How AVO information can be practically incorporated in full waveform inversion

Kris Innanen*
*University of Calgary, CREWES; k.innanen@ucalgary.ca

Introduction

We analyze the quasi-Newton updates of the previous poster, in order to understand their **computational cost**, and their ability to **diagnose ill-posedness** and **suppress cross-talk**. This analysis shows the continuous - discrete formulation to naturally incorporate seismic AVO information in the FWI update. We use exact expressions for all the ingredients of the 1st iteration of a FWI step, including the data, to compare a given quasi-Newton update to an analytic “perfect” update.

Analytic ingredients in AVO terms

To calculate the elements g_κ, g_ρ, $H_{\rho\kappa}$, etc. from the formulas of the previous poster, we require the Green’s functions for the background medium, and an expression for the residuals δP. If the data consist of a reflection from a single interface at depth z_1 (Figure 1), these are

$$\delta P^*(k_g, \omega | s_\kappa^{(0)}, s_\rho^{(0)}) = -R(\theta)\frac{e^{-i2q_g z_1}}{i2q_g},$$

for wavenumbers q_g (Figure 2). In AVO-familiar terms, we can write

$$R(\theta) \approx -\frac{1}{4}\frac{1}{\cos^2\theta}\left(\frac{\delta s_\kappa}{s_{\kappa 0}}\right) - \frac{1}{4}\cos^2\theta\left(\frac{\delta s_\rho}{s_{\rho 0}}\right),$$

to first order in the ideal updates δs_κ and δs_ρ. These forms imply we consider the problem in the (k_g, ω) domain, and models constrained to vary with depth.

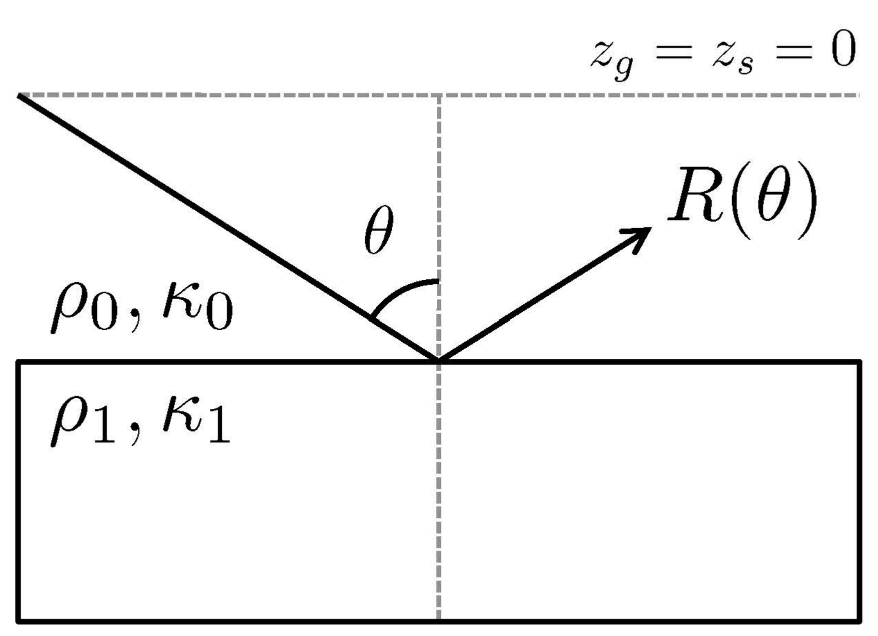

Figure: 1. Geometry of a reflection from a horizontal planar acoustic interface.

Analytic ingredients in AVO terms

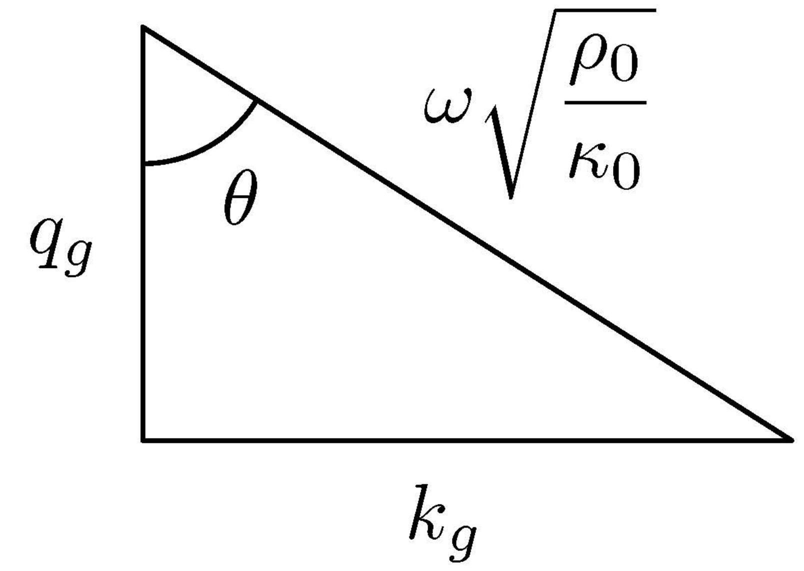

Figure: 2. Geometry of harmonic plane wavenumbers.

Results

The ingredients are assembled in the calculation of gradients and Hessian elements. Integrals over ω are analytically evaluated, and sums over k_g are transformed into sums over angle θ.

Gradient-based step

Taking the ρ update as an example, given two angles of data, the gradient-based step is within a multiplicative factor of

$$g_\rho^{(0)}(z) \approx \frac{c_0}{32}\left[\cos^3\theta_1 + \cos^3\theta_2\right]\left(\frac{\delta s_\rho}{s_{\rho 0}}\right) S(z - z_1) + \frac{c_0}{32}\left[\frac{1}{\cos\theta_1} + \frac{1}{\cos\theta_2}\right]\left(\frac{\delta s_\kappa}{s_{\kappa 0}}\right) S(z - z_1), \quad (1)$$

where the $S(z - z_1)$ are steps at z_1.

— Criticisms —

A perfect step would be equal or proportional to $\delta s_\rho S(z - z_1)$. The gradient as a proxy for this update evidently comes up short, as it

- **depends on which angles are used**;
- **is blind to ill-posedness**;
- **is strongly impacted by cross-talk**.

The presence of θ_1 and θ_2 clinches the first point. Regarding the second point, recalling that two parameters are not constrained by one angle, and seeing that the gradient can be made with θ_1 as easily as (θ_1, θ_2), we find no math check that such a gradient is bogus. Finally, the gradient is a linear combination of the ideal updates for **both** parameters, δs_κ and δs_ρ, which means it is unprotected from parameter cross-talk.

Results continued

Parameter-type step

The parameter-type update (see the previous poster), with no extra computation, is based on the gradients too, but through the partial Hessian it mixes and weights them slightly differently for each parameter (Figure 3a). The κ parameter - type quasi - Newton step is, for instance,

$$-\frac{1}{2c_0^2}\left(\frac{\delta s_\kappa}{s_{\kappa 0}}\right) S(z - z_1). \quad (2)$$

The off-diagonal elements of the 2×2 matrix exactly subtracts from the update the cross-talk term (as seen in equation 2). The determinant is zero if insufficient data angles are provided. The diagonal weights suppress all remaining angle-dependent terms.

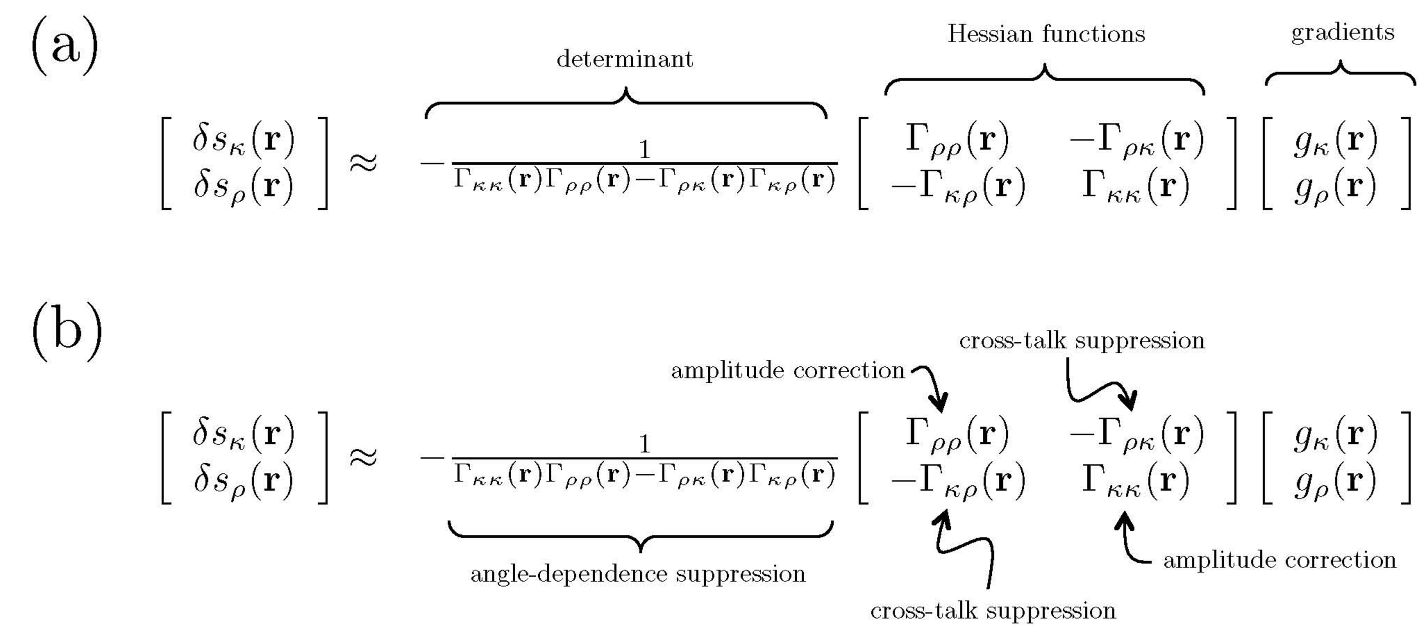

Figure: 3. The parameter-type update. (a) Its components. (b) Its tasks.

Having observed these items we can generate a “map” of the parameter-type quasi-Newton step, as depicted in Figure 3b.

AVO and FWI

A FWI Newton update involves the inversion of a large $NM \times NM$ block matrix, for N pixels and M parameters. Our formulation permits a quasi - Newton update to retain diagonal elements of the N pixel part, but full on- and off-diagonal parts for the M parameters, suppressing cross-talk, and analysis of FWI in the language of AVO.

References

- Please see the report for a full reference list.