Hagedoorn's +/- method and interferometric refraction imaging

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Introduction

Hagedoorn's plus/minus method involves the calculation of sums and differences of first arrivals in refraction data sets. These quantities can be used to estimate subsurface velocities and the depths of curved refractors. Since seismic interferometry is, in a sense, a formal way of adding and subtracting traveltimes, it follows that the +/- method should be expressible in the form of an interferometric procedure. An algorithm for refractor imaging, expressed in terms of interferometric calculations on forward and reverse shot records, is derived heuristically and tested with a simple synthetic data set.

The plus-minus method

Hagedoorn's plus/minus method is applied to refraction problems involving undulating refractors, whose depth is to be determined as a function of lateral position (along with the velocities above and below the refractor). In Figure 1 is a rough schematic diagram of the survey and its +/- interpretation. A suite of geophones is arrayed between two shot points, which we will refer to as F for forward and R for reverse (referring to the directions the wave energy is shot into the geophones).

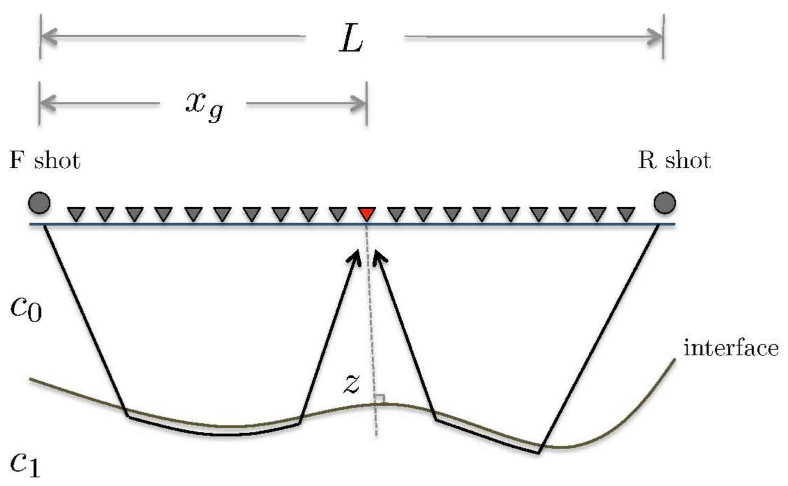

Figure: 1. Schematic diagram for +/- method.

The method involves the idea of *delay times*:

$$\delta_{xg} = \frac{z(x_g)\cos\theta_c}{c_0}, \quad (1)$$

where θ_c is the critical angle and z is the refractor depth normal to the interface beneath the geophone x_g. The forward shot then has the traveltime equation

$$\tau_F(x_g) = \frac{x_g}{c_1} + \delta_F + \delta_{xg}, \quad (2)$$

and the reverse shot has

$$\tau_R(x_g) = \frac{L - x_g}{c_1} + \delta_R + \delta_{xg}. \quad (3)$$

Hagedoorn's plus term is defined as

$$\tau^+(x_g) = \tau_F(x_g) + \tau_R(x_g) - \tau_{rc}, \quad (4)$$

where τ_{rc} is the *reciprocal time*, or the travel time from one shot to the other. Hagedoorn's minus term is similarly defined to be

$$\tau^-(x_g) = \tau_F(x_g) - \tau_R(x_g) - \tau_{rc}. \quad (5)$$

We have that

$$\tau^+(x_g) = 2\delta_{xg}. \quad (6)$$

Finally, substituting equation (1) into equation (7), we see that by scaling the plus term we may generate a continuous estimate of the normal depth to the interface as a function of x_g:

$$z(x_g) = \left(\frac{c_0}{2\cos\theta_c}\right)\tau^+(x_g). \quad (7)$$

Interferometric formulation

Let the refraction data from the forward and reverse shots be $D^F(x_g, t|x_F)$ and $D^R(x_g, t|x_R)$ respectively. Further, consider

$$\hat{D}^R(x_g, t|x_R) = \int dt' D^R(x_g, t'|x_R)\delta(t' + \tau_{rc}) \quad (8)$$

to be the reverse shot, delayed by τ_{rc}. Our interest is in the sum of first arrival travel times at common x_g. Notice that if we form

$$\begin{aligned} D^+(x_g, t|x_F, x_R) &= \int d\tau D^F(x_g, \tau|x_F)\hat{D}^R(x_g, t-\tau|x_R) \\ &= D^F(x_g, t|x_F) \otimes \hat{D}^R(x_g, -t|x_R), \end{aligned} \quad (9)$$

i.e., correlate D^F with the time reverse of $\hat{D}^R$, the first arrivals in the result will have times that correspond to the desired output. So, D^+ is a direct calculation of Hagedoorn's plus term. We will refer to D^+ as the "plus-field". This is a sequence of 1D interferometric quantities, in the spirit of the original form due to Claerbout in 1968.

Synthetic model

We will test the idea with a synthetic data set taken over the refractor in Fig. 2.

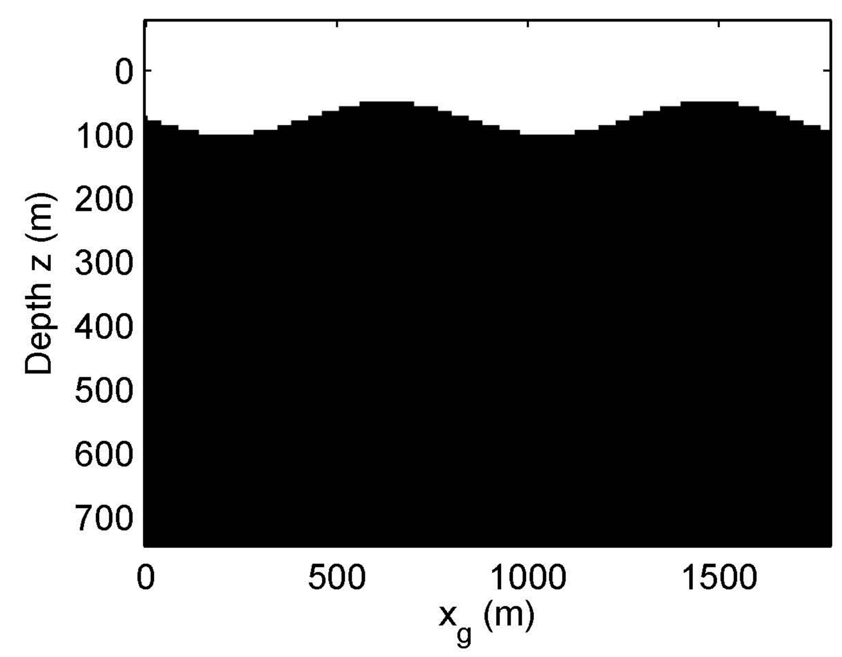

Figure: 2. Refractor model.

Synthetic data

Data measured over the model in Fig. 2. are shown in Fig. 3.

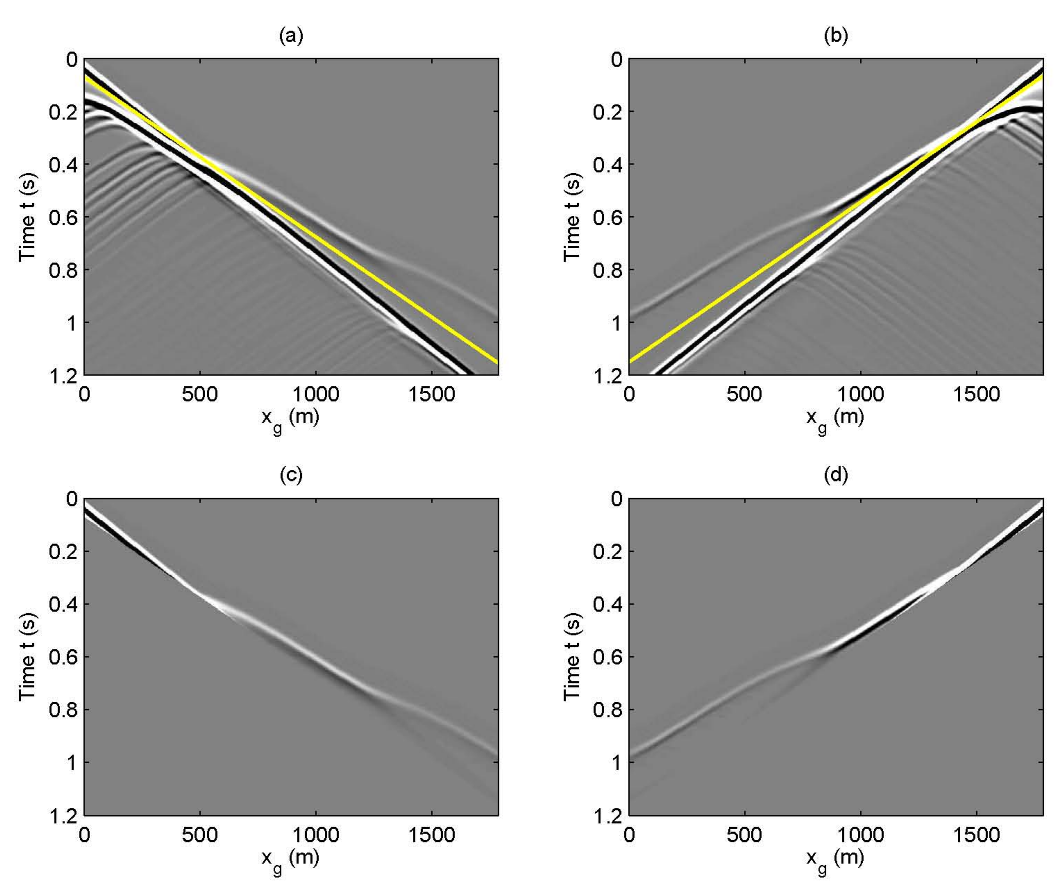

Figure: 3. Refraction data: (a-b) full forward and reverse shots; (c-d) muted.

The forward and reverse shots with all events, including the direct, reflected and refracted arrivals are shown in Fig. 3a-b. A linear mute is illustrated in yellow. The forward and reverse shots after muting are shown in Fig.3c-d.

Plus and minus fields

We next take the forward and reverse shots illustrated in Fig. 3c-d and use them as input to the interferometric calculations embodied by e.g., equation (9)—please refer to the full report for details. In Figure 4a the plus field is illustrated; in Figure 4b the minus field is illustrtated.

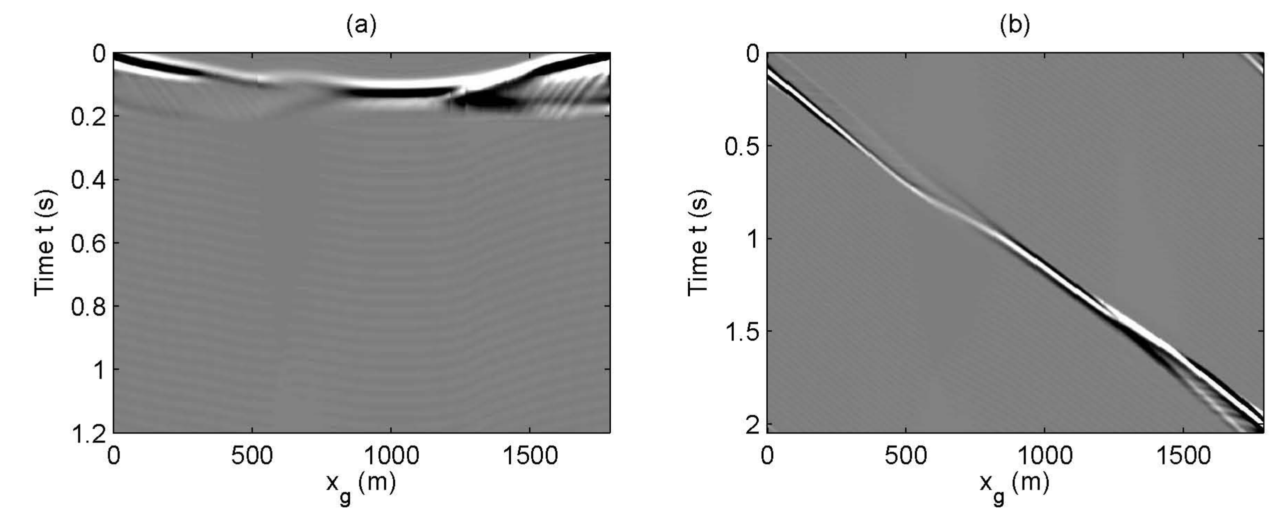

Figure: 4. The plus field (a) and the minus field (b).

Refraction images from the plus field

We use the processed data in Figure 4a to form a refraction image, by scaling the time axis to depth as per equation (7). The result is shown in Figure5a, with the actual model repeated for comparison next to it Figure 5b. The plus-minus window is judged to lie roughly between $x_g = 500$m and $x_g = 1500$m.

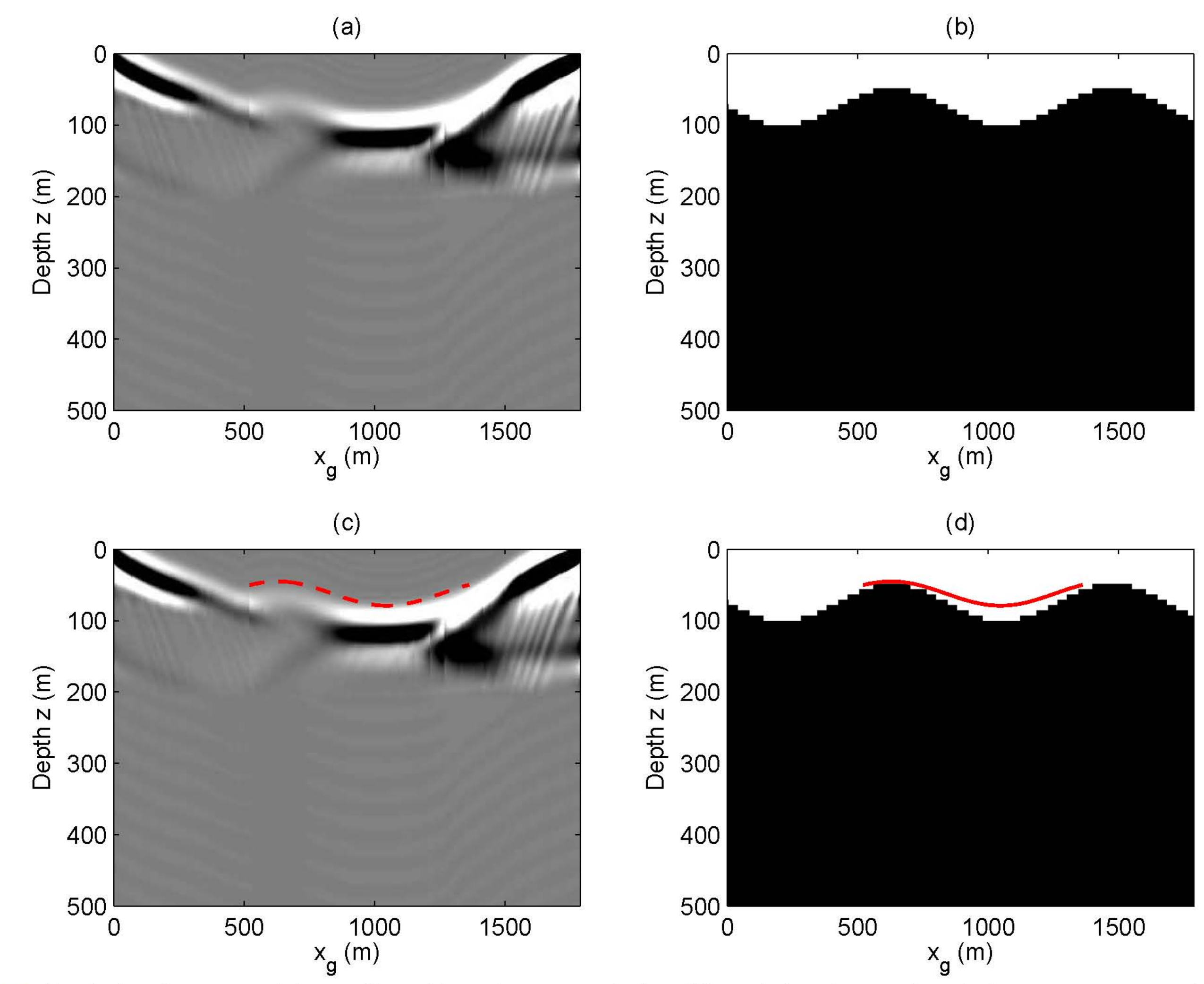

Figure: 5. Interferometric refraction image (a) with picks in red; picks compared to actual model (b).

Conclusions

An interferometric calculation of the "plus field" forms an image when the depth is scaled properly. One interpretation is that in calculating the plus field, we construct a notional fixed-offset reflection data set, in which a wave propagates a distance of $2z(x_g)$ with the vertical slowness $\cos\theta_c/c_0$. And, in scaling the time axis to a depth axis, we perform a rudimentary migration of that data set through this medium.

Bibliography

Please see the corresponding CREWES report for a full bibliography.