A hybrid tomography method for crosswell seismic inversion

Danping Cao, Wenyuan Liao*

University of Calgary, wliao@ucalgaruy.ca

Abstract

The full waveform inversion (FWI) is in general better than the ray tomography method; however a reliable initial model is usually required to ensure the success of seismic inversion. In this work we designed a cascade-like hybrid tomography technology to solve the crosswell seismic inversion problem. We start from the Radon transform based back projection (BP) method, which produces a smooth initial velocity model. Next, the Linear Iterative Reconstruction (LIR) method is adopted to update this initial model, then it is further improved by the nonlinear Gradient-based Eikonal Traveltime Tomography (GETT) method. The velocity model reconstructed from the previous multi ray tomography methods is sufficiently reliable to serve as the initial model for the computational intensive waveform tomography method, from which the accurate velocity model is obtained. The numerical example shows that this hybrid tomography method has great potential in reconstructing accurate acoustic velocity or other high-resolution reservoir characterization for crosswell seismic inversion. It is noticeable that this hybrid tomography method is able to obtain an accurate velocity model even when the recorded seismic data is in poor coverage at spatial and the signal-to-noise ratio is low.

Methodology

The workflow of the hybrid tomography technology is shown in the following figure. It is composed of multi technologies to reconstruct the velocity model from crosswell seismic data. Based on this workflow, we give an introduction and the implementation to each of the four components of the hybrid method.

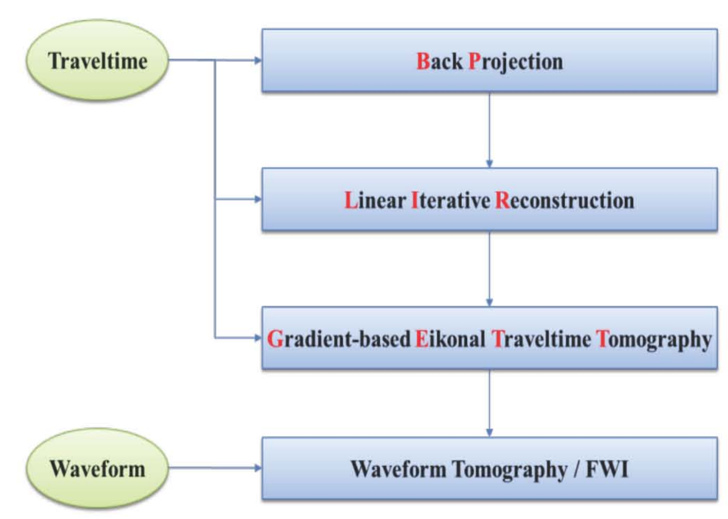

Figure: Flow chart of the hybrid tomography method

Back Projection tomography

The back projection (BP) is the dual transform of the Radon transform, which has been used as the theory basis for computerized tomography (CT). BP is the simplest concept and the easiest method in terms of implementation. In fact, the back projection result is not the original parameters distribution but a smoothed-out version of it. Nowadays, BP has been replaced by the improved filtered back-projection (FBP) method, which has been widely implemented in commercial scanners. In brief, BP is an operation that computes the average value of the projected values (or Radon transforms) using the following formula:

$$s(x, y) = \frac{1}{\pi}\int_0^{\pi} t(x\cos\theta + y\sin\theta, \theta)\, d\theta, \qquad (1)$$

where s is the slowness, which is the reciprocity of the velocity, t is the traveltime of the seismic ray that passes through the point (x, y), θ is the angle of the seismic ray. The advantage of BP reconstruction is the efficiency. Moreover, BP method does not need an initial model, which makes it a perfect starter of our hybrid tomography method, especially when a prior information of the model is not available.

Linear Iterative Reconstruction tomography

The Linear Iterative Reconstruction (LIR) method refers to all iterative algorithms that solve the large-scale sparse linear algebra matrix to reconstruct the image. The linear tomography problem can be expressed as $\mathbf{G}\cdot\mathbf{s} = \mathbf{T}$, where the matrix $\mathbf{G}$ is composed of entries g_{ij} which is the length of $i-th$ ray that crosses the $j-th$ cell. The vectors s and T represent the slowness and observed traveltime respectively. The matrix $\mathbf{G}$ is a large-scale sparse matrix, which is usually solved by the iterative reconstruction methods. The LIR is the family of iterative algebraic methods for solving this tomography problem including ART, SIRT, LSQR and CG, to name a few. The widely used simultaneous iterative reconstruction technique (SIRT) can be expressed as

$$\mathbf{s^{n+1}} = \mathbf{s^n} + \lambda_n \frac{\mathbf{1}}{\mathbf{N}}\sum_{i=1}^{N} \frac{\mathbf{t_i} - <\mathbf{g_i}, \mathbf{s^{n,i}}>}{\|\mathbf{g_i}\|^2}\mathbf{g_i}, \qquad (2)$$

where n is the iteration number, λ_n is a parameter chosen for the iteration step length control.

Gradient-based Eikonal Traveltime tomography

The nonlinear Gradient-based Eikonal Traveltime Tomography (GETT) method reconstructs the velocity model through solving the inverse problem constrained by the Eikonal equation. The misfit function of the inverse problem is defined as the difference between the observed data and the simulated traveltime from the Eikonal equation

$$E_{geet}(m) = \frac{1}{2}(\mathbf{t}_{obs} - \mathbf{t}_{cal}(m))^T(\mathbf{t}_{obs} - \mathbf{t}_{cal}(m)), \qquad (3)$$

where $\mathbf{t}_{obs}$ is the observed first-arrival traveltime, $\mathbf{t}_{cal}(m)$ is the predicted traveltime based on the current velocity model m.

Full Waveform tomography

FWI uses all the waveform information of the seismic data. It is a PDE-constrained optimization problem, in which we minimize the least squares difference between the observed data and the synthetic seismogram

$$E_{FWI}(m) = \frac{1}{2}(\mathbf{d}_{obs} - \mathbf{d}_{cal}(m))^T(\mathbf{d}_{obs} - \mathbf{d}_{cal}(m)), \qquad (4)$$

where $\mathbf{d}_{obs}$ is the recorded seismic data, $\mathbf{d}_{cal}(m)$ is the predicted seismic data at the designed shot-receivers position.

In summary, the initial model for the nonlinear GETT method is provided by the LIR method, which takes the result of BP as its initial model. The result of GETT is then used as the starting model for FWI.

Numerical Example

A 2D model on $100m \times 100m$ domain is used to test our hybrid method. With the spatial sample interval of 1 meter, the model size is 10201. In this test, the observations of 11 shots (marked with red star) and 101 receivers (marked with blue circle) have been made, as shown in the following Figure (a). From Figure (b), we can see that the ray density is uneven as there are more seismic rays crossing in the middle part of the domain. It is obvious that the blue region corresponding to low ray density is difficult to recover with the ray tomography method.

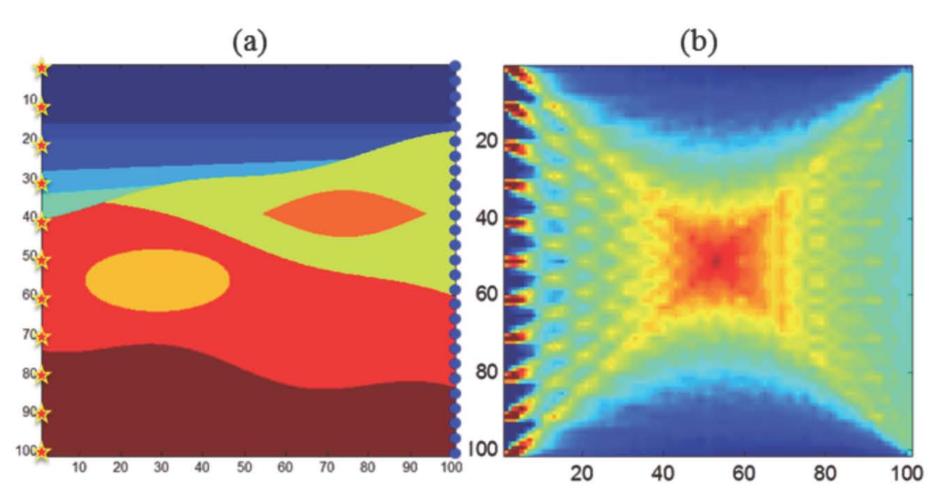

Figure: True velocity model (a) and ray density of the data (b)

One shot recording for this model is shown in the following Figure:

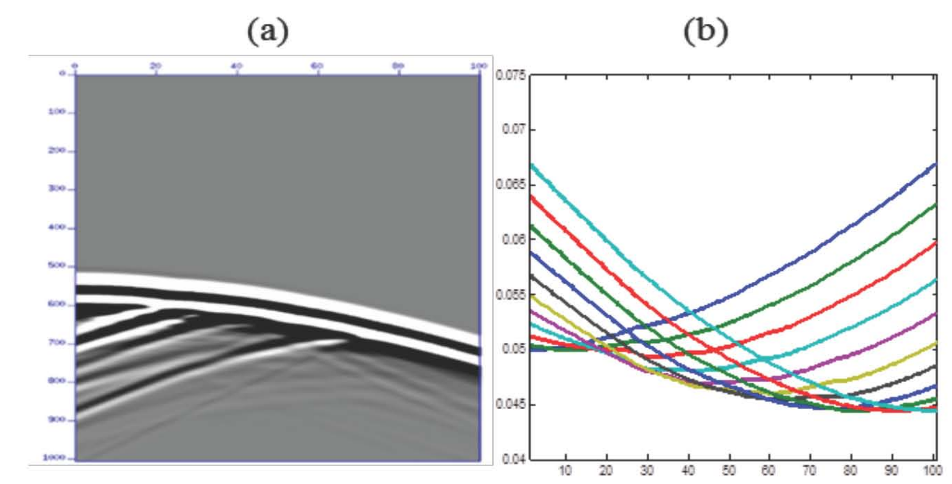

Figure: Observed waveform of 1 shot (a) and First arrival traveltime of 11 shots (b)

We now show the results obtained by different tomography methods.

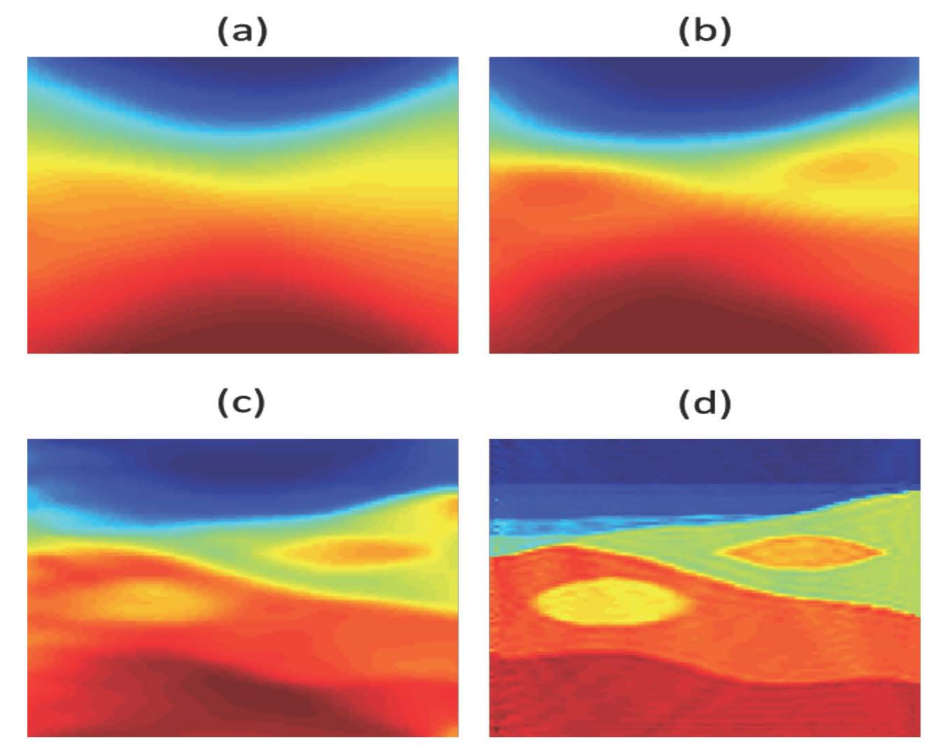

Figure: (a): Result by BP method; (b) Result by LIR based BP; (c) Result by GETT based on LIR; (d) Result by FWI based on GETT

To show the improvement in computational efficiency, the square error between the true model and the recovered velocity by different methods are plotted and compared in the following figure.

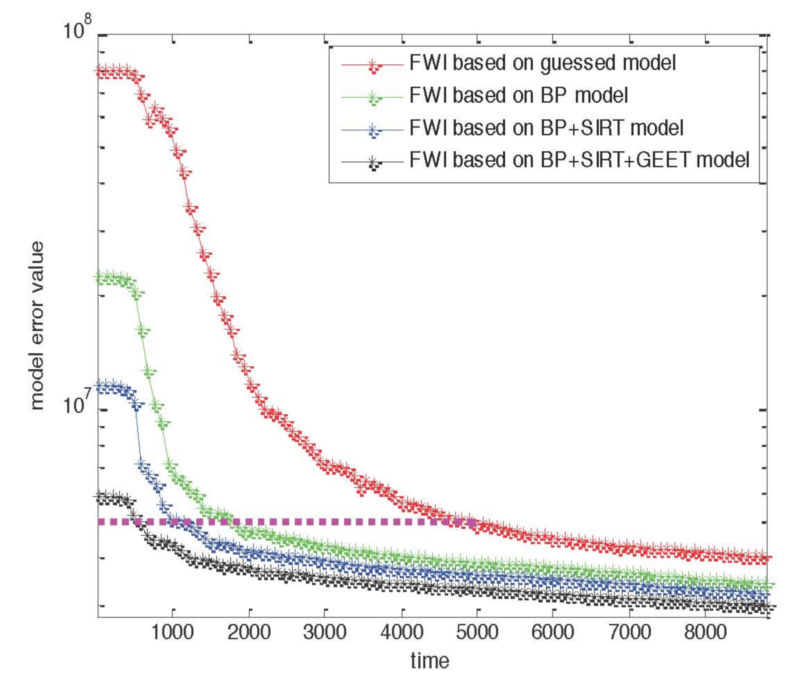

Figure: The square errors vs CPU times for different methods

Conclusions

A hybrid seismic tomography technology is designed to solve the crosswell seismic inversion problem, based on the analysis and comparison of different waveform tomography and traveltime tomography technologies. By carefully exploiting the tradeoff between accuracy and efficiency of each component, an integrated workflow of optimal performance is obtained. With the combination of these methods, the cheaper method that delivers less accurate result is used as the initial model feeder for the method that is computationally more expensive to obtain more accurate result. Numerical results demonstrated that this hybrid tomography method is cost-effective and accurate.