A collision theory of seismic waves applied to elastic VSP data

Kris Innanen* and Kevin Hall*
*University of Calgary, CREWES; k.innanen@ucalgary.ca

Introduction

We have been assembling a theory for seismic waves which is based on particles with well - defined momenta and masses colliding inelastically. Here we make early arguments towards applying this picture to elastic VSP data, including such processes as P-S conversions. We choose a candidate reflection in the Husky Cold Lake 3C walkaway VSP data set, as discussed by Hall et al. in the 2012 CREWES Report, which has clear evidence of P-P interactions and P-S conversions, and report our initial attempts to validate the collision model.

Inelastic hits, elastic waves

In the scalar case we decided that of the two equations deriving from the scalar boundary conditions, one,

$$T = 1 + R,$$

was in the collision framework a statement of conservation of mass. "After" the collision, the aggregate particle has mass T, the sum of the masses 1 and R "before" the collision. The other equation

$$T = \left(\frac{c_1}{c_0}\right) 1 - \left(\frac{c_1}{c_0}\right) R,$$

was a statement of conservation of momentum.

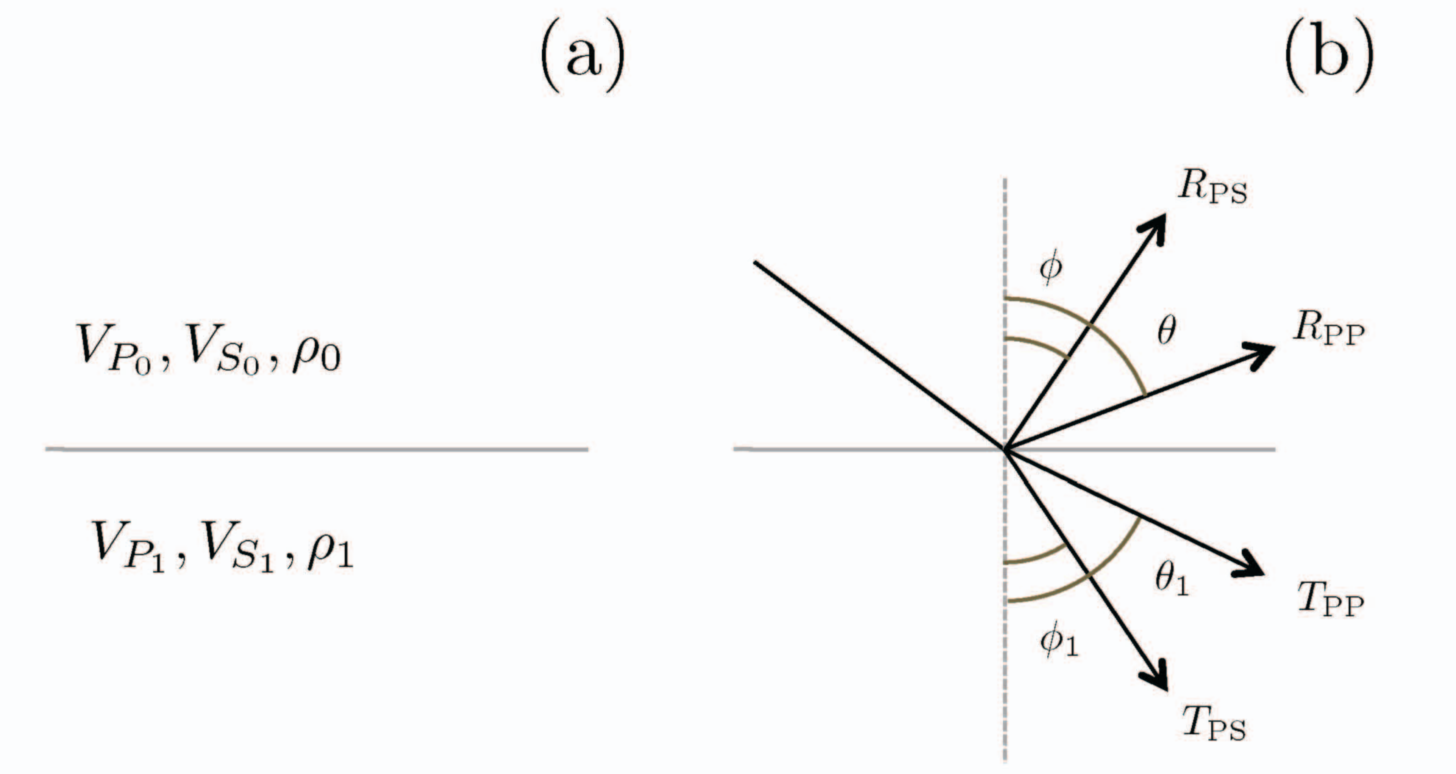


Figure: 1. The elastic particle model for an incident P-wave. (a) A horizontal planar boundary. (b) An incident P wave excites reflected and transmitted P and S waves.

Inelastic hits, elastic waves

Generalizing to elastic waves for an incident P-wave (Figure 1), we have two mass relations:

$$T_{PP} = \frac{\cos(\theta - \phi_1)}{\cos(\theta_1 - \phi_1)} - \frac{\cos(\theta + \phi_1)}{\cos(\theta_1 - \phi_1)} R_{PP} + \frac{\cos(\phi + \phi_1)}{\cos(\theta_1 - \phi_1)} R_{PS},$$

and

$$T_{PS} = \frac{\sin(\theta - \theta_1)}{\cos(\theta_1 - \phi_1)} + \frac{\sin(\theta + \theta_1)}{\cos(\theta_1 - \phi_1)} R_{PP} + \frac{\cos(\phi + \theta_1)}{\cos(\theta_1 - \phi_1)} R_{PS},$$

and two momentum relations:

$$\begin{bmatrix} T_{PP} \\ T_{PS} \end{bmatrix} = -\left(\frac{V_{S_0}}{V_{S_1}}\right)\left(\frac{\rho_0}{\rho_1}\right) \mathrm{M}^{-1}\mathrm{N},$$

where

$$\mathrm{M} = \begin{bmatrix} \sin\phi_1 \cos\theta_1 & (1/2 - \sin^2\phi_1) \\ \gamma_1(1/2 - \sin^2\phi_1) & -\sin\phi_1\cos\phi_1 \end{bmatrix}$$

and

$$\mathrm{N} = \begin{bmatrix} \sin\phi\cos\theta(1 - R_{PP}) & -(1/2 - \sin^2\phi)R_{PS} \\ \gamma_0(1/2 - \sin^2\phi)(1 + R_{PP}) & -\sin\phi\cos\phi R_{PS} \end{bmatrix}.$$

These relations must describe the collision pictured in the snapshots in Figures 2a-d (black horizontal line). Three events approach each other from t_1 to t_2, collide at t_3, and depart as two events at and beyond t_4.

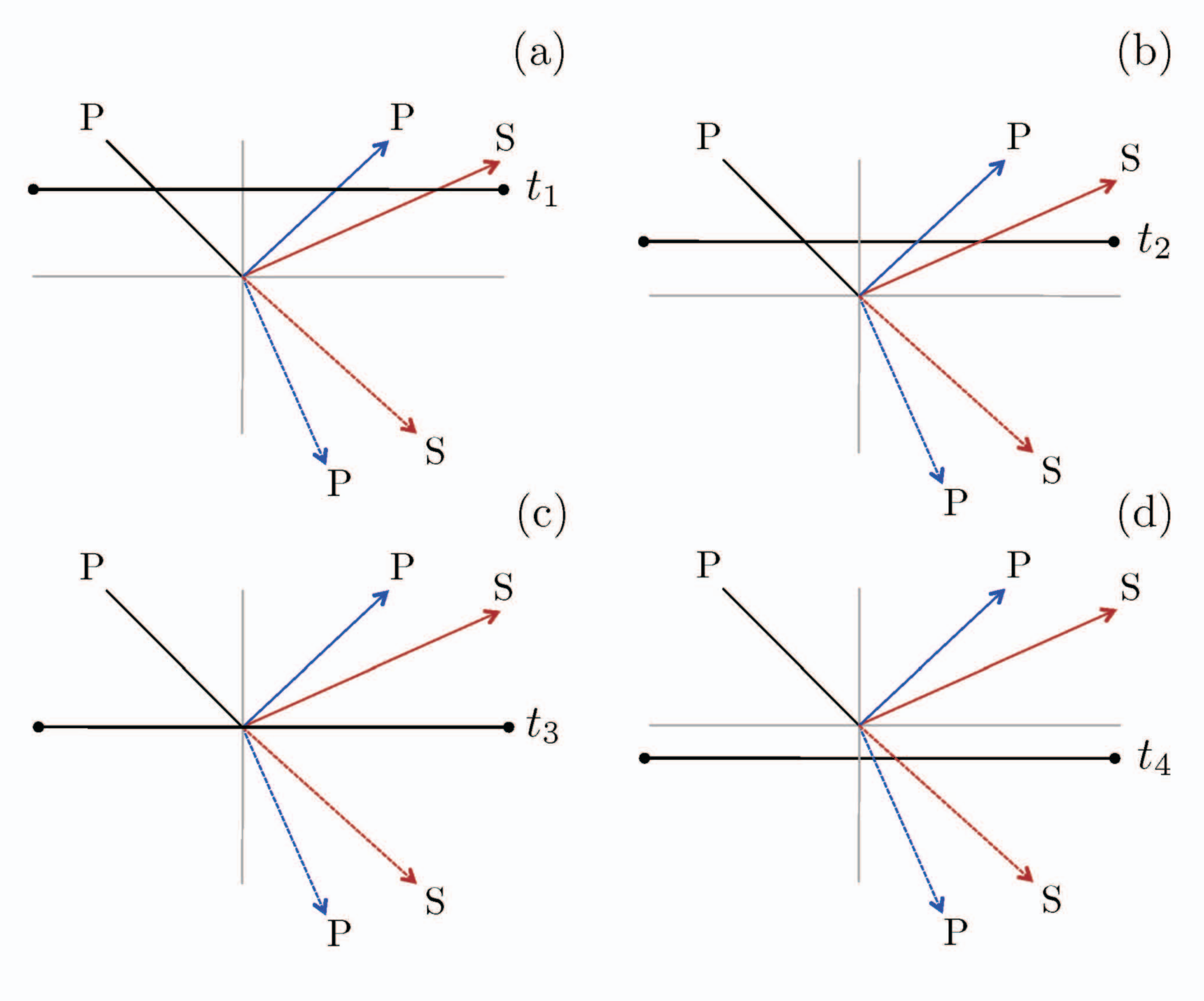
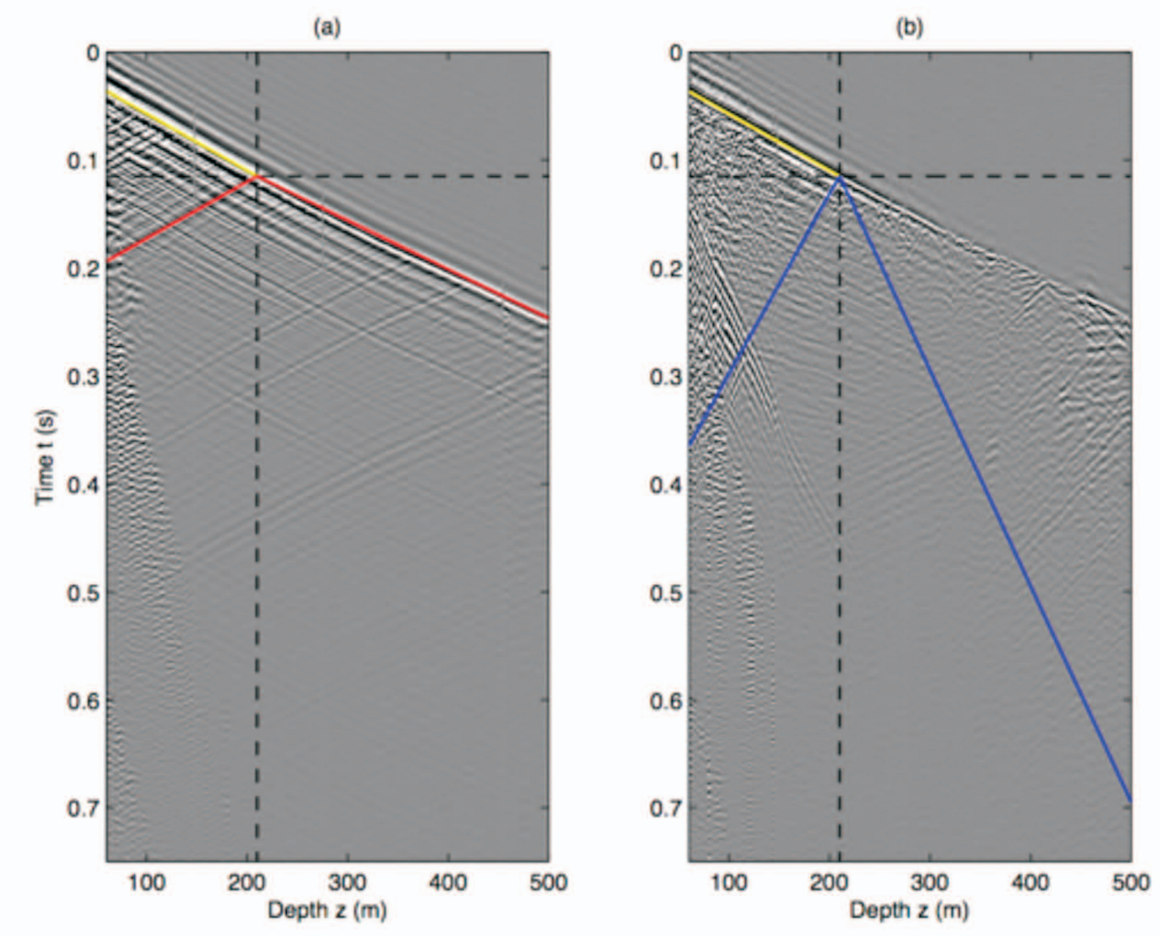


Figure: 2. (a)-(d) Four "moments" during the seismic particle collision process.

Husky 2011 3C VSP example

The Husky 2011 3C VSP data set (Hall et al., 2012) vertical component (Figure 3a) and radial component (3b) are canvassed for a suitable PP (yellow + red) and PS (yellow + blue) interaction.



"Movie frames" from just prior to the interaction (vertical component) are plotted in Figure 4a-d. PP reflection is indicated with vertical dashed line. We are currently matching amplitudes and drift rates against mass and momenta equations.

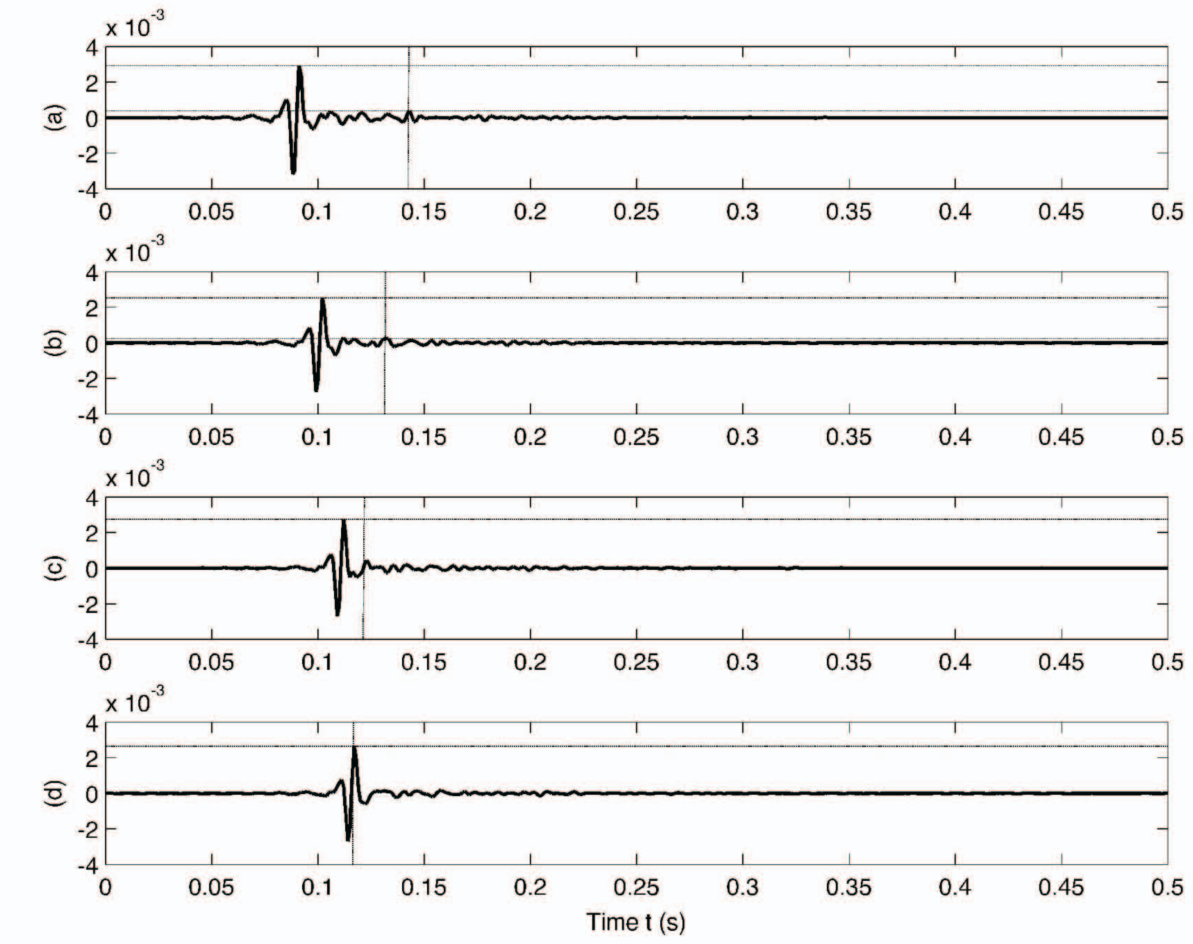


Figure: 4. (a)–(d) Frames of the collision process extracted from the vertical component VSP data set, for depth "moments" leading up to the point of collision. The reflected event is indicated with dashed lines.

References

- Please see the report for a full reference list.