Anelastic reflection, scattering and sensitivity analysis

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Introduction

We write down anelastic AVO approximations for problems involvingelastic and anelastic incidence media. Variations in the anelastic properties of the Earth are expressed in terms of reflectivity- and relative change-type quantities. R_{PP}, R_{PS} and R_{SS} coefficients are each investigated. Further, we frame the full anelastic scattering problem, focusing on: transformation of anelastic scattering potentials to the P-, Sv-, and Sh- potential domain, and the consequences of moving from an elastic reference/anelastic perturbation model to an anelastic reference/anelastic perturbation model.

Anelastic scattering and reflection

We treat the anelastic AVO problem for two cases: (i) the incidence medium is elastic (Figure 1a); (ii) the incidence medium is anelastic (Figure 1b). We treat the scattering problem for four possible scenarios: an incoming and outgoing P-wave (Figure 2a), an incoming P-wave, outgoing S-wave (Figure 2b), an incoming S-wave, outgoing P-wave (Figure 2c), and an incoming and outgoing S-wave (Figure 2d). The anelastic problem also subdivides into homogeneous/inhomogeneous waves (Figure 3).

Schemes

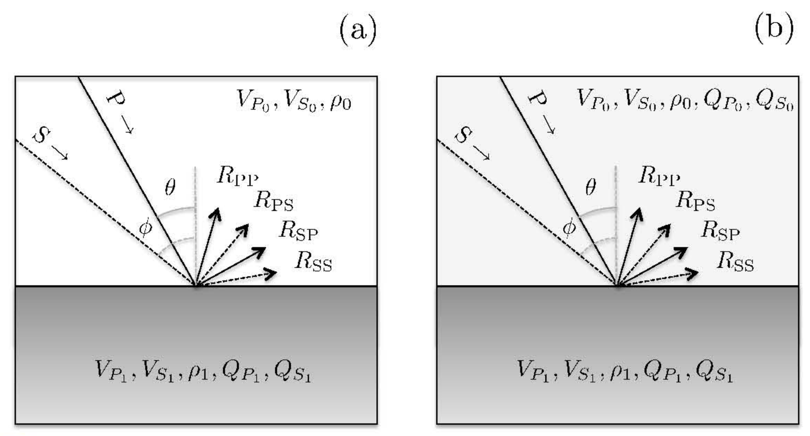

Figure: 1. Schematic diagram of anelastic AVO.

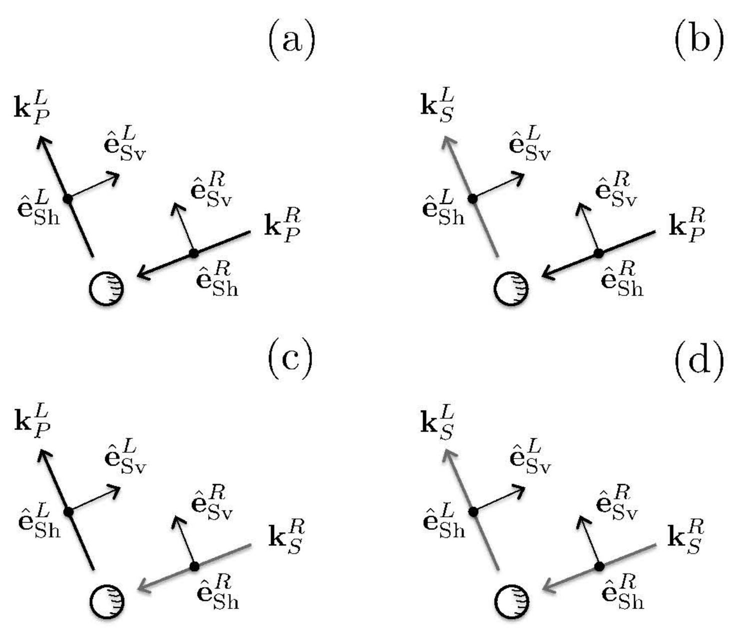

Figure: 2. Configurations for anelastic scattering.

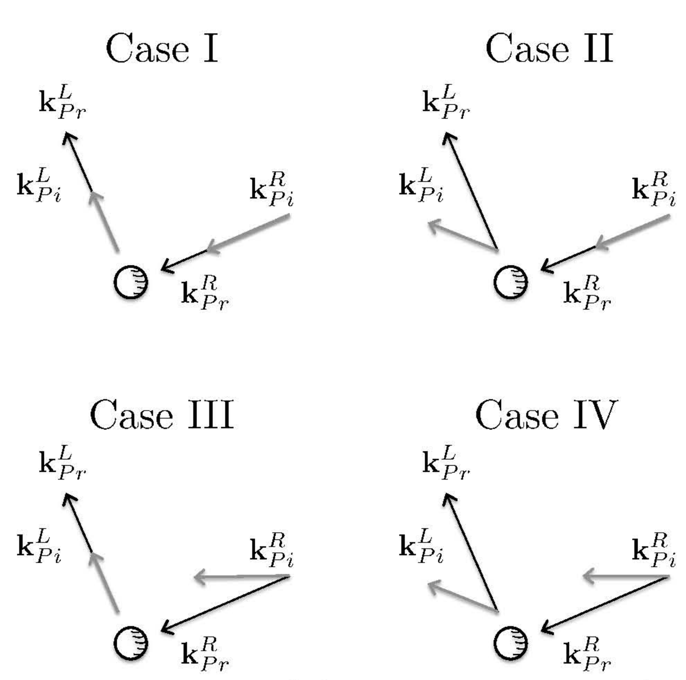

Figure: 3. Homogeneous vs. inhomogeneous anelastic scattering.

Anelastic scattering theory

The anelastic Born model of data is

$$D(\mathbf{r},\mathbf{r}_s,\omega) \approx \int_V d\mathbf{r}' G_L(\mathbf{r},\mathbf{r}',\omega)\mathcal{V}(\mathbf{r}',\omega)G_R(\mathbf{r}',\mathbf{r}_s,\omega) \quad (1)$$

where

$$\mathcal{V}(\mathbf{r}',\omega) = \sum_i \mathcal{A}_i^L(\mathbf{r}',\omega)\left[\frac{\Delta\chi_i}{\chi_i}\right](\mathbf{r}')\mathcal{A}_i^R(\mathbf{r}',\omega). \quad (2)$$

Substituting anelastic terms for the scattering operator we derive

$$V^{AE} = \begin{bmatrix} V^{AE}_{PP} & V^{AE}_{PSh} & V^{AE}_{PSv} \\ V^{AE}_{ShP} & V^{AE}_{ShSh} & V^{AE}_{ShSv} \\ V^{AE}_{SvP} & V^{AE}_{SvSh} & V^{AE}_{SvSv} \end{bmatrix} = \begin{bmatrix} V^{AE}_{PP} & 0 & V^{AE}_{PSv} \\ 0 & V^{AE}_{ShSh} & 0 \\ V^{AE}_{SvP} & 0 & V^{AE}_{SvSv} \end{bmatrix}, \quad (3)$$

where elements are interpretable in terms of P-P, P-Sv, etc. scattering interactions. The elements particular to the anelastic problem are

$$V^{AE}_{PPqp}(\mathbf{r},\omega) = 2F_P(\omega)\rho_0\left(\frac{V_{P_0}^2}{\omega}\right)^2\frac{\Delta Q_P}{Q_P}|\mathbf{k}_P^L|^2|\mathbf{k}_P^R|^2, \quad (4)$$

and

$$V^{AE}_{PPqs}(\mathbf{r},\omega) = -2F_S(\omega)\rho_0\left(\frac{V_{P_0}}{\omega}\right)^2 V_{S_0}^2\frac{\Delta Q_S}{Q_S}|\mathbf{k}_P^L\times\mathbf{k}_P^R|^2. \quad (5)$$

Here $F_P = i/2 - (1/\pi)\log(\omega/\omega_P)$ and $F_S = i/2 - (1/\pi)\log(\omega/\omega_S)$.

Example anelastic AVO approximations

Three example linearized anelastic AVO approximations are

$$R^A_{SS}(\phi,\omega) \approx -\frac{1}{2}\left(1-7\sin^2\phi_A\right)\frac{\Delta V_S}{V_S} - \frac{1}{2}\left(1-4\sin^2\phi_A\right)\frac{\Delta\rho}{\rho} - \frac{1}{2}Q_{S_0}^{-1}F_S(\omega)\left(1-7\sin^2\phi_A\right)\frac{\Delta Q_S}{Q_S}, \quad (6)$$

and

$$R^E_{PP}(\theta,\omega) \approx \frac{1}{2}(1+\tan^2\theta)\frac{\Delta V_P}{V_P} + \frac{1}{2}\left[1-4\left(\frac{V_{S_0}}{V_{P_0}}\right)^2\sin^2\theta\right]\frac{\Delta\rho}{\rho} - 4\left(\frac{V_{S_0}}{V_{P_0}}\right)^2\sin^2\theta\frac{\Delta V_S}{V_S} + \frac{1}{2}(1+\tan^2\theta)\frac{\Delta Q_P}{Q_P} + 4\left(\frac{V_{S_0}}{V_{P_0}}\right)^2\sin^2\theta\frac{\Delta Q_S}{Q_S}, \quad (7)$$

and

$$R^E_{SS}(\phi,\omega) \approx -\frac{1}{2}\left(1-7\sin^2\phi\right)\frac{\Delta V_S}{V_S} - \frac{1}{2}\left(1-4\sin^2\phi\right)\frac{\Delta\rho}{\rho} + \frac{1}{2}\left(1-7\sin^2\phi\right)\frac{\Delta Q_S}{Q_S}, \quad (8)$$

Sensitivities

The Born model leads to sensitivity calculations for anelastic full waveform inversion, e.g.,

$$\delta PP_{qp}(\mathbf{r}_g,\mathbf{r}_s,\omega) = \int_V d\mathbf{r}' G_P(\mathbf{r}_g,\mathbf{r}',\omega)V^{AE}_{PPqp}(\mathbf{r},\omega)G_P(\mathbf{r}',\mathbf{r}_s,\omega) = 2F_P(\omega)\rho_0\left(\frac{\omega}{V_{P_0}}\right)^2\int_V d\mathbf{r}' G_P(\mathbf{r}_g,\mathbf{r}',\omega)\delta Q_P(\mathbf{r}')G_P(\mathbf{r}',\mathbf{r}_s,\omega), \quad (9)$$

where $\delta Q_P = \Delta Q_P/Q_P$. Setting $\delta Q_P(\mathbf{r}') = \delta Q_P\delta(\mathbf{r}-\mathbf{r}')$, we finally have

$$\frac{\partial PP_{qp}(\mathbf{r}_g,\mathbf{r}_s,\omega)}{\partial Q_P(\mathbf{r})} = 2F_P(\omega)\rho_0\left(\frac{\omega}{V_{P_0}}\right)^2 G_P(\mathbf{r}_g,\mathbf{r},\omega)G_P(\mathbf{r},\mathbf{r}_s,\omega). \quad (10)$$

Numerical examples

Example approximate AVO curves follow (in blue), in particular as compared to exact curves (black), and the curves which would have been constructed under the elastic assumption (red).

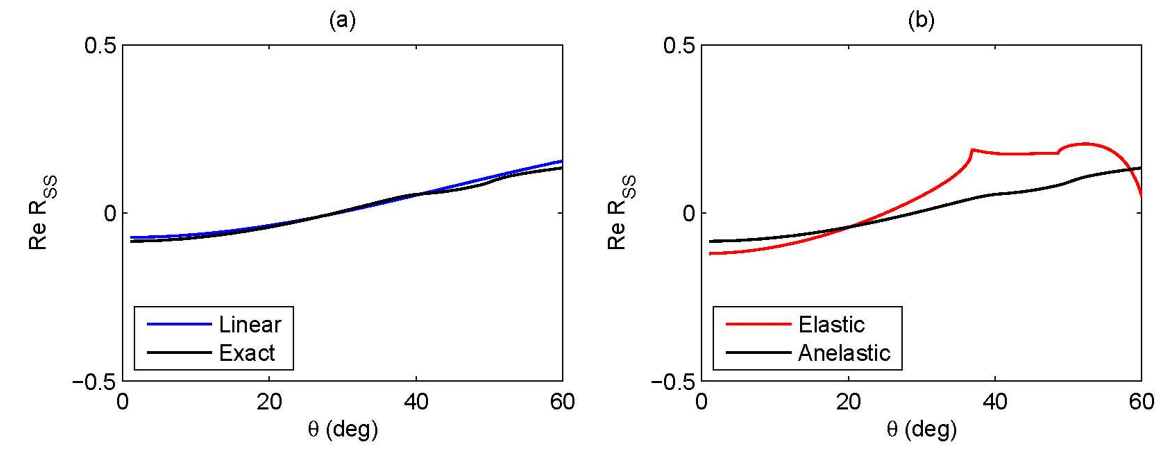

Figure: 4. R_{SS} in terms of relative change parameters.

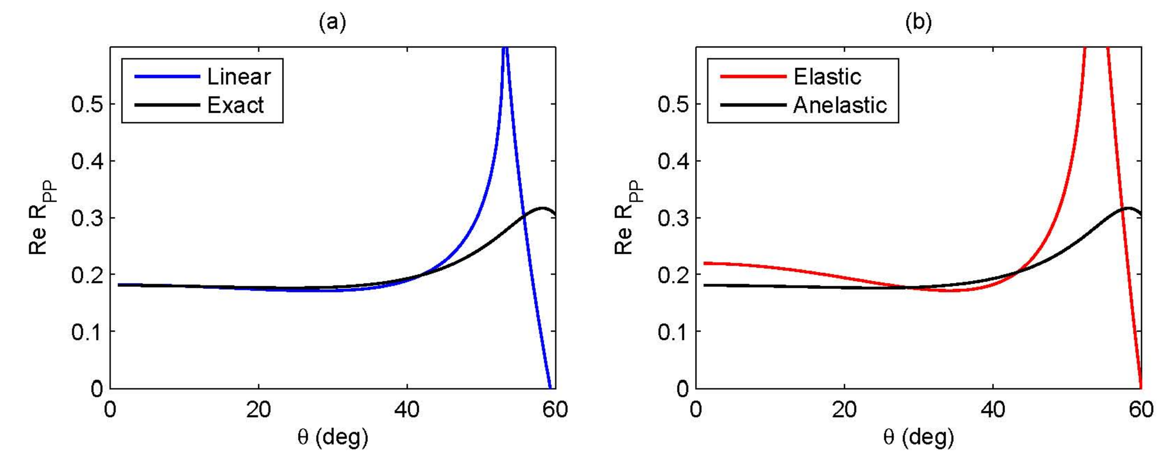

Figure: 5. R_{PP} in terms of reflectivity parameters.

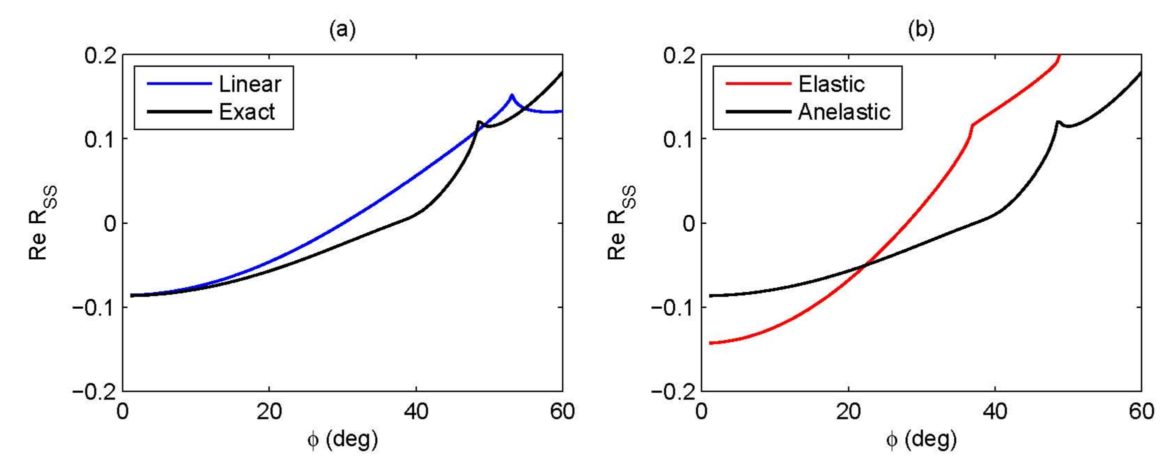

Figure: 6. R_{SS} in terms of reflectivity parameters.

Conclusions

Characterization of the scattering problem for anelastic waves has been carried out in full anelastic regime, focusing on some key issues: transformation of anelastic scattering potentials to the P-, Sv-, and Sh- potential domain, and the consequences to that transformation of moving from an elastic reference/anelastic perturbation model to an anelastic reference/anelastic perturbation model.

We use the scattering potentials thus derived to produce sensitivity kernels for full waveform inversion iterates wherein V_P, V_S, ρ updates are carried out in elastic target determination, and Q_P and Q_S updates are added in anelastic target determination.

Key next steps are: (1) use the anelastic reference medium framework to further understand the ability of homogeneous waves to scattering into inhomogeneous waves, (2) use the same framework to discuss scattering of Borcherdt's Type-I and II anelastic S-waves, (3) use the anelastic sensitivities to analyze potential gradient-based and Newton iterates of full waveform inversion based thereupon.

Bibliography

Please see the corresponding CREWES report for a full bibliography.