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UNIVERSITY OF CALGARY

Anelastic attenuation and anisotropy in seismic data: modeling and imaging

by

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A THESIS

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Abstract

Anisotropy and attenuation are critical to the modeling and analysis of seismic amplitude, phase, and traveltime data. Neglect of either of these phenomena, which are often both operating simultaneously, diminishes the resolution and interpretability of migrated images. In order to obtain an accurate image of subsurface earth, it is necessary to include both anisotropy and attenuation in inversion procedures as subsurface materials are far from being isotropic or elastic. In this thesis, I address attenuation and anisotropy problems such as modeling of wave propagation and compensating for attenuation effects in seismic images. To develop such analysis, I derive a time domain viscoacoustic constant-Q wave equation in isotropic and anisotropic media using an un-split field PML's scheme that is practically efficient and accurately simulates the constant-Q attenuation behavior and developing an Q-compensated reverse-time migration approach to compensate for attenuation effects in seismic images during migration. First, I investigate the accuracy of the new approach of viscoacoustic wave equation based on constant-Q theory. Most importantly, this approach separates attenuation and dispersion operators that allow us to mitigate both amplitude attenuation and phase dispersion effects in seismic imaging. This equation is the key modeling engine for seismic migration. Second, I present a method to improve image resolution by mitigating attenuation effects. I develop a Q-compensated reverse-time migration imaging approach (Q-RTM) and demonstrate this approach using different synthetic models. In Q-RTM, amplitude compensation happens within the migration process through manipulation of attenuation and phase dispersion terms in the time domain differential equations. Particularly, the back-propagation operator is constructed by reversing the sign only of the amplitude loss operators, but not the dispersion-related operators, a step made possible by reformulating the absorptive equations such that the two appear separately. Numerical results further verify that this Q-RTM approach can effectively improve the resolution of seismic images, particularly beneath high-attenuation zones. Finally, I extend the viscoacoustic wave equations from isotropic media to transversely isotropic (TI) media. For imaging application, I discuss the stability condition and the artifacts of shear wave triplications. The results indicate that the stable anisotropic reverse time migration is accessible by taking off anisotropy around the selected high gradient points of tilt angle in areas of rapid changes in the symmetry axes.

Preface

The current PhD thesis is written in a manuscript-style format based three submitted papers and two technical reports. The full citation for chapter 2 is: Ali Fathalian, Kristopher A. Innanen, 2D finite-difference modeling and imaging of viscoacoustic wave propagation using a PML absorbing boundary condition, CREWES Research Report, Volume 28 (2016). The full citation for chapter 3 is: Ali Fathalian, Kristopher A. Innanen, Approximating constant-Q reverse time migration in the time domain, submitted in Geoscience, Springer. The full citation for chapter 4 is: Ali Fathalian, Daniel O. Trad, Kristopher A. Innanen, An approach for attenuation-compensating multidimensional constant-Q viscoacoustic reverse time migration, submitted in Geophysics. The full citation for chapter 4 is: Ali Fathalian, Kristopher A. Innanen, Viscoacoustic VTI and TTI wave equations and their application for anisotropic reverse time migration: Constant-Q approximation, CREWES Research Report, Volume 29 (2017). The full citation for chapter 4 is: Ali Fathalian, Daniel O. Trad, Kristopher A. Innanen, Viscoacoustic reverse time migration in TTI media with attenuation compensation, submitted in Geophysics.

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Dedication

To Sahar.

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Chapter 1

Introduction

Seismic techniques currently being used for seismology and exploration geophysics problems usually are based on non-attenuating acoustic, elastic and anisotropic wave propagation because of complexity in attenuating media. Anelastic properties of the subsurface media cause phase dispersion and amplitude loss of seismic wave, especially in high-attenuation areas such as regions involving gas. To improve the accuracy of seismic methods in attenuating media, we must add the attenuation effects to seismic methods and enhance the resolution of the migrated image. In this dissertation, we develop seismic modelling, inversion, and imaging in attenuating media for isotropic and anisotropic structures to improve the accuracy of seismic exploration techniques.

1.1 Attenuation and dispersion of seismic waves

Seismic waves are subjected to the anelasticity and complicated structure of the Earth which modify the propagating waveforms. These modifications usually include an amplitude loss and many types of amplitude and phase variations because of the heterogeneity of the elastic structure. Amplitudes of seismic signals are reduced, and their waveforms are distorted during wave propagation due to the inelasticity of the Earth materials. The reduction of amplitude is also associated with an energy loss carried by the wave, which is usually described as attenuation and is an important characteristic in modern seismology which needs to be studied.

The most common way to account for attenuation effects in seismic modeling approaches is the phenomenological dimensionless quantity Q, which is called the quality factor. This quantity is expected to lump together all the mechanisms of attenuation without considering their detail. A general definition of Q is given by the ratio of the maximum energy stored during a cycle to the energy loss during the cycle (Aki and Richards, 2002). Kolsky (1956) and Lomnitz (1957) present a linear description of the absorption that could account for the observed frequency independence. O'connell and Budiansky (1978) define Q in terms of the mean stored energy and energy loss during a cycle.

Based on linear theory of wave propagation, there is a wide range of mathematical definitions of the Q model to describe the attenuation and the attendant dispersion. To approximate a specific viscoelastic model with a certain Q-versus-frequency relation an array of standard linear solids in parallel can be used. A viscoelastic model consisting of a series of standard linear solids in parallel can nearly approximate a constant Q over a specified frequency range. Since attenuation and dispersion are related through a Kramers-Kronig relation (Futterman, 1962), the constant-Q model also yields an equally realistic dispersion relation. The nearly constant- model given by (Futterman, 1962) is independent of frequency for frequencies above a certain characteristic value, which can be chosen low enough to be outside our interest frequency range. Nearly constant Q (NCQ) models include at least one parameter that is in some way related to the range of frequencies over which the model gives Q nearly independent of frequency. For all of NCQ models (Lomnitz, 1957; Futterman, 1962; Strict, 1967; Liu et al., 1976) the physical implications of the cutoff parameters are different, which can be chosen quite arbitrarily.

Kjartansson (1979) presents a linear description of attenuation with Q exactly independent of frequency, without any cut-offs. This constant-Q model is completely specified by two parameters, e.g., Q and phase velocity at an arbitrary reference frequency. The simplicity of the constant-Q description allows the derivation of exact analytical expressions for the various frequency domain properties such as phase velocity, and attenuation coefficient, that is efficient over any range of frequencies with any positive value of Q. However, when the frequency range is restricted, and the losses are small, the results obtained from the nearly constant-Q model approach the same limit as those obtained from the constant-Q model.

1.2 Compensation of seismic attenuation

Seismic attenuation of real earth causes waveform distortion and energy loss of seismic wave. Seismic attenuation in the laboratory and the field is widely observed in rocks. In earth media, however, high-attenuation areas are found in gas hydrocarbon fields (Dvorkin and Mavko, 2006; Bear et al., 2008) that are located underneath gas clouds. Generally, high attenuation areas cause significant loss of signal strength and bandwidth. In such regions, the waves that travel through them are attenuated and reflected events have lower amplitude and frequency content, especially beneath the strongly attenuating area. To accurately simulate wave propagation in real media, attenuation and associated dispersion effects should be taken into account in seismic modelling approaches, to restore the high frequencies to enhance the seismic resolution, and to correct the phases related to arrival timings of reflections.

The compensation of attenuation is addressed by many authors to model acoustic attenuation effects during wave propagation. In general, attenuation compensation in geophysics can be approximately classified into two classes: seismic record-based compensation and propagation-based compensation. The earlier classification of compensation methods includes time-varying deconvolution (Clarke, 1968; Griffiths et al., 1977; Margrave et al., 2011), time-variant spectral whitening (Yilmaz, 2001) and inverse Q filtering (Hargreaves and Calvert, 1991; Wang, 2002). These record-based compensation methods are directly carried out on attenuated seismic records in the time or frequency domain. Inverse Qfiltering method is the earliest efforts at compensating for attenuation loss in seismic data. These methods partially correct for the attenuation loss that occurs during wave propagation, and therefore, Q compensation is required during migration.

Propagation-based compensation methods include Q-compensated one-way wave equation migration (Dai and West, 1994; Mittet et al., 1995; Yu et al., 2002; Mittet, 2007; Zhang et al., 2012), Q-compensated reverse time migration (Zhang et al., 2010; Zhu et al., 2014; Sun et al., 2016) and Q-compensated Gaussian beam migration (Bai et al., 2016). Since amplitude absorption and phase dispersion occur during wave propagation, it is more physically consistent to compensate these effects during migration (Zhang et al., 2010; Zhu et al., 2014; Sun et al., 2015). However, the high-frequency ambient noise cause instability and affects the amplitude compensation during migration (Sun and Zhu, 2015; Xue et al., 2016).

1.3 Anisotropy

Anisotropy is an essential concept in geophysical exploration. Seismic anisotropy is used to describe the directional dependence of seismic wave speed in subsurface media in the Earth and is defined as the directional variation of seismic properties (Crampin, 1966, 1989). Seismic anisotropy has become increasingly more important with long offset seismic data with greater angles of incidence (the angle-dependence of velocity is more evident), wide azimuth, amplitude versus offset (AVO) quantitative analysis, and anisotropic migration. For interpretation and processing of these data, understanding of anisotropy in the subsurface is required. The assumption of isotropy is still prevalent because of the inability to measure enough parameters in the field to fully characterize the anisotropic elasticity tensor that is a needed input to many processing, inversion and interpretation algorithms. The isotropic acoustic assumption can lead to low resolution and misplaced images of subsurface structures. To overcome this problem, for imaging of data characterized by strong anisotropic effects, an anisotropic migration method is required to obtain a significant improvement in image quality, accuracy, and positioning of reflectors.

Fine-layering and parallel cracks in the structure of rocks can induce anisotropic effects in the wave propagation. For imaging of these structures, the vertical transversely isotropic (VTI) reverse-time migration (RTM) is a useful simplification because the medium is a stack of thin isotropic layers that induce apparent anisotropic effects. However, for imaging under a steeply dipping anisotropic overburden such as shale masses overlying dipping salt flanks, the symmetry axis is most likely to be tilted, and the VTI assumption may not be satisfied. Ignoring the tilted symmetry axis can lead to image blurring and mispositioning of salt flank and degrades and distorts the base of the salt and subsalt images (Zhang et al., 2011). In order to image anisotropic media, pseudo-acoustic RTM and pure acoustic RTM methods are two ways to achieve wave reverse time migration (RTM). Alkhalifah (1998) first proposed the pseudo-acoustic equation for transversely isotropic (TI) media by setting the shear wave velocity along the symmetry axis to be zero using the acoustic TI approximation. Based on Alkhalifah's pseudo-acoustic approximation, Zhou et al. (2006b); Du et al. (2008) and Duveneck et al. (2008) developed and simplified the pseudo-acoustic equation to account for VTI media giving expressions that are more convenient for numerical implementation. The non-vertical but locally variable assumption is extended these developments from VTI to TTI (Zhou et al., 2006b; Zhang and Zhang, 2008; Fletcher et al., 2008). This allows imaging algorithms to contain anisotropy because of spatially variable structures.

Tilting the symmetry axis from the vertical causes numerical dispersion and instability. None of the transversely isotropic acoustic wave equations mentioned above are really free of shear waves (Grechka et al., 2004), because these equations set the value of the shear wave velocity to zero only along on the symmetry axis. The instability problem is often alleviated using smoothing the anisotropic parameters before modeling or migration (Zhang and Zhang, 2008). Instead of model smoothing, Yoon et al. (2010) proposed an approach to reduce the instability by setting $\varepsilon = \delta$ in regions with rapid dip angle variations. However, differences in the anisotropic parameter models still produce shear wave artifacts. To avoid the unwanted shear wave mode, a number of different approaches have been proposed to model the pure P-wave mode (Etgen and Brandsberg-Dahl, 2009; Liu et al., 2009; Pestana et al., 2011) for the VTI case.

1.4 Contributions

The novel contribution of this dissertation is to combine attenuation and anisotropy theory into practical seismic techniques. The contributions are the following:

1. An unsplit-field PML formulation for the viscoacoustic wave equation in the time domain using a series of standard linear solid mechanisms is derived. The unsplit-field formulation is mostly reflection free regardless of the incidence angle and frequency and provides excellent results with less computational cost and does not have the instability problem.

2. A novel viscoacoustic constant-Q wave equation that is efficient and accurately simulates constant-Q attenuation behaviour.

3. A Q-RTM approach to mitigate for attenuation effects in reverse-time migration for isotropic and anisotropic structures.

1.5 Thesis overview

In this thesis, I focus on understanding the attenuation effects on seismic wave propagation and finding a practical solution to address the attenuation effects in seismic images. The main objective of this thesis is to develop a collection of workable and practically affordable numerical methods for seismic exploration in attenuating media from isotropic and anisotropic structures. The thesis is organized as follows.

Chapter 2 presents the unsplit-field PML formulation for the 2D viscoacoustic wave equation in the time domain with definition of attenuation and the quality factor and review the attenuation models for seismic modelling in attenuating media, i.e., the Standard Linear Solid (SLS).

Chapter 3 investigates the efficiency of attenuation modeling approaches based on the SLS model. It is shown that the single SLS model is the most efficient and accurate in practical seismic applications using the unsplit-field wave equation explained in chapter 2.

Chapter 4 presents a new approach of the viscoacoustic wave equation in the time domain that is based on constant-Q theory. In this chapter, it is shown how to implement seismic imaging in attenuating media and how to improve the image resolution by compensating attenuation effects. The proposed formulation is tested for 2D and 3D example during the forward modeling and backward wave propagation. The results show that this approach can compensate for the attenuation effects during time-reverse modeling.

Chapter 5 presents time-domain approximate constant-Q wave propagation involving a series of standard linear solid (SLS) mechanisms in anisotropic media. The unsplit-field viscoacoustic wave equations that were explained in chapter 4 are extended from isotropic media to transversely isotropic (TI) media, including VTI and TTI media. For imaging applications, the stability condition and the artifacts of shear wave triplications are discussed. Results show that a stable anisotropic reverse time migration is accessible by taking off anisotropy around the selected high gradient points in areas of rapid changes in the symmetry axes.

Chapter 6 contains a time-domain viscoacoustic RTM imaging algorithm (Q-RTM) in tilted TI media based on a series of standard linear solid mechanisms, which mitigates attenuation

and dispersion effects in migrated images. I show the basic principle of Q-RTM by different synthetic models. Numerical tests on synthetic data illustrate with sufficiently accurate Q and velocity models, the TTI Q-RTM can produce better images than isotropic RTM, especially in areas with anisotropy, attenuation and strong variations of dip angle. Chapter 7 contains a summary and conclusions of this thesis.

Chapter 2

An unsplit-field PML formulation for the 2D viscoacoustic wave equation in the time domain

2.1 Abstract

Simulation of wave propagation in a constant-Q viscoacoustic medium is an important problem, for instance within Q-compensated reverse-time migration. Such wave propagation can be modelled with a finite difference scheme by introducing a series of standard linear solid mechanisms, and it can be carried out within a computationally tractable region by making use of perfectly-matched layer (PML) boundary conditions. An efficient un-split field PML scheme is derived in this chapter by introducing appropriate auxiliary variables and their associated partial differential equations. The scheme is tested on a homogeneous velocity model and a modified Marmousi velocity model. I validate and examine the response of this equation by using it within a reverse time migration scheme.

2.2 Introduction

The crux of the trade-off between introducing boundary artifacts and computing within tractably small domains is the management of boundary conditions. Absorbing boundary conditions (ABCs) are often used in numerical simulations of wave propagation in unbounded problems to absorb outgoing waves. There are a number of ABCs in common use in finite-difference modelling of acoustic and elastic wave propagation. The introduction of a lossy material layer to attenuate fields near the computational boundary was discussed by Cerjan et al. (1985) and Levander (1985). The ABCs of Clayton and Engquist (1977), which are based on a paraxial approximation of solutions of the elastic wave equation, have been effective in many applications but with this approach artificial reflections are still identifiable at the edges of the computational domain. Although these ABCs are successful in many applications, they provide only limited absorption to waves in a specific range of incidence

angles and frequencies. Besides, none of these ABCs can be applied to problems where a dipping interface intersects the outer boundary.

The perfectly matched layer (PML) was introduced by Berenger (1994) for numerical simulation of electromagnetic wave propagation. In the PML approach, rather than applying an absorbing boundary condition, an absorbing boundary layer is introduced on the edges of the grid. The wave is attenuated by the absorption and decays exponentially when it enters the absorbing layer. The PML approach absorbs very effectively over a wide range of angles, and is relatively insensitive to frequency, and for these reasons is widely applied. Berenger's original formulation is referred to as a split-field PML, because it splits the variables into two independent parts in the PML region. Later formulations, such as the uniaxial PML (UPML), employ within the PML region an ordinary wave equation with a combination of artificial anisotropic absorbing materials, and have been regularly applied because of their simplicity and efficiency. PML applications for more complex elastic media (Chew and Liu, 1996), poroelastic media (Zeng and Liu, 2001), and anisotropic media (Bécache et al., 2003) have been analyzed, and variants such as convolutional PML (Kuzuoglu and Mittra, 1996) and multi-axial PML (Meza-Fajardo and Papageorgiou, 2008), and hybrids (Li et al., 2017), have been devised to further suppress boundary reflections. PML is mostly reflection free regardless of the incidence angle and frequency (Chew and Liu, 1996; Zeng and Liu, 2001; Festa and Nielsen, 2003). Although there are weak reflections associated with the discretization, PML provides excellent results with less computational cost and does not have the instability problem (Mahrer, 1986).

Attenuation affects the amplitude and phase of propagating seismic wave, especially below high-attenuation areas, which may lead to weak accuracy and incorrect position of reflectors in a migrated image. In this chapter, the PML ABC is extended to the acoustic wave equation in attenuation media and an unsplit-field PML formulation for the 2-D viscoacoustic wave equation is derived. I describe the PML for viscoacoustic medium and give numerical results using test simulations. I use two synthetic examples to test the accuracy of the proposed method in application to modeling and imaging in attenuating media.

2.3 Attenuation model for seismic wave propagation

In reflection seismology, wave propagation is described commonly by using an acoustic or an elastic wave equation. The main assumption is that the seismic wave propagates in the heat-insulated media, i.e, it will continue infinitely without the attenuation. However, the Earth is generally anelastic in nature and due to internal friction, seismic waves lose energy as they propagate. The attenuation of seismic waves is due to three effects: geometric spreading, intrinsic attenuation, and scattering. Intrinsic (viscoelastic) attenuation is energy lost to heat and internal friction during the passage of an elastic wave.

The wavenumber in attenuating media, which in the acoustic case is $k = \omega/c_0$, where c_0 is the phase velocity and ω is the angular frequency, takes on additional real and imaginary terms. A plane wave propagating in the x direction, for instance, becomes $\exp(-\alpha x) \exp(ikx)$, where $\alpha(\omega)$ is the attenuation coefficient. This coefficient is related to the seismic quality factor Q by

$$\alpha(\omega) = \frac{\omega}{2c_0 Q},\tag{2.1}$$

where ω is the angular frequency, and c_0 is the wavespeed. A common form for the real and imaginary parts of α , which is associated with a causal, linear impulse response and a Q which is approximately constant on the seismic frequency band (Aki and Richards, 2002) arises with the replacement of $k = \omega/c_0$ by

$$K = \frac{\omega}{c_0} \left[1 + \frac{i}{2Q} - \frac{1}{\pi Q} \ln \left(\frac{\omega}{\omega_r} \right) \right], \qquad (2.2)$$

where ω_r is a reference frequency at which the wave field propagates with the phase velocity c_0 . This implies a complex velocity

$$v(\omega) = \frac{\omega}{K} = c_0 \left[1 + \frac{i}{2Q} - \frac{1}{\pi Q} \ln\left(\frac{\omega}{\omega_r}\right) \right]^{-1}, \qquad (2.3)$$

whose real part describes the wave dispersion:

$$v_p(\omega) = c_0 \left[1 + \frac{1}{\pi Q} \ln \left(\frac{\omega}{\omega_r} \right) \right].$$
(2.4)

These altered aspects of wave propagation generate the gross properties of the viscoacoustic wave, such as its amplitude losses, but also its more subtle features, such as phase changes

during propagation, and characteristic frequency dependence in the reflection coefficient. The reflection coefficient for viscoacoustic medium can be written as (Appendix A)

$$R(z,\sigma,\omega) \simeq \frac{i\omega}{2c_0\cos\theta_0} \bigg\{ \alpha(z) - 2\zeta(z)F(\omega) + \beta(z) \bigg[1 + \cos\sigma \bigg] \bigg\},$$
(2.5)

where $\theta_0 = \sigma/2$ is the incidence angle, σ is the opening angle, $\alpha(z)$ is the wavespeed perturbation, $\beta(z)$ is the density perturbation, $\zeta(z) = 1/Q$ is the attenuation parameter, and $F(\omega) = i/2 - 1/\pi [ln(\omega/\omega_r)].$

For this nearly constant Q model (Aki and Richards, 2002), the attenuation coefficient is linear with frequency ω , and the phase velocity v_p is slightly dependent on frequency. In Figures 2.1a and 2.1b the phase velocity and reflection coefficient over a 120Hz band are plotted for different Q values. Q has a strong influence on velocity dispersion, which for low Q values (e.g., Q=20) becomes a dominant feature of wave propagation (Figure 2.1a). Especially at low Q, there will appear in the waveform greater phase delays at lower frequencies. The spectra of reflection coefficients (Figure 2.1b), meanwhile, are suggestive that reflection strengths deviate from their acoustic counterparts as ω deviates from ω_r , weakening towards lower frequencies. The attenuative reflection coefficient approaches its acoustic counterpart as $k \longrightarrow k_r$; the variability of R with f increases away from the reference frequency. Numerical viscoacoustic simulations must predict both these overt and subtle features of wave-medium interaction. Because dispersion relations like that in equation 2.3 lead to complex forms in the time domain, viscoelastic/viscoacoustic models involving loworder stress and strain derivatives are usually adopted for time-domain numerical solutions. Nearly constant Q models can be simulated with suitable parallel arrangements of viscoelastic standard-linear solid spring-dashpot elements (Liu and Tao, 1997; Day and Minster, 1984; Carcione et al., 1988a; Carcione, 2007a).

2.4 Complex stretching of coordinates

Constructing a perfectly matched layer is based on analytic continuation of the spatial coordinate to the complex domain using stretching functions inside the PML region (Chew and Liu, 1996; Zeng and Liu, 2001; Liu and Tao, 1997; Teixeira and Chew, 2000). This amounts to an analytic continuation of the wave equation into complex coordinates, replacing propa-



Figure 2.1: Illustrate the frequency-dependence of viscoacoustic (a) phase velocity and (b) reflection coefficient R(f) for different value of Q (Q = 20 and Q = 100).



Figure 2.2: Illustrate an absorbing layer is placed adjacent to the edges of the computational region—a perfect absorbing layer would absorb outgoing waves without reflections from the edge of the absorber. There are three different PML absorbing regions ,PML for x direction (d(y) = 0), PML for y direction (d(x) = 0), and PML in the corners, both damping parameters d(x) and d(z) are positive, and either d(z) = 0 or d(x) = 0 inside the PML region in the x and z directions.

gating (oscillating) waves by exponentially decaying waves. In a 2D frequency-domain wave equation the x and z coordinates can be transformed into $\tilde{x}(x)$ and $\tilde{z}(z)$ complex coordinates, which inside the physical domain reduce to x and z, and outside the physical domain suppress reflections. Assuming that the region close to and inside the PML is linear and homogeneous (Figure 2.2), x and z appear in the differential equations only as a partial derivative. The wave equation is in this way expressed in complex coordinates, where ∂x and ∂z are replaced by $\partial \tilde{x}$ and $\partial \tilde{z}$ respectively. The complex stretch functions are defined as

$$s(x) = \frac{\partial \tilde{x}(x)}{\partial x},$$

$$s(z) = \frac{\partial \tilde{z}(z)}{\partial z}.$$
(2.6)

Since the stretch functions are complex functions of x and z, they can alternatively be expressed in terms of the damping parameter and scaling factor:

$$s(x) = b(x) \left[1 + i \frac{d(x)}{\omega} \right], \qquad (2.7)$$
$$s(z) = b(z) \left[1 + i \frac{d(z)}{\omega} \right],$$

where d is the damping coefficient in the PML region, b is the scaling factor (corresponding to stretching when b > 1 or compressing when 0 < b < 1), and ω is the temporal frequency.

In the interior region (i.e., the physical domain), the damping parameters are set to d(x) = 0 and d(z) = 0, and the scaling factors are set to b(x) = 1 and b(z) = 1, while in the PML region, the damping parameters are d(x) > 0 and d(z) > 0 and the scaling factors can differ from one. To observe the effect of the complex stretching of coordinates, I consider viscoacoustic wave propagation in a homogeneous volume. The viscoacoustic medium considered here is characterized by the constant velocity model, where the size of the grid is 401 × 401. The source signature is a zero-phase Ricker wavelet with a central frequency of 25 Hz. The grid spacing in the x and z directions is 4m. Figure 2.3 shows the snapshots at t = 26 s. I employ a standard stretch function, i.e, d(x), and d(z) > 0 and b(x) = 1, and b(z) = 1 in the PML region, which leads to

$$s(x) = \left[1 + i\frac{d(x)}{\omega}\right],$$

$$s(z) = \left[1 + i\frac{d(z)}{\omega}\right].$$
(2.8)

In Figure 2.3b the wave is plotted in the PML region under exponential damping. The wave is increasingly damped as the thickness of PML is increased (see Figures 2.3c and 2.3d).

2.5 Unsplit PML formulation for viscoacoustic wave equation

The 2D viscoacoustic wave field can be solved for through a system of first-order differential equations in terms of the particle velocities and stresses. In linear viscoelasticity, the basic



Figure 2.3: Effect of complex coordinate stretching on a 2D viscoacoustic wave propagation. (a) Wave propagating without PML, (b), (c) and (d) wave propagating into a PML with different thicknesses (the shaded region) when the damping parameters are grater than zero (d(x), d(z) > 0) and scaling factor are equal to one (b(x) = 1, b(z) = 1).

hypothesis is that the value of the stress tensor depends upon the time-history of the strain tensor. The viscoelastic hypothesis can be expressed as (Christensen, 1982)

$$\sigma = G(t) \ast \dot{\varepsilon},\tag{2.9}$$

where G(t) is the relaxation function and the symbol * denotes time convolution. The equations describing wave propagation in viscoacoustic media can be derived in terms of the stress relaxation function. Assuming pressure $P = -\sigma$ is viscoacoustic and σ is the stress, the equation of deformation (Hook's law) is written as (Robertsson et al., 1994)

$$\frac{\partial P}{\partial t} = -M_R \left(\nabla \cdot \mathbf{u} \right) \left[1 - \sum_{\ell=1}^L \left(1 - \frac{\tau_{\epsilon\ell}}{\tau_{\sigma\ell}} \right) \right]$$

$$-M_R \left[\sum_{\ell=1}^L \frac{1}{\tau_{\sigma\ell}} \left(1 - \frac{\tau_{\epsilon\ell}}{\tau_{\sigma\ell}} \right) e^{-t/\tau_{\sigma\ell}} \right] H(t) * \left(\nabla \cdot \mathbf{u} \right),$$
(2.10)

where $\tau_{\varepsilon \ell}$ and $\tau_{\sigma \ell}$ denote strain and stress relaxation times for the ℓ th mechanism, L is the number of relaxation mechanisms for a standard linear solid model, M_R is the relaxed modulus of the medium (Pipkin, 1986), H(t) is the Heaviside function, and $\mathbf{u} = \{u_x, u_z\}$ represents the particle velocity vector. Newton's second law completes the full description of wave propagation in a viscoacoustic medium. The linearized equation of motion in an anelastic medium can be written as

$$\frac{\partial u_x}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial x},$$

$$\frac{\partial u_z}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial z},$$
(2.11)

where $u_x(x,t)$ and $u_z(x,t)$ are the particle velocity components in the x- and z-directions respectively, P(x,t) is pressure wavefield, and $\rho(x)$ is density. Equations 2.10 and 2.11 together describe the deformation in a viscoacoustic medium. Equation 2.10 is expensive to solve by numerical modeling because of the associated convolution operation. The convolution terms are eliminated by introducing a memory variable term r_{ℓ} (Robertsson et al., 1994). Equation 2.10 thereby reduces to

$$\frac{\partial P}{\partial t} = -M_R \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) \left[1 - \sum_{\ell=1}^L \left(1 - \frac{\tau_{\varepsilon\ell}}{\tau_{\sigma\ell}} \right) \right] - \sum_{\ell=1}^L r_\ell, \tag{2.12}$$

where r_{ℓ} , which are referred to as memory variables (Carcione et al., 1988a), satisfy

$$\frac{\partial r_{\ell}}{\partial t} = -\frac{1}{\tau_{\sigma\ell}} r_{\ell} + \rho c_p^2 \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) \frac{1}{\tau_{\sigma\ell}} \left(1 - \frac{\tau_{\varepsilon\ell}}{\tau_{\sigma\ell}} \right), 1 \le \ell \le L,$$
(2.13)

For one relaxation mechanism (L = 1), which is sufficient for practical purposes (Blanch et al., 1995), the first-order linear differential equations for a 2D viscoacoustic medium can be written as

$$\frac{\partial P}{\partial t} = -K \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) \left(\frac{\tau_{\varepsilon}}{\tau_{\sigma}} \right) - r, \qquad (2.14)$$

$$\frac{\partial u_x}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial x},\tag{2.15}$$

$$\frac{\partial u_z}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial z},\tag{2.16}$$

$$\frac{\partial r}{\partial t} = -\frac{1}{\tau_{\sigma}}r + K\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}\right)\frac{1}{\tau_{\sigma}}\left(1 - \frac{\tau_{\varepsilon}}{\tau_{\sigma}}\right).$$
(2.17)

where K represents the bulk modulus of the medium. The stress and strain relaxation parameters, τ_{ε} and τ_{σ} , are related to the quality factor Q and the reference angular frequency ω as (Robertsson et al., 1994)

$$\tau_{\sigma} = \frac{\sqrt{1 + 1/Q^2} - 1/Q}{\omega}, \qquad (2.18)$$
$$\tau_{\varepsilon} = \frac{1}{\omega^2 \tau_{\sigma}}.$$

where ω is the central frequency of the source wavelet.

In order to introduce the PML boundary for such viscoacoustic waves, the first-order linear differential equations are modified using the complex coordinate stretching approach. In the frequency domain, derivative operators are replaced as follows

$$\partial_x \to \left[1 + \frac{id(x)}{\omega} \right] \partial_x, \qquad (2.19)$$
$$\partial_z \to \left[1 + \frac{id(z)}{\omega} \right] \partial_z.$$

By applying the complex coordinate stretching to the first-order linear differential equations 2.14-2.17 in the frequency domain I obtain

$$-i\omega \left[1 + \frac{id(x)}{\omega}\right] \left[1 + \frac{id(z)}{\omega}\right] \tilde{P} = -K \left[\left(1 + \frac{id(x)}{\omega}\right) \frac{\partial \tilde{u}_x}{\partial x} + \left(1 + \frac{id(z)}{\omega}\right) \frac{\partial \tilde{u}_z}{\partial z}\right] \left(\frac{\tau_{\varepsilon}}{\tau_{\sigma}}\right) - r,$$
(2.20)

$$\frac{\partial}{\partial t} \left[1 + \frac{id(x)}{\omega} \right] \tilde{u_x} = -\frac{1}{\rho} \frac{\partial \tilde{P}}{\partial x} \to -i\omega \left[1 + \frac{id(x)}{\omega} \right] \tilde{u_x} = -\frac{1}{\rho} \frac{\partial \tilde{P}}{\partial x}, \tag{2.21}$$

$$\frac{\partial}{\partial t} \left[1 + \frac{id(z)}{\omega} \right] \tilde{u_z} = -\frac{1}{\rho} \frac{\partial \tilde{P}}{\partial z} \to -i\omega \left[1 + \frac{id(z)}{\omega} \right] \tilde{u_z} = -\frac{1}{\rho} \frac{\partial \tilde{P}}{\partial z}, \tag{2.22}$$

$$-i\omega \left[1 + \frac{id(x)}{\omega}\right] \left[1 + \frac{id(z)}{\omega}\right] \tilde{r} = K \left[\left(1 + \frac{id(x)}{\omega}\right) \frac{\partial \tilde{u}_x}{\partial x} + \left(1 + \frac{id(z)}{\omega}\right) \frac{\partial \tilde{u}_z}{\partial z}\right] \frac{1}{\tau_\sigma} \left(1 - \frac{\tau_\varepsilon}{\tau_\sigma}\right) - \frac{1}{\tau_\sigma} r,$$
(2.23)

where \tilde{u}_x , \tilde{u}_z , \tilde{P} , and \tilde{r} are the temporal Fourier transforms of u_x , u_z , P, and r, respectively. To calculate the unsplit-field PML formulations these equations must be transformed back to the time domain. Unsplit field formulations use the physical field variables along with extra auxiliary variables that are typically needed to obtain the time-domain equations from the frequency-domain equations. In the split-field PML formulations, the velocity, and pressure fields are split into two independent parts based on the spatial derivative terms in the original equations in two space dimensions, but the unsplit-field PML formulation did not split the physical variables but instead augmented the wave equations with additional terms. Equations 2.20-2.23 are transformed back to time domain to get the unsplit-field PML formulations

$$\frac{\partial P}{\partial t} = -K \left[\frac{\partial (u_x + d(z)u_x^{(1)})}{\partial x} + \frac{\partial (u_z + d(x)u_z^{(1)})}{\partial z} \right] \left(\frac{\tau_{\varepsilon}}{\tau_{\sigma}} \right)$$
(2.24)
$$- \left[d(x) + d(z) \right] P - d(x) d(z) p^{(1)} - r,$$

$$\frac{\partial u_x}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial x} - d(x)u_x, \qquad (2.25)$$



Figure 2.4: The constant velocity model.

$$\frac{\partial u_z}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial z} - d(z)u_z, \qquad (2.26)$$

where the auxiliary variables $u_x^{(1)}$, $u_z^{(1)}$, $P^{(1)}$, and $r^{(1)}$ are the time-integrated components for velocity, pressure and memory variable fields. They are defined as

$$\frac{\partial r}{\partial t} = -\frac{1}{\tau_{\sigma}}r + K \left[\frac{\partial(u_x + d(z)u_x^{(1)})}{\partial x} + \frac{\partial(u_z + d(x)u_z^{(1)})}{\partial z}\right] \frac{1}{\tau_{\sigma}} \left(1 - \frac{\tau_{\varepsilon}}{\tau_{\sigma}}\right) \qquad (2.27)$$
$$- \left[d(x) + d(z)\right]r - d(x)d(z)r^{(1)}.$$

$$u_x^{(1)}(X,t) = \int_{-\infty}^t u_x(X,t')dt',$$

$$u_z^{(1)}(X,t) = \int_{-\infty}^t u_z(X,t')dt',$$

$$P^{(1)}(X,t) = \int_{-\infty}^t P(X,t')dt',$$

$$r^{(1)}(X,t) = \int_{-\infty}^t r(X,t')dt'.$$
(2.28)

High frequency components of the wavefield will be generated during the wave propagation and lead to instability. Adding a regularization term $(\varepsilon/c_p(\partial_t \nabla))$ during reverse time



Figure 2.5: The snapshots of 2-D viscoacoustic simulations using the PML absorbing layer model with $\delta = 20$ m, $\delta = 40$ m, and $\delta = 80$ m. The snapshots are the norm of the velocity at t =0.28 s.


Figure 2.6: Velocity dispersion (a) and amplitude loss effects(b) at the same simulation configuration. i) Acoustic wavefield, ii) viscoacoustic wavefield (Q=100), iii) viscoacoustic wavefield (Q=50), and iv) viscoacoustic wavefield (Q=20). The value of v = 2500 m/s and $\rho = 1200 \ g/m.^3$.

propagation is one way to eliminate this high frequency instability. I construct a regularized viscoacoustic equation as:

$$\frac{\partial P}{\partial t} = -K \left[\frac{\partial (u_x + d(z)u_x^{(1)})}{\partial x} + \frac{\partial (u_z + d(x)u_z^{(1)})}{\partial z} \right] \left(\frac{\tau_{\varepsilon}}{\tau_{\sigma}} \right)$$

$$-\varepsilon \rho c_p \frac{\partial}{\partial t} \left[\frac{\partial (u_x + d(z)u_x^{(1)})}{\partial x} + \frac{\partial (u_z + d(x)u_z^{(1)})}{\partial z} \right] - \left[d(x) + d(z) \right] P - d(x)d(z)p^{(1)} - r_{\varepsilon}$$
(2.29)

where ε is a small positive regularization parameter.

Several formulations of unsplit-field PML exist (Abarbanel and Gottieb, 1997; Sacks et al., 1995; Turkel and Yefet, 1988; Fan and Liu, 2001; Zhou, 2005) based on uniaxial PML and complex coordinate stretching methods. These formulations are well-posed. However, it is necessary to look for a simple and systematic approach to derive well-posed PML formulations so that PML methods can easily be applied to spatially complex media. Fan and Liu (2001) proposed an unsplit-field PML formulation in Cartesian coordinates for Maxwell's equations in a lossy medium.

2.6 Numerical results

A perfectly matched layer (PML) is an artificial absorbing layer introduced to aid in the numerical, often finite-difference (FD), solution of wave equations. The PML truncates computational regions to simulate open boundaries, in such a way that waves outbound from the interior of the computational region are absorbed and little wave energy is reflected back into the interior. In the interior, the PML equations are the same as the original wave equations. In the absorbing layers, Collino and Tsogka (Collino and Tsogka, 2001) introduce the damping parameter d(x) based on a theoretical reflection coefficient as

$$d(x) = d_0 \left(\frac{x}{\delta}\right)^2, \qquad (2.30)$$

where δ is the width of the PML layer, d_0 is the maximum damping parameter, which is a function of the theoretical reflection coefficient (Chew and Liu, 1996; Collino and Tsogka, 2001; Kucukcoban and Kallivokas, 2011), (Appendix B),

$$d_0 = \log\left(\frac{1}{R}\right)\frac{3c_P}{2\delta},\tag{2.31}$$



Figure 2.7: Depth traces shown at four different time steps. The solid black lines, solid red lines, and dashed blue lines represent the acoustic and viscoacoustic wave propagation with Q=100 and Q=20 respectively. The viscoacoustic waveforms have smaller amplitude and greater wavelength and shifted phase due to velocity dispersion.



Figure 2.8: (a) Marmousi velocity model, and (b) Q model.



Figure 2.9: Shot record from acoustic simulation (a), and viscoacoustic simulation (b) using PML absorbing boundary condition. Attenuation effects and dispersion of the events highlighted with the blue arrow.

where c_p represents the *P* wave velocity, and *R* represents the theoretical reflection coefficient. I examine the numerical character of the solutions of the constant-Q wave equation as created using the unsplit-field PML boundary approach. I first consider propagation of waves in a homogeneous model. The viscoacoustic medium considered here is characterized by the constant velocity model presented in Figure 2.4, where the size of the grid is 251×251 . The source is located at the point (500 m, 12 m), and the source signature is a zero-phase Ricker wavelet with a central frequency of 25 Hz. The grid spacing in the *x* and *z* directions is 4 m. An unsplit-field PML absorbing boundary condition is applied to the sides and bottom of the model. In Figure 2.5 snapshots of the 2D viscoacoustic wavefield using the unsplit-field PML absorbing layers are plotted.

The results indicate that, though significant wave energy outbound at the boundaries is absorbed in the PML layer, some artificial reflections are still visible. By expanding the computational grid by the size of the PML, I can remove these artificial reflections, since the effectiveness of the PML depends on its size and absorption. Also the time step used in the simulation can affect the effectiveness of the PML. For a small time step, the wave spends more time in the PML layer and is more significantly absorbed. In Figures 2.6a and 2.6b I show the effect of attenuation on amplitude and phase of a propagating seismic wave in a homogeneous medium with a background velocity of 2500 m/s for different values of quality factor (Q=20, Q=50 and Q=100). In Figure 2.7 depth profiles of the wave field extracted at four different times are plotted. The solid blue lines, dashed lines, and solid red lines represent the acoustic wave, and two examples of the viscoacoustic wave, with Q=20 and Q=100respectively. There are two main effects visible, reduced amplitude and phase shift due to dispersion. The phases do not match, and mismatch increases with depth because of velocity dispersion in the attenuating media. As the wave begins to propagate, the amplitudes for the acoustic and viscoacoustic cases are very similar, but as the propagation time increases and the wave reaches greater depth, its amplitude is strongly attenuated especially for the case Q=20. For moderate attenuation values (Q=100), as shown by the black dashed curves in Figure 2.7, the attenuation effects at different depths are still significant compared to the blue curves that represent the acoustic case (no attenuation).

In the second example, I consider wave propagation within the acoustic Marmousi model.



Figure 2.10: The layered velocity model (a) and Q model (b).



Figure 2.11: The layered velocity model (a) and Q model (b).

A portion of the velocity model, 6 km wide and 3 km in depth, is illustrated in Figure 2.8a. A shot is positioned at 500 m distance and a depth of 12 m. From this point a wave with a time dependence given by a zero-phase Ricker wavelet with a centre frequency of 25 Hz propagates into the model. I position an array of receivers at the same depth, 4 m apart. The Q model includes constant quality factors with different values, while the background model has Q = 100 (Figure 2.8b). The FD synthetic data for acoustic and viscoacoustic cases using equations 2.24-2.27 are shown in Figure 2.9. The first-order pressure-velocity viscoacoustic wave equation using PML absorbing boundary condition is used to compute the synthetic seismograms. The staggered-grid FD solver has 2nd-order accuracy in time and 4th-order accuracy in space. The shots in Figure 2.9 include first arrivals, multiples, reflections, refractions and diffractions. The viscoacoustic simulation exhibits reduced amplitude (particularly multiples) and shifted phase due to velocity dispersion, as highlighted by the blue arrow.

Next, the accuracy of the unsplit PML formulation within viscoacoustic RTM imaging for the layered model is tested (Figure 2.10). The model grid dimensions are 401×401 , the grid size is 4 m × 4 m, and the background quality factors is 100. The sampling interval is 0.4 ms and the recording length is 2 s. I use as the source a zero-phase Ricker wavelet with a centre frequency of 25 Hz. Perfectly matched layer (PML) absorbing boundary conditions are used to attenuate the reflections from an artificial boundary. Figures 2.11a and 2.11b compare the RTM images for acoustic RTM (reference) and acoustic approximations with attenuation. The viscoacoustic RTM image exhibits weaker amplitudes. Attenuation affects the amplitudes and the phases of viscoacoustic RTM because the algorithm does not include the correct physics (attenuation). For viscoacoustic RTM, without compensating for amplitude loss, the image amplitudes and positions will not be accurate for the reflectors beneath the strongly attenuating layers. To mitigate the amplitude loss and velocity dispersion in the next chapters, I introduced the new approach of the viscoacoustic wave equation in the time domain.

2.7 Conclusions

I have derived an unsplit-field PML formulation for the 2D viscoacoustic wave equation in the time domain using a series of standard linear solid mechanisms. The advantage of the unsplit-field formulation is mostly reflection free regardless of the incidence angle and frequency and provides excellent results with less computational cost and does not have the instability problem. I provided a detailed formulation of this viscoacoustic for constant and complex velocity models. Numerical results demonstrate that the unsplit-field viscoacoustic wave equation models the approximate constant-Q attenuation. I validate and test the response of unsplit-field equation by using it within a reverse time migration scheme.

Appendix A

A complex, frequency dependent reflection coefficient

Viscoacoustic processing and inversion stands to gain a very powerful tool in the inverse scattering series but it will do so using information that it can glean from the reflection coefficients. The reflection coefficient for an acoustic wave field on a contrast in wavespeed (from c_0 to c_1), density (from ρ_0 to ρ_1) and Q (from ∞ to Q_1), is

$$R = \frac{\rho_1 c_1 \cos \theta_0 - \rho_0 c_0 \cos \theta}{\rho_1 c_1 \cos \theta_0 + \rho_0 c_0 \cos \theta}.$$
(A.1)

where $\theta_0 = \sigma/2$ and θ are the incidence and the reflection angle respectively. The perturbation parameters can be defined as

$$\alpha(z) = 1 - \frac{c_0^2}{c_1^2},\tag{A.2}$$

$$\beta(z) = 1 - \frac{\rho_0}{\rho_1}.$$
 (A.3)

The reflection coefficient can be written as a power series in perturbation parameters (Fathalian and Innanen, 2015).

$$R \simeq \frac{1}{2\left(1 + \cos\sigma\right)} \left(\alpha(z) + \beta(z)\left(1 + \cos\sigma\right)\right). \tag{A.4}$$

I consider three variants on the acoustic case, each utilizing wavenumbers which permit attenuation to be modelled in addition to acoustic behaviour. This requires moving away from the acoustic $k_0 = \omega/c_0$, and adopting for the actual medium:

$$k(z) = \frac{\omega}{c(z)} \left[1 + \frac{1}{Q(z)} F(k) \right] = \frac{\omega}{c(z)} \left[1 + \zeta(z) F(k) \right]. \tag{A.5}$$

where $\frac{F(k)}{Q(z)}$, a complex number, is the spatial distribution of an attenuation parameter which instills absorption and dispersion character into the wavefield. The first corresponds to media in which both wavespeed contrasts and attenuation contrasts are permitted:

$$\alpha(z) = 1 - \frac{c_0^2}{c^2(z)} \left[1 + \zeta(z)F(k) \right]^2$$

$$\simeq 1 - \frac{c_0^2}{c^2(z)} \left[1 + 2\zeta(z)F(k) \right].$$
(A.6)

Substitute A.6 into A.5. The reflectivity function can be written as

$$R(z,\sigma,k) \simeq \frac{1}{2\left(1+\cos\sigma\right)} \left[\left[1 - \frac{c_0^2}{c^2(z)} \left(1 + 2\zeta(z)F(k) \right) \right] + \beta(z) \left(1 + \cos\sigma \right) \right], \quad (A.7)$$

and include the standard perturbation on the wavespeed profile c(z) in terms of $\alpha(z)$, producing

$$R(z,\sigma,k) \simeq \frac{1}{2\left(1+\cos\sigma\right)} \left[\left[1 - \left(1-\alpha(z)\right) \left(1+2\zeta(z)F(k)\right) \right] + \beta(z)\left(1+\cos\sigma\right) \right] (A.8)$$
$$\approx \frac{1}{2\left(1+\cos\sigma\right)} \left[\alpha(z) - 2\zeta(z)F(k) + \beta(z)\left(1+\cos\sigma\right) \right].$$

dropping all terms quadratic and higher in the perturbation parameters. The normal derivative of scattering potential (V) can be related to reflectivity function (Stolt and Weglein, 2012).

$$R \propto \frac{\partial V}{\partial n} = \hat{\mathbf{n}} \cdot \nabla V \tag{A.9}$$

where $\hat{\mathbf{n}}$ is a unit vector in the direction pointing downward normal to the boundary. From the viscoacoustic scattering potential, the normal derivative is

$$\frac{\partial \upsilon}{\partial n}(z,\sigma,k) = \frac{1}{\omega^2} \frac{\partial V}{\partial n} = \frac{\delta\left(\hat{\mathbf{n}}.(z-z_r)\right)}{\rho_0 c_0^2} \left[\alpha(z) - 2\zeta(z)F(k) + \beta(z)\left(1+\cos\sigma\right)\right]. \quad (A.10)$$

where $v = V/\omega^2$. Comparing the reflectivity function (Equation A.7) and the normal derivative of scattering potential (Equation A.9), I can write

$$R(z,\sigma,k) \simeq \frac{c_0^2 \rho_0}{2\left(1+\cos\sigma\right)} \left[\frac{\delta\left(\hat{n}.(z-z_r)\right)}{\rho_0 c_0^2} \left[\alpha(z) - 2\zeta(z)F(k) + \beta(z)\left(1+\cos\sigma\right)\right]\right] (A.11)$$
$$= \frac{c_0^2 \rho_0}{4\cos^2\theta_0} \left[\frac{\partial v}{\partial n}(z,\sigma,k)\right],$$

for non-converted waves, the normal derivative of scattering potential can replace with $-(2i\omega\cos\theta/c_0)v$ (Stolt and Weglein, 2012). Hence, the reflection coefficient is

$$R(z,\sigma,k) \simeq \frac{i\omega c_0 \rho_0}{2\cos\theta_0} \bigg(\upsilon(z,\sigma,k) \bigg), \tag{A.12}$$

Finally, I have

$$R(z,\sigma,k) \simeq \frac{i\omega}{2c_0\cos\theta_0} \bigg[\alpha(z) - 2\zeta(z)F(k) + \beta(z)\bigg(1 + \cos\sigma\bigg) \bigg].$$
(A.13)

Appendix B

Damping parameter choice

I consider the classical stretch function, i.e, d(x), and d(z) > 0 and b(x) = 1, and b(z) = 1in the PML region. For this case the stretch function simplifies as

$$s(x) = [1 + i\frac{d(x)}{\omega}],$$

$$s(z) = [1 + i\frac{d(z)}{\omega}].$$
(B.1)

There is no rigorous methodology for choosing the damping coefficient, but the polynomial functions are used (Chew and Liu, 1996; Kucukcoban and Kallivokas, 2011). The damping parameter can expressed as polynomial degree n:

$$d_j(x) = \begin{cases} 0, & 0 \le x \le x_0 \\ d_0 \left(\frac{(x-x_0)n_j}{\delta}\right)^n, & x_0 \le x \le x_0 + \delta \end{cases}$$

where d_0 is constant that refer to the maximum of damping coefficient, δ is the thickness of the PML, n is the polynomial degree, and n_j is the jth component of the outward normal to the interface between the PML region and the Physical domain. For one-dimensional wave propagation, d_0 can be written as a function of amplitude reflection coefficient due to the reflection from the PML absorbing boundary condition.

$$d_0 = \log(\frac{1}{R})\frac{3V_P}{2\delta}.\tag{B.2}$$

Chapter 3

Approximating constant-Q reverse time migration in the time domain

3.1 Abstract

I investigate the effects of approximating constant-Q on reverse time migration (RTM) in attenuating medium by introducing a series of standard linear solid (SLS) mechanisms and making use of perfectly-matched layer (PML) boundary conditions. To consider the effects of the number of relaxation mechanisms (L), numerical and analytical solution of the wave equation for a homogeneous and complex medium are compared. In weak attenuation (Q = 100), numerical solutions using a series of SLS relaxation mechanisms and analytical solutions agree very well, and the acoustic and viscoacoustic RTM images have similar artifacts and amplitudes in the shallow layers and have comparable results in the deeper layers. In strong attenuation (Q = 20), when the wave reaches greater depth, the error of numerical solutions using a single SLS mechanism increases and the viscoacoustic RTM images are not so accurate. In this case, I found that the numerical solutions using three SLS relaxation mechanisms are accurate for both weak and strong attenuation. Although the errors of numerical solutions using a single SLS relaxation increase with increasing depth, the results are still useful for practical application and has less computational costs in time and memory.

3.2 Introduction

The anelastic nature of the earth causes amplitude loss and phase distortion of seismic waves and can decrease the resolution of migration images. The quality factor Q quantifies the attenuating and dispersive effects in an attenuative medium. Thus, it is essential to use the Q model during seismic inversion and imaging (Quan and Harris, 1997; Xin et al., 2009; Cavalca et al., 2011). There are various models to estimate and compensate for anelastic effects in seismic exploration. McDonal et al. (1958) perform the constant-Q model, i.e., the attenuation coefficient is considered to be approximately linear with frequency. There is physical evidence that attenuation is almost linear with frequency (therefore Q is constant) in many frequency bands. Constant-Q models provide an efficient parameterization of seismic attenuation in rocks and can improve the seismic inversion by reducing the number of parameters. Kjartansson (1979) used a linear model for attenuation of waves with Q independent of frequency, without any cut-offs, which is mathematically simple and completely specified by phase velocity and Q. Although McDonal et al. (1958) and Kjartansson (1979) provided a linear attenuation model, Scott-Blair (1951) discusses this idea for the first time. Kjartansson's constant-Q model is mathematically simpler than any model with nearly constant Q as a spectrum of the general standard linear solid (GSLS) models (Carcione, 2007a). This model is popular in many seismic applications, especially in the frequency domain, although it is complex in the time domain (Carcione, 2008). Instead of constant-Q model, an efficient method based on GSLS is developed to approximate constant Q over over a specified frequency band in time domain (Liu et al., 1976; Carcione et al., 1988a,b,c), which called the nearly constant-Q model (Casula et al., 1992; Carcione, 2007a). In seismic, it is important that the material rheology gives causal behaviour and approximate a constant Q factor in the seismic frequency band. Liu et al. (1976) found that the experimental observation of wave propagation through the earth can be explained using viscoelastic rheology with multiple relaxation mechanisms. They illustrated that constant Q value and a dispersion relation which qualitatively explains differences in seismic-wave velocities in different frequency ranges can be obtained with a suitable choice of material parameters. In all nearly constant-Q models, as well as the models used by Liu et al. (1976), at least there is one parameter that is related to a range of frequencies over which the model gives Q nearly independent of frequency. However, the physical implications of the cutoff parameters are different between models (Lomnitz, 1957; Futterman, 1962; Strict, 1967).

The nearly constant-Q model has been applied to many different problems. Käser et al. (2007) used the nearly constant-Q model to introduce a new numerical method to solve the heterogeneous anelastic, seismic wave equations with arbitrary high order accuracy in space and time on 3-D unstructured tetrahedral meshes. Xu and McMechan (1998) reparameterized relaxation mechanisms from relaxation time to relaxation frequency in Q fitting, and present a new 3-D viscoelastic wave equations with composite memory-variables to represent Q. Hestholm et al. (2006) presented an improved Q-parameterization scheme for viscoelastic modelling in an finite-difference (FD) simulation of a moving source along a path on the topographic surface near a valley surrounding a prominent irregular hill. The number of relaxation mechanisms (L) causes increased computational cost. Hence, a key issue in the nearly constant-Q method is the selection of the appropriate number of mechanisms. Three SLS (L=3) is considered to be accurate for weak and strong attenuating mediums in geophysical prospecting and global seismology problems. Emmerich and Korn (1987) developed a 2-D finite difference algorithm for scalar wave propagation based on the generalized standard linear solid (GSLS) model and illustrated the L=3 is accurate. Savage et al. (2010) found that an SLS with three mechanisms (L=3) is close to constant Q-value in frequency, and showed the frequency band becomes wider with large L. Day and Minster (1984) presented the best Pade approximation to a constant Q (standard linear solid (SLS)) by introducing a set of internal or memory variables at each stress node point and described an excellent approximation to a constant-Q in a frequency band with several SLS mechanisms. This method increased the computational cost, concerning both memory and time, of simulation of viscoelastic wave propagation because it requires several SLS mechanisms. Blanch et al. (1995) introduced a method for modeling constant-Q as a function of frequency using a single parameter to yield an excellent constant-Q approximation. The viscoelastic-description used in this method yields savings in computations and memory requirements. Although the results of single SLS relaxation mechanism are acceptable for practical applications (Blanch et al., 1995) but they reported the result in the narrow range of frequencies.

In this chapter, I investigate the simulation of wave propagation in attenuation medium within approximating constant-Q using an unsplit-field viscoacoustic wave equation in the time domain. To consider the accuracy of the number of relaxation mechanisms (L), I compare numerical and analytical solution of the wave equation for a homogeneous and complex medium over a frequency band (5-125 Hz). The reduced amplitude and distorted dispersion of seismic waves caused by attenuation always affect the resolution of migrated images. The Q-RTM studies for complex structures mainly used time-domain anelastic wave equations based on the SLS model (Deng and McMechan, 2007, 2008; Cheng et al., 2015). Their equa-

tion is based on the constant-Q model with single relaxation mechanisms at the reference frequency. In our study, the effect of the number of mechanisms on the RTM images over the range of frequencies is investigated.

This chapter is organized as follows. The background research is described in the first section, and then the attenuation model is presented in the second section. In the third section, I introduce the unsplit-field PML formulation of the approximate constant-Q viscoacoustic wave equation. Numerical results on synthetic data are presented in the fourth section.

3.3 Attenuation model

The 2D viscoacoustic wave field can be solved for through a system of first-order differential equations in terms of stresses and particle velocities. In linear viscoelasticity, the basic hypothesis is that the value of the stress tensor depends upon the time-history of the strain tensor. The viscoelastic hypothesis can be expressed as (Christensen, 1982, 2012)

$$\sigma = G(t) * \dot{\varepsilon},\tag{3.1}$$

where G(t) refer to the relaxation function and the symbol * denotes time convolution. The equations describing wave propagation in viscoacoustic media can be derived in terms of the stress relaxation function. The constant-Q model for attenuation is linear with frequency and is obtained by transforming Equation 3.1 to the frequency domain as

$$\sigma(\omega) = M(\omega)\varepsilon(\omega), \tag{3.2}$$

where $M(\omega)$ is the complex relaxation modulus, and ω is the angular frequency. The frequency-dependent Q is defined as (Carcione et al., 1988b; Ben-Menahem and Singh, 2012)

$$Q(\omega) = Re[M(\omega)]/Im(M(\omega)), \qquad (3.3)$$

where Re and Im are the real and imaginary parts, respectively. The frequency-dependent phase velocity is

$$v_p(\omega) = (Re[\sqrt{\rho/M(\omega)}])^{-1}, \qquad (3.4)$$

where ρ is the medium density.

I consider the generalized standard linear solid model (GSLS) to obtain a nearly constant

quality factor (Liu et al., 1976) over the frequency range. The complex modulus of a GSLS can be expressed in the frequency-domain as

$$M(\omega) = M_R \left[1 - L + \sum_{l=1}^{L} \frac{1 + \omega \tau_{\varepsilon l}}{1 + \omega \tau_{\sigma l}} \right], \qquad (3.5)$$

where M_R is the relaxed modulus, and $\tau_{\sigma l}$ and $\tau_{\varepsilon l}$ are the stress and strain relaxation times given by

$$\tau_{\sigma l} = \frac{\sqrt{1 + 1/Q_{0l}^2} - 1/Q_{0l}}{\omega}, \qquad (3.6)$$
$$\tau_{\varepsilon} = \frac{1}{\omega^2 \tau_{\sigma l}}.$$

Where ω is the centre angular frequency of the relaxation peak and Q_{0l} is the minimum quality factors. Experiments have shown that earth materials have constant Q over a limited range of frequency (Bourbie et al., 1987). Therefore the quality factor is usually considered to be constant in the exploration frequency bandwidth. For generalized standard linear solid model (GSIS), the quality factor is

$$Q = \frac{Re[M(\omega)]}{Im[M(\omega)]} = \frac{1 + \sum_{l=1}^{L} \frac{\omega^2 \tau_{\sigma_l}^2}{1 + \omega^2 \tau_{\sigma_l}^2} \tau}{\sum_{l=1}^{L} \frac{\omega \tau_{\sigma_l}}{1 + \omega^2 \tau_{\sigma_l}^2} \tau},$$
(3.7)

where $\tau = (\tau_{\varepsilon l}/\tau_{\sigma l}) - 1$ (Blanch 1995). By applying the approximation $\tau_{\varepsilon l} \approx \tau_{\sigma l}$ the dissipation factor (Q^{-1}) for single mechanism can be obtained as (Bourbie et al., 1987):

$$Q^{-1} = \frac{\omega(\tau_{\varepsilon} - \tau_{\sigma})}{1 + \omega^2 \tau_{\sigma} \tau_{\varepsilon}},\tag{3.8}$$

With the assumption $\tau_{0l} = (\tau_{\sigma l} \tau_{\varepsilon l})^{1/2}$ and $Q_{0l} = 2\tau_{0l}/\tau_{\varepsilon l} - \tau_{\sigma l}$ (Casula et al., 1992) the seismic quality factor Q for a GSLS can be obtained as:

$$Q = Q_0 L \left(\sum_{l=1}^{L} \frac{2\omega\tau_{0l}}{1+\omega^2\tau_{0l}^2} \right)^{-1},$$
(3.9)

The almost constant quality factor is the quality factor at the centre of the frequency band, $Q(\omega_0) = \bar{Q}$, thus

$$\bar{Q} = Q_0 L \left(\sum_{l=1}^{L} \frac{2\omega_0 \tau_{0l}}{1 + \omega_0^2 \tau_{0l}^2} \right)^{-1}.$$
(3.10)



Figure 3.1: The dissipation factor of single standard linear solid (a) and three SLS mechanisms (b). In this case $\bar{Q} = 100$, the system is composed of L relaxation peaks of the maximum value of Q_0^{-1} each, and equally distributed in the $\log(\omega)$ scale.



Figure 3.2: The dissipation factor of single standard linear solid (a) and three SLS mechanisms (b). In this case $\bar{Q} = 20$, the system is composed of L relaxation peaks of the maximum value of Q_0^{-1} each, and equally distributed in the $\log(\omega)$ scale.



Figure 3.3: The effect of increasing number of mechanisms on minimum quality factor for (a) constant Q = 100, and (b) constant Q = 20.

Using Equation 3.10, I consider the dissipation factor for different values of constant Q over a broad frequency range (between 5Hz and 125Hz). The dissipation factor of the standard linear solid represents a single relaxation peak at $\omega_0 = 1/\tau_0$ (Figures 3.1a). A constant-Q model can be constructed with a parallel connection of standard linear solid elements. Figures 3.1b shows the dissipation factor for 3 single standard linear elements. The curve is composed of 3 single mechanisms, each with maximum dissipation factor Q_0^{-1} . Similarly, in Figure 3.2, the dissipation factor for single and three pairs of relaxation mechanisms for the strong attenuation is displayed. The effect of increasing the number of mechanisms on minimum quality factor shown in Figure 3.3.

To investigate the accuracy of a series of single SLS mechanisms, the dissipation factor (Q^{-1}) and phase velocity of one, three, and five SLS mechanisms compare with the theoretical model. The reference velocity is 2.5 km/s, and the reference band is 5 – 125 Hz. For Q = 100, the three SLS fits the theoretical model curves very well in the central frequency. Note, the one, three, and five SLS have a good approximation to the phase velocity and dissipation factor around the reference frequency (Figure 3.4). In Figure 3.5 the dissipation factor and phase velocity of five, three and one SLS mechanisms for the strong attenuation case are compared with the theoretical model. The three SLS mechanisms fit the theoretical model curves in the central frequency, and the single and five SLS mechanisms have a good approximation to the phase velocity and dissipation factor around the center frequency. In Figures 3.6a and 3.6b the percent error of phase velocity in a range of frequency for weak and strong attenuation media are displayed. In strong attenuating media, the error is more significant than the weak attenuating media.

3.4 Viscoacoustic wave equation

The unsplit-field PML equations of the GSLS model based on the viscoacoustic medium theory was derived by applying the complex coordinate stretching to the first-order linear differential equations (Robertsson et al., 1994) in the frequency domain and transforming back to the time domain as (Fathalian and Innanen, 2016)



Figure 3.4: Illustrate the dissipation factor (a) and phase velocity (b) of constant Q = 100. The black line corresponds to constant Q = 100, red line corresponds to one SLS ($Q_0 = 100$), blue line corresponds to three SLS ($Q_0 = 59$), and cyan line corresponds to five SLS ($Q_0 = 65$).



Figure 3.5: Illustrate the dissipation factor (a) and phase velocity (b) of constant Q = 20. The black line corresponds to constant Q = 20, red line corresponds to one SLS ($Q_0 = 20$), blue line corresponds to three SLS ($Q_0 = 11.9$), and cyan line corresponds to five SLS ($Q_0 = 13.1$).



Figure 3.6: Comparison of phase velocity error of different relaxation mechanisms for (a) Q=100, and (b) Q=20.

$$\partial_t u_x = -(1/\rho)\partial_x P - d(x)u_x, \qquad (3.11)$$

$$\partial_t u_z = -(1/\rho)\partial_z P - d(z)u_z, \qquad (3.12)$$

$$\partial_t P = -K[\partial_x(u_x + d(z)u_x^{(1)}) + \partial_z(u_z + d(x)u_z^{(1)})] \left[1 - \sum_{\ell=1}^L \left(1 - \frac{\tau_{\varepsilon\ell}}{\tau_{\sigma\ell}}\right)\right] \quad (3.13)$$
$$- \left[d(x) + d(z)\right] P - d(x)d(z)p^{(1)} - \sum_{\ell=1}^L r_\ell,$$

$$\partial_t r_{\ell} = -(1/\tau_{\sigma})r_{\ell} + K[\partial_x(u_x + d(z)u_x^{(1)}) + \partial_z(u_z + d(x)u_z^{(1)})]\frac{1}{\tau_{\sigma l}} \left(1 - \frac{\tau_{\epsilon l}}{\tau_{\sigma l}}\right) \quad (3.14)$$
$$- [d(x) + d(z)]r_l - d(x)d(z)r_l^{(1)}.$$

where $u_x^{(1)}$, $u_z^{(1)}$, $P^{(1)}$, and $r^{(1)}$ are the time-integrated components for velocity, pressure, and memory variable fields. Elements $\tau_{\sigma\ell}$ and $\tau_{\varepsilon\ell}$ refer to the stress and strain relaxation times of the ℓth mechanism. Equations 3.11-3.14 include the system of first-order linear differential equations of 2D viscoelastic wave propagation with L sets of standard linear solids. The number of equations to solve for the 2D viscoacoustic wave equation is 3+L (table 1). The memory variables increase the computational cost for solving viscoacoustic wave equation. It is evident that the computational time of FD simulation with the single relaxation mechanisms is less than higher relaxation mechanisms, but the accuracy of approximating constant-Q in attenuation media is essential as well. To determine the right number of relaxation mechanisms accuracy of the constant-Q model is investigated.

2D viscoacoustic wave equation			
Number of relaxation	Hook's law	Equation of mo-	Memory variables
mechanisms (L)		tion	
L=1	$\frac{\partial P}{\partial t}$	$\frac{\partial u_x}{\partial t}, \frac{\partial u_z}{\partial t}$	$\frac{\partial r_1}{\partial t}$
L=3	$\frac{\partial P}{\partial t}$	$\left \frac{\partial u_x}{\partial t}, \frac{\partial u_z}{\partial t} \right $	$\frac{\partial r_1}{\partial t}, \frac{\partial r_2}{\partial t}, \frac{\partial r_3}{\partial t}$
L=5	$\left \begin{array}{c} \frac{\partial P}{\partial t} \end{array} \right $	$\left \begin{array}{c} \frac{\partial u_x}{\partial t}, \frac{\partial u_z}{\partial t} \end{array} \right $	$\left \frac{\partial \tilde{r}_1}{\partial t}, \frac{\partial \tilde{r}_2}{\partial t}, \frac{\partial \tilde{r}_3}{\partial t}, \frac{\partial r_4}{\partial t}, \frac{\partial r_5}{\partial t} \right $

Table 3.1: 2D viscoacoustic wave equation for different relaxation mechanisms (L).



Figure 3.7: (a) Snapshots showing an expanding wavefront at different time step for an acoustic medium (left panels), and attenuation media (Q = 100) with three mechanisms (L = 1, L = 3, and L = 5). (b) Depth traces from of FD and analytical solution of Figure (a) showing the effect of attenuation on the amplitude and phase of propagating wave with three different SLS.



Figure 3.8: (a) Snapshots showing an expanding wavefront at different time step for an acoustic medium (left panels), and attenuation media (Q = 20) with three mechanisms (L = 1, L = 3, and L = 5). (b) Depth traces from of FD and analytical solution of Figure (a) showing the effect of attenuation on the amplitude and phase of propagating wave with three different SLS.

3.5 Numerical experiments

In order to investigate the accuracy of the SLS mechanisms I compare numerical solutions with the analytical solution. To obtain the analytical solution I used the viscoacoustic Green's function in homogeneous medium is given in Carcione et al. (1988b). I examine the numerical character of the solutions of the constant-Q wave equation as created using the unsplit-field PML boundary approach. I consider the propagation of waves in a homogeneous model with a constant velocity model and a grid size of 1001×1001 . The source, located at the point (2000 m, 2000 m), is a zero-phase Ricker wavelet with a central frequency of 25 Hz. The grid spacing in the x and z directions is 4 m. An unsplit-field PML absorbing boundary condition is applied to the sides and bottom of the model to reduced artificial reflections (Berenger, 1994; Chew and Liu, 1996; Zeng and Liu, 2001; Bécache et al., 2003). In Figure 3.7a snapshots of the 2D viscoacoustic wavefield with different mechanisms are plotted. There are two main visible effects, reduced amplitude and phase shift due to dispersion. There is a phase mismatch that increases with depth because of velocity dispersion in the attenuating media. For weak attenuation values (Q=100), as shown in Figure 3.7b, the attenuation effects at different depths for a single and series of SLS mechanisms (L = 1, L)L = 3, and L = 5) are still significant compared to the black curves that represent the analytical solutions. The results show that the single SLS mechanism also yields comparable results when the propagation time increases and the wave reaches greater depth. In the strong attenuation case (Q=20), the amplitudes of the SLS mechanisms are very similar when the wave begins to propagate (Figure 3.8a). In this case, the amplitudes are strongly attenuated by increasing the propagation time. Figure 3.8b show comparisons between the FD results and the analytical solutions when Q=20. The numerical and analytical solutions also match very well. At greater depth, the single SLS mechanism is not so accurate when compared wit the L=3 result.

3.6 Synthetic model

In the first example, I consider the accuracy of nearly constant-Q wave propagation by series of standard linear solid (SLS) for RTM images with attenuation for a layered model using



Figure 3.9: A layered model: (a) true velocity model (b) migration velocity model



Figure 3.10: Shot record (left) and the S spectrum (right) of the shot record of the layered model with background attenuation (Q = 100) from acoustic simulation (a), viscoacoustic (L = 1) simulation (b), viscoacoustic (L = 3) simulation (c), and difference between viscoacoustic data (L = 1) and viscoacoustic data (L = 3) (d) using PML absorbing boundary condition.



Figure 3.11: The RTM images of the layered model with background attenuation (Q = 100) (a) acoustic RTM (reference), (b) viscoacoustic RTM (L = 1), (c) viscoacoustic RTM (L = 3) and (d)viscoacoustic RTM (L = 5).



Figure 3.12: Depth slices from Figure 11 showing the effect of attenuation and comparison of acoustic (black solid line), viscoacoustic L = 1 (solid red line), viscoacoustic L = 3 (solid blue line), and viscoacoustic L = 5 (dashed black line).



Figure 3.13: Shot record (left) and the S spectrum (right) of the shot record of the layered model with background attenuation (Q = 20) from acoustic simulation (a), viscoacoustic (L = 1) simulation (b), viscoacoustic (L = 3) simulation (c), and difference between viscoacoustic data (L = 1) and viscoacoustic data (L = 3)(d) using PML absorbing boundary condition.



Figure 3.14: The RTM images of the layered model with background attenuation (Q = 20) (a) acoustic RTM (reference), (b) viscoacoustic RTM (L = 1), (c) viscoacoustic RTM (L = 3) and (d)viscoacoustic RTM (L = 5).


Figure 3.15: Depth slices from Figure 14 showing the effect of attenuation and comparison of acoustic (black solid line), viscoacoustic L = 1 (solid red line), viscoacoustic L = 3 (solid blue line), and viscoacoustic L = 5 (dashed black line).

a time-space domain FD method. The synthetic data are migrated by using the acoustic RTM and viscoacoustic RTM with a different number of mechanisms (L = 1, L = 3, and L = 5). Figure 3.9a, and 3.9b shows the true and migration velocity models. The model grid dimensions are 401×451, and the grid size is 4 m ×4 m. The sampling interval is 0.8 ms, and the recording length is 2 s. Figures 3.10a, 3.10b, and 3.10c (left panel) show the FD synthetic data with a series of SLS mechanisms (L = 1, and L = 3) when Q=100. The viscoacoustic simulation exhibits reduced amplitude and shifted phase due to velocity dispersion.

I can see that the viscoacoustic data are very similar, while the single SLS mechanism has some error compared with the three relaxation mechanisms. I know that in the attenuation media the reflection wave energy decay with the increase depth, so the frequency bandwidth becomes narrow, the high frequencies decay more and the dominant frequencies move to low frequencies (Figures 3.10a, 3.10b, and 3.10c (right panel)). The difference waveforms and S spectrum between the viscoacoustic data (L = 1) and viscoacoustic data (L = 3) are shown in Figure 3.10d. For weak attenuation values, as the wave begins to propagate, the viscoacoustic (L = 3) and viscoacoustic (L = 1) data are very similar. When the wave reaches greater depth, the single SLS mechanism is still significant compared to the three SLS result (Figure 3.10d). The black arrows show the difference between single SLS and three SLS mechanisms data. At the shallow layers, the single SLS and Three SLS agree very well. At the deep layers, I can see that the single SLS result is comparable with the three SLS result. For weak attenuation, i.e., Q = 100, the RTM images are shown in Figure 3.11, which includes the acoustic RTM without attenuation (reference) (Figure 3.11a), the viscoacoustic RTM with a different number of mechanisms (Figures 3.11b, 3.11c, and 3.11d) results. The attenuation affects the amplitudes and the phases of the propagating waves, effects which are not taken into account in the acoustic RTM image. In Figure 3.12 I compare the depth traces of acoustic and viscoacoustic RTM data with a different number of SLS mechanisms. The acoustic and viscoacoustic RTM images have similar artifacts and amplitudes in the shallow layers. At the deep layers, I can see that the single SLS mechanism has a comparable result with the three, and five SLS mechanisms.

Figures 3.13a, 3.13b, and 3.13c (left panel) show the snapshot of acoustic and viscoacoustic

(Q = 20) with a series of SLS mechanisms (L = 1, and L = 3). In strong attenuating media, the difference between single SLS and three SLS mechanisms is more obvious than the weak attenuating media (Figure 3.13d (black arrows indicate the difference between L=1 and L=3 mechanisms)). However, the single SLS mechanism has some error compared with the three relaxation mechanisms in the higher depth with the strong attenuation. In this case, the frequency bandwidth becomes narrower than the weak attenuating media, so the dominant frequency moves to low frequency (Figures 3.13a, 3.13b, and 3.13c (right panel)). For the strong attenuation value, i.e., Q = 20, the RTM images of the single, three, and five SLS mechanisms are shown in Figure 3.14 and compare with the acoustic case results. The viscoacoustic RTM images have very weak amplitudes in the deeper layers with strong attenuation (Figures 3.14b, 3.14c, 3.14d). In Figure 3.15 I show a comparison of amplitudes between acoustic and viscoacoustic data. The three and five SLS mechanisms results agree very well together, while the single SLS mechanism is not so accurate. Although in the deeper layers the error of single SLS mechanism increases, the results are still useful for practical application.

In the second example, I consider the more complex Marmousi model. Figure 3.16a and 3.16c shows the true velocity and Q models, respectively, used for forward modeling. The migration velocity and Q models are shown in Figure 3.16b and 3.16d, used for back propagate modeling. I position an array of shots and receivers at a constant depth of 12 m. From this point, a wave with a time dependence given by a zero-phase Ricker wavelet with a center frequency of 15 Hz propagates into the model. The Q model includes constant quality factors with different values, while the background model has Q = 100 (Figure 3.16b). In Figures 3.17a, 3.17b, and 3.17c (left panel) the FD synthetic data of acoustic and viscoacoustic with a series of SLS mechanisms (L = 1, and L = 3) are displayed. The shots include first arrivals, multiples, reflections, refractions, and diffractions. The viscoacoustic simulation exhibits reduced amplitude (particularly multiples) and shifted phase due to velocity dispersion. I can see that the viscoacoustic data are very similar at shallow layers, but in the deeper layers, the single SLS mechanism has some error compared with the three relaxation mechanisms (Figure 3.17d). The difference between single SLS and three SLS mechanisms is increased at the deep layers. To consider the attenuation effects on



Figure 3.16: The Marmousi models: (a) true velocity model, (b) migration velocity model, (c) true Q model, and (d) migration Q model.



Figure 3.17: Shot record (left) and the S spectrum (right) of the shot record of the Marmousi model from acoustic simulation (a), viscoacoustic (L = 1) simulation (b), viscoacoustic (L = 3) simulation (c), and difference between viscoacoustic data (L = 1) and viscoacoustic data (L = 3) (d) using PML absorbing boundary condition.



Figure 3.18: Comparison among images from (a) acoustic RTM, (b) viscoacoustic (L=1) RTM, (c) viscoacoustic (L=3) RTM, and (d) viscoacoustic (L=5) RTM.



Figure 3.19: Depth slices from Figure 18 showing the effect of attenuation and comparison of acoustic (black solid line), viscoacoustic L = 1 (solid red line), viscoacoustic L = 3 (solid blue line), and viscoacoustic L = 5 (dashed black line).

the reflection wave energy, the S spectrum of acoustic and viscoacoustic data are displayed in Figures 3.17a, 3.17b, and 3.17c (right panel). The frequency bandwidth becomes narrow ,i.e., the high frequencies decay and the dominant frequencies move to low frequencies. In Figure 3.18 the viscoacoustic RTM images of the single, three, and five SLS mechanisms are compared with the acoustic case results. The viscoacoustic and acoustic RTM images are similar for the shallow layers, while the viscoacoustic RTM images have weak amplitudes at the deeper layers especially below the layers with the strong attenuation. In Figure 3.19 I show a comparison of amplitudes between acoustic and viscoacoustic data. I note that there is the significant error from single SLS mechanism when the attenuation is strong. In exploration seismology, it is implausible to find large anomalies with strong attenuation, so the single SLS is usually sufficient in practice.

3.7 Conclusions

Time-domain approximate constant-Q wave propagation involving a series of standard linear solid (SLS) mechanisms is investigated. I found that the numerical results and analytical solutions using single and a series of standard linear solid (SLS) mechanisms in the weak attenuating media agree very well. In strong attenuating media, the error of numerical solutions using a single SLS mechanism increases and the viscoacoustic RTM images is less accurate. Although modeling of a single SLS relaxation mechanism is still useful for practical applications and faster than three and five SLS mechanisms, the three SLS relaxation mechanism is accurate for both weak and strong attenuation over a broad frequency range.

Chapter 4

An approach for attenuation-compensating multidimensional constant-Q viscoacoustic reverse time migration

4.1 Abstract

Simulation of wave propagation in a constant-Q viscoacoustic medium is an important problem, for instance within Q-compensated reverse-time migration. Processes of attenuation and dispersion influence all aspects of seismic wave propagation, degrading resolution of migrated images. To improve image resolution, I present a new approach for the numerical solution of the viscoacoustic wave equation in the time domain, and develop an associated viscoacoustic reverse time migration (Q-RTM) method. The main feature of the Q-RTM approach is compensation of attenuation effects in seismic images during migration by separation of amplitude attenuation and phase dispersion terms. Because of this separation, I am able to compensate the amplitude loss effect in isolation, the phase dispersion effect in isolation, or both effects concurrently. In the Q-RTM implementation, an attenuationcompensated operator is constructed by reversing the sign of the amplitude attenuation, and a regularized viscoacoustic wave equation is invoked to eliminate high frequency instabilities. The scheme is tested on a layered model and a modified acoustic Marmousi velocity model. I validate and examine the response of this approach by using it within a reverse time migration scheme adjusted to compensate for attenuation. The amplitude loss in the wavefield at the source and receivers due to attenuation can be recovered by applying compensation operators on the measured receiver wavefield. 2D and 3D numerical tests focus on the amplitude recovering and resolution of the Q-RTM images as well as interface locations. Improvements in all three of these features beneath highly attenuative layers are evident.

4.2 Introduction

Attenuation is an increasingly indispensable component of wavefield simulation in seismic exploration and monitoring applications. It is a key element in many recent instances of data modeling, reverse time migration (RTM), and full waveform inversion (FWI). Especially in applications like RTM and FWI, the trade-off between computational efficiency (in particular, being able to calculate inside tractably small computational domains) and simulation accuracy (in particular, avoiding computational artifacts such as boundary reflections and numerical dispersion) is important and difficult to manage. This chapter formulates a tool for viscoacoustic wavefield simulation that addresses this trade-off in a direct and novel manner, and validates it with synthetic attenuation-compensating RTM examples. Processes of attenuation and dispersion influence all aspects of seismic wave propagation, degrading resolution, but, at the same time, introducing unique data variations that can be quantitatively analyzed. Both the positive and the negative features of attenuation have influenced recent amplitude-variation-with-offset (AVO) studies (Chapman et al., 2006; Innanen, 2011; Wu et al., 2014) and FWI developments (Hicks and Pratt, 2001; Hak and Mulder, 2011; Kamei and Pratt, 2013; Métivier et al., 2015; Yang et al., 2016; Keating and Innanen, 2017). The full viscoelastic wave propagation problem (Carcione, 2007a), whether or not in association with anisotropy (Cerveny and Psencik, 2005a,b; Zhu and Tsvankin, 2006; Vavryčuk, 2008), is quite complex, and involves a set of homogeneous and inhomogeneous wave modes that propagate and scatter in unique ways (Borcherdt, 2009). However, many of the important aspects of compressional-wave attenuation are captured within the viscoacoustic approximation, which I will make in this chapter.

Reduced amplitude and phase dispersion of seismic waves in attenuating media affect the resolution of migrated images. To improve the resolution, one must correct for these attenuation effects. The earliest efforts to mitigate the attenuation losses in seismic data involved using an inverse Q-filtering method based on 1D wave back-propagation (Bickel and Natarajan, 1985; Hargreaves and Calvert, 1991; Wang, 2002). Although these methods partially correct for the attenuation losses, they are limited to 1D Q models and the inverse filter is unstable (Wang, 2002, 2006; Zhang and Ulrych, 2007).

In the context of prestack depth migration, several studies of attenuation compensation have examined using ray-tracing methods (Xin et al., 2008; Xie et al., 2009) and one-way waveequation migration methods in the frequency domain (Dai and West, 1994; Mittet et al., 1995; Yu et al., 2002; Valenciano et al., 2011; Zhang et al., 2012).

A key application of wave propagation simulation tools is RTM, both for its own merits and/or as part of the construction of FWI. I will use analysis of RTM solutions as our primary means of validating our viscoacoustic simulations. It has been pointed out that, within the back-propagation and modelling components of RTM, attenuation and dispersion can be compensated in a largely full-waveform consistent manner. Zhang et al. (2010), for instance, uses the dispersion relation of Kjartansson (1979) to formulate a pseudo-differential wave equation with separable the effects of amplitude loss and velocity dispersion. They apply a regularization process to stabilize the back-propagating wavefield and suppress highfrequency noises. Suh et al. (2012) employ a viscoacoustic VTI approximation and compensates for amplitude losses within the resulting imaging procedure. The same instability problem during back-propagation of the receiver wavefield as in Zhang et al. (2010) affects the resolution of images. To avoid amplifying high-frequency noise, they apply a high-cut filter to the receiver wavefield. Bai et al. (2013) derive a similar approach for attenuation compensation in RTM. They use a new viscoacoustic wave equation defined without any memory variable. To avoid the challenges of stabilising the wave propagation, Fletcher et al. (2012) proposed applying separate amplitude and phase filters to twice-extrapolated source and receiver wavefields to compensate for amplitude and phase effects. Dutta and Schuster (2014) adopted a least-squares RTM (LSRTM) approach for attenuation compensation based on an standard linear solid (SLS) model and its adjoint operator (Blanch et al., 1995) with a simplified stress-strain relation. Zhu and Harris (2014) introduced a constant-Q viscoacoustic wave equation with separated fractional Laplacians, and applied it to the problem of Q-compensated RTM. In this method, the fractional orders are related to Q, and the average value of the spatially varying orders for numerical simulation is used. The average scheme is unsuitable for relatively sharp Q contrasts and just works for smoothly heterogeneous Q models. However, the amplification of high-frequency components during compensation for attenuation effects usually generates instability. Bai et al. (2013) and Zhu and Harris (2014)

used a low-pass filter before or during wavefield extrapolate in RTM to eliminate instability. The low-pass filtering must be used for the whole model, and this is not useful for severely variational regions.

Following Bai et al. (2013) approach, here I derive a new approach to the solution of the viscoacoustic wave equation within attenuating media in the time domain based on an SLS model. I present the formulation for attenuation media that describes constant-Q wave propagation and contains independent terms for phase dispersion and amplitude attenuation. Our approach leads to a methodology for Q-RTM that can compensate for attenuation effects in the migrated images. The attenuation effects are compensated for in the reconstructed wavefield by reversing the sign of the amplitude loss operator and an unchanged sign of dispersion operator. In our new approach, there is no need for any extra memory variables, unlike the traditional viscoacoustic wave equations. I employ two synthetic examples to test the accuracy of the proposed method in application to imaging and demonstrate that Q-RTM compensates in the receiver wavefields when using the cross-correlation imaging condition. I apply the source normalized crosscorrelation imaging condition to construct the image in RTM because of its advantages, which include better imaging quality at the deeper reflectors and reliable image amplitudes that represent the reflectivity of the mode with the correct scaling and sign. To suppress high-frequency noise during extrapolation, I add a regularization term to the viscoacoustic wave equation. Q-RTM images show that the amplitudes below highly attenuative layers are more accurately recovered in this way, with the reflectors imaged at the correct locations.

This chapter is organized as follows. First, I describe the basic methodology of Q-RTM, and introduce our formulation of the approximate constant-Q viscoacoustic wave equation. Next, I discuss our modified formulation of Q-RTM and its treatment of the compensation of attenuation effects. Numerical results on 2D and 3D synthetic data are presented in the last section.



Figure 4.1: Schematic diagram of wave propagation in an attenuating medium. (a) Forward modeling and (b) RTM extrapolation. The back-propagated wavefield in reverse time from the receiver is attenuated: $R^A(x, z, t) = R(x, z, t)e^{-\alpha X_{down}}e^{-\alpha X_{up}}$. Here R refers to the wavefield in the acoustic case. After attenuation compensation, the receiver wavefield will be $R^C(x, z, t) = R^A(x, z, t)e^{+\alpha X_{up}}$.

4.3 Imaging condition in attenuating media

RTM is a three-step procedure, involving (a) forward propagation of a wavefield through an appropriate velocity model, (b) back propagation of the measured data through the same model, and (c) superposition of both using a suitable imaging condition. In our case, the source normalized cross-correlation imaging condition is most suitable, within which only the backward receiver wavefield requires compensation. Similarly to the acoustic case (Whitmore and Lines, 1986), imaging conditions can be developed for the 2D viscoacoustic case. The imaging condition for the acoustic case is:

$$I(x,z) = \frac{\int_{t} S(x,z,t)R(x,z,t)dt}{\int_{t} S^{2}(x,z,t)dt}.$$
(4.1)

where, I(x, z) is the migration result at the position (x, z), S(x, z, t) represents the timedomain forward propagated wavefield from the source, and R(x, z, t) represents the timedomain receiver wavefield that is back-propagated in reverse time from the receiver. The auto-correlation of the source on the right acts to compensate for illumination. In viscoacoustic media, the amplitude decreases as a function of the attenuation coefficient, α , and the propagation distance, X, exponentially $e^{-\alpha X}$ (Mittet et al., 1995; Zhu et al., 2014). The wave-path from source to receiver includes both downgoing and upgoing waves, so the amplitude of waves is reduced by the factor $e^{-\alpha X_{down}}e^{-\alpha X_{up}}$ (Figure 4.1a). Thus the receiver wavefield at any point in imaging space is of the form

$$R^{A}(x, z, t) = R(x, z, t)e^{-\alpha X_{down}}e^{-\alpha X_{up}},$$
(4.2)

where $R^A(x, z, t)$ is the receiver wavefield in an attenuating medium (Figure 4.1b). The measured receiver wavefield should be corrected to compensate for amplitude loss in the wavefield at the receivers. For extrapolating the receiver wavefield from receiver to reflector, I apply the compensation factor $e^{+\alpha X_{up}}$ (Figure 4.1b). The receiver wavefield amplitude $R^A(x, z, t)$ at the reflector is compensated to $R^C(x, z, t)$ as

$$R^{C}(x, z, t) = R^{A}(x, z, t)e^{+\alpha X_{up}},$$
(4.3)

With the compensated receiver wavefield, I apply the source normalized cross-correlation imaging condition at each reflection point:

$$I^{C}(x,z) = \frac{\int_{t} S^{A}(x,z,t) R^{C}(x,z,t) dt}{\int_{t} (S^{A}(x,z,t))^{2} dt}.$$
(4.4)

where $S^A(x, z, t) = S(x, z, t)e^{-\alpha X_{down}}$ is the source wavefield at any point X = (x, z) in imaging space. Substituting the compensated receiver wavefield $R^C(x, z, t)$ into equation 4, the imaging condition because of time-independent of attenuation factors can be corrected as

$$I^{C}(x,z) = \frac{\int_{t} [S(x,z,t)e^{-\alpha X_{down}}][e^{+\alpha X_{up}} e^{-\alpha X_{down}}e^{-\alpha X_{up}}R(x,z,t)]dt}{\int_{t} e^{-2\alpha X_{down}}(S(x,z,t))^{2}dt} = \frac{\int_{t} S(x,z,t)R(x,z,t)dt}{\int_{t} (S(x,z,t))^{2}dt},$$
(4.5)

where the compensated image from the source normalized crosscorrelation imaging condition is theoretically equivalent to that produced within the acoustic case. In zero-lag cross-correlation imaging condition Zhu and Harris (2014), Q-RTM must compensates for attenuation effects in both source and receiver wavefields. In this method, the wavelets of source and receiver are mismatched at reflection point, and the reflectivity values have a loss because the regularisation is applied to ensure stability. I found that source normalized



Figure 4.2: Frequency-dependence of phase velocity in attenuation media.

cross-correlation imaging condition more suitable, and only backward receiver wavefield is needed to compensated. There is no regularization in forward modeling, so in principle the forward and backward wavefields and their respective wavelets match well at the reflector and the reflectivity values are less attenuated.

4.4 Viscoacoustic wave equation

In viscoacoustic media, the wavenumber (k) takes on additional real and imaginary terms. These act to attenuate the wave and alter its phase velocity. A plane wave propagating in the x direction, for instance, becomes $\exp(-\alpha x) \exp(ikx)$, where $\alpha(\omega)$ is the attenuation coefficient. This coefficient is related to the seismic quality factor Q by

$$\alpha(\omega) = \frac{\omega}{2c_0 Q},\tag{4.6}$$

where ω is the angular frequency, and c_0 is the wave speed. Aki and Richards (2002) note that many approaches result in combined absorption and dispersion pairs that, in the context of Q, and on a reasonable seismic frequency band, amount to the replacement of the wavenumber $k = \omega/c_0$ by

$$K = \frac{\omega}{c_0} \left[1 + \frac{i}{2Q} - \frac{1}{\pi Q} \ln\left(\frac{\omega}{\omega_r}\right) \right], \qquad (4.7)$$

where ω_r is a reference frequency at which the wavefield propagates with phase velocity c_0 . This implies a complex velocity

$$v(\omega) = \frac{\omega}{K} = c_0 \left[1 + \frac{i}{2Q} - \frac{1}{\pi Q} \ln\left(\frac{\omega}{\omega_r}\right) \right]^{-1}, \qquad (4.8)$$

whose real part describes the wave dispersion:

$$v_p(\omega) = c_0 \left[1 + \frac{1}{\pi Q} \ln \left(\frac{\omega}{\omega_r} \right) \right].$$
(4.9)

These altered aspects of wave propagation generate the gross properties of the viscoacoustic wave, such as its amplitude losses, but also its more subtle features, such as phase changes during propagation, and characteristic frequency dependence in the reflection coefficient. For this nearly constant Q model, the attenuation coefficient is linear with frequency ω , and the phase velocity v_p is slightly dependent on frequency. A requirement in theory for materials satisfying the linear attenuation assumption is that the reference frequency ω_r is a finite (arbitrarily small but nonzero) cut-off on the absorption. According to Kolsky (1956) and Futterman (1962), I am free to choose reference frequency ω_r following the phenomenological criterion that it be small compared with the lowest measured frequency ω in the frequency band. To obtain the better approximate the constant-Q model, I assume that the reference frequency $\omega_r = \omega_0$ is the central frequency of the source and I force Q of the single peak at that frequency to be Kjartansson's Q (Kjartansson, 1979). The frequency-dependence of phase velocity for a reference velocity ($c_0=2.5$ km/s) and two Q values are illustrated in Figure 4.2. Q has a strong influence on velocity dispersion, which for low Q values (e.g., Q=20) becomes a dominant feature of wave propagation (Figure 4.2). Especially at low Q, there will appear greater phase delays in the waveform at lower frequencies. Numerical viscoacoustic simulations must predict both these overt and subtle features of the wave-medium interaction. Nearly constant Q models can be simulated with suitable parallel arrangements of viscoelastic standard-linear solid spring-dashpot elements (Liu et al., 1976; Day and Minster, 1984; Carcione et al., 1988b; Carcione, 2007b).

Based on this constant-Q dispersion relation, the viscoacoustic wave equation, as a set of first-order differential equations in terms of the particle velocities and stress, describes the dispersion and attenuation effects (Robertsson et al., 1994; Aki and Richards, 2002; Christensen, 2012). The 2D velocity-stress formulation of the viscoacoustic wave equation is expressed as

$$\partial_t u_x = -\rho^{-1} \partial_x P + f_x, \tag{4.10}$$

$$\partial_t u_z = -\rho^{-1} \partial_z P + f_z, \tag{4.11}$$

$$\partial_t P = -K \left(\partial_x u_x + \partial_z u_z \right) \left(\tau_{\varepsilon} \tau_{\sigma}^{-1} \right) - r, \qquad (4.12)$$

$$\partial_t r = -\tau_{\sigma}^{-1} r + K \left(\partial_x u_x + \partial_z u_z \right) \tau_{\sigma}^{-1} \left(1 - \tau_{\varepsilon} \tau_{\sigma}^{-1} \right).$$
(4.13)

where $u_x(x,t)$ and $u_z(x,t)$ are the particle velocity components in the x- and z-directions respectively, P(x,t) is pressure wavefield, $\rho(x)$ is density, r is a memory variable, f_x and f_z are the body force components, K represents the bulk modulus of the medium, τ_{ε} and τ_{σ} are related to the quality factor Q and the reference angular frequency ω as (Robertsson et al., 1994)

$$\tau_{\sigma} = \frac{\sqrt{1 + 1/Q^2} - 1/Q}{\omega}, \qquad (4.14)$$
$$\tau_{\varepsilon} = \frac{1}{\omega^2 \tau_{\sigma}}.$$

In attenuative media, the wave experiences two main effects, reduced amplitude, and phase distortion due to dispersion. By separation between the amplitude attenuation and phase dispersion effects, wave propagation can be simulated in three scenarios, i.e., only the amplitude loss effect, only the phase dispersion effect, or both effects concurrently. In this chapter, I present an approach for the solution of the viscoacoustic wave equation in the time domain to explicitly separate phase dispersion and amplitude attenuation. I first apply the Fourier transform to the first-order linear differential equations 4.10, 4.11, 4.12, and 4.13 in the time domain to obtain the frequency domain viscoacoustic wave equation:

$$i\omega\tilde{u}_x = -\rho^{-1}\partial_x\tilde{P} + \tilde{f}_x,\tag{4.15}$$

$$i\omega\tilde{u}_z = -\rho^{-1}\partial_z\tilde{P} + \tilde{f}_z,\tag{4.16}$$

$$i\omega\tilde{P} = -K\left(\partial_x\tilde{u}_x + \partial_z\tilde{u}_z\right)\left(\tau_{\varepsilon}\tau_{\sigma}^{-1}\right) - \tilde{r},\tag{4.17}$$

$$i\omega\tilde{r} = -\tau_{\sigma}^{-1}\tilde{r} + K\left(\partial_x\tilde{u}_x + \partial_z\tilde{u}_z\right)\tau_{\sigma}^{-1}\left(1 - \tau_{\varepsilon}\tau_{\sigma}^{-1}\right).$$
(4.18)

From equation 4.18, the memory variable in the frequency domain can be calculated as a function of the particle velocity and the relaxation time

$$\tilde{r} = K \left(\partial_x \tilde{u}_x + \partial_z \tilde{u}_z\right) \frac{\tau_{\sigma}^{-1} \left(1 - \tau_{\varepsilon} \tau_{\sigma}^{-1}\right)}{\left(i\omega + \tau_{\sigma}^{-1}\right)}.$$
(4.19)

By substituting equation 4.19 into equation 4.17, the memory variable equation is removed. The new first-order viscoacoustic wave equation in the frequency domain without source components is

$$i\omega\tilde{P} = -K\left(\partial_x\tilde{u}_x + \partial_z\tilde{u}_z\right)\left[\left(\tau_{\varepsilon}\tau_{\sigma}^{-1}\right) + \frac{\tau_{\sigma}^{-1}(1 - \tau_{\varepsilon}\tau_{\sigma}^{-1})}{(i\omega + \tau_{\sigma}^{-1})}\right],\tag{4.20}$$

which can be re-written, after some algebraic manipulation (see Appendix C), as

$$i\omega\tilde{P} = -K\left(\partial_x\tilde{u}_x + \partial_z\tilde{u}_z\right)\left[\frac{(\omega^2\tau_\varepsilon\tau_\sigma + 1)}{\omega^2\tau_\sigma^2 + 1} + i\frac{(\omega\tau_\varepsilon - \omega\tau_\sigma)}{\omega^2\tau_\sigma^2 + 1}\right].$$
(4.21)

When transformed back to the time domain, equation 3.1 produces a viscoacoustic wave equation that maintains the approximate constant-Q attenuation and dispersion behaviours during wave propagation. To apply this equation within RTM, I write viscoacoustic forward and backward extrapolation as:

$$\partial_t P = -K \left(\partial_x u_x + \partial_z u_z \right) \left[\kappa_1(2/A) + i\kappa_2(2/AQ) \right], \tag{4.22}$$

where $A = \sqrt{1 + 1/Q^2} - 1/Q)^2 + 1$. The quantities 2/A and 2/AQ are dispersion-dominated and amplitude-attenuation-dominated operators, respectively. The only difference between viscoacoustic and acoustic wave equations is the complex- valued term 2/A + i2/AQ. The coefficients κ_1 and κ_2 are constants of unit magnitude, whose signs are important for the forward and backward extrapolation. Note that when $Q \to \infty$, the dispersion-dominated operator goes to 1 and the amplitude-loss-dominated operator disappears, i.e., the viscoacoustic case approaches to the acoustic case.



Figure 4.3: Wavefield snapshots in (a) acoustic, (b) lossy-dominated, (c) Dispersive-dominated, and (d) viscoacoustic media.

To demonstrate the decoupling of the velocity dispersion and amplitude loss, I consider a homogeneous model with a background velocity of 2500 m/s and quality factor Q=10.



Figure 4.4: Comparison between the FD and analytical solutions. Q is 20.

A snapshot of the acoustic reference wavefield is shown in Figure 4.3a. The wavefront is indicated by the dashed yellow line. The amplitude-loss simulation (imaginary part of equation 4.22) is shown in Figure 4.3b. Compared with the acoustic case, the amplitude is attenuated, but the phases are the same. In Figure 4.3c the phase dispersion simulation (real part of equation 4.22) is shown. The phase has a shift, and the amplitude is similar to the acoustic case. In Figure 4.3d the viscoacoustic wavefield is displayed. The reduced amplitude and shifted phase are visible compared with the acoustic reference wavefield. Thus, using the viscoacoustic wave equation formulation decoupling attenuation and dispersion terms, I can compensate the amplitude loss and dispersion in images separately.

In order to evaluate the accuracy of our method based on the SLS mechanism, I compare numerical solutions with the analytical solution. To obtain the analytical solution I used the viscoacoustic Green's function in a homogeneous medium is given in Carcione et al. (1988b). Figure 4.4 presents depth profiles of the wavefield extracted at two different times with Q =20. The numerical and analytical solutions agree very well.

4.5 Viscoacoustic reverse time propagation

Reverse-time migration reconstructs the receiver wavefield through backward propagation of the measured seismic data from the receiver. When the wave propagates in attenuating media, the amplitudes of the back propagation waves are reduced, and needs to be amplified. In equation 4.22, the constant κ_2 simulates the reduction of amplitudes in forward propagation. By reversing the sign of the amplitude attenuation term ($\kappa_2 = -1$) in the viscoacoustic wave equation, I can compensate for the amplitude loss. The viscoacoustic wave equation also contains a dispersion term that affects the phase during wave propagation. The sign of this term ($\kappa_1 = 1$) does not need to be changed.

For the backward modeling, the viscoacoustic wave equations with compensation of attenuation effects ($\kappa_1 = 1$ and $\kappa_2 = -1$) can be written as

$$\partial_t P = -K \left(\partial_x u_x + \partial_z u_z \right) \left[\kappa_1(2/A) - i\kappa_2(2/AQ) \right].$$
(4.23)

The phase-only viscoacoustic wave equation is

$$\partial_t P = -K \left(\partial_x u_x + \partial_z u_z \right) \left[(2/A) \right]. \tag{4.24}$$

If I want to compensate for loss only and ignore the dispersion effects, I can retain the imaginary part of equation 3.3, so the loss-dominated wave equation for back-propagation with attenuation compensation will be

$$\partial_t P = -K \left(\partial_x u_x + \partial_z u_z \right) \left[-(2/AQ) \right]. \tag{4.25}$$

Combining equations 4.10, 4.11, and 4.23 (with t replaced by -t), I obtain the viscoacoustic backward modeling equation. For the backward modeling, I solve equation 4.23 to extrapolate the receiver wavefield by time reversing the measured data $R(\mathbf{X}_r, t)$ at the receivers with a boundary condition. I have

$$P(\mathbf{X}_r, t) = R(\mathbf{X}_r, T - t).$$
(4.26)

where T and $\mathbf{X}_r = (x_r, z_r)$ represent the maximum recording time and the receiver location respectively.

Instabilities appear when compensating for absorption during extrapolation. The highfrequency components of the wavefield travel faster than low-frequency counterparts and arrive at the receivers earlier. When the receiver wavefield is reverse extrapolated, the high-frequencies again propagate faster, and this leads to instability. A basic approach for suppressing the high-frequency noise is to use a low-pass filter (Suh et al., 2012; Bai et al., 2013; Zhu et al., 2014) in the wavenumber domain. This method is not suitable for regions of high spatial variability due to the difficulty of choosing a single appropriate filter. Here, I propose a regularization term ($\varepsilon/c_p(\partial_t \nabla)$) as an alternative way to suppress reverse time propagation high frequency instability (Campbell Hetrick et al., 2008). I construct a regularized viscoacoustic equation based on equation 4.23:

$$\partial_t P = -K \left(\partial_x u_x + \partial_z u_z \right) \left[\kappa_1(2/A) - i\kappa_2(2/AQ) \right] + \varepsilon \rho c \partial_t \left(\partial_x u_x + \partial_z u_z \right), \tag{4.27}$$

where ε is a small positive regularization parameter.

4.6 2D synthetic examples

In this section, I first test our proposed approach to Q-RTM using a layered model and then using the complex Marmousi model. Figure 4.5 illustrates the true and migration velocity



Figure 4.5: (a) True velocity model, (b) migration velocity model, (c) true Q model, and (d) migration Q model.



Figure 4.6: Synthetic shot gathers in (a) an acoustic and (b) a viscoacoustic medium. Viscoacoustic data are attenuated because reflections are travelling through the high-attenuation zone.



Figure 4.7: Time-frequency spectrum of the 240th channel of (a) the acoustic (reference) and (b) the viscoacoustic shot records respectively. The frequency bandwidth in (b) becomes narrower in comparison to the reference time-frequency spectrum. (c) Spectrum of acoustic (solid) and viscoacoustic (dashed) shot gather data. The loss in high-frequency content is obvious in the viscoacoustic case.



Figure 4.8: Comparison of (a) acoustic RTM (reference), (b) acoustic RTM with viscoacoustic data, and (c) Q-RTM with viscoacoustic data. The black arrows point to the reflectors beneath the high attenuation zones where improvements from Q-RTM can be seen.



Figure 4.9: Comparisons of three traces of RTM images between the acoustic (Figure 7a), the noncompensated image (Figure 7b), and the compensated image (Figure 8). The solid red line refers to the reference (acoustic) trace, the solid green line refers to the noncompensated trace, and the dashed blue line indicates the compensated trace.

and Q models for the first test. In Figure 4.5c, the Q model contains two high attenuation anomalies. The model grid dimensions are 401×551 , with grid size 4 m×4 m. The sampling interval is 0.4 ms and the recording length is 2 s. As the source I use a zero-phase Ricker wavelet with a central frequency of 25 Hz. Perfectly matched layer (PML) absorbing boundary conditions are used to attenuate the reflections from the artificial computational boundaries (see Appendix B). FD synthetic data for the acoustic and viscoacoustic media are simulated by solving a set of first-order differential equations (4.10, 4.11, and 4.22). Examples are plotted in Figure 4.6. The viscoacoustic simulation exhibits reduced amplitude and shifted phase due to velocity dispersion. In Figure 4.7 the time-frequency spectra of the 240th traces of acoustic and viscoacoustic shot records respectively are compared. These spectra are computed using a numerical S-transform (Stockwell et al., 1996). In the spectra, we observe that the reflection energy in the viscoacoustic case decreases with depth, with the frequency bandwidth becoming narrower, the highest frequency reduced, and the dominant frequency moving towards lower frequency. The loss in frequency content with time is obvious for the viscoacoustic data from Figure 4.7c. To demonstrate the effect of attenuation, I apply viscoacoustic RTM to the viscoacoustic data set. I generate the Q-RTM image shown in Figure 4.8 and compare it with the acoustic RTM image as reference. The velocity and Q models are first smoothed from the true models and then used for migration. In acoustic RTM with viscoacoustic data (noncompensated RTM) (Figure 4.8b), there are three reflectors in the RTM-image and some regions with amplitude losses. These are indicated by black arrows. The high-attenuation anomaly causes a reduction in wave amplitude, so migrating the attenuated data produces an anomalously weak estimate of the reflectivity. To solve the illumination problem of viscoacoustic RTM images, I apply attenuation compensation during wave propagation using the new Q-RTM approach (equation 4.23). The compensated image using viscoacoustic RTM is plotted in Figure 4.8c. The result is an improved RTM image with recovered amplitudes of the reflectors at the dip depths compared with the reference image in Figure 4.8a. After compensation, the events have corrected amplitudes and corrected phases. To verify that the reflectors are migrated to the correct position I compare the image traces at different offsets. Figure 4.9 presents comparisons of three traces from the RTM images (Figure 4.8) at distances 0.58 km, 1.1 km, and 1.68 km. The viscoacoustic traces (solid green) have shifted phases and reduced amplitudes. The compensated traces (dashed blue) match much more closely in amplitude and phase the reference traces (solid red).



Figure 4.10: The Marmousi models: (a) true velocity model, (b) migration velocity model, (c) true Q model, and (d) migration Q model.

In the second example, I consider the more complex Marmousi model. Figure 4.10 illustrates the actual and migration velocity models and the corresponding true and migration Q models. In the Q model, there are some regions that attenuate waves traveling through them, such that reflections with weaker amplitudes for the deeper layers result, especially beneath strongly attenuating layers. The model grid dimensions are 281×801 and the grid size is 10 m×10 m. I model 80 sources positioned at a depth of 30 m using a zero-phase Ricker wavelet with a central frequency of 15 Hz. The sampling interval rate is 0.4 ms, and the recording length is 3 s. Example synthetic data for both acoustic and viscoacoustic



Figure 4.11: (a) Shot record from acoustic simulation; (b) shot record from viscoacoustic simulation. The viscoacoustic simulation exhibits reduced amplitude and shifted phase due to velocity dispersion.



Figure 4.12: Time-frequency spectrum of example traces from (a) acoustic simulation (reference), and (b) viscoacoustic simulation without compensation. (c) Spectrum of acoustic (solid) and viscoacoustic (dashed) shot gather data. Note that the high-frequency content in viscoacoustic case is more attenuated than acoustic case.



Figure 4.13: RTM images of the Marmousi model. (a) Acoustic RTM (reference); (b) acoustic RTM with viscoacoustic data; and (c) compensated RTM. The dashed yellow and red rectangles show regions of poor illumination caused attenuation in the upper areas. In comparison, the RTM image is clearer, and the reflector positions are more accurate than those of the acoustic RTM image.

cases are plotted in Figure 4.11. The shots include first arrivals, multiples, reflections, refractions and diffractions. The viscoacoustic simulation exhibits reduce amplitudes and phase distortions due to velocity dispersion (Figure 4.11), and the frequency bandwidth becomes narrow (Figure 4.12). The spectrum of acoustic and viscoacoustic shot gather data are plotted in Figure 4.12c. The high-frequency content is more attenuated in the viscoacoustic data than in the acoustic data.

The RTM images are plotted in Figure 4.13. This includes (a) the acoustic RTM without attenuation (reference case), (b) the acoustic RTM with viscoacoustic data (noncompensated case), and (c) the compensated RTM result using viscoacoustic RTM equations. The



Figure 4.14: Magnified views of the dashed red rectangle section of RTM-images in Figure 4.13 $\,$



Figure 4.15: Comparison of two traces extracted from the RTM images in Figure 4.14 at the horizontal offset of x = 3.35 km and x = 4.2 km. The solid red line corresponds to the reference image (acoustic RTM), solid green line corresponds to acoustic RTM with viscoacoustic data, and blue dash line corresponds to compensated RTM.



Figure 4.16: Magnified views of the dashed yellow rectangle section of RTM-images in Figure 4.15



Figure 4.17: Comparison of two traces extracted from the RTM images in Figure 4.16 at the horizontal offset of x=4.6 km and x=5.6 km. The solid red line corresponds to reference image (acoustic RTM), solid green line corresponds to noncompensated RTM, and blue dash line corresponds to compensated RTM.
reference RTM image (Figure 4.13a) has similar artifacts and amplitudes in the shallow layers compared with the noncompensated RTM image, but the noncompensated case in Figure 4.13b has very weak amplitudes in the deeper layers especially beneath the layers with strong attenuation. The attenuation impacts the amplitudes and the phases of the propagating waves, effects which are not taken into account in the acoustic RTM image. The Q-compensated RTM image is shown in Figure 4.13c. After compensation, the reflectivity amplitudes match the reference image (acoustic case), and the image resolution is improved. To verify the compensated RTM result, I show in Figures 4.14 and 4.16 magnified views of the dashed red and yellow rectangles that are illustrated in Figure 4.13. For the red rectangle, it is evident that the reflectors are improved, and the amplitude and phase in Figure 4.14c (compensated case) is comparable to the reference image in Figure 4.14a. Figure 4.15 shows comparisons of two traces from Figure 4.14 for different horizontal distances (3.35 km and 4.2 km). The non-compensated traces have shifted phase and reduced amplitudes. After compensation, the traces (dashed blue line) have improved phase signature and amplified amplitudes compared to the reference (solid red line).

In Figure 4.16, I show the enlarged images from the yellow rectangle, which include the top of the anticline structures (black arrows) in the right part of the model (Figure 4.13). The compensated RTM in Figure 4.16c is a significantly improved RTM image, with better illumination of the anticline structures. Comparisons of two traces, from Figure 4.16 for different offsets (4.6 km and 5.6 km) are plotted in Figure 4.17. The compensated traces recover the reflector amplitudes (blue dashed line) especially near the anticline structures (hydrocarbon reservoirs); the results are comparable to the reference image.

4.7 3D synthetic example in complex media

The synthetic 3D SEG/EAGE salt model (Aminzadeh et al., 1997) offers an opportunity to test the Q-RTM equation in a 3D setting (see Appendix E). I chose a small portion of this model that contains thin sand layers and a salt body with a complex topology. The true and migration velocity models with a Q anomaly are plotted in Figures 4.18 and 4.19. The Q model contains a high attenuation anomaly with Q=20. The data set is generated by



Figure 4.18: Velocity cubes in the 3D SEG/EAGE salt models: (a) true velocity; (b) migration velocity.



Figure 4.19: Q models: (a) true Q; (b) migration Q.



Figure 4.20: Synthetic data example. (a) shot record in the acoustic approximation; (b) crossline shot record in the acoustic approximation; (c) inline and (d) shot records in the viscoacoustic case.



Figure 4.21: Reference acoustic image by applying acoustic RTM on acoustic data.



Figure 4.22: Acoustic RTM image with viscoacoustic data (noncompensated). It is obvious the attenuation effect especially for Salt flanks under the high-attenuation zone.



Figure 4.23: Viscoacoustic RTM image (compensated). RTM image is clear, and the position is accurate from compensated viscoacoustic RTM, compared with the acoustic RTM image.

finite differences with a Ricker source with central frequency 15 Hz. The grid spacings for the three directions are 20 m, and the sampling interval is 1 ms. Figure 4.20 shows the horizontal component of acoustic and viscoacoustic data respectively. The data from the viscoacoustic simulation shows reduced amplitude and shifted phase compared to those of the acoustic case beneath the strongly attenuating zones. I apply acoustic and viscoacoustic RTM imaging to viscoacoustic seismic data. The RTM images include the RTM of viscoacoustic data without compensation (Figure 4.21), the compensated viscoacoustic RTM (Figure 4.22), and the reference RTM results (Figures 4.23). For viscoacoustic RTM, without compensating for amplitude loss, the image amplitudes and positions are inaccurate for the reflectors beneath the strongly attenuating layers, and salt flanks in the image are shifted down relative to the reference depth. After compensation, in the viscoacoustic RTM, the migrated amplitudes of layers and salt flanks are more accurate by resolved than in the non-compensated RTM images, and the reflectors are imaged at the correct locations compared with the reference images.

4.8 Discussion

I present and analyze a Q-RTM approach that models the phase dispersion and amplitude attenuation independently and compensates for attenuation effects in the reconstructed wavefield by reversing the sign of loss operators. The main issue affecting the resolution of RTM images during the attenuation compensation is the stability of wave propagation. The data are often contaminated with high-frequency noise which lead to instability. To suppress the reverse time propagation high-frequency instability I add a regularization term to the viscoacoustic wave equation. Our approach is suitable for strongly heterogenous regions because I can select the regularization parameter for each layer of model separately. I point out that the source normalized cross-correlation imaging condition is particularly suitable for Q-RTM images. In this imaging condition, only backward receiver wavefield require compensation; the source wavefield remains attenuated. The compensated image from the source normalized crosscorrelation imaging condition is theoretically equivalent to that obtained using the imaging condition for the acoustic case. This Q-RTM approach will also be suitable for other related problems such as viscoacoustic full waveform inversion (Q-FWI). The adjoint of the attenuation operator causes the amplitude of the back-propagated wavefield to be attenuated more. This is not favourable for the FWI gradient computation. Our Q-RTM approach would tend to compensate the phase dispersion, and amplitude losses during wavefield back-propagation and also provide a strategy for updating the velocity and attenuation separately.

The number of first-order equations to simulate the viscoacoustic wave propagation depend on the number of mechanisms (Fathalian and Innanen, 2017), which affects the computational cost of Q-RTM. Hence, the simulation of Q-RTM requires more computational time than acoustic RTM. Because our approach does not require any extra memory variables in the viscoacoustic wave equation, computational cost can be reserved for a large number of mechanisms.

4.9 Conclusions

I have presented a viscoacoustic RTM imaging algorithm based on the time-domain constant-Q wave propagation that can correct the attenuation and dispersion effects in migrated images. The amplitude loss and phase dispersion in the source and receivers wavefields can be recovered by applying compensation operators on the measured receiver wavefield. The phase dispersion and amplitude attenuation operators in Q-RTM approach are separated, and the compensation operators are constructed by reversing the sign of the attenuation operator without changing the sign of the dispersion operator. I constructed a regularized equation to suppress the reverse time propagation high-frequency instabilities and to improve the resolution of images. Numerical tests in 2D and 3D on synthetic data illustrate that this Q-RTM approach can improve the image resolution, particularly beneath areas with strong attenuation.

Appendix C

Viscoacoustic wave equation in the frequency domain

The first-order viscoacoustic wave equation in the frequency domain without source components and memory variable is

$$i\omega\tilde{P} = -K\left(\partial_x\tilde{u}_x + \partial_z\tilde{u}_z\right)\left[\left(\tau_{\varepsilon}\tau_{\sigma}^{-1}\right) + \frac{\tau_{\sigma}^{-1}\left(1 - \tau_{\varepsilon}\tau_{\sigma}^{-1}\right)}{\left(i\omega + \tau_{\sigma}^{-1}\right)}\right],\tag{C.1}$$

The expression inside the bracket can be simplified as

$$\begin{pmatrix} \tau_{\varepsilon}\tau_{\sigma}^{-1} \end{pmatrix} + \frac{\tau_{\sigma}^{-1}\left(1 - \tau_{\varepsilon}\tau_{\sigma}^{-1}\right)}{\left(i\omega + \tau_{\sigma}^{-1}\right)} = \frac{\left(\tau_{\varepsilon}\tau_{\sigma}^{-1}\right)\left(i\omega + \tau_{\sigma}^{-1}\right) + \tau_{\sigma}^{-1}\left(1 - \tau_{\varepsilon}\tau_{\sigma}^{-1}\right)}{\left(i\omega + \tau_{\sigma}^{-1}\right)}$$

$$= \frac{i\omega\tau_{\varepsilon}\tau_{\sigma}^{-1} + \tau_{\varepsilon}\tau_{\sigma}^{-2} + \tau_{\sigma}^{-1} - \tau_{\varepsilon}\tau_{\sigma}^{-2}}{\left(i\omega + \tau_{\sigma}^{-1}\right)}$$

$$= \frac{i\omega\tau_{\varepsilon}\tau_{\sigma}^{-1} + \tau_{\sigma}^{-1}}{\left(i\omega + \tau_{\sigma}^{-1}\right)}$$

$$= \frac{i\omega\tau_{\varepsilon}\tau_{\sigma}^{-1} + \tau_{\sigma}^{-1}}{\left(i\omega + \tau_{\sigma}^{-1}\right)}$$

$$(C.2)$$

I can multiply both top and bottom of expression by $-i\omega + \tau_{\sigma}^{-1}$ (the conjugate of $i\omega + \tau_{\sigma}^{-1}$):

$$\frac{i\omega\tau_{\varepsilon}\tau_{\sigma}^{-1} + \tau_{\sigma}^{-1}}{\left(i\omega + \tau_{\sigma}^{-1}\right)} = \frac{i\omega\tau_{\varepsilon}\tau_{\sigma}^{-1} + \tau_{\sigma}^{-1}}{\left(i\omega + \tau_{\sigma}^{-1}\right)} \times \frac{\left(-i\omega + \tau_{\sigma}^{-1}\right)}{\left(-i\omega + \tau_{\sigma}^{-1}\right)} = \frac{\omega^{2}\tau_{\varepsilon}\tau_{\sigma}^{-1} + i\omega\tau_{\varepsilon}\tau_{\sigma}^{-2} - i\omega\tau_{\sigma}^{-1} + \tau_{\sigma}^{-2}}{\omega^{2} + \tau_{\sigma}^{-2}} = \frac{\tau_{\sigma}^{-2}\left(\omega^{2}\tau_{\varepsilon}\tau_{\sigma} + i\omega\tau_{\varepsilon} - i\omega\tau_{\sigma} + 1\right)}{\tau_{\sigma}^{-2}\left(\omega^{2}\tau_{\varepsilon}^{2} + 1\right)} = \frac{\omega^{2}\tau_{\varepsilon}\tau_{\sigma} + i\omega\tau_{\varepsilon} - i\omega\tau_{\sigma} + 1}{\omega^{2}\tau_{\sigma}^{2} + 1} = \frac{\left(\omega^{2}\tau_{\varepsilon}\tau_{\sigma} + 1\right)}{\omega^{2}\tau_{\sigma}^{2} + 1} + i\frac{\left(\omega\tau_{\varepsilon} - \omega\tau_{\sigma}\right)}{\omega^{2}\tau_{\sigma}^{2} + 1}$$
(C.3)

After these algebra manipulations, the viscoacoustic wave equation in the frequency domain can be written as

$$i\omega\tilde{P} = -K\left(\partial_x\tilde{u}_x + \partial_z\tilde{u}_z\right)\left[\frac{\left(\omega^2\tau_{\varepsilon}\tau_{\sigma} + 1\right)}{\omega^2\tau_{\sigma}^2 + 1} + i\frac{\left(\omega\tau_{\varepsilon} - \omega\tau_{\sigma}\right)}{\omega^2\tau_{\sigma}^2 + 1}\right].$$
(C.4)

Appendix D

Unsplit-field PML formulation

The crux of the trade-off between introducing boundary artifacts and computing within tractably small domains is the management of boundary conditions. Wave propagation can be modelled with a finite difference scheme by introducing a series of standard linear solid mechanisms, and it can be carried out within a computationally tractable region by making use of perfectly-matched layer (PML) boundary conditions. An efficient un-split field PML scheme is derived by introducing appropriate auxiliary variables and their associated partial differential equations as (Fathalian and Innanen, 2016)

$$\partial_t u_x = -\rho^{-1} \partial_x P - d(x) u_x, \tag{D.1}$$

$$\partial_t u_y = -\rho^{-1} \partial_y P - d(y) u_y, \tag{D.2}$$

$$\partial_t u_z = -\rho^{-1} \partial_z P - d(z) u_z, \tag{D.3}$$

$$\partial_t P = -K \left[\frac{\partial \left(u_x + d(z) u_x^{(1)} \right)}{\partial x} + \frac{\partial \left(u_z + d(x) u_z^{(1)} \right)}{\partial z} \right] \left[\kappa_1 \left(2/A \right) + i \kappa_2 \left(2/AQ \right) \right] \quad (D.4)$$
$$- \left[d(x) + d(z) \right] P - d(x) d(z) p^{(1)},$$

where d(x) and d(z) are the damping coefficients in the PML region. In the interior region (i.e., the physical domain), the damping parameters are set to d(x) = 0 and d(z) = 0, and in the PML region, the damping parameters are d(x) > 0 and d(z) > 0. The auxiliary variables $u_x^{(1)}$, $u_z^{(1)}$, and $P^{(1)}$ are the time-integrated components for velocity and pressure fields. They are defined as

$$u_{x}^{(1)}(X,t) = \int_{-\infty}^{t} u_{x}(X,t')dt',$$

$$u_{z}^{(1)}(X,t) = \int_{-\infty}^{t} u_{z}(X,t')dt',$$

$$P^{(1)}(X,t) = \int_{-\infty}^{t} P(X,t')dt',$$
(D.5)

Appendix E

3D viscoacoustic wave equation

The Viscoacoustic velocity/stress equations in 3D can be written as

$$\partial_t u_x = -\rho^{-1} \partial_x P + f_x, \tag{E.1}$$

$$\partial_t u_y = -\rho^{-1} \partial_y P + f_y, \tag{E.2}$$

$$\partial_t u_z = -\rho^{-1} \partial_z P + f_z, \tag{E.3}$$

$$\partial_t P = -K \bigg(\partial_x u_x + \partial_y u_y + \partial_z u_z \bigg) \bigg[\kappa_1 \bigg(2/A \bigg) + i \kappa_2 \bigg(2/AQ \bigg) \bigg], \tag{E.4}$$

where

$$A = \sqrt{1 + 1/Q^2} - 1/Q)^2 + 1.$$
 (E.5)

To obtain the 3D backward equation for attenuation compensation, the sign of the loss operator must be changed in the reconstructed wavefield.

Chapter 5

Viscoacoustic VTI and TTI wave equations and their application for anisotropic reverse time migration: Constant-Q approximation

5.1 Abstract

In this chapter, I investigate the simulation of viscoacoustic wave propagation and reverse time migration (RTM) in transversely isotropic (TI) media, vertical TI (VTI) and tilted TI (TTI). Such wave propagation can be modeled with a finite difference scheme by introducing a series of standard linear solid (SLS) mechanisms, and it can be carried out within a computationally tractable region by making use of perfectly-matched layer (PML) boundary conditions. The viscoacoustic wave equation for VTI and TTI mediums is derived using the wave equation in anisotropic media by setting the value of shear wave velocity to zero. Using the TI approximation and ignoring all spatial derivatives of the anisotropic symmetry axis direction lead to instabilities in some areas of the model with rapid variations in the symmetry axis direction. A solution to this problem is proposed that involves using a selective anisotropic parameter matching technique to reduce the instability in regions with rapid dip angle variation. After correcting for the eects of anisotropy, TTI RTM in attenuating media can produce a more accurate image than isotropic RTM, especially in areas with anisotropy and rapid variations of dip angle.

5.2 Introduction

To consider anisotropic media, the isotropic acoustic assumption for seismic processing and imaging affects the resolution and reflector locations of subsurface structures (Zhou et al., 2006a). Therefore, it is necessary to focus on the anisotropy and viscosity for complex media to obtain a significant improvement in image resolution and positioning. There are two ways to consider an anisotropic medium, the pseudo-acoustic wave equation and the pure acoustic wave equation. Alkhalifah (1998, 2000) derived the pseudo-acoustic wave equation from the dispersion relation by setting the shear-wave velocity along the anisotropy symmetry axis to be zero. To reduce the computational time, based on the pseudo-acoustic approximation, Zhou et al. (2006b) and Duveneck et al. (2008) developed and simplified the pseudo-acoustic wave equation into two coupled second- order partial differential equations to account for VTI media. Although the VTI wave equation is used to image structures which have similar properties with VTI media (Crampin, 1984), it may not be satisfied in anisotropic dipping layers. The TTI equations have been derived from VTI equations by assuming the symmetry axis is non-vertical and locally variable (Fletcher et al., 2008; Zhang and Zhang, 2008). The TI wave equations with the zero value of SV wave's velocity on the axis symmetry can not remove the effect of the residual shear wave, so an instability occurs. Fletcher et al. (2009) proposed new equations by adding non-zero S-wave velocity terms to solve the problem. To stabilize wave propagation and reduce shear wave artifacts the anisotropy model can be smoothed before numerical simulation, and $\varepsilon = \delta$ can be set in the regions around source and areas with a high symmetry axis gradient (Zhang and Zhang, 2008; Yoon et al., 2010). Usually, when investigating the RTM images, the focus is on the anisotropy or viscosity. In this work, I focus on both anisotropy and viscosity to obtain accurate RTM images. I investigate the simulation of wave propagation within the constant-Q approximation in the time domain and derive a viscoacoustic wave equation for VTI and TTI media. This chapter is organized as follows. In the first section, I describe the research background, and then the anisotropic model is presented. I introduce the split-field PML formulation of the approximate constant-Q viscoacoustic wave equation for VTI and TTI media in the second section. Numerical results on synthetic data are presented in the fourth section.

5.3 Viscoacoustic wave equation in anisotropic media

In this section, I derive systems of first-order differential equations regarding the particle velocities and stresses that describe the propagation of waves in anisotropic media. The starting point for driving viscoacoustic wave equations is Hooke's law, with the elastic tensor, together with the equation of motion in anisotropic media with $V_S = 0$ (Alkhalifah, 2000). I derive the viscoacoustic VTI equation directly from acoustic VTI media, and then the viscoacoustic TTI wave equations will be calculated by introducing a rotation of the coordinate system.

5.3.1 Viscoacoustic VTI media equation

In the 2D case, the first order acoustic VTI wave equations can be written as (Duveneck et al., 2008)

$$\sigma_H = \rho V_P^2 \left[(1+2\varepsilon)\varepsilon_{11} + \sqrt{1+2\delta}\varepsilon_{33} \right], \qquad (5.1)$$

$$\sigma_V = \rho V_P^2 \left[\sqrt{1+2\delta}\varepsilon_{11} + \varepsilon_{33} \right],$$

where σ_H and σ_V represent the horizontal and vertical stress components respectively. ε and δ are Thomsen parameters, and the ε_{ij} are diagonal elements of the strain tensor. The first order differential equations of acoustic VTI media can be obtained by taking a time derivative of the stress-strain relationship given in eq.5.1 and combining the result with the equations of motion

$$\partial_t u_x = (1/\rho) \partial_x \sigma_H, \tag{5.2}$$
$$\partial_t u_z = (1/\rho) \partial_z \sigma_V,$$

$$\partial_t \sigma_H = \rho V_P^2 \left[(1+2\varepsilon) \partial_x u_x + \sqrt{1+2\delta} \partial_z u_z \right], \qquad (5.3)$$
$$\partial_t \sigma_V = \rho V_P^2 \left[\sqrt{1+2\delta} \partial_x u_x + \partial_z u_z \right],$$

where $\partial_t \varepsilon_{ij} = \partial_{xi} u_i$ has been used, and u_x , and u_z are components of the particle velocity vector. Equations for a viscoacoustic anisotropic medium can be obtained by modifying eqs.5.2 and 5.3. The viscoacoustic wave equation in anisotropic media for a series of SLS can be written as

$$\partial_t u_x = (1/\rho) \partial_x \sigma_H, \tag{5.4}$$
$$\partial_t u_z = (1/\rho) \partial_z \sigma_V,$$

$$\partial_t \sigma_H = \rho V_P^2 \left[\left(1 + 2\varepsilon \right) \left[\left[1 - \sum_{\ell=1}^L \left(1 - \frac{\tau_{\varepsilon\ell}}{\tau_{\sigma\ell}} \right) \right] \partial_x u_x - \sum_{\ell=1}^L r_{H\ell} \right] + \sqrt{1 + 2\delta} \partial_z u_z \right], \quad (5.5)$$
$$\partial_t \sigma_V = \rho V_P^2 \left[\sqrt{1 + 2\delta} \partial_x u_x + \left[1 - \sum_{\ell=1}^L \left(1 - \frac{\tau_{\varepsilon\ell}}{\tau_{\sigma\ell}} \right) \right] \partial_z u_z - \sum_{\ell=1}^L r_{V\ell} \right],$$

where $r_{H\ell}$, and $r_{V\ell}$, are memory variables for the horizontal and vertical stress components (Carcione et al., 1988a). These memory variables satisfy

$$\partial_t r_{H\ell} = -\frac{1}{\tau_{\sigma\ell}} r_{H\ell} + \rho V_p^2 \left(\partial_x u_x \right) \frac{1}{\tau_{\sigma\ell}} \left(1 - \frac{\tau_{\varepsilon\ell}}{\tau_{\sigma\ell}} \right), \tag{5.6}$$
$$\partial_t r_{V\ell} = -\frac{1}{\tau_{\sigma\ell}} r_{V\ell} + \rho V_p^2 \left(\partial_z u_z \right) \frac{1}{\tau_{\sigma\ell}} \left(1 - \frac{\tau_{\varepsilon\ell}}{\tau_{\sigma\ell}} \right) . 1 \le \ell \le L,$$

The stress and strain relaxation parameters, τ_{ε} and τ_{σ} , are related to the quality factor Qand the reference angular frequency ω as (Robertsson et al., 1994)

$$\tau_{\sigma} = \frac{\sqrt{1 + 1/Q^2} - 1/Q}{\omega}, \qquad (5.7)$$
$$\tau_{\varepsilon} = \frac{1}{\omega^2 \tau_{\sigma}}.$$

where ω is the central frequency of the source wavelet.

In order to introduce the PML boundary for viscoacoustic waves, the first-order linear differential equations are modified using the complex coordinate stretching approach. In the frequency domain, derivative operators are replaced as follows

$$\partial_x \to \left[1 + \frac{id(x)}{\omega} \right] \partial_x, \tag{5.8}$$
$$\partial_z \to \left[1 + \frac{id(z)}{\omega} \right] \partial_z.$$

By applying the complex coordinate stretching to the first-order linear differential equations 5.4, and 5.5 in the frequency domain I obtain

$$-i\omega \left[1 + \frac{d(x)}{-i\omega}\right] \tilde{u}_x = \frac{1}{\rho} \partial_x \tilde{\sigma_H}, \tag{5.9}$$

$$-i\omega \left[1 + \frac{d(z)}{-i\omega}\right] \tilde{u}_z = -\frac{1}{\rho} \partial_z \tilde{\sigma_V}, \qquad (5.10)$$

$$-i\omega\left[1+\frac{d(x)}{-i\omega}\right]\left[1+\frac{d(z)}{-i\omega}\right]\tilde{\sigma}_{H} = \rho V_{P}^{2}\left[\left(1+2\varepsilon\right)\left[\left[1-\sum_{\ell=1}^{L}\left(1-\frac{\tau_{\varepsilon\ell}}{\tau_{\sigma\ell}}\right)\right]\times\left(1+\frac{d(z)}{-i\omega}\right)\partial_{x}\tilde{u}_{x}-\sum_{\ell=1}^{L}r_{H\ell}\right]+\sqrt{1+2\delta}\left(1+\frac{d(x)}{-i\omega}\right)\partial_{z}\tilde{u}_{z}\right],\quad(5.11)$$

$$-i\omega\left[1+\frac{id(x)}{\omega}\right]\left[1+\frac{id(z)}{\omega}\right]\tilde{\sigma}_{V} = \rho V_{P}^{2}\left[\sqrt{1+2\delta}\left(1+\frac{d(z)}{-i\omega}\right)\partial_{x}\tilde{u}_{x}+\left[1-\sum_{\ell=1}^{L}\left(1-\frac{\tau_{\varepsilon\ell}}{\tau_{\sigma\ell}}\right)\right]\left(1+\frac{d(z)}{-i\omega}\right)\partial_{z}\tilde{u}_{z} - \sum_{\ell=1}^{L}r_{H\ell}\right],\quad(5.12)$$

where \tilde{u}_x , \tilde{u}_z , $\tilde{\sigma}_H$, and $\tilde{\sigma}_V$ are the temporal Fourier transforms of u_x , u_z , σ_H , and σ_V , respectively. To calculate the split-field PML formulations, these equations must be transformed back to the time domain. In the split-field PML formulations, the velocity, and pressure fields are split into two independent parts based on the spatial derivative terms in the original equations in two space dimensions. For one relaxation mechanism (L = 1), which is sufficient for practical purposes (Blanch et al., 1995), equations 5.9, 5.10, 5.11, and 5.12 are transformed back to the time domain to get the split-field PML formulations

$$\partial_t u_x = \frac{1}{\rho} \partial_x \sigma_H - d(x) u_x, \qquad (5.13)$$

$$\partial_t u_z = \frac{1}{\rho} \partial_z \sigma_V - d(z) u_z, \tag{5.14}$$

$$\partial_t \sigma_H = \rho V_P^2 \left[(1+2\varepsilon) \left[\left(\frac{\tau_\varepsilon}{\tau_\sigma} \right) \left[\partial_x (u_x + d(z)u_x^{(1)}) \right] - r_H \right] + \sqrt{1+2\delta} \left[\partial_z (u_z + d(x)u_z^{(1)}) \right] - (d(x) + d(z))\sigma_H - d(x)d(z)\sigma_H^{(1)}, \quad (5.15)$$

$$\partial_t \sigma_V = \rho V_P^2 \left[\sqrt{1 + 2\delta} \left[\partial_x (u_x + d(z)u_x^{(1)}) \right] + \left(\frac{\tau_\varepsilon}{\tau_\sigma}\right) \left[\partial_z (u_z + d(x)u_z^{(1)}) \right] - r_V \right] - (d(x) + d(z))\sigma_V - d(x)d(z)\sigma_V^{(1)}, \quad (5.16)$$

where the auxiliary variables $u_x^{(1)}$, $u_z^{(1)}$, $\sigma_H^{(1)}$, and $\sigma_V^{(1)}$ are the time-integrated components for velocity, pressure and memory variable fields. To avoid the high-frequency effect on reverse time propagation, a regularization approach must be considered. I construct a regularized equation based on equations 5.15, and 5.16 in viscoacostic VTI media

$$\partial_t \sigma_H = \rho V_P^2 \left[(1+2\varepsilon) \left[\left(\frac{\tau_\varepsilon}{\tau_\sigma} \right) \left[\partial_x (u_x + d(z)u_x^{(1)}) \right] - r_H \right] \right. \\ \left. + \sqrt{1+2\delta} \left[\partial_z (u_z + d(x)u_z^{(1)}) \right] \right] - \left[\epsilon \rho V_P \sqrt{1+2\varepsilon} \left[\partial_t (u_x + d(z)u_x^{(1)}) \right] \right] \\ \left. - (d(x) + d(z))\sigma_H - d(x)d(z)\sigma_H^{(1)}, \quad (5.17) \right] \right]$$

$$\partial_t \sigma_V = \rho V_P^2 \left[\sqrt{1 + 2\delta} \left[\partial_x (u_x + d(z)u_x^{(1)}) \right] + \left(\frac{\tau_{\varepsilon}}{\tau_{\sigma}} \right) \left[\partial_z (u_z + d(x)u_z^{(1)}) \right] - r_V \right] \\ - \left[\epsilon \rho V_P \left[\partial_t (u_z + d(x)u_z^{(1)}) \right] \right] - (d(x) + d(z))\sigma_V - d(x)d(z)\sigma_V^{(1)}.$$
(5.18)

where ϵ is a small positive regularization parameter.

I examine the numerical character of the solutions of the wave equation for viscoacoustic VTI media as created using the split-field PML boundary approach. I first consider propagation of waves in a homogeneous model. The anisotropic viscoacoustic medium considered here is characterized by the constant velocity model with $V_P = 2500$ m/s, and a grid size of 651×651. One source, located at the point (1300 m, 1300 m), consists of a zero-phase Ricker wavelet with a central frequency of 25 Hz. The grid spacing in the x and z directions is 4 m, and the Thomsen anisotropic parameters $\varepsilon = 0.2$ and $\delta = 0.05$. In Figure 5.1 the 0.5 s snapshots of the 2D anisotropic viscoacoustic wavefield are plotted. The wavefront of P wave shows an elliptical shape because of anisotropic effect.

The shear waves are generated by the source (Figure 5.1). For acoustic and viscoacoustic media, these shear waves have to regarded as artifacts(Alkhalifah, 2000; Grechka et al., 2004). These artifacts are only generated in an elliptic media ($\varepsilon \neq \delta$), but they can be suppressed by designing a small smoothly tapered circular region with $\varepsilon = \delta$ around the source (Figure 5.2).

In Figure 5.3 I show the attenuation effects in anisotropic media for different values of quality factor (Q=infinity, Q=100, Q=20 and Q=10). The attenuation affects the seismic wave energy, and the phase velocity changes with direction. In Figure 5.4 depth profiles of the wave field extracted at t = 0.5 s are plotted. The solid black line, solid blue line,



Figure 5.1: 2D wavefield snapshots in a viscoacoustic VTI medium with $\varepsilon = 0.2$, and $\delta = 0.05$. (a) σ_H , (b) σ_V , (c) u_x , and (d) u_z . Shear wave artifacts are generated by ignoring elastic waves in equations 5.15 and 5.16.



Figure 5.2: 2D wavefield snapshots with suppression of source-generated shear wave artifacts.



Figure 5.3: The 0.5s snapshots with four different quality factors in a VTI medium: (a) Q = infinity, (b)Q = 100, (c) Q = 20, and (d) Q = 10.



Figure 5.4: Depth traces of FD showing the comparison trace number 370 of viscoacoustic VTI medium with different Q.

dashed red line, and dashed black line represent the acoustic wave, and three examples of the viscoacoustic wave, with Q = 100, Q = 20 and Q = 10 respectively. There are two main effects visible, reduced amplitude and phase shift due to dispersion.

5.3.2 Viscoacoustic TTI media equation

The VTI medium is only valid for simple geologic formations. In anticline structures and thrust sheets where sediments are steeply dipping, the VTI medium approximation is not useful because of non-vertical symmetry axis of the medium. In such areas, it is better to use the tilted transversely isotropic (TTI) media. One way to calculate the TTI equations is to locally rotate the coordinate system of VTI medium. The rotation matrix as function of the polar angle and azimuth angle is defined as

$$\mathbf{R} = \begin{pmatrix} \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta\\ -\sin\varphi & \cos\varphi & 0\\ \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \end{pmatrix}$$
(5.19)

where θ represent the tilt angle and φ represent the azimuth of tilt for the TTI symmetry axis. The spatial derivatives in a rotated coordinate system can be written as

$$\begin{pmatrix} \partial_{x'} \\ \partial_{y'} \\ \partial_{z'} \end{pmatrix} = \mathbf{R} \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix}$$
(5.20)

where primed refer to the rotated coordinate system. Substituting eq.5.20 into eq.5.4 and eq.5.5, the 2D viscoacoustic wave equation in anisotropic TTI media for a series of SLS can be written as

$$\partial_t u_x = (1/\rho) \partial_{x'} \sigma_H, \tag{5.21}$$
$$\partial_t u_z = (1/\rho) \partial_{z'} \sigma_V,$$

$$\partial_t \sigma_H = \rho V_P^2 \left[\left(1 + 2\varepsilon \right) \left[\left[1 - \sum_{\ell=1}^L \left(1 - \frac{\tau_{\varepsilon\ell}}{\tau_{\sigma\ell}} \right) \right] \partial_{x'} u_x - \sum_{\ell=1}^L r_{H\ell} \right] + \sqrt{1 + 2\delta} \partial_{z'} u_z \right], \quad (5.22)$$
$$\partial_t \sigma_V = \rho V_P^2 \left[\sqrt{1 + 2\delta} \partial_{x'} u_x + \left[1 - \sum_{\ell=1}^L \left(1 - \frac{\tau_{\varepsilon\ell}}{\tau_{\sigma\ell}} \right) \right] \partial_{z'} u_z - \sum_{\ell=1}^L r_{V\ell} \right],$$

where $\partial_{x'}$, and $\partial_{z'}$ are the first order differential operators in the rotated coordinate system aligned with the symmetry axis:

$$\partial_{x'} = \cos\theta\cos\varphi\partial_x - \sin\theta\partial_z, \qquad (5.23)$$
$$\partial_{z'} = \cos\varphi\sin\theta\partial_x + \cos\theta\partial_z,$$

For one relaxation mechanism (L = 1), I apply the complex coordinate stretching to the first-order linear differential equations in the frequency domain. To calculate the split-field PML formulations these equations transformed back to the time domain. The split-PML viscoacoustic wave equations in TTI medium become



Figure 5.5: Snapshots at t = 0.5s with four different quality factors in a TTI medium: (a) Q = infinity, (b) Q = 100, (c) Q = 20, and (d) Q = 10.

$$\partial_t u_x = \frac{1}{\rho} (\cos\theta\cos\varphi\partial_x - \sin\theta\partial_z)\sigma_H - d(x)u_x, \qquad (5.24)$$

$$\partial_t u_z = \frac{1}{\rho} (\cos\varphi \sin\theta \partial_x + \cos\theta \partial_z) \sigma_V - d(z) u_z, \qquad (5.25)$$

$$\partial_t \sigma_H = \rho V_P^2 \left[(1+2\varepsilon) \left[\left(\frac{\tau_\varepsilon}{\tau_\sigma} \right) \left[(\cos\theta\cos\varphi\partial_x - \sin\theta\partial_z)(u_x + d(z)u_x^{(1)}) \right] - r_H \right] + \sqrt{1+2\delta} \left[(\cos\varphi\sin\theta\partial_x + \cos\theta\partial_z)(u_z + d(x)u_z^{(1)}) \right] - (d(x) + d(z))\sigma_H - d(x)d(z)\sigma_H^{(1)}, \quad (5.26)$$

$$\partial_t \sigma_V = \rho V_P^2 \left[\sqrt{1 + 2\delta} \left[(\cos\theta\cos\varphi\partial_x - \sin\theta\partial_z)(u_x + d(z)u_x^{(1)}) \right] + \left(\frac{\tau_\varepsilon}{\tau_\sigma}\right) \left[(\cos\varphi\sin\theta\partial_x + \cos\theta\partial_z)(u_z + d(x)u_z^{(1)}) \right] - r_V \right] - (d(x) + d(z))\sigma_V - d(x)d(z)\sigma_V^{(1)}, \quad (5.27)$$

Similarl to the VTI medium, regularization must be applied because of high frequency instability. I construct a regularization equation based on the split-field PML equation in TTI medium:

$$\partial_t \sigma_H = \rho V_P^2 \left[(1+2\varepsilon) \left[\left(\frac{\tau_\varepsilon}{\tau_\sigma} \right) \left[(\cos\theta\cos\varphi\partial_x - \sin\theta\partial_z)(u_x + d(z)u_x^{(1)}) \right] - r_H \right] \right. \\ \left. + \sqrt{1+2\delta} \left[(\cos\varphi\sin\theta\partial_x + \cos\theta\partial_z)(u_z + d(x)u_z^{(1)}) \right] \right] \\ \left. - \left[\epsilon\rho V_P \sqrt{1+2\varepsilon} \left[\partial_t \left[(\cos\theta\cos\varphi\partial_x - \sin\theta\partial_z)(u_x + d(z)u_x^{(1)}) \right] \right] \right] \\ \left. - \left(d(x) + d(z) \right) \sigma_H - d(x)d(z)\sigma_H^{(1)}. \right]$$
(5.28)

$$\partial_t \sigma_V = \rho V_P^2 \left[\sqrt{1 + 2\delta} \left[(\cos\theta\cos\varphi\partial_x - \sin\theta\partial_z)(u_x + d(z)u_x^{(1)}) \right] \\ + \left(\frac{\tau_{\varepsilon}}{\tau_{\sigma}}\right) \left[(\cos\varphi\sin\theta\partial_x + \cos\theta\partial_z)(u_z + d(x)u_z^{(1)}) \right] - r_V \right] \\ - \left[\epsilon \rho V_P [\partial_t [(\cos\varphi\sin\theta\partial_x + \cos\theta\partial_z)(u_z + d(x)u_z^{(1)})]] \right] - (d(x) + d(z))\sigma_V - d(x)d(z)\sigma_V^{(1)}.$$
(5.29)



Figure 5.6: Depth traces of FD showing the comparison trace number 370 of viscoacoustic TTI medium with different Q.



Figure 5.7: P-wave velocity model

where ϵ is a small positive regularization parameter.

Figure 5.5 shows time snapshots of viscoacoustic wave propagation in a 2D homogenous anisotropic medium. The anisotropic viscoacoustic medium considered here is characterized by a constant velocity model with $V_P = 2500$ m/s, and a grid size of 651×651 . The source is located at the center of the model, and the source signature is a zero-phase Ricker wavelet with a central frequency of 25 Hz. The grid spacing in the x and z directions is 4 m, and the Thomsen anisotropic parameters are $\varepsilon = 0.2$ and $\delta = 0.05$. Figure 5.5 corresponds to an axis of symmetry tilting at 45. The compressional P wavefront is approximately ellipsoidal and the shear wave artifacts generated in anelliptic media are suppressed at the source by designing a small smoothly tapered circular region with $\varepsilon = \delta$ around the source. However, to avoid the numerical instability in TTI media, I apply the viscoacoustic wave equation and simply set the shear wave velocity along the tilted symmetry axis to zero. In Figure 5.6 I show the effect of attenuation on amplitude and phase of a propagating seismic wave in a homogeneous anisotropic medium for different values of quality factor (Q=infinity, Q=100, Q=20 and Q=10). The attenuation affects the seismic wave energy, and the phase velocities of waves in all directions are different. The amplitude is reduced and there is phase shift due to dispersion. Also, in attenuating media the reflection wave energy decays with the increase of the depth. Therefore, with increasing depth, the frequency bandwidth becomes narrower, and the dominant frequency moves to the low frequency.



Figure 5.8: Transversely isotropic velocity model. (a) Thomsen's ϵ model, (b) Thomsen's δ model, (c) Tilted dip angle along the tilted symmetry-axis, and (d) Q model.

5.4 Synthetic RTM example

To verify the accuracy of the viscoacoustic wave equation in TTI media, the 2D data set modeled in an inhomogeneous TTI velocity model (Duveneck and Bakker, 2011) is tested (Figure 5.7). There are some dipping anisotropic layers in the velocity model that terminate against the salt body. The Q and anisotropy models are shown in Figure 5.8. The rapid variation of the tilt angle affected the TTI RTM images. The model grid dimensions are 700×1200 , and the grid size is $6.25 \text{ m} \times 6.25 \text{ m}$. The sampling interval is 0.8 ms, and the recording length is 6 s. I use as the source a zero-phase Ricker wavelet with a center frequency of 10 Hz. To remove the S-wave artifacts the source is located in the isotropic part of the



Figure 5.9: The wavefield snapshots without using the elliptical approximation ($\varepsilon = \delta$).

model, i.e., $\varepsilon = \delta$. In Figure 5.9 snapshots of the viscoacoustic wavefield from the forward modeling simulation are plotted. The wavefield shows the instability around the salt area because of rapid variation of the tilt angles. Using the TI approximation and ignoring all spatial derivatives of the anisotropic symmetry axis direction leads to instabilities in areas of the model with rapid variations in the symmetry axis direction (Duveneck and Bakker, 2011). The instability appears at later times and can be partially solved by smoothing of the model.

Smoothing the model helps in some models, but it is not useful for any models. However, Yoon et al. (2010) show that some spots with high symmetry axis gradient produce large instabilities blowing up the amplitudes of the wavefield (Figure 5.9). In Figure 5.10a the gradient of theta is displayed. I can pick up the high gradient points by filtering the gradient of theta with a given threshold (Figure 5.10b). In areas with instability, the anisotropy can be taken off around the selected high gradient points by setting $\varepsilon = \delta$. The stable forward modeling snapshot is plotted in Figure 5.11.

Figure 5.12 shows the reverse-time migration results obtained using the viscoacoustic TTI



Figure 5.10: (a) The gradient of the tilted dip angle theta. (b) The filtered gradient by a given threshold.



Figure 5.11: The anisotropy effect in tilted dip angle with rapid variation is reduced by setting $\varepsilon = \delta$ around the high gradient points.



Figure 5.12: Anisotropic reverse-time migration in attenuation medium. (a) VTI RTM, and (b) TTI RTM.

equations (Figure 5.12b) and, for comparison, using viscoacoustic VTI wave equations (Figure 5.12a). For the VTI migration, θ is set to zero, but other parameters are the same as the TTI model. The images of the dipping layers at the salt flank are mispositioned in the VTI RTM image because of the presence of anisotropy. TTI RTM for both the salt body and dipping layers that terminating against the salt body is more accurate than VTI RTM.

To verify the accuracy of the proposed approach, I do a second test on the Marmousi model (Figure 5.13). In Figure 5.14, the true and migration Q models are illustrated. The Q model contains high attenuation anomalies. The two Tompson parameters, ε and δ , are shown in Figure 5.15. The symmetry axis is constant and tilted at 45°. Figure 5.16 shows the comparison of the RTM images showing that for this model the TTI RTM is more accurate than conventional isotropic and VTI RTM.

5.5 Conclusions

Time-domain approximate constant-Q wave propagation involving a series of standard linear solid (SLS) mechanisms is investigated. The wave equations have been extended from isotropic media to transversely isotropic (TI) media including VTI and TTI media. The stability condition and the artifacts of shear wave triplications for imaging have been discussed. Results show that a stable anisotropic reverse time migration is achievable by taking anisotropy off around the selected high gradient points in areas of rapid changes in the symmetry axes. The TTI RTM image is more accurate than the VTI RTM and isotropic RTM images, especially in areas with strong variations of dip angle along the tilted symmetry-axis. The application of anisotropic equations to 3D RTM and field data and reduce computational time remains a challenge.



Figure 5.13: The Marmousi models: (a) true velocity model, (b) migration velocity model



Figure 5.14: The Marmousi models: (a) true Q model, and (b) migration Q model.


Figure 5.15: Transversely isotropic Marmousi model. (a) Thomsen's ϵ model, (b) Thomsen's δ model.



Figure 5.16: Anisotropic reverse-time migration in attenuation medium for the Marmousi model. (a) Isotropic RTM image, (b) VTI RTM image, and (c) TTI RTM image.

Chapter 6

Viscoacoustic reverse time migration in tilted transversely-isotropic media with compensation for attenuation and dispersion

6.1 Abstract

Anisotropy and absorption are critical to the modeling and analysis of seismic amplitude, phase, and traveltime data. Neglect of either of these phenomena, which are often both operating simultaneously, degrades the resolution and interpretability of migrated images. However, a full accounting of anisotropy and anelasticity is computationally complex and expensive. One strategy for accommodating these aspects of wave propagation while keeping cost and complexity under control is to do so within an acoustic approximation. I set up a procedure for solving the time-domain viscoacoustic wave equation for tilted transversely isotropic (TTI) media, based on a standard linear solid model, and, from this, develop a viscoacoustic reverse time migration (Q-RTM) algorithm. In this approach, amplitude compensation occurs within the migration process through a manipulation of attenuation and phase dispersion terms in the time domain differential equations. Specifically, the back-propagation operator is constructed by reversing the sign only of the amplitude loss operators, but not the dispersion-related operators, a step made possible by reformulating the absorptive TTI equations such that the two appear separately. The scheme is tested on synthetic examples to examine the capacity of viscoacoustic RTM to correct for attenuation, and the overall stability of the procedure..

6.2 Introduction

Attenuation and anisotropy are increasingly indispensable components of wavefield simulation in seismic exploration and monitoring applications. They are especially important in modern seismic amplitude modelling and reverse time migration (RTM) procedures. The approximate use of the isotropic acoustic approximation in situations involving significant anisotropy and attenuation leads to resolution degradation and mispositioning of reflectors within images (Zhou et al., 2006a). In these cases, including anisotropy and viscosity in our physical models is essential; however, the computational expense involved in the use of full elastodynamic equations (i.e., those needed for a proper treatment of anisotropy and viscoelasticity) motivates approximate wave formulations which remain as close as possible to the acoustic case.

Transverse isotropic media with a vertical symmetry axis (VTI) are appropriate for thin and approximately horizontal layering or fracturing (Crampin, 1984). Tilted transverse isotropy (TTI) is derived from VTI equations by assuming the symmetry axis is non-vertical and locally variable (Fletcher et al., 2008; Zhang and Zhang, 2008); this formulation is suitable for aligned fractures with a symmetry axis lying between vertical and horizontal. Zero-valued S_V wave velocities on the symmetry axis within the pseudo-acoustic approximation lead to instabilities in the TTI equations associated with the appearance of a residual shear wave. To stabilize wave propagation, and reduce shear wave artifacts, the anisotropic model parameters may be smoothed before numerical simulation. Further, imposing the elliptical anisotropy approximation, i.e., $\varepsilon = \delta$, in regions near the source and with a large symmetry axis gradients (Zhang and Zhang, 2008; Yoon et al., 2010) can be effective in suppressing artifacts. Alternatively, Fletcher et al. (2009) proposed adding a non-zero S-wave velocity term to the TTI equations. These methods, though effective for reducing artifacts, change the wave propagation kinematics, and leave artificial shear-wave components in the P-wave simulation. As a more complete way of eliminating shear-wave artifacts, Etgen and Brandsberg-Dahl (2009), Song et al. (2011), Fowler and Lapilli (2012), Zhan et al. (2012), and Song et al. (2013) proposed spectral methods.

As with anisotropy, anelastic attenuation of the wavefield during propagation, when unaccounted for, reduces image resolution. This amplitude loss can be mitigated through inverse Q processing, wherein an equivalent data set, absent attenuation and dispersion, is estimated (Bickel and Natarajan, 1985; Hargreaves and Calvert, 1991; Wang, 2006; Innanen and Lira, 2010), but these tend to have been developed for simple media and can produce inconsistent results in regions of complex geology. Waveform inversion methods accounting for attenuation (Hicks and Pratt, 2001; Malinowski et al., 2011; Kamei and Pratt, 2013; Métivier et al., 2015; Plessix et al., 2016; Keating and Innanen, 2019) in principle accommodate media of any complexity, however these are built up with a focus on simultaneous estimation of multiple properties, rather than the creation of a well-focused reflectivity image. Dai and West (1994), Mittet et al. (1995), Yu et al. (2002) and Mittet (2007) formulated migration methodologies containing Q compensation using one-way frequency domain wave operators. Within ray-based migration, Traynin et al. (2008) and Xie et al. (2009) set up pre-stack Kirchhoff inverse-Q migration schemes, wherein amplitudes and bandwidth are both recovered beneath gas zones. In all of these methods, amplitude attenuation and phase dispersion are coupled in the underlying wave equations and operators. When coupled, amplitude attenuation can be compensated-for within both source-side and receiver-side wavefields, but phase dispersion cannot. Fletcher et al. (2012) introduced amplitude and phase filters to be applied separately to source and receiver wavefields, allowing amplitude and phase effects to be corrected. Dutta and Schuster (2014) employed a least-squares RTM (LSRTM) approach for attenuation compensation based on a standard linear solid (SLS) model and its adjoint operator (Blanch and Symes, 1995) with a simplified stress-strain relation. Zhu and Harris (2014) introduced a constant-Q viscoacoustic wave equation with separate fractional Laplacians, and applied it to the problem of Q-compensated RTM. Based on the dispersion relation in a linear viscoacoustic medium (Kjartansson, 1979), Zhang et al. (2010) presented a viscoacoustic wave equation using the pseudo-differential operator in the time domain for isotropic media and applied it in reverse time migration. Along similar lines, Bai et al. (2013) derived a new viscoacoustic wave equation in which memory variables are avoided, and used this as the basis for carrying out attenuation compensation within RTM.

The imaging problems of interest in this chapter are those in which both TTI and attenuation/dispersion operate simultaneously. Suh et al. (2012) extended the viscoacoustic wave equation for anisotropic media, and set out a VTI viscoacoustic reverse time migration algorithm. However, to date, most analyses and algorithms have tended to focus on only one of either anisotropy or viscosity. In this chapter, building on the work reported by Fathalian and Innanen (2018), beginning with a SLS model, I formulate a time-domain anisotropic viscoacoustic wave equation. This equation describes constant-Q wave propagation and contains independent terms for phase dispersion and amplitude attenuation, such that through appropriate sign reversal attenuation and dispersion can be individually affected in the modelled wavefield. This TTI viscoacoustic wave equation is used as the basis for a Q-RTM methodology that can corrects for both amplitude attenuation and phase dispersion in the migrated images. Two simulated data sets are employed to examine the accuracy of the imaging method. In particular, I demonstrate that, when combined with the cross-correlation imaging condition, Q-RTM compensates for amplitude attenuation and phase dispersion in the receiver wavefields.

6.3 TTI-viscoacoustic equations

In this section, I first derive the viscoacoustic wave equation in TTI media, then set up viscoacoustic RTM based on the constant-Q model. This Q-RTM is a three-step procedure, involving (a) forward propagation of a wavefield through an appropriate velocity model, (b) back propagation of the measured data through the same model, and (c) superposition of both using an imaging condition. In our case, the source normalized cross-correlation imaging condition is found to be most suitable. Within this formulation only the back-propagated receiver wavefield requires compensation.

6.3.1 Viscoacoustic wave equation in TTI media

I adopt an attenuation model which is linear in frequency (Kjartansson, 1979), implying a frequency-independent Q. In Appendix A the 2D first order VTI viscoacoustic wave equations based on this attenuation model are shown to be

$$\partial_t u_x = \frac{1}{\rho} (\cos\theta\cos\varphi\partial_x - \sin\theta\partial_z)\sigma_H, \tag{6.1}$$

$$\partial_t u_z = \frac{1}{\rho} (\cos\varphi \sin\theta \partial_x + \cos\theta \partial_z) \sigma_V, \tag{6.2}$$

$$\partial_t \sigma_H = \rho V_P^2 \left[(1 + 2\varepsilon) \left[\left(\frac{\tau_\varepsilon}{\tau_\sigma} \right) \left[(\cos \theta \cos \varphi \partial_x - \sin \theta \partial_z) u_x \right] - r_H \right] + \sqrt{1 + 2\delta} \left[(\cos \varphi \sin \theta \partial_x + \cos \theta \partial_z) u_z \right] \right],$$
(6.3)

and

$$\partial_t \sigma_V = \rho V_P^2 \left[\sqrt{1 + 2\delta} \left[(\cos \theta \cos \varphi \partial_x - \sin \theta \partial_z) u_x \right] + \left(\frac{\tau_{\varepsilon}}{\tau_{\sigma}} \right) \left[(\cos \varphi \sin \theta \partial_x + \cos \theta \partial_z) u_z + \right] - r_V \right],$$
(6.4)

where $u_x(x,t)$ and $u_z(x,t)$ are the particle velocity components in the x- and z-directions respectively, σ_H and σ_V are the horizontal and vertical stress components respectively, P(x,t)is the pressure field, ρ is the density, r is a memory variable, ε and δ are Thomsen parameters, K is the bulk modulus, θ is the tilt angle, and φ is the azimuth of tilt for TTI symmetry axis. The memory variables for horizontal and vertical stress, $r_{H\ell}$ and $r_{V\ell}$ (Carcione et al., 1988a), satisfy

$$\partial_t r_{H\ell} = -\frac{1}{\tau_{\sigma\ell}} r_{H\ell} + \rho V_p^2 \left[(\cos\theta\cos\varphi\partial_x - \sin\theta\partial_z)u_x \right] \frac{1}{\tau_{\sigma\ell}} \left(1 - \frac{\tau_{\varepsilon\ell}}{\tau_{\sigma\ell}} \right), \tag{6.5}$$
$$\partial_t r_{V\ell} = -\frac{1}{\tau_{\sigma\ell}} r_{V\ell} + \rho V_p^2 \left[(\cos\varphi\sin\theta\partial_x + \cos\theta\partial_z)u_z \right] \frac{1}{\tau_{\sigma\ell}} \left(1 - \frac{\tau_{\varepsilon\ell}}{\tau_{\sigma\ell}} \right), \quad 1 \le \ell \le L.$$

The stress and strain relaxation parameters, τ_{ε} and τ_{σ} , are related to the quality factor Qand the reference angular frequency ω through (Robertsson et al., 1994)

$$\tau_{\sigma} = \frac{\sqrt{1 + 1/Q^2} - 1/Q}{\omega}, \qquad (6.6)$$
$$\tau_{\varepsilon} = \frac{1}{\omega^2 \tau_{\sigma}},$$

where ω is the central frequency of the source wavelet.

In attenuating media, a wave undergoes two primary changes, a reduced amplitude and a phase shift due to dispersion. When simulating attenuative wave propagation one may wish to include only the amplitude loss effect, only the phase dispersion effect, or both. In our approach, I explicitly separate phase dispersion and amplitude attenuation. The Fourier transforms of the first-order linear differential equations 6.1–6.5 are

$$\tilde{u}_x = \frac{1}{\rho} (\cos\theta\cos\varphi\partial_x - \sin\theta\partial_z)\tilde{\sigma}_H, \tag{6.7}$$

$$\tilde{u}_z = \frac{1}{\rho} (\cos\varphi\sin\theta\partial_x + \cos\theta\partial_z)\tilde{\sigma}_V, \tag{6.8}$$

$$i\omega\tilde{\sigma}_{H} = \rho V_{P}^{2} \left[(1+2\varepsilon) \left[\left(\frac{\tau_{\varepsilon}}{\tau_{\sigma}} \right) \left[(\cos\theta\cos\varphi\partial_{x} - \sin\theta\partial_{z})\tilde{u}_{x} \right] - \tilde{r}_{H} \right] + \sqrt{1+2\delta} \left[(\cos\varphi\sin\theta\partial_{x} + \cos\theta\partial_{z})\tilde{u}_{z} \right] \right],$$
(6.9)

$$i\omega\tilde{\sigma}_{V} = \rho V_{P}^{2} \left[\sqrt{1+2\delta} \left[(\cos\theta\cos\varphi\partial_{x} - \sin\theta\partial_{z})\tilde{u}_{x} \right] + \left(\frac{\tau_{\varepsilon}}{\tau_{\sigma}} \right) \left[(\cos\varphi\sin\theta\partial_{x} + \cos\theta\partial_{z})\tilde{u}_{z} \right] - \tilde{r}_{V} \right],$$

$$(6.10)$$

$$i\omega\tilde{r}_{H\ell} = -\frac{1}{\tau_{\sigma\ell}}\tilde{r}_{H\ell} + \rho V_p^2 \left(\left(\cos\theta\cos\varphi\partial_x - \sin\theta\partial_z\right)\tilde{u}_x \right) \frac{1}{\tau_{\sigma\ell}} \left(1 - \frac{\tau_{\varepsilon\ell}}{\tau_{\sigma\ell}} \right), \tag{6.11}$$

$$i\omega\tilde{r}_{V\ell} = -\frac{1}{\tau_{\sigma\ell}}\tilde{r}_{V\ell} + \rho V_p^2 \left((\cos\varphi\sin\theta\partial_x + \cos\theta\partial_z)\tilde{u}_z \right) \frac{1}{\tau_{\sigma\ell}} \left(1 - \frac{\tau_{\varepsilon\ell}}{\tau_{\sigma\ell}} \right), \tag{6.12}$$

where $1 \leq \ell \leq L$. From equations 6.11 and 6.12, the memory variables in the frequency domain can be calculated as a function of the particle velocity and the relaxation time

$$\tilde{r}_{H\ell} = \rho V_p^2 \left(\left(\cos \theta \cos \varphi \partial_x - \sin \theta \partial_z \right) \tilde{u}_x \right) \frac{\tau_{\sigma}^{-1} \left(1 - \tau_{\varepsilon} \tau_{\sigma}^{-1} \right)}{\left(i\omega + \tau_{\sigma}^{-1} \right)}, \tag{6.13}$$

$$\tilde{r}_{V\ell} = \rho V_p^2 \left(\left(\cos \varphi \sin \theta \partial_x + \cos \theta \partial_z \right) \tilde{u}_z \right) \frac{\tau_{\sigma}^{-1} \left(1 - \tau_{\varepsilon} \tau_{\sigma}^{-1} \right)}{\left(i\omega + \tau_{\sigma}^{-1} \right)} \right).$$
(6.14)

Equations 6.13-6.14 can be used to eliminate the memory variables from equations 6.9-6.10, leading to the new first-order viscoacoustic system

$$i\omega\tilde{\sigma}_{H} = \rho V_{P}^{2} \left[\left(1 + 2\varepsilon\right) \left[\left(\frac{\tau_{\varepsilon}}{\tau_{\sigma}} - \frac{\frac{1}{\tau_{\sigma}}(\frac{\tau_{\varepsilon}}{\tau_{\sigma}} - 1)}{i\omega + \frac{1}{\tau_{\sigma}}}\right) \left[\left(\cos\theta\cos\varphi\partial_{x} - \sin\theta\partial_{z}\right)\tilde{u}_{x} \right] \right] + \sqrt{1 + 2\delta} \left[\left(\cos\varphi\sin\theta\partial_{x} + \cos\theta\partial_{z}\right)\tilde{u}_{z} \right] \right],$$

$$(6.15)$$

$$i\omega\tilde{\sigma}_{V} = \rho V_{P}^{2} \left[\sqrt{1+2\delta} \left[(\cos\theta\cos\varphi\partial_{x} - \sin\theta\partial_{z})\tilde{u}_{x} \right] + \left(\frac{\tau_{\varepsilon}}{\tau_{\sigma}} - \frac{\frac{1}{\tau_{\sigma}}(\frac{\tau_{\varepsilon}}{\tau_{\sigma}} - 1)}{i\omega + \frac{1}{\tau_{\sigma}}} \right) \left[(\cos\varphi\sin\theta\partial_{x} + \cos\theta\partial_{z})\tilde{u}_{z} \right] \right],$$

$$(6.16)$$

With some algebraic manipulation, these equations simplify to

$$\begin{split} i\omega\tilde{\sigma}_{H} = \rho V_{P}^{2} \left[(1+2\varepsilon) \left[\left(\frac{(\omega^{2}\tau_{\varepsilon}\tau_{\sigma}+1)}{\omega^{2}\tau_{\sigma}^{2}+1} + i\frac{(\omega\tau_{\epsilon}-\omega\tau_{\sigma})}{\omega^{2}\tau_{\sigma}^{2}+1} \right) \left[(\cos\theta\cos\varphi\partial_{x}-\sin\theta\partial_{z})\tilde{u}_{x} \right] \right] \\ + \sqrt{1+2\delta} \left[(\cos\varphi\sin\theta\partial_{x}+\cos\theta\partial_{z})\tilde{u}_{z} \right] \right], \end{split}$$
(6.17)

and

$$i\omega\tilde{\sigma}_{V} = \rho V_{P}^{2} \left[\sqrt{1+2\delta} \left[(\cos\theta\cos\varphi\partial_{x} - \sin\theta\partial_{z})\tilde{u}_{x} \right] + \left(\frac{(\omega^{2}\tau_{\varepsilon}\tau_{\sigma} + 1)}{\omega^{2}\tau_{\sigma}^{2} + 1} + i\frac{(\omega\tau_{\epsilon} - \omega\tau_{\sigma})}{\omega^{2}\tau_{\sigma}^{2} + 1} \right) \left[(\cos\varphi\sin\theta\partial_{x} + \cos\theta\partial_{z})\tilde{u}_{z} \right] \right],$$

$$(6.18)$$



Figure 6.1: 2D wavefield snapshots in TTI media using (a) acoustic, (b) amplitude loss, (c) dispersive, and (d) viscoacoustic data with $\varepsilon = 0.2$, $\delta = 0.05$, and $\theta = 45^{\circ}$.

which may be transformed back to the time domain, producing the constant-Q viscoacoustic TTI equations underlying our imaging algorithms:

$$\partial_t \sigma_H = \rho V_P^2 \left\{ (1+2\varepsilon) \left[\left(a_1 \frac{2}{A} + ia_2 \frac{2}{AQ} \right) \left[(\cos\theta\cos\varphi\partial_x - \sin\theta\partial_z)u_x \right] \right] + \sqrt{1+2\delta} \left[(\cos\varphi\sin\theta\partial_x + \cos\theta\partial_z)u_z \right] \right\},$$
(6.19)

$$\partial_t \sigma_V = \rho V_P^2 \bigg\{ \sqrt{1 + 2\delta} \left[(\cos\theta\cos\varphi\partial_x - \sin\theta\partial_z)u_x \right] \\ + \left(a_1 \frac{2}{A} + ia_2 \frac{2}{AQ} \right) \left[(\cos\varphi\sin\theta\partial_x + \cos\theta\partial_z)u_z \right] \bigg\},$$
(6.20)

where $A = \sqrt{1 + 1/Q^2} - 1/Q)^2 + 1$. The quantities 2/A and 2/AQ are dispersiondominated operator and attenuation-dominated operators, respectively. The coefficients a_1 and a_2 , which can take on values of +1, 0 or -1, have been formally introduced to permit



Figure 6.2: (a) True velocity model, (b) True Q model, (c) migration velocity model, and (d) migration Q model.

inclusion, suppression and/or reversal of the processes of attenuation and dispersion. In the acoustic limit $Q \to \infty$, the dispersion-dominated operator lapses to 1 and the attenuation-dominated vanishes.

To illustrate the decoupling of velocity dispersion and amplitude losses in numerical simulations based on these equations, I consider a homogeneous model with a background velocity of 2500 m/s and quality factor Q=10. The source is located in the center of the model, and the source signature is a zero-phase Ricker wavelet with a central frequency of 25 Hz. The grid spacing in the x and z directions is 4 m, the Thomsen parameters are set to $\varepsilon = 0.2$ and $\delta = 0.05$, and the symmetry axis is tilted at 45°. In the four panels of Figure 6.1 the top right corner of the model is plotted with a snapshot of the compressional P wavefront overlain. The wavefront is approximately ellipsoidal; the shear wave artifacts common to such modelling in elliptic media are suppressed by positioning a small, smoothly-tapered circular region with $\varepsilon = \delta$ around the source. To avoid numerical instability I apply also set the shear wave velocity along the tilted symmetry axis to zero.

A snapshot of a reference (acoustic) wavefield is plotted in Figure 6.1a. The dashed yellow line touches the leading edge of the reference wavefront. The amplitude-loss simulation $(a_1 = 0 \text{ and } a_2 = 1)$ is plotted in Figure 6.1b. Compared with the acoustic case, the amplitude is attenuated, but the phases are the same. The phase dispersion simulation $(a_1 = 1 \text{ and } a_2 = 0)$ is plotted in Figure 6.1c. The phase has a shift, and the amplitude is similar to the acoustic case. In Figure 6.1d the viscoacoustic wavefield calculated with equations 6.19 and 6.20 is plotted. The reduced amplitude and shifted phase (relative to the acoustic reference wavefield) are visible. I conclude that through use of the TTI viscoacoustic wave equations, and by decoupling attenuation and dispersion terms, I can accommodate amplitude losses and dispersion separately. This will be made use of in the reverse-time migration algorithms derived next.

6.3.2 TTI-viscoacoustic wave propagation in reversed time

In RTM, a receiver wavefield is reversed in time, and then propagated through the velocity model to each image point. The act of forward propagation of a time-reversed field, which undoes all (but only) time-reversible phenomena of wave propagation, leads to the require-



Figure 6.3: Transversely isotropic layered velocity model. (a) Thomsen's ε model, (b) Thomsen's δ model.

ment for special treatment of attenuation. Consider an attenuative and dispersive medium, in which low frequencies propagate more slowly than high, i.e., in which low-frequency waveform components arrive at sensors later than their high-frequency counterparts. If a trace containing such a waveform is time-reversed, its low-frequency components precede its highfrequency components. To focus at an image point in depth, these components must arrive simultaneously, requiring the low-frequency components (which depart from their re-injection point earlier) to travel more slowly than the high (which depart later). I note that this is accomplished by forward propagation using the original equations. In contrast, in the same trace, the process of propagation has broadened and attenuated the waveform. Time-reversal has no impact on this, and upon forward propagation with the original equations the attenuation is aggravated rather than corrected. Summarizing, when absorption is included in wave modelling, both reversible and irreversible phenomena are introduced, and each must be addressed differently in RTM.

With attenuation and dispersion separated in the viscoacoustic equations, this issue can be easily addressed. The sign of a_2 controls attenuation. To arrange for amplitude compensation, I reverse its sign, i.e., set $a_2 = -1$. Conversely, for correction of dispersion the sign of the associated coefficient remains unchanged: $a_1 = 1$. The resulting viscoacoustic TTI wave equations are

$$\partial_t \sigma_H = \rho V_P^2 \left[(1 + 2\varepsilon) \left[(a_1(2/A) - ia_2(2/AQ)) \left[(\cos\theta\cos\varphi\partial_x - \sin\theta\partial_z)u_x \right] \right] \\ + \sqrt{1 + 2\delta} \left[(\cos\varphi\sin\theta\partial_x + \cos\theta\partial_z)u_z \right] \right],$$
(6.21)

$$\partial_t \sigma_V = \rho V_P^2 \left[\sqrt{1 + 2\delta} \left[(\cos \theta \cos \varphi \partial_x - \sin \theta \partial_z) u_x \right] + (a_1(2/A) - ia_2(2/AQ)) \left[(\cos \varphi \sin \theta \partial_x + \cos \theta \partial_z) u_z \right] \right],$$
(6.22)

Using the phase dispersion part, the phase only viscoacoustic TTI wave equation will be

$$\partial_t \sigma_H = \rho V_P^2 \left[(1+2\varepsilon) \left[(a_1(2/A)) \left[(\cos\theta\cos\varphi\partial_x - \sin\theta\partial_z)u_x \right] \right] + \sqrt{1+2\delta} \left[(\cos\varphi\sin\theta\partial_x + \cos\theta\partial_z)u_z \right] \right],$$

$$\partial_t \sigma_V = \rho V_P^2 \left[\sqrt{1+2\delta} \left[(\cos\theta\cos\varphi\partial_x - \sin\theta\partial_z)u_x \right] + (a_1(2/A)) \left[(\cos\varphi\sin\theta\partial_x + \cos\theta\partial_z)u_z \right] \right],$$
(6.23)
$$(6.24)$$

To compensate for amplitude attenuation only, I keep the imaginary part of equation 6.19 and 6.20, so that the viscoacoustic TTI wave equation for back-propagation become

$$\partial_t \sigma_H = \rho V_P^2 \left[(1+2\varepsilon) \left[(-a_2(2/AQ)) \left[(\cos\theta\cos\varphi\partial_x - \sin\theta\partial_z)u_x \right] \right] + \sqrt{1+2\delta} \left[(\cos\varphi\sin\theta\partial_x + \cos\theta\partial_z)u_z \right] \right],$$

$$\partial_t \sigma_V = \rho V_P^2 \left[\sqrt{1+2\delta} \left[(\cos\theta\cos\varphi\partial_x - \sin\theta\partial_z)u_x \right] + (-a_2(2/AQ)) \left[(\cos\varphi\sin\theta\partial_x + \cos\theta\partial_z)u_z \right] \right],$$
(6.25)
(6.26)

The viscoacoustic back propagation equation can be obtained by combining equations 6.25-6.26 and equations 6.1 and 6.2.

6.3.3 Imaging condition

I also require a suitable imaging condition for Q-compensated RTM, which in this case is the source-normalized cross-correlation condition (Claerbout, 1971; Chattopadhyay and McMechan, 2008; Fathalian and Innanen, 2018):

$$I(x,z) = \frac{\int_{t} s(x,z,t)r^{c}(x,z,t)dt}{\int_{t} s^{2}(x,z,t)dt},$$
(6.27)

where s(x, z, t) is the source wavefield at the position (x, z) in image space, and $r^{c}(x, z, t)$ is the compensated receiver wavefield. The source auto-correlation in the denominator of the right hand side compensates for illumination variations.

6.4 Numerical examples

I examine the numerical response of this approach to TTI Q-RTM, first with a layered model and then with an augmented Marmousi model. In Figures 6.2a and 6.2b the true and migration (smooth) velocity models for the layered example, with a Q anomaly included, are plotted. The model grid dimensions are 401×501 , with a grid size of 4×4 m. The Thomsen anisotropic parameter models are plotted in Figure 6.3. In the simulated data, the sampling interval is 0.8 ms, and the recording length is 1.5 s. A zero-phase Ricker wavelet with a center frequency of 25 Hz is adopted for the source, and perfectly matched layers (PML) are used on all boundaries. Because of the non self-consistent nature of the equations of acoustic



Figure 6.4: Reference snapshots of the source wavefield, the receiver wavefield, and the RTM image at 0.5, 0.62, and 0.79 s. The acoustic data in TTI media are extrapolated for the receiver wavefield using the acoustic RTM.



Figure 6.5: Snapshots of the source wavefield, the receiver wavefield, and the RTM image at 0.5, 0.62, and 0.79 s. The viscoacoustic data are extrapolated for the receiver wavefield using acoustic RTM.



Figure 6.6: Snapshots of the source wavefield, the receiver wavefield, and the RTM image at 0.5, 0.62, and 0.79 s. The viscoacoustic data are extrapolated for the receiver wavefield using Q-RTM to compensate for the amplitude during extrapolating.



Figure 6.7: Comparison among from (a) acoustic RTM (reference), (b) acoustic RTM with viscoacoustic data, and (c) Q-RTM with viscoacoustic data. The compensated case agree with the reference image very well. The right panels show the reference trace (solid line), non-compensated trace (dashed line), and compensated trace (dash-dotted line)at the horizontal 1 km. The compensated case agree with the reference image very well.



Figure 6.8: Comparisons of traces extracted from the RTM images in Figure 7 at the horizontal offset of x = 0.7 km. The solid line refers to the reference (acoustic) trace, the dashed line refers to the non-compensated evidence, and the dash-dotted line indicates the compensated trace.

anisotropy, fictitious shear waves are generated at the source (Alkhalifah, 2000; Grechka et al., 2004). However, such waves are not generated in elliptic media, so by designing a small smoothly tapered circular region with $\varepsilon = \delta$ around the source, following Duveneck et al. (2008), I avoid these artifacts.

To analyze the Q-RTM developed in the previous section in action, I plot example snapshots of the source and receiver wavefields, as well as final RTM images, in both the attenuating media defined above, and also within reference models which are non-attenuating but otherwise identical. Figure 6.4 shows the reference snapshot results using acoustic RTM at different time step. To obtain the RTM image the acoustic data are extrapolated for the receiver wavefields. In Figure 6.5 acoustic RTM is applied to viscoacoustic data. The receiver wavefield exhibits reduced wave amplitude, while the source wavefield is comparable to the reference result. Consequently, the RTM images exhibit reduced amplitudes and resolution. The TTI Q-RTM is next applied to the viscoacoustic data. In Figure 6.6 associated snapshots of source and receiver wavefields, and RTM images, are plotted. Here the amplitudes within both the source wavefield and the receiver wavefield are magnified. though the receiver wavefield is still weaker than the reference field because of incomplete compensation. The TTI Q-RTM images are, however, properly compensated and are comparable to the reference images. I next zoom in on the viscoacoustic RTM image in Figure 6.7, and compare it with the reference acoustic RTM image. When the acoustic RTM is carried out on viscoacoustic data (i.e., non-compensated RTM), as seen in Figure 6.7b, the reflector images have significantly weaker amplitudes than those generated with purely acoustic data. The TTI Q-RTM approach (i.e., equations 6.19 and 6.20) exhibits compensated images (Figure 6.7c) with correct amplitudes and phases. The right panel of each subfigure shows the corrected amplitude and phase. In Figure 6.8, I zoom in on events and compare the depth traces of RTM data. The non-compensated trace (dashed line) has a shifted phase and a reduced amplitude, while the compensated trace (dash-dotted line) have amplitude and phase comparable to the acoustic case (solid line).

In the second example, I add attenuation and anisotropy to some of the layers of the more complex Marmousi model. In Figure 6.9 the actual and migration velocity models and the corresponding true and migration Q models are plotted; in Figure 6.10 the associated Thomsen parameter models are plotted. I produce a synthetic viscoacoustic TTI data set by setting the tilt angle to 45°. As before, shear wave artifacts are avoided by setting up small smoothly tapered circular regions with $\varepsilon = \delta$ around the source positions (Duveneck et al., 2008). The model grid dimensions are 281×701, and the grid size is 10×10 m. 50 sources are positioned along the surface at a depth of 30 m, and a zero-phase Ricker wavelet with 15 Hz center frequency is adopted. The sampling interval rate is 0.4 ms, and the recording length is 3s.

Within this model I generate both acoustic reference and viscoacoustic (actual) synthetic data sets; shot records from an example shot position are plotted in Figures 6.11a and 6.11b. Spectra computed from these shot records, plotted in Figure 6.11c, illustrate the attenuation in the viscoacoustic data as compared to that of the acoustic case.

The RTM image for the purely acoustic RTM algorithm applied to data without attenuation (i.e., the reference case) is plotted in Figure 6.12a. The acoustic RTM algorithm applied to viscoacoustic data (i.e., the non-compensated case) is plotted in Figure 6.12b. Finally, the compensated RTM image created using the new TTI Q-RTM algorithm is plotted in Figure 6.12c. The reference RTM image (Figure 6.12a) has similar amplitudes in the shallower layers, when compared with the non-compensated RTM image, but in the non-compensated case (Figure 6.12b) weak amplitudes in the deeper layers, especially beneath the layers with strong attenuation, are evident. The compensated RTM image (Figure 6.12c) has recovered reflector amplitudes at the dipping deeper layers, for which the image amplitudes compare well with those of the reference case. To verify that the reflectors have been migrated to the correct positions, I include image profiles from a fixed lateral position in the right panels in Figure 6.12. The non-compensated trace (solid red line) has a shifted phase and a reduced amplitude. The compensated traces (solid green line) have amplitude and phase comparable to the reference case (solid blue line). I view these examples as evidence that the proposed Q-RTM approach in TTI media are well-posed and generate well-resolved and artifact-free images when both anisotropic and viscous processes are active for a surface data set.



Figure 6.9: The Marmousi models: (a) true velocity model, (b) true Q model, (c) migration velocity model, and (d) migration model.



Figure 6.10: Transversely isotropic Marmousi velocity model. (a) Thomsen's ε model, (b) Thomsen's δ model.

6.5 Discussion

This is to our knowledge the first implementation of Q-RTM in TTI media within the acoustic approximation. To make it work, it is important that the phase dispersion and amplitude attenuation appear, and are treated, independently in the mathematical formulation of the wave propagation. When this is true, compensation for attenuation in the reconstructed wavefield can be accomplished by reversing the sign of the loss operator in the back propagation equation. The main issue affecting the resolution of RTM images involving attenuation compensation is the stability of wave propagation given noise in the input (i.e., time-reversed data). Generally data contain high-frequency noise; when, as in all attenuation compensation algorithms, high frequency components of the data are amplified, the boosted noise can dominate in the image and cause instabilities. To manage these instabilities within the Q-RTM, the amplitude compensation operator is modified with a low-pass filter, such that data are not amplitude-boosted above a certain set frequency.

A separate concern in TTI Q-RTM are the instabilities arising in areas of the model with rapid variations in the direction of the symmetry axis (Duveneck and Bakker, 2011). Such instabilities can sometimes be solved by smoothing the model, but not always: Yoon et al. (2010) showed that regions containing high symmetry axis gradient values produce large instabilities, leading to uncontrolled increases in the amplitudes of the wavefield. In an earlier work, I observed that numerical stability can be ensured by, first, detecting high gradient points, and then modifying the model slightly by including small regions around the selected high gradient points, and within these enforcing $\varepsilon = \delta$ (Fathalian and Innanen, 2018).

The source-normalized cross-correlation imaging condition is particularly suitable for Q-RTM images. With this imaging condition, only the back propagated receiver wavefield requires compensation; the source wavefield remains attenuated. The compensated image from the source-normalized cross-correlation imaging condition is theoretically equivalent to that obtained using the imaging condition when the data are purely acoustic.

The TTI Q-RTM approach I have formulated is potentially also suitable for other related problems, such as viscoacoustic / anisotropic full waveform inversion (FWI). In FWI, the

adjoint back-propagated wavefield should be expected to have similar issues in the FWI gradient computation as those addressed here. The independent compensation for phase dispersion and amplitude losses during wavefield back-propagation may provide a strategy for separately updating the velocity attenuation and velocity in FWI.

The number of first-order equations needed to simulate viscoacoustic wave propagation depends on the number of mechanisms chosen for the SLS Q-model (Fathalian and Innanen, 2017). Therefore, the computational cost of Q-RTM is higher than acoustic RTM. On the other hand, the current approach does not require extra memory variables for wave simulation.

6.6 Conclusions

I have presented a time-domain viscoacoustic RTM imaging algorithm in tilted TI media based on a series of standard linear solid mechanisms, which mitigates attenuation and dispersion effects in migrated images. The wave equations have been extended from isotropic media to tilted TI media. The amplitude loss and phase dispersion in the source and receivers wavefields can be recovered by applying compensation operators on the measured receiver wavefield. The phase dispersion and amplitude attenuation operators in Q-RTM approach are separated, and the compensation operators are constructed by reversing the sign of the attenuation operator without changing the sign of the dispersion operator. With sufficiently accurate Q and velocity models, the TTI Q-RTM can produce better images than isotropic RTM, especially in areas with anisotropy, attenuation and strong variations of dip angle. Numerical tests on synthetic data illustrate also that this approach can improve the image resolution beneath areas with strong attenuation.



Figure 6.11: (a) Shot record from acoustic simulation; (b) shot record from viscoacoustic simulation. The viscoacoustic simulation exhibits reduced amplitude because of reflections travelling through the high-attenuation zone. (c) Spectrum of acoustic (solid) and viscoacoustic (dashed) shot gather data. The loss in high-frequency content with time is obvious in the viscoacoustic case.



Figure 6.12: Comparison among (a) acoustic RTM (reference), (b) acoustic RTM with viscoacoustic data, and (c) Q-RTM with viscoacoustic data. The right panels show the reference trace (black line), non-compensated trace (dashed red line), and compensated trace (dashed green line)at the horizontal 4.2 km. The compensated case agree very well with the reference image.

Chapter 7

Summary and Future Work

In this thesis, seismic techniques that can be applied in geological media with both anisotropy and attenuation is presented. I discuss the problem of modeling, with constant-Q wave propagation, and the issue of migration, with Q-compensated reverse-time migration (Q-RTM). Within our framework, I derived an unsplit-field PML formulation for the viscoacoustic wave equation in the time domain using a series of standard linear solid mechanisms. This formulation is mostly reflection free regardless of the incidence angle and frequency, provides excellent results with less computational cost and does not have the instability problem. To verify the accuracy of the standard linear solid model (SLS) to approximate constant Q in the time domain, I compared the numerical FD results against the analytical solutions that show a similar accuracy between one and three SLS mechanisms. I found that FD simulations using a single SLS mechanism are sufficiently accurate and suitable for most practical applications. However, in the unsplit-field formulation based on SLS mechanisms, the amplitude loss and phase dispersion couples together, and it is difficult to use for compensation in the inverse problems. To solve this limitation, I derive a new approach to the solution of the viscoacoustic wave equation within attenuating media in the time domain that describes constant-Q wave propagation and contains independent terms for phase dispersion and amplitude attenuation. Simulation of wave propagation with this approach is accurate and stable for complex structures in attenuation media. In this formulation, there is no need for any extra memory variables, unlike the traditional viscoacoustic wave equations, and thus it is more cost-effective in computational costs.

I have presented a viscoacoustic RTM imaging algorithm based on the time-domain constant-Q wave propagation that can compensate for attenuation effects in migrated images. The amplitude loss and phase dispersion in the source and receivers wavefields can be recovered by applying compensation operators on the measured receiver wavefields when using the source normalized crosscorrelation imaging condition. I constructed a regularized equation to suppress the reverse time propagation high-frequency instabilities and to improve the resolution of images. I found that the Q-RTM imaging approach for 2D and 3D synthetic data can improve the image resolution, particularly beneath areas with strong attenuation.

I extended the new approach of the viscoacoustic wave equation from isotropic media to transversely isotropic (TI) media, including VTI and TTI media. The stability condition and the artefacts of shear wave triplications have been discussed. I found that the stable anisotropic reverse time migration is accessible by taking off anisotropy around the selected high gradient points of tilt angle in areas of rapid changes in the symmetry axes. From the numerical results, it is obvious the TTI Q-RTM can produce better images than isotropic RTM, especially in areas with anisotropy, attenuation and strong variations of dip angle along the tilted symmetry axis with many details about geological layers and structures.

In geophysical research, the application of viscoacoustic/viscoelastic wave equation for both isotropic and anisotropic media to 3D RTM and field data and computational cost remains a challenge. The work presented in this dissertation can be extended to solve these problems for practical applications using pure P-wave modeling in attenuation media and anisotropic tomography using VTI/TTI pure P-wave equation.

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