Reflection and Transmission coefficients for VTI media.

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Summary

Presently, we obtain the reflection(R) and transmission(T) coefficients in the plane wave domain in behalf of its importance for numerical computations. To do this, effective ray parameter has been used to express classical R and T coefficients in the plane wave domain from phase angle domain. Now following the Graebner's approach we obtain R and T coefficients in terms of Thomson's parameters as these parameters are essential for understanding the seismic waves signatures in the anisotropic media. Further, as the dependency of the reflected waves amplitude on offset has proven to be a valuable exploration tool for direct hydrocarbon detection, we have demonstrated that anisotropy does have considerable influence on the reflection coefficient of seismic waves. Thus conventional AVO analysis needs to be modified in the presence of the anisotropy on either side of the reflecting boundary. A test of accuracy of Rüger's approximation illustrates that we should deal with the more exact algorithm so that the scanty of the accuracy could be avoided.

Theory

R and *T* coefficients for anisotropic media can be written as

$$R_{SH} = \frac{\rho_1 {\beta_{01}}^2 q_1 - \rho_2 {\beta_{02}}^2 q_2}{\rho_1 {\beta_{01}}^2 q_1 + \rho_2 {\beta_{02}}^2 q_2}, \quad (1)$$

and

$$T_{SH} = 2\frac{\rho_1 {\beta_{01}}^2 q_1}{\rho_1 {\beta_{01}}^2 q_1 + \rho_2 {\beta_2}^2 q_2}, \quad (2)$$

where ρ and β are the density and the vertical shear wave velocity and q_1, q_2 are the vertical slownesses in the incident and refracted medium and written as

$$q_1 = \sqrt{{\beta_{01}}^{-2} - p_l^2\,(2\,\gamma_1 + 1)}, \quad (3)$$

and

$$q_2 = \sqrt{{\beta_{02}}^{-2} - p_l^2\,(2\,\gamma_2 + 1)}, \quad (4)$$

where γ is the Thomson's parameter and p_l is the effective ray parameter and can be computed as

$$p_l = \left|\hat{\mathbf{p}} \times \hat{\mathbf{a}}\right| \sqrt{p_1^2 + p_2^2 + q^2}. \quad (5)$$

P-SV waves Reflection and Transmission Coefficients.

- Following Graebner's approach, the stress and strain relationship ($\tau = \mathbf{c}\epsilon$) can be written as

$$\begin{bmatrix} \tau_{xx} \\ \tau_{yy} \\ \tau_{zz} \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} A & A-2N & F & 0 & 0 & 0 \\ A-2N & A & F & 0 & 0 & 0 \\ F & F & C & 0 & 0 & 0 \\ 0 & 0 & 0 & L & 0 & 0 \\ 0 & 0 & 0 & 0 & L & 0 \\ 0 & 0 & 0 & 0 & 0 & N \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ 2\,\epsilon_{yz} \\ 2\,\epsilon_{zx} \\ 2\,\epsilon_{xy} \end{bmatrix}, \quad (6)$$

where $\epsilon_{ij} = \frac{1}{2}(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$, and $i, j = x, y, z$ or 1,2,3.

P-SV waves Reflection and Transmission Coefficients.

The τ_{ij} are the stresses, the ϵ_{ij} are the strains, the u_i are the components of particle displacement and the *A*, *C*, *F*, *L* and *N* are the elastic constants.

- R and T coefficients are obtained in terms of the elastic constant by solving the system of equations given by **Sx**=**b** where

$$\mathbf{S} = \begin{bmatrix} l_{\alpha_1} & m_{\beta_1} & -l_{\alpha_2} & -m_{\beta_2} \\ m_{\alpha_1} & -l_{\beta_1} & m_{\alpha_2} & -l_{\beta_2} \\ a_1 & b_1 & a_2 & b_2 \\ c_1 & d_1 & -c_2 & -d_2 \end{bmatrix}, \quad (7)$$

$$\mathbf{x} = [r_{pp},\ r_{ps},\ t_{pp},\ tps]',$$

and

$$\mathbf{b} = [-l_{\alpha_1},\ m_{\alpha_1},\ L_1\,(q_1\,l_{\alpha_1} + p_l m_{\alpha_1}),\ -p_l l_{\alpha_1} F_1 - q_{\alpha_1} m_{\alpha_1} C_1]'. \quad (8)$$

- The components of **S** and **b** are known in terms of the vertical and horizontal slownesses and elastic constant used by Graebner.
- Using the elastic stiffness matrix to describe VTI media

$$c_{VTI} = \begin{bmatrix} c_{11} & c_{11} - 2\,c_{66} & c_{13} & 0 & 0 & 0 \\ c_{11} - 2\,c_{66} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}, \quad (9)$$

and Thomson's parameters, we come up with a relationship as

$$A = \rho\alpha_0^2(1 + 2\epsilon),\ C = \rho\alpha_0^2,\ L = \rho\beta_0^2,\ N = \rho\beta_0^2,$$

and

$$F = \rho\sqrt{\left({\alpha_0}^2 - {\beta_0}^2\right)\left((2\,\delta + 1)\,{\alpha_0}^2 - {\beta_0}^2\right)} - \rho\,{\beta_0}^2. \quad (10)$$

Examples

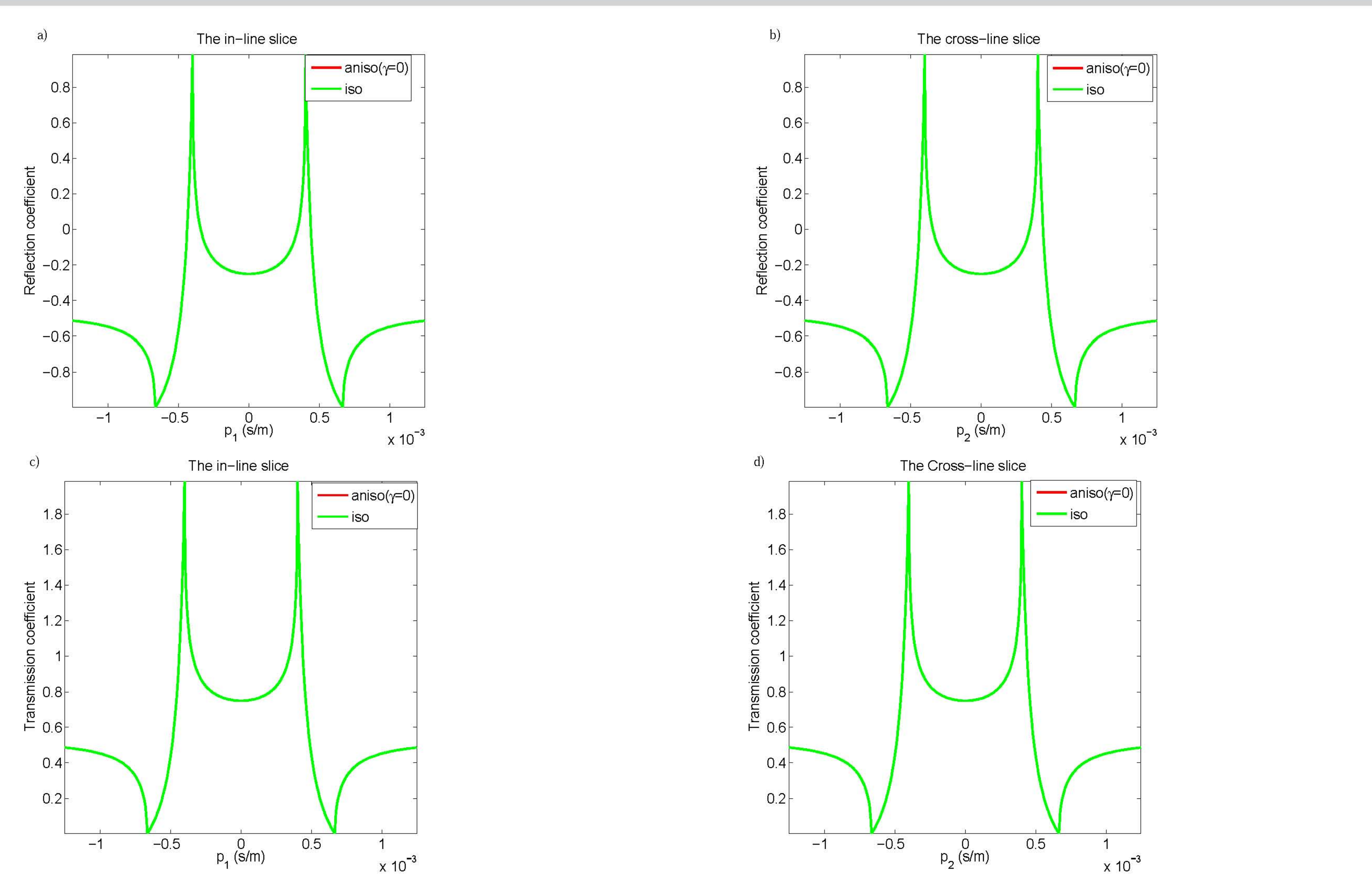

Fig.1: In-line, Cross-line slices of the (a), (b) reflection and (c),(d) transmission coefficients for an isotropic medium. The authentication of proposed approach is demonstrated.

Examples

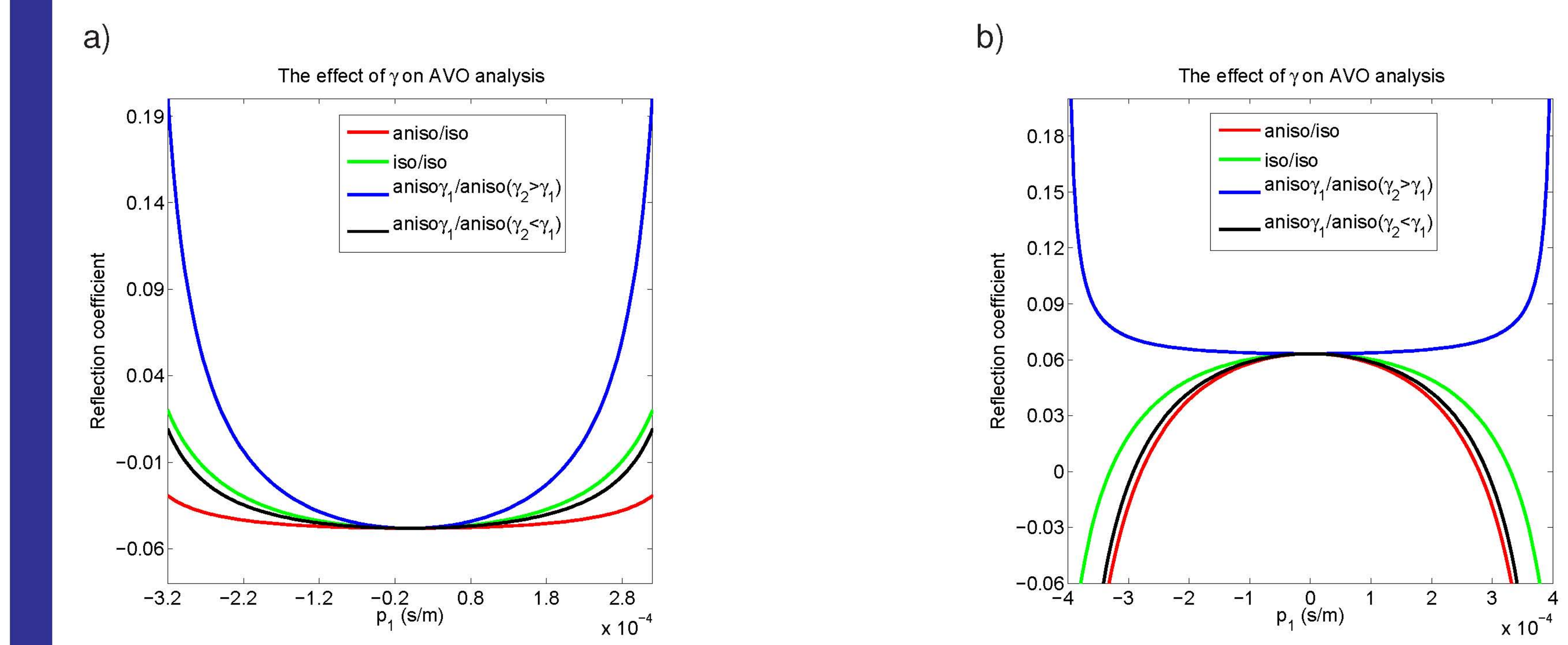

Fig.2: The effect of γ on AVO analysis for different -different interfaces as shown for (a) model1 (b) model2.

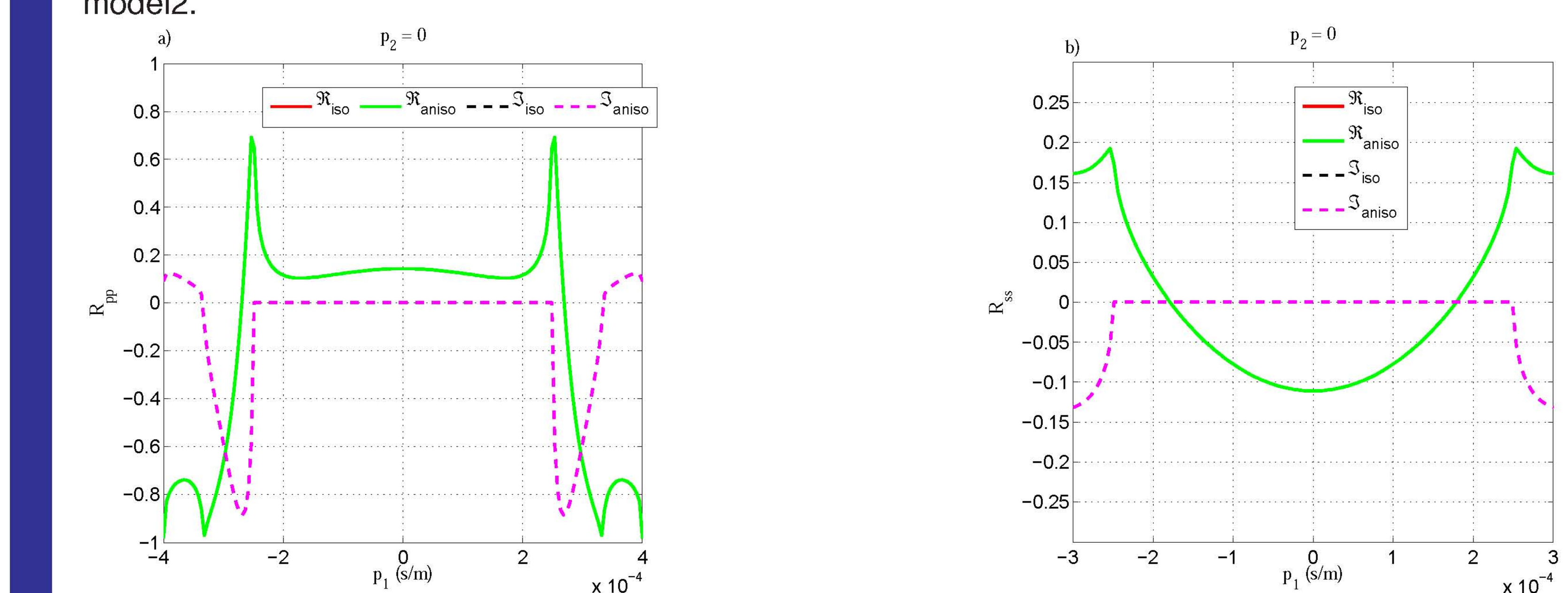

Fig.3: Real and Imaginary part of R coefficients for (a) P-P (b) SV-SV cases for isotropic media. The authentication of the proposed approach is illustrated here.

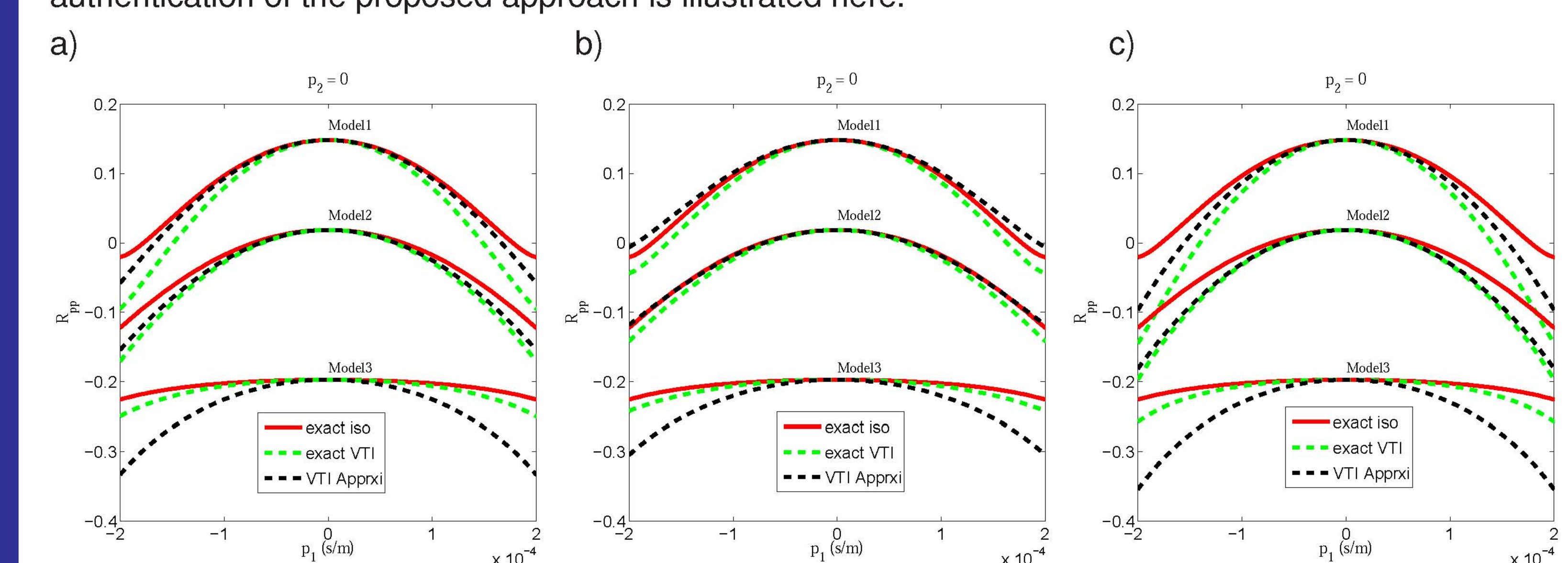

Fig.4: The effect of ϵ on the AVO analysis of the three models characterized by Class 1, 2 and 3 type of Gas sand anomaly. δ is considered constant while ϵ possesses (a) 0.133 (b) 0 and (c) 0.233 values, respectively.

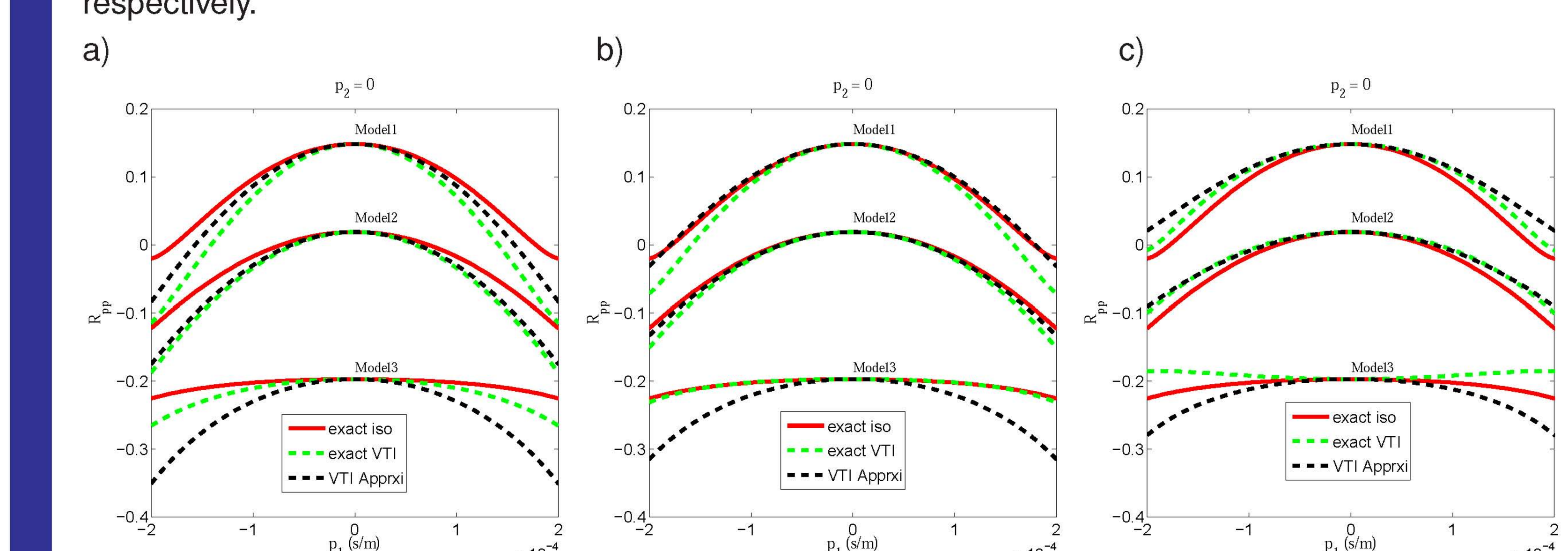

Fig.5: The effect of δ on the AVO analysis of same models as considered before in Fig.4 . ϵ is considered constant while ϵ possesses (a) 0.12 (b) 0 and (c) 0.240 values, respectively.

Acknowledgments

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