the full waveforms and true reflections. (d)-(f) show the procedure of reflections extraction using KL transform

the previous example (Figure 5.20c). The reflections are again obtained after the primary waves are eliminated. The results are plotted in Figure 5.21f. Most of the reflection energy is separated, with some exceptions near 2ms, which is visible when compared with Figure 5.21c. Also, some noise is introduced before 2 ms according to 5.21f. The KL transform method is, therefore, observed to be sensitive to vertical changes in the formation parameters.

Table 5.3: Parameters of a three-layer model outside the borehole.

	$V_f(m/s)$	$V_P(m/s)$	$V_S(m/s)$	$\rho(g/cm^3)$
Borehole	1500	-	-	1.00
formation 1	-	3350	1814	2.24
formation 2	-	4550	2674	2.56
formation 3	-	5000	2800	2.70

Finally, we apply a more complex perturbation to V_P and V_S in the formation 1. The new V_P

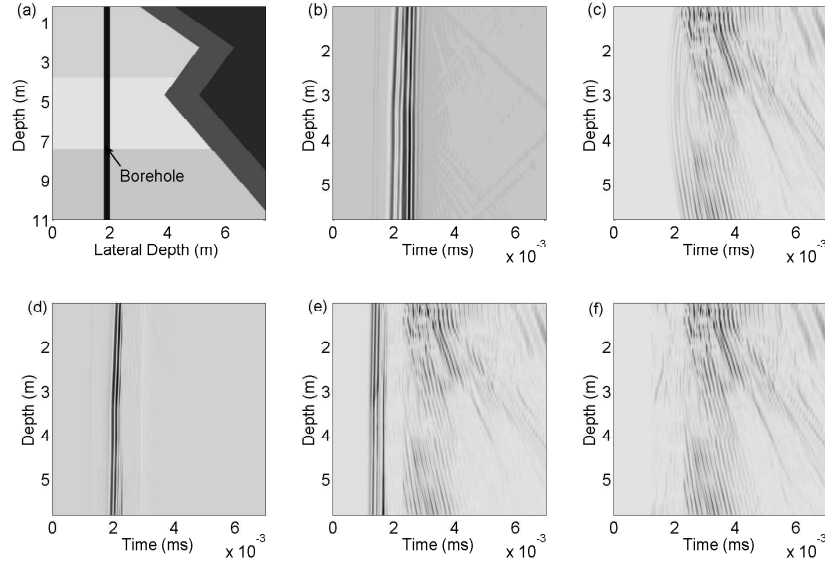


Figure 5.21: Sonic waveform extraction from a three-layer model, the adjacent formation outside the borehole is a vertically three-layer formation. (a) shows a schematic picture of this model, the source-receiver distribution is the same as Figure 5.20a. (b) and (c) are the full waveforms and true reflections. (d)-(f) show the procedure of reflections extraction using KL transform

and V_S exhibit a linear growth with the depth (see Figure 5.22a). In Figure 5.22b and c, the full waveforms and true reflections recorded by the receiver are plotted. In Figure 5.22d, reflection signals separated using the KL transform procedure are illustrated. We observe strong contamination of the result, as compared with Figure 5.22c.

For situations with strongly irregular changes of parameter distributions, the primary wave energy will also undergo irregular changes, and this nonstationarity impedes the effectiveness of the KL transform method. If a case of this kind is encountered, we recommend a more restrictive processing time window be used to most of the principle energy and exclude other modes. In the mean time, a division of the processing depth range should be enforced corresponding to abrupt changes of the lithology, ranges within which primary modes are maximally linear will reduce nonstationarity and the associated issues. Most importantly, according to the difference between the homogeneous model in Figure 5.20a and the laminated inhomogeneous models in Figure 5.22a, we

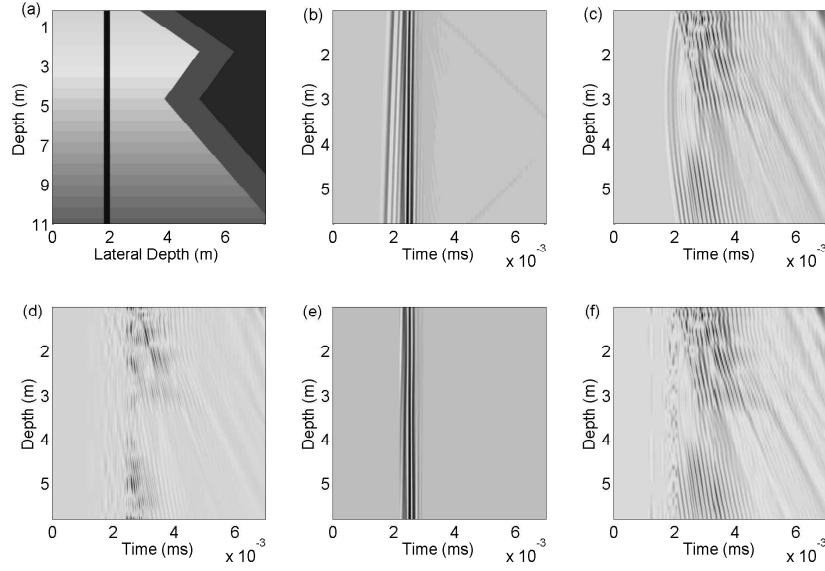


Figure 5.22: Sonic waveform extraction from a three-layer model, the adjacent formation outside the borehole is a vertically laminated formation with the V_P and V_S linearly increasing with the depth. (a) shows a schematic picture of this model, the source-receiver distribution is the same as Figure 5.20a. (b) and (c) are the Full waveforms and True reflections. (d) shows the reflections directed extracted by KL transform, the reflections are severely contaminated by unwanted signals. (e)-(f) show the procedure of reflections extraction using KL transform with primary modes straightened

provide a time shift for each waveform to linearize the principle wave mode according to its energy before applying the KL transform procedure. Figure 5.22e illustrates the extracted Stoneley wave after the Stoneley mode is linearized. Figure 5.22f illustrates the extracted reflections with a mode linearization procedure before KL transform. Compared with 5.22d, we observe a considerable improvement.

5.6 Conclusions

Sonic reflection logging is theoretically capable of providing a clear view of structures outside the well site. However, reflections are usually small in amplitude relative to other observed wave modes, and are often indiscernible. The KL transform was investigated as a means to separate

reflections from this variety of directly propagating wave events in acoustic reflection well logging data. Based on the simple assumption that significant energy differences characterize each signal component, the direct P- and S waves as well as the Stoneley wave can be effectively removed, as evidenced in synthetic testing. Comparisons with recent methods (i.e., the MSTC method) carried out both with synthetic and laboratory data, confirm that the KL transform is capable of providing reflection signal estimates with less noise and higher overall precision. It is particularly effective as a tool to enhance acoustic borehole imaging results, wherein these now-separated upgoing and downgoing acoustic reflections are further distinguished from each other based on the acoustic array selection.

Additional, but relatively simple, processing is required for practical application to field data. Particularly, a high pass filter should be applied at the outset to mitigate Stoneley signals(primary Stoneley wave that follows the water wave, the Stoneley “ringing” after the primary Stoneley wave caused by the firing of the transmitters while fully coupled to the bore hole and the Stoneley reflections), to make up for the inability of the KL transform to isolate Stoneley signals once and for all. With this additional step in place, field imaging results determined from the upgoing and downgoing reflections display a number of very promising features.

Chapter 6

Azimuth ambiguity elimination for borehole imaging using 3D borehole RTM scheme

6.1 Introduction

Images of geological structures away from the borehole can be derived from recorded data from borehole acoustic measurements by applying seismic imaging schemes (Hornby, 1989; Li et al., 2002). Monopole acoustic imaging has produced positive results in delineating near borehole structures (Fortin et al., 1991; Coates et al., 2000; Li et al., 2002). However, one of the weaknesses of the omni-directional monopole acoustic prototype is that it only measures the acoustic pressure and is therefore insensitive to reflector azimuth.

The azimuth ambiguity has been an issue ever since the beginning of borehole acoustic reflection imaging. The problem is conceptually similar to that encountered when 2D processing and imaging methods are applied to lines of seismic reflection data over 3D geological structures. 2D methods assume all upward-propagating waveforms originate immediately below the source/receiver line, whereas in reality out-of-plane effects can be significant.

In order to mitigate this directional ambiguity, the dipole acoustic reflection imaging was developed (Tang et al., 2003; Tang, 2004; Tang and Patterson, 2009; Bolshakov et al., 2011). In dipole methods, dispersive flexure waves, whose velocity at the cutoff frequency equals the S-wave velocity, are analyzed. These data, given the deviation angle of the well bore and the tool azimuthal angle, can determine the azimuth of the structures outside the borehole after migration (Tang et al., 2003). Tang et al. (2007) applied this technique to dipole S-wave log data. He also developed a method to extract the shear wave reflection signals which were then used to get the S-wave imaging.

Li (Li et al., 2014a) applied the blind signal separation method into the synthetic horizontal data from Sonic scanner tool to get the separated reflections from different reflectors. However, the amplitude information is not conserved in the output of this method. Rougha (Al Rougha et al., 2005), Yamamoto (Yamamoto et al., 2000) and Haldorsen (Haldorsen et al., 2006) developed a 3D assembly of hydrophones on the logging tool, where 4 or 8 omnidirectional hydrophones are located azimuthally around the tool (Sonic Scanner tool developed by Schlumberger), in order to sense azimuthal information from hydrophones towards different directions. However, 4 or 8 migrations are needed for evenly spaced hydrophone stations. (Haldorsen et al., 2010; Li et al., 2013).

Reverse time migration (RTM) is not a new seismic prestack depth migration method. It was first introduced in the late 1970s (Hemon, 1978) and shows promising imaging capabilities (Baysal et al., 1983; Whitmore et al., 1983; McMechan, 1983; Loewenthal and Mufti, 1983). Because of its high computational cost, three dimensional (3-D) prestack RTM was not yet available until recent years (Yoon et al., 2003). In the application of borehole acoustic reflection imaging, the 2-D borehole RTM in isotropic medium is first introduced in 2014 (Li et al., 2014b).

In this chapter, we first simulate recorded waveforms at the 8 evenly spaced hydrophones using a staggered-grid finite difference method. Then, a borehole 3D RTM scheme is developed and validated using the reflection signals extracted from the simulated waveforms. To compare its performance against the standard methods discussed above, the 2-D synthetic data measured in two horizontal wells is also simulated and migrated using a 2-D borehole RTM scheme.

6.2 Borehole reverse-time migration

RTM involves solving the two-way wave equation for both the source wave field and receiver wave field, and is significantly more complicated than the one-way wave equation based migration methods. As a result, RTM methods have the ability to migrate any type of multiples (surface and internal) to their correct location in the subsurface, can handle multi-pathing, image turning waves

and steep dips. RTM can be divided into four parts: (1) Forward simulation of the source wave field, in which we calculate the wave field of the source propagating along the positive time axis and save the wave field at every time step; (2) Backward extrapolation of the receiver wave field from the maximum record time and save the wave field at every time step; (3) Application of an imaging condition to get image results from the saved forward simulated wave field and backward extrapolated wave field; (4) Superposition of the imaging result from every migration time step.

Though the basic steps of RTM are also valid in the borehole environment, the borehole RTM differs from that used in seismic. First, the borehole wall itself acts as the first reflector after the transmitter has been fired. The received reflection signals derived from energy that escaped from the borehole are affected by the borehole wall. The forward extrapolation must include simulation of the wave field in the borehole environment in order to take the borehole influence into consideration. Therefore, in order to apply FDM, the elastic parameters in the vicinity of the borehole and formation interface should be set to ensure the accuracy of the wave field propagation. Second, the frequency used in the reflection well logging is considerably higher. Generally, the central frequency of the field data can reach up to 10-15 kHz. The source frequency that used in forward extrapolation should be close to that of the received signals. The closer it is to the received signals. The better the imaging result will be (Yang, Chang and Liu 2010).

6.3 Forward and backward wavefield propagation

The forward wavefield propagation can be simulated by including a source term within the normal stress terms. For a dipole source simulation, two monopole sources with opposite phase are positioned close to the borehole wall. Likewise, the backward wavefield simulation can also be

obtained by a similar scheme, except equation (2.12) and equation (2.14) should be modified as,

$$\begin{aligned}
V_x^{n-\frac{1}{2}} &= V_x^{n+\frac{1}{2}} - \frac{\Delta t}{\rho} (\delta_x \sigma_{xx}^n + \delta_y \sigma_{xy}^n + \delta_z \sigma_{xz}^n) \\
V_y^{n-\frac{1}{2}} &= V_y^{n+\frac{1}{2}} - \frac{\Delta t}{\rho} (\delta_x \sigma_{xy}^n + \delta_y \sigma_{yy}^n + \delta_z \sigma_{yz}^n) \\
V_z^{n-\frac{1}{2}} &= V_z^{n+\frac{1}{2}} - \frac{\Delta t}{\rho} (\delta_x \sigma_{xz}^n + \delta_y \sigma_{yz}^n + \delta_z \sigma_{zz}^n)
\end{aligned} \tag{6.1}$$

and,

$$\begin{aligned}
\sigma_{xx}^n &= \sigma_{xx}^{n+1} - \Delta t [c_{11} \delta_x V_x^{n+1/2} + (c_{11} - 2c_{66}) \delta_y V_y^{n+1/2} + c_{13} \delta_z V_z^{n+1/2}] \\
\sigma_{yy}^n &= \sigma_{yy}^{n+1} - \Delta t [(c_{11} - 2c_{66}) \delta_x V_x^{n+1/2} + c_{11} \delta_y V_y^{n+1/2} + c_{13} \delta_z V_z^{n+1/2}] \\
\sigma_{zz}^n &= \sigma_{zz}^{n+1} - \Delta t [c_{13} \delta_x V_x^{n+1/2} + c_{13} \delta_y V_y^{n+1/2} + c_{33} \delta_z V_z^{n+1/2}] \\
\sigma_{yz}^n &= \sigma_{yz}^{n+1} - \Delta t c_{44} [\delta_y V_z^{n+1/2} + \delta_z V_y^{n+1/2}] \\
\sigma_{xz}^n &= \sigma_{xz}^{n+1} - \Delta t c_{44} [\delta_x V_z^{n+1/2} + \delta_z V_x^{n+1/2}] \\
\sigma_{xy}^n &= \sigma_{xy}^{n+1} - \Delta t c_{66} [\delta_x V_y^{n+1/2} + \delta_y V_x^{n+1/2}]
\end{aligned} \tag{6.2}$$

And the received reflections are treated as source signals added in the normal stress terms during wavefield backward propagation.

6.4 Imaging condition

Standard imaging conditions are based on crosscorrelation or deconvolution of the reconstructed wavefields (Claerbout, 1971). Most imaging conditions are designed to be applied in the acoustic case. In acoustic RTM, therefore, wavefield reconstruction is carried out with the acoustic wave

equation using the recorded scalar data as boundary conditions. In contrast, for elastic RTM, reconstruction is done with the elastic wave equation, using the recorded vector data as boundary conditions. Elastic RTM has the same logical structure as acoustic RTM: the source and receiver wavefield reconstruction and imaging condition application. The source and receiver wavefields are reconstructed by forward and backward propagation in time with an FD scheme.

6.5 Simulation and Comparison with 2D borehole RTM

In this section, 2D synthetic data sets are first analyzed to examine and expose some features of the problem of azimuthal ambiguity. The velocity model used to generate the 2D synthetic data includes a horizontal well, where a borehole horizontally locates in the middle. A corresponding 3D synthetic model with two interfaces parallel to a vertical borehole is also generated.

In Figure 6.1, a water-filled well horizontally penetrates into a fast formation (formation in red, with its V_p and V_s being 4000 m/s, 2300 m/s, respectively). The influence of the sonic tool is also taken into consideration in this model, where we set the V_p and V_s velocity and the density of the tool as 5860 m/s, 3300 m/s, 7850 kg/m^3 , respectively. An interface with a dip angle of 15° on the top of the model (formation in yellow, with its V_p and V_s being 3000 m/s, 1800 m/s, respectively). Let the sonic tool move from left to right at a starting poing of ($x=2 \text{ m}$, $z=6 \text{ m}$) and set the distance from the source to the first receiver as 3.27 m, with two arrays (each array has 13 hydrophones with a spacing of 0.15 m) of receivers sitting on both sides of the borehole wall. A data set of all together 40 shots is thus generated with a total recording time of 0.01 s (recording time sample is $5e-7 \text{ s}$).

In Figure 6.2, reflection signals of the synthetic model recorded by upper and lower receiver arrays are present respectively in Figure 6.2a and Figure 6.2b (the energy of reflection signal in Figure 6.2b is multiplied by 5 times). Although the interface is on the upper layer of the model and the borehole fluid and acoustic tool act as obstacles, preventing the reflections from the upper interface being received by the lower array of the receivers, there is still considerable reflection energy

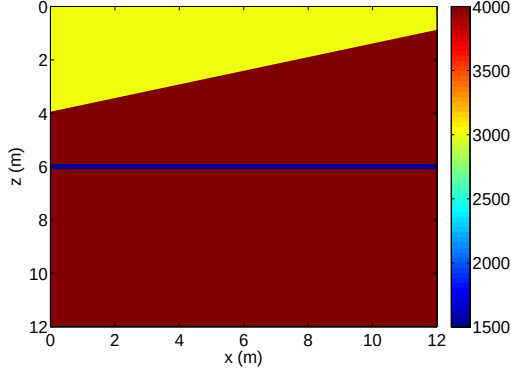


Figure 6.1: The synthetic model of a horizontal well filled with water (blue) horizontally penetrates into a fast formation (red). A dip interface locates on the top of the model.

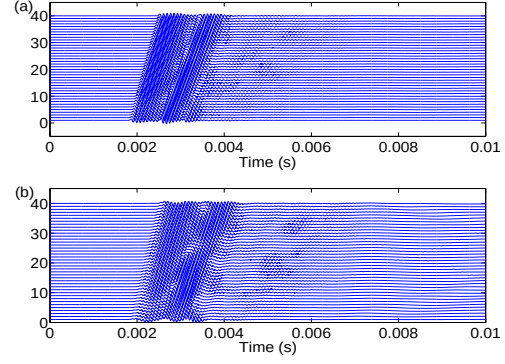


Figure 6.2: The reflection signals of the synthetic model: (a) The reflections recorded by the upper array of receivers; (b) the reflections from the lower array of receivers.

received by the lower receivers, which will inevitably have a negative influence on the imaging result. As a result, only reflections from the upper receiver array are used to the next migration step. The imaging result is shown in Figure 6.5. Because of the intrinsically azimuthal ambiguity in 2D environment, the borehole RTM cannot tell which side the reflections are coming from. The reflection energy is focused on both sides of the borehole, which produces an artifact reflector positioned opposite to the true interface, with the measurement surface acting as a symmetry plane.

In Figure 6.3, a dipping layer with interfaces on both sides of the horizontal geometrical model are present. The parameters of formations for the upper and middle layers are the same as those in Figure 6.1. The V_p and V_s velocities in the lower layer are $4500m/s$ and $2600m/s$ respectively. The borehole radiation mechanisms and receiver response (source, receiver arrays and the correspondent parameters such as offset and receivers spacing) are the same as the previous model. In Figure 6.4, reflection signals of the synthetic model are illustrate(reflections recorded by the upper and lower array of receivers are illustrated in 6.4a and b, respectively,the energy of reflection signal in (b) is multiplied by 5). Because of the differences in the geometry and positions of the two interfaces, the reflection energy received by the lower array of receivers is smaller than that received by the upper ones. Nevertheless, by inspection of the diagram it is clear that the lower receivers can

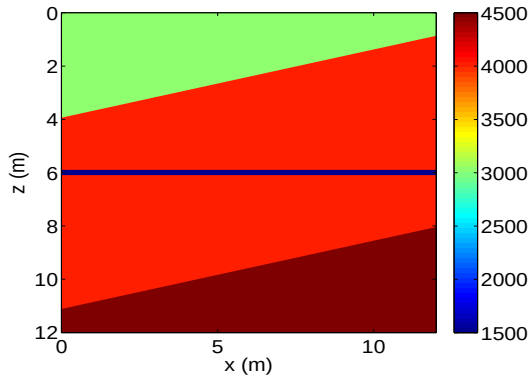


Figure 6.3: The synthetic model of a horizontal well filled with water (blue) horizontally penetrates into a fast formation (red).

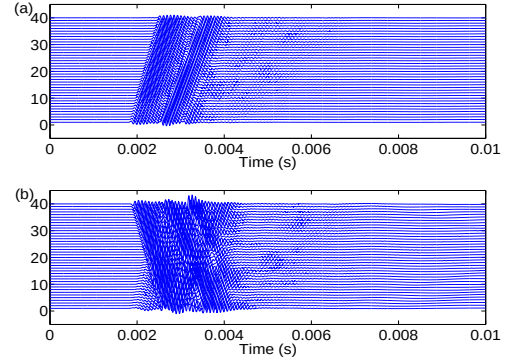


Figure 6.4: The reflection signals of the synthetic model: The reflections recorded by the upper array of receivers (a) and lower array of receivers (b).

still receive the reflection signals from the upper interface. The imaging result is shown in Figure 6.6. The two interfaces are now distributed on both sides.

Tang (2004) successfully calculated the strike of the reflector outside the borehole using the shear wave directional information. However, this technique is only available in the presence of a dipole source. Wang et al. (2015) found the arrival times of reflections in different azimuth receivers of the monopole tool are different, based on which, he successfully determined the strike of the reflector. However, in principle with the correct sensing device and sufficiently realistic physical assumptions, a more direct way of resolving azimuthal ambiguity, by applying a 3D borehole RTM is applied to the reflections recorded by 8 omnidirectional hydrophones located azimuthally around the tool.

In Figure 6.7, the formation between the two reflectors is a slow VTI formation whose elastic

parameters are,

$$c_{VTI} = \begin{bmatrix} 23.87 & 15.33 & 9.79 & 0 & 0 & 0 \\ 15.33 & 23.87 & 9.79 & 0 & 0 & 0 \\ 9.79 & 9.79 & 15.33 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.77 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.77 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.27 \end{bmatrix}. \quad (6.3)$$

The interface in brown is 2 m away from the well, whereas the interface in gray on the other side is 2.5 m away from the borehole. The formation outside the two interfaces is isotropic ($V_p = 4000\text{m/s}$, $V_s = 2300\text{m/s}$). A dipole source with a central frequency of 2000 Hz emits energy towards the x axis. The 8 receiver arrays are evenly spaced around the well, with 20 hydrophones in each array. The distance between the nearest hydrophone to the dipole source is 1 m and the hydrophone spacing of each array is 0.15 m. The total recording time is 1 ms with a time sample of $5\ \mu\text{s}$. Figure 6.8 shows a cross-section profile of the 3D model in x-z plane. Following the work flow proposed by Li et al. (2014b), the snapshots of the forward wavefield propagation in x-z plane from borehole fluid to the formation outside the borehole are shown in Figure 6.9, with the time increasing from 1.5 ms to 5.25 ms. When the wavefield propagates to the interfaces, the reflections on both sides of the borehole are generated and thereafter propagate back from the interfaces to the borehole. The arrival times of the reflections on both sides are different. The imaging result for one shot of the 3D model are then shown in Figure 6.10. The two interfaces are focused in the correct locations. The one on the left side denotes the horizontal cross section of the brown interface in Figure 6.7. The center of the well is in position ($x=80$, z), with a H-PML layer of 15 grid points outside each surface of the model, the distance between left reflector to the center of the well is 40 grids or 2 m; the reflector on the right side is 50 grids or 2.5 m away from the borehole.

To make a comparison, a monopole source is applied in the next synthetic model, where 8 receiver arrays are evenly spaced around the well, with 20 hydrophones in each array. In Figure

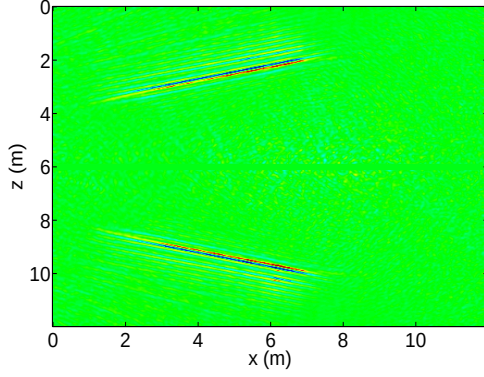


Figure 6.5: Imaging result of the upper interface model, shown in Figure 6.5. The upper structure is the real interface, however, the lower structure is the artificial reflector caused by azimuthal ambiguity.

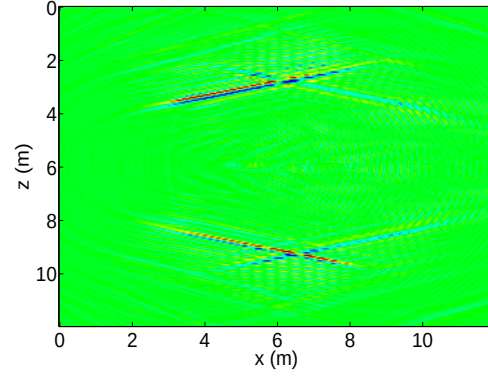


Figure 6.6: Imaging result of the model with interfaces on both sides of the well. The artificial reflectors caused by azimuthal ambiguity are present both in upper and lower parts of the well.

6.11, an interface in brown is 2 m away from the well with a strike perpendicular to x axis, whereas the interface in gray on the other side has a strike of 45° from the borehole. The elastic parameters of the formations outside the borehole are the same as those in previous model. To better illustrate the geometrical aspects of this solution, a cross-sectional profile of the 3D model in x - z plane is shown in Figure 6.9 shows. The distance between the nearest hydrophone to the monopole source is 1 m and the hydrophone spacing of each array is 0.15 m. The imaging result for all together 15 shots of the 3D model is plotted in Figure 6.13. The two interfaces are also focused in the right locations, which demonstrates the 3D borehole RTM can solve the azimuthal ambiguity even with a monopole source.

6.6 Conclusions

Most of the methods and principles for borehole acoustic reflection imaging, to date, are fundamentally 2D. Within such a framework, the distance and dip angles of the structures such as vugs and fractures outside a borehole can then be delineated by means of borehole migration and imaging. However, azimuth information concerning the structures away from the borehole is not used

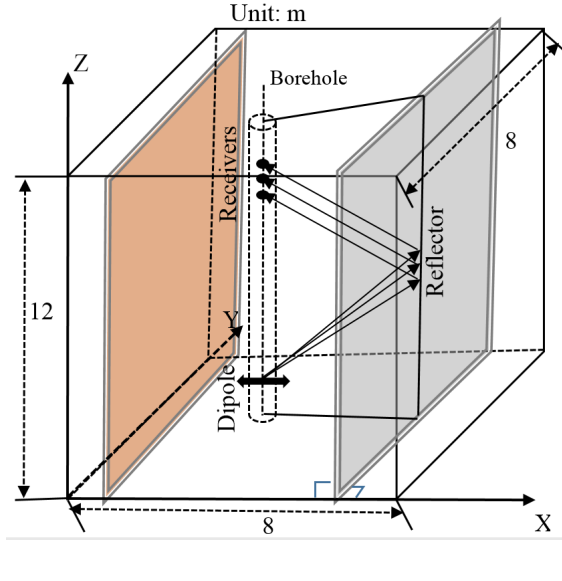


Figure 6.7: The 3D profile of the VTI model with a dipole source and 8 evenly spaced hydrophones.

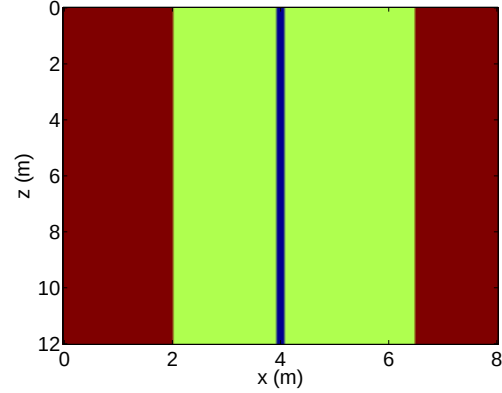


Figure 6.8: A cross-section profile of the 3D model in x-z plane.

in these approaches and consequently structures associated with a single azimuth appear spread out across all azimuths.

Given tools for 3D modelling and reverse-time migration (such as those developed in this thesis), and supported by sensing instrumentation with the intrinsic ability to distinguish the directionality of the incoming field, this issue is in principle possible to overcome. In this chapter, this ambiguity resolution is examined using two horizontal borehole models. A 3D borehole RTM methodology is proposed in this chapter, taking the advantage a borehole logging device design involving 8 omnidirectional hydrophones evenly spaced around the borehole to receive reflections from all directions. The imaging results of the 3D synthetic model show the azimuthal ambiguity problem can be resolved in this way; in fact, the conclusion holds true even with a monopole source.

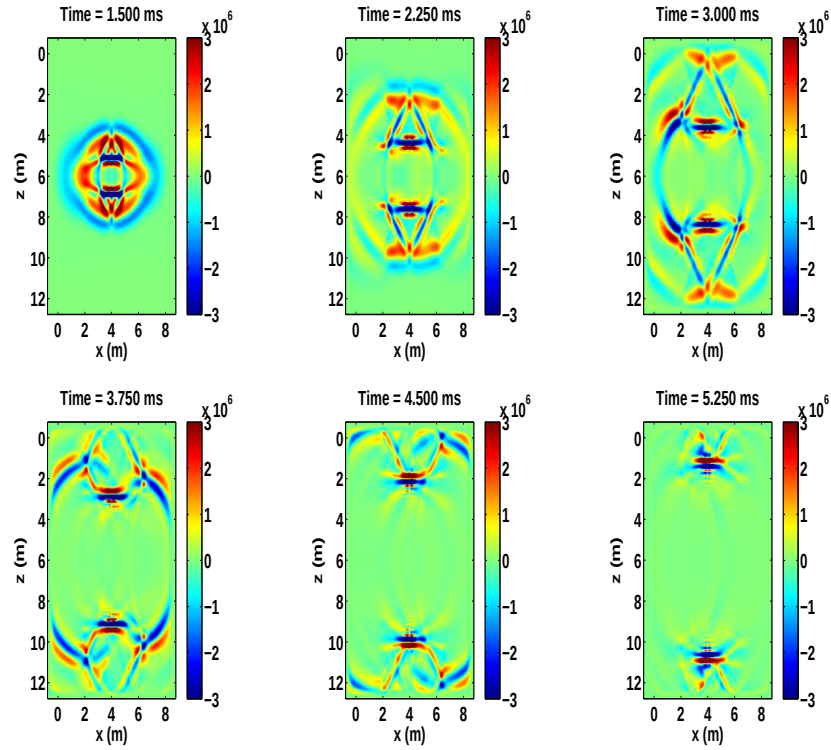


Figure 6.9: The snapshots of the forward wavefield propagation in x-z plane from borehole fluid to the formation outside the borehole with the time increasing from 1.5 ms to 5.25 ms.

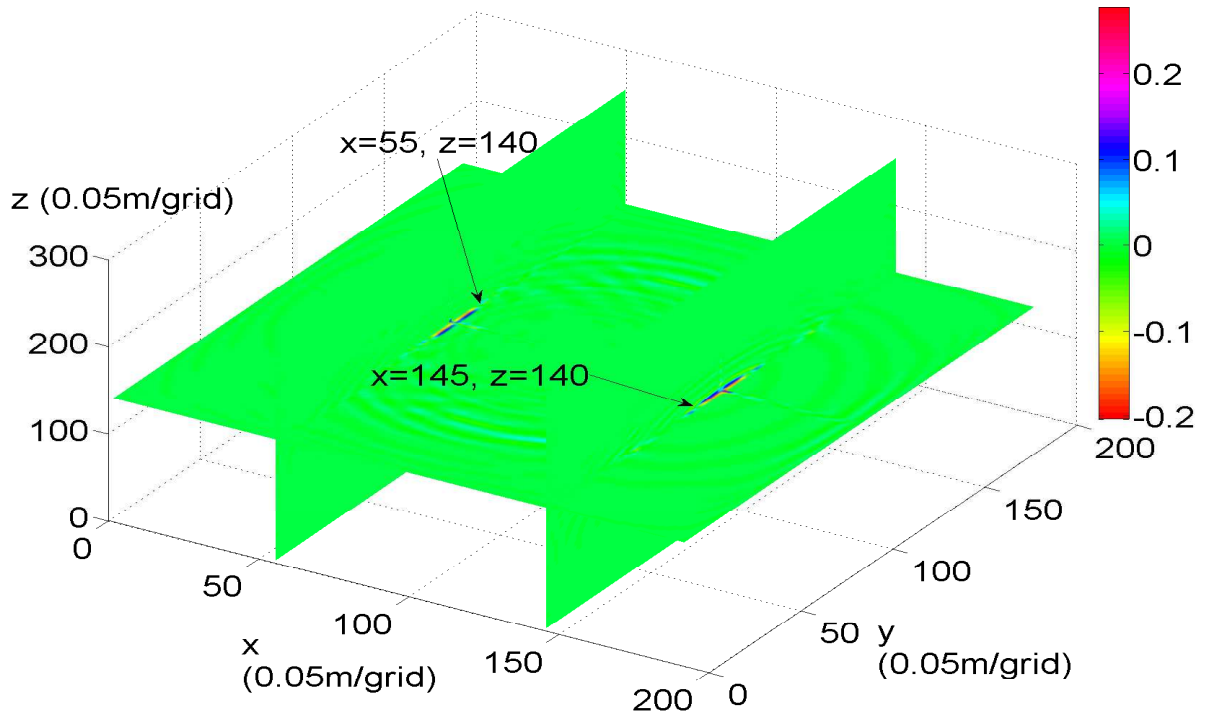


Figure 6.10: The imaging result for one shot of the 3D model.

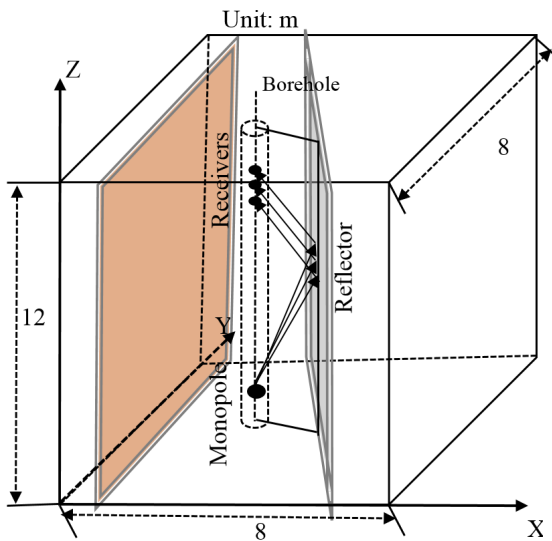


Figure 6.11: The 3D profile of the VTI model with a monopole source and 8 evenly spaced hydrophones.

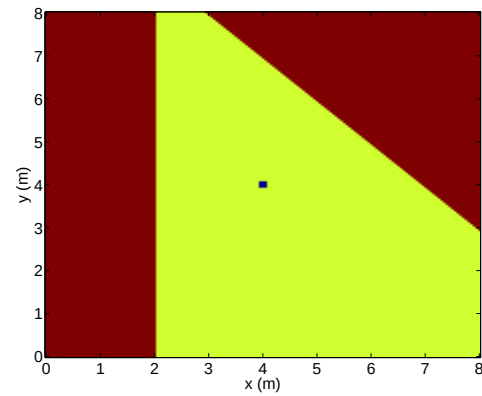


Figure 6.12: A cross-section profile of the 3D model in x - z plane.

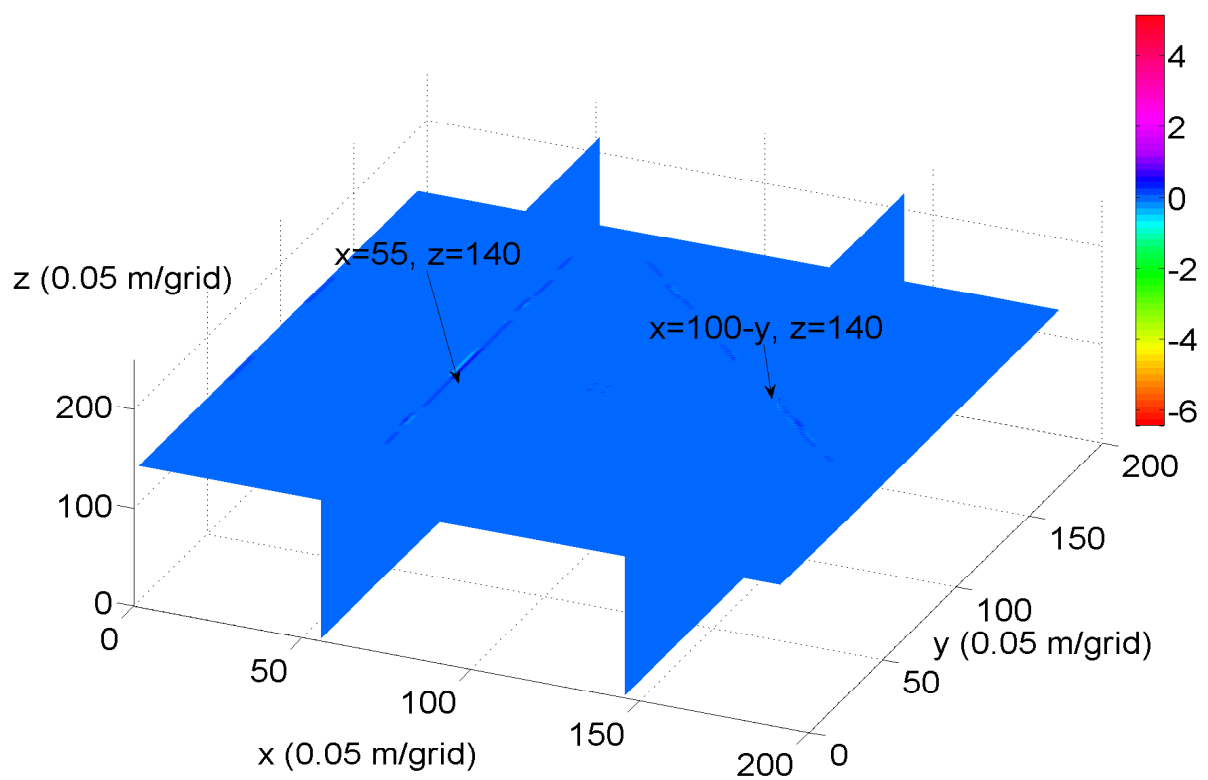


Figure 6.13: The imaging result for one shot of the 3D model.

Chapter 7

Summary and Future study

7.1 Summary

In this thesis, a three-dimensional staggered-grid FDM is developed to simulate wavefield propagation in both isotropic and anisotropic media. At and around borehole fluid and formation boundaries, the harmonic average method is applied to the elastic moduli related to shear stress tensors during the stress components update. For suppression of the artificial boundary reflections, standard PML, C-PML and M-PML methods can be employed. A hybrid PML based on the C-PML and M-PML is proposed in this thesis. Comparisons with the C-PML and M-PML, indicate the new H-PML performs superior in both isotropic and anisotropic media.

The 3D elastic staggered-grid finite difference method is applied to the investigation of wavefield simulation for a directional dipole source. In an isotropic medium, the reflections observed at four evenly spaced receivers around the borehole show an angular dependence related to the geometry of the reflector. Furthermore, a transition is detected between the SH-SH reflection and SV-SV reflection with the increase of offsets. Analysis of the relationships between the borehole wavefield reception, radiation and reflection of S-S reflected signals show that the maximum S-S reflected amplitude occurs when the incident angle of S wave reaches its critical value (when total reflection occurs). Based on the cross-plot of maximum amplitude versus receiver offsets, the offset of maximum amplitude can be found, and used to determine the total travel distance. Both the distance between the borehole and the reflector and the critical angle can therefore be calculated. As a result, the shear wave velocity of the second layer outside a borehole can be obtained according to Snell's law.

In the VTI medium, the received waveforms recorded by the receivers is different from the isotropic medium. The SH-SH reflection coefficient in the VTI medium is introduced and used

to calculate the relationship between the incident angle and reflected amplitude. As a result, the maximum value of the received SH amplitude occurs when the wave propagates to the interface with a critical angle.

For modeling the propagation of seismic waves using FDM, different wave modes (P-, SV- and SH- wave) are to be simulated simultaneously, which causes the crosstalk from the interference of different modes. This crosstalk reduces the precision of imaging condition during migration and impedes the determination of formation parameter gradients during time domain full waveform simulation (FWI). The independent simulation of decoupled wave modes is applied to reduce this kind of crosstalk. In Chapter 3, analysis of the SH- reflection wave indicates the SH energy is strongest among the others. The SH- reflection can be used in the migration and imaging step to obtain structure information such as azimuth. Furthermore, the crosstalk caused by the interference from other modes is expected to be reduced in further time domain full wave form inversion (FWI). A temporal fourth-order scheme for solving the elastic SH wave equations in VTI media is proposed, which in the numerical analysis presented here suppresses the wrap-around and Gibbs' artifacts that have been observed in other methodologies when waves propagate through heterogeneous formations—especially in the presence of large and abrupt changes in the medium properties.

Before applying borehole RTM to the problem of acoustic reflection imaging logging, the process of reflection extraction is discussed in this thesis. The KL transform was investigated as a means to separate reflections from this variety of directly propagating wave events in acoustic reflection well logging data. Based on the simple assumption that significant energy differences characterize each signal component, the direct P- and S waves as well as the Stoneley wave can be effectively removed, as evidenced in synthetic testing. Comparisons with recent methods (i.e., the MSTC method) carried out both with synthetic and laboratory data, confirm that the KL transform is capable of providing reflection signal estimates with less noise and higher overall precision. It is particularly effective as a tool to enhance acoustic borehole imaging results, wherein these now-

separated upgoing and downgoing acoustic reflections are further distinguished from each other based on the acoustic array selection. When processing field data, a high pass filter should be applied at the outset to mitigate Stoneley signals, to make up for the inability of the KL transform to fully isolate Stoneley signals. With this additional step in place, field imaging results determined from the upgoing and downgoing reflections display a number of very promising features.

As a key procedure, the determination of the confidence level is discussed by analyzing the relationship between the confidence level and the energy distribution of signals transmitted by the source in the borehole. Additionally, to address the change of relative amplitude of each mode with the depth, I also tested the KL transform method by using homogeneous and heterogeneous models, respectively. For heterogeneous media, a more restrictive processing time window is suggested be used to most of the principle energy and exclude other modes. In the mean time, a division of the processing depth range should be enforced corresponding to abrupt changes of the lithology, ranges within which primary modes are maximally linear will reduce nonstationarity and the associated issues.

Given tools for 3D modelling and reverse-time migration (such as those developed in this thesis), and supported by sensing instrumentation with the intrinsic ability to distinguish the directionality of the incoming field, the azimuth ambiguity is in principle possible to overcome by a 3D borehole RTM methodology. The imaging results of the 3D synthetic model show the azimuthal ambiguity problem can be resolved in this way; in fact, the conclusion holds true even with a monopole source.

7.2 Future study

Accurate and efficient wave field simulation in 3D isotropic and anisotropic media using dipole source can help determine formation parameters such as P- and S- wave velocities. In this thesis, the SH- wave velocity is determined by analyzing the SH- reflection wave. The qP- and qSV- wave velocities can in principle also be determined by the same procedure. This is an important subject

for future research.

Pure SH wave simulation is proposed, involving a new PSTD method. Pure qP- and qSV-wavefield simulation can be carried out using the same method. The decoupled wavefield simulation can reduce the crosstalk introduced by elastic wavefield imaging condition. This is also an important area of ongoing and future research.

The 3D elastic wavefield forward modeling and migration can further be used in full waveform inversion (FWI) in time domain. In the first step of FWI, the conventional method of gradient calculation in time domain requires the computation and storage of the synthetic wavefield at each time step to correlate with the adjoint wavefield, which has a high memory cost. On top of this, poor input/output (I/O) of recorded data dramatically increases the total running time of the algorithm, further decreasing computational efficiency. The random boundary condition method that uses an increasingly random velocity region by replacing the conventional damped region is a possible method to reduce the high memory cost and time consuming of data I/O. For anisotropic media, a new random boundary condition based on the change of elastic constants in the damping layer can be developed based on the finite difference tools created in this thesis. Pursuing this is also an important area of ongoing and future research.

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