Modelling migration, and inversion using Linear Algebra

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Abstract

Modelling migration and inversion can all be accomplished using Linear Algebra. Key to these processes is the diffraction array that is multidimensional. Two dimensional poststack migrations require a fourth order diffraction array. Current processing practices for Least-Squares analysis require a diffraction array that is two dimensional (a matrix), with one dimensional vectors for the reflectivity and seismic data. These matrices and vectors can be derived from multidimensional data by helical unwrapping. The field of Multilinear Algebra may allow the data to retain their multidimensional arrays, but require defining processes such as a two dimensional transpose of a multi dimensional array. Modelling, migration, and inversion are demonstrated using very small dimensions using MATLAB software.

Diffractions

a. $$T^2 = T_0^2 + \frac{4x^2}{v^2}$$

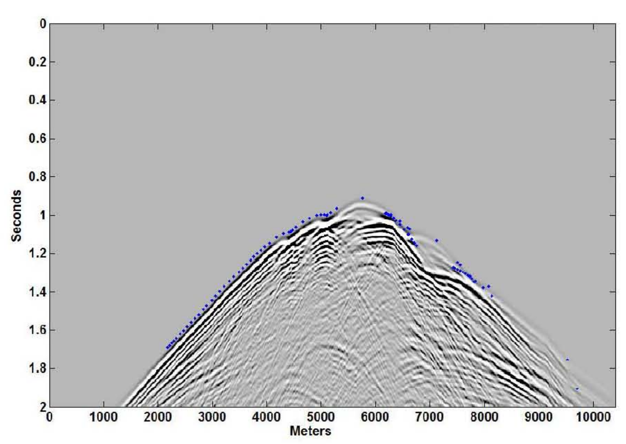
b.

a) [3 2 2 2 3] time or sample number

b) [0.2 0.5 1.0 0.5 0.2] spatial amplitude of the diffraction

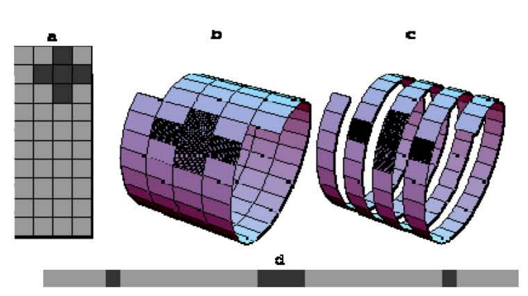
c.

$$\begin{bmatrix} . & . & . & . & . \\ . & 0.5 & 1.0 & 0.5 & . \\ 0.2 & . & . & . & 0.2 \\ . & . & . & . & . \end{bmatrix}$$



The matrix form of a diffraction contains much more information than the alternate forms. A Kirchhoff migration (matched filter) can then recover all the scattered energy.

Helical unwrapping (Claerbout)



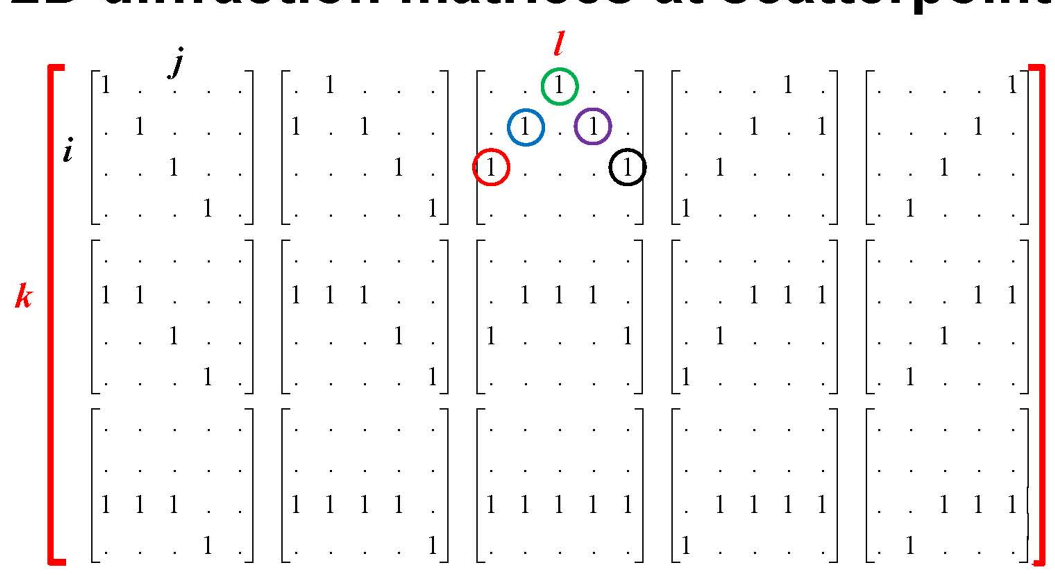
2D Reflectivity

$$\mathbf{r} = \begin{matrix} 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

1D Reflectivity

$\mathbf{r}_v$ = [0 0 0 2 0 0 0 0 0 0 0 0 0 4 0]^T

2D diffraction matrices at scatterpoints



Diffraction matrices at scatterpoints

$\mathbf{D}\mathbf{v}^T$ = D_{11} D_{21} D_{31} D_{21} D_{22} D_{23} D_{31} D_{32} D_{33} D_{41} D_{42} D_{43} D_{51} D_{52} D_{53}

2D Diffraction matrix

$\mathbf{D}_{2D}$ =

1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	1	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	1	0	1	1	0	0	0	0	0	0	0
0	0	1	0	0	1	0	0	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	1	0	1	1	0	0	0	0
1	1	1	0	0	1	0	0	1	0	0	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	1	1	0	0	1	0	1	1	0
0	0	0	1	1	1	0	0	1	0	0	1	0	0	1
1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	1	1	0	0	1	0
0	0	0	0	0	0	1	1	1	0	0	1	0	0	1
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0

Modelled seismic in 2D matrix

$$\mathbf{s}_v = \mathbf{D}_{2D}\mathbf{r}_v$$

$\mathbf{s}$ =

0	2	0	0	0
2	0	2	4	4
0	0	4	2	0
0	4	0	0	2

Migrated data in 2D matrix

$$\mathbf{m} = \mathbf{D}_{2D}^T\mathbf{s}_v$$

$\mathbf{m}$ =

4	10	4	6	12
6	8	6	10	16
4	8	6	6	10

Least squares inverted data

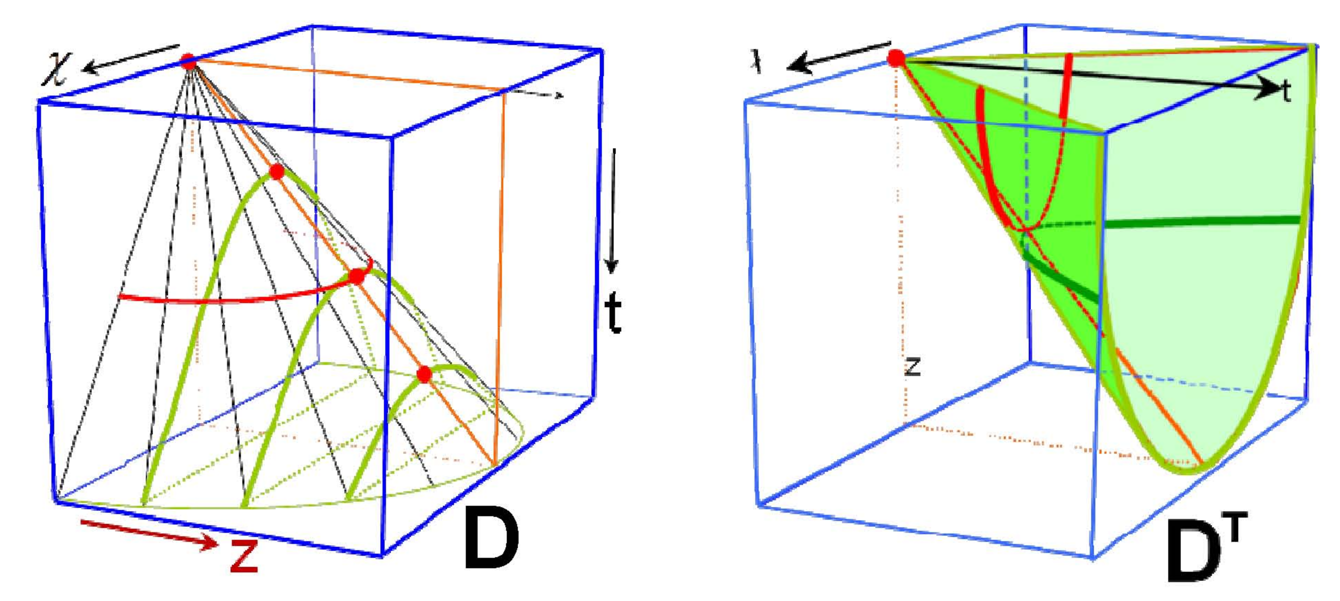
$$\mathbf{r}_v = \left(\mathbf{D}_{2D}^T\mathbf{D}_{2D}\right)^{-1}\mathbf{D}_{2D}^T\mathbf{s}_v$$

Inv =

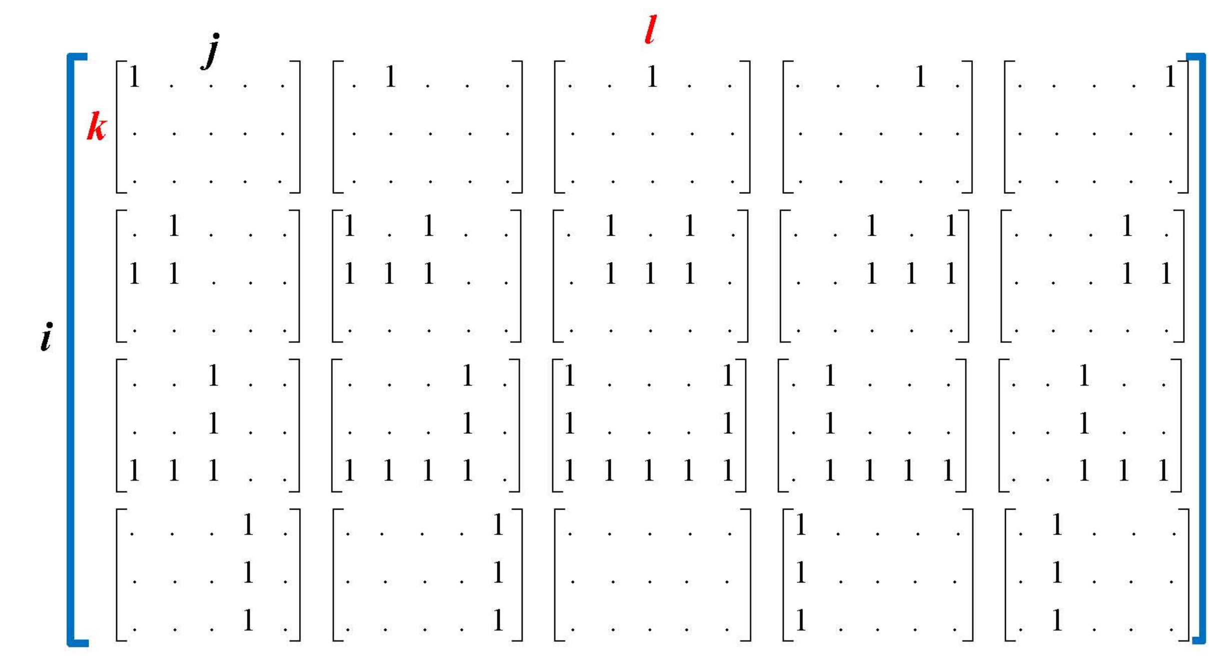
-2.2204e-016	2.0000e+000	2.6645e-015	-6.6613e-016	-6.6613e-016
-8.4377e-015	9.4369e-015	-4.4409e-015	-1.7764e-015	4.0000e+000
1.0936e-014	-8.4377e-015	-1.5543e-015	8.8818e-016	2.8866e-015

Multilinear Algebra

Diffractions in 3D volume



2D transpose of the 4D diffraction array



Comments and Conclusions

1. Linear algebra can be used to process seismic data.
2. Requires unwrapping.
3. Multilinear Algebra <==> data seismic?
4. Defining the format of the data may be difficult.
5. Data storage an issue.
6. Not if already using Kirchhoff wavefield migration.
7. Are algorithms available?
8. $\mathbf{r}$ =[$I\ J$], $\mathbf{d}$=[$K\ L$], $\mathbf{D}$ = [$I\ J\ K\ L$]:
9. $I = 3, J = 5, K = 4, K = 5$, $\mathbf{D}$ = [300]
10. Small 2D, $I = 1000$, $J = 200$, $K = 1000$, $L = 1500$, $\mathbf{D}$ = [$3*10^{11}$]
11. Include bytes, 3D seismic, and prestack: $\mathbf{D}$ = [10^{17}]

(Tera byte = 10^{12} bytes, Petabyte = 10^{15} bytes, Exabyte is 10^{18} bytes.)

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