A particle/collision model of seismic data

Kris Innanen (k.innanen@ucalgary.ca)

ABSTRACT

It can occasionally be a valuable exercise to take a familiar phenomenon and observe it from a different point of view. For instance, in the right frame of reference, waves in a VSP experiment seem to act like particles, drifting freely or accelerating in a potential, and thereafter colliding, sticking together, and spontaneously disintegrating. In fact all of the important properties of simple wave phenomena (phase velocities, propagation directions, amplitudes, reflection and transmission coefficients) are correctly captured by speaking of the experiment entirely in terms of a system of colliding particles (with well-defined, though notional, masses, velocities, momenta). In the same framework, a seismic *event*, meaning a coherent arrival of wave energy to which we assign a well-defined history of propagation, reflection, and transmission, can be represented in one of two ways. Either a single particle, whose "world-line" is free to move both forwards and backwards in time, or several particles interacting through a specified set of the aforementioned productions and annihilations.

Reflection & transmission as mass/momentum conserving collisions

Let us take a zero-offset VSP experiment, and imagine that we experience it occurring in an unusual way. Rather than seeing it as a set of seismic amplitudes, available to us over a defined depth interval, evolving as we step forward through time, let us take the reverse view. Let us imagine we perceive the same experiment as a set of amplitudes over a defined time interval, evolving as we step forward through depth. Pretend, in other words, that we experience depth the way we normally experience time, and vice versa.

Consider a subsurface with a single reflector at depth z_1, separating media with velocities c_0 above and c_1 below. Receivers are at depths z_g within the well. The response at z_g is, if $z_g < z_1$, a direct downgoing wave of amplitude 1 and an upgoing wave of amplitude R, and, if $z_g > z_1$, a downgoing wave of amplitude T.

Suppose we were to make a movie out of the results of this experiment. Each frame is a seismic trace, i.e., the wave field over the full time interval at a given z_g. We create a suite of frames by letting z_g vary from 0 up to some z_{max}. We arrange them from smallest z_g to largest, and run it. FIG. 1 contains five of these frames, illustrating the general look of the movie. Because the direct wave takes longer to arrive as z_g grows, the disturbance on the left appears to drift to the right with a uniform apparent velocity (i.e., the rate of motion along the time axis per unit change in depth). The reflection from z_1 arrives at maximally late times when z_g is small, and hence the right-most arrival appears to drift to the left. When $z_g = z_1$ the two arrivals coincide. For $z_g > z_1$, only one disturbance, the transmitted wave, remains, and since it propagates downward, it retains the right-moving motion of the direct wave, though, since $c_1 \neq c_0$, it does so at a different apparent velocity.

To the eye, the movie in FIG. 1 appears to depict two objects, one large and one small, moving towards each other at equal and opposite rates. They appear to collide at the apparent time $z_g = z_1$, whereafter, stuck together, they proceed together in the same direction that the original larger object had, but more slowly. If we were to put two carts of different masses on a track and set them on a collision course, and if we could arrange to have them lock together upon colliding, we might expect to see something like FIG. 1 happen.

The masses and velocities of two *carts* obey clear rules, to wit, their masses and momenta before and after the collision are conserved. To what extent do the "particles" in the VSP movie follow rules of this kind?

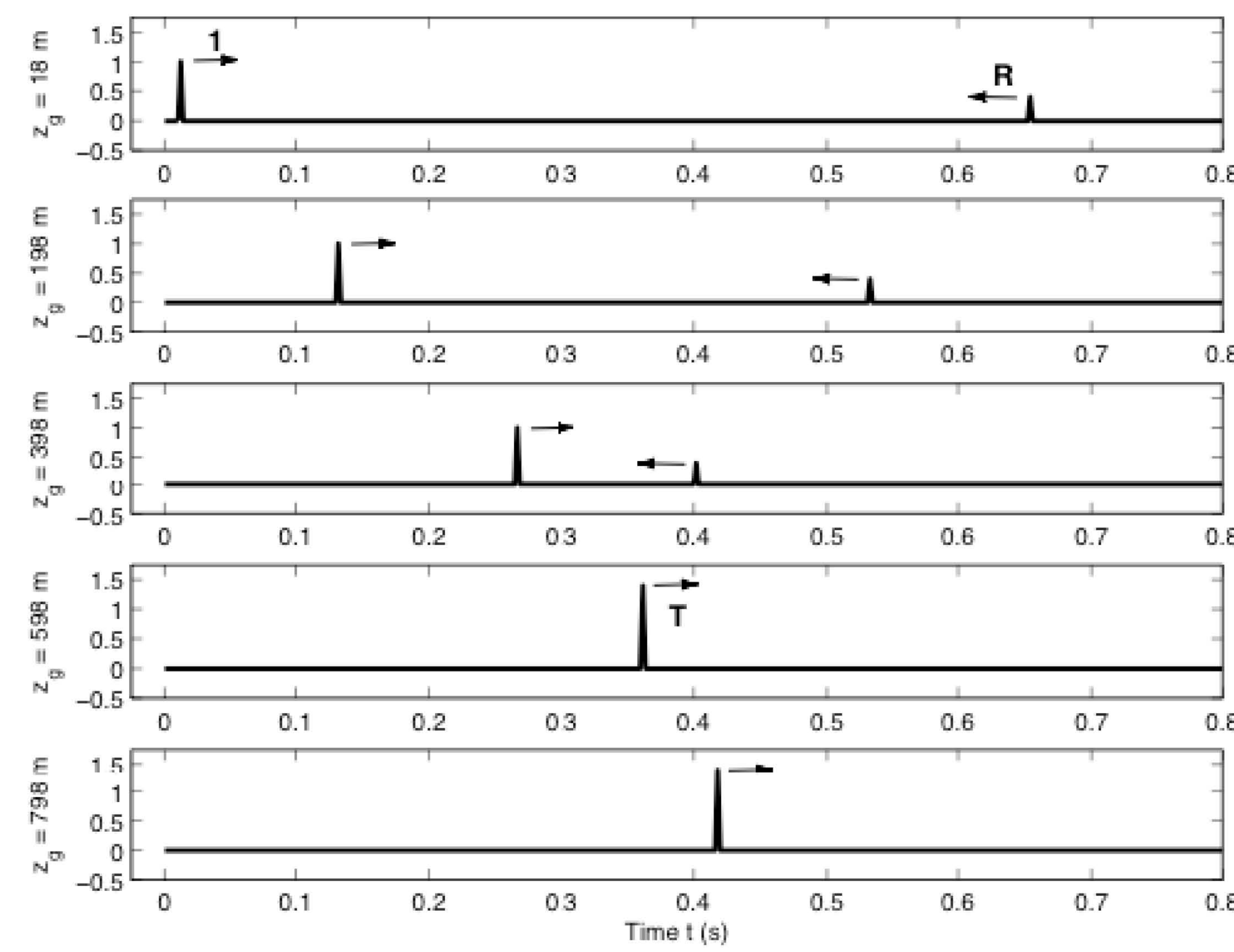

FIG. 1. Collision view of VSP data; 5 snapshots at increasing depth. Bottom two panels are below z_1=500m.

The particles with amplitudes 1, R, and T drift with apparent velocities $v_D=1/c_0$, $v_R=-1/c_0$ and $v_T=1/c_1$ respectively in this view; let us furthermore assign them masses equal to their amplitudes: $m_D=1$, $m_R=R$ and $m_T=T$. If we make this assignment we find that the principles of conservation of mass and momentum provide us with the correct wave relationships typically generated by continuity of the field and its derivative across the boundary:

$$m_\mathrm{D} + m_\mathrm{R} = m_\mathrm{T} \quad \Rightarrow \quad 1 + R = T$$

$$m_\mathrm{D} v_\mathrm{D} + m_\mathrm{R} v_\mathrm{R} = m_\mathrm{T} v_\mathrm{T} \quad \Rightarrow \quad \left(\frac{1}{c_0}\right) + R\left(-\frac{1}{c_0}\right) = T\left(\frac{1}{c_1}\right)$$

Disintegrations & particles moving in a potential

The same rules correctly express all processes of reflection and transmission in media with multiple interfaces, provided momentum- and mass-conserving *distintegrations* are permitted also (please see the report for more details). Two complications of the seismic problem that cause the particles to appear to accelerate, as opposed to drift freely, are (1) a smooth velocity variation c(z), and (2) offset. A possible way to accommodate this type of behaviour in a collision model is to see these influences as *potentials* affecting the motion of the particles. If $v(z_g)$ is the apparent velocity varying with depth, but z_g is interpreted as an apparent time variable, then the whole function is, in essence, expressing

$$\dot{x}(\tau)$$

Where x is the location of the arrival on the trace, and we replace z_g with τ to remind us that it is acting as time. This may be differentiated and integrated to determine the potential energy V a particle must have in order to undergo these changes in apparent velocity, thus incorporating background velocity changes and offset into the model for a direct wave of amplitude P_0:

$$V(x) = P_0 \int_0^x \ddot{x}(x')dx'$$

Seismic events

The particle/collision view provides a means for the classification of seismic events based on characteristic sets of particle interactions. In FIG. 2 we focus on a portion of the paths of wave energy in a two interface model, corresponding to a transmitted multiple. The particle "movie" can be created by sweeping a horizontal line across this diagram. In the movie, particles interact as illustrated in FIG. 3.

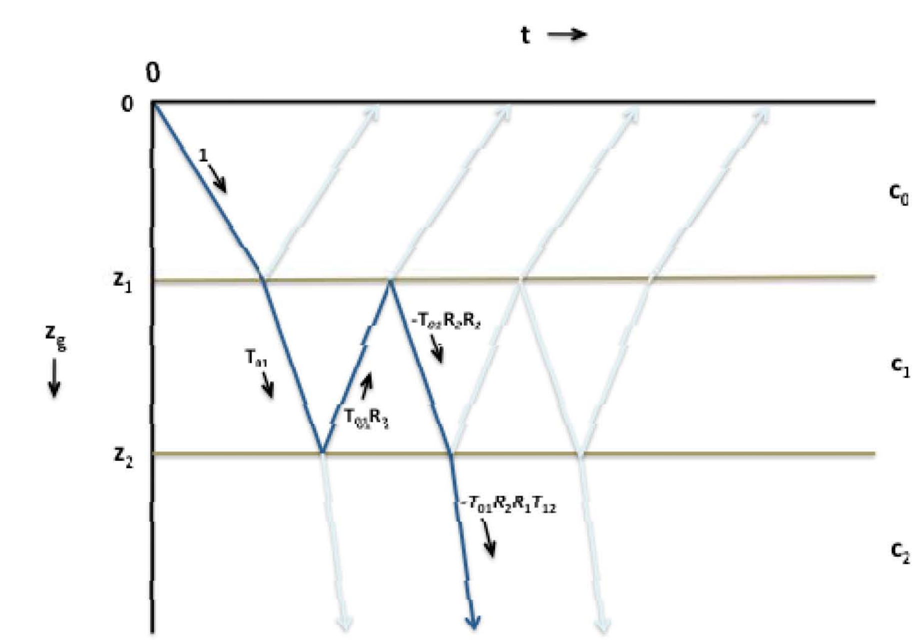

FIG. 2. An event is an arrival with a definite propagation history. It can be represented with a line diagram of the wave path over the course of its various reflections and transmissions. Depicted here is a first order multiple. The event seen over the totality of t and z_g appears as a single entity which travels sometimes forwards and sometimes backwards in apparent time z_g. If the event is seen with z_g growing from small to large values, it instead appears as three interacting particles.

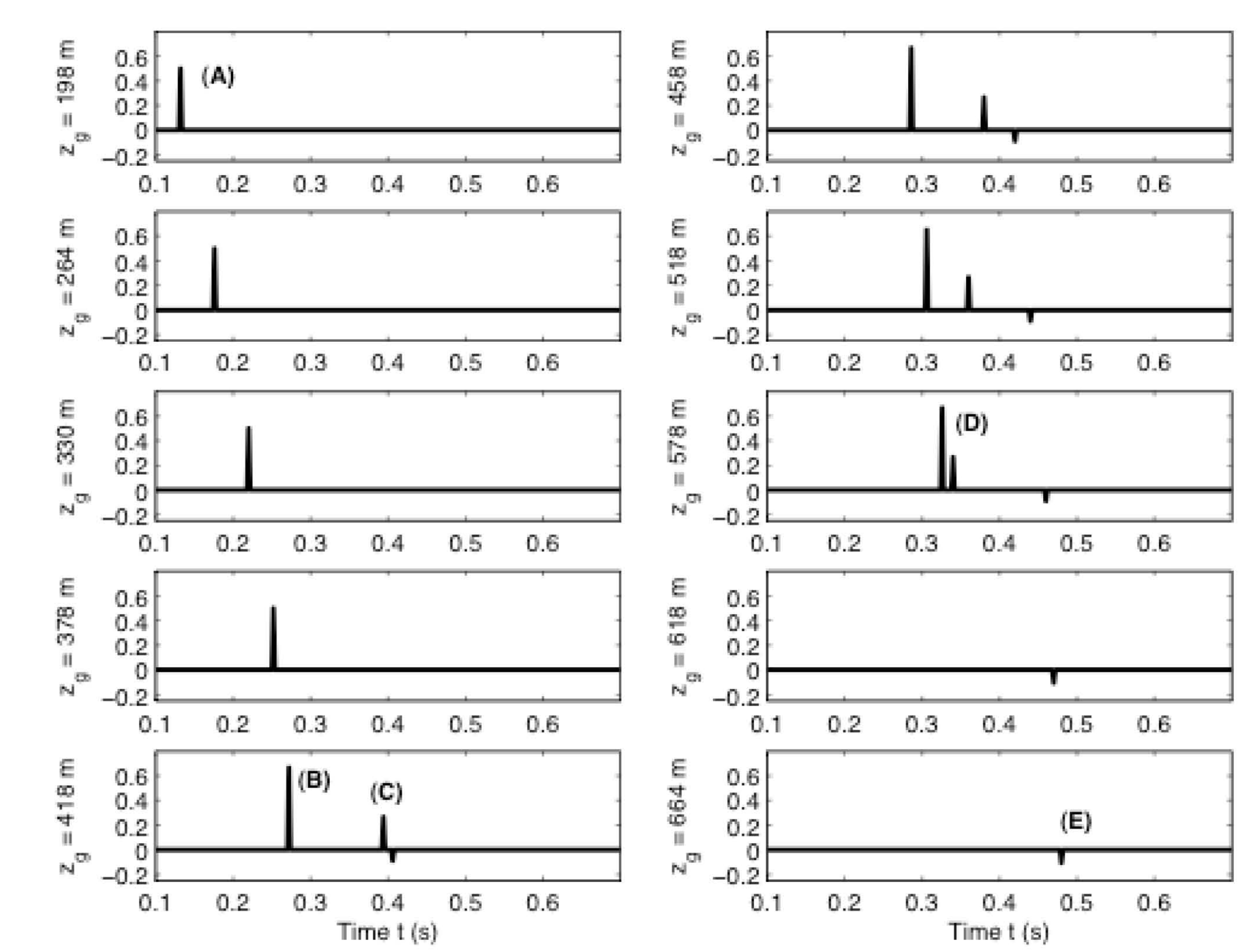

FIG. 3. Frames from the movie of the three particles whose interaction is characteristic of a first order transmitted multiple. In FIG. 2 we saw the multiple as a single entity, a sequence of line segments depicting propagation forwards and occasionally backwards in apparent time. To view it entirely in terms of particles colliding, we constrain time (z_g) to move forwards, hence we do not discern partially time-reversed processes as straightforwardly as this. Instead, we perceive it as three interacting particles, all moving forwards in time, but in different directions. In this Figure are ten snapshots of the collision model movie. At $z_g<z_1$ (A), the multiple consists of a single particle drifting to the right. At time $z_g=z_1$, it decelerates (B) and there is a spontaneous production of two more particles (C), one drifting to the right and one to the left. Two of these approach one another, ultimately colliding and annihilating (D) at time $z_g=z_2$. The right-going particle continues to drift in that direction (E).

CONCLUSIONS

It may be useful to discuss more complex wave interactions such as conversions within a particle/collision framework. Also, the elements of the model seem well-suited for a discussion of the nature of reverse time migration and its various imaging conditions.