Amplitude calibration of a fast S-transform algorithm

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ABSTRACT

Time-frequency decomposition methods provide a means to estimate the local spectrum of seismic events. The S-transform is a time-frequency decomposition technique with a wide range of applications in seismic signal analysis but has been under-used due to high computational cost. A fast, non-redundant S-transform (FST) is presented. The ability of the FST to provide high fidelity estimates of the amplitudes of frequency dependent reflection coefficients is tested in this paper. Where the FST cannot provide accurate estimates of amplitudes, we calibrate by normalizing the unit impulse response of the algorithm. Due to the low resolution at low frequency nature of the FST algorithm, the spectra of individual events will interfere with each other and impede the ability of the FST to estimate amplitudes at low frequencies. We propose a set of standards for the fidelity of local spectra as a function of proximity in time to other events.

How the Fast S-transform works

Consider a signal in time, g(t). Taking the Fourier transform of g(t) yields G(ω). Now we build a matrix G(ω+ ω′) such that every row is the Fourier transform of g(t), G(ω), shifted by a frequency increment ω′. Now, W(ω, ω′) is a matrix which is populated by the Fourier transform of Gaussian window functions. Then the α-domain is obtained by multiplying every row of G(ω+ ω′) with every row of W(ω, ω′). Finally, we segment in the α-domain, with large segments for high frequencies and small segments at low frequencies, assign one frequency in each segment to be the representative frequency for each segment and apply an inverse Fourier transform to each segment to obtain the S-domain. Figure 1 shows how the S-domain is constructed from the α-domain for an example time signal.

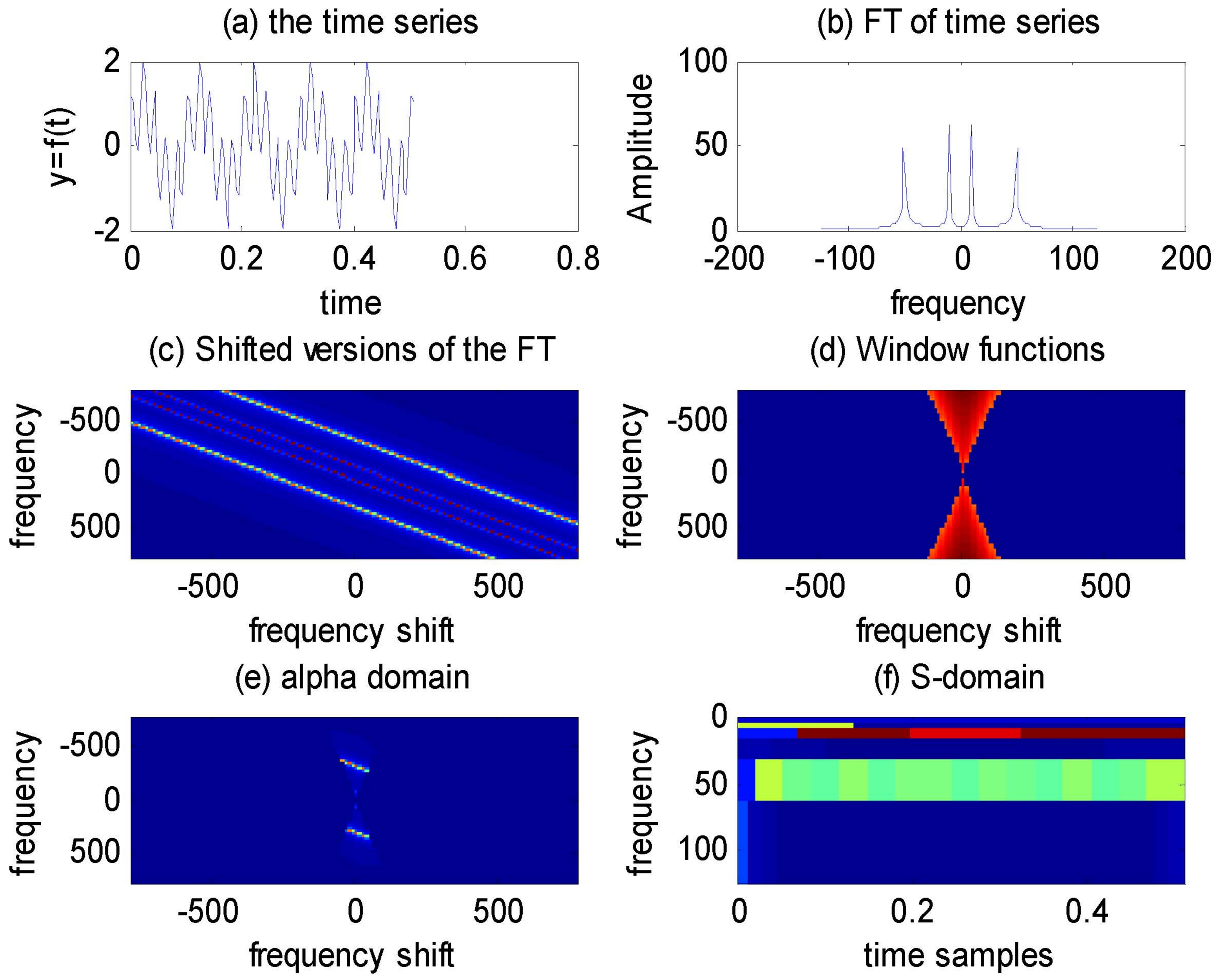

FIG. 1. Construction of S-domain from α-alpha domain. (a) is an example time signal which is the sum of two harmonics of 10 Hz and 50 Hz. (b) is the Fourier transform of the time signal. (c) is the matrix which is populated by shifted versions of (b). (d) is the matrix which contains the Fourier transform of the window functions. (e) is the α-domain obtained by multiplying (c) with (d). (f) is the S-domain obtained by taking the inverse Fourier Transform of the α-domain.

Calibration of the FST amplitudes

Figure 2 shows the unit impulse response of the FST algorithm before calibration. In figure 2(a) the unit impulse is plotted in time. Figure 2(b) shows the S-transform and in figure 2(c) we see the spectrum of the unit impulse picked from the S-transform. The spectrum of the unit impulse is not flat constant spectrum as we would expect but rather we see that the amplitudes increase with frequency in a stair case trend. Figure 3 shows the dependence of the unit impulse response on position of the impulse in time. Figure 4 shows the unit impulse response of the FST after amplitude calibration. As can be seen in figure 4(c) the spectrum is a flat constant at unity as we would expect

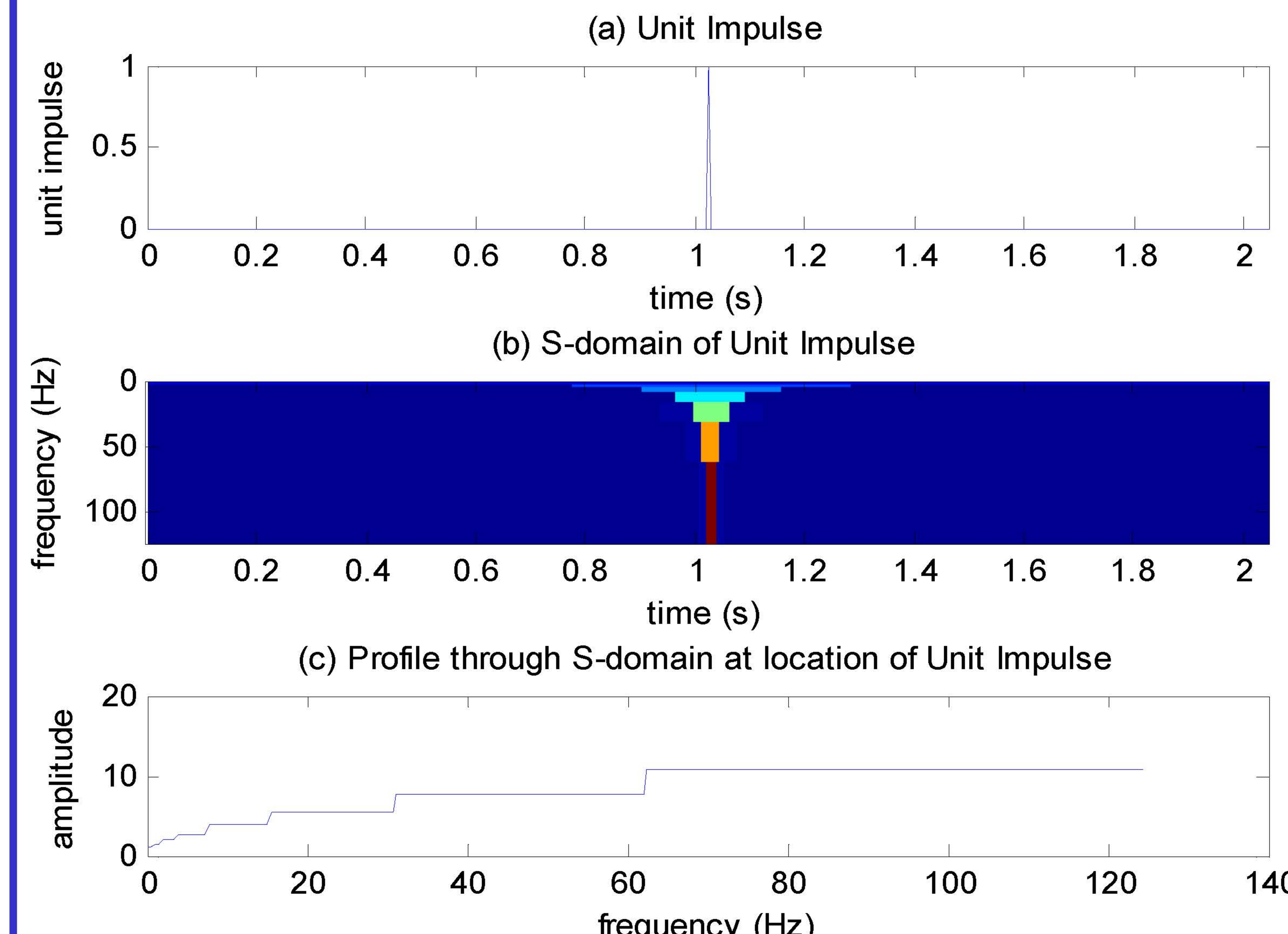

FIG. 2. UIR of FST before any corrections. (a) is a unit impulse input into the FST. (b) is it's S-domain and (c) is a frequency profile through the S-domain at the time sample of the unit impulse.

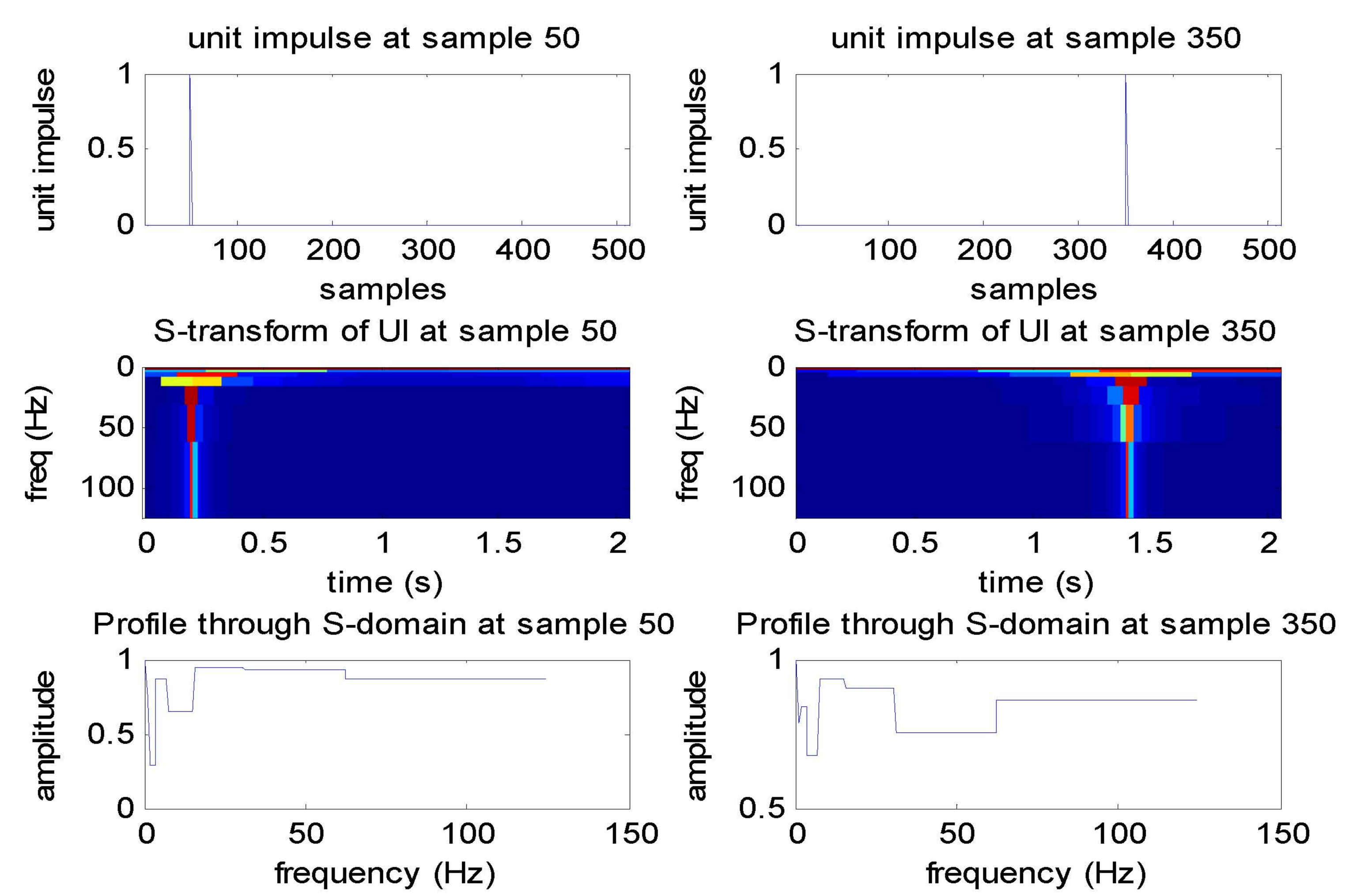

FIG. 3 Dependence of the position of the unit impulse on the UIR of the FST algorithm. The top two panels show two unit impulses which were input into the FST at 50th and 350th time sample respectively. The middle two panels show their respective S-domains and the bottom two panels show their frequency profiles through the S-domain.

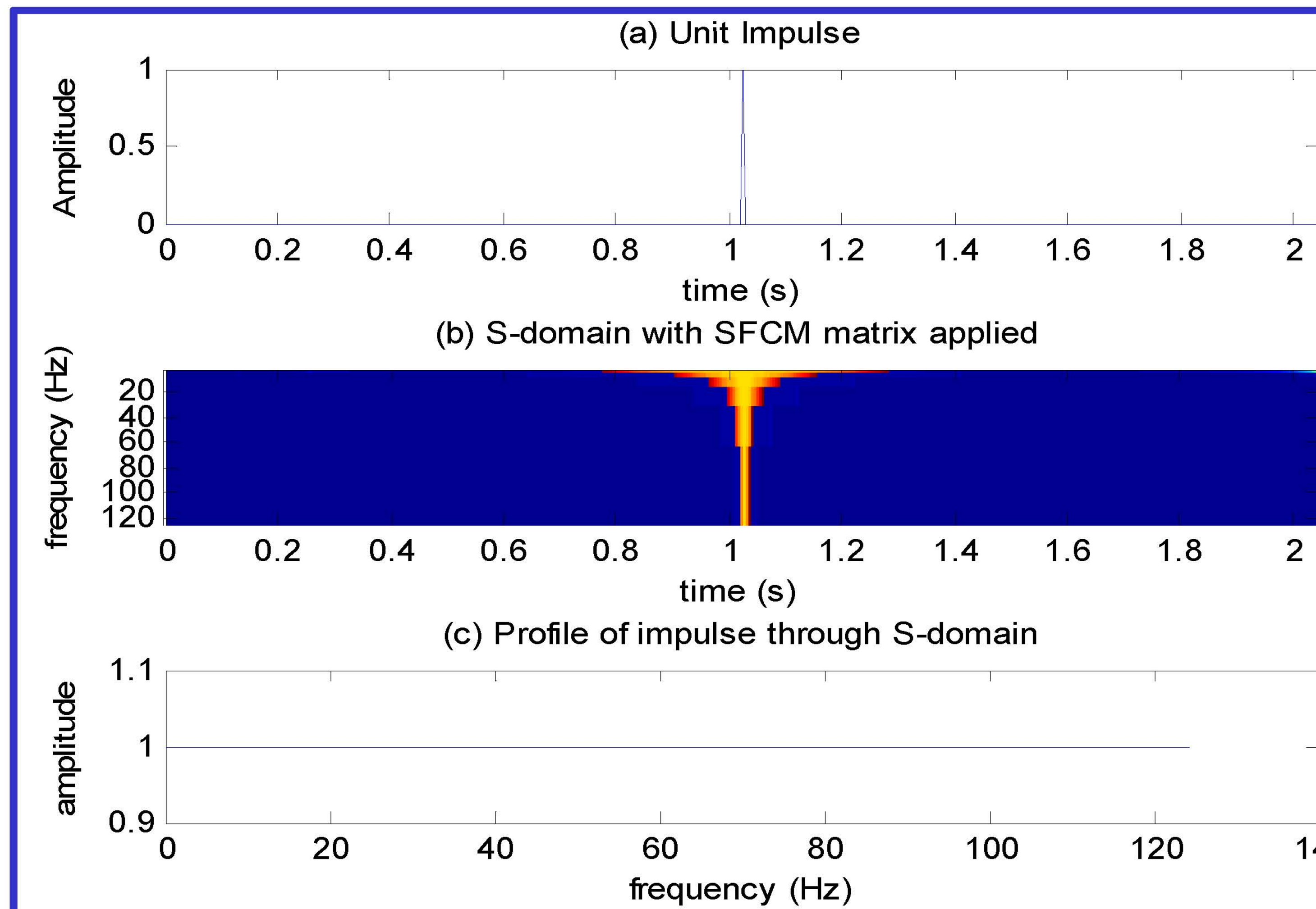

FIG. 4. Unit Impulse Response of FST after calibration. In (a) we have a unit impulse in time. In (b) we have the calibrated S-transform of the unit impulse and in (c) we see the frequency profile of the unit impulse through the S-domain

Limits of the amplitude calibration

Figure 5 shows that the spectra of proximal events interfere at low frequencies. Figure 6 quantifies trust zones as a function of proximity

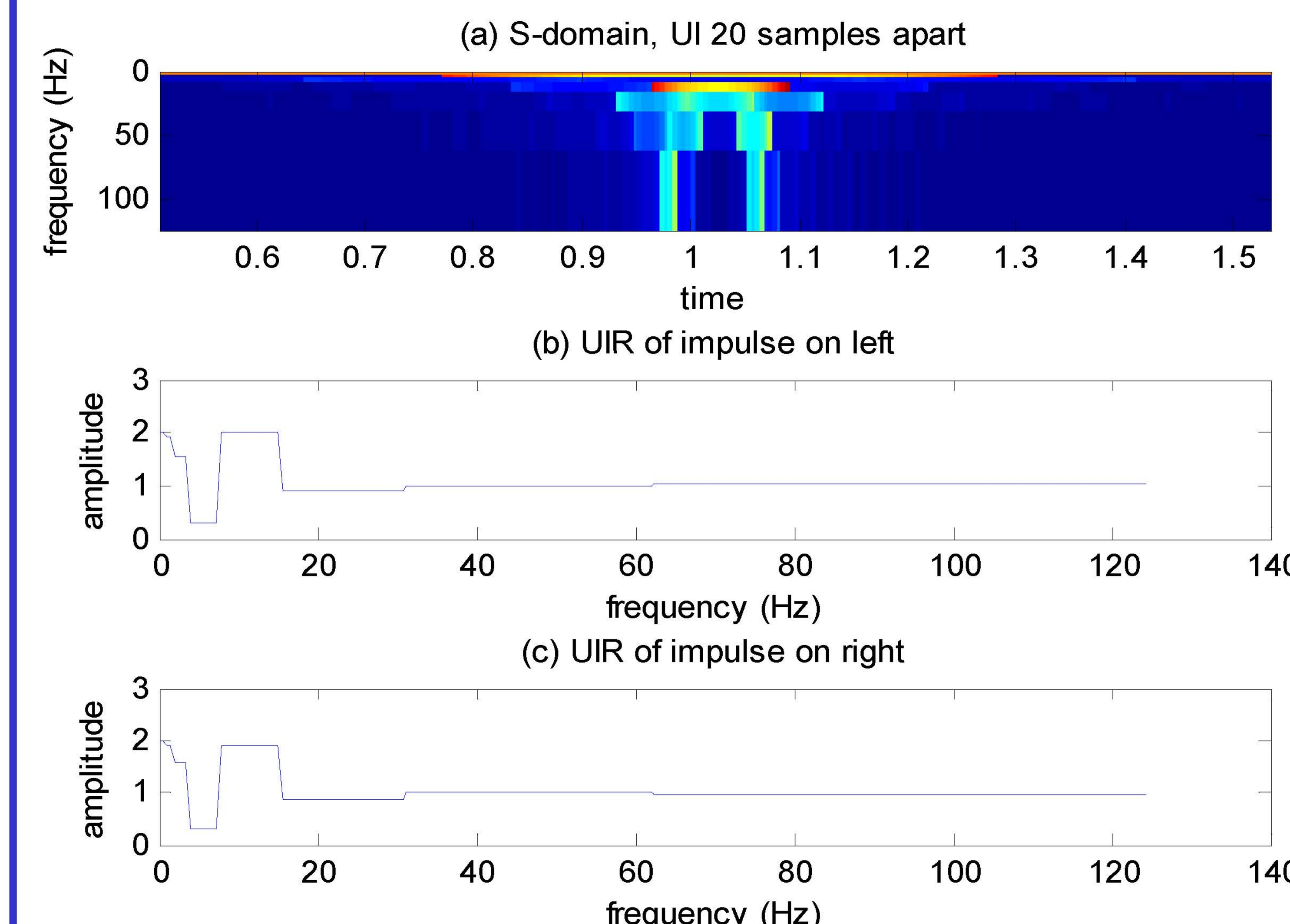

FIG. 5. S-domain of two unit impulses 20 samples apart. (a) shows the S-domain of the two unit impulses input into the FST simultaneously. (b) and (c) show their amplitudes picked from the S-domain. It is clear from (b) and (c) that the spectra of these two unit impulses interfere with each other for frequencies below roughly 17 Hz (define as the cutoff frequency). It is therefore important that we define the fidelity of an events spectrum in the S-domain as a function of proximity to other events.

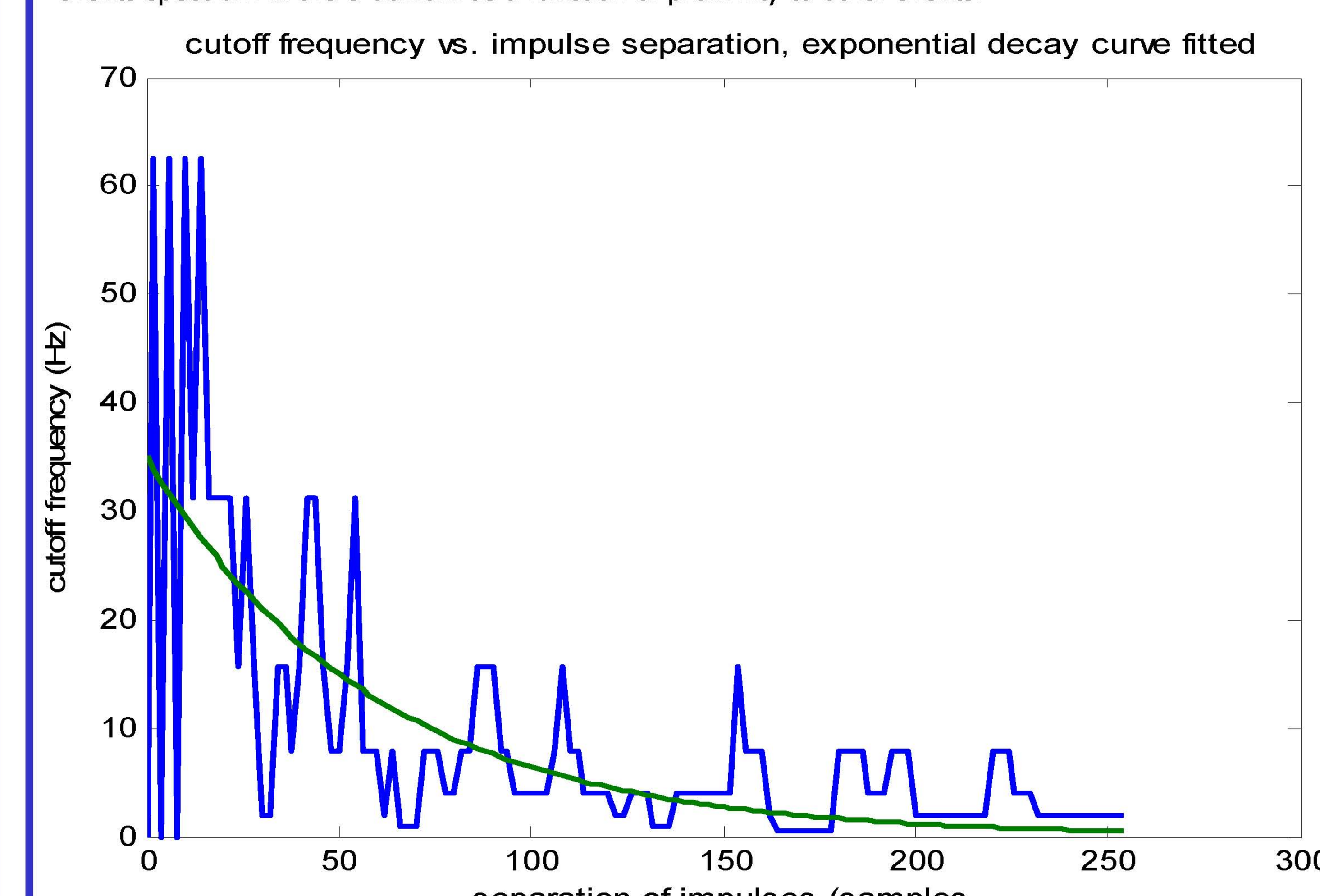

FIG 6. cutoff frequency vs. impulse separation(blue curve). The green curve is an exponential decay which was fit to the data. The green curve defines trust zones as a function of proximity to other events.