The relationship between Lipschitz exponents and Q: synthetic data tests

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Introduction

In seismic signal analysis, regions of abrupt change, often considered expressions of underlying singularities within a given function contain considerable amount of a signal's information (Innanen, 2003). Applying the continuous wavelet transform enables us to obtain the modulus maxima from seismic data and estimate the Lipschitz exponents which in turn allows us to measure the local regularity of functions and differentiate the intensity profile of different edges (Mallat and Zhong, 1992).

It is generally understood that due to absorption, the energy of seismic waves propagating through an anelastic medium would dissipate over a given distance. The overall effect of seismic attenuation is described by the dimensionless quality factor Q, with studies in seismic data processing concentrating either on modelling, estimation or compensation (Innanen, 2003). In practical terms, estimation and compensation can hope to dramatically enhance the resolving power of seismic data.In this paper we discuss numerical implementation of the continuous wavelet transform and estimation of the associated Lipschitz exponent (α) and the possibility of establishing an empirical relation between a function's regularity and the quality factor Q.

Numerical Implementation

I. Implementation of CWT and extraction of modulus maxima

Applying continuous wavelet transform to a given function say f and obtaining the modulus maxima at each scale s leads to the following relation (Mallat and Zhong, 1992),

$$|W_s f(x)| \le A s^{\alpha} \quad (1)$$

where $|W_s f(x)|$ is the modulus maxima of the function $f(x)$ at various scales $s=2^j$. Linearising equation (1) by taking logarithm of both sides provides the following,

$$\log_2 |W_s f(x)| \le \log_2 A + \alpha \log_2(s) \quad (2)$$

Most signal structures could be described as smoothed functions with an underlying singularity (Mallat and Zhong, 1992; Innanen, 2003). Such a function would have to be modelled as a delta function smoothed by a Gaussian with variance σ^2.

As a result, one would have to minimise the following non-linear objective function

$$\phi(A,\alpha,\sigma)=\sum_{i,j=1}^{n}[\log_2|a_i|-\log_2(A)-j+\frac{\alpha-1}{2}(\log_2(\sigma^2+2^{2j})]^2 \quad (3)$$

II. Estimation of the Lipschitz exponent: Linear model

We start with a single event, represented by a delta function as our input signal where the Lipschitz regularity of such an event is equal to −1.

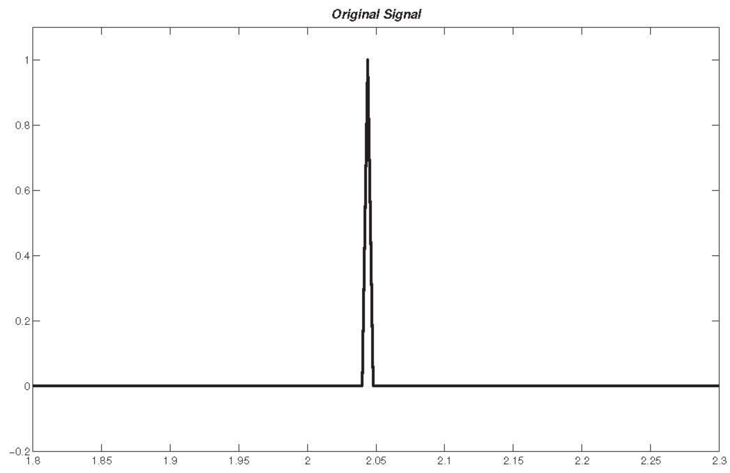

FIG. 1. Original signal.

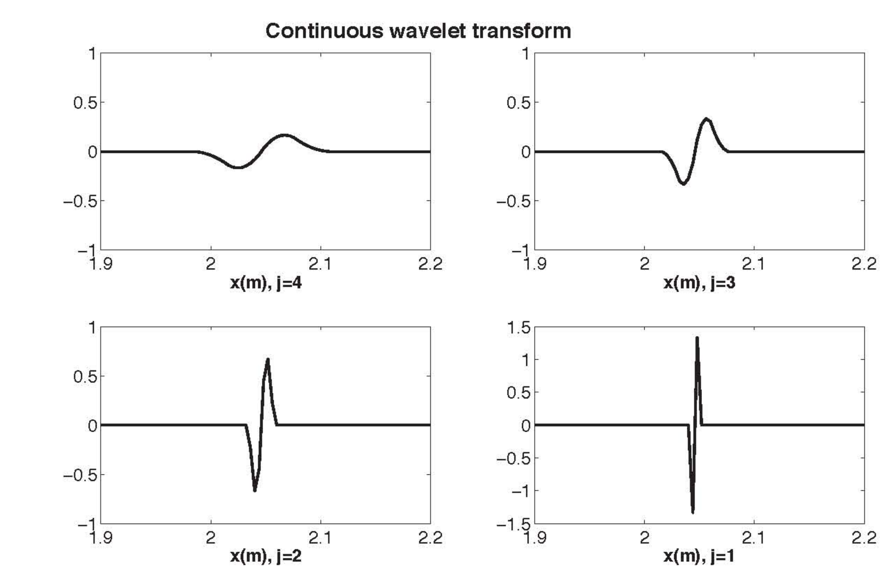

FIG. 2. Corresponding wavelet transform.

Computing the slope based on (2) yields an estimate for α which is equal to −0.999. The intercept produces the value for A, which is equal to 2.33. Additionally one could estimate α and A by forming the objective function in order to estimate A and α. Using this method, we obtain a value of 2.44 for A and a value of −1.0004 for α.

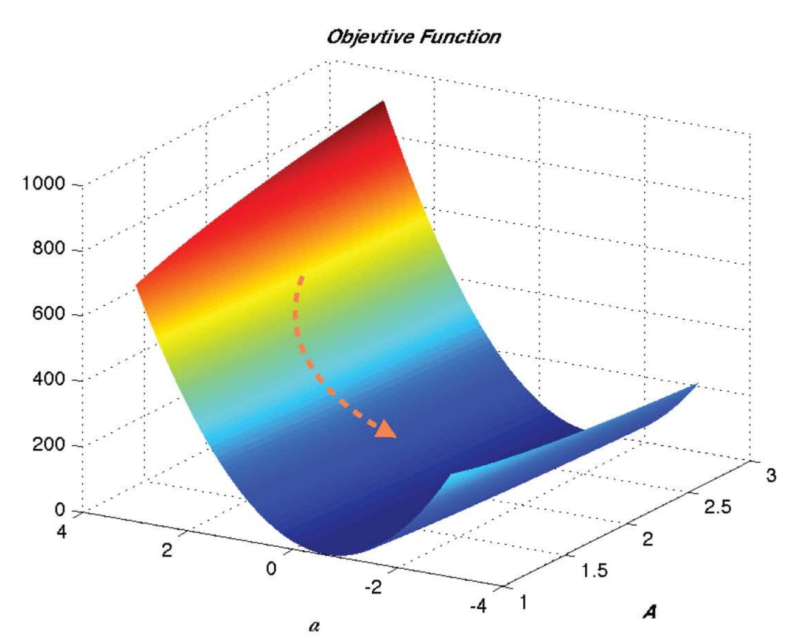

FIG. 3. Two parameter objective function. The, x and y axes represent the values for α and A respectively.

III. Estimation of the Lipschitz exponent: non-Linear model

To asses the effects of absorption on a function's regularity we apply the CWT on a seismic trace and subsequently form and minimise the objective function given in equation (3).

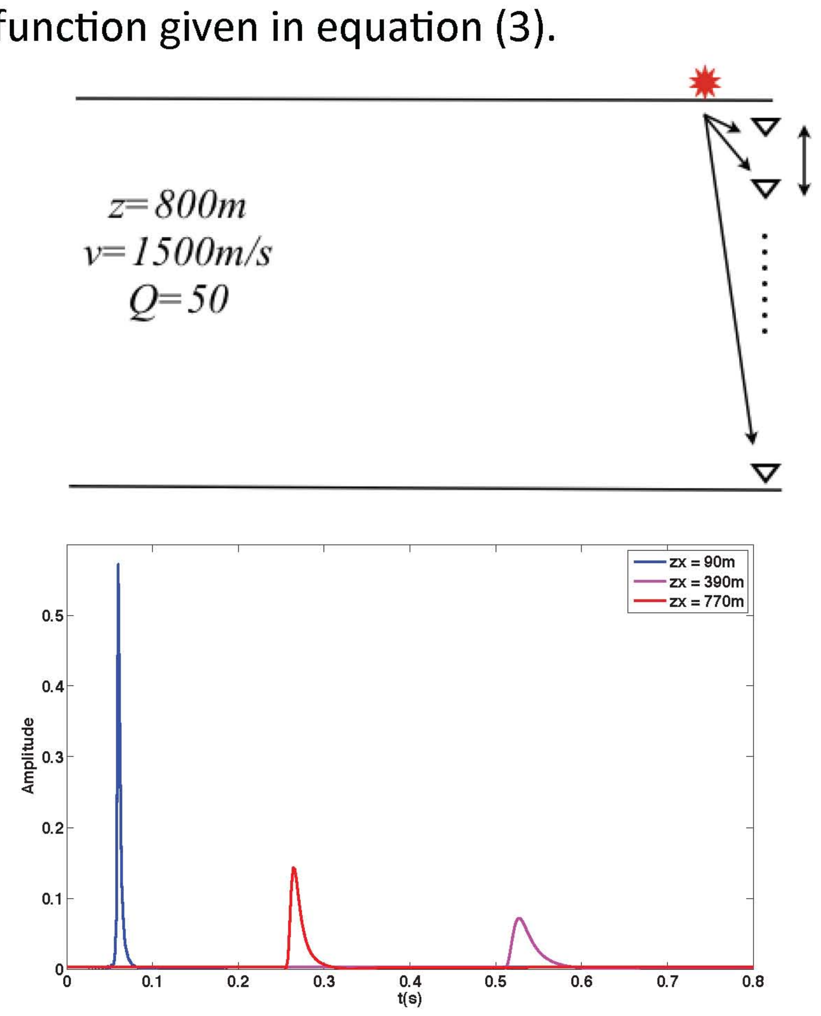

FIG. 4. Amplitude of arrivals *vs* time for three receivers located at *z1 = 90m*, *z2 = 390m* and *z3 =770m* with *Q=50.*

Given the resemblance of the first arrival to a delta function, we expect a Lipschitz value close to −1. For each event, the corresponding A and α values are obtained using the steepest descent method.

	Guess 1	Estimate	Guess 2	Estimate	Guess 3	Estimate
α	-2	-0.3500	-1	-0.3547	0	-0.3724
A	1	2.0860	2	2.1105	3	2.2306
σ	0.003	0.0003	0.01	0.0095	0.1	-0.0294
∇g		2.8735		2.8725		2.8723

IV. Scale cut-off approach

Given the non-linear nature of our model, the steepest descent is time-consuming and inefficient. As an alternative to the steepest descent, one may consider linearising the problem by analysing the dominant behaviour of α and imposing a threshold on the scales.

The problem of visual bias complicates this approach. In addition, this approach should be expected to encounter significant accuracy issues for combination of low Q and large propagation distances

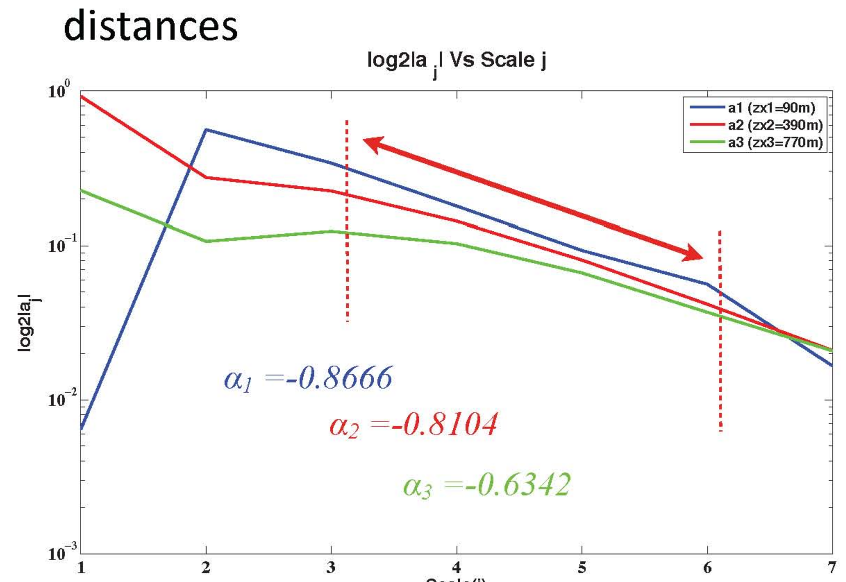

FIG. 5. Plot of log2(a) vs scale for each arrival for receivers located at depth *z1 = 90m*, *z2 = 390m* and *z3 = 770m.*

Relation between α and Q

Plotting α values versus corresponding receiver depth for various values of Q provides illustrate a proportional dependency between α and depth and an inverse relation between α and Q.

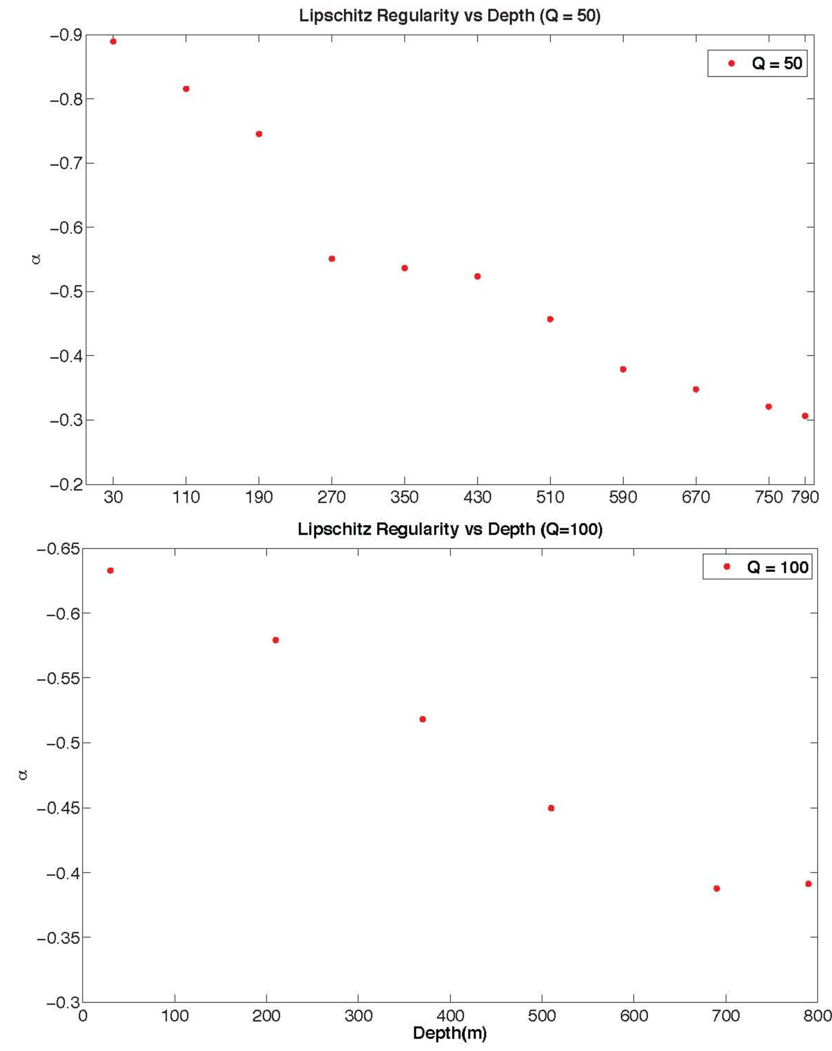

FIG. 6. Plot of α *vs* depth for Q=50 and Q=100

Conclusion

An accurate estimation of the Lipschitz regularity of a seismic trace is regarded as a highly desirable goal. For a single event, resembling a delta function or a Heaviside function, one could use a linear model, based on a functions smoothness at each scale and estimate the Lipschitz exponent by finding the slope or by forming and minimising an objective function. However due to absorption, one would have to model a given pulse as delta function convolved with a Gaussian which in effect leads to a non-linear model. Thus, in order to estimate the Lipschitz exponent, one could use the steepest descent or some sort of thresholding method in order to simplify the problem.

The results illustrate a relation between absorption and a functions decay, which could be used to establish an empirical relation between the associated Lipschitz exponent and Q. It would be of particular interest to test the model on field data and analyse the presence of noise, primaries, with varying Q values on the non-linear model.

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