Wave propagation and interacting particles continued: plane waves at oblique incidence

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Introduction

We continue the development of a model of seismic data based on the idea of particles, or groups of particles, undergoing collision and disintegration interactions—rather than seismic wave events reflecting and transmitting. Here we admit plane waves which propagate obliquely with respect to the spatial axis along which they are observed (which we have thus far fixed to be the depth axis, e.g., the well in a VSP experiment). We consider both harmonic and transient waves. Acoustic and multi parameter problems demand the inclusion of additional particles in order that boundary conditions are honoured.

Plane waves in multi dimensions

The key conceptual hurdle when extending to multiple spatial dimensions lies in the problem of switching the roles of space and time. When there was only one space axis, doing so did not involve any real "decisions". Now, if in addition to the z axis there is an x, or even an x and a y axis also, which one do we switch with t? We will deal with this by continuing to examine the wave along a single, preferred space axis, in spite of the fact that it will now vary along several. This observation axis, which will usually be the z axis in this paper, will be the one that is switched with the time axis.

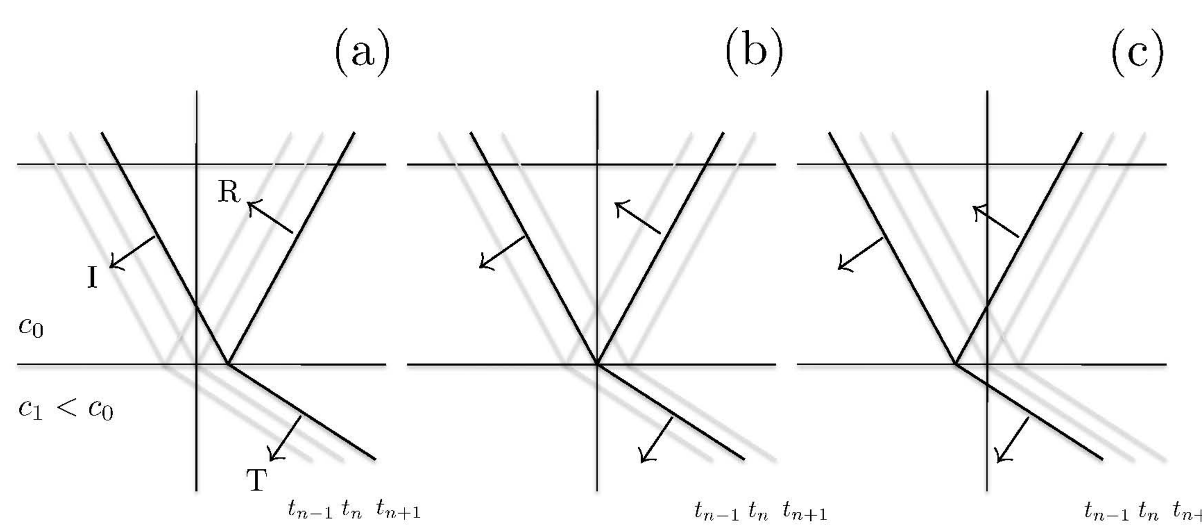

Figure: 1. Choosing a single "observational" space axis on which to focus attention.

The particle model

The 2D-3D picture still involves, implicitly, two coordinate axes: an axis along which waves are observed propagating, and a time axis, regardless of the number of actual spatial dimensions. Thereafter, interchanging the depth and time dimensions, the properties of waves in these environments are correctly predicted using the particle model.

Events

Let a wave of amplitude 1 move down and to the left towards a boundary. At the boundary it reflects with amplitude R and transmits with amplitude T. We choose $c_1 < c_0$, so the transmitted wave's path steepens in comparison to the incident wave. How does the wave appear to an observer with access to measurements along the vertical well? For z values above the single reflector the movie would look like

what is illustrated in Figure 2. Beginning with Figure 2a, i.e., $z = 0$, we see two events, the incident wave at an early time (or zero time) and the reflected wave at a relatively late time. By depth z_1 (Figure 2b) the two events have drifted towards each other, and even further towards each other by depth z_2 (Figure 2c). For z values below the reflector, i.e.,

$z > z_I$, the wave consists of the single transmitted event, moving to greater times as depth increases (Figure 3a–c).

Events above an interface

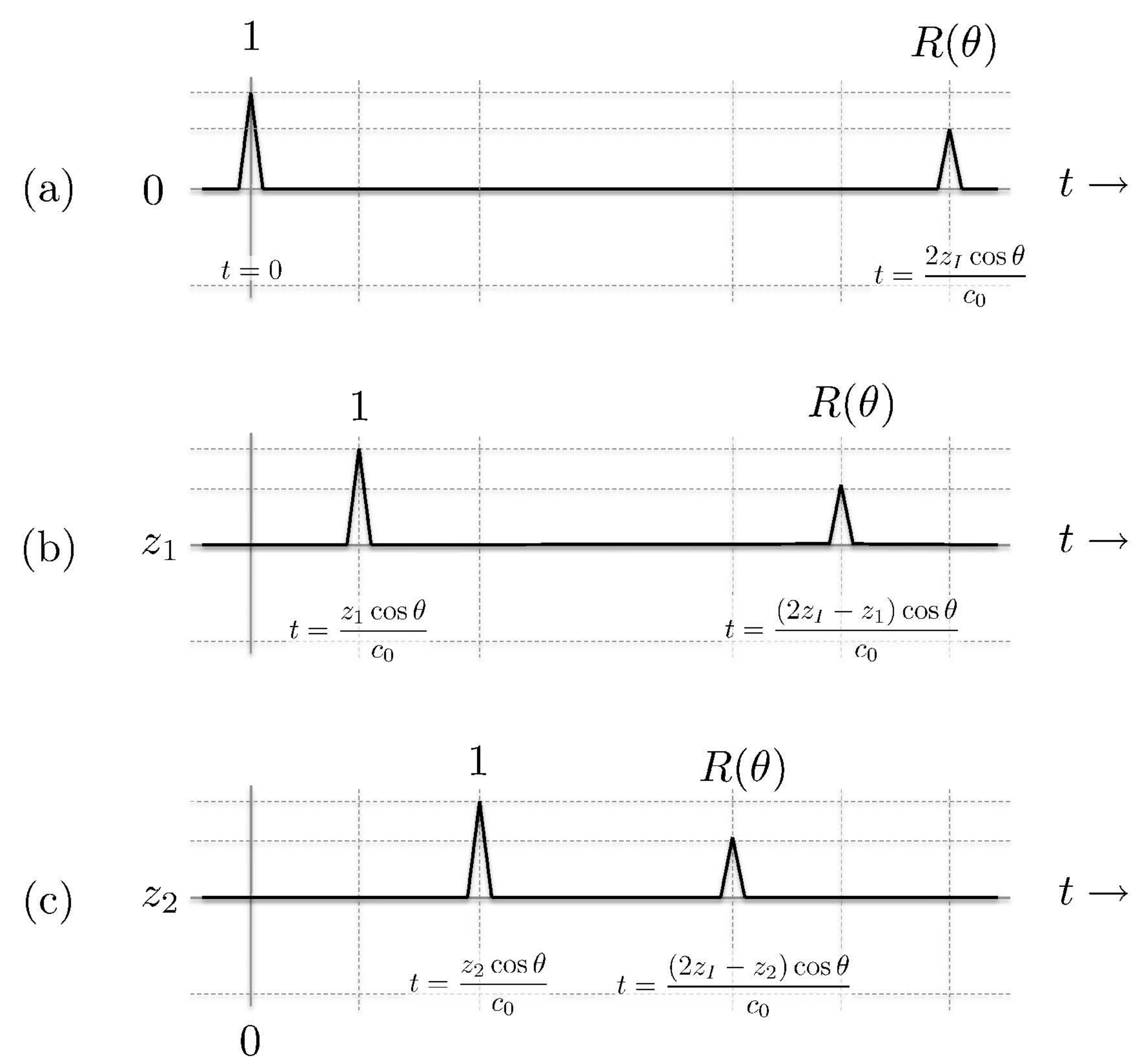

Figure: 2. As depth increases, the particles approach each other.

Events below the interface

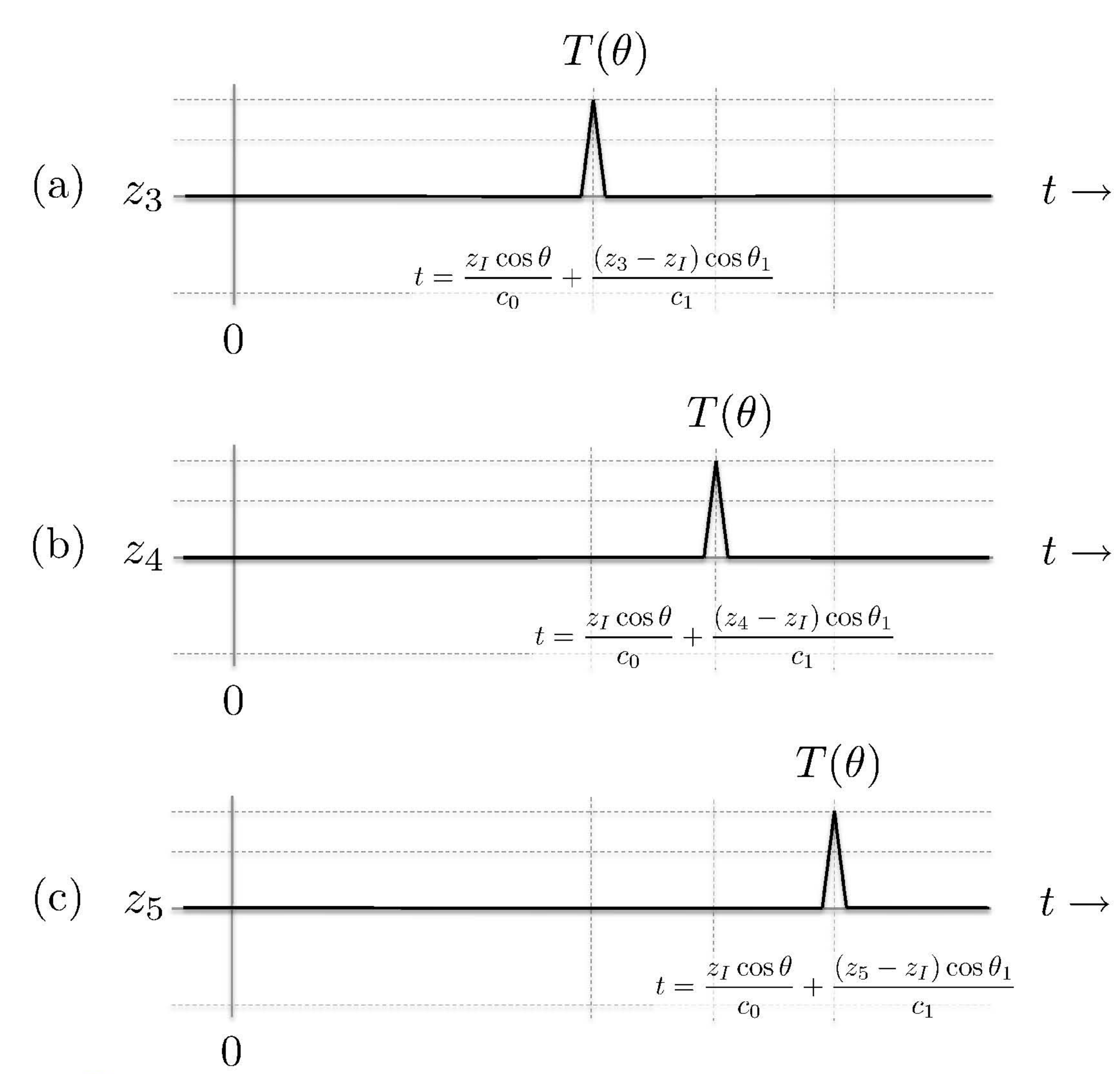

Figure: 3. They collide anelastically and continue propagating.

Definitions

We set the notional masses of these two particles to be equal to their plane wave amplitudes, thus:

$$m_I = 1, \quad m_R = R(\theta), \quad m_T = T(\theta). \tag{1}$$

We will incorporate these masses in conservation of mass and momentum calculations.

Results

The particles 1 and $R(\theta)$ meet at $z = z_I$, and thereafter are replaced by the single transmitted event. Before the collision, the total mass and momenta of the particles are

$$m_I + m_R = 1 + R(\theta), \quad m_I \times v_I + m_R \times v_R = 1 \times \frac{\cos\theta}{c_0} + R(\theta) \times \left(-\frac{\cos\theta}{c_0}\right). \tag{2}$$

After the collision, the same totals are

$$m_T = T(\theta), \quad m_T \times v_T = T(\theta) \times \frac{\cos\theta_1}{c_1}. \tag{3}$$

Hence, if it was true that our particle/collision model correctly captured the effects of boundary conditions on plane waves, then it would have to be true that

$$1 + R(\theta) = T(\theta), \tag{4}$$

and

$$[1 - R(\theta)]\frac{\cos\theta}{c_0} = T(\theta)\frac{\cos\theta_1}{c_1}, \tag{5}$$

or

$$\frac{c_0 \cos\theta_1}{c_1 \cos\theta} = \frac{1 - R(\theta)}{1 + R(\theta)}. \tag{6}$$

Since these are well-known results of wave theory, in this sense the model is validated for plane waves.

Multiparameter problems

When we move to a multi-parameter problem the boundary conditions are are of the form

$$P|_\Omega^+ = P|_\Omega^-, \quad \rho_0^{-1}P'|_\Omega^+ = \rho_1^{-1}P'|_\Omega^-. \tag{7}$$

We may extend the model by introducing a *new particle* to carry off additional momentum (Figure 4).

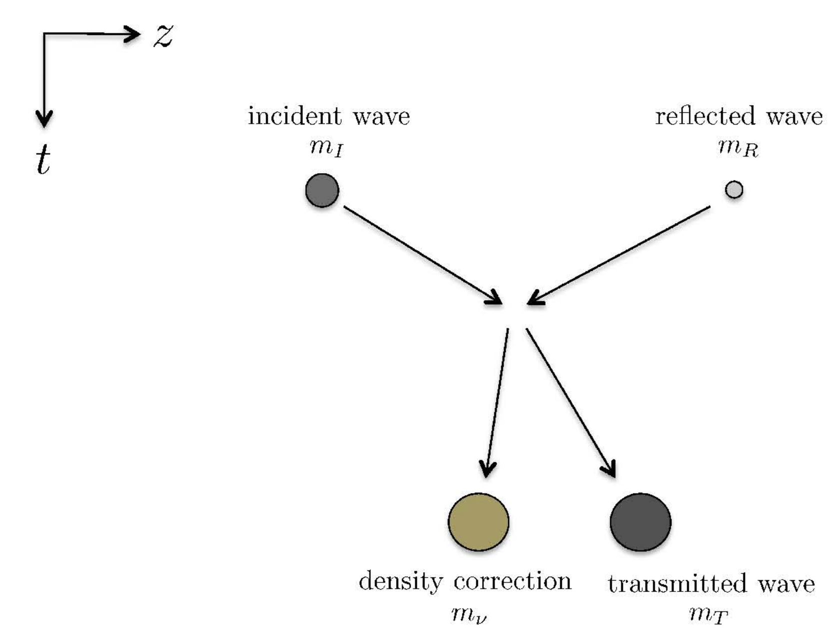

Figure: 4. Multiparameter problems must involve additional particles.

Conclusions

Transient or harmonic scalar plane waves, propagating at arbitrary angles with respect to a chosen observation axis (e.g., the depth axis in the case of VSP data), can be treated within the particle model. This is a significant increase in the scope of the model, admitting any multidimensional wave that can be analyzed into plane harmonic waves.

Bibliography

Please see the corresponding CREWES report for a full bibliography.