# An analysis of time-lapse phase shifts using perturbation theory

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## Introduction

Perturbation theory can be used as a powerful theoretical approach to model and invert seismic data, including 4D time-lapse data (Innanen 2008, Zhang 2006, Weglein et al. 2003). Time-lapse difference data between the baseline and monitor surveys indicate the change in the amplitude and travel time of the seismic trace. In perturbation theory, these difference data are considered as the scattered wavefield data, where the baseline survey and the monitoring survey are the reference medium and the perturbed medium respectively. Innanen et al. (2013) have pointed out that this analysis requires a representation of difference data as an expansion in terms of both the time-lapse perturbation and the baseline medium properties.

Thore and Hubans (2012) and Williamson (2007) make use of a model based formula which takes into account time-shifts of events of interest which occur because of time-lapse changes in the P-wave velocity of the overburden. This formula is a highly nonlinear, since the shift of a $\Delta V_P/V_P$ interface is given by the sum of $\Delta V_P/V_P$ itself from the surface to the interface. We suspect that our perturbation theory for linear and/or nonlinear time-lapse seismic inversion, is a natural framework for deriving such nonlinear shift formulas.

In this paper we begin the process of analyzing phase issues in nonlinear time-lapse perturbation theory. We will not in this paper arrive at processing or inversion formulas, but merely continue to study the basics of the modelling equations. Our aim is to report next year on the ability of the framework to provide practical algorithms.

## Theory

We will consider two seismic experiments involved in a time-lapse survey, the baseline survey, followed by a monitoring survey. The acoustic medium is one-dimensional, varying in depth only, with a normal incident plane source. We begin with a one dimensional constant density acoustic wave equation for the baseline wavefield which is the reference wavefield:

$$L_0 G_0 = \delta(z - z_s), \qquad LG = \delta(z - z_s). \tag{1}$$

where $L_0$, and L represent the differential operator describing wave propagation in the reference and perturbed mediums, and $G_0$ and G for the corresponding Green's functions. The source is a pulse which is presented by a delta function at $z = z_s$. Based on the Lippmann-Schwinger equation, $G_0$ and $G$ are related as:

$$G - G_0 = G_0(L_0 - L)G. \tag{2}$$

If we define the perturbation as $V = L_0 - L$, and scattered wavefield or time-lapse difference data as $\psi = G - G_0$, iterating the Lippmann-Schwinger equation back into itself generates:

$$\psi = \sum_{n=1}^{\infty} G_0 (V G_0)^n. \tag{3}$$

An integral form corresponding to equation (2) is:

$$\psi(z_g, z_s; k) = \int_{-\infty}^{\infty} G_0(z_g, z'; k) k^2 \alpha(z') G(z', z_s; k) dz'. \tag{4}$$

where $k = \frac{\omega}{c_0}$, and $V = k^2\alpha$, and perturbation defined as:

$$\alpha(z) = 1 - \frac{c_0^2}{c^2(z)}. \tag{5}$$

$C_0$ and C are the homogenous reference and perturbed velocities.
Iterating $G(z, z_s; k)$ back into $G_0(z, z_s; k)$ as in equation (3), we can form the scattering series for the difference data:

## Theory continued

$$\begin{aligned}
\psi(z_g, z_s; k) &= \int_{-\infty}^{\infty} G_0(z_g, z'; k) k^2 \alpha(z') G_0(z', z_s; k) dz' \\
&+ \int_{-\infty}^{\infty} G_0(z_g, z'; k) k^2 \alpha(z') dz' \int_{-\infty}^{\infty} G_0(z', z''; k) k^2 \alpha(z'') G_0(z'', z_s; k) dz'' \\
&+ \int_{-\infty}^{\infty} G_0(z_g, z'; k) k^2 \alpha(z') dz' \int_{-\infty}^{\infty} G_0(z', z''; k) k^2 \alpha(z'') \\
&\int_{-\infty}^{\infty} G_0(z'', z'''; k) k^2 \alpha(z''') G_0(z''', z_s; k) dz''' + \dots \\
&= \psi_1 + \psi_2 + \psi_3 + \dots
\end{aligned} \tag{6}$$

## A structural perturbed time-lapse problem

In this study, we define the medium at the time of the baseline survey as a single interface in a constant density acoustic medium whose depth and rock properties change before the time of the monitoring survey (Figure 1).

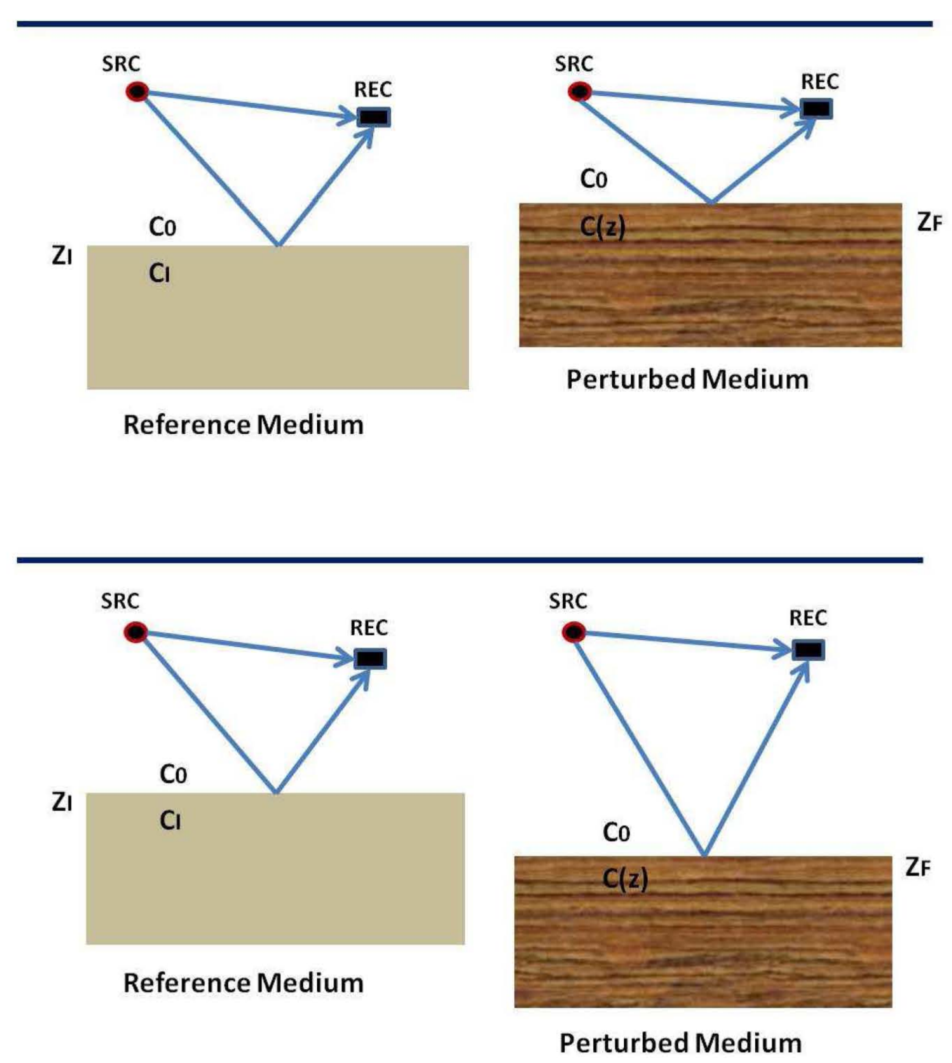


Figure 1: Illustration of a structural perturbed time-lapse problem

The Green's function for the reference medium is:

$$G_0(z_g, z_g, \omega) = \frac{e^{ik|z_g - z_s|}}{i2k} + R_I \frac{e^{ik(z_I - z_g)} e^{ik(z_I - z_s)}}{i2k}. \tag{7}$$

where $k = \frac{\omega}{c_0}$, $R_I = (c_I - c_0)/(c_I + c_0)$. The first term in this equation represents the direct wavefield propagating from the source to the receiver, and the second term is the reflection from the interface at location $z_I$ (Figure 1). We form scattering series in terms of the perturbation and Green's function as in equation (6), and categorize them based on their order in the perturbation, $\alpha(z)$. Considering only zero offset, $z_s = z_g = 0$, the first, second, and third order terms in the perturbation are:

$$\psi_1(0, 0; k) = \int_{-\infty}^{\infty} G_0(0, z'; k) k^2 \alpha(z') G_0(z', 0; k) dz',$$

$$\begin{aligned}
\psi_2(0, 0; k) &= -\frac{1}{4} \int_{-\infty}^{\infty} e^{i2kz'} \left(\frac{1}{2}\right) \left[\alpha^2(z') + \frac{d}{dz}\alpha(z') \int_{-\infty}^{z'} \alpha(z'') dz''\right] dz' \\
&- \frac{R}{2} e^{i2kz_I} \left( (-ik) \int_{-\infty}^{\infty} \alpha(z') dz' \int_{-\infty}^{z'} \alpha(z'') dz'' - \frac{1}{2} \int_{-\infty}^{\infty} dz' \right. \\
&\left. \left[\alpha^2(z') + \frac{d}{dz}\alpha(z') e^{i2kz'} \int_{-\infty}^{z'} \alpha(z'') e^{-i2kz''} dz''\right] \right).
\end{aligned}$$

## A structural perturbed time-lapse problem continued

$$\begin{aligned}
\psi_3(0, 0; k) &= -\left(\frac{1}{32}\right) \int_{-\infty}^{\infty} e^{i2kz'} \frac{d^2}{dz^2} \left[\alpha(z') \left(\int_{-\infty}^{z'} \alpha(z'') dz''\right)^2\right] dz' \\
&- \frac{R}{2} e^{i2kz_I} (ik)^2 \left(\frac{1}{2}\right) \left[ \left(\int_{-\infty}^{\infty} \alpha(z') dz'\right)^2 \int_{-\infty}^{z'} \alpha(z'') dz'' \right. \\
&- \frac{1}{2ik} \int_{-\infty}^{\infty} \alpha(z') dz' \int_{-\infty}^{\infty} \alpha(z'')^2 dz'' \\
&+ \frac{1}{(2ik)^2} \int_{-\infty}^{\infty} \alpha(z') dz' \int_{-\infty}^{\infty} \alpha(z'') \frac{d}{dz} \alpha(z'') dz'' \\
&\left. + \frac{1}{(2ik)^2} \int_{-\infty}^{\infty} \alpha(z') dz' \int_{-\infty}^{\infty} e^{-i2kz''} \alpha(z'') dz'' \int_{z''}^{\infty} e^{i2kz''} \frac{d^2}{dz^2} \alpha(z'') dz'' \right] \\
&- \frac{R}{2} e^{i2kz_I} \frac{(ik)^2}{4} \left[ -\frac{1}{2ik} \int_{-\infty}^{\infty} \alpha(z') dz' \int_{-\infty}^{z'} \alpha^2(z'') dz'' \right. \\
&+ \frac{1}{(2ik)^2} \int_{-\infty}^{\infty} \alpha(z') dz' \int_{-\infty}^{z'} \alpha(z'') \frac{d}{dz} \alpha(z'') dz'' \\
&\left. + \frac{1}{(2ik)^2} \int_{-\infty}^{\infty} \alpha(z') dz' \int_{-\infty}^{z'} e^{-i2kz''} \alpha(z'') dz'' \int_{z''}^{\infty} e^{i2kz''} \frac{d^2}{dz^2} \alpha(z'') dz'' \right]
\end{aligned} \tag{8}$$

The contribution of the first, second, and third order terms to the time-lapse difference data is illustrated in Figure 2 for the upward migration of the interface between cap rock and the reservoir.

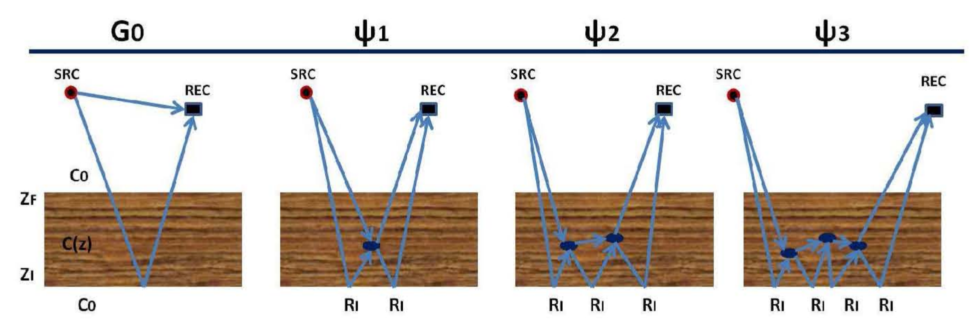


Figure 2: Illustration of the geometry of the time-lapse difference field for the Green's function, $\psi_1$, $\psi_2$ and $\psi_3$ (first, second, and third order terms).

## Discussion

Setting the baseline survey as a reference wavefield encounters two particular difficulties. The reference medium is as complicated as the perturbed medium. Another concern is due to the reflected data in the baseline survey which are absent in the reference medium for a standard scattering method. The wavefield describing the reflected wave in the baseline survey introduces new extra terms in the first and higher order approximations for the difference data which are the function of the combination effects of the perturbation in the monitor survey and the reflectivity at the interface in the baseline survey.

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